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## CENIER DISCUSSION PAPER NO. 515

MANAGERIAL QUAITITY IN CENIRAIIZED VERSUS DECENIRAITZED
ECONOMIC SYSTEMS

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# MANAGERIAL QUALITX IN CENTRALIZED VERSUS DECENTRALIZED ECONOMIC SYSTEMS 

ABSTRACT

In this paper, we ask, what are the dynamic consequences of a greater centralization or decentralization of decision-making authority (to appoint managers) on the quality of managers who are actually appointed? The central result we obtain is that there is a greater variability (over time) in the quality of managers in a more centralized system. An intuitive reason underlying this result is that though a highly capable decision maker has large beneficial effects on the managerial choices within a centralized system (because this decision maker wields greater authority in such a system), a highly incapable decision maker placed in the same position has correspondingly large deleterious effects. Our analysis also investigates the consequences of the above differences in managerial quality on the outputs of centralized versus decentralized systems.

One of the most important tasks facing the leadership of any organization is the choice of their successors and subordinates. Tenared faculty spend days, sometimes months, deciding on whether particular individuals should be admitted into their ranks. The committees of the Board of Directors assigned to the task of choosing the corporation's next president are among the most powerful and important within the corporation. Corporate presidents, in turn, spend a significant proportion of their own time in selecting the upper echelons of the management.

The effort and contentionsness which often goes into this process suggests that it has important consequences, both for the organization and for the individuals who are selected or not selected. This is because it is recognized that there are large differences in individuals' abilities, and that the abilities of those in leadership and managerial positions inevitably affect the performance, and possibly the survival, of the organization.

Our objective in the present paper is to ask the question, how does centralization or decentralization of decision-making authority affect the quality of the managers who are actually selected? It is natural to pose this question in a dynamic setting because the quality of current managers is not only influenced by the quality of past managers but it, in turn, affects the quality of future managers.

The main result of this paper is that there is a greater variability (over time) in the steady-state quality of managers in more centralized system compared to a less centralized system. This result is based on a specific definition of what we mean by a "more" or "1ess" centralized system: the system $s^{\prime \prime}$ is called more centralized than the system $s$ if one or more of those managerial slots which are meant for subordinates in the system $s^{\prime \prime}$ are endowed with independent decision-making authority in the system s.

An intuitive reason underlying the above result is as follows. If the decision-making authority is more centralized (that is, if one or more managerial positions are endowed with greater authority to select future managers) then high1y capable decision-makers have greater beneficial effects on the managerial choices within the system. By the same token, highly incapable managers placed in the same positions have greater deleterious effects on the managerial choices within such a system. The overall effect of greater centralization, therefore, is to induce a greater variability, over time, in the system's managerial quality.

A natural next question which arises is what is the implication of the above relationship between the degree of centralization and the managerial quality on the outputs of centralized versus decentralized economic systems? Since the relationship between the capability of managers and the organization's output can take several different forms, it is not surprising that no general answers are available to this question; the answer depends, in particular, on the concavity or the convexity of the relationship between managers' ability and organizational output. We have therefore investigated in detail one special case in which managers
choose projects (in addition to choosing their successors and subordinates). In this case we show that: If the managerial screening of projects is tight (that is, if less than half of the projects get accepted), and if the fraction of projects accepted by good and bad managers is identical, then the expected steady-state profit in a decentralized system is larger than that under a centralized system.

The paper is organized as follows. We first illustrate the main results through a highly stylized model in which there are two managers in a centralized as well as in a decentralized economic system (Section $I$ ). We then show in Section II that these results hold under more general specifications of centralized and decentralized systems. The implications of managerial quality on the output and performance of alternative systems are analyzed in Section III. In the concluding section, we briefly discuss some of the caveats of our analysis as well as some of the possible ways in which the analysis can be extended.

## I. A Simple Mode 1

There are two types of managers; those with high and low abilities; for brevity, they are referred as "good" and "bad" managers respectively. We focus here on the managerial ability in selecting future managers. If a high ability manager selected a future manager, then $q_{1}$ denotes the probability that he will select a high ability manager, and $1-q_{1}$ denotes the probability that a low ability person will be selected. The corresponding probabilities for a low ability manager are $q_{2}$ and $1-q_{2}$. Unless stated otherwise, we assume that $1>q_{1}>q_{2}>0$; that is, while neither type of manager is perfect, each type has some ability to select
high ability managers. We treat $q$ 's as exogenously specified parameters; the issue of endogeneity of these probabilities is discussed later.

We consider here two types of organizations: a polyarchy and a hierarchy, denoted respectively by the superscript $s=P$ and H. ${ }^{1}$ Each organization consists of two managers. In a polyarchy, each manager selects his own successor. In contrast, in a hierarchy, the higher level manager selects both his own successor as well as that of the lower level bureancrat. Clearly, the decision making authority is more centralized in a hierarchy. If $G$ and $B$ represent a good and a bad manager respectively, then the four possible combinations of managerial abilities to be found are: $\{G G, G B, B G, B B\}$. Denote these four possible managerial configurations (or managerial states) of a system by $i=1,2,3$ and 4 respectively.

If $\mathbf{Q}_{\mathbf{i j}}^{S}$ denotes the transition probability from the state $i$ to the state $j$ in the system $s$, then the transition matrices for a polyarchy and a hierarchy, respectively, are
(1)

$$
Q^{P}=\left\{\begin{array}{llll}
q_{1}^{2} & q_{1}\left(1-q_{1}\right) & q_{1}\left(1-q_{1}\right) & \left(1-q_{1}\right)^{2} \\
q_{1} q_{2} & q_{1}\left(1-q_{2}\right) & q_{2}\left(1-q_{1}\right) & \left(1-q_{1}\right)\left(1-q_{2}\right) \\
q_{1} q_{2} & q_{2}\left(1-q_{1}\right) & q_{1}\left(1-q_{2}\right) & \left(1-q_{1}\right)\left(1-q_{2}\right) \\
q_{2}^{2} & q_{2}\left(1-q_{2}\right) & q_{2}\left(1-q_{2}\right) & \left(1-q_{2}\right)^{2}
\end{array}\right.
$$

(2) $\quad Q^{H}=\left\{\begin{array}{llll}q_{1}^{2} & q_{1}\left(1-q_{1}\right) & q_{1}\left(1-q_{1}\right) & \left(1-q_{1}\right)^{2} \\ q_{1}^{2} & q_{1}\left(1-q_{1}\right) & q_{1}\left(1-q_{1}\right) & \left(1-q_{1}\right)^{2} \\ q_{2}^{2} & q_{2}\left(1-q_{2}\right) & q_{2}\left(1-q_{2}\right) & \left(1-q_{2}\right)^{2} \\ q_{2}^{2} & q_{2}\left(1-q_{2}\right) & q_{2}\left(1-q_{2}\right) & \left(1-q_{2}\right)^{2}\end{array}\right.$

If the row vector $\pi^{s}=\left\{\pi_{1}^{s}, \pi_{2}^{s}, \pi_{3}^{s}, \pi_{4}^{s}\right\}$ denotes the equilibrium (steadystate) probabilities of the four managerial states in the system $s$, then $\pi^{s}$ is characterized by

$$
\begin{align*}
& \pi^{s}=\pi^{s} Q^{s}, \text { where }  \tag{3}\\
& \sum_{i} \pi_{i}^{s}=1
\end{align*}
$$

and $\pi_{i}^{s} \geq 0$. The economic interpretation of $\pi_{i}^{s}$ is obvious: it represents the frequency with which the managerial configuration $i$ will be observed (over time) in the system $s$, when the system perpetuates itself in a steady-state fashion.

A simple procedure to obtain the equilibrim probabilities is as follows. First, one of the four equations in (3) is redundant because $\pi_{i}^{s}$ sum up to unity according to (4); the equation to be dropped thas may be chosen at will. Second, note that $\pi_{2}^{s}=\pi_{3}^{s}$. For $s=H$, this is obvious from inspection of (2) and (3). For $s=P$, (1) and (3) yield $\left(\pi_{2}^{P}-\pi_{3}^{P}\right)\left(1-q_{1}+q_{2}\right)=0$, which implies that $\pi_{2}^{P}=\pi_{3}^{p}$, since $1-q_{1}+q_{2}>0$. If $\pi_{2}^{s}=\pi_{3}^{s}$ is substituted into (3) then one more equation, for either $\pi_{2}^{s}$ or $\pi_{3}^{s}$, can be dropped. The remaining two equations of (3), along with (4), can be solved to yield the following for a polyarchy.
(5)

$$
\begin{aligned}
& \mathbf{\pi}_{1}^{\mathbf{P}}=q_{2}^{2} / \mathbf{a}^{\mathbf{P}}, \mathbf{\pi}_{2}^{\mathbf{P}}=\pi_{3}^{\mathbf{P}}=q_{2}\left(1-q_{1}\right) / a^{\mathbf{P}}, \quad \text { and } \\
& \pi_{4}^{\mathbf{P}}=\left(1-q_{1}\right)^{2} / \mathbf{a}^{\mathbf{P}} .
\end{aligned}
$$

where $\bar{q}=q_{1}-q_{2}$, and $a^{P}=(1-\bar{q})^{2}$. The equilibrium probabilities for a hierarchy are

$$
\begin{align*}
& \pi_{1}^{H}=q_{2}\left(q_{2}+q_{1} \bar{q}\right) / a^{H}, \pi_{2}^{H}=\pi_{3}^{H}=q_{2}\left(1-q_{1}\right)(1+\bar{q}) / a^{H}, \text { and }  \tag{6}\\
& \pi_{4}^{H}=\left(1-q_{1}\right)\left[1-q_{2}(1+\bar{q})\right] / a^{H}
\end{align*}
$$

where $\mathbf{a}^{\mathbf{H}}=1-\bar{q}$.
To verify that the above probabilities are positive, first note that $1>\bar{q}>0$ and, hence, $a^{H}>0$ and $a^{P}>0$. Now, looking at (5) and (6), it is obvious that all of these expressions, except that for $\pi_{4}^{H}$ are positive. Further, the numerator of $\pi_{4}^{H}$ can be expressed as: $\left(1-q_{1}\right)\left[q_{2}\left(1-q_{1}\right)+\left(1-q_{2}\right)^{2}\right]>0$. Thas $\pi_{i}^{s}>0$. It then follows from (4) that $\pi_{i}^{s}<1$.

Note that the difference in individuals' abilities is critical to the present model since, otherwise, the degree of centralization has no impact on the distribution of managerial abilities which will emerge in the system. This can be verified by substituting $q_{1}=q_{2}=q$ (that is, $\bar{q}=0$ ) into (5) and (6), which yields: $\pi_{1}^{s}=q^{2}, \pi_{2}^{s}=\pi_{3}^{s}=q(1-q)$ and $\pi_{4}^{s}=(1-q)^{2}$.

Effect of Individuals' Abilities on Systems' Managerial Quality: In our mode1, a higher $q_{1}$ and $q_{2}$ means that an individual manager, good and bad respectively, has higher ability in selecting managers. To ascertain the effect of a worsening or an improvement in the individuals' abilities on the managerial quality in a system, we obtain the derivatives of $\pi_{i}$ 's
with respect to $q$ 's. This yields: $\partial \pi_{1}^{s} / \partial q_{k}>0$, and $\partial \pi_{4}^{s} / \partial q_{k}<0$, for $k=1$ and $2 .^{2}$ The following conclusion emerges.

PROPOSITION 1: If individual managers are more capable in choosing
future managers, then the probability that an economic system (centralized or decentralized) has all high ability managers is higher, and the probability that the system has all low ability managers is lower.

Clearly, both of these results are what we would have expected. Note, however, that the effect of individuals' abilities on the probability of having a mixed managerial configuration (the configaration with good as well as bad managers) is not in general predictable.

Comparison of Managerial Quality in Alternative Systems: To compare the probabilities of various managerial configarations in the two economic systems, we obtain the following from (5) and (6).

$$
\begin{equation*}
\pi_{1}^{\mathrm{P}}-\pi_{1}^{\mathrm{H}}=\pi_{4}^{\mathrm{P}}-\pi_{4}^{\mathrm{H}}=-\delta, \quad \text { and } \quad \pi_{2}^{\mathrm{P}}-\pi_{2}^{\mathrm{H}}=\pi_{3}^{\mathrm{P}}-\pi_{3}^{\mathrm{H}}=\delta, \tag{7}
\end{equation*}
$$

where $\delta=q_{2}\left(1-q_{1}\right) \bar{q}^{2} /(1-\bar{q})^{2}>0$. The qualitative implications of expression (7) are summarized below.

PROPOSITION 2: The probability that all managers are of high ability, and the probability that all managers are of 10 w ability, are lower in a more decentralized system, whereas the probability that there is a mixture of abilities among managers is higher in a more decentralized system.

Moreover, note from (7) that: The difference in the probabilities of alternative managerial configurations between a polyarchy and a hierarchy are identical in magnitude.

Let the random number $m$ denote the namber of good managers in an
economic system, and let $\pi^{s}(m)$ denote the steady-state probability associated with there being $m$ good managers in the system s. Then

$$
\begin{equation*}
\pi^{s}(0)=\pi_{4}^{s}, \quad \pi^{s}(1)=2 \pi_{2}^{s}, \quad \text { and } \quad \pi^{s}(2)=\pi_{1}^{s} \tag{8}
\end{equation*}
$$

Also, let $\lambda^{s}$ be the average number of good managers, and $V^{s}$ be the variance in the number of good managers in the system $s$; that is

$$
\lambda^{s}=\sum_{m=0}^{2} \pi^{s}(m) m, \text { and } V^{s}=\sum_{m=0}^{2} \pi^{s}(m) m^{2}-\left(\lambda^{s}\right)^{2} . \quad \text { Using (5), (6) and (8), }
$$

then, we find that $\lambda^{P}-\lambda^{H}=0$, and $V^{P}-V^{H}=-2 \delta<0$. These conclusions are summarized below.

PROPOSITION 3: Although the alternative economic systems under consideration have the same average number of good managers, the variance in

## the number of good managers is lower in a more decentralized system.

A qualitatively analogous result is: The distribation of the number of good managex in a polyarchy is a mean preserving improvement over the corresponding distribution in a hierarchy. ${ }^{3}$

The intuitive reason underlying the above results is that the beneficial effects of a high ability manager as well as the damaging effects of a low ability manager are more pronounced (in affecting the system's overall managerial choices) in a hierarchy compared to those in a polyarchy. A good (or bad) manager in a polyarchy improves (or worsens) the choice of his own successor, but he has no impact on the choices being made by other independent decision-making units. In contrast, whether the current hierarch (the leader of a hierarchy) is good or bad affects not only the choice of his own successor but also that of the future subordinates. This difference between the two systems generates a
dynamic process which resalts in a hierarchy exhibiting a greater tendency towards the extremes of managerial abilities.

## II. More General Centralized versus Decentralized Systems

The results obtained earlier are robust not only to the size of an economic system, but also to the precise degree of centralization within an economic system. To demonstrate this, we begin by defining what we mean in this paper by a "more centralized" or a "more decentralized" system. Consider an economic system consisting of an arbitrary number of hierarchies of different sizes. The size of a hierarchy means here the number of managers within the hierarchy, one of whom is the hierarch (the boss) and others are subordinates. The current hierarch in a given hierarchy has the authority to appoint his own successor as well as those of his subordinates, but he has no influence on the choice of managers in any other hierarchy. Such an economic system could also be called a polyarchy of hierarchies, and its features have a closer resemblance to a typical economy than those of the polar cases of a pure hierarchy (where there is a single boss in the entire economy) or a pure polyarchy (where everyone is his own boss).

Next, consider the following hypothetical experiment. Start with a given economic system (that is, a given polyarchy of hierarchies) and break up one or more of its constituent hierarchies into smaller hierarchies. We call the latter system more decentralized because one or more of those managerial slots which were meant for subordinates in the previous system are now endowed with independent decision-making authority. Our objective in this section then is to compare any two economic
systems (with an arbitrarily specified structure of decision-making authority) one of which is more decentralized. Such comparisons, it should be emphasized, do not depend on whether decentralization is or is not feasible in any one of the two alternative systems under consideration.

Let $n_{M}(s)$ denote the number of hierarchies of size $M$ in the economic system $s$, where $M \geq 1$, and it is an integer, and let $N(s)=\sum_{M} M n_{M}(s) d e-$ note the total number of managers in the system $s$. Then, according to our definition, the system $s$ is more decentralized than the system $s^{\prime}$. if $N(s)=N\left(s^{\prime}\right)=N$, and if

$$
\begin{equation*}
\sum_{M \leq J} M_{M}(s) \geq \sum_{M \leq J} M_{M}\left(s^{\prime}\right) \tag{9}
\end{equation*}
$$

for all $J \geq 1$, and the strict inequality holds in (9) for at least one $J$. In the simple polyarchy examined in Section $I_{\text {, }}$ for instance, $n_{1}=2$, and $\mathbf{n}_{M}=0$ for $M \neq 1$. This polyarchy is clearly more decentralized than the corresponding hierarchy, where $n_{2}=1$, and $n_{M}=0$, for $M \neq 2$.

An analogons representation of a more decentralized system is as follows. Let $f_{M}(s)=M n_{M}(s) / N$ denote the fraction of the total namber of managers in the system $s$ who belong to one of the hierarchies of size M. If $f_{M}(s)$ is treated as the density of a discrete distribution defined over positive integer values of $M$, then the expression (9) implies that the system $s$ is more decentralized than the system $s^{\prime}$ if the density $f_{M}(s)$ is ärst-order stochastic worsening of the density $f_{M}\left(s^{\prime}\right)$.

For later use, we define the average size of hierarchies in the economic system sas

$$
\begin{equation*}
h(s)=\sum_{M} M^{2} n_{M}(s) / N \tag{10}
\end{equation*}
$$

where the weights used in calculating the average size of hierarchies are the numbers of managerial slots in hierarchies of different sizes. It is obvious then that the average size of hierarchies must necessarily be smaller in a more decentralized system. That is, if the economic system $s$ is more decentralized than the system $s^{\prime}$, then

$$
\begin{equation*}
h(s)<h\left(s^{\prime}\right) \tag{11}
\end{equation*}
$$

The reverse, on the other hand, need not hold. That is, if two economic systems have different average sizes of hierarchies, then it is not always the case that one of them is more centralized or decentralized compared to the other.

Now consider a hierarchy of size $M$, in isolation from all other constituents of the economic system. Recall that the random variable m denotes the number of good managers. Within the hierarchy under consideration, m takes integer values ranging from 0 to M. The density of $m$ in this hierarchy is denoted by $g(m \mid M)$, and it is

$$
\begin{align*}
& g(m \mid M)=\sum_{k=1}^{2} z_{k} b\left(m, M, q_{k}\right), \text { where }  \tag{12}\\
& z_{1}=q_{2} /(1-\bar{q}), z_{2}=1-z_{1}=\left(1-q_{1}\right) /(1-\bar{q}), \tag{13}
\end{align*}
$$

and $b\left(m, M, q_{k}\right)=\left[\begin{array}{l}M \\ m\end{array}\right] q_{k}^{m}\left(1-q_{k}\right)^{M-m}$ represents the binomial density of $m$ successes out of $M$ trials when $q_{k}$ is the probability of success.

The derivation of (12) is highly intuitive. A critical element in the succession process in a hierarchy is the selection of the hierarch because the capabilities of other individaals within the hierarchy do not
influence the succession process. ${ }^{4}$ Now, focussing on the selection of hierarchs (that is, the next period's hierarch is chosen by the carrent hierarch, and so on), it turns out that $z_{1}$ is the (steady-state) probability that the hierarch is a good manager, and $z_{2}$ is the probability that the hierarch is a bad manager. ${ }^{5}$ Further, it is obvious that the binomial density $b\left(m, M, q_{1}\right)$ represents the probability that mood managers are chosen when the hierarch is a good manager, and $b\left(m, M, q_{2}\right)$ is the corresponding probability when the hierarch is a bad manager.

Straightformard combination of these probabilities yields (12).
Next, denote the average number of good managers in the above hierarchy of size $M$ by $\lambda(M)$, and the variance of this number by $V(m)$. Then, using (12), (13), and the standard properties of the binomial variate, it can be ascertained that

$$
\begin{equation*}
\lambda(M)=M z_{1}, \quad \text { and } \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
V(M)=M^{2} z_{1} z_{2} \bar{q}^{2}+M \sum_{k=1}^{2} z_{k} q_{k}\left(1-q_{k}\right) \tag{15}
\end{equation*}
$$

Our interest here is in characterizing the distribution of managerial quality within the economic system as a whole. If $\pi(m \mid s)$ denotes the probability density associated with the state in which there are mad managers in the system $s$, then $\pi(m \mid s)$ for various m's are obtained from the convolution of the densities (12). For instance

$$
\begin{align*}
& \pi(0 \mid s)=\prod_{M}\left\{\sum_{k=1}^{2} z_{k} b\left(0, M, q_{k}\right)\right\}^{n_{M}(s)}, \text { and }  \tag{16}\\
& \left.\pi(N \mid s)=\prod_{M} \sum_{k=1}^{2} z_{k} b\left(M, M, q_{k}\right)\right\}^{n_{M}(s)} \tag{17}
\end{align*}
$$

denote the probabilities associated with the polar states of the system s where, respectively, none of the managers in the system is good, and none of the managers in the system is bad. It is straightforward to verify that $\partial \pi(0 \mid s) / \partial q_{k}<0$, and $\partial \pi(N \mid s) / \partial q_{k}>0$, for $k=1$ and 2 . It can also be shown that $\pi\left(m \mid s^{\prime}\right)>\pi(m \mid s)$, for $m=0$ and $N .^{6}$ Propositions 1 and 2 thus hold within the more general setup of the present section.

Finally, let $\Lambda(s)$ and $\operatorname{Var}(s)$ respectively denote the average number of good managers, and the variance in the number of good managers in the economic system s. Since the mean or the variance of a sum of independent random variables is the same as the sum of their respective means or Variances, it follows from (10), (14) and (15) that

$$
\begin{equation*}
\Lambda(s)=\mathrm{Nz}_{1}, \text { and } \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\operatorname{Var}(s)=\left[z_{1} z_{2} \bar{q}^{2} h(s)+\sum_{k=1}^{2} z_{k} q_{k}\left(1-q_{k}\right)\right] N \tag{19}
\end{equation*}
$$

An immediate consequence of (19) is the following result.

PROPOSITION 4. Var $\left(s^{\prime}\right) \zeta \operatorname{Var}(s)$, if $h\left(s^{\prime}\right) \zeta h(s)$. That is, the variance in the number of good managers in an economic system is smaller if the average size of hierarchies within the system is smaller.

Therefore, recalling our definition of decentralization, it follows from (18) and (19) that though a more decentralized system has the same average number of good managers, it has a smaller variance in the number of good managers. Proposition 3 is thas established for the general economic systems under consideration.

What Proposition 4 says, in addition, is that even when two alternative systems are not strictly comparable to one another, in the sense
that one of them is not more centralized or decentralized than another (based on our definition), it is still possible to infer the difference between the two systems concerning the variance in the number of good managers. Moreover, such an inference is possible based solely on the average size of hierarchies in each of the two system.

## III. Comparison of Outputs

The relationship between the quality of managers and the output (or the performance) of an economic system is a complicated one. It depends not only on the distribution of authority within the system, and on the set of tasks which the managers are supposed to perform, but also on what kinds of positive and negative externalities are exerted by good and bad managers on one another. In this section, we examine these aspects Within the context of the simple model of Section $I$.

First consider the case where the (expected) aggregate output of both economic systems is the same if they have the same number of good managers. Then, from a standard result in the theory of stochastic dominance, and from the observation made earlier that the distribution of the number of good managers in a hierarchy is a mean preserving worsening of the corresponding distribution in a polyarchy, it follows that: The steady-state output in a polyarchy is larger (smaller) than that in a hierarchy if the output is concave (convex) in the number of good managers.

The relative performance of a hierarchy is weakened further if the yardstick of comparison is not the expected output but the expected atility of the output, and if the utility function is posited to display some
risk aversion. In fact, even when output is convex in the number of good managers, if the utility function is sufficiently concave in output, the expected utility under a polyarchy could exceed that in a hierarchy.

Although the assumption that a system's output is concave in the number of good managers (that is, the output of a system with one good manager and one bad manager is greater than half of the outputs of two systems, one of which has two good managers and the other has two bad managers) might appear reasonable, this is not always the case. If the task of the manager is to develop new projects or ideas, then there is a presumption that variance helps: it is only the best that has a significant value. This presumption is also consistent with the view that effective organizations are highly fragile, and that small changes in the tail of ability distribution within an economic system may have large effects on the overall performance of the system. In these cases, not only is the future mix of managerial abilities in a system sensitive to its current mix, but also the system's future performance is particularly sensitive to the position of the individual(s) within the system who are currently the best. On the other hand, if there is sufficient redundancy within an economic system, then its performance may be relatively insensitive to having a limited number of incompetent managers, and a significant deterioration in the system's performance might arise only when incompetency exceeds a certain level. In such cases, the relationship between the managerial quality and the system's performance will be concave.

In the rest of this section, we analyze a specific example in which the tasks of managers are explicitly defined. Managers select projects
(from a large set of available projects), in addition to selecting future managers. For simplicity, we assume that there are only two types of projects; good projects, yielding an (expected net) profit $x$; and bad projects, yielding a profit -x. Half the projects are of each type. Bad managers are assumed not to have any discriminating ability; that is, they randomly accept a fraction $p_{2}$ of the projects. Good managers are better not only in choosing future managers but also projects; they accept a good project with probability $p_{1}^{1}$ and a bad project with probability $p_{1}^{2}$, where $p_{1}^{1}>p_{2}>p_{1}^{2}$. The fraction of projects which a good manager accepts is denoted by $p_{1}$; clearly, $p_{1}=\left(p_{1}^{1}+p_{1}^{2}\right) / 2$.

In a hierarchy, a project is accepted only if both managers accept the project. In contrast, in a polyarchy, a project is accepted if any one of the two independent managers accept it. Thus, a polyarchy is more decentralized not only in the selection of successors, but also in the selection of projects. ${ }^{7}$ Let $Y_{i}^{s}$ denote the profit of the system $s$ under the managerial state $i$, where it will be recalled that $i=\{1,2,3,4\}$ correspond respectively to the managerial states \{GG, GB, BG, BB\}. Then, $Y_{i}^{s}$ are as follows ${ }^{8}$

$$
\begin{align*}
& Y_{1}^{P}=2\left(1-p_{1}\right), \quad Y_{2}^{P}=Y_{3}^{P}=1-p_{2}, \text { and } \quad Y_{4}^{P}=0  \tag{20}\\
& Y_{1}^{H}=2 p_{1}, \quad Y_{2}^{H}=Y_{3}^{H}=p_{2}, \quad \text { and } \quad Y_{4}^{H}=0 . \tag{21}
\end{align*}
$$

Denote the steady state profit in the system $s$ by $\mathbf{Y}^{\mathbf{S}}=\sum_{i} \pi_{i}^{S} Y_{i}^{S}$, and let $\Delta Y=Y^{P}-Y^{H}$ denote the difference between the profits of the two systems. Then (7), (20) and (21) yield

$$
\begin{equation*}
\Delta Y=2 \pi_{1}^{P}\left(1-2 p_{1}\right)+2 \pi_{2}^{P}\left(1-2 p_{2}\right)-2 \delta\left(p_{1}-p_{2}\right) \tag{22}
\end{equation*}
$$

Note that the project selection abilities of managers affect the relative profit performance only through parameters $p_{1}$ and $p_{2}$, which denote the fractions of projects accepted by a good versus a bad manager. In the central case where both types of managers accept the same fraction of projects (that is, $p_{1}=p_{2}=p$ ), the screening of projects (by both managers) is "tight" or "slack" depending simply on whether p is smaller or larger than one-half. It follows from (22) then that: If the fraction of projects approved by good and bad managers is identical, then the steady-state profit in a polyarchy is larger (smaller) than that in a hierarchy, provided the screening of projects by managers is tight (s1ack).

## IV. Concluding Remarks

We have long been aware that political decisions-decisions undertaken by governments, concerning resource allocations as we 11 as those Which circumscribe the actions which various individuals can or cannot undertake-have vast repercussions; if decisions are made well, much of the society benefits; if made badly, much of the society suffers. In other words, the quality of public decision-making is a public good, one of the most important public goods. ${ }^{9}$

Similarly, we have increasingly become aware in the past fifteen years that, even in private organizations, the actions undertaken by one individual or organization has important externalities on others. This is in part because of the incompleteness of markets and imperfectness of information. When the manager of a firm misallocates the firm's resources, he suffers; but so do his stockholders, his suppliers, and often
his customers. It has increasingly become recognized that the management of joint stock companies (and indeed, with limited liability, virtually all firms) is a quasi-public good.

Once we recognize that some individuals are better decision-makers than others (that is, individuals are not only fallible, but the degree of fallibility differs across individuals), and that the managerial quality of individuals has important repercussions on the performance of economic systems, then it is natural to ask, what are the determinants of a system's managerial quality? In this paper, we have singled ont and analyzed one particular determinant, namely, the degree of centralization or decentralization of the decision-making authority to appoint future managers. Our central result suggests that greater centralization leads to a greater variability in the system's managerial quality. We end this paper by briefly discussing some of the important aspects from which our analysis has abstracted. This discussion is meant to be suggestive rather than exhaustive.

First, our model does not fully reflect the continum of influence relationships which might exist within an organization. For example, our assumption that the choices of future managers within a hierarchy are made solely by the boss overstates the degree of authority that the bosses typically have, and understates the influence that subordinates typically exercise on the choices of not only their own successors but also their boss's successor. An opposite bias is reflected in our assumption that a subordinate can exercise a veto (similar to one that his boss can exercise) in the choice of projects that a hierarchy undertakes. Though these assumptions can be justified under particular types of cost
and benefits of exerting influence (including the technologies for communicating and interpreting information), it should be apparent that an explicit analysis of these costs and technologies will suggest a range of influence relationships (as well as intra-organizational specialization in different types of decision-making) within different types of organizations, and that the nature of influence relationships which are more likely to emerge may depend, in turn, on the architecture of the economic system (for instance, on the degree of centralization) to which an organization belongs.

Second, we have assumed that the nature of errors made by an individual does not depend on the characteristics of the economic system to which he belongs. This assumption appears to be an appropriate first approximation, in the sense that it focusses only on the inherent abilities of individuals. It does, however, abstract from the fact that the nature of an individual's error does in part depend on what information he chooses to collect, and that the costs and benefits of collecting different types of information depend, in turn, on the architecture of the economic system. ${ }^{10}$

Third, we have abstracted from the role that incentives might play in influencing individuals' fallibility. We have assumed that each manager makes the best decision he can, and that some managers are better at decision-making than others. We believe that there is a great deal of trath in this perspective: it is frequently no more difficult to make a good decision than to make a bad decision. Moreover, to the extent that an individual's fallibility in selecting managers might be related to the incentives he faces, it may not be possible to achieve a significant
amelioration in these incentives through the usual method of assigning appropriate property rights. This is because the consequences of incompetent decision-making are often not recognized until years after the retirement of the individual who had chosen an incompetent successor; and by then he has sold his shares or, in the case of older decision-makers (who typically outnumber younger persons in leadership positions), has died. In other words, most of the consequences which follow upon the choice of an incompetent successor are borne not by the decision maker, but by the contemporaries and the successors of the incompetent successor.

Finally, and most importantly, the present analysis has abstracted from the role and the consequences of natural selection-the fact that the economic environment often influences the survival probabilities of different individuals and organizations. In a sequel to the present paper (1986b) we have analyzed these aspects by extending the framework developed here: natural selection leads to the elimination of some capable managers, and it allows for the survival of some incapable managers. The magnitude and the consequences of these alternative types of limitations of natural selection depend not only on certain salient properties of the natural selection process, but also on the degree of centralization or decentralization within the economic system. This analysis suggests that the overall effect of natural selection is likely to be more beneficial within a more decentralized system, than within a more centralized economic system.

## REFERENCES

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## FOOTNOTES

${ }^{1}$ We have used concepts such as a polyarchy and a hierarchy in a previous paper [Sah and Stiglitz (1986a)] but in a very different context. The focus there was on fallibility of homogeneous individuals concerning the choice of projects, and on the static consequences of such fallibility on organizations' performance.
${ }^{2}$ Specifically, $\partial \pi_{1}^{P} / \partial q_{1}=2 q_{2}^{2} /(1-\bar{q})^{3}>0$,
$\partial \pi_{1}^{P} / \partial q_{2}=2 q_{2}\left(1-q_{1}\right) /(1-\bar{q})^{3}>0, \partial \pi_{4}^{P} / \partial q_{1}=-2 q_{2}\left(1-q_{1}\right) /(1-\bar{q})^{3}<0$, and $\partial \pi_{4}^{P} / \partial q_{2}=-2\left(1-q_{1}\right)^{2} /(1-\bar{q})^{3}<0$. The signs of the relevant dexivatives of $\pi_{1}^{H}$ and $\pi_{4}^{H}$ can be analogously established.
${ }^{3}$ This is established by demonstrating that $\sum_{c \leq m} \sum_{j \leq c}\left\{\pi^{\mathbf{H}}(j)-\pi^{\mathbf{P}}(j)\right\} \geq 0$ for all m , where $c^{\prime}$ s are nonnegative integers, and the strict inequality holds for at least one $m$. See Rothschild and Stiglitz (1970) for the definition of mean preserving changes in a distribution.

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${ }^{4}$ This assumption exaggerates the typical asymmetry of authority between the hierarch and the subordinates within a hierarchy. Some of the issues underlying such an assumption are briefly discussed later.
${ }^{5}$ In the steady-state, $z_{1}=z_{1} q_{1}+z_{2} q_{2}$, because $q_{1}$ (respectively, $q_{2}$ ) is the probability of selecting a good manager as the next period's hierarch if the carrent hierarch is a good (respectively, bad) manager. Using $z_{2}=1-z_{2}$, the preceding expression can be solved to yield (13).
${ }^{6}$ Proof: (16) can be reexpressed as: $1 n \pi(0 \mid s)=N \sum_{M} f_{M}(s) \ln t(M)$, where $t(M)=\left\{\sum_{k=1}^{2} z_{k}\left(1-q_{k}\right)^{M}\right\}^{1 / M}$. Since $t$ is non-decreasing in $M$, a firstorder stochastic worsening in the "density" $f_{M}(s)$ must lower $\ln \pi(0 \mid s)$. Hence $\pi(0 \mid s)<\pi\left(0 \mid s^{\prime}\right)$. An analogous argument shows that $\pi(N \mid s)<\pi\left(N \mid s^{\prime}\right)$.
${ }^{7}$ We have assumed here that the centralization of decision-making authority in one dimension (selection of projects) is correlated with that in another dimension (selection of managers). Such an assumption may not always be appropriate.
${ }^{8}$ In the expressions for $Y_{i}^{s}$, we have suppressed a constant of proportionality $\operatorname{Tx}\left(p_{1}^{1}-p_{1}^{2}\right)$, where $T$ is the number of available projects. It is assumed that, in a polyarchy, half of the projects go initially to each of the two managers, those rejected by one manager get passed along to the other manager, and the same project is not reviewed more than once by any one manager. Thas, in the GG polyarchy, the probability that a good projected is accepted is $p_{1}^{1}\left(2-p_{1}^{1}\right)$, and that a bad project is accepted is $p_{1}^{2}\left(2-p_{1}^{2}\right)$. The profit is $Y_{1}^{P}=T x\left[p_{1}^{1}\left(2-p_{1}^{1}\right)-p_{1}^{2}\left(2-p_{1}^{2}\right)\right]$, which is reexpressed as in (20). In a GG hierarchy, the probability that a good project is accepted is $\left(p_{1}^{1}\right)^{2}$, and that a bad project is accepted is $\left(p_{1}^{2}\right)^{2}$. The profit is $Y_{1}^{H}=T_{x}\left[\left(p_{1}^{1}\right)^{2}-\left(p_{1}^{2}\right)^{2}\right]$, which is restated as in (21). Other expressions in (20) and (21) can be analogonsiy derived. For additional details, in the simpler context of managers with homogeneous abilities, see Sah and Stiglitz (1986a).
${ }^{9}$
${ }^{9}$ The extensive 1iterature on public choice which has emphasized the prob1em of how preferences can or cannot be aggregated, often misses the point that mach of pub1ic discourse on good or bad governance is concerned not with values but with the abilities of those to be entrusted to govern.
${ }^{10}$ See Sah and Stiglitz (1986a) for an analysis of endogenous errors in the context of project selection by homogeneous individuals.

