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## Promotion and Mandatory Retirement

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CENTER DISCUSSION PAPER NO. 491

PROMOTION AND MANDATORY RETIREMENT

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#### **ABSTRACT**

In this paper, a firm maximizes profits over choices of wage schedules, hiring schedules, pension schedules and mandatory retirement ages in a model with turnover costs and a productivity function which depends upon position and experience. It is shown that firms may have reason to institute a mandatory retirement age and that they can accomplish the same goal through proper uses of wage and pension schedules.

Steve Stern, "Promotion and Mandatory Retirement."

## Promotion and Mandatory Retirement\*

### Section I: Introduction

Over the last twenty years, there has been much discussion in the media and in Congress concerning mandatory retirement. Congress has progressively pushed back the minimum mandatory retirement age. At this time, almost all employees are protected against mandatory retirement until the age of seventy by the Age Discrimination in Employment Act (ADEA). The major exemption in the ADEA is for employees in bona-fide executive positions. A firm is allowed to force executives to retire so that:

- 1) the firm can bring in "new blood" to maintain the inflow of new ideas,  
and
- 2) the firm can provide younger employees with promotion opportunities.

There are those in Congress who are now suggesting a total ban on mandatory retirement.

Many arguments have been suggested for the existence of mandatory retirement:

- 1) Employees become less productive in their sixties, and the productivity of individual workers is difficult to measure. Age is used as a proxy for productivity, and employees are fired when their estimated productivity is below their wage.
- 2) Employees become less productive in their sixties, and they prefer to retire with a "gold watch" at a common age than to be fired or receive a wage reduction individually after being identified as less productive.
- 3) Mandatory retirement makes it easier for firms to comply with affirmative action requirements.

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\*The author wishes to thank Bill Johnson, Paul Schultz, Joyce Cooper and workshop participants at Yale University and the University of Virginia for helpful comments. All remaining errors are mine.

- 4) Both the firm and the employee benefit if some portion of wage payments are deferred until late in the employee's career. This implies that an employee will be paid more than his marginal product late in his career. He must be terminated at some point so that the net present value of his lifetime wage payments equals the net present value of his lifetime marginal product.
  
- 5) Mandatory retirement creates room for promotion of younger employees.

The first and second arguments assume that older employees become less productive. The problem with these arguments is that they require the firm to fire employees with a positive marginal product rather than to just lower their wages. Some suggest that setting an arbitrary mandatory retirement date improves morale relative to lowering wages. However, it is not clear that firing an employee under any conditions improves morale relative to lowering his wage. Furthermore, there is some evidence [see Clark, Kreps and Spengler (1978)] that older employees are not less productive on average. The screening problem may present an explanation for mandatory retirement if there is adverse selection [see Greenwald (1979) for example]. But this is a potential problem at all ages. Why aren't younger employees also subject to a screening problem with adverse selection?

Mandatory retirement may increase the effect of affirmative action programs. However, to the extent that affirmative action programs are short term programs, this argument does not present a good reason on which to base long term decisions. Besides, mandatory retirement existed decades before affirmative action programs did.

The moral hazard argument for mandatory retirement has been suggested by

Lazear in a series of papers. Lazear (1979) suggests that the employer-employee relationship is fraught with moral hazard problems which can be solved optimally by deferring some portion of wage payments until late in an employee's career while maintaining the present value of an employee's stream of earnings. Both the firm and its employee prefer such an arrangement. However, this means that the employee will be paid more than his marginal product late in his life and will retire later than he would have had wage payments not been deferred. Thus, a mandatory retirement program becomes necessary to force the employee to retire at his optimal retirement age. Lazear (1983) shows that early retirement benefits may be viewed as severance pay to employees who are earning more than their marginal product. The value of the early retirement benefit is equal to the value of a normal retirement pension plus any rents the employee would have earned had he stayed at the firm until the mandatory retirement age.

There are many problems with Lazear's analysis. First of all, the high incidence of early retirement is not consistent with his analysis. Lazear (1983) shows that the firm may offer early retirement benefits to employees to induce them to retire early. However, this argument has no explanatory power since both the firm and its employees are indifferent between the employee retiring early or at the mandatory retirement age. In fact, the existence of adverse selection would make early retirement benefits unprofitable to the firm. Furthermore, employees who retire early in Lazear's model should find another job. In the real world, many early retirees do not find new jobs.

Second, the purpose of deferring wage payments is to induce employees not to "shirk" or "steal from the firm." If an employee is caught shirking, he is fired immediately and forfeits any future rents. The return to the employee of shirking is a stock. If it were not a stock, no one would ever have

incentive to shirk whether or not wages were deferred. But, since it is a stock, at some point arbitrarily close to the mandatory retirement age, all employees should shirk because the value of a finite stock is always greater than the value of a finite flow over a short enough time period.

A pension program can be instituted with pensions payable only to employees who reach the mandatory retirement age. This would provide the firm with a method of preventing employees from shirking arbitrarily close to the mandatory retirement age. However, only benefits that are available to only those who leave the firm at the mandatory retirement age serve this role. These are called supplementary benefits and do not receive the same tax advantages as regular benefits covered by ERISA. Only 7% of firms in a large sample provided supplementary benefits.<sup>1</sup>

In Lazear's model, the firm could replace a mandatory retirement age with a "recontracting age" at which point wages would be readjusted to the employee's marginal product. The optimal mandatory retirement age is set so that if employees were given a chance to recontract they would decline to do so. However, there is much evidence that many workers accept other jobs at lower pay when forced to retire from their career job [see Schulz (1985), and Gustman and Steinmeier (1984)].

The focus of this paper is the promotions argument. It is suggested that the firm may force its older employees to retire in order to open up promotion possibilities for younger employees. In a survey of firm managers, it was found that 67% of managers felt that "mandatory retirement [was] necessary to create job openings and promotion opportunities for younger people."<sup>2</sup> This reason was cited more frequently than any other reason.

The production function of the firm is such that the marginal productivity of each employee net of training costs depends upon both the

employee's experience and his position. Employees are promoted through the hierarchy of the firm. Promotion is based upon seniority or experience. Many papers in the economics literature discuss why promotion would be based upon seniority. These include Carmichael (1983), Ioannides and Pissarides (1983) and Rosen (1982). Most of these papers have a comparative advantage argument in them. Actually, it is not necessary for any of the results of this model for promotion to be based on seniority. The firm maximizes profits subject to choices of a wage schedule, a pension program, a hiring schedule and a mandatory retirement age. It is shown that the firm chooses a finite mandatory retirement age in many cases. Furthermore, it is shown that pensions may play a role in subverting the spirit of ADEA but play no other role in firm policy when there are perfect capital markets and no tax distortions. Furthermore, it is shown that the relationship between marginal product and wage may be very tenuous in management positions. Finally, it is shown that firms may discriminate against older potential employees because it is difficult to recuperate hiring costs.

It is assumed that there is a firm with a hierarchy of jobs. Jobs are ranked by productivity and then dispersed to employees in order of experience. The position each employee gets is a function of what percentage of the other employees have less experience. The wage is quoted as a function of position and experience.

The distribution of experience at a firm is a function of the exit rate from the firm at all ages, the hiring rate at all ages and the mandatory retirement age. If the exit rate rises, the hiring rate falls or the mandatory retirement age falls, then employees rise more quickly through the hierarchy of the firm.

Each employee decides when to leave the firm by comparing the present



value of staying at the firm to opportunities outside of the firm. It is assumed that the distribution of alternative opportunities can be summarized in a sufficient statistic which is the reservation value. If the value of staying is less than the reservation value, then all employees leave immediately. When the average value of staying is greater than the average value of leaving, the exit behavior of employees depends upon unspecified characteristics of the market.

It can be shown that under many diverse assumptions about the exit rate and wage schedule, the firm will maximize the present value of a prospective employee's staying by instituting a mandatory retirement program. This occurs even though the prospective employee knows he will be retired at the mandatory retirement age.

However, the goal of the firm is not to maximize the net present value of staying for its youngest employees. First of all, it is interested in the present value of staying for all its employees. Secondly, it is really only interested in maximizing its own profits. To the extent that maximizing the net present value of staying for its employees contributes to maximum profits, it follows the interests of its employees. But there will be some competing interests between the firm and its employees.

The firm maximizes profits over choices of wage schedules, pension schedules, hiring schedules and mandatory retirement ages. It takes into account how its choices affect the exit behavior of its employees which in turn affects profits. The exit behavior of employees and the hiring schedule determine promotion possibilities, the productivity of its employees and the absolute size of the firm. In deciding upon a mandatory retirement age, the firm considers the wage and marginal product of its oldest employees. But it also considers how its oldest employees' leaving will affect the exit rate of

its other employees, and it discounts the marginal product of its oldest employees by how effectively younger employees can replace the oldest employees.

## Section II: The Firm's Problem

The firm has a production function such that it divides up its labor force into a continuum of positions referenced by  $G \in [0,1]$ , i.e., 100% of the labor force has a position worse than  $G$ . The productivity of an employee with general experience  $t$  in position  $G$  is  $\rho(G,t)$  where  $\rho_1 \geq 0$  and  $\rho_2 \geq 0$ .  $\rho(G,t)$  is determined by the exogenous production function.  $\rho(G,t)$  is net of any training costs.

The firm has a work force with a distribution of experience,  $H(t)$ .  $H(t)$  is determined by the exit rate of employees from the firm and the rate at which employees are hired for different positions.

Let  $\lambda(t)$  be the exit rate of employees with  $t$  years of experience. Then the survivor probability for employees who joined the firm with no experience,  $E(t)$ , is:

$$(2.1) \quad E(t) = \exp\left\{-\int_0^t \lambda(u)du\right\}$$

and the survivor probability for employees who joined the firm with  $s$  years of experience is  $E(t)/E(s)$ .

Let  $Z(t)$  be the cumulative distribution function for the number of employees hired with  $t$  years of experience. For example, if all employees are hired with no experience, then  $Z(t) = 1, \forall t \geq 0$ . The density function for

the distribution of employees by experience,  $H(t)$ , is:

$$(2.2) \quad h(t) = \frac{\int_0^t E(t)Z'(s)ds/E(s)}{\int_0^\tau \int_0^u E(u)Z'(s)dsdu/E(s)} = \frac{E(t)D(t)}{\int_0^\tau E(u)D(u)du}$$

where  $D(t) = \int_0^t \frac{Z'(s)}{E(s)}$  and  $\tau$  is the first age at which all employees have retired. The numerator is the proportion of employees that started  $t-s$  years ago who are still working for the firm, summed over  $s$ . The denominator is the proportion of employees that started  $u-s$  years ago who are still working for the firm, summed over  $u$  and  $s$ .<sup>3</sup>

The firm promotes employees strictly on the basis of seniority. It does this because more experienced employees have a comparative advantage in senior positions. The firm's promotion policy implies that  $G = H(t)$ .

The firm must choose a personnel policy consisting of four components. The first component is the wage schedule,  $w(G,t)$ . Since  $G = H(t)$ ,  $w(G,t) = w(H(t),t) = W(t)$ . The second component is a pension schedule,  $p(t)$ .  $p(t)$  is the present value of the stream of benefits an employee would receive from the firm if he left after accumulating  $t$  years of experience.  $W(t)$  and  $p(t)$  constitute total compensation paid to employees.

The third component is the hiring schedule,  $Z(t)$ . If the firm promotes only from within,  $Z(t) = 1 \quad \forall t \geq 0$  and  $Z'(t) = 0 \quad \forall t > 0$ . If the firm hires from without for position  $G = H(t)$ , then  $Z'(H^{-1}(G)) > 0$ .

The last component is a mandatory retirement age,  $\tau$ , at which all remaining employees are forced to leave the firm. It is possible that  $\tau$  is large enough so that all employees have voluntarily retired by  $\tau$ . The focus of this paper is the determination of  $\tau$ , the mandatory retirement age.

The goal of the firm is to maximize long term profits,  $\int_0^\infty e^{-rs}L(W,p,Z,\tau)ds$  where  $L(W,p,Z,\tau)$  is the profit earned by the firm at time

s.  $L(W,p,Z,\tau)$  depends upon the wage schedule,  $W$ , the pension schedule,  $p$ , the hiring schedule,  $Z$ , and the mandatory retirement age,  $\tau$ , in effect at time  $s$ . If the firm is in a steady state, then  $L$  does not depend upon time; maximizing long term profits is equivalent to maximizing instantaneous profits,  $L$ .

It is necessary to determine what instantaneous profits are. Let:

$$(2.3) \quad D(t) = \int_0^t Z'(s)ds/E(s)$$

so that  $E(t)D(t)$  is the number of employees with  $t$  years of experience. Instantaneous profits made on employees with  $t$  years of experience are:

$$(2.4) \quad [\rho(H(t),t) - W(t) - \lambda(t)p(t)]E(t)D(t) .$$

The firm also hires some new employees with  $t$  years of experience and incurs hiring costs of  $S^*(t)$ . Thus total instantaneous profits are:

$$(2.5) \quad L(w,p,Z,\tau) = \int_0^\tau [(\rho(H(t),t) - W(t) - \lambda(t)p(t))E(t)D(t) - S^*(t)Z'(t)]dt - p(\tau)E(\tau) .$$

### Section III: Employee Behavior

The firm needs to maximize  $L(W,p,Z,\tau)$  over choices of  $W$ ,  $p$ ,  $Z$  and  $\tau$ . However, it is constrained by how its decisions affect the exit rate of its employees and the cost of hiring new employees. If employees consider the

value of leaving the firm to be greater than the value of staying, they will leave. Furthermore, a new employee will only join the firm if the value of accepting a job is greater than the value of rejecting it.

Let  $V(t)$  be the average value to a worker with  $t$  years of experience of being employed at the firm. Let  $V^*(t)$  be the average value to a worker with  $t$  years of experience of not being employed at the firm. If workers are homogeneous, then the value of staying and leaving for each worker are  $V(t)$  and  $V^*(t)$  respectively. If workers are heterogeneous with respect to outside opportunities, then the values of staying and leaving vary by worker and  $V(t)$  and  $V^*(t)$  are only sufficient statistics for the distribution of values of staying and leaving.

The value of leaving the firm,  $V^*(t)$ , is a function of market wages, the cost of search, the value of leisure and the value of any income contingent on not working (e.g., Social Security payments and unemployment insurance). It is assumed that at some senior age,  $V^*(t)$  increases rapidly. This represents the cost of foregone Social Security payments and the increasing disutility of work caused by failing health.

The value of staying at the firm,  $V(t)$ , is a function of future wage and pension payments,  $V^*(t)$ , the exit rate and the mandatory retirement date:

$$(3.1) \quad V(t) = \left[ \int_t^{\tau} e^{-r(u-t)} (W(u) + \lambda(u)(V^*(u) + p(u))) E(u) du \right. \\ \left. + e^{-r(\tau-t)} E(\tau)(V^*(\tau) + p(\tau)) \right] / E(t)$$

which satisfies the differential equation:

$$(3.2) \quad V'(t) = rV(t) - W(t) - \lambda(t)(V^*(t) + p(t) - V(t)) .$$

Equation (3.2) states that  $V(t)$  changes over experience by the return on future wages minus the wages paid at  $t$  and the value of lost opportunities that occurred at  $t$ . Whether  $V(t)$  and  $V^*(t)$  are the values of staying and leaving or just the average values of staying and leaving, equation (3.2) holds.

Exit rates are determined by each employee deciding when is the optimal time for him to leave the firm. If employees are homogeneous with respect to the value of staying and leaving, then there is some time,  $t^*$ , such that the exit rate is  $\lambda(t) = 0$  for  $t < t^*$  and  $\lambda(t^*) = \infty$ . If employees are heterogeneous, then  $\lambda(t)$  is a function of  $V(t)$  and  $V^*(t)$ :

$$(3.3) \quad \lambda(t) = \Delta(V(t), V^*(t)) .$$

It is assumed that  $\Delta_1 \leq 0$  and  $\Delta_2 \geq 0$ , that there is some  $V^*(t)$  such that if  $V(t) < V^*(t)$  then  $\lambda(t) = \infty$ , and that  $\lambda(t) \geq 0$  when  $V(t) < \infty$ .

The cost of hiring employees with  $t$  years of experience,  $S^*(t)$  is also a function of  $V(t)$  and  $V^*(t)$ :

$$(3.4) \quad S^*(t) = S(V(t), V^*(t)) .$$

It is assumed that  $S_1 \leq 0$  and  $S_2 \geq 0$ . In other words, the greater the value of joining the firm is relative to not joining, the less it costs the firm to find new employees.

The firm's problem can be written as:

$$(3.5) \quad \max_{W, p, Z, \tau} L(W, p, Z, \tau) = \int_0^{\tau} [(\rho(H(t), t) - W(t))$$

$$- \lambda(t)p(t))E(t)D(t) - S(V(t),V^*(t))Z'(t)]dt$$

$$- p(\tau)E(\tau)$$

subject to

$$(3.6) \quad H'(t) = h(t) ,$$

$$(3.7) \quad D'(t) = Z'(t)/E(t) ,$$

$$(3.8) \quad \frac{h'(t)}{h(t)} = \frac{D'(t)}{D(t)} + \frac{E'(t)}{E(t)} ,$$

$$(3.9) \quad -E'(t)/E(t) = \lambda(t) ,$$

$$(3.10) \quad \lambda(t) = \Delta(V(t),V^*(t)), \text{ and}$$

$$(3.11) \quad V'(t) = rV(t) - W(t) - \lambda(t)(V^*(t) + p(t) - V(t)) .$$

This is a standard calculus of variations problem which can be solved with the standard techniques.

In the next three sections, this problem is solved in increasing generality. Section IV contains a simple case in order to build intuition. Section V adds enough detail to allow for a discussion of the role of pensions and turnover costs. Section VI is the most general case and allows for a discussion of discrimination in hiring against older workers.

Section IV: Homogeneous Employees

Assume that all employees face the same  $V(t)$  and  $V^*(t)$  schedule. This implies that there is some time,  $\tau$ , such that:

$$(4.1) \quad \lambda(t) = \begin{cases} 0 & \text{if } t < \tau \\ \infty & \text{if } t = \tau. \end{cases}^4$$

$\tau$  may be the age when  $V(t) < V^*(t)$  for the first time or it may be the mandatory retirement age. In this example, the firm does not need an explicit mandatory retirement age; it can induce all employees to retire at any particular age by just reducing total compensation enough so that it is in each employee's interest to leave.

Furthermore, assume that:

$$(4.2) \quad S^*(t) = \begin{cases} S^* & t = 0 \text{ and } V(0) \geq V^*(0) \\ \infty & \text{otherwise} \end{cases}$$

It is obvious that the firm should only promote from within, i.e.,  $Z(t) = 1 \forall t \geq 0$ . Furthermore, the firm should set total compensation for each cohort so that  $V(t) = V^*(t)$  until some time,  $\tau$ , that it wants employees to leave. Let  $W^*(t)$  be the total minimum compensation necessary to keep  $V(t) \geq V^*(t)$ .

Since  $V(t) = V^*(t)$  until  $\tau$ , all employees remain with the firm until  $\tau$  and then leave. Thus the survivor function is:



$$(4.3) \quad E(t) = \begin{cases} 1 & t < \tau \\ 0 & t \geq \tau. \end{cases}$$

This implies that the distribution of employees by experience is

$$H(t) = t/\tau \text{ for } t \in [0, \tau].$$

The firm has already picked the optimal hiring schedule and total compensation schedule. It only needs to pick an optimal mandatory retirement age:

$$(4.4) \quad \max_{\tau} L(\tau) = \int_0^{\tau} (\rho(\frac{t}{\tau}, t) - W^*(t))dt - S^*$$

The optimal mandatory retirement age is at the age when:

$$(4.5) \quad \rho(1, \tau) - W^*(\tau) = \frac{1}{\tau} \int_0^{\tau} \frac{t}{\tau} \rho_1(\frac{t}{\tau}, t) dt$$

which is equivalent to:

$$(4.6) \quad W^*(\tau) = \int_0^{\tau} \rho_2(\frac{t}{\tau}, t) dt + \int_0^{\tau} \frac{1}{\tau} \rho(\frac{t}{\tau}, t) dt .$$

Equation (4.6) says that the firm should set the mandatory retirement age at the age when the total compensation necessary to keep the oldest employee is equal to the average product of all of its employees plus the gains to a more experienced workforce. The necessary second order condition is that  $W^*(t)$  is rising faster than the right hand side of equation (4.6) at  $\tau$ .

The first and second order conditions for a finite mandatory retirement age should be met if  $W^*(t)$  rises fast enough. Furthermore, the optimal

mandatory retirement age is likely to be at age 65 if  $V^*(t)$  rises discontinuously at that age because of the earnings test for collecting Social Security benefits.

If  $\rho_2 = 0$ , i.e., there are no productivity gains to experience, then the mandatory retirement age is the age when total compensation is equal to average product. This occurs even though the oldest employee's product is  $\rho(1)$  which is greater than the average product. Since all employees would be as effective as the oldest in the top position, the loss of the top employee is only the average product. When  $\rho_2 > 0$ , there is an added cost to firing the oldest employee. However, there is still no reason why  $W^*(\tau)$  is equal to the product of the oldest employee.

Up until now it has been assumed that the firm hires one unit of employees. Another possible reasonable assumption would be that it maintains a total workforce of one unit. Let employees be hired at the rate  $\alpha$ . The total workforce is then:

$$(4.7) \quad \alpha \int_0^\tau E(t) dt$$

which in this example is equal to  $\alpha\tau$ . In order to maintain a workforce of size 1, the firm must hire new employees at the rate,  $\alpha = \frac{1}{\tau}$ . The firm's maximization problem becomes:

$$(4.8) \quad \max_{\tau} L(\tau) = \frac{1}{\tau} \left[ \int_0^\tau \rho\left(\frac{t}{\tau}, t\right) - W^*(t) \right] dt - S^*$$

which has an optimum when:

$$(4.9) \quad W^*(\tau) - \int_0^\tau \frac{1}{\tau} W^*(t) dt = \int_0^\tau \rho_2\left(\frac{t}{\tau}, t\right) dt + \frac{1}{\tau} S^* .$$

The difference between the highest wage and the average wage must equal the gains to increased experience plus the reduction in hiring costs. The rest of the intuition from the problem holding the number of employees hired fixed is the same.

### Section V: Heterogeneous Employees

Assume that employees are heterogeneous with respect to outside opportunities. Thus:

$$(5.1) \quad \lambda(t) = \Delta(V(t), V^*(t)).$$

$\Delta(V(t), V^*(t))$  is common knowledge but any particular employee's value of leaving is unknown to the firm. Further, assume that:

$$(5.2) \quad S^*(t) = \begin{cases} S(V(0), V^*(0)) & t = 0 \\ \infty & t > 0. \end{cases}$$

Thus  $Z(t) = 1 \quad \forall t \geq 0$ .

The firm maximizes instantaneous profits by solving a calculus of variations problem which is developed below. Let  $y(t)$  be the set of dependent variables:

$$(5.3) \quad y(t) = [H(t), W(t), E(t), \lambda(t), V(t), h(t), p(t)].$$

Let:

$$(5.4) \quad \Omega(t, y(t), y'(t)) = [\rho(H(t), t) - W(t) - \lambda(t)p(t)]E(t) .$$

Then the firm maximizes:

$$(5.5) \quad L(W, p, \tau) = \int_0^{\tau} \Omega(t, y(t), y'(t)) dt \\ - S(V(0), V^*(0)) - p(\tau)E(\tau)$$

subject to:

$$(5.6) \quad H'(t) = h(t) ,$$

$$(5.7) \quad h'(t)/h(t) = E'(t)/E(t) ,$$

$$(5.8) \quad -E'(t)/E(t) = \lambda(t) ,$$

$$(5.9) \quad \lambda(t) = \Delta(V(t), V^*(t)), \text{ and}$$

$$(5.10) \quad V'(t) = rV(t) - W(t) - \lambda(t)(V^*(t) + p(t) - V(t))$$

and the terminal conditions:

$$(5.11) \quad H(0) = 0 , \quad H(\tau) = 1 , \quad E(0) = 1 , \quad \text{and} \quad V(\tau) = V^*(\tau) .$$

This problem can be written in Lagrangian form as:

$$\begin{aligned}
(5.12) \quad \phi(t, \chi(t), \chi'(t)) = & \Omega(t, \chi(t), \chi'(t)) \\
& + \delta_1(t)[H'(t) - h(t)] \\
& + \delta_2(t)[E'(t) + \lambda(t)E(t)] \\
& + \delta_3(t)[h'(t) + \lambda(t)h(t)] \\
& + \delta_5(t)[\lambda(t) - \Delta(V(t), V^*(t))] \\
& + \delta_6(t)[V'(t) - rV(t) + W(t) + \lambda(t)(V^*(t) + p(t) - V(t))] .
\end{aligned}$$

First order conditions for an interior solution<sup>5</sup> are  $\partial\phi/\partial\chi = (d/dt)(\partial\phi/\partial\chi')$ :<sup>6</sup>

$$(5.13) \quad \rho_1(H, t)E = \delta_1' \quad (H) ,$$

$$(5.14) \quad E = \delta_6 \quad (W) ,$$

$$(5.15) \quad \rho(H, t) - W - \lambda p + \delta_2 \lambda = \delta_2' \quad (E) ,$$

$$(5.16) \quad -pE + \delta_2 E + \delta_3 h + \delta_5 + \delta_6(V^* + p - V) = 0 \quad (\lambda) ,$$

$$(5.17) \quad -\delta_5 \Delta_1(V, V^*) - \delta_6(r + \lambda) = \delta_6' \quad (V) ,$$

$$(5.18) \quad -\delta_1 + \delta_3 \lambda = \delta_3' \quad (h) , \text{ and}$$

$$(5.19) \quad \lambda E = \delta_6 \lambda \quad (p) .$$

Note that the first order conditions for  $W$  and  $p$ , equations (5.14) and (5.19), are equivalent whenever  $\lambda > 0$ , i.e., whenever any employee collects a pension. Since there are perfect capital markets, the firm and its employees are indifferent between the same sets of wage and pension schedules. Given any optimal wage and pension schedule, there is a continuum of wage and pension schedules that are as good. One of them sets  $p(t) = 0$ . Thus, without loss of generality,  $p(t)$  is set equal to zero.

The remaining equations can be reduced to four equations in  $H$ ,  $\lambda$ ,  $V$  and  $W$ :

$$(5.20) \quad H''/H' = -\lambda ,$$

$$(5.21) \quad \lambda = \Delta(V, V^*),$$

$$(5.22) \quad V' = rV - W - \lambda(V^* - V) , \text{ and}$$

$$(5.23) \quad V = V^* - \frac{1}{AH'r} \int_t^\tau [c(u) - W(u)] AH' du - \frac{r}{\Delta_1(V, V^*)}$$

where  $A$  is a constant of integration equal to:

$$(5.24) \quad A = \int_0^\tau E(u) du$$

and  $c(t) = \int_0^t \rho_2(H, s) ds$  is the cumulative value of experience. Initial and terminal conditions that determine the optimal  $\tau$  are:

$$(5.25) \quad H(\tau) = 1 ,$$

$$(5.26) \quad V(\tau) = V^*(\tau) ,$$

$$(5.27) \quad E(\tau) = 0$$

$$(5.28) \quad \Delta_1(V(\tau), V^*(\tau)) = -\infty ,$$

$$(5.29) \quad V(0) + \int_0^{\tau} [\rho(H, u) - W]Edu = V^*(0) - \frac{r}{\Delta_1 V(0), V^*(0)}$$

$$(5.30) \quad 1 = -S_1(V(0), V^*(0))$$

Equations (5.27) and (5.28) imply that the firm should set a wage schedule so that all employees retire voluntarily by the mandatory retirement age. Technically, no employees should be forced to retire. However, this only means that the firm should offer older employees such a low wage that effectively they are forced to retire. This is analogous to the well known discussion of the distinction between quits and layoffs.

Equation (5.29) states that the total value of the firm-employee relationship, the value of the job to the employee plus the profits made on employees, must equal the value of alternative opportunities with an adjustment for turnover. This equation determines the optimal mandatory retirement age as long as the wage necessary to keep any employees rises with age after some age,  $t$ .

Equation (5.30) states that at the optimum, a small increase in the value of a job at time zero should be just offset by the reduced cost of search.

The ADEA prevents the firm from imposing a mandatory retirement age

before 70 and from lowering its older employees' wages solely because of age. However, ADEA says nothing about pensions. In fact the early retirement benefits provided by most pension plans have the same effect as reduced wages; both can cause employees to retire before the earliest allowed mandatory retirement age. Thus, a restriction on reducing wages is irrelevant unless there is also a restriction on total compensation.

### Section VI: Promotion from Outside

The firm's problem written in equations (3.5) through (3.11) now is considered in its complete generality. However, pension benefits are set equal to zero since only total compensation matters.<sup>7</sup> The problem is rewritten for the reader's convenience:

$$(6.1) \quad \max_{W, Z, \tau} L(W, Z, \tau) = \int_0^{\tau} [(\rho(H(t), t) - W(t))E(t)D(t) - S(V(t), V^*(t))Z'(t)] dt$$

subject to:

$$(6.2) \quad H'(t) = h(t) ,$$

$$(6.3) \quad D'(t) = Z'(t)/E(t) ,$$

$$(6.4) \quad \frac{h'(t)}{h(t)} = \frac{D'(t)}{D(t)} + \frac{E'(t)}{E(t)} ,$$

$$(6.5) \quad -E'(t)/E(t) = \lambda(t) ,$$



$$(6.6) \quad \lambda(t) = \Delta(V(t), V^*(t)) , \text{ and}$$

$$(6.7) \quad V'(t) = rV(t) - W(t) - \lambda(t)(V^*(t) - V(t))$$

with terminal and initial conditions:

$$(6.8) \quad H(0) = 0 , \quad H(\tau) = 1 , \quad E(0) = 1 , \quad V(\tau) = V^*(\tau) , \quad Z(\tau) = 1.$$

Let: "

$$(6.9) \quad \chi(t) = [H(t), W(t), E(t), \lambda(t), V(t), Z(t), D(t), h(t)]$$

and:

$$(6.10) \quad \Omega(t, \chi(t), \chi'(t)) = [\rho(H(t), t) - W(t)]E(t)D(t) \\ - S(V(t), V^*(t))Z'(t) .$$

Then the Lagrangian equation is:

$$(6.11) \quad \phi(t), \chi(t), \chi'(t) = \Omega(t, \chi(t), \chi'(t)) \\ + \delta_1(t)[H'(t) - h(t)] \\ + \delta_2(t)[E'(t) + \lambda(t)E(t)] \\ + \delta_3(t)[h'(t) - \frac{D'(t)}{D(t)} h(t) + \lambda(t)h(t)]$$

$$\begin{aligned}
& + \delta_4(t) \left[ D'(t) - \frac{Z'(t)}{E(t)} \right] \\
& + \delta_5(t) [\lambda(t) - \Delta(V(t), V^*(t))] \\
& + \delta_6(t) [V'(t) - rV(t) + W(t) \\
& + \lambda(t)(V^*(t) - V(t))] .
\end{aligned}$$

In theory, necessary first order conditions can be taken and solved. In order to determine the optimal mandatory retirement age, initial and terminal conditions must be taken. But terminal conditions are only valid if the interior solution to equation (6.11) is a true optimum. In fact, the optimal solution must have a corner. There is some age,  $t^* < \tau$ , at which all employees have been hired;  $Z(t^*) = 1$  and  $Z'(t) = 0 \quad \forall t > t^*$ . It can be shown that  $t^*$  is at the point where:

$$\begin{aligned}
(6.31) \quad S(V(t^*), V^*(t^*)) &= \int_{t^*}^{\tau} \frac{H'(s)}{H'(t^*)} [\rho(H(s), s) - W(s)] ds \\
&- \int_{t^*}^{\tau} \frac{H'(s)}{H'(t^*)} \int_0^s \rho_2(H(x), x) dx ds .
\end{aligned}$$

At  $t^*$ , hiring costs must be equal to the average product of employees with at least  $t^*$  years of experience adjusted for gains to experience. After  $t^*$ , hiring costs are greater than can be earned by employees hired at that age.

The firm is only willing to hire new employees older than  $t^*$  if it can pay them a lower wage than an employee with the same experience already hired. This idea is similar to models of discrimination against women because

of higher turnover costs. See Barnes and Jones (1974) or Salop and Salop (1976).

Since  $S(V(t), V^*(t)) > 0$  for all  $t$  and the right hand side of equation (6.31) approaches zero as  $t^*$  approaches  $\tau$ , there must be a  $t^* < \tau$  at which equation (6.31) is satisfied. At this age, the firm solves a problem similar to that solved in Section V. Thus, the optimal mandatory retirement age already has been characterized for this section.

### Section VII: Conclusions

A model of the firm has been presented in which the productivity of its employees depends on their positions as well as their experience. Each employee leaves the firm when it is optimal for him to do so. It has been shown that:

- 1) The optimal mandatory retirement age is a function of the reservation value function, the productivity schedule and the increase to average productivity of having an older workforce.
- 2) When capital markets are perfect, for any optimal wage and pension schedule, there is another wage schedule with no pension benefits that is as good for both the firm and its employees. This occurs because employees can save as effectively as the firm.
- 3) Pensions may play a role in inducing employees to retire when there is a) a ban on mandatory retirement and also b) restrictions on

lowering older employees' wages. The firm can reduce the total value of employment effectively either by reducing wages or by reducing the present value of pensions. The firm reduces the present value of pensions to its oldest employees by providing large early retirement benefits. The reduction of the present value of pensions for older employees has the same effect on employee retirement behavior as wage reductions would have had they not been illegal.

- 4) The wage schedule depends more on an employee's opportunities outside of the firm than on his marginal product. It is only in equilibrium that wage may equal marginal product.
- 5) Our Social Security system causes firms to make the mandatory retirement age 65 and causes many employees to retire before age 65. The benefits test and early benefits make this happen.
- 6) Firms discriminate against potential older employees because it is difficult to regain hiring costs. This is even true when the added benefit of having an older workforce is considered.

Unfortunately, there are some basic questions that this model does not address. These include:

- 1) Why do some firms have a mandatory retirement age while others do not?

- 2) Why is the incidence of mandatory retirement correlated with firm size?
- 3) Why is the incidence of mandatory retirement correlated with the incidence of pension programs?

The answers to these questions depend upon the form of the exit rate function, the productivity function and the search cost function.

The most intriguing question concerning mandatory retirement is why the great majority of firms with a mandatory retirement program have a mandatory retirement age of 65. Both Lazear (1979) and this paper suggest that the Social Security earnings test causes this. But, it is not clear that such a result would follow if employees were heterogeneous. For example, in a model with heterogeneous ability we might observe the existence of a tenure age as exists in universities and many law firms or even multi-tiered tenure structures as exists in the armed forces. This is a topic for future research.

It is too early to derive any policy implications from this model. It is clear that older workers are discriminated against both because they are fired at a somewhat arbitrary age independent of their ability to work and because they have a difficult time finding new jobs. This model presents some reasons that firms discriminate against older workers. It implies that it may be Pareto optimal to allow for such discrimination. However, there are many firms that have no mandatory retirement age. Before evaluating the value of mandatory retirement programs we also must understand why some firms do not have mandatory retirement programs.

## Footnotes

1. See Kotlikoff and Smith (1983) for this data.
2. William Mercer-Mendinger, Inc., "Employer Attitudes Toward Mandatory Retirement," New York, 1977, page 6.
3. For example, let  $Z(t) = 1, t \geq 0$  and:

$$(2.2a) \quad \lambda(t) = \begin{cases} \infty & \text{if } t = \tau \\ 0 & \text{if } t < \tau \end{cases}$$

Then  $E(t) = 1$  if  $t < \tau$  and  $E(t) = 0$  if  $t \geq \tau$ .  $h(t)$  is equal to:

$$(2.2b) \quad h(t) = 1/\int_0^\tau E(u)du = \frac{1}{\tau}$$

and  $H(t) = t/\tau$ , which is the uniform distribution with bounds  $[0, \tau]$ .

4. The flavor of the results would not change if  $\lambda(t) = \begin{cases} \lambda \geq 0 & t < \tau \\ \infty & t = \tau \end{cases}$ .

5. The optimal solution to the firm's problem must be an interior solution between 0 and  $\tau$ . The only variables that could possibly have corner solutions are  $\lambda$ ,  $E$ ,  $W$ ,  $p$  and  $V$ . If  $\lambda(t) = \infty$  then  $t = \tau$ .  $\lambda(t) > 0$  when  $V(t) < \infty$  by assumption.  $V(t)$  can never fall below  $\underline{V^*(t)}$  and  $V(t)$  can only diverge to  $\infty$  if future total compensation diverges to infinity. But the firm would lose money by providing such high total compensation.

6. The independent variable,  $t$ , is implicit.

7. Another reason to drop pensions is that it does not make sense to pay two employees who leave at the same time the same pension benefit if they started at different times. To correct for this problem is too difficult and adds no insight.

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