

Eckstein, Zvi; Stern, Steven; Wolpin, Kenneth I.

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ECONOMIC GROWTH CENTER

YALE UNIVERSITY

P.O. Box 1987, Yale Station  
27 Hillhouse Avenue  
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 439

FERTILITY CHOICE IN A GROWTH MODEL WITH LAND: MALTHUS RECONSIDERED

Zvi Eckstein

Steven Stern

Kenneth I. Wolpin

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**Notes:** Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

## 1. Introduction

This paper presents a neoclassical growth model designed to explore the robustness of Malthus's pessimistic conjecture about the inevitability of a subsistence steady state to alterations in his fundamental postulates. Although Malthus's argument is well known, it is best to be as precise as was Malthus himself. He wrote

I think I may fairly make two postulata. First, that food is necessary to the existence of man. Secondly, that the passion between the sexes is necessary and will remain nearly in its present state. . . Assuming then my postulata as granted, I say, that the power of population is indefinitely greater than the power in the earth to produce subsistence for man.<sup>1</sup>

Critics of Malthus have tended to concentrate on his failure to predict "exogenous" technical change that would augment the productive power of the land to overcome the diminishing returns that are due to the fixity of land.<sup>2</sup> Yet, such an explanation of the rising living standards that dramatically deny the Malthusian hypothesis is vacuous unless the conditions for the existence and adoption of technical change are explicated as well. We do not in this paper attempt to rectify this shortcoming, except insofar as endogenous capital accumulation is considered analogous to technical change. Instead, we focus on the second of the two postulates, namely on exogenous fertility.

We utilize the Samuelson (1958) - Diamond (1965) overlapping generations growth model as a neoclassical paradigm within which to analyze Malthusian

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<sup>1</sup>Thomas Robert Malthus, An Essay on the Principle of Population, first published 1789, (Penguin Books, England 1979)p.70-71.

<sup>2</sup>Some recent economic literature that has dealt with the Malthusian hypothesis in the context of economic growth is surveyed by Pitchford (1974).

assumptions and results. We add to that model a fixed factor of production that is essential to the production of food--land. We also allow the single good to be stored and used in future period production. Given these assumptions, both Malthus's concern about the population pressure on land as well as the issue of technical change and capital accumulation as they possibly impinge upon Malthus's contention can be treated within this framework.

There are a number of studies (see e.g. Pitchford) which treat population growth, and thus fertility, as endogenous either in the sense that population growth is assumed to be related to capital per-capita and/or consumption per capita, or in the sense that population directly enters a social welfare criterion and is optimally "chosen". In this paper, as in Eckstein and Wolpin (1982) (see also Razin and Ben-Zion (1975)) we depart from this tradition by assuming that fertility is a choice of the individual, i.e., the manner in which population changes over time is behaviorially determined. This seemingly minor modification turns out to have major consequences for economic growth when there is a fixed factor of production.

The paper is organized as follows. In the next section we describe the preference structure and the technology of an economy with land. Then, in Section 3 we show that Malthus's assumptions and conclusions are consistent within a neoclassical growth model. In other words, if fertility is not subject to individual choice, and land is fixed and essential for production, then per capita consumption converges to a subsistence level (zero) independent of the organization of the economy, planned or decentralized. However, if land is not essential, as for example in constant elasticity of substitution production functions with an elasticity of

substitution that is greater than unity, Malthus's results do not emerge. We also show an example of a decentralized Malthusian economy which exhibits the long-run subsistence outcome. In Section 4 we permit individuals to choose the quantity of children to bear, balancing the psychic benefits of children against an exogenous child-rearing cost. We demonstrate that in a model where land is essential, there exists a competitive solution in which the fertility rate converges to unity, that is, to zero population growth. The Malthusian outcome is thus seen to depend heavily on the notion that fertility is not a choice at the individual level, even in a decentralized economy.<sup>3</sup> Section 5 summarizes and briefly discusses the relationship of the fixed factor growth model to growth models with exhaustible resources.

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<sup>3</sup>Malthus did recognize avenues through which fertility (or the number of surviving offspring) would be affected by economic growth. For example, age at marriage might respond to income, but in Malthus's view only a limited extent, and other forms of deliberate fertility control were assumed to be unimportant. Population change occurred mainly through the response of mortality to economic circumstance. In any case, even if for Malthus population growth was related to economic activity, it was only through a technical, non-behavioral mechanism, which as we have already noted is the predominant assumption of neoclassical growth models with "endogenous" population.

## 2. A Growth Model with Land and Changing Population.

Technology is represented by a constant returns to scale aggregate production function  $F(K, L, R)$  where  $K$  is capital,  $L$  is labor, and  $R$  is land, such that  $f(k, r) = F\left(\frac{K}{L}, 1, \frac{R}{L}\right)$  where  $k = \frac{K}{L}$  and  $r = \frac{R}{L}$ . The single good can either be consumed or stored as capital for next period consumption. Capital depreciates at rate  $\delta$  in storage and production. Land cannot be directly consumed and does not depreciate in production. Individuals live for three periods, as infants who make no decisions in the first period, as workers ("young") in the second period, and finally as retired ("old") in the third period. In the second period, individuals supply one unit of labor and decide upon life cycle consumption (savings). Individuals are assumed to enjoy parenthood and they decide upon the quantity of own children in the second period of life. Children are costly to bear and rear; each child born at time  $t$  consumes  $e$  units of the good.

The representative individual of generation  $t$  has lifetime utility function

$$(2.1) \quad V(C_1(t), C_2(t), n(t+1))$$

where  $C_1(t)$  is the consumption of a member of generation  $t$  at period  $i + 1$  of the individual's life ( $i=1, 2$ ), and  $n(t+1)$  is the number of

children (fertility) of each member of generation  $t$ <sup>4</sup>. The utility function satisfies the usual concavity and differentiability conditions with respect to all variables. To ensure that  $C_1$  and  $C_2$  are never optimally zero, the utility function is assumed to satisfy the following condition

$$(2.2) \quad \frac{V_1(C_1, C_2)}{V_2(C_1, C_2)} \rightarrow \infty \quad (0) \quad \text{as} \quad C_1(C_2) \rightarrow 0$$

At time  $t$  the economy consists of  $N(t+1)$  infants,  $N(t)$  young and  $N(t-1)$  old. The economy begins at  $t=1$  with  $N(0)$  old and  $N(1)$  young as initial conditions. Each of the initial old is endowed with  $K(1)$  units of capital and  $\frac{R}{N(0)}$  units of land, where  $R$  is the aggregate fixed stock of land. Since all individuals are assumed to be alike, there are  $N(t) = n(t)N(t-1)$  young at each period  $t \geq 1$ . Each of the old at time  $t$  owns  $K(t)$  units of capital and  $R(t) = \frac{R}{N(t-1)}$  units of land. Since each young supplies one unit of labor, the number of workers at time  $t$  is  $N(t) = L(t)N(t-1)$  with  $L(t) = n(t)$  the number of workers per old at time  $t$ .

Consumption possibilities for the economy at time  $t$  is given by<sup>5</sup>

$$(2.3) \quad C_1(t) + \frac{C_2(t-1)}{n(t)} + en(t+1) = f(k(t), \frac{R}{N(t)}) - n(t+1)k(t+1) + (1-\delta)k(t)$$

<sup>4</sup> Alternatively one can view  $n(t+1)$  as the number of surviving children given a fixed and known child mortality rate, i.e., as the net fertility rate.

<sup>5</sup> Aggregate consumption expenditures must equal aggregate output less net savings, i.e.

$$N(t)C_1(t) + N(t-1)C_2(t-1) + en(t+1)N(t) = N(t-1)F(K(t), L(t), R(t)) - N(t)K(t+1) + (1-\delta)N(t-1)K(t)$$

Dividing by  $N(t)$  yields equation (2) in the text recognizing that

$$f(k(t), \frac{R}{N(t)}) = \frac{1}{n(t)} F(K(t), n(t), R(t)).$$

An allocation  $\{C_1(t), C_2(t-1), n(t+1), K(t-1)\}$  is feasible for all  $t \geq 1$  if equation (2.2) is satisfied for non-negative values of  $C_1(t)$ ,  $C_2(t-1)$  and  $n(t+1)$ . A stationary allocation (steady state) is defined such that

$$\lim_{t \rightarrow \infty} C_1(t) = C_1 \geq 0, \quad \lim_{t \rightarrow \infty} C_2(t) = C_2 \geq 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} n(t) = n \geq 0. \quad \text{Thus,}$$

at the steady state, consumption possibilities and lifetime utility are independent of the time index  $t$ .

### 3. A Malthusian Economy

Malthus in his essay on the Principle of Population made three assumptions: (a) the marginal product of labor is decreasing; (b) land is an essential factor of production and its quantity is fixed; and (c) the population growth rate is not related to individual choice.

Consider, then, the economy described above, however, with exogenous constant population growth, i.e.,  $n(t) = n > 1$ .<sup>6</sup> Without loss of generality we assume that fertility is costless ( $e=0$ ). Given that framework we can demonstrate the validity of the main proposition of Malthus, namely that under the preceding assumptions the economy must reach or approach subsistence consumption. To do that define the essentiality of land in production by the following condition:<sup>7</sup>

$$(3.1) \quad \lim_{r \rightarrow 0} f(k_0, r) = 0 \quad \text{for all } \infty > k_0 > 0.$$

<sup>6</sup>For the case when  $n(t)$  changes over time, see the later discussion in this section.

<sup>7</sup>This is a conventional definition of essentiality as in the literature on exhaustible resources, e.g., Solow (1974).



Proposition: In the Malthusian economy in which land is essential in production consumption per capita approaches or reaches zero (subsistence). If capital per capita monotonically decreases, consumption per capita approaches zero; otherwise consumption is zero in a finite time.

Proof: From the feasibility constraint (2.3) consumption is positive only if

$$(3.2) \quad C_1(t) + \frac{C_2(t-1)}{n} = f(k(t), \frac{R}{N(0)n^t}) - n k(t+1) + (1-\delta)k(t) > 0$$

From (3.2) it is clear that if  $k(t+1) < \frac{1-\delta}{n} k(t)$  then consumption is positive for  $k(t) > 0$ . However, if  $k(t) \rightarrow 0$  then since  $f(\dots) \rightarrow 0$  as  $t \rightarrow \infty$ , per capita consumption must also go to zero. If  $k(t) \rightarrow k > 0$ , then eventually  $k(t+1) > \frac{1-\delta}{n} k(t)$  and consumption is negative (given the essentiality of land).

Thus,  $f(\dots)$  approaches zero unless  $k(t)$  is monotonically increasing. Given that  $n > 1$  and  $0 < \delta < 1$ , so that  $n + \delta - 1 > 0$ , it is sufficient to prove that

$$(3.3) \quad f(k(t), \frac{R}{N(0)n^t}) - k(t) (n+\delta-1) < 0 \text{ for some } t > 1.$$

From constant returns to scale of  $F(\dots)$  we know that for any  $\lambda > 1$ ,  $f(\lambda k(T), r) < f(\lambda k(T), \lambda r) < \lambda f(k(T), r)$ . Hence, for any  $t > T$

letting  $\lambda = \frac{k(t)}{k(T)}$ , and  $r = \frac{R}{N(0)n^t}$ , it follows that

$$(3.4) \quad f(k(t), \frac{R}{N(0)n^t})/k(t) < f(k(T), \frac{R}{N(0)n^t})/k(T).$$

Since the right hand side of (3.4) approaches zero as  $t \rightarrow \infty$  given the essentiality of land (3.1), the left hand side must also approach zero. Then, there must exist a time  $T^*$  at which for all  $t > T^*$ ,

$$f(k(t), \frac{R}{N(0)n^t})/k(t) < (n+\delta-1). \text{ Thus, for all } t > T^*,$$

$$f(k(t), \frac{R}{N(0)n^t}) < (n+\delta-1)k(t). \text{ Q.E.D.}$$

It is first of all important to emphasize that the proposition is derived only from the feasibility condition and is therefore independent of the nature of economic organization. Neither a planner nor a Walrusian auctioneer could alter the outcome. It is also useful to recognize that there exists a somewhat stronger version of the proposition which permits an interaction between the net fertility rate and economic activity as some might argue is closer to Malthus's intention. Suppose that instead of a constant population growth rate, we consider a given sequence of net fertility rates  $\{n(t)\}_{t=0}^{\infty}$  which could be, for example, a sequence corresponding to any particular sequence of per-capita income. Further assume that  $n(t) > 1$  for all  $t$  and that  $\lim n(t)$  converges to a value greater than one. If we define  $\bar{n} = \min_{t=0}^{\infty} \{n(t)\}$ , then the proof of the proposition carries over exactly for this value of  $\bar{n}$  and therefore clearly holds for the given sequence of population growth rates.

Malthus defined subsistence consumption to be that level at which population was stable. Although as just noted, we have simplified by assuming that population growth is independent of consumption, it is easy to accommodate this notion of subsistence. To do so, define subsistence consumption to be a value,  $\epsilon > 0$ , such that for all levels of consumption per capita below  $\epsilon$ , population is constant. Our proposition would then imply that the economy will reach this subsistence level of consumption in a finite time and will remain there as long as population is constant.

The proposition stated above is not trivial given the existence of capital in the model. If there is no capital, then the Malthusian result follows even if land is not essential, i.e.,  $\lim_{R \rightarrow 0} F(0, L, R) > 0$ , as long as the marginal product of labor is, after some point, declining.

The above proposition shows that even with endogenous capital accumulation, the economy converges to subsistence consumption if land is fixed in quantity and is essential for production, and fertility is exogenously given at greater than the replacement rate. Malthus's pessimism is not necessarily due to a misunderstanding of the process of capital accumulation nor to an inability to foresee technical change, for even costless technical change has to be sustained at an average rate which is higher than the exogenous rate of population growth in order to prevent the eventual decline in consumption.

Our proposition gives sufficient conditions for the Malthusian result. In order to understand the importance of these conditions, we focus now on two examples. First, it is straightforward to see that if land is not essential in production, as in the case of the CES production function with an elasticity of substitution greater than 1, the Malthusian result does not follow since  $\lim_{r \rightarrow 0} f(k, r) > 0$ . For example, if land and capital are perfect substitutes in production, the model is asymptotically equivalent to the standard growth model.

Second, in Section 4, we use the Cobb-Douglas example with essential land to demonstrate that if fertility is a choice of the economic agents, then there exists an equilibrium path for the decentralized economy in which the steady state coincides with zero population growth. As such, we show the necessity of condition (c), given at the beginning of this section, for the Malthusian result.

#### A Malthusian Decentralized Economy: An Example

To make the transition to the example in Section 4 and to give some basis for comparison, we first characterize the decentralized Malthusian economy. The problem of a young person at generation  $t$  who is born at  $t-1$  is to maximize

$$(3.5) \quad V(C_1(t), C_2(t))$$

for all  $t \geq 1$ , subject to

$$(3.6) \quad C_1(t) = W(t) - K(t+1) - P(t)R(t+1)$$

$$(3.7) \quad C_2(t) = F(K(t+1), L(t+1), R(t+1)) - W(t+1)L(t+1) + (1-\delta)K(t+1) + P(t+1)R(t+1)$$

by choice of  $K(t+1)$ ,  $R(t+1)$  and  $L(t+1)$ . Each of the young of generation  $t$  saves  $K(t+1)$  units of the single consumption good for use in production at time  $t+1$  and purchases  $R(t+1)$  units of land for the same purpose at price per unit  $P(t)$ . At time  $t$ , each young supplies exactly one unit of labor and receives as a wage  $W(t)$  units of the consumption good. At time  $t+1$  each of the old of generation  $t$  hires  $L(t+1)$  units of labor for production using the accumulated capital  $K(t+1)$  and purchased land  $R(t+1)$ , and consumes the net of labor cost production, the non-depreciated quantity of capital, and the revenues from selling the non-depreciated land.

The first-order necessary condition for a maximum are

$$(3.8) \quad -V_1 + [F_1 + (1-\delta)]V_2 \leq 0 \quad \text{with} = \text{if } K(t+1) > 0$$

$$(3.9) \quad [F_2 - W(t+1)]V_2 \leq 0 \quad \text{with} = \text{if } L(t+1) > 0$$

$$(3.10) \quad -P(t)V_1 + [F_3 + P(t+1)]V_2 \leq 0 \quad \text{with} = \text{if } R(t+1) > 0$$

Observe that (3.9) implies that the real wage is equal to the marginal product of labor, and that, if  $K(t+1)$  and  $R(t+1)$  are positive then (3.8) and (3.10) together imply that the net rate of return on capital is equal to the rate of return on land.

In addition to the existence of non-negative values of  $K(t+1)$ ,  $L(t+1)$ ,  $R(t+1)$ ,  $W(t)$  and  $P(t)$  which satisfy (3.8) - (3.10), a perfect foresight competitive equilibrium requires that land and labor markets clear. The equality between labor demand and labor supply is given by

$$(3.11) \quad L(t)N(t-1) = N(t)$$

and that between the demand for land and its exogenously given stock by

$$(3.12) \quad R(t+1)N(t) = R$$

We consider the example where capital is fully depreciated in production ( $\delta=1$ ), where the utility function is log additive

$$(3.13) \quad V(C_1(t), C_2(t)) = \beta_1 \ln C_1(t) + \beta_2 \ln C_2(t)$$

and where production is Cobb-Douglas

$$(3.14) \quad F(K(t+1), L(t+1), R(t+1)) = AK(t+1)^{\alpha_1} L(t+1)^{\alpha_2} R(t+1)^{1-\alpha_1-\alpha_2} \\ = AL(t+1)k(t+1)^{\alpha_1} r(t+1)^{1-\alpha_1-\alpha_2}$$

$$\text{with } k(t) = \frac{K(t)}{L(t)} \quad \text{and } r(t) = \frac{R(t)}{L(t)}. \quad 8$$

<sup>8</sup>The Cobb-Douglas example is used extensively in the exhaustible resource literature because it satisfies the essentiality condition. We adopt it here for the same reason.

Using the first-order conditions (3.8) - (3.10), the budget constraints (3.6) - (3.7) and the market clearing conditions (3.11) - (3.12), an equilibrium path for capital per capita can be shown to be

$$(3.15) \quad \log k(t+1) = a(t) + \alpha_1 \log k(t)$$

where  $a(t) = a - (1-\alpha_1-\alpha_2)t \log n$ .<sup>9</sup> Notice that if  $1-\alpha_1-\alpha_2 = 0$ , i.e., if there is no fixed factor one gets the conventional equilibrium path for capital per capita. However, with a fixed factor it is apparent from (3.15) that  $k(t)$  must eventually converge to zero since  $\lim_{t \rightarrow \infty} a(t) = -\infty$ , as must consumption per-capita as shown previously. But notice that if  $a(t) > 0$ ,  $k(t)$  rises, which may be the case for small  $t$ . Thus, there exists a competitive equilibrium in the decentralized Malthusian economy that is characterized by eventual immiseration, although "short run" growth could also be observed.

#### 4. Endogenous Fertility in a Decentralized Malthusian Growth Model: An Example

In this section we consider a simple modification of the Malthusian model of the previous section. We assume, as in section 2, that children are costly to bear and rear ( $e > 0$ ) and that individuals enjoy parenthood and choose the number of children in the second period of their life. The utility function is thus given by

$$(4.1) \quad V(C_1(t), C_2(t), n(t+1))$$

and the first period budget constraint reflects the cost of rearing the  $n(t+1)$  children,  $en(t+1)$ . We again consider the log additive form

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<sup>9</sup> $a = \log \gamma + (1-\alpha_1-\alpha_2)[\log R - \log N(0)]$  where  $\gamma = \frac{\theta \alpha_1}{n[\theta + A(1-\alpha_1-\alpha_2)]}$  and

$\theta$  is a constant that appears in the equilibrium price process

$P(t) = \theta r(t) \frac{1-\alpha_1-\alpha_2}{k(t)} \alpha_1$ . A full discussion of the solution method is delayed until the next section.

for  $V$ , with  $\beta_3$  the coefficient corresponding to  $\log n(t+1)$ , and the Cobb-Douglas production function.

In addition to the first-order conditions derived in section 3, the additional necessary condition determining fertility is given by

$$(4.2) \quad -V_1 e + V_3 \leq 0 \quad \text{with } = \text{ if } n(t+1) > 0.$$

Algebraic manipulation of the first-order conditions, the budget constraint and market clearing relationships yield for this example

$$(4.3) \quad n(t+1)k(t+1) + P(t)R(t+1) = \left(\frac{\beta_2}{\beta_1 + \beta_2 + \beta_3}\right) A\alpha_2 k(t)^{\alpha_1} r(t)^{1-\alpha_1-\alpha_2} = sW(t)$$

where  $s = \frac{\beta_2}{\beta_1 + \beta_2 + \beta_3}$  is the saving rate and  $W(t)$  is the equilibrium wage rate that is equal to the marginal product of labor.

The fertility rate is given by

$$(4.4) \quad n(t+1) = \frac{1}{e} \left(\frac{\beta_3}{\beta_1 + \beta_2 + \beta_3}\right) A\alpha_2 k(t)^{\alpha_1} r(t)^{1-\alpha_1-\alpha_2} = \frac{\beta_3}{\beta_2 e} sW(t)$$

and is thus seen to be a constant fraction of first period income. Fertility, and thus population growth, is greater the lower is

the cost of children and the greater their psychic benefit.

Notice that these two equations contain three unknowns,  $n(t+1)$ ,  $k(t+1)$ , and  $P(t)$ .<sup>10</sup> Substituting (4.4) into (4.3) yields

$$(4.5) \quad \frac{\beta_3}{\beta_2 e} sW(t)k(t+1) + P(t) \frac{R}{N(t)} = sW(t)$$

An equilibrium for this economy consists of a time path for  $\{P(t), k(t+1), n(t+1)\}_{t=1}^{\infty}$  that satisfies (4.3) and (4.4) and the initial conditions.<sup>11</sup> Suppose that  $P(t)$  is conjectured to be of the form

$$(4.6) \quad P(t) = \theta sW(t) \frac{N(t)}{R}.$$

Then, for a constant  $\theta$  the solution for  $k(t)$  is<sup>12</sup>

$$(4.7) \quad k(t) = (1-\theta) \frac{\beta_2}{\beta_3} e \quad 0 < \theta < 1$$

and that for the population growth rate

$$(4.8) \quad n(t+1) = \frac{1}{e} \left( \frac{\beta_3}{\beta_1 + \beta_2 + \beta_3} \right) A \alpha_2 [(1-\theta) \frac{\beta_2}{\beta_3} e]^\alpha r(t)^{1-\alpha} = Cr(t)^{1-\alpha}.$$

We have thus found an equilibrium path for the economy characterized by constant capital per capita.

<sup>10</sup>Recall that  $R(t+1) = \frac{R}{N(t)}$  and  $N(t) = n(t)n(t-1)n(t-2)\dots n(1)N(0)$

<sup>11</sup>It is quite possible that there exist multiple equilibria particularly since there is no initial condition for the price of land.

<sup>12</sup>Using (3.8) and (3.10) it can be shown that there exists a unique  $\theta$ ,  $0 < \theta < 1$ , that satisfies  $s\alpha_2 \theta^2 + (1-\alpha_1-\alpha_2s)\theta - (1-\alpha_1-\alpha_2) = 0$ .



Further, rewriting (4.8) as

$$(4.9) \quad n(t+1) = c \left( \frac{R}{N(0)} \right)^{1-\alpha_1-\alpha_2} \left[ \frac{1}{n(1)n(2)\dots n(t)} \right]^{1-\alpha_1-\alpha_2} = n(2)^{(\alpha_1+\alpha_2)t-2}$$

it is apparent that since  $\alpha_1 + \alpha_2 < 1$ , population growth or the fertility rate converges to unity. Thus, the competitive equilibrium is characterized by zero population growth in the steady state. If  $n(2)$  is bigger than unity then convergence is from above while if  $n(2)$  is less than unity convergence is from below. Whether  $n(2)$  is above or below unity depends upon the given level of  $n(1)$  and the other parameter values. For example, the lower the cost of children ( $e$ ) the higher will be the fertility rate at each point along the path. Hence, if initially the cost of children is low, then along the competitive equilibrium path capital per capita is constant, population declines and income per capita ( $w(t)$ ) decreases.<sup>13</sup> Since in the stationary equilibrium  $n = 1$ , consumption per capita has a positive finite steady state level. Thus, when population growth is endogenous, there exists a competitive equilibrium which avoids the Malthusian outcome.

<sup>13</sup>In Eckstein and Wolpin (1982) it is shown that an equilibrium path with a decreasing fertility rate and increasing income per capita can be generated in a model where there is a time cost for children, but where all productive factors are variable.

## 5. Concluding Remarks

We have shown by example that the Malthusian result of subsistence consumption given unchecked population growth and a fixed factor of production can be avoided if individuals choose their level of fertility within a decentralized economic environment in which children are costly to bear and rear. Indeed, there was shown to exist a competitive solution in which population growth is zero in the steady state. Thus, Malthus's postulate that fertility is uncontrollable is no less important than the assumption that food is necessary for survival. What is most remarkable is not that fertility control undermines the usual result, since effective external fertility control, say through government intervention in the form of forced sterilization, could obviously do so, but rather that the decentralized economy can lead to a non-subsistence steady state given individual fertility control.

We have also demonstrated that exogenous fertility is a necessary condition for the Malthusian outcome. With exogenous fertility a decentralized economy eventually vanishes possibly even in a finite time, although the path is likely to be Pareto optimal. No redistribution of resources between generations can prevent this outcome. It is also likely that the allocation in the example with endogenous fertility is efficient. Hence, whether we should be pessimistic or optimistic about prospects for long run per capita

consumption depends upon our assumptions about the course of technology and human fertility. It would seem to us an open question as to whether our example can be generalized. When fertility is endogenous and land is essential in production, can per capita consumption in a decentralized economy, under any circumstances, approach the subsistence level?

There are obvious parallels between a Malthusian fixed factor model and a growth model with an exhaustible resource. Clearly, a fixed factor impinges less on growth possibilities than does an exhaustible resource and in the latter case an economy with a growing population could obviously not be supported without consumption per capita being driven to subsistence. Solow (1974) has demonstrated that with an exhaustible resource, zero population growth is feasible in that a positive steady state consumption level can be sustained with an appropriate rate of capital accumulation. The feasibility condition in the overlapping generations model with an exhaustible resource is merely the discrete time analog of the continuous time formulation of Solow (1974). One would therefore expect the same result to hold. Moreover, the neoclassical growth model formulation of Solow and the decentralized economy formulation here in the presence of an exhaustible resource can be shown to have identical first-order conditions and thus identical solutions. Properties of this solution have not as yet been derived and remain for further research.

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