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Working Paper

Exchange Risk, Political risk and Macroeconomic Equilibrium

Center Discussion Paper, No. 388

Provided in Cooperation with:
Economic Growth Center (EGC), Yale University

Suggested Citation: Eaton, Jonathan; Turnovsky, Stephen J. (1981) : Exchange Risk, Political risk and Macroeconomic Equilibrium, Center Discussion Paper, No. 388, Yale University, Economic Growth Center, New Haven, CT

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EXCHANGE RISK, POLITICAL RISK AND MACROECONOMIC EQUILIBRIUM

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September 1981

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I. INTRODUCTION

With the advent of increased exchange rate flexibility, the role of policy and the transmission of economic disturbances under flexible rates have become topics of considerable interest. Fleming (1962) and Mundell (1963), in their early analysis of the theory of policy in open economies, emphasized the importance of the degree of capital mobility. Subsequently, a number of authors have analyzed the implications of the degree of capital mobility for exchange rate and income determination in open economies; see for example Kouri and Porter (1974), Branson (1975, 1979), Girton and Henderson (1976), and Turnovsky (1976). The polar case of perfect capital mobility has received most attention.

Capital is perfectly mobile when investors consider domestic bonds, denominated in domestic currency, and foreign bonds, denominated in foreign currency, to be perfect substitutes. Two factors may cause perfect substitutability to break down. One is exchange risk: the values of the two bonds are defined in terms of different currencies and the exchange rate at the date of maturation is uncertain. The second is default risk: investors may perceive that foreign bonds are more subject to the risk of default than are domestic bonds. For one thing, in the event of default the investor must pursue his claim through a foreign legal system, which is likely to be costly. Aliber (1973) terms differences in default risk arising from the "foreignness" of assets as "political risk".

Where domestic and foreign securities are assumed to be only imperfect substitutes, it is important to distinguish the degree of imperfect mobility due to these two sources of risk. While this distinction has been noted in optimal portfolio models developed by authors such as Kouri (1976) and Adler and Dumas (1977), it has not been brought out in the current macro literature. The objective of the present paper is to investigate the macroeconomic consequences of this distinction and to show how it may be crucial in assessing the effects of various macroeconomic disturbances under varying degrees of capital mobility. We find that well-known results about...
the effects of increased capital mobility on the efficacy of monetary policy are correct when the increased mobility is due to a reduction in exchange risk, but not necessarily so when it is due to a reduction in default risk.

To carry out this analysis we derive the underlying behavioral relationships from an optimizing model of portfolio allocation and speculation in a stochastic context in which both types of risk are present. Such a model is first developed for a single risk averse investor in Section 2. A number of authors have developed financial models of currency speculation and foreign investment; see e.g. Solnik (1973), Grauer, Litzenberger and Stehle (1976), Kouri (1976), Adler and Dumas (1977), Roll and Solnik (1977), Frankel (1979), Fama and Farber (1979), Dornbusch (1980), Stein (1980). We find it useful to provide our own derivation, since other models either consider only exchange risk or make very restrictive assumptions about the interaction of exchange rates and prices.

Section 3 uses these relationships, under the assumption that investors are identical in each country, to formulate a general equilibrium macroeconomic model. We assume that the economy we consider is small, so that it treats certain characteristics of the foreign economy as parametric. Section 4 carries out certain comparative static exercises with this model. Specifically, it considers some important relationships between the available policy instruments and endogenous variables, and shows how these are affected by the degree of capital mobility in the two senses in which this term is defined. In particular, we show that, as long as there is a possibility of default, there exists an independent role for forward market intervention. In the limiting case of no default risk, when domestic and foreign bonds are perfect substitutes on a covered basis, this independent role ceases and forward market intervention becomes equivalent to bond market intervention. Secondly, the comparative statics of changes in the degree of capital mobility are discussed. Apart from the results on the
effectiveness of monetary policy noted above, we also consider how varying degrees of capital mobility affect the response of the domestic economy to foreign shocks.

2. OPTIMAL INVESTMENT AND PORTFOLIO BEHAVIOR

Consider an open economy in which individuals plan over a two period horizon, one period of which is devoted to working, the other to retirement. During period $t$ members of the working generation earn labor income which they allocate between current consumption and savings, where savings can take the form of additions to holdings of domestic bonds, foreign bonds, and domestic money. Assuming, for simplicity and without loss of generality, an initial wealth of zero, the budget constraint during this period may be expressed as

\[(2.1) \quad W_t = Y_t - C_t\]

where

\[(2.2) \quad W_t = (M_t + B^d_t + E^s_tE^f_t)/P_t\]

and $W_t =$ real wealth at time $t$,
$Y_t =$ real income at time $t$,
$C_t =$ real consumption at time $t$,
$M_t =$ holdings of domestic money at time $t$,
$B^d_t =$ holdings of domestic bonds, denominated in domestic currency, at time $t$,
$B^f_t =$ holdings of foreign bonds, denominated in foreign currency, at time $t$,
$E^s_t =$ spot price of foreign currency in terms of domestic currency, at time $t$,
$P_t =$ domestic price level at time $t$.

In period $t$ a working individual may also take a position in the forward market, but since this does not involve financial commitments during that period, this does not appear in the constraint (2.1).
Domestic bonds pay a nominal interest rate $i_t$, while foreign bonds pay $i^*_t$. The nominal yield on domestic money is zero. We also assume that foreign bonds are subject to default with probability $\pi$. This risk, which we take to be exogenous, may be assumed to reflect "political" factors abroad, and applies symmetrically between the domestic and foreign economies. Forward contracts, like domestic bond holdings, are not subject to default risk.

Under these assumptions, real resources available for consumption during period $t+1$, the retirement period, are given by

$$W_{t+1} = \frac{(1+i_t)B^d_t + Q_{t+1}(1+i^*_t)E^S_{t+1}B^f_t + M_t + (E^S_{t+1} - E^f_t)S_t}{P_{t+1}}$$

where $E^f_t$ is the one-period forward price of foreign currency, $S_t$ is the forward purchase of foreign currency at the end of period $t$. The variable $Q_{t+1}$ is a dummy variable, taking on the value 1 if default on foreign bonds does not occur, and 0 otherwise. According to (2.3), real wealth available at the end of period $t+1$ consists of: domestic bonds, including the interest earned on them; the stock of foreign bonds valued at the end of period exchange rate, provided that there is no default, or zero otherwise; the stock of money; and the gains made through speculation in the forward market; all four of these quantities deflated at the end of period price $P_{t+1}$.

For convenience, we shall define the share of wealth allocated to the three assets by

$$(2.4a) \quad \lambda^M = \frac{(M_t/P_t)}{W_t}$$

$$(2.4b) \quad \lambda^F = \frac{(E^S_{t+1}B^f_t/P_t)}{W_t}$$

$$(2.4c) \quad \lambda^D = \frac{(B^d_t/P_t)}{W_t} = 1 - \lambda^M - \lambda^F$$

while analogously the real purchases of forward currency, as a ratio of wealth, can be defined by

$$(2.4d) \quad \lambda^S = \frac{(E^f_{t+1}S_t/P_t)}{W_t}$$
We shall assume that the individual's objective can be expressed in terms of the following two period utility function

\[ V = U^1(C_t, M_t/P_t) + U^2(W_{t+1}) \]  

That is, individuals are assumed to derive utility from consumption in period \( t \) and also from the consumption of their wealth in period \( t+1 \).

In addition, following Fama and Farber (1979), for example, we assume that holdings of real money balances facilitate current consumption, and hence yield direct utility. While the procedure of introducing money into a utility function is a much argued one, because in our subsequent analysis we take domestic bonds to be riskless, it is necessary in order to derive a well defined demand function for money. Finally, note that, for simplicity, the utility function is assumed to be additively separable over time.

The individual's problem is to choose \( C_t, \lambda^M, \lambda^F, \) and \( \lambda^S \) where we require \( C_t, \lambda^M, \lambda^F, 1 - \lambda^M - \lambda^F = \lambda^D > 0 \) to maximize the expected value of (2.5) subject to the budget constraints (2.1), (2.3). We shall assume that at the time these variables are chosen, \( Q_{t+1}, P_{t+1} \) and \( E^s_{t+1} \) are random variables.

Individuals anticipate that \( Q_{t+1} \) will equal one with probability \( \pi \) and zero with probability \( (1 - \pi) \). The domestic price level and the exchange rate are assumed to be generated by the stochastic processes

\[ P_{t+1} = p_t + u^P_{t+1} \]  
\[ e^s_{t+1} = e^s_t + u^s_{t+1} \]

where \( p_t = \ln P_t, \ e^s_t = \ln E^s_t \), and the random variables \( u^P_{t+1}, u^s_{t+1} \) are assumed to be normally distributed

\[ u^P_{t+1} \sim N(\rho, \sigma_p^2) \]  
\[ u^s_{t+1} \sim N(\varepsilon, \sigma_s^2) \]
and \( \sigma_{sp} \) denotes the correlation between \( u_{t+1}^D \) and \( u_{t+1}^S \). In general, these random variables are functions of random variables of both domestic and foreign origin impinging on the economy. If, for example, exact purchasing power parity (PPP) obtains, and the foreign price level is rising non-stochastically at rate \( \rho^* \), then \( \varepsilon = \rho - \rho^* \) and \( \sigma_s^2 = \sigma_p^2 = \sigma_{sp} \). We assume that \( Q_{t+1} \) is independent of both \( u_{t+1}^D \) and \( u_{t+1}^S \); default risk on foreign bonds is independent of exchange risk and stochastic price movements.

In order to derive explicit expressions for the optimal holdings of assets we approximate the expectation of the objective function (2.5). To do this it is convenient to expand (2.5) in a second order Taylor series around \( W_t \) and take conditional expectations at time \( t \) to yield

\[
E_t(V) = U^1(C_t, \lambda^M W_t) + U^2(W_t) + U^2'(W_t)E_t(W_{t+1} - W_t) + \frac{1}{2}U^{2''}(W_t)E_t(W_{t+1} - W_t)^2
\]

Next, substituting the relationships (2.4) - (2.7) into the expression for wealth in retirement years, (2.3), we obtain

\[
W_{t+1} - W_t = \left[\left\{\left(1-\lambda^M-\lambda^F\right)(1+i_t^*) + \lambda^M + \lambda^F Q_{t+1} (1+i_t^*) E_t^S / E_t^S + \lambda^S (E_t^S - E_t^f) / E_t^f\right] P_t / P_{t+1} - 1 \right] W_t
\]

Under the above specifications of the stochastic processes generating \( P_t \) and \( E_t^S \), we may obtain approximations for the rates of return on money, domestic bonds, foreign bonds, and speculation, from which the corresponding means and variances can be calculated. These are presented in Table 1 below. These approximations assume that \( i, i^*, \rho, \varepsilon, \sigma_p^2, \sigma_s^2, 1-\pi \) and \( f_t^S - e_t^S \) are sufficiently small to allow us to treat the product of any two of them as approximately zero. We define the real rate of return on speculation as \( (E_t^S - E_t^f) / E_t^f (P_t / P_{t+1}) \).
Table 1

Real Rates of Return on Assets and Speculation

<table>
<thead>
<tr>
<th>Asset</th>
<th>Real Rate of Return</th>
<th>Expected Return</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money</td>
<td>$-u^p_{t+1}$</td>
<td>$-\rho$</td>
<td>$\sigma_p^2$</td>
</tr>
<tr>
<td>Domestic Bonds</td>
<td>$i_t - u^p_{t+1}$</td>
<td>$i_t - \rho$</td>
<td>$\sigma_p^2$</td>
</tr>
<tr>
<td>Foreign Bonds</td>
<td>$Q_{t+1}(i^<em>+u^S_{t+1} - u^p_{t+1}) - (1-Q_{t+1})\pi(i^</em>+\epsilon-\rho) - (1-\pi)\quad\pi(\sigma_s^2+\sigma_p^2-\sigma_{sp}) + (1-\pi)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speculation</td>
<td>$u^s_{t+1} - f_t$</td>
<td>$\epsilon - f_t$</td>
<td>$\sigma_s^2$</td>
</tr>
</tbody>
</table>

Substituting the expressions for the real rates of return into (2.9) we obtain the approximation

(2.9')

$$W_{t+1} - W_t = \left\{i_t(1-\lambda^M - \lambda^F) + [Q_{t+1}(i_t^* + u^S_{t+1}) - (1 - Q_{t+1})]^\lambda^F + (u^S_{t+1} - f_t)^\lambda^S - [1 - \lambda^F(1 - Q_{t+1})]u^P_{t+1}\right\}_t$$

Taking conditional expectations at time $t$ yields

(2.10)

$$E_t(W_{t+1} - W_t) = \left\{i_t(1-\lambda^M - \lambda^F) + [\pi(i^* + \epsilon) - (1 - \pi)]^\lambda^F + (\epsilon - f_t)^\lambda^S - \rho\right\}_t$$

Next, squaring (2.9') and taking conditional expectations we derive

(2.11)

$$E_t(W_{t+1} - W_t)^2 = \left\{\pi^2_s + (1-\pi)\right\}^\lambda^F + \sigma_s^2(\lambda^S)^2 + \sigma_p^2\sigma_s^2(\lambda^F + \lambda^S)^2 + 2[\pi\sigma_s^2\lambda^F\lambda^S - \sigma_{sp}(\pi\lambda^F + \lambda^S)]\right\}_t^2$$

where all second or higher order terms are ignored as being negligible.

Maximizing (2.8) subject to (2.1), incorporating (2.10) and (2.11), leads to the following first order optimality conditions

(2.12a)

$$U^M_i - U^2_i i_t = 0$$

(2.12b)

$$\lambda^F = \frac{(\Delta r_t/R) + \pi\sigma_{sp} - \pi\sigma_s^2\lambda^S}{\pi\sigma_s^2 + (1-\pi)}$$
(2.12c) \[ \lambda^S = \frac{(\Delta e_t/R) + \sigma_{sp} - \pi \sigma^2_F}{\sigma_s^2} \]

(2.12d) \[ U'_C + E\left(U^{2'}_t \frac{dW_{t+1}}{dC_t}\right) = 0 \]

where \[ \Delta x_t = \pi(i^*_t + \epsilon) - (1 - \pi) - i_t \]

measures the difference between the expected nominal interest rates on foreign and domestic bonds;

\[ \Delta e_t = \epsilon - f_t = e^{s}_{t+1,t} - e^f_t \]

is the expected excess of the spot rate in period \( t+1 \) over the forward rate, where throughout the paper we shall use the notation \( x_{t+i,t} \) to denote the expectation of \( x \) formed at time \( t \) for time \( t+i \); and

\[ R = -U''W_t/U^{2'} \]

is the coefficient of relative risk aversion. The optimality conditions with respect to money and consumption, (2.12a) and (2.12b), are straightforward and need not be discussed further.

Of particular interest is the optimality condition for speculation, (2.12c). The three terms in the numerator of this expression correspond to three motives for taking a position in the forward market. The first of these derives from currency speculation. The term \( \Delta e_t \) represents the expected gain from buying one unit of foreign currency in terms of domestic currency. If \( R = 0 \) (investors are risk neutral) or if \( \sigma_s^2 = 0 \) (there is no exchange risk), a non-infinite speculative demand for forward exchange requires that the forward rate in period \( t \) represent an unbiased prediction of the spot rate in period \( t+1 \). If \( R \neq 0 \) and \( \sigma_s^2 = 0 \), a bias may emerge.

The second motive is to hedge against inflation risk. In the particular case in which exact PPP obtains and only the domestic price level is uncertain, then \( \sigma_s^2 = \sigma_{sp}^2 \). In this case, the second term of (2.12c)
divided by the denominator equals unity. The individual sells his entire wealth forward to hedge against inflation risk.

The third motive derives from forward cover. In this case the individual sells a fraction of the total value of his foreign bonds forward equal to the probability of no default on these bonds. In the case in which there is no default risk, he covers the entire holdings of these bonds by a position in the forward market.

Solving equations (2.12b) and (2.12c) simultaneously, one can obtain the following expression for the demand for foreign bond holdings in terms of the covered interest differential:

\[
\lambda^F = \max \left\{ \frac{\Delta c_t / R}{(1 - \pi)}, 0 \right\}
\]

where

\[
\Delta c_t \equiv \pi(i^*_t + f_t) - (1 - \pi) - i_t
\]

denotes the expected covered interest differential. In deriving (2.13) from (2.12b) and (2.12c) all terms of second order of smallness are ignored.

When \( \pi = 1 \), (2.13) reduces to \( \Delta c_t / 0 \); in the absence of default risk the covered interest differential must be zero in order for the demand for foreign bonds to remain finite. The condition \( \Delta c_t = 0 \) then reduces to the familiar covered interest parity (CIP) condition

\[
i_t = i^*_t + f_t
\]

In this case \( \lambda^S \) and \( \lambda^F \) cannot be determined independently from (2.12b) and (2.12c). Rather, these two equations determine the net open position in foreign bonds

\[
\lambda^F + \lambda^S = \frac{\Delta e_t / R + \sigma sp}{\sigma_s^2}
\]

In this case it follows from CIP and the underlying stochastic process determining the exchange rate that \( \Delta e_t = e^S_{t+1,t} - e^S_t + i^*_t - i_t \), so that the relevant interest rate determining the demand for foreign bonds is the uncovered interest differential.
In the presence of default risk, however, the expected covered interest differential may deviate from zero since domestic and covered foreign bonds no longer represent perfect substitutes. In this case, to the first order of approximation, the demand for foreign bonds depends on the covered interest differential, the probability of default and the degree of relative risk aversion. We have repressed second order terms which include, among other things, the variance of the exchange rate. This term enters because, in the presence of default risk, forward cover cannot eliminate exchange risk entirely.

More specifically, the demand for foreign bonds will be affected negatively by the variance of the exchange rate. From (2.12c) the investor obtains forward cover on a fraction $\pi$ of his total foreign bond holdings. If default risk does not occur he is subject to exchange risk on the remaining $1-\pi$. If it does occur he must nevertheless honor a forward sale of foreign currency in amount of $\pi F_t W_t$. Uncertainty in the real value of these magnitudes rises with the variance of the exchange rate. However, this relationship is of second order of magnitude so does not appear in (2.13).

A set of equations analogous to (2.12) may be derived for a representative investor in the foreign country. In particular, if we denote the corresponding magnitudes with an asterisk, we obtain, parallel to (2.12c), (2.13), respectively,

$$(2.12c') \quad \lambda^S* = \frac{-\Delta e_t / R^* - \sigma s p^* - \pi \sigma^2 F^*}{\sigma^2 s}$$

$$(2.13') \quad \lambda^F* = \max \left\{ \frac{\Delta c_t^* / R^*}{(1-\pi)}, 0 \right\}$$

where $$\Delta c_t^* = \pi (i_t - f_t) - (1 - \pi) - i_t^*$$

Here we define $\lambda^S*$ as foreign purchases of domestic currency forward and $\lambda^F*$ as foreign purchases of domestic bonds, each as shares of foreign
real wealth. But for simplicity we assume that foreigners perceive
the default risk on domestic bonds to be \( \pi \).

3. FORMULATION OF A GENERAL EQUILIBRIUM MACRO MODEL

So far we have derived the optimal bond and forward position of
a single optimizing individual who takes prices, interest rates and
exchange rates as given. We now assume that this individual is representative
of all portfolio maximizers in his own economy to characterize conditions
for equilibrium in the money markets, forward market, and bond markets.
Finally, we introduce assumptions about behavior in the commodity market that
yield conditions for equilibrium in that market as well. Together,
equilibrium conditions in these four markets constitute a general equilibrium
macroeconomic system. To simplify the analysis we make the assumption that
the domestic economy is small relative to the rest of the world so that
it views the foreign price level and interest rate as parametric.

3.1 Money Market Equilibrium

Money market equilibrium in each country requires that

\[
\lambda_M \frac{W_t}{P_t} = \frac{M_t}{P_t}
\]

(3.1)

\[
\lambda_{M*} \frac{W^*_t}{P^*_t} = \frac{M^*_t}{P^*_t}
\]

(3.2)

where \( W_t \) now denotes domestic real national financial wealth and \( M_t \) the
domestic money supply. An asterisk denotes parallel quantities abroad.

Consider the domestic money market. The demand for money is
defined implicitly by equation (2.12a). This equation may be combined
with (2.1) and (3.1) to describe domestic money market equilibrium by

\[
\frac{M_t}{P_t} = \lambda_M (i_t, C_t, Y_t)(Y_t - C_t)
\]

(3.3)

It is interesting to note that given \( C \), the demand for money depends only
upon the domestic interest rate \( i \) and domestic output \( Y \). Any other factors
which influence \( \lambda_M(.) \) do so through their influence on \( C \), via the optimality
condition (2.12d). Since our analysis does not purport to contribute
anything new to the demand for money, we shall simply assume that (3.3)
can be adequately approximated by the log-linear relationship
where \( m_t \equiv \ln M_t \), \( P_t \equiv \ln P \), \( Y_t \equiv \ln Y_t \). In writing (3.4) we are in effect assuming that the critical elements influencing \( C \) are \( Y \) and \( i \).

### 3.2 Bond Market Equilibrium

Equilibrium in the markets for domestic and foreign bonds requires that

\[
(1 - \lambda^M - \lambda^F)W_t + \lambda^F e^{SP*W_t/P_t} = B_t/P_t \tag{3.5}
\]

\[
\lambda^F W_t/(E_t^P*) + (1 - \lambda^M - \lambda^F)W_t = B_t/P_t \tag{3.6}
\]

where \( B_t \) and \( B^* \) denote the supplies of domestic and foreign bonds respectively.

Expressing the expected covered interest rate differential as \( \Delta c_t = \pi(i^* + e^f_t - e^s_t - i_t) - (1 - \pi)i_t \), the real domestic demand for foreign bonds, \( \lambda^F W_t \), obtained by combining (2.13) with (2.1), is given by

\[
\lambda^F W_t = \lambda^F(i^* + e^f_t - e^s_t - i_t)(Y_t - C_t) \tag{3.7}
\]

which we approximate by the log-linear relationship

\[
\lambda^F W_t \approx \omega_1(i^*_t + e^f_t - e^s_t - i^*_t) + \omega_2 i_t + \omega_3 Y_t \tag{3.7'}
\]

The term \( \omega_1 \), capturing the substitution effects between covered foreign and domestic bonds, is positive, and as default risk tends to zero, \( \omega_1 \rightarrow \infty \). The sign of \( \omega_2 \) is ambiguous, since it incorporates interest rate effects on savings and hence on \( W_t \). If savings is interest inelastic, then (2.13) implies that \( \omega_2 \leq -\omega_1 \). We therefore assume that \( \omega_2 < 0 \). Thirdly, assuming that \( 0 < \frac{\partial C_t}{\partial Y_t} < 1 \), the income effect is positive, so that \( \omega_3 > 0 \).

Finally, we should note that, as expression (2.13) indicates, foreign bond demand depends upon \( \pi \) as well. By assuming that this magnitude remains constant, it may be suppressed from the functions.

The real domestic demand for domestic bonds, \( (1 - \lambda^M - \lambda^F)W_t \) is obtained residually from equations (2.1), (3.3) and (3.7). Analogously,
the following log-linear approximation is adopted

\[(3.8) \quad (1 - \lambda^M - \lambda^F)w_t \approx -\omega_1 (i_t^* + e_t^f - e_t^s - i_t) + \omega_2 i_t + \omega_3 y_t \]

We have assumed already that \(\omega_1 > 0\). Unless \(\partial C_t / \partial i_t\) is highly positive, it seems reasonable to assume that the demand for domestic bonds varies positively with the domestic interest rate so that \(\omega_2^D > 0\). Since both \(\lambda^M w_t\) and \(\lambda^F w_t\) are assumed to be positive functions of \(y_t\), the coefficient \(\omega_3^D\) is indeterminate and so no sign restriction is imposed.⁶

The real supply of domestic bonds is given by \(B_t / p_t\). Letting \(b_t \equiv \ln B_t\), and recalling the definition of \(p_t\), we adopt the linear approximation⁹

\[(3.9) \quad \frac{B_t}{p_t} \approx \omega_4 (b_t - p_t) + \omega_5 \]

where \(\omega_4 \equiv (\bar{B}_t / \bar{p}_t)\), the average stock of domestic bonds divided by the average price level, and \(\omega_5 \equiv \omega_4 [1 - \ln \omega_4]\). We shall normalize units so that \(\ln \omega_4 = 1\) and \(\omega_5 = 0\).

To simplify the analysis we shall restrict ourselves to the case in which \(\lambda^F > 0\) while \(\lambda^F^* = 0\). From (2.13) and (2.12') observe that this case is more likely to arise when, among other things, the domestic price level is more highly correlated with the exchange rate than is the foreign price level. Combining (3.8) and (3.9), then, we obtain as a condition for equilibrium in the domestic bond market

\[(3.10) \quad -\omega_1 (i_t^* + e_t^f - e_t^s - i_t) + \omega_2 i_t + \omega_3 y_t \approx \omega_4 (b_t - p_t) \]

Note that solving (3.10) for \(i_t\) yields

\[(3.11) \quad i_t = \frac{\omega_1 (i_t^* + e_t^f - e_t^s) - \omega_3 y_t + \omega_4 (b_t - p_t)}{\omega_1 + \omega_2} \]

In the limiting case in which \(\omega_1 \to \infty\), covered interest parity, \(i_t = i_t^* + e_t^f - e_t^s\), obtains. Otherwise, an increase in the covered foreign interest rate has a less than proportional effect on the domestic interest rate, while the
the domestic interest rate responds positively to changes in the real bond supply and ambiguously to changes in income.

3.3 Forward Market Equilibrium

Equilibrium in the forward exchange market is described by

\[ E_t^F S^* P^{*W_t}/P_t = \lambda_s W_t + G_t/P_t \]

where \( G_t \) denotes the net government purchases of foreign currency in the forward market translated to domestic currency units at the forward exchange rate. Substituting the expressions for \( \lambda_s \) and \( \lambda_s^* \) from (2.12c) and (2.12c') into (3.12) we obtain

\[ \frac{\Delta e_t}{\sigma_s} \left( W_t \frac{A_t}{R_t} + \frac{A_t^F}{R^*_t} \right) + \pi(\lambda^F A_t^F - \lambda^F W_t) \]

\[ + \frac{1}{\sigma_s^2} (W_t \sigma_{sp} + A_t^F \sigma_{sp}^*) + \frac{G_t}{P_t} = 0 \]

where

\[ A_t^F = E_t^F P^{*W_t}/P_t \]

that is, the nominal value of foreign wealth translated to domestic currency units at the forward rate and deflated by the domestic price level.

The first term of (3.13) corresponds to the net speculative demand for foreign currency forward. Its sign is given by \( \Delta e_t \) and its absolute magnitude depends negatively on the variance of the exchange rate and on the domestic and foreign coefficients of risk aversion. The second term denotes the net demand for foreign currency for arbitrage. The sign of this term depends on whether the domestic economy is a net importer or exporter of bonds. The third term of (3.13) is the net demand for foreign currency forward as an inflation hedge. Its sign depends upon the relative magnitudes of the correlation between the spot exchange rate and the price levels in each country. The fourth term is simply the government position in the market.

The equilibrium condition (3.13) enables us to determine the
relationship between the forward rate and the expected future spot rate.
Specifically it can be seen that the forward rate is more likely to
overpredict the expected future spot rate \(\Delta e_t < 0\) if: (1) the domestic
price level is more correlated with the exchange rate than is the foreign price
level, i.e. \(\sigma_{sp} > -\sigma_{sp^*}\), when the countries are of equal size in terms of wealth;
(2) the domestic country is larger in terms of wealth, while price
variability is the same; (3) assets denominated in domestic currency
exceed national wealth.\(^{10}\) Given these factors, the degree of overprediction
will be greater, the greater the degree of exchange rate variability \(\sigma_s^2\)
and the degrees of relative risk aversion in each country.

Since we treat \(\sigma_s^2, \sigma_{sp}, R\) and \(R^*\) as constants and assume that
\(\lambda F^* = 0\), we may represent the condition for forward market equilibrium,
incorporating (3.7) and the definition of \(\Delta e_t\), as

\[
(3.14) \quad -\gamma(e_{t+1}^s - e_t^s) + \pi[\omega_1(i_t^* + e_t^s - e_t^s) + \omega_2 i_{t^*} + \omega_2 y_{t^*}] = \lambda(g_t - p_t)
\]

where \(g_t \equiv \ln G_t\) and \(\lambda \equiv \tilde{c}_t / \tilde{P}_t\). Here we have used a log-linear approximation
to \(G_t / P_t\) similar to the one applied to \(B_t / P_t\) above.

### 3.4 Commodity Market Equilibrium

We assume that the prices of some tradable commodities are established
in auction markets while the prices of other tradable and nontradable
commodities are established by contract in the previous period. The prices
of the first type of commodities are hence flexible within the period, while
the prices of the latter group are fixed. The prices that are flexible
are determined by the international law of one price. Thus

\[
(3.15) \quad p_t^A = p_t^* + e_t^s
\]

where \(p_t^A\) denotes the logarithm of the price index of the flexibly-priced
commodities. Prices that are established by contract, we assume, are
set at the price level expected to obtain in the relevant period, as of
the previous period. Our assumption here parallels the contract theory of wage determination of Gray (1976), Fischer (1977) and Phelps and Taylor (1977). Thus, we have

\[ (3.16) \quad p^C_t = p_{t-1} \]

where the superscript C denotes the contract-priced goods. The overall price index is therefore

\[ (3.17) \quad p_t = \delta(p^{*}_t + e^{S}_t) + (1-\delta)p_{t-1} \]

where \( \delta \) is the share of the flexibly-priced commodities. As long as \( p^{*}_t \) and \( e^{S}_t \) are imperfectly correlated, an increase in \( \delta \) raises the correlation between the domestic price level and the exchange rate, \( \sigma_{sp} \).

Finally, we assume that the deviation of output from its natural level \( \tilde{y} \) is determined by the unanticipated component of the domestic price level. That is,

\[ (3.18) \quad y_t = \theta(p_t - p_{t-1}) \quad \theta \geq 0 \]

where we normalize units by setting \( \tilde{y} = 0 \). Such a specification follows from the contract theory of wage determination referred to above.\(^{11}\)

Together, the conditions for money market equilibrium, (3.4), bond market equilibrium, (3.10), forward market equilibrium, (3.14), and the price and output equations, (3.17) and (3.18), constitute a complete macroeconomic model of a small open economy. The system of equations determines equilibrium values of the five endogenous variables \( p_t, i_t, e^{S}_t, e^{f}_t \) and \( y_t \) in terms of the expectations \( e^{S}_{t+1, t}, p_{t-1} \) and the exogenous variables, \( m_t, b_t, g_t, i^*_t \) and \( p^*_t \), as well as the structural parameters of the system. To close the model we assume that expectations are formed rationally; i.e., that price and exchange rate expectations are formed on the basis of the model itself and all available information.

We now use the model to contrast the effects of two different ways in which capital can become more mobile. On the one hand, exchange risk
or aversion to exchange risk may fall. This will be reflected by an increase in the parameter $\gamma$. As a consequence, the forward rate will become more closely tied to the expectation of the future spot rate. On the other hand, default risk or aversion to default risk may fall, raising the parameter $w_1$. As a consequence, the domestic interest rate will be determined more closely by the covered interest parity condition. We show that the way in which capital mobility is increased has important implications for the response of the economy to a monetary disturbance.

4. COMPARATIVE STATICS

In the model contained in equations (3.4), (3.10), (3.14), (3.17) and (3.18), individuals' expectations about the future influence their current behavior. As is well known (c.f. Taylor (1977)), when expectations are formed rationally, such models have non-unique solutions. There is only one solution to this model, however, for which the exchange rate and the price level remain bounded into the infinite future. We focus on this solution to the model.

Our purpose in solving the model is to analyze the response of the economy to changes in the policy variables $m_t$, $b_t$ and $g_t$ and also to changes in the foreign variables $i_t^*$ and $p_t^*$. Disturbances in the exogenous variables may be distinguished by a number of characteristics: (a) their expected permanence or transience, (b) the extent to which they are anticipated or unanticipated, (c) whether they occur immediately or are announced in advance. In principle, we could analyze the effects of many specific types of disturbances. We shall limit ourselves, however, to two types. First, we will consider the effect on impact of disturbances that are (a) unannounced, (b) unanticipated, and (c) expected to last only one period; second, we will consider the effects in steady state of changes in the exogenous variables that are perceived as permanent.

To analyze the effect of a temporary, unanticipated disturbance in period $t$ we may assume, without loss of generality, that
m_{t,t} = b_{t,t} = g_{t,t} = p^*_t, t = i^*_t = 0, \tau > t \text{ implying that } e^s_{t+1,t} = 0, \\
\text{while } m_{t,\tau} = b_{t,\tau} = g_{t,\tau} = p^*_t, t = i^*_t = 0, \tau < t \text{ which implies that } \\
P_{t,t-1} = 0. \text{ The reduced form expressions for the endogenous variables are easily obtained by solving the five equations of the system with these assumptions. However, the expressions are messy and we do not report them explicitly.}

To analyze the steady-state effects of permanent changes in the exogenous variables we set \( p_{t,t-1} = p_t = \bar{p} \) and \( e^s_{t+1,t} = e^s_t = \bar{e}^s \), where \( \bar{x} \) denotes the steady state value of \( x \). Again the expressions are tedious and are not reported.

4.1 Money Market, Bond Market and Forward Market Intervention

The government can affect the domestic economy by changing the supply of money, the stock of domestic bonds, and its forward market position. Manipulation of each instrument impinges initially on different markets and has differential effects on domestic income, the price level, and the interest rate. The three policy instruments are linearly independent except in some special cases. We summarize the most important relationships between policy instruments and endogenous variables:

1. The effect, on impact, of an unanticipated, transitory increase in \( m_t, b_t \) or \( g_t \) is expansionary. As is well known, increasing \( m_t \) or \( b_t \) raises nominal wealth above its desired level at initial commodity prices. Prices that are flexible rise, inducing an expansion in income. An increase in \( g_t \), government holdings of foreign exchange forward, raises \( e^f_t \) and hence the rate of return on covered foreign bonds. Individuals' demand for money and domestic bonds falls so that a higher price level is required to equate demand to the existing nominal stocks. For similar reasons, the steady state effect of permanent increases in \( m_t, b_t \) and \( g_t \) is to raise \( p_t \) and \( e^s_t \); \( y_t \) remains fixed at its long-run natural level.

2. A permanent increase in \( m_t \) lowers the domestic and covered foreign interest rates, while a permanent increase in \( b_t \) or \( g_t \) does the opposite;
\( b_t \) has relatively more impact on the domestic rate while \( g_t \) has relatively greater influence on the covered foreign rate.

3. In the limit as the probability of default goes to zero, \( \omega_1 \to \infty \), and covered interest parity obtains. In this case \( b_t \) and \( g_t \) affect all domestic variables in the same linear proportions, \( \omega_4 b_t + \lambda g_t \). In other words, bond market and forward market intervention become equivalent in their effects on all endogenous variables; either one is therefore redundant.

4. In the limit as either the degree of risk aversion or exchange risk goes to zero, \( \gamma \to \infty \) and forward market intervention ceases to have an effect on any endogenous variable. Any forward market intervention by the monetary authority is exactly offset by the actions of speculators.

5. In the limit as both exchange risk and default risk go to zero, (or else risk aversion goes to zero) both \( \gamma \to \infty \) and \( \omega_1 \to \infty \) and neither bond market nor forward market intervention can have any effect. The familiar proposition that fiscal policy (i.e. a change in the government deficit unaccompanied by changes in the money supply) is ineffective under flexible rates and perfect capital mobility (see Mundell (1963)) is true only if perfect capital mobility is interpreted to mean an absence of default risk and exchange risk. Otherwise, fiscal policy is still effective.

4.2 Capital Mobility, the International Transmission of Disturbances and the Role of Policy

In terms of our model, an increase in capital mobility can be interpreted to mean either (1) a diminution of default risk or aversion to default risk (an increase in \( \omega_1 \)) or (2) a diminution in exchange risk or aversion to exchange risk (an increase in \( \gamma \)). While a number of studies have emphasized the importance of the degree of capital mobility for both the efficacy of policy and the international transmission of disturbances, no distinction has been made in this literature, to our knowledge, between the two ways in which it can occur.

In fact, an increase in capital mobility in either sense tends to increase the sensitivity of the domestic economy to foreign disturbances.
The response of the domestic interest rate to a change in the foreign interest rate, whether it is transitory \( \frac{\partial i_t}{\partial i_t^*} \) or permanent \( \frac{\partial i_t}{\partial i_t^*} \) increases with an increase in either \( \omega_1 \) or \( \gamma \). Likewise, the response of the domestic price level to a transitory increase in the foreign price level \( \frac{\partial p_t}{\partial p_t^*} \) increases with both \( \omega_1 \) and \( \gamma \), while the steady-state response \( \frac{\partial p}{\partial p^*} \) is always zero.

Increased capital mobility in its two different forms, however, can have very different implications for the effects of domestic policies. For example, a reduction in exchange risk or aversion to exchange risk always increases the steady-state responses of the exchange rate and price level to permanent changes in the domestic money supply. This is because as \( \gamma \) increases, the steady state forward and spot rates move closer together, and the covered foreign exchange rate becomes less flexible, as does the domestic interest rate. As a result, more of the change in the money supply must be reflected in a higher price level and exchange rate and less in a lower interest rate.

By contrast, a reduction in aversion to default risk has an ambiguous effect on the steady-state relationship between the money supply and price level. An increase in \( \omega_1 \) raises or lowers the term \( \frac{\partial p}{\partial m} \) as

(4.1) \[ a_2 \gamma [\omega_4 (\gamma - \pi \omega_2) - \lambda \omega_2] < 0 \]

The reason for the ambiguity is as follows. An increase in \( \omega_1 \) always ties the domestic interest rate more closely to the covered foreign interest rate. When \( \gamma \) is large the covered and uncovered foreign rates are close, so that a reduction in default risk ties the domestic interest rate more closely to the uncovered foreign rate as well, so that it is less flexible. As with a reduction in exchange risk, then, more of a change in money supply must be absorbed by the price level rather than by the interest rate.

When \( \gamma \) is small, however, the covered and uncovered foreign rates can diverge, and the increase in \( m \) can be accompanied by a fall in \( i^f \) and \( e^f - e^s \). If \( \omega_2^F \) is large then the fall in \( i^f \) raises the demand for covered foreign bonds.
The consequent increase in the sale of foreign exchange forward from arbitrage requires, for forward market equilibrium, offsetting speculation; \( e^s - e^f \) must consequently rise to elicit this speculation. The interest rate on covered foreign bonds, \( i^* + e^f - e^s \), therefore falls, inducing a further fall in the domestic interest rate \( i \). Increased capital mobility in this sense can therefore make the domestic interest rate more flexible. In other words, an increase in the degree of capital mobility, in the sense of domestic and covered foreign bonds becoming closer substitutes, can reduce the rise in the price level and increase the change in the interest rate resulting from a monetary expansion.

5. CONCLUSION

Exchange risk and political risk both diminish the mobility of capital. Macroeconomic analyses of open economies have emphasized the importance of the degree of capital mobility but have not, typically, distinguished between these two impediments of mobility. In fact, they impinge on the economy in different ways. On the one hand, exchange risk creates a divergence between the forward exchange rate and the spot rate expected to prevail when the forward contract matures, in turn creating a divergence between the expected rates of return on covered and uncovered foreign bonds. Exchange risk does not, however, affect the relationship between the covered foreign interest rate and the domestic interest rate. Political risk, on the other hand, weakens this second relationship, but does not affect the first.

In the absence of both exchange risk and political risk, i.e. if capital is perfectly mobile, changes in the supply of domestic bonds or in the government's forward position have no effect on the domestic economy. Only changes in the money supply have an impact. Introducing exchange risk alone makes bond market and forward market intervention effective, although their effects are not linearly independent. They are essentially
equivalent and one or the other is redundant. Alternatively, introducing
default risk alone gives efficacy to bond market intervention, but not
to forward market intervention. Finally, if both exchange and default
risk are present, then forward market, bond market, and money market
intervention all have independent effects on the economy.

An increase in capital mobility due to the diminution of the effects
of either type of risk increases the sensitivity of the domestic interest
rate to the foreign interest rate and the sensitivity of the domestic
price level to the foreign price level. An increase in capital mobility
in the form of an increase in the elasticity of speculation increases the
steady state effect of a change in the money supply on the domestic price
level. However, an increase in capital mobility that takes the form of
decreasing the divergence between the domestic and foreign covered interest
rates has an ambiguous effect on the relationship between money and the
price level.
The "life cycle" context is convenient in that it allows us to ignore the correlation between earned income in the second period and the rates of return on assets. Introducing nonwealth income in the second period poses no conceptual problems, but clutters the analysis. See Frankel (1979) for a model which does introduce other sources of income and discusses their consequences.

We assume that domestic bonds and money are outside assets; i.e., they constitute net wealth. Frankel (1979) has shown that if no nominally denominated assets are net wealth, the forward rate equals the expectation of the future spot rate. Even if bonds do not constitute net wealth, however, a bias emerges as long as money is considered to be part of wealth.

See Kenen (1965) for an early analysis of the forward market incorporating potential default. Political risk may derive from the threat of exchange controls as well as from the threat of outright debt repudiation. In either case the risk may not be the loss of the total net worth of the investment. When the threat derives from exchange controls, the expected value of net worth may even be preserved, but if payment is delayed, forward cover obtained for the investment will no longer be appropriate. For our purposes it is the risk that repayment not occur on the date for which the forward cover is arranged, thereby exposing the investor to exchange risk, that is relevant. Our analysis could easily be modified to incorporate a preservation of some or all net worth in the event of default. See Eaton and Gersovitz (1981) for a model in which the probability of default is endogenous.

We shall let primes denote derivatives and shall denote partial derivatives by corresponding lower case subscripts.

The first and third motives are equivalent to what Kenen (1965) terms the demand for forward cover by speculators and arbitrageurs. Kenen also identifies a third source as demand by traders, which does not seem to be the exact equivalent to our demand as an inflation hedge.

A common specification of imperfect capital mobility in the macro literature is to specify the demand for bonds as being proportional to the uncovered interest differential. It is evident from our analysis that procedure is consistent with portfolio maximization only in the absence of default risk.

For the case in which the utility function is logarithmic, C will depend only upon Y.

Expressions (3.3), (3.7) and (3.8) implicitly define a savings function, which will be highly nonlinear in form. Lucas (1975) follows a similar procedure of defining all assets demand functions explicitly in log-linear form, with a savings function being defined residually.

The approximation in (3.9) is obtained as follows

$$\frac{B_t}{p_t} \approx \frac{B}{p} + \frac{B}{p} \left( \frac{B_t - B}{B} \right) - \frac{B}{p} \left( \frac{p_t - \bar{p}}{\bar{p}} \right) = \omega 4 \left( 1 + \frac{B_t - \bar{B}}{B} - \frac{p_t - \bar{p}}{p} \right)$$

Now

$$\frac{B_t - \bar{B}}{\bar{B}} \approx \ln \left( 1 + \frac{B_t - \bar{B}}{\bar{B}} \right) = \ln B_t / \bar{B} = b_t - \ln \bar{B}.$$
Similarly \[ \frac{p_t - \bar{p}}{\bar{p}} \approx p_t - \ln \bar{p} \] so that

\[ \frac{B_t}{\bar{p}_t} \approx \omega_4 [1 + (b_t - \ln \bar{B}) - (p_t - \ln \bar{p})] \]

\[ = \omega_4 (1 - \ln \omega_3) + \omega_4 (b_t - p_t). \]

10. Adding the domestic money market equilibrium condition (3.1) to the domestic bond market equilibrium condition (3.5) yields:

\[ \lambda F^{*} E^{*} P^{*} W^{*}/P_t - \lambda F W = (M_t + B_t)/P_t - W_t \]

Thus

\[ \lambda F^* A^*_t - \lambda F W_t = (M_t + B_t)/P_t + \lambda F^* A^*_F - W_t. \]

Foreigners' holdings of domestic bonds must be adjusted by the forward premium in calculating the real level of assets denominated in domestic currency.

11. Equation (3.18) may be obtained in the following way: Assume that all wages are determined by one-period ahead contracts so that we may set \( \omega_t = p_{t-1} \), where \( \omega_t \) is the logarithm of the wage in period \( t \). In the contract-price sector the wage divided by the output price is constant as are output and employment. Thus we may set \( y^C_t = 0 \), where \( y^C_t \) is the logarithm of the output of the contract-price sector. In the flexible-price sector the wage divided by the output price is inversely proportional to the price level. Thus output and employment expand when prices are high and conversely. Defining \( y^A_t \) as the output of the flexible-price sector, we have

\[ y^A_t = \theta (p^A_t - p_{t-1}) \quad \theta' > 0 \]

or, substituting (3.15) and (3.17),

\[ y_t = \theta (p_t - p_{t-1}) \]

where \( \theta = \theta'/\delta \).

12. Elsewhere (Eaton and Turnovsky (1980)) we discuss how these three forms of intervention are related by the balance-sheet constraint of the combined monetary-fiscal authority.


