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## Determination of Fertility, Child Quality, and Child Survival in Guatemala

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CENTER DISCUSSION PAPER NO. 332

DETERMINATION OF FERTILITY, CHILD QUALITY, AND CHILD SURVIVAL IN GUATEMALA

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## Introduction

A leading topic of analysis among economic demographers has been the relationship of child mortality and fertility. Significant correlations between declining child mortality rates and declining fertility rates--often with a lag of several years--has led most researchers to postulate a causal link between the two variables. Models designed to analyze this linkage included biological supply models emphasizing the influence of breast-feeding and economic models emphasizing the role of individual preferences and demands.

The biological supply effect can occur with the death of a currently breast-fed child. When the mother's lactation hormone is no longer produced, the mother's period of post-partum sterility tends to be shortened, and her fecundity can increase as a result. The importance of this effect depends primarily on the age of the mother, the age of the child at death, the intensity of breast-feeding, and perhaps the mother's health and nutrition (Schultz, 1976).

Demand studies of the effects of child mortality on fertility fall into two groups (O'Hara, 1972). The first group of studies emphasizes the demand for surviving children rather than the demand for births. In these studies, the impact of mortality is measured with the "replacement factor"; community and individual experience with child mortality determine the number of births necessary to achieve a particular desired family size. Optimal desired family size is determined by comparing the expected costs and benefits of children. This approach was used by Schultz (1969) with Puerto Rican data, Schultz and DaVanzo (1970) in East Pakistan, and DaVanzo (1970) in Chile. These studies were regional analyses and did not examine household behavior.

In the second group of demand studies, exogenous changes in child mortality can alter desired family size and the investment in children by affecting the costs and benefits associated with fertility. The effect of a decline in mortality on fertility depends on the expected costs and benefits of a child as evaluated by the parents before conception. The costs of children at every age are defined as foregone consumption of other goods, and returns include the psychic as well as monetary income derived from children. A decline in mortality increases the probability of survival at every age causing expected costs and expected returns to rise. If returns rise faster with age than costs, then a decline in child mortality increases the net return to children and induces parents to purchase more "child services." These services can be in the form of numbers of children or quality of children. If quantity and quality are good substitutes, then a decline in mortality could reduce births enough to offset the change in mortality (O'Hara, 1972).

Ben-Porath (1976) modified this analysis by including child survival as an exogenous trait used in the production of child quality. Assuming that child survival and other goods are unrelated in the production of quality, then an increase in desired family size is associated with an increase in child survival if the demand for children is price elastic.

The fertility reaction to child mortality occurs through hoarding and replacement. Hoarding is the response to expected mortality and occurs when replacement is not possible. With hoarding, assuming risk neutrality, actual fertility is greater than the number of surviving children expected at the end of the childbearing period. Replacement is the response to actual mortality and can occur if the mother is young enough at the deaths of her children to produce additional births. Cross-section micro mortality data can only measure the replacement effect (Ben-Porath, 1976) and a num-

ber of studies have attempted to do so. In these studies, the unit of analysis is the household. Ben-Porath (1976) and Ben-Porath & Welch (1975) measured significant effects of child mortality on the stopping probabilities of Israeli and Bangladesh women. The replacement effect was significant and occurred quickly; a weak response would have been expected if hoarding dominated. Rutstein (1974) found significant positive replacement effects in Taiwan using the parity progression ratio as a dependent variable. He included an index of attitudes to measure expectations of mortality and found that an expectation of future mortality increased fertility. Heer and Wu (1976) analyzed the effects of community mortality and replacement on fertility in Taiwan and concluded that perceptions of child survival were positively influenced by community child survival.

In most populations studied, some replacement response was measured but, in general, the response was not complete. The implication is that a reduction in child mortality is accompanied by a lagged fertility decline but that the fertility decline is less than the mortality decline. Population growth rates tend to rise (Preston, 1975).

In all the papers cited, mortality has been treated as an exogenous variable included in the demands for children and child quality. No theory of the determinants of infant mortality has been promulgated with the exception of Williams (1974). Williams hypothesized that child survival and family size were determined simultaneously in the household. Demand for births was a function of income, prices, and survival while demand for survival was a function of income, prices and fertility. The empirical analysis of mortality found that most independent variables did not contribute significantly to explaining child survival. The major problem with this analysis

is that simultaneous equations are not derivable from the theory. The equations are not ordinary demand functions and the estimated coefficients cannot be interpreted as income and price effects.

A large number of empirical studies have investigated the determinants of mortality without analyzing the problem theoretically. Repetto (1978) estimated a simultaneous model of crude birth and death rates across countries and measured statistically significant effects of fertility on mortality and of mortality on fertility. This approach has the same interpretation problem, however, as the Williams' model and in addition uses poorly suited measures of the demographic outcomes at the family level. Additional papers by Russell (1974), Scrimshaw, Taylor, and Gordon (1968), and Sloan (1971) have estimated significant correlations between mortality and fertility, birth weight, birth order, literacy, and health care.

These studies suggest that child mortality is determined to some extent by parental inputs and is not totally exogenous. If the survival of a child is at least partially a choice variable, then estimating fertility demand equations as functions of the degree of survival produces coefficients on the remaining price and income variables which cannot be interpreted as pure price and income effects. In this case, the correct procedure is to estimate a system of demand equations, including the demand for survival, in which the quantity demanded is a function only of income and price variables. In this paper, I develop a model of the demand for child services in which fertility, child quality, and survival are choice variables. Each commodity is produced in the home with inputs of time and goods and services. Demand functions are derived from the model and estimated using data from Guatemala. Significant coefficients on the variables in the survival equation provide some support for the hypothesis that survival is a choice variable.

### Theoretical Model

The model is a variation of DeTray (1973) and Becker and Lewis (1973). The household faces a two-good utility function; the goods which the household can choose to consume are child services (C) and all other household commodities (Z).

$$U = U (C, Z) \quad (1)$$

The Z commodity is produced within the home with inputs of parental time ( $t_{H,Z}$  and  $t_{W,Z}$  where H is the husband and W is the wife), child time ( $t_{C,Z}$ ), and market goods ( $X_Z$ ) subject to the efficiency of the parents in production ( $E_H$  and  $E_W$ ).

$$Z = Z (t_{W,Z}, t_{H,Z}, t_{C,Z}, X_Z; E_W, E_H)$$

The C commodity is a function of three home-produced commodities - the number of live births (N), the child survival rate to adulthood (S), and child quality (Q). To simplify the interpretation of child quality, it is assumed that Q is an average quality of all births. N, S, and Q are produced in the home with a vector of inputs of parental time  $t_{H,j}$  and  $t_{W,j}$  ( $j = N, S, Q$ ) and market goods ( $X_j$ ) subject to efficiency of the parents in the production of each good ( $E_H$  and  $E_W$ ). Parents have acquired a stock of skills which are used to augment the production of N, S, Q, and Z. The skills utilized in production of N ( $E_{H,N}$ ,  $E_{W,N}$ ) are identical to the skills used in the production of S ( $E_{H,S}$ ,  $E_{W,S}$ ), Q ( $E_{H,Q}$ ,  $E_{W,Q}$ ) and Z ( $E_{H,Z}$ ,  $E_{W,Z}$ ). In this paper, children's time is assumed to be a minor input in the production of N, S, and Q relative to parental time and is, therefore, excluded from the production functions.

The production functions for C, N, S, and Q are summarized below:

$$C = C(N, S, Q) \quad (3)$$

$$N = N(t_{H,N}, t_{W,N}, X_N; E_H, E_W) \quad (4)$$

$$S = S(t_{H,S}, t_{W,S}, X_S; E_H, E_W) \quad (5)$$

$$Q = Q(t_{H,Q}, t_{W,Q}, X_Q; E_H, E_W) \quad (6)$$

In developing countries, the economic contribution of children is an important component of family income; total earned income includes the wage contribution of parents as well as children. Total income (Y) is the sum total of unearned income (B), the husband's wage rate times his market time ( $W_H t_{H,L}$ ), the wife's wage rate times her market time ( $W_W t_{W,L}$ ), and the wage rate of children times the average market time of all surviving children ( $W_C t_{C,L}^{NS}$ ).

$$Y = B + W_W t_{W,L} + W_H t_{H,L} + W_C t_{C,L}^{NS} \quad (7)$$

Household income is spent on market goods used in the production of N, S, Q, and Z. The complete income constraint is:

$$Y = B + W_W t_{W,L} + W_H t_{H,L} + W_C t_{C,L}^{NS} = P_Z X_Z + P_N X_N + P_S X_S + P_Q X_Q \quad (8)$$

$P_N$ ,  $P_S$ ,  $P_Q$ , and  $P_Z$  are the prices of market goods used in the production of N, S, Q, and Z respectively.

Time in the home is purchased by sacrificing time in the market. The opportunity cost of time spent in home production, including leisure time, is the market wage. Market time ( $t_{1,L}$ ) can be rewritten in terms of full time ( $T_1$ ) and the time spent in home production. All  $W_1 t_{1,j}$  terms are added to the sum of market purchases to form the full unobservable prices of Z and



C ( $\Pi_Z$  and  $\Pi_C$ ). The final full income constraint can be written by income source of expenditure per commodity.

$$\begin{aligned} I &= B + W_H T_H + W_H T_H + W_C T_C^{NS} \\ &= \Pi_Z Z + \Pi_C C \end{aligned} \quad (9)$$

Full income is the sum of unearned income and the economic contribution of all earners in the family working full time at their current market wage and is spent either on purchases of market goods or on purchases of time inputs to C and Z.

The production function for child services is assumed to be linear homogeneous. I am assuming for simplicity that some positive quantities of N, S, and Q are produced by the household so that child services cannot be produced without surviving children and a minimum investment in quality. With this specification, a change in the demand for child services can be expressed as the sum of weighted changes in prices and income.

$$EC = EI - [K_C \eta_{CI} + (1-K_C) \sigma] E\Pi_C + (1-K_C) (\sigma - \eta_{CI}) E\Pi_Z \quad (10)$$

where E is the Allen notation for a percentage change operator (Allen, 1938),  $K_C$  is the share of full income expended on C,  $\sigma$  is the elasticity of substitution of C for Z, and  $\eta_{CI}$  is the income elasticity of demand for child services. In this two-good world, C and Z are substitutes ( $\sigma > 0$ ) and  $\eta_{CI}$  is positive only if child services is a normal good.

The production functions for N, S, Q, and Z are also linear homogeneous. The equations for the changes in the shadow prices of C, N, S, Q, and Z are presented below:

$$E\Pi_C = \alpha_N E\Pi_N + \alpha_S E\Pi_S + \alpha_Q E\Pi_Q \quad (11)$$

$$E\Pi_Z = \alpha_{t_H,Z} E\Pi_H + \alpha_{t_W,Z} E\Pi_W + \alpha_{t_C,Z} E\Pi_C \\ + \alpha_{X_Z} E\Pi_Z - \mu_{Z,E_H} E(E_H) - \mu_{Z,E_W} E(E_W) \quad (12)$$

$$E\Pi_j = \alpha_{t_H,j} E\Pi_H + \alpha_{t_W,j} E\Pi_W + \alpha_{X_j} E\Pi_j \\ - \mu_{j,E_H} E(E_H) - \mu_{j,E_W} E(E_W) \\ j = N, S, Q \quad (13) - (15)$$

$$\text{where } \alpha_j = \frac{\Pi_{j,j}}{\Pi_C} \text{ and } \alpha_{i,j} = \text{the share of expenditure in } j$$

accounted for by 1.

Household utility (1) is maximized subject to the full income constraint (9) and the production function constraints. A series of three demand equations follow for N, S, and Q given in reduced form in Appendix equations (A.18), (A.19), and (A.20). The demand equations for live births, child survival, and child quality are log-linear functions of unearned income, wage rates, parental efficiency, and the prices of goods used in the production of N, S, Q, and Z. The equations can be estimated consistently, in log-linear form, using ordinary least squares. The elasticities are the coefficients of the independent variables.

No a priori predictions about the signs on the coefficients can be presented without certain assumptions. Assuming that child services is a normal good, the coefficients on EN, ES, and EQ are indeterminate unless

$2\eta_{CI} \frac{W^T NS}{C^T C^T I}$  is less than one. This is likely if the children's share in full

income is small or  $\eta_{CI}$  is small or both are valid. For example, if the children's income share is .25, then  $\eta_{CI}$  must be less than 2 if the coefficient is to be positive. I will assume, for simplicity, that the coefficient is positive so that the effects of the exogenous variables can be analyzed. First, the elasticities of N, S, and Q with respect to unearned income (B) are positive and equal because the production function of child services is linear homogeneous. Second, if  $\sigma$  is larger than  $\eta_{CI}$ , the elasticities of N, S, and Q with respect to  $P_Z$  are positive and equal and the elasticities of N, S, and Q with respect to  $W_C$  are positive and equal.

Third, the effect of a change in  $W_W$  or  $W_H$  on the demand for j ( $j = N, S, Q$ ) depends on the relative magnitudes of the pure income effect and the four substitution effects. The pure income effect is positive if C is normal, but the substitution effects depend on the substitutability of N, S, Q, and Z and the relative factor shares. No predictions of the signs on the coefficients are possible without restrictive assumptions on the size of factor shares and the elasticities of substitution. The effect of changes in  $E_W, E_H, P_N, P_S, P_Q$  on j are also indeterminate without assumptions on substitutability.

In summary, a system of demand equations for child services is specified utilizing production and consumption relationships embodied in these demands. A model of household choice is developed to justify the choice of determinants of demand equations that include the components of child services--the number of live births, the rate of child survival, and child quality.

These three commodities are functions of parental time, market goods, and parental efficiency. The model suggests the need to estimate the demand equations for these three commodities in log-linear form.

$$\begin{aligned}\ln N = & \alpha_1 \ln B + \alpha_2 \ln T_H + \alpha_3 \ln T_W + \alpha_4 \ln T_C + \alpha_5 \ln W_H + \alpha_6 \ln W_W \\ & + \alpha_7 \ln W_C + \alpha_8 \ln P_N + \alpha_9 \ln P_S + \alpha_{10} \ln P_Q + \alpha_{11} \ln P_Z + \alpha_{12} \ln E_H \\ & + \alpha_{13} \ln E_W\end{aligned}$$

$$\begin{aligned}\ln S = & \gamma_1 \ln B + \gamma_2 \ln T_H + \gamma_3 \ln T_W + \gamma_4 \ln T_C + \gamma_5 \ln W_H + \gamma_6 \ln W_W \\ & + \gamma_7 \ln W_C + \gamma_8 \ln P_N + \gamma_9 \ln P_S + \gamma_{10} \ln P_Q + \gamma_{11} \ln P_Z + \gamma_{12} \ln E_H \\ & + \gamma_{13} \ln E_W\end{aligned}$$

$$\begin{aligned}\ln Q = & \delta_1 \ln B + \delta_2 \ln T_H + \delta_3 \ln T_W + \delta_4 \ln T_C + \delta_5 \ln W_H + \delta_6 \ln W_W \\ & + \delta_7 \ln W_C + \delta_8 \ln P_N + \delta_9 \ln P_S + \delta_{10} \ln P_Q + \delta_{11} \ln P_Z + \delta_{12} \ln E_H \\ & + \delta_{13} \ln E_W\end{aligned}$$

Given the assumptions above, the implications of the model are that:

(1) the pure income elasticities of N, S, and Q ( $\alpha_1, \gamma_1, \delta_1$ ) are positive and equivalent across equations; (2) the elasticities of demand for N, S, and Q with respect to  $P_Z$  and  $W_C$  ( $\alpha_7, \gamma_7, \delta_7$  and  $\alpha_{11}, \gamma_{11}, \delta_{11}$ ) are positive and equal; and (3) the elasticities of demand with respect to parental wages, efficiency, and the prices of N, S, and Q are indeterminant without assumptions about the relative factor shares and the substitutability of inputs.

### The Data

The data source is the 1974-1975 Longitudinal Guatemala survey conducted by the Rand Corporation in cooperation with the Institute for Nutrition in Central America and Panama (INCAP). The purpose of the survey was to analyze fertility behavior in rural and "semi-rural" Guatemala.

The survey was conducted in five villages within 70 miles of Guatemala City. Four villages were classified as rural or agricultural communities in the department of El Progreso; one village was semi-rural in which a larger percentage of the adult population worked primarily in Guatemala City. All individuals questioned for the survey were Ladinos, of Spanish descent. The rural villages were sites of a previous study of nutrition, physical growth, and mental development. The remaining village was added in 1974 to increase the size of the surveyed sample.

Households containing a male head, a female head and their children are the relevant observations for this study. The maximum number of usable households in the survey is 1238.

### Endogenous Variables

The model includes three endogenous variables: the number of live births (N), child survival (S), and child quality (Q). If all families are complete, N is measured as the total number of children born alive and S is the proportion of these births which survived. Data were collected, however, on many women who were probably capable of having additional children. Restricting the analysis to a sample of women whose fertility must be completed introduces two problems. First, the number of observations is substantially reduced, and the precision of the estimators is lowered. Second, inaccurate reporting of fertility is often said to be more common in a sample of older (Boulier & Rosenzweig, 1978).

Several procedures have been adopted to analyze or include in the analysis households with incomplete fertility. One of the most common procedures has been to condition fertility on the wife's age at marriage [Afzal, Khan, and Chaudhry (1976)] or marriage duration [Harman (1970)]. These models are misspecified because they condition on an economic choice variable - marriage - rather than on an exogenous biological one. Age at marriage and childbearing are simultaneously determined functions of socioeconomic variables. The problem is aggravated in developing countries, where consensual marriage is as common as legal marriage, and the switch from consensual to legal marriage is frequently observed. Thus, estimates of coefficients in a fertility regression conditioned on an endogenous variable such as marriage are, in general, biased estimates of population parameters.

The Boulier-Rosenzweig (1978) duration ratio-DRAT is a second procedure for adjusting the cumulative fertility of incomplete families.

DRAT is the ratio of the number of children ever born divided by a woman's "natural" fertility from her date of marriage. The variable is useful in analyzing fertility within marriage under the assumption that contraception begins immediately after marriage. DRAT can then be the dependent variable in an analysis of cumulative fertility, and implies the effectiveness of the contraceptive regime.

Cumulative fertility can also be standardized for the age-fecundity relationship using a natural fertility schedule beginning at age 0 rather than at the endogenous age of marriage. This procedure is preferable to the use of DRAT where marriage is not well defined, as in Guatemala, and interest is in fertility and not marital fertility. A second, simple biological adjustment has been to include an age variable directly in the fertility regression. Assuming that fertility is a linear function of a woman's age, age can be included in the regression of fertility on socioeconomic variables (Ben-Porath, (1973), Gardner (1973), Snyder (1974)). The efficacy of this procedure depends on the validity of the assumption that age and fertility are linearly related -- an assumption which cannot be accepted when the fertility of married women of all ages is analyzed. A plot of cumulative fertility against mother's age in rural Guatemala reveals a slightly ogive or S-shaped pattern of growth. This pattern was also discerned by Coale and Trussell (1974) in 43 different countries. The procedure used in this paper is to estimate the demand for children constrained by a non-linear biological supply function.

A simple, estimable functional form of this general S-shape is the negative exponential distribution.

$$N = \frac{N^c}{[1 - e^{-\gamma(t-a_0)}]^3} \quad (1)$$

where  $N$  is the cumulative supply of births at the end of childbearing;  $N^c$  is the current number of births;  $t$  is the mother's current age; and  $a_0$  is her age at the beginning of childbearing. For simplicity, it is assumed that  $a_0$  is constant across women and known or iteratively fit.

Rewriting in logarithms, equation (1) becomes:

$$\ln N^c = \ln N + \ln \left[ 1 - e^{-\gamma (t - a_0)} \right]^3 \quad (2)$$

Several sources of error are possible in estimating this function from observations on  $N^c$  and  $t$ . First, the function is easy to estimate because it has only one parameter. However, this puts excessive weight on this one parameter to fully describe the cumulative fertility schedule. Second, the assumption that  $a_0$  is constant and known across women is an oversimplification.  $a_0$  is technically the beginning of childbearing or the onset of menstruation, and this is influenced by heredity as well as health and nutrition. If there is considerable variation in  $a_0$  in the population, then failure to estimate  $a_0$  can lower the precision of the estimated coefficients. If  $a_0$  is correlated with the socioeconomic determinants of fertility, the estimates are biased.

The demand for completed fertility ( $N$ ) is a log-linear function of a vector of wage, price, and income variables ( $X$ ).

$$\ln N = \beta \ln X + \ln c \quad (3)$$

Substituting equation (3) into equation (2) results in the final estimating equation for births.



$$\ln N^c = \beta \ln X + \ln \left[ 1 - e^{-\gamma(t-a_0)} \right]^3 + \ln \epsilon \quad (4)$$

The demand for births can now be estimated using the current number of births in all households for married women with one or more children adjusted according to the age pattern of fertility observed in the sample and fit to  $\gamma$  given  $a_0$ . The function is fit to the observed pattern of fertility growth inferred from the cross age sample rather than being adjusted to a natural fertility schedule derived primarily from European registration materials. The demand for births does not depend upon endogenous variables such as the duration of marriage or the age of marriage in its estimation, although the actual onset of childbearing may have an endogenous component.

To measure child quality (Q), an education variable is constructed. The quality variable places the greatest weight on the responses of the younger children (Birdsall, 1979). The ratio of each child's actual schooling ( $EDC_{ijk}$ ) to the sample average for his age and sex ( $\overline{EDC}_{jk}$ ) is computed. These ratios are summed across children of a mother, and the sum is divided by the number of responding children (n).

$$QUALITY = \frac{\sum_{i=1}^n \left[ (EDC_{ijk} / \overline{EDC}_{jk}) \right]}{n}$$

j = ages 7 to 18

k = male, female

To measure child survival, life tables are constructed from all of the sample's pregnancy history data. The age intervals are one-year intervals rather than five-year intervals to reduce the censoring bias. Within each interval, the proportion dead ( ${}_nq_x$ ) is calculated as the number of

children who died within the interval divided by the number exposed to death. The number exposed is computed as the number entering the interval minus one-half of the number lost--those not completing the interval but not reporting a death.

The proportion surviving,  $({}_n p_x)$ , is  $1 - {}_n q_x$ .  ${}_1 p_x$  is a conditional probability or the probability of surviving through the interval  $x$  to  $x + 1$  given survival to age  $x$ . From these figures, the cumulative survival rates--the proportion surviving from age zero to age  $x$ --are derived as the product of all  ${}_1 p_x$  computed for all intervals between 0 and  $x + 1$ .

The household's expected survival rate is the sum of the probability of survival of each child to the interview date divided by the number of live births. The household's actual survival rate is the ratio of the number of children alive at the interview to the number of live births. The ratio of actual survival to expected survival is the variable used to measure the survival demanded by the household.

### Exogenous Variables

Unearned income is the value of nonhuman wealth from which services could potentially be derived. The values of high, average, and low quality land, durable, producer goods including the home, and livestock are obtained from a 1974 survey of responsible individuals in each of the five villages. The quantities of land, durables, and livestock are weighted by these values. Total nonhuman wealth of the household is the sum of the value of agricultural land, non-agricultural land, durable goods, and livestock. Because wealth is estimated from single period data, this variable can produce unexpected results. Wealth accumulation is partly the result of past savings decisions. With only a single period of cross-sectional data, wealth is reported by individuals at different periods in their life cycles (Smith, 1973). The wealth variable, therefore, does not represent a permanent or uniform lifetime measure that might influence lifetime reproductive demands. Wealth may require an age standardization which would partially adjust for this life cycle effect.

Efficiency of the parents in the production of N, S, and Q is measured as the number of years of schooling of the male and female household heads. Prices of goods used exclusively in the production on N, S, and Q are either unavailable or unobservable. In this cross-section, price variation is likely to be inconsequential because of the propinquity of villages to Guatemala City. However, dummy variables for village location are included to measure the effect of distance from Guatemala City. TOWN 1, TOWN 2, TOWN 3, and TOWN 4 are the four rural communities. TOWN 1 is the farthest from Guatemala City--approximately 80 kilometers; TOWN 2, TOWN 3, and TOWN 4 are approximately 30 to 40 kilometers from the city.

Sixty percent of all male household heads, seven percent of all female household heads, and ten percent of all children between the ages of 7 and 18 in the survey are engaged in wage employment. The majority of males not working for wages are self-employed farmers. The females not working for wages are primarily engaged in home production. To limit the analysis to families with both heads and children reporting wages reduces the sample size dramatically and produces inconsistent estimates of population parameters (Olsen, 1977). Estimates of the value of time of non-wage earners are necessary.

The most common procedure has been to estimate labor demand functions from the samples of working men and women and to impute a wage to all individuals based on the estimated coefficients. This procedure is justified in cases in which sample selection bias is not an acute problem. In addition to increasing the sample size, wage imputation produces consistent estimators if errors in variables is a problem, and it frees variables of the influence of transitory fluctuation (Schultz, 1975).

Imputed wages are derived from earnings functions, the functional form of which is derived by Mincer (1974). The natural log of wages is expected to increase with labor market experience at a decreasing rate and to increase with formal training. The log of the daily wage for male heads and female heads is regressed on post schooling experience, experience squared, schooling, and village location.<sup>1</sup> Wages are expected to be lower in rural communities and to be lower the farther the village is from

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<sup>1</sup>Post schooling experience is equal to current age-schooling-7.

Guatemala City. Table 1 presents the estimated earnings functions for male and female heads.<sup>2</sup>

In the male head regression, the log of wages does increase at a decreasing rate with experience and increases with schooling. In addition, residence in a rural village reduces the log of wages, and the town farthest from Guatemala City (TOWN1) has the smallest expected wage. All coefficients, with the exception of experience, are significant at the five percent level.

In the female head regression, the log of wages increases with additional schooling and increases at a decreasing rate with experience. Wages are higher in the rural communities. However, only schooling is significantly different from zero. An F-test comparing regressions with and without the set of town dummy variables indicated no significant contribution of village location to the results.

These estimates can be biased estimates of the value of time of non-wage earners if the households excluded from the regressions do not constitute a random subsample of the entire sample of households. Systematic differences can exist between wage-earners and non-wage-earners with the same experience and training.

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<sup>2</sup> An earnings function was also estimated for children between the ages of 7 and 18. Less than 10 percent of all children between these ages reported a wage, and the only variable of significance was a dummy variable for sex of the respondent. No further attempt was made to estimate a wage for children.

Table 1: Earnings Functions for Husbands and Wives.<sup>a, b</sup>

Independent Variables	Dependent Variables	
	Log of Husband's Wage	Log of Wife's Wage
Intercept	.607 (.158)	-.830 (.411)
Experience	.011 (.009)	.040 (.025)
Experience squared divided by 1,000	-.24 (.13)	-.00050 (.00036)
Education	.091 (.0098)	.158 (.033)
TOWN1	-.788 (.079)	.032 (.262)
TOWN2	-.274 (.077)	.270 (.338)
TOWN3	-.263 (.081)	.0066 (.391)
TOWN4	-.338 (.108)	.668 (.493)
F	45.29	3.77
R <sup>2</sup>	.366	.248
Sample Size	556	87

<sup>a</sup>Standard errors are in parentheses.

<sup>b</sup>The sample includes only husbands and wives reporting wage income.

To test and correct for sample selection bias, a Tobit model (Heckman, 1974) or a probit model (Olsen, 1977) can be estimated. The Tobit approach is preferable only if the same model determines both the number of hours devoted to off-farm labor and the decision to work off the farm.<sup>3</sup> This assumption is not supported by the Guatemalan data, and the probit procedure is, therefore, adopted.<sup>4</sup> Estimating equations for the reservation wage and the offered wage of males are derived. Equations are not estimated for females because of the small sample size. The schooling variable proxies the woman's market value of time as well as her non-market productive efficiency.

A likelihood ratio test for the presence of selection among males is significant. However, in these selection models, the assumption of normality of residuals is crucial. The results are very sensitive to the presence or absence of normality. If the normality assumption is incorrect, the likelihood test can still be significant, but the significance is not due to selectivity bias (Olsen, 1979).

The residuals of the probit equation are tested for non-normality, and a likelihood ratio test supports the hypothesis that the residuals are non-normal. The probit procedure is rerun adjusting for non-normality. The test of no selection given the non-normal distribution produces a chi-squared of 3.1426 which is only marginally significant. The hypothesis of selectivity bias is, therefore, not supported once the distribution of

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<sup>3</sup>Labor force participation is defined as work in the wage-earning sector. Non-participation in the labor force usually implies self-employment on the respondent's farm.

<sup>4</sup>For additional detail on this procedure, see Anderson (1979).

the errors is changed. The coefficients derived from the probit selection model are not used to impute a value of time to males. The least squares coefficients are consistent estimates of the value of time for wage earners, and the reservation wage coefficients for non-wage earners are derived from the probit model of labor force participation. The experience variable is used to identify the model (Heckman, 1976).<sup>5</sup> These coefficients are presented in Table 1 and are used to impute wages to all males in the sample.

<sup>5</sup>The computing program depended upon estimating a just-identified model. Both experience and experience squared could not be omitted. However, the lack of significance on experience squared in the reservation wage equation suggests that it is essentially excluded from the estimation.

Table 2: Wage offer and reservation wage equations for males<sup>a</sup>

Independent Variables	Dependent Variables	
	Offered Wage	Reservation Wage
Constant	.77197	.78379
Experience	.001698	-
Experience squared divided by 1,000	-.11282	-.08285
Education	.080246	.078692
TOWN1	-.73651	-.72126
TOWN2	-.30846	-.29308
TOWN3	-.24605	-.26359
TOWN4	-.33107	-.30563
Wealth divided by 1,000	-	.00370
Sample Size	644	1238

<sup>a</sup>The estimates are derived from a full sample of husbands.



### Empirical Results

Variables used in the analysis are defined in Table 3 with summary statistics. The estimates of the non-linear fertility demand equation are presented in Table 4. The log of children ever born (LCEB) to women age 14 to 50 is regressed on education, wealth, the husband's wage, and village location and is adjusted for the mother's current age.<sup>6</sup> The wage variable used in the column 1 regression is computed imputing a wage derived from the wage offer and reservation wage equations in Table 2. The wage variable used in the column 2 regression is imputed directly from the earnings function in Table 1. Selectivity bias is not observable is either imputed wage variable, but the variable derived from wage offer and reservation wage equations corrects for censoring in the sample. The column 3 regression contains no male wage variable.

The wage elasticities computed for both equations 1 and 2 are negative but less than one. The elasticity reported in equation 2 is approximately one-third the size of the elasticity in equation 1. Not adjusting for truncation in the wage equation does reduce the size of the wage effect but does not change the direction of the effect or the signs of any of the remaining variables.

The wealth effect in all three equations is positive as expected

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<sup>6</sup>I derived a second measure of children ever born from data in the census. Women aged 14 to 89 were included in this sample. The results were different in many respects from the results reported here. Sign reversals were recorded for the town variables and male education. I ran the regressions a third time for women over the age of 50 and found large standard errors on all variables and signs reversed. The respondent in the census was primarily the female head of household although nine percent of the questions were answered by the male head. The respondent in the retrospective data was any women who had had at least one pregnancy and was less than 50. The census data and retrospective data appear to be two different samples which can account for the discrepancy in results. In addition, more reporting errors are expected within an older population.

Table 3 Summary statistics for variables used in the regressions.

Variables	Definition	Mean	Standard Deviation	Minimum	Maximum
LCEB	Log of children ever born	1.279	.718	0	2.773
QUALITY	Child quality	1.344	1.268	0	15.5
SURVIVAL	Ratio of expected child survival to actual survival	1.030	.226	0	1.306
EPH	Schooling of husband	2.562	2.780	0	15.0
EPW	Schooling of the wife	1.865	2.413	0	15.0
WEALTH divided by 1,000	Value of land, durables, live-stock	1.110	1.862	0	16.368
LNWHIM	Male wage imputed from reservation wage	.692	.406	-.319	1.981
LNWHIM2	Male wage imputed from labor demand	.691	.440	-.542	2.064
TOWN 1	Santo Domingo-rural	.127			
TOWN 2	Cornacoste-rural	.148			
TOWN 3	Esiritu Santo-rural	.147			
TOWN 4	San Juan-rural	.093			

Table 3: Non-linear regression of logarithm of children ever born.<sup>a, b</sup>

INDEPENDENT VARIABLES	(1)	(2)	(3)
Intercept	3.139 (.280)	2.751 (.263)	2.610 (.193)
EPH	.0734 (.039) [.189]	.012 (.038) [.032]	-.008 (.008) [-.021]
EPW	-.030 (.009) [-.057]	-.030 (.009) [-.057]	-.029 (.009) [-.054]
WEALTH	.007 (.011) [.007]	.005 (.011) [.005]	.003 (.010) [.004]
LNWHIM	-.982 (.454) [-.982]	-	-
LNWHIM2	-	-.232 (.399) [-.232]	-
TOWN 1	-.824 (.342)	-.280 (.324)	-.082 (.060)
TOWN 2	-.272 (.147)	-.040 (.121)	.037 (.057)
TOWN 3	-.291 (.131)	-.100 (.120)	-.025 (.062)
TOWN 4	-.308 (.158)	-.075 (.150)	.018 (.065)
AGEW	.051 (.010)	.046 (.010)	.044 (.009)
Sample Size	638	638	657

<sup>a</sup>Standard errors are in parentheses.

<sup>b</sup>Elasticities are in brackets.

although the standard errors are larger than the coefficients. The wealth effect is larger in equation 1 than in either equations 2 or 3.

The husband's schooling is a significant positive determinant of CEB when the wage effect is also estimated. One explanation is that schooling does increase the husband's efficiency in production causing him to produce more children if everything else is held constant. In equation 3 excluding the wage, the education elasticity is negative because it includes the negative effect of wages. Education can also be interpreted as a wealth effect or representing tastes against children.

The wife's schooling is negative and significant in all three equations as expected. The education elasticity is approximately  $-.05$  in all equations. This schooling effect includes both a wage effect and an efficiency effect. If the number of years of schooling of the wife were to double from the sample mean of 1.9 years, then live births would fall by 5 percent.

One surprising result is the negative effect of residence in the rural villages on fertility in equations 1 and 2. Holding constant within village differences in education, wealth, and wages, a between village effect appears to be significant. These four villages, however, are not typical rural villages in Guatemala because of the establishment of health and nutrition programs within the communities. All four villages received free health care in local clinics and free nutritional guidance on demand while two of the villages (Town 2 and Town 3) were given high protein-calorie supplements. With such intervention, a between village effect is reasonable. Although the programs were primarily designed to reduce mortality, if survival and fertility are substitutes, then a decline in fertility might be expected due to the presence of this program.

In addition, contraceptive information could have been dispensed at the centers causing fertility to fall.<sup>7</sup>

The estimation of child quality is presented in Table 5. A large number of households did not respond to questions on the education of their children so that the sample size fell.<sup>8</sup>

All variables in Table 5 are statistically significant, with the exception of nonhuman wealth. The husband's wage elasticity is negative and greater than one in both equations 1 and 2. A one percent increase in his wage induces a decline in child quality of between two and three percent.

The wealth effect is not statistically significant, and the elasticity evaluated at the mean is small. An increase in wealth has little effect on quality.

Parental education is a positive determinant of child quality. The male education elasticity in column 1 exclusive of the wage effect is approximately .6. The female education effect is small but inclusive of her wage effect. When compared with the male education elasticity in column 3 which includes his wage effect, the female education elasticity is larger.

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<sup>7</sup>It is difficult to draw conclusions from coefficients on regional dummy variables because of the lack of variability within the variables; in this case, I only have five pieces of information. I ran the fertility equation again excluding the dummy variables and tested the hypothesis that the coefficients on the town variables were zero. The hypothesis was rejected. However, the coefficients on EPH and LNWHIM did change signs but were no longer statistically significant because the imputed wage is picking up the town effect. Improved wealth effects were expected, but this was not confirmed by the data. The quality and survival regressions were also run without town variables with similar results.

<sup>8</sup>Quality was estimated for all children regardless of sex. Two additional regressions were run separately for male and female children. The differences were so slight between the two groups that I am not reporting those results.

Table 5: Estimation of child quality. <sup>a,b</sup>

INDEPENDENT VARIABLES	(1)	(2)	(3)
Constant	3.735 (.407)	2.911 (.321)	1.414 (.125)
EPH	.332 (.059) [.633]	.254 (.054) [.484]	.006 (.026) [.011]
EPW	.043 (.027) [.060]	.047 (.027) [.065]	.047 (.028) [.065]
WEALTH divided by 1,000	-.005 (.003) [-.004]	-.011 (.028) [-.009]	-.025 (.029) [-.020]
LNWHIM	-3.655 (.594) [-2.720]	-	-
LNWHIM2	-	-2.452 (.466) [-1.825]	-
TOWN 1	-3.045 (.485)	-2.285 (.423)	-.301 (.176)
TOWN 2	-1.289 (.239)	-.837 (.200)	-.210 (.161)
TOWN 3	-1.302 (.230)	-.974 (.207)	-.321 (.167)
TOWN 4	-1.648 (.263)	-1.315 (.243)	-.532 (.194)
F	7.490	6.180	3.270
R <sup>2</sup>	.094	.070	.036
Sample Size	579	579	614

<sup>a</sup>Standard errors are in parentheses.

<sup>b</sup>Elasticities evaluated at the sample means are in brackets.

In all equations, residence in a rural village lowers child quality investment. Several explanations of this result are possible. First, the opportunity cost of children is likely to be higher in the rural villages because children are employed in farm labor. If the price of schooling, measured as time lost in home employment, is higher, then substitution away from quality is expected. Second, fewer schooling facilities are available in the rural areas and are probably of lower quality. Again, no definitive conclusions are possible from regional shift variables.

The results of the child survival estimation are presented in Table 6. First, the male wage elasticities are positive and significant, suggesting that the production of survival is goods-intensive relative to the father's time. Second, the wealth effect is positive as expected but very small and not statistically significant.

The husband and wife's education elasticities are both positive when the husband's wage is not held constant. Changes in the husband's schooling increase child survival by a larger percentage than changes in the wife's schooling, although neither effect is large. Holding the husband's wage constant, the husband's schooling elasticity becomes negative.

Coefficients on the town dummy variables are positive in equations 1 and 2 and contribute significantly to the explanatory power of the regressions. The significant between-village effect is surprising but is possibly attributable to the nutrition intervention. The programs were established to lower child survival, and this result supports the contention that the program was successful in achieving this goal (Delgado, et al., 1978).

Table 6: Regressions of child survival.<sup>a,b</sup>

INDEPENDENT VARIABLES	(1)	(2)	(3)
Intercept	.699 (.111)	.695 (.111)	.991 (.019)
LNWHIM	.413 (.154) [.413]	-	-
LNWHIM2	-	.425 (.157) [.425]	-
WEALTH divided by 1,000	.002 (.005) [.002]	.002 (.005) [.002]	.003 (.005) [.003]
EPH	-.026 (.014) [-.065]	-.031 (.015) [-.076]	.010 (.004) [.024]
EPW	.004 (.004) [.008]	.005 (.004) [.009]	.004 (.004) [.007]
TOWN 1	.279 (.118)	.312 (.129)	-.032 (.027)
TOWN 2	.088 (.054)	.077 (.050)	-.041 (.026)
TOWN 3	.176 (.050)	.179 (.051)	.064 (.028)
TOWN 4	.137 (.058)	.150 (.061)	.003 (.030)
F	4.620	4.640	4.520
R <sup>2</sup>	.055	.056	.046
Sample Size	638	638	657

<sup>a</sup>Standard errors are in parentheses.

<sup>b</sup>Elasticities are in brackets.



### Conclusions

A derived demand model is developed in which child survival is explicitly included as a choice variable. The model is estimated using data from Guatemala and the results support the hypothesis that survival is not completely exogenous to the household. The estimation of fertility and child quality also produced significant results which, in general, support previous research.

If the husband's wage is excluded from the regressions, then additional schooling by the husband or wife reduces fertility demand but raises the demands for quality and child survival. The schooling elasticities are less than one in absolute value, and the wife's schooling elasticities are larger than the husband's elasticities. Additional schooling by the husband, holding his value of time constant, increases the demands for fertility and quality but significantly reduces the demand for survival. The wife's schooling elasticities remain unchanged.

The results indicate that the wage imputation procedure used can be important in deriving the correct wage elasticities. If the husband's value of time is derived from wage offer and reservation wage equations which adjust for censoring in the sample, a one percent increase in the value of time decreases fertility by one percent, decreases quality by three percent, and increases survival by .4 percent. The signs on the wage coefficients remain unchanged if the value of time is imputed from earnings functions, but the sizes of the elasticities change significantly in the fertility and quality regressions. The wage elasticity is only - .2 in the fertility regression and - 1.8 in the quality regression. No difference is observable in the survival regression. Although selectivity bias was not a problem in estimating a wage effect, censoring is important in determining the magnitude of this effect.

The results provide some indication that nutritional and health intervention in several of the rural villages alters behavior. After holding education, wealth, and wages within the communities constant, fertility is lower and survival higher in these rural villages. This contradicts previous research on urban-rural differences in fertility and survival although most studies have failed to control for within village differences. The result does encourage further investigation into these communities using more detailed data on access to the nutrition programs and on the provision of child health inputs in the control villages. These data were collected but are not yet in the public domain.

Finally, one prediction of the model is not supported. Unearned income elasticities are predicted to be positive and equal in all equations. Using a wealth index to measure unearned income, positive elasticities are estimated for fertility and survival, but a negative elasticity is estimated for quality. The elasticities are approximately equal in absolute value. The wealth variable is not a very good measure of unearned income but is the best that can be derived from this data set. Before rejecting the model because of this result, the equations should be re-estimated using better data on wealth.

## Appendix

### Derivation of the model

Household utility,  $U=U(C,Z)$ , is maximized subject to a full income constraint,  $I=\pi_Z Z + \pi_C C = B + W_W T_W + W_H T_H + W_C T_C NS$  and the production functions for  $C$ ,  $N$ ,  $S$ ,  $Q$ , and  $Z$ :

$$C = C(N, S, Q) \quad (A.1)$$

$$N = N(t_{H,N}, t_{W,N}, X_N; E_H, E_W) \quad (A.2)$$

$$S = S(t_{H,S}, t_{W,S}, X_S; E_H, E_W) \quad (A.3)$$

$$Q = Q(t_{H,Q}, t_{W,Q}, X_Q; E_H, E_W) \quad (A.4)$$

$$Z = Z(t_{H,Z}, t_{W,Z}, t_{C,Z}, X_Z; E_H, E_W) \quad (A.5)$$

where:

$C$  = child services

$Z$  = all other household commodities

$N$  = quantity of live births

$S$  = survival rate of live births

$Q$  = quality of live births

$t_{H,j}$  = husband's time in the production of  $j$  ( $j = N, S, Q, Z$ )

$t_{W,j}$  = wife's time in the production of  $j$

$t_{C,Z}$  =  $C$ 's time in the production of  $Z$

$X_j$  = goods used in the production of  $j$

$E_H$  = husband's technological efficiency

$E_W$  = wife's technological efficiency

$\pi_k$  = full price of  $k$  ( $k=Z, C$ )

$B$  = nonearnings income

$W_i$  = wage rate of  $i$  ( $i = \text{Husband (H), Wife (W)}$ )

$T_i$  = full time of  $i$

$I$  = full income

Under the assumption that all production functions are linear homogeneous, the following expressions are derived from Allen (1938):

$$EC = \eta EI - \left[ K_C \eta_{CI} + (1-K_C)\sigma \right] E\Pi_C + (1-K_C)(\sigma - \eta_{CI}) E\Pi_Z \quad (A.6)$$

$$EC = \alpha_N EN + \alpha_S ES + \alpha_Q EQ \quad (A.7)$$

$$E\Pi_C = \alpha_N E\Pi_N + \alpha_S E\Pi_S + \alpha_Q E\Pi_Q \quad (A.8)$$

$$E\Pi_N = \alpha_{t_H, N}^{EW_H} + \alpha_{t_W, N}^{EW_W} + \alpha_{X_N}^{EP_N} - \mu_{N, E_H} E(E_H) \quad (A.9)$$

$$- \mu_{N, E_W} E(E_W)$$

$$E\Pi_S = \alpha_{t_H, S}^{EW_H} + \alpha_{t_W, S}^{EW_W} + \alpha_{X_S}^{EP_S} - \mu_{S, E_H} E(E_H) \quad (A.10)$$

$$- \mu_{S, E_W} E(E_W)$$

$$E\Pi_Q = \alpha_{t_H, Q}^{EW_H} + \alpha_{t_W, Q}^{EW_W} + \alpha_{X_Q}^{EP_Q} - \mu_{Q, E_H} E(E_H) \quad (A.11)$$

$$- \mu_{Q, E_W} E(E_W)$$

$$E\Pi_Z = \alpha_{t_H, Z}^{EW_H} + \alpha_{t_W, Z}^{EW_W} + \alpha_{t_C, Z}^{EW_C} + \alpha_{X_Z}^{EP_Z} \quad (A.12)$$

$$- \mu_{Z, E_H} E(E_H) - \mu_{Z, E_W} E(E_W)$$

where:

$\eta_{CI}$  = income elasticity of demand for child services

$K_C$  = share of full income expended on child services =  $\frac{\Pi_C C}{I}$

$1-K_C = \frac{\Pi_Z Z}{I}$

$\sigma$  = elasticity of substitution of C for Z

$\Pi_m$  = full price of m (m = c, N, S, Q, Z)

$P_j$  = price of goods used in the production of j (j = N, S, Q, Z)

$$\alpha_N = \frac{\Pi_N N}{\Pi_C C}$$

$$\alpha_S = \frac{\Pi_S S}{\Pi_C C}$$

$$\alpha_Q = \frac{\Pi_Q Q}{\Pi_C C}$$

$\alpha_{i,j}$  = share of expenditure in j accounted for by i

$\mu_{j,b}$  = partial elasticity of j with respect to b  
(b =  $E_H, E_W$ )

The percentage change in full income (I) is the weighted sum of its parts. The expression for EI is:

$$\begin{aligned} EI = & EB (B/I) + ET_W \left( \frac{W_W^T W}{I} \right) + ET_H \left( \frac{W_H^T H}{I} \right) \\ & + ET_C \left( \frac{W_C^{CT} C}{I} \right) + EW_W \left( \frac{W_W^T W}{I} \right) + EW_H \left( \frac{W_H^T H}{I} \right) \\ & + EW_C \left( \frac{W_C^{CT} C}{I} \right) + EN \left( \frac{W_C^{CT} C}{I} \right) + ES \left( \frac{W_C^{CT} C}{I} \right) \end{aligned} \quad (A.13)$$

#### Derivation of the EN Equation

Rewriting (A.7) in terms of EN yields:

$$EN = \frac{EC}{\alpha_N} - \frac{\alpha_S}{\alpha_N} ES - \frac{\alpha_Q}{\alpha_N} EQ \quad (A.14)$$

The elasticity of substitution of Q for N ( $\sigma_{NQ}$ ) is the percentage change in  $Q/N$  divided by the percentage change in the ratio of prices,  $\Pi_N/\Pi_Q$ .

$$\sigma_{NQ} = \frac{EQ - EN}{E\Pi_N - E\Pi_Q}$$

Rewriting in terms of EQ yields:

$$EQ = \sigma_{NQ} (E\Pi_N - E\Pi_Q) + EN \quad (A.15)$$

The elasticity of substitution of S for N ( $\sigma_{NS}$ ) is the percentage change in S/N divided by the percentage change in the ratio of prices,  $\Pi_N/\Pi_S$ .

$$\sigma_{NS} = \frac{ES - EN}{E\Pi_N - E\Pi_S}$$

Rewriting in terms of ES yields:

$$ES = EN + \sigma_{NS} (E\Pi_N - E\Pi_S) \quad (A.16)$$

Equation (A.13) is substituted into (A.6) for EI

$$\begin{aligned} EC = \eta_{CI} \left[ EB \frac{B}{I} + ET_W \frac{W^T_W}{I} + ET_H \frac{W^T_H}{I} \right. \\ \left. + ET_C \frac{W^T_C^{NS}}{I} + EN \frac{W^T_C^{NS}}{I} + ES \frac{W^T_C^{NS}}{I} \right] \\ - [K_C \eta_{CI} + (1-K_C)\sigma] E\Pi_C + (1-K_C) (\sigma - \eta_{CI}) E\Pi_Z \end{aligned} \quad (A.17)$$

Equations (A.17), (A.15) and (A.16) are substituted for EC, EQ, and ES in (A.14) and the resulting equation is solved for EN. Equations (A.8) - (A.12) are then substituted for  $E\Pi_C$ ,  $E\Pi_N$ ,  $E\Pi_S$ ,  $E\Pi_Q$ , and  $E\Pi_Z$ . The final demand function for EN is the following:

$$\begin{aligned}
 & EN \left\{ 1 - 2\eta_{CI} \frac{W_{CC}^{TNS}}{I} \right\} \tag{A.18} \\
 & = EB \left\{ \eta_{CI} \frac{B}{I} \right\} \\
 & + ET_W \left\{ \eta_{CI} \frac{W_{WW}^{TW}}{I} \right\} \\
 & + ET_H \left\{ \eta_{CI} \frac{W_{HH}^{TH}}{I} \right\} \\
 & + ET_C \left\{ \eta_{CI} \frac{W_{CC}^{TNS}}{I} \right\} \\
 & + EW_W \left\{ \eta_{CI} \frac{W_{WW}^{TW}}{I} + \alpha_{t_w, N} \left[ \sigma_{NS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_Q \sigma_{NQ} - \alpha_S \sigma_{NS} - \alpha_N \right. \right. \\
 & \quad \left. \left. (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\} \\
 & + \alpha_{t_w, S} \left[ \alpha_S \sigma_{NS} - \sigma_{NS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & + \alpha_{t_w, Q} \left[ \alpha_Q \sigma_{NQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right] + \alpha_{t_w, Z} (1 - K_C) (\sigma - \eta_{CI}) \Big\} \\
 & + EW_H \left\{ \eta_{CI} \frac{W_{HH}^{TH}}{I} + \alpha_{t_H, N} \left[ \alpha_{NS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_Q \sigma_{NQ} \right. \right. \\
 & \quad \left. \left. - \alpha_S \sigma_{NS} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\} \\
 & + \alpha_{t_H, S} \left[ \alpha_S \sigma_{NS} - \sigma_{NS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & + \alpha_{t_H, Q} \left[ \alpha_Q \sigma_{NQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right] + \alpha_{t_H, Z} (1 - K_C) (\sigma - \eta_{CI}) \Big\} \\
 & + EW_C \left\{ \eta_{CI} \frac{W_{CC}^{TNS}}{I} + \alpha_{t_e, Z} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + EP_N \left\{ \alpha_{x_N} \left[ \sigma_{NS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_Q \sigma_{NQ} - \alpha_S \sigma_{NS} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + EP_S \left\{ \alpha_{x_S} \left[ \alpha_S \sigma_{NS} - \sigma_{NS} \eta_{CI} \frac{W_{C^T C}^{T NS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\} \\
 & + EP_Q \left\{ \alpha_{x_S} \left[ \alpha_Q \sigma_{NQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\} \\
 & + EP_Z \left\{ \alpha_{x_Z} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + E(E_W) \left\{ -\mu_{N, E_W} \left[ \sigma_{NS} \eta_{CI} \frac{W_{C^T C}^{T NS}}{I} - \alpha_Q \sigma_{NQ} - \alpha_S \sigma_{NS} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right. \\
 & \quad - \mu_{S, E_W} \left[ \alpha_S \sigma_{NS} - \sigma_{NS} \eta_{CI} \frac{W_{C^T C}^{T NS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & \quad - \mu_{Q, E_W} \left[ \alpha_Q \sigma_{NQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & \quad \left. - \mu_{Z, E_W} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + E(E_H) \left\{ -\mu_{N, E_H} \left[ \sigma_{NS} \eta_{CI} \frac{W_{C^T C}^{T NS}}{I} - \alpha_Q \sigma_{NQ} - \alpha_S \sigma_{NS} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right. \\
 & \quad - \mu_{S, E_H} \left[ \alpha_S \sigma_{NS} - \sigma_{NS} \eta_{CI} \frac{W_{C^T C}^{T NS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & \quad - \mu_{Q, E_H} \left[ \alpha_Q \sigma_{NQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \\
 & \quad \left. - \mu_{Z, E_H} (1 - K_C) (\sigma - \eta_{CI}) \right\}
 \end{aligned}$$

The equations for ES and EQ are derived similarly.



$$\begin{aligned}
 & E_S \left\{ 1 - 2 \eta_{CI} \frac{W_C^T C^{NS}}{I} \right\} & (A.19) \\
 & = EB \left\{ \eta_{CI} \frac{B}{I} \right\} \\
 & + ET_W \left\{ \eta_{CI} \frac{W_W^T W}{I} \right\} \\
 & + ET_H \left\{ \eta_{CI} \frac{W_H^T H}{I} \right\} \\
 & + ET_C \left\{ \eta_{CI} \frac{W_C^T C^{NS}}{I} \right\} \\
 & + EW_W \left\{ \eta_{CI} \frac{W_W^T W}{I} + \alpha_{t_{W,N}} [\alpha_N \sigma_{SN} + \sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} \right. \\
 & \quad \left. - \alpha_N (K_C \eta_{CI} + (1-K_C)\sigma)] + \alpha_{t_{W,S}} [\sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} \right. \\
 & \quad \left. - \alpha_Q \sigma_{SQ} + \alpha_N \sigma_{SN} - \alpha_S (K_C \eta_{CI} + (1-K_C)\sigma)] \right. \\
 & \quad \left. + \alpha_{t_{W,Q}} [\alpha_Q \sigma_{SQ} - \alpha_Q (K_C \eta_{CI} + (1-K_C)\sigma)] \right. \\
 & \quad \left. + \alpha_{t_{W,Z}} (1-K_C) (\sigma - \eta_{CI}) \right\} \\
 & + EW_H \left\{ \eta_{CI} \frac{W_H^T H}{I} + \alpha_{H,N} [\alpha_N \sigma_{SN} + \sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} \right. \\
 & \quad \left. - \alpha_N (K_C \eta_{CI} + (1-K_C)\sigma)] + \alpha_{t_{H,S}} [\sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} \right.
 \end{aligned}$$

$$\begin{aligned}
 & - \alpha_Q \sigma_{SQ} + \alpha_N \sigma_{SN} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) ] \\
 & + \alpha_{t_H, Q} [\alpha_Q \sigma_{SQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) ] \\
 & + \alpha_{t_H, Z} (1 - K_C) (\sigma - \eta_{CI}) \} \\
 & + E W_C \left\{ \eta_{CI} \frac{W_C T_C^{NS}}{I} + \alpha_{t_C, Z} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + E P_N \left\{ \alpha_{X_S} [\alpha_N \sigma_{SN} - \sigma_{SN} \eta_{CI} \frac{W_C T_C^{NS}}{I} \right. \\
 & \quad \left. - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) ] \right\} \\
 & + E P_S \left\{ \alpha_{X_S} \left[ \sigma_{SN} \eta_{CI} \frac{W_C T_C^{SN}}{I} - \alpha_Q \sigma_{SQ} - \alpha_N \sigma_{SN} \right. \right. \\
 & \quad \left. \left. - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \right] \right\} \\
 & + E P_Q \left\{ \alpha_{X_Q} [\alpha_Q \sigma_{SQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) ] \right\} \\
 & + E P_Z \left\{ \alpha_{X_Z} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + E(E_W) \left\{ - \mu_{N, E_W} [\alpha_N \sigma_{SN} + \sigma_{SN} \eta_{CI} \frac{W_C T_C^{SN}}{I} - \alpha_N (K_C \eta_{CI} \right. \\
 & \quad \left. + (1 - K_C) \sigma) ] - \mu_{S, E_W} [\sigma_{SN} \eta_{CI} \frac{W_C T_C^{SN}}{I} - \alpha_Q \sigma_{SQ} \right. \\
 & \quad \left. + \alpha_N \sigma_{SN} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) ] \right. \\
 & \quad \left. - \mu_{Q, E_W} [\alpha_Q \sigma_{SQ} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) ] \right. \\
 & \quad \left. - \mu_{Z, E_W} (1 - K_C) (\sigma - \eta_{CI}) \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + E (E_H) \{ - \mu_{N,E_H} [\alpha_N^{\sigma_{SN}} + \sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} - \alpha_N (K_C \eta_{CI} \\
 & + (1-K_C) \sigma)] - \mu_{S,E_H} [\sigma_{SN} \eta_{CI} \frac{W_C^T C^{SN}}{I} \\
 & - \alpha_Q^{\sigma_{SQ}} + \alpha_N^{\sigma_{SN}} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma)] \\
 & - \mu_{Q,E_H} [\alpha_Q^{\sigma_{SQ}} - \sigma_Q (K_C \eta_{CI} + (1 - K_C) \sigma)] \\
 & - \mu_{Z,E_H} (1 - K_C) (\sigma - \eta_{CI}) \}
 \end{aligned}$$

(A.20)

$$\begin{aligned}
 & \text{EQ} \left\{ 1 - 2 \eta_{CI} \frac{W_C^T NS}{I} \right\} \\
 & - \text{EB} \left\{ \eta_{CI} \frac{B}{I} \right\} \\
 & + \text{ET}_W \left\{ \eta_{CI} \frac{W_W^T}{I} \right\} \\
 & + \text{ET}_H \left\{ \eta_{CI} \frac{W_H^T}{I} \right\} \\
 & + \text{ET}_C \left\{ \eta_{CI} \frac{W_C^T NS}{I} \right\} \\
 & + \text{EW}_W \left\{ \eta_{CI} \frac{W_W^T}{I} + \alpha_{t_W, N} [\alpha_N \sigma_{QN} - \sigma_{QN} \eta_{CI} \frac{W_C^T NS}{I} \right. \\
 & \quad \left. - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma)] + \alpha_{t_W, S} [\alpha_S \sigma_{QS} - \sigma_{QS} \eta_{CI} \frac{W_C^T NS}{I} \right. \\
 & \quad \left. - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma)] + \alpha_{t_W, Q} [\sigma_{QN} \eta_{CI} \frac{W_C^T NS}{I} \right. \\
 & \quad \left. + \sigma_{QS} \eta_{CI} \frac{W_C^T NS}{I} - \alpha_N \sigma_{QN} - \alpha_S \sigma_{QS} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) \right. \\
 & \quad \left. + \alpha_{t_W, Z} (1 - K_C) (\sigma - \eta_{CI}) \right\} \\
 & + \text{EW}_H \left\{ \eta_{CI} \frac{W_H^T}{I} + \alpha_{t_H, N} [\alpha_N \sigma_{QN} - \sigma_{QN} \eta_{CI} \frac{W_C^T NS}{I} \right. \\
 & \quad \left. - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma)] + \alpha_{t_H, S} [\alpha_S \sigma_{QS} \right.
 \end{aligned}$$

$$\begin{aligned}
 & - \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) ] \\
 & + \alpha_{t_H, Q} [ \sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} + \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} \\
 & - \alpha_N \sigma_{QN} - \alpha_S \sigma_{QS} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) ] \\
 & + \alpha_{t_H, Z} (1 - K_C) (\sigma - \eta_{CI}) \} \\
 & + EP_N \{ \alpha_{X_N} [ \alpha_N \sigma_{QN} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \\
 & - \sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] \} \\
 & + EP_S \{ \alpha_{X_S} [ \alpha_S \sigma_{QS} - \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) \\
 & - \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] \} \\
 & + EP_Q \{ \alpha_{X_N} [ \sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} + \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} \\
 & - \alpha_N \sigma_{QN} - \alpha_S \sigma_{QS} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma) ] \} \\
 & + EP_Z \{ \alpha_{X_Z} (1 - K_C) (\sigma - \eta_{CI}) \} \\
 & + E(E_W) \{ - \mu_{N, E_W} [ \alpha_N \sigma_{QN} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma)
 \end{aligned}$$

$$\begin{aligned}
 & - \sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] - \mu_{S, E_W} [\alpha_S \sigma_{QS} - \alpha_S (K_C \eta_{CI} \\
 & + (1 - K_C) \sigma) - \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] \\
 & - \mu_{Q, E_W} [\sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} + \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} \\
 & - \alpha_N \sigma_{QN} - \alpha_S \sigma_{QS} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma)] \\
 & - \mu_{Z, E_W} (1 - K_C) (\sigma - \eta_{CI}) \} \\
 & + E(E_H) \left\{ - \mu_{N, E_H} [\alpha_N \sigma_{QN} - \alpha_N (K_C \eta_{CI} + (1 - K_C) \sigma) \right. \\
 & - \sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] - \mu_{S, E_H} [\alpha_S \sigma_{QS} \\
 & + \alpha_S (K_C \eta_{CI} + (1 - K_C) \sigma) - \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} ] \\
 & - \mu_{Q, E_H} [\sigma_{QN} \eta_{CI} \frac{W_{CC}^{TNS}}{I} + \sigma_{QS} \eta_{CI} \frac{W_{CC}^{TNS}}{I} \\
 & - \alpha_N \sigma_{QN} - \alpha_S \sigma_{QS} - \alpha_Q (K_C \eta_{CI} + (1 - K_C) \sigma)] \\
 & - \mu_{Z, E_H} (1 - K_C) (\sigma - \eta_{CI}) \}
 \end{aligned}$$

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