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**Working Paper**

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Center Discussion Paper, No. 325

**Provided in Cooperation with:**
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*Suggested Citation:* Warr, Peter G. (1979) : Shadow Pricing Rules for Non-Traded Commodities, Center Discussion Paper, No. 325, Yale University, Economic Growth Center, New Haven, CT

This Version is available at:
https://hdl.handle.net/10419/160252

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New Haven, Connecticut

CENTER DISCUSSION PAPER NO. 325

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October 1979

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## I. INTRODUCTION

Four themes can be detected in much of the large literature on benefit-cost analysis to emerge in the last decade. The first is that market prices are presumed to be distorted, whether because of undesirable governmental interventions or the absence of optimal interventions, a problem that is usually claimed to be most serious in the less-developed countries. The second is that there is assumed to be a central agency of the government whose task is to determine welfare maximizing shadow prices, discount rates, etc. for use in project evaluation throughout the public sector, and occasionally in the private sector as well. This agency has relatively unrestrained powers in the exercize of this task, but essentially no powers to influence the governmental tax policies, etc. that are responsible for, or could eliminate, the distortions in market prices. Consequently, it must treat existing market distortions as given in its welfare maximizing exercize.

The third theme is that the literature attempts to develop "rules" for guiding this agency in its task which consist, ideally, at least, of principles for deriving the optimal set of shadow prices from observable, or potentially observable, data. Finally, there is the theme that this aim is best achieved by relating production in the public sector to international trade. The simplest and most widely accepted result to emerge from this literature is that, given the usual "small country" assumption, the relative shadow prices of commodities traded internationally

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should be set at their relative international (border) prices. This result has been found to hold regardless of the existence of (non-prohibitive) tariffs, government budgetary constraints, or distortions in the markets for non-traded commodities and regardless of the precise form of the welfare function being maximized.[1] There has been much less agreement on the appropriate principles for guiding the shadow pricing of non-traded commodities. Numerous seemingly conflicting rules are to be found.

The present paper attempts to clarify the issues involved by analyzing a particularly simple general equilibrium model, seemingly the simplest model possible which captures the essence of the problems involved. Sections II and III attempt to clarify the relationships between the various shadow pricing rules advocated in the literature and the conditions under which they are correct. The aim is not to derive new benefit-cost rules but to clarify the existing ones within a simple unified treatment. The paper then asks, in Section IV, how the shadow pricing rules derived from this and other, similar, analyses would be applied in practice, particularly when, as much of the literature suggests, the shadow prices obtained are to have wide application within the public sector, which is itself large in many less-developed countries. The question of how sufficient information is to be generated in practice to apply the shadow pricing rules advocated for non-traded commodities has been ignored by most of the literature and proves to raise severe problems. This is so even in seemingly minimal models. In Section V the paper then turns to examine the implications of some alternative shadow pricing procedures which, while not "optimal" in a world of costless information, nevertheless offer greater prospect of being informationally feasible.

## II. DERIVATION OF THE OTPIMAL SHADOW PRICE

### *Details of the Model*

The economy consists of a single consumer and two firms, one "private" and the other "public". There are three commodities. Commodities e and i are traded internationally at prices which are given for the economy concerned, the first being an export good and the second an import good, while commodity n is non-traded. Commodities e and n are consumed domestically, but commodity i is not consumed. It is a fully imported intermediate good, not produced domestically. Commodity e is produced in the public firm, using commodity n as an input, while commodity n is produced in the private firm using commodity i as an input. The consumer's utility function is $U = U(c_e, c_n)$, where $c_e$ and $c_n$ denote the consumption of commodities e and n, respectively.[2] This function is assumed to be quasi-concave and twice differentiable with $U_e, U_n > 0$. The public firm's production function is $x_e = g(x_n)$ where $x_e$ and $x_n$ denote respectively the public firm's output of commodity e and its use of commodity n as an input. The private firm's production function is $y_n = f(y_i)$, where $y_n$ and $y_i$ denote respectively the private firm's output of commodity n and its use of commodity i as an input. The functions g and f are assumed to be twice differentiable with $g', f' > 0$ and $g'', f'' < 0$. The variables $c_e$, $c_n$, $x_e$, $x_n$, $y_e$ and $y_i$ are all constrained to be non-negative.[3]

The international prices of commodities e and i are normalized at unity, so the trade balance constraint for the domestic economy can be written

$$c_e \leq x_e - y_i. \tag{1}$$

Equivalently, the imports of commodity i cannot exceed the net exports of commodity e. There is also a physical balance constraint which applies

to commodity n, namely

$$c_n \leq y_n - x_n. \tag{2}$$

The consumption of commodity n cannot exceed the difference between the private firm's production and the public firm's usage of that commodity. The domestic market prices of commodities e, n and i are denoted $p_e$, $p_n$ and $p_i$, except that units of measurement are chosen such that $p_e \equiv 1$.

The private firm maximizes its profits and the consumer maximizes his utility, each treating market prices parametrically. Assuming interior solutions, as we do throughout this paper, this implies that

$$f'(y_i) = p_i/p_n$$

and

$$U_n/U_e = p_n.$$

Any tax revenue is turned over to the consumer in lump-sum form along with the profits of the private firm and any profits of the public firm. Any losses incurred by the public firm are financed by lump-sum taxes on the consumer. This simplifying assumption avoids complications arising from a government budgetary constraint, but will be relaxed later in the paper.

The public firm attempts to maximize "shadow" profit, using the shadow prices given it by a "project planner", treating these shadow prices parametrically. These shadow prices are denoted $s_e$ and $s_n$ (commodity i is neither an output nor an input of the public firm), except that we normalize again by setting $s_e \equiv 1$. This implies that

$$g'(x_n) = s_n.$$

The project planner's task is to set $s_n$, the shadow price of the non-traded commodity, so as to maximize the consumer's utility. This is the only control variable the project planner possesses; in particular,

he has no control over the government's tax policy and must treat the existence of any distortionary taxes as given. Our concern in this paper is with how he should go about this task.

*Derivation from an Optimization Model*

Consider first the welfare maximization problem in the absence of any tax distortions. This "first-best" problem is simply

$$\max U(c_e, c_n) \text{ subject to (1) and (2).}$$

The first-order conditions for a maximum are

$$U_n/U_e = 1/f' \tag{3}$$

and

$$1/f' = g'. \tag{4}$$

These imply that

$$s_n = p_n \quad \text{and} \quad p_i = 1.$$

We now introduce a tariff on imports of commodity i at the proportional rate t, so that $p_i = 1+t$. No explanation is offered for the existence of this tariff. It is to be regarded as a purely distortionary intervention which must, nevertheless, be taken as given for the purposes of shadow pricing. This assumption is central, because all of the problems discussed in this paper would vanish if this tariff were eliminated. The basic assumption is one of a government with discrete areas of control, where the distortions created by one branch create problems for the welfare-maximizing tasks of another.

We now have

$$U_n/U_e = (1+t)/f', \tag{5}$$

which violates (3). The "second-best" welfare maximization problem is now

$$\max U(c_e, c_n) \text{ subject to (1), (2) and (5).}$$

Deriving the first order conditions for this problem we now obtain the result that

$$s_n = \frac{p_n + t\lambda R_n}{1 + t\lambda R_e} = p_n + t\lambda\left[\frac{R_n - p_n R_e}{1 + t\lambda R_e}\right], \qquad (6)$$

where $\lambda = -1/(R_n f' - R_e + Q)$, $Q = (1+t)f''/(f')^2$ and $R_e = \partial(U_n/U_e)/\partial c_e$, etc. Even in an extraordinarily simple model like the present one, the expression for the optimal shadow price of a non-traded commodity in the presence of a market distortion is surprisingly complicated. It is obvious, simply by inspection of (6), that its informational requirements are substantial.

## III. COMPARATIVE STATIC INTERPRETATION OF THE OPTIMAL SHADOW PRICE

We now consider whether, and in what sense, the optimal shadow price derived above is consistent with the various shadow pricing rules advocated in the literature.

*Market Behavior Interpretation and the "Weighted-Average" Rule*

First, we derive a more interesting, and more useful, form of (6) which substitutes the derivatives of the private firm's supply relation and the consumer's demand relation for the terms $R_e$, $R_n$ and Q in (6). This has the substantial advantage that relationships observable in market behavior are substituted for the unobserved first and second derivatives of production and utility functions. The resulting expression proves, on rearrangement, to be the well-known "weighted-average" formula derived by Harberger (1969 and 1971).

The equation $f'(y_i) = (1+t)/p_n$ must hold for all $p_n$. Differentiating it with respect to $p_n$ is therefore legitimate and gives

$$Q = -1/Y_{in} = -(1+t)/(p_n Y_{nn}), \tag{9}$$

where $Y_{in} \equiv dy_i/dp_n$ and $Y_{nn} \equiv dy_n/dp_n$. Similarly, the equations $R(c_e, c_n) = p_n$, where $R(c_e, c_n)$ denotes the consumer's marginal rate of substitution, $U_n/U_e$, and the budget constraint $c_e + p_n c_n = M$, where M denotes the consumer's lump-sum income, must hold for all $p_n$ and M. Substituting the demand relations $c_e = C_e(p_n, M)$ and $c_n = C_n(p_n, M)$ into these equations, differentiating with respect to $p_n$ and M, and solving for $R_n$ and $R_e$ we obtain[5]

$$R_n - p_n R_e = C_{nn}^{-1} \tag{10}$$

where $C_{nn} \equiv dc_n/dp_n$. Substituting this into (6) gives, on rearranging,

$$s_n = p_n \frac{C_{nn}}{(C_{nn} - Y_{nn})} - \frac{p_n}{(1+t)} \frac{Y_{nn}}{(C_{nn} - Y_{nn})} \tag{11}$$

$$= p_n \left[ \frac{r/(1+t) - 1}{r - 1} \right], \tag{12}$$

where $r \equiv Y_{nn}/C_{nn}$. This is precisely the Harberger "weighted average" formula and is clearly a vastly more useful expression for the optimal shadow price than (6).

The intuitive meaning of (11) is straight-forward and is illustrated in Figure 1. The consumer's demand relation and the private sector supply relation are marked $C_n(p_n)$ and $Y_n(p_n)$, respectively.[6] Aggregate demand is of course the consumer's demand plus public sector demand and the market price is determined by the intersection of the aggregate demand and private sector supply schedules. Consider a one unit increase in public demand for good n, from $x_n^0$ to $x_n^1$. This forces a rise in $p_n$ from $p_n^0$ to $p_n^1$, which causes consumption to fall from $c_n^0$ to $c_n^1$ and production to rise from $y_n^0$ to $y_n^1$. Together, these effects sum to the increased public demand. The marginal social cost of the fall in consumption is indicated by the consumer's willingness to pay, the market price, $p_n$. This accounts for the first term in (11). For a discrete change this gives the left-handed shaded area under the demand relation. The marginal *private* cost of the increase in production is also $p_n$, the good's supply price, but not all ot this is a social cost. Part of it is simply a transfer of tariff revenue to the government induced by the increased imports of good i. The marginal *social* cost is the payment to foreigners for increased imports of good i, namely $dy_i/dy_n = 1/f' = p_n/(1+t)$. This accounts for the second term in (11) and the right-handed shaded area under the schedule $Y_n(p_n/(1+t))$ in Figure 1. This schedule represents the marginal social cost of producing good n which is its marginal private cost, $p_n$, minus the tariff revenue generated per unit of good n produced, $tp_n/(1+t)$. This schedule also represents what the supply relation for good n *would be* in the absence of a distortion in the market for good i.

The optimal shadow price of the non-traded good reflects the marginal social cost of drawing the good into the public sector. This is given by a "weighted average" of the good's market price and marginal social cost of production, the weights reflecting the proportions in which additional public demand is satisfied by a fall in consumption and a rise in production, respectively. These proportions are indicated by the relative slopes of the demand and supply relations.

*The Government Revenue Rule*

This rule focuses on the effect that the public use or production of a good has on total government revenue. Its use in benefit-cost analysis has been advocated by Harberger (1971) and Boadway (1975) and has its origin in a classic paper by Hotelling (1938). It states that the shadow price of a commodity is what we will call its "government revenue effect", consisting of its producer price minus (plus) the effect on total tax revenue of a unit increase in its net use (production) in public projects. It is shown below that this rule is correct, provided that the only distortions present are tax-induced, and provided that the numeraire commodity is shadow priced similarly. In particular, if the numeraire commodity is traded, as in the present case, and is valued at its international price, then the correct version of this rule is that the shadow price of a non-traded commodity is its government revenue effect *relative* to that of the numeraire commodity.

We will show that this rule gives a result identical with (6). To show this it is convenient to differentiate equations (1), (2) and (5) with respect to $x_e$ and $x_n$. This gives the system

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -f' \\ R_e & R_n & Q \end{bmatrix} \begin{bmatrix} \partial c_e/\partial x_e & \partial c_e/\partial x_n \\ \partial c_n/\partial x_e & \partial c_n/\partial x_n \\ \partial y_i/\partial x_e & \partial y_i/\partial x_n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \qquad (13)$$

The effect of changing $x_n$ on total tax revenue is simply $t\partial y_i/\partial x_n$. Now, substituting from the above system, the government revenue effect of commodity n is given by

$$p_n - t\partial y_i/\partial x_n = p_n + t\lambda R_n, \tag{14}$$

where $\lambda$ is defined as in (6). It is obvious, by comparison with (6), that this is only the numerator of the optimal shadow price. But the shadow price of the numeraire commodity, as given by this rule, is $1 + t\partial y_i/\partial x_e$. Substituting again from (13) we obtain

$$\frac{p_n - t\partial y_i/\partial x_n}{1 + t\partial y_i/\partial x_e} = \frac{p_n + t\lambda R_n}{1 + t\lambda R_e}, \tag{15}$$

which is identical with (6).[7]

The intuitive interpretation of this result is best seen from the left-hand side of (15). From the duality properties of non-linear programming models, we known that[8]

$$s_n = -\frac{\partial U/\partial x_n}{\partial U/\partial x_e} \tag{16}$$

The tariff distortion causes too little of good i to be imported. To the extent that public demand for good n forces up the market price, $p_n$, it induces additional private sector production, and hence additional imports of good i. Each additional unit of $y_i$ so imported raises national income by $p_i = 1+t$, but costs only its international price, unity, the net gain being the magnitude of the tax, t. Consequently, $t\partial y_i/\partial x_n$ measures the indirect benefit from increasing $x_n$ and should be subtracted from $p_n$ in forming the numerator of (16). Similarly, increasing $x_e$ generates "foreign exchange" earnings which the consumer will spend partly on consumption of good e and partly on good n. The latter effect pushes up $p_n$ which in turn increases $y_i$, as before. Consequently, $t\partial y_i/\partial x_e$ must be *added* to the "foreign exchange" return from producing a unit of good e in forming the denominator of (16).

The government revenue rule has an important, and previously undeveloped, implication. It implies that, when the correct shadow prices are being employed, any project losses incurred at market prices will at least be matched by the indirect effects of the project on tax revenue.[9] In other words, provided the project makes non-negative profits at shadow prices, no additional taxes or increases in existing tax rates need be introduced to finance the losses the project incurs, if any, at market prices. It should be stressed that this result holds only when the distortions in market prices are tax-induced. It is nevertheless important, since previous authors have thought it necessary, in analyzing this case, to assume that any project losses could be financed by the imposition of non-distorting taxes, and have argued that relaxing this strong assumption would necessitate changes in the shadow prices calculated.[10]

First, consider a small marginal project that is viable at shadow prices. We represent this project by a pair of numbers $(dx_e, dx_n)$ representing an output of good e and input of good n such that

$$dx_e - s_n dx_n \geq 0 \tag{17}$$

Let $T = ty_i$ denote total tax revenue. Then, from (15),

$$dx_e(1 + \partial T/\partial x_e) - dx_n(p_n - \partial T/\partial x_n) \geq 0 \tag{18}$$

and

$$dx_e \frac{\partial T}{\partial x_e} + dx_n \frac{\partial T}{\partial x_n} \geq p_n dx_n - dx_e \tag{19}$$

The right-hand side of (19) is the project losses at market prices and the left-hand side is its indirect effect on tax revenue. The project is (at least) self-financing. If the project exactly breaks even at shadow prices, then both (17) and (19) become strict equalities, and the project is exactly self-financing.

Turning to the large scale application of shadow pricing in the public sector, it is now easy to show that any losses incurred by that sector at market prices must at least be matched by the indirect tax effects of public production. This follows directly from the concavity of $g(\cdot)$. This implies that $x_e - g'(x_n)x_n \gtrless 0$. So $g'(x_n) = s_n$ implies $x_e - s_n x_n \gtrless 0$, and the argument proceeds as before. Similarly, the marginal unit of public production, if just exactly viable at shadow prices, generates enough tax revenue indirectly to match exactly its impact on public sector losses at market prices. There is no need to impose additional taxes to finance project losses, provided that projects are viable at shadow prices. Alternatively, the existence of a government budgetary constraint does not imply the necessity to revise shadow prices, in the case where market price distortions are tax-induced.

*The Final Consumption Rule*

This rule focuses on the effect that public use or production of a good has on the value of final consumption. It appears to have its origin in Meade (1955) and its use is advocated, in general terms, in Dasgupta, Marglin and Sen (1972). The rule states that the shadow price of a commodity is given by the effect of a unit increase in its net production, or a unit reduction in its net use, in the public sector on the value of final consumption at consumer prices. We will call this the commodity's "final consumption effect", and we will see below that this rule is correct, provided, as above, that the numeraire commodity is shadow priced similarly. This result is obtained easily from equation system (13). The final consumption effects of commodities n and e are given by

$$-(\partial c_e/\partial x_n + p_n \partial c_n/\partial x_n) \quad \text{and} \quad \partial c_e/\partial x_e + p_n \partial c_n/\partial x_e$$

Substituting from (13), the ratio of these expressions is identical with (6), a result which is obvious from (16).

*The "Foreign Exchange Equivalent" Rule*

This rule focuses on the connection between the production of non-traded commodities and the domestic country's foreign exchange earnings.[11] Its use is advocated in the influential writings of Little and Mirrlees (1969, 1972a, 1972b and 1974), in Bruno (1962 and 1967), and in virtually all of the domestic resource cost literature, such as Balassa and Schydlowsky (1968), Bruno (1972) and Krueger (1972). It has been applied extensively to benefit-cost analysis and effective resource cost calculations in many countries. In the case of a non-traded input used by a public project and produced elsewhere (say, in the private sector), the rule states that the shadow price of this good is the marginal cost, in terms of traded inputs valued at their international prices and non-traded inputs valued at their respective shadow prices, of supplying the good from this source. In the literature this has come to be called the good's "foreign exchange equivalent".[12]

The shadow prices of the good's non-traded inputs are obtained by similarly breaking them down into their respective inputs, ultimately giving their shadow prices in foreign exchange terms. This, and the existence of primary factors and labour, creates computational difficulties which will nevertheless not arise in the present analysis.[13] The construction of the model is such that the meaning of the "foreign exchange equivalent" rule is simple and unambiguous. There is only a single variable input used in the production of the non-traded commodity and that input is traded. The "foreign exchange equivalent" of the non-traded commodity as given by this rule is simply

$$dy_i/dy_n = 1/f' = p_n/(1+t) \qquad (20)$$

By comparison with (6) and (11) it is obvious that this differs from the optimal shadow price.

This is seen most clearly by reference to equation (11). Increased public sector demand implies a rise in $p_n$. Suppose that consumers do not respond to this price rise. Then $C_{nn} = 0$ and (11) collapses to the foreign exchange equivalent rule. Alternatively, referring to (12),

$$\lim_{r \to \infty} s_n = p_n/(1+t) . \tag{21}$$

The Little-Mirrlees foreign exchange equivalent rule can be seen as the limiting case of the optimal shadow pricing rule for a non-traded commodity where all adjustment in non-traded goods markets to an increased public sector demand occurs on the production side. This will occur, strictly speaking, only if demand is completely inelastic or supply is infinitely elastic.[14]

*The Market Price Rule*

This rule simply evaluates commodities at their market prices, regardless of the existence of market distortions. Its use is recommended by Rudra (1972), Weckstein (1972) and, in the case of non-traded commodities, as an approximation to the shadow prices given by the final consumption rule, by Dasgupta, Marglin and Sen (1972) and Dasgupta (1972).[15] This rule is immediately seen to be the opposite limiting case from the Little-Mirrlees rule, since, from (12),

$$\lim_{r \to 0} s_n = p_n . \tag{22}$$

The market price rule corresponds to the limiting case of the optimal shadow pricing rule where all adjustment to increased public sector demand for a non-traded commodity occurs on the consumption side. Indeed, it is easily seen from (12) that for $t > 0$ and for any specified value of $p_n$ (noting that $r \leq 0$),

$$p_n \geq s_n \geq p_n/(1+t) . \tag{23}$$

Strong criticism of shadow pricing rules which rest, explicitly or implicitly, on *approximations* to "optimal" rules would, if based solely on the kind of theoretical analysis presented so far, be unfair. Though the point is not always made explicitly, many of the authors concerned have clearly viewed the practical problems of attempting to implement "optimal" rules as being prohibitive. Nevertheless, it is fair to say that these writings have typically lacked any systematic discussion of what the optimal rules would amount to, of precisely what the practical problems are that prevent their implementation, or of why the particular approximation rules they recommend are considered superior to other feasible approximations. We now turn to these issues.

## IV. PROBLEMS OF APPLICATION

While the comparative static interpretation of the optimal shadow pricing rules is of interest, it still leaves the central informational questions unanswered. How is sufficient information to be generated in practice to apply these rules; and if the informational problems are probitive, what can be done instead? We now examine these issues with the aid of an extensive set of numerical examples. This serves both to illustrate the nature of the problems involved in shadow pricing and to provide a convenient vehicle for studying the efficacy of alternative means of dealing with them. This is done by exploring the welfare implications of alternative shadow pricing strategies within the context of log-linear production functions and Cobb-Douglas utility functions. Numerical examples of this kind enable a number of interesting conceptual experiments to be performed and these can be quite helpful in obtaining a feeling for the quantitative significance of some of the issues involved. While it would obviously be unscientific to assert generality for the numerical results obtained, examples of this kind can be valuable in showing the kinds of numerical outcomes that emerge when seemingly "reasonable" assumptions are made; it is orders of magnitude and directions of effects, rather than precise numerical results, that are of most interest.

### 1. *The Numerical Examples*

We assume the following functional relationships:

$$g(x_n) = x_n^{\alpha}, \quad f(y_i) = by_i^{\beta}$$

and

$$U(c_e, c_n) = c_e^{\gamma} c_n^{1-\gamma}.$$

Given these functional assumptions, four parameters characterize the state of technology and consumer tastes:[16] $\alpha$, $\beta$, $\gamma$ and b. The parameters $\alpha$, $\beta$ and $\gamma$

are constrained to lie in the interval (0, 1) and $b > 0$. Table 1 presents the complete equilibrium solutions for the model for the specific case $\alpha = \beta = \gamma = \frac{1}{2}$ and $b = 1$. Column (1) presents the solution to the first-best optimization problem characterized by equations (3) and (4). There is, of course, no tariff on commodity i in this case. Column (2) presents to solution to the second-best optimization problem characterized by equations (5) and (6), where the tariff on commodity i is fixed at $t = 1$. This numerical example, $\alpha = \beta = \alpha = \frac{1}{2}$ and $b = t = 1$, will henceforth be referred to as Numerical Example I. For comparison, columns (3) and (4) present the equilibrium solutions when the public sector uses the market price, $p_n$, and the foreign exchange equivalent price, $p_n/(1+t)$, as shadow prices. The solutions represented by the remaining columns of Table 1 will be explained later in the paper.[17]

To examine the degree to which the numerical results obtained reflect the particular parametric assumptions embodied in Numerical Example I we perform extensive parametric variations. The set of parametric values employed will be called Parameter Set A. It consists of three subsets, $A_1$, $A_2$ and $A_3$. Parameter Set $A_1$ has $\alpha$ and $\beta$ independently taking the values (0.1, 0.3, 0.5, 0.7, 0.9) with $\gamma$, b and t held fixed at the values given in Numerical Example I. Parameter Set $A_2$ has $\alpha$ and $\beta$ constrained to be equal and this common value and $\gamma$ independently take the values (0.1, 0.3, 0.5, 0.7, 0.9), while b and t are fixed at unity, as before. Parameter Set $A_3$ has $\alpha = \beta = \gamma = \frac{1}{2}$ as in Numerical Example I and b and t take the values (0.5, 0.75, 1, 1.25, 1.5) and (0.25, 0.5, 1, 1.5, 2), independently. Each of Parameter Sets $A_1$, $A_2$ and $A_3$ has 25 elements, each element being a quintuple $(\alpha, \beta, \gamma, b, t)$. The union of these sets is Parameter Set A and their intersection is Numerical Example I.

## 2. *Application to Marginal Projects*

It is important to distinguish between the informational and adjustment problems of shadow pricing when benefit-cost analysis is seen, on the one hand, simply as a way of evaluating small marginal projects on an infrequent basis, and on the other as a tool for widespread application within the public sector.[18] The implications of benefit-cost analysis for overall resource allocation are, in the first case, small by definition, but in the second case they are potentially very considerable. While we have seen that the same shadow pricing "rules" apply in the two cases, the problems encountered in their application are far greater in the second case than in the first.

Suppose that initially the public sector is basing its production decisions on market prices. The optimal shadow price is now to be estimated using the rule given by (12) for use in a small marginal project. Let $r^o$ denote the correct value of r at this point, and $\hat{r}$ denote the estimate of $r^o$ which is in fact fed into (12). Obviously, errors will, in practice, be made in $\hat{r}$. Indeed, obtaining greater precision in the estimation of $r^o$ will entail costs and it will not be rational to invest in this information gathering activity beyond the point at which the expected marginal benefits of the information gathered equal its marginal costs. This could well mean that *no* resources should be invested in collecting information for the estimation of $r^o$, but in any case it is clear that it would virtually never be optimal, even if it were possible, to eliminate all error in the value of $\hat{r}$ which is in fact fed into (12). How sensitive is the resulting shadow price to errors in $\hat{r}$?

Consider the elasticity of $s_n$ to $\hat{r}$, evaluated at the point $\hat{r} = r^o$. In the case of Numerical Example I, this elasticity is 0.14. A ten per

cent error in $\hat{r}$ gives a 1.4 per cent error in $s_n$. Allowing the parameters $\alpha$, $\beta$, $\gamma$, b and t to vary across Parameter Set A gives values of this elasticity ranging from 0.0005 to 0.17. We must conclude that, for this class of example at least, the estimated value of $s_n$ is not particularly sensitive to errors in $\hat{r}$ in the case of a small marginal project. The reason for this is clear on inspection of (12). Since r appears in both the numerator and denominator with the same sign, changing the value of r has only a small effect on the overall expression.

Suppose now that the true value of r is completely unknown and that no estimation of it is feasible. We have already seen that the shadow price given by (12) is bounded, on the one hand by the market price, $p_n$, and on the other by the foreign exchange equivalent, $p_n/(1+t)$. Can we say which of these is likely to be the better approximation? We investigate this by computing[19]

$$m^o = (s_n^1 - p_n^o)/(p_n^o - p_n^o/(1+t)). \qquad (21)$$

The superscript "zero" denotes evaluation of the variable concerned at the solution where the public sector is initially shadow pricing commodity n at its market price. Clearly $m^o$ potentially takes value between zero and unity. The closer $m^o$ is to unity, the better is the foreign exchange equivalent as an approximation of $s_n^1$, while $m^o$ close to zero indicates that the market price is a close approximation. If $m^o = 0.5$, then $s_n^1$ is midway between the two. It is easily verified that this case corresponds to $r^o = -1$. In the case of Numerical Example I, $m^o = 0.77$, but performing parametric variations over Parameter Set A we find values of $m^o$ ranging from 0.16 to 0.98. *Either* the market price *or* the foreign exchange equivalent can be the better approximation to the appropriate shadow price, and no broad generalizations could conceivably be justified. There is really no alternative to estimating the value of $r^o$.

*3. Problems of Large Scale Application*

We now consider the application of the shadow pricing rule given by (12) as an instrument for moving the economy from some non-optimal position to the second-best optimum. For simplicity we will suppose the initial position to be one where the public sector is shadow pricing commodity n at its market price. In the case of Numerical Example I, this initial solution, and the solution aimed for, are described in columns (3) and (2) of Table 1, respectively. We can think of this occurring either in a single step or, more plausibly, iteratively. To move from the initial position to the second-best optimum in a single step it is necessary to estimate the right hand side of (12), not at its currently observable value, but at the value it takes at the second-best optimum. We will denote values at the latter solution by the superscript (*).

The initial value of $p_n$, $p_n^o$, is directly observable and the initial value of r, $r^o$, can in principle be estimated. But the values of these variables at the solution aimed for, $p_n^*$ and r*, are what must be fed into (12) to move directly to that solution, and these typically will differ from their initial values. For example, in the case of Numerical Example I, the values of $s_n$ at the initial position and at the second-best optimum are $s_n^o = 0.8206$ and $s_n^* = 1.0075$, respectively.[20] Obviously, the empirical determination of $s_n^*$ is a sizeable task. In practice, errors will be made; indeed, it is difficult to avoid the view that in practice estimates of $s_n^*$ would be based largely on guesswork. Denote an estimate of $s_n^*$ by $\hat{s}_n^*$. It is clear from (12) that $s_n^o < p_n^o$ and $s_n^* < p_n^*$. Furthermore, on *a priori* grounds, $s_n^* < p_n^o$, so for values of $\hat{s}_n^*$ such that $s_n^* \leq \hat{s}_n^* \leq p_n^o$ welfare will at least not be reduced relative to the initial

position. The danger is of choosing $\hat{s}_n^* < s_n^*$. This leads to "overshooting", $x_n > x_n^*$. How sensitive is the potential welfare gain from shadow pricing to errors of this sort?

Consider the value of $\hat{s}_n^*$ such that $\hat{s}_n^* < s_n^*$ and the welfare level obtained from the use of this shadow price is the same as that obtained from the use of the initial market price, $p_n^o$. Call this value $\bar{s}_n^o$. Then for $\hat{s}_n^* < \bar{s}_n^o$, welfare is reduced relative to the use of the market price. How large an error in $\hat{s}_n^*$ is required for this to occur? In the case of Numerical Example I, $\bar{s}_n^o$ corresponds to a 20% underestimate of $s_n^*$. Turning to Parameter Sets $A_1$, $A_2$ and $A_3$, the percentage errors in $\hat{s}_n^*$ required to reduce welfare relative to the initial position fall in the intervals (1, 40), (5, 28) and (8,25), respectively. Seemingly very small errors in the estimation of the optimal shadow price can lead to welfare outcomes that are worse than the use of unadjusted market prices. It is not good enough to say that $s_n^*$ can be estimated "more or less". In this class of examples, at least, a relatively high degree of precision in the estimation of the optimal shadow price is required to support the presumption that its use will raise welfare rather than reduce it.

The informational problems of moving to the second-best optimum in a single step are severe. It seems almost inevitable that the use of shadow pricing rules to achieve the second-best optimum would have to proceed iteratively, using only currently observable data at each step. The most obvious iterative process is the following. Initially, $s_n^o = p_n^o$. We estimate the value of $r^o$ at this point and from (12) compute $s_n^1$ using these data. This causes $p_n$ and r to adjust to the values $p_n^1$ and $r^1$. We re-estimate $s_n$ from (12) giving $s_n^2$, etc. So, assuming no errors are made,

$$s_n^{\tau+1} = p_n^{\tau}\left[\frac{r^{\tau}/(1+t)-1}{r^{\tau}-1}\right], \quad \tau = 0, 1, \ldots \qquad (24)$$

where $\tau$ denotes planning time. Aside from the obvious possibility of

error at each step there is the further problem that, even if no errors are made at any single step, the process need not converge. This process is analyzed in Warr (1978), for a somewhat different model, but since the ideas involved are similar, the analysis need not be repeated. The question is whether non-convergence is a problem in this model.

Convergence occurs in Numerical Example I, but of the fifty parametric combinations contained in Parameter Sets $A_1$ and $A_2$, non-convergence occurs in nine cases. It does not occur in $A_3$. The point is that if the optimum is to be approached iteratively, as above, non-convergence is a practical possibility that cannot be dismissed. In such cases, even though no errors are made in the iterative application of the rule given by (12), the adjustment process this generates does not lead the economy towards, but further away from, the solution aimed for, reducing welfare at each step. Of course this problem can also occur with other, "non-optimal", shadow pricing rules, as well. The essence of the difficulty is that information on the right hand side of (12) flows to project planners on a discontinuous basis. If continuous and instantaneous adjustment of shadow prices were possible the problem discussed here would not arise; but this is obviously impracticable.

Finally, quite aside from the possibility of introducing errors into the process and the possibility of non-convergence, there remains the obvious fact that iterative adjustment processes take time and that they involve adjustment costs. Provided the process is convergent, the optimum is approached, not directly, but by alternating iteratively around it, the iterations becoming successively closer. Obviously, substantial resource reallocations must occur over time, and these are costly.[21] The welfare gains ultimately achieved, discounted to the present, must be compared

with the discounted adjustment costs of reaching that solution, along with the opportunity costs of the skilled manpower etc. required for such an exercize. It becomes less and less clear that this is an activity that makes practical sense. We now turn to examine some possible alternative shadow pricing procedures which avoid some of these problems.

## V. ALTERNATIVE APPROACHES TO SHADOW PRICING

We now consider some alternative shadow pricing procedures, all of which have been advocated, in one form or another, in the benefit-cost analysis literature. First, we consider simply the first iteration of the iterative process described above. Second, we suppose that an "adjustment factor" is estimated at the initial position and then applied as a constant adjustment to the market price. Next, it is assumed that shadow prices are estimated from an economy-wide programming model, and finally we compare the use of the unadjusted market price with the use of the Little-Mirrlees foreign exchange equivalent shadow price.

### *1. Single Iteration Results*

Suppose the rule given by (12) is applied by measuring the numerical magnitude of the right hand side of that expression at some initial position and then applying this shadow price throughout the public sector, rather than simply for a "small" marginal project. We will assume, as before, that the "initial position" is one in which the market price of commodity n is being used as its shadow price. Consequently, this procedure amounts simply to the first iteration of the iterative mechanism described above. What is the welfare outcome from this procedure?

Consider the change in welfare resulting from the application of this shadow price, rather than the initial market price. We present these welfare effects as a percentage of the welfare gain to be achieved from moving from the same initial position to the second-best optimum. In the case of Numerical Example I this percentage welfare effect is 12.8. The equilibrium solution resulting is presented in column (5) of Table 1. Turning to Parameter Sets $A_1$, $A_2$ and $A_3$, the percentage welfare effects fall in the intervals (-2019, 99.9), (-386, 98.3) and (9.3, 26.7).

Obviously, a negative percentage welfare effect indicates that the single iteration procedure reduces welfare. Of the 25 parametric combinations considered in each of Parameter Sets $A_1$ and $A_2$, welfare falls in 11 and 13 cases, respectively. Welfare rises in each case of Parameter Set $A_3$. A surprising feature of these results is that the welfare effects from a single iteration of applying the rule given by (12) can be negative, even though the repeated application of this iterative process leads ultimately to convergence on the second-best optimum. Clearly, the once-and-for-all application on a large scale of the "optimal" shadow pricing rule using currently observable data may be welfare increasing or substantially welfare reducing, but is is not a procedure that can be recommended with any confidence.

2. *Estimating a Constant Adjustment Factor*

A second obvious alternative is to estimate the bracketed term in (12) at an initial position and thereafter to apply this term as a constant adjustment factor to the (currently observed) market price. Let the initial value of the bracketed term be $K^o$. Then this shadow pricing rule is simply $s_n = p_n K^o$. Obviously, since the adjustment factor is measured only once, the informational problems of applying this rule are substantially less than those encounted with the "optimal" rule above. The equilibrium this procedure leads to is not the second-best optimum since, in general, $K^o \neq K^*$. Nevertheless, in this class of examples the welfare effects of applying this rule are reasonably impressive.

As before, we compare the welfare effects from moving from the initial position (use of the market price as a shadow price) to the equilibrium resulting from the application of this rule with those

of moving to the second-best optimum as above. In Numerical Example I, the percentage welfare effect is 99.2. (See column (6) of Table 1.) In Parameter Sets $A_1$, $A_2$ and $A_3$ the percentage welfare effects fall in the intervals (96.5, 99.93), (94.3, 99.98) and (97.7, 99.5), respectively. In every case, the welfare effects are positive and superior to those of the single iteration application of the optimal rule, and in most cases, substantially so. What this means is that $K^o$ and $K^*$ are reasonably close in this class of examples.[22] Furthermore, as we have seen, the estimated value of $K^o$ is relatively insensitive to errors in the estimation of $r^o$. The generality of this result seems worthy of further exploration, but the procedure of estimating a constant adjustment factor to apply to the market price of a non-traded good seems promising for practical purposes.

*3. Shadow Prices from Programming Models*

Another, quite different, shadow pricing procedure recommended in the literature, is to construct a non-linear programming model of the economy and to compute the prices associated with a first-best optimum.[23] These prices are then used as shadow prices for benefit-cost analysis, even though there are in fact fixed market distortions. This procedure avoids the substantial programming problem of incorporating market distortions satisfactorily into the model, but its use rests on the assumption that the shadow prices associated with first-best and second-best optima are numerically similar, a proposition that is by no means obvious when market distortions are significant. Nevertheless, in this class of examples at least, this assumption is a very good one. The shadow prices associated with first-best and second-best optima are not identical, but they are very close.

Denoting the shadow price associated with the first-best optimum by $s_n^{**}$ we consider the ratio $(s_n^{**} - s_n^*)/(s_n^* - p_n^o)$. In Numerical Example I this ratio is 0.022, while in Parameter Sets $A_1$, $A_2$ and $A_3$ it falls in the intervals (0.00008, 0.15), (0.00001, 0.25) and (0.007, 0.04) respectively. In almost all cases, $s_n^*$ and $s_n^{**}$ are very close. Comparing the welfare effects of applying this shadow pricing procedure with those of applying the optimal (second-best) shadow price as above, the percentage welfare effect in Numerical Example I is 99.7. (See column (7) of Table 1.) Those occurring in Parameter Sets $A_1$, $A_2$ and $A_3$ fall in the intervals (96.2, 99.99), (91.5, 99.99) and (99.4, 99.86), respectively. In every case, almost all of the welfare gains that are achievable from the use of the optimal second-best shadow price can be achieved with this procedure.

These impressive results assume, of course, that the non-linear programming model from which shadow prices are computed incorporates the *correct* values of the parameters of the model. In practice, some errors would obviously be made. How sensitive are the welfare gains to be achieved from applying this procedure to errors in the parametric assumptions underlying the computed shadow prices? We focus on the parameter b. Let the true value of b be $\bar{b}$ and the estimated value of $\bar{b}$, which is actually fed into the non-linear programming model, be $\hat{b}$. We will assume that all the other parameters of the model, $\alpha$, $\beta$ and $\gamma$ are estimated without error. The question is how much $\hat{b}$ must differ from $\bar{b}$ for the welfare gains potentially to be achieved from applying this shadow pricing procedure, starting from the initial use of market prices, to be eroded.

Either an underestimate or an overestimate of $\bar{b}$ can give this result. In Numerical Example I, where, of course, $\bar{b} = 1$, values of

$\hat{b}$ of 0.65 and 1.37 lead to the same welfare outcome as the use of market prices in the public sector. The percentage errors that these values represent are not particularly great. Obviously, errors in excess of these lead to welfare losses. Turning to Parameter Sets $A_1$, $A_2$ and $A_3$, the percentage overestimates of $\bar{b}$ that lead to the same welfare outcome as the use of market prices fall in the intervals (8, 65), (4, 43) and (13, 51), respectively. The percentage underestimates of $\bar{b}$ giving the same outcome are similar. The point is that quite small errors in $\hat{b}$ can give welfare outcomes that are worse than the use of market prices, even though a large proportion of the welfare gains potentially obtainable from shadow pricing can be achieved using the correct parametric value. These errors are in many cases well within the accepted tolerance limits of econometric estimation. It can hardly be assumed, then, that the shadow prices obtained in practice from programming models will be welfare-increasing. The resulting shadow prices, and the welfare effects following from their use, can be highly sensitive to errors in the parametric estimates that are fed into the programming models.

### 4. *Market Price Versus Foreign Exchange Equivalent*

Finally, we ask whether it is possible to rank the welfare outcomes resulting from the use of market prices as shadow prices for non-traded commodities on the one hand, and the use of the Little and Mirrlees foreign exchange equivalent shadow prices on the other. In the present model, neither procedure presents any informational difficulties so this question is certainly of interest in a world in which the informational problems of applying the optimal rules are considered prohibitive. Does one of these rules dominate the other? Unfortunately, neither is dominant.

Consider the change in welfare resulting from persuance of the foreign exchange equivalent rule until equilibrium is achieved, starting from the initial use of the market price. As before, we will compare this welfare effect with the welfare gain from adoption of the optimal (second-best) shadow price. In Numerical Example I this percentage welfare effect is 88. In Parameter Sets $A_1$, $A_2$ and $A_3$, however, these percentage welfare effects fall in the intervals (-3703, 99.98), (-5429, 99.99) and (83.4, 92.1), respectively. Either the market price rule or the foreign exchange equivalent rule may be vastly superior to the other. Of the 25 parametric cases in each of Parameter Sets $A_1$, $A_2$ and $A_3$, the market price rule is superior in 7, 7 and zero cases, respectively. The important point is that no overall generalizations are possible as to which rule is superior.

## VI. SUMMARY AND CONCLUSION

The literature on benefit-cost analysis abounds with "rules" for the shadow pricing of non-traded commodities. This paper has attempted to explore the issues involved within the context of a simple general equilibrium model illustrated by extensive numerical examples. It is argued that while several of the rules advocated prove to be equivalent and correct, the most operationally useful of these, within the context of the simple model being analysed, is due to Harberger. When shadow pricing is being applied widely throughout a large public sector, however, which numerous authors (not including Harberger) clearly intend, its informational problems are greatly compounded. The data necessary for the estimation of the optimal shadow prices are not (locally) observable and the welfare gains potentially obtainable from the use of the correct shadow prices can be eroded by quite small errors in the shadow prices estimated.

The efficacy of alternative means of dealing with these problems are explored in the paper. Two of these, the estimation of constant adjustment factors to be applied to market prices and the estimation of shadow prices from "first-best" non-linear programming models are shown to have desirable properties within a broad class of numerical examples. Nevertheless, the welfare gains potentially obtainable from the latter exercize are shown to be quite sensitive to errors in the parametric assumptions underlying the programming exercize. Two other shadow pricing rules commonly advocated for non-traded commodities, the use of unadjusted market prices and the use of "foreign exchange equivalent" shadow prices, are shown to be incorrect, whether these shadow prices are to be used on a small or a large scale. Furthermore, it is shown to be impossible to generalize as to which of these is likely to be the better approximation to the optimal shadow price.

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FOOTNOTES

* This paper has benefited from the author's discussions with W.M. Corden and R.M. Parish and the comments of an anonymous referee, who are not responsible for the views presented or any errors. Portions of the research were conducted while the author was a Visiting Fellow, Research School of Social Sciences, Australian National University. Computational assistance was received from Janet Atkins and Edgar Wilson.

1. Much of the credit for this important result must be assigned to the pioneering work of Little and Mirrlees (1969). See also Joshi (1972), Corden (1974), Dasgupta and Stiglitz (1974), Findlay and Wellisz (1976), Warr (1977b) and Srinivasan and Bhagwati (1978). An important exception to the rule is the case of binding quantitative restrictions. See Warr (1977a).

2. There may well be other factors of production used in both firms, but these factors are assumed to be specific to the firm concerned and immobile between firms. Hence, they will not affect the analysis of this paper. Also, the single consumer's utility function may be interpreted as a social utility function where the individual consumers have identical homothetic preference maps. In this case U must also be homothetic

3. Some hypothetical names for these commodities may be helpful. Goods e, n and i may be thought of as "cheese", "milk" and "feed grains", respectively Milk is produced using imported feed grains (and other specific factors) in the private firm. The public firm is a cheese factory producing that good for export using milk as an input. Milk is non-traded due to its high transport costs and both milk and cheese are consumed domestically.

4. For convenience, the total derivative notation is used in this discussion, but the partial derivative notation would be equally correct.

5. Our assumption that the single consumer is the sole income recipient is important here, since $dM/dp_n = c_n$ . So the slope of the demand relation, $C_{nn} = dc_n/dp_n = \partial C_n(p_n,M)/\partial p_n + c_n \partial C_n(p_n,M)/\partial M$ . This is the slope of the *income compensated* demand function. Relaxing our assumption of a single consumer and allowing different income recipients to have different tastes would complicate this interpretation.

6. Note that since all prices other than $p_n$, namely $p_i$ and $p_e$ are *in fact* fixed in this model, the usual partial equilibrium *ceteris paribus* assumption is unnecessary, and hence Figure 1 depicts a general equilibrium analysis. Figure 1 owes much to the author's discussions with R.M. Parish.

7. The denominator of (15) may be interpreted as the shadow price of foreign exchange in units of domestic currency and the numerator as the "shadow price" of commodity n, if one wishes, and many authors proceed in this way. But it is then necessary to compute two shadow prices, rather than one. This is inconvenient because both expressions are more complex than (15), having a common complex denominator. It is simpler, and sufficient, to take their ratio as in (15).

8. More precisely, introducing the variables $v_n$ and $v_e$ such that $c_n = y_n - x_n + v_n$ and $c_e = x_e - y_i + v_e$, then $s_n = (\partial U/\partial v_n)/(\partial U/\partial v_e)$.

9. This discussion has benefited greatly from conversations with W.M. Corden.

10. See, for example, Srinivasan and Bhagwati (1978, p. 114). This issue is also discussed in Corden (1974, pp. 390-392) and in Dasgupta and Stiglitz (1974, pp. 28-29). In Warr (1977b) it is shown that the existence of a government budgetary constraint does not affect the shadow pricing of traded commodities subject to tax distortions. The present discussion extends that result.

11. The use of the words "foreign exchange" is, strictly speaking, inappropriate in models in which money is not present. Nevertheless, this has become common usage and does little harm.

12. In Findlay and Wellisz (1976), Srinivasan and Bhagwati (1978) and Warr (1977a and 1978) it is shown that the "foreign exchange equivalent" rule, appropriately interpreted, is correct for the valuation of a non-traded factor of production. In all these models the set of consumed goods is a subset of the set of internationally traded goods, so that increasing "foreign exchange" earnings is equivalent to an outward shift in the consumption possibility set. Intuitively, the result presented here shows that when there are non-traded consumption goods, this equivalence breaks down.

13. See Dasgupta (1972) for a useful discussion of this.

14. This point also has implications for the "domestic resource cost" literature which also values non-traded commodities by breaking them down into their inputs by input-output methods, assuming adjustment to occur solely on the supply side.

15. Nevertheless, this is virtually the *only* instance in which the Rudra-Weckstein and Dasgupta-Marglin-Sen recommendations on shadow pricing coincide.

16. The choice of a linear homogeneous utility function has the added advantage that it also serves as a true quantity index in consumption space. See Lloyd (1975 and 1978).

17. In the last row of Table 1 the utility outcomes of the various shadow pricing strategies are expressed as indices, denoted W, with free trade at 100.

18. The latter is the clear intention of several authors, including those of the two most influential studies, Little and Mirrlees (1969 and 1974) and Dasgupta, Marglin and Sen (1972). These authors envisage widespread application of benefit cost analysis in a large public production sector and sometimes, through the control of government approvals, in pricate sector projects as well.

19. Initially, $s_n^o = p_n^o$. A new value of $s_n$ is then calculated from (12) using $p_n^o$ and $r^o$ as data. To distinguish it from $s_n^o$ we denote this value in (21) by $s_n^1$.

20. In this case $p_n^o = 1.3389$, $r^o = -3.4289$, $p_n^* = 1.7143$ and $r^* = -4.7015$.

21. In Warr (1978) it is shown that a "damped" adjustment of shadow prices can always be devised which will convert non-convergent iterative processes into convergent ones. It is clear that damped adjustment can also reduce the adjustment costs occurring in convergent iterative processes.

22. In Numerical Example I, $K^o = 0.6139$, and $K^* = 0.5877$.

23. See, for example, Bacha and Taylor (1971) and the references cited there.

TABLE 1: SOLUTIONS FOR NUMERICAL EXAMPLE 1

| Solution | First-best | Second-best | Market price | Foreign exchange equivalent | Single iteration | Constant adjustment factor | First-best price |
|---|---|---|---|---|---|---|---|
| $x_e$ | .5000 | .4963 | .3735 | .5586 | .6093 | .4856 | 0.5000 |
| $x_n$ | .2500 | .2463 | .1395 | .2901 | .3113 | .2358 | 0.2500 |
| $y_n$ | .5000 | .4286 | .3347 | .4641 | .5278 | .4200 | .4316 |
| $y_i$ | .2500 | .1837 | .1120 | .2154 | .2786 | .1764 | .1863 |
| $c_e$ | .2500 | .3126 | .2615 | .3232 | .3307 | .3092 | .3137 |
| $c_n$ | .2500 | .1823 | .1952 | .1740 | .1565 | .1842 | .1816 |
| $p_i$ | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| $p_n$ | 1 | 1.7144 | 1.3385 | 1.8565 | 2.1114 | 1.6799 | 1.7266 |
| $s_n$ | 1 | 1.0074 | 1.3386 | .9283 | .8206 | 1.0297 | 1 |
| U | .25000 | .23872 | .22592 | .23717 | .22756 | .23862 | .23869 |
| W | 100 | 95.488 | 90.368 | 94.868 | 91.024 | 95.448 | 95.476 |
| Column number | (1) | (2) | (3) | (4) | (5) | (6) | (7) |

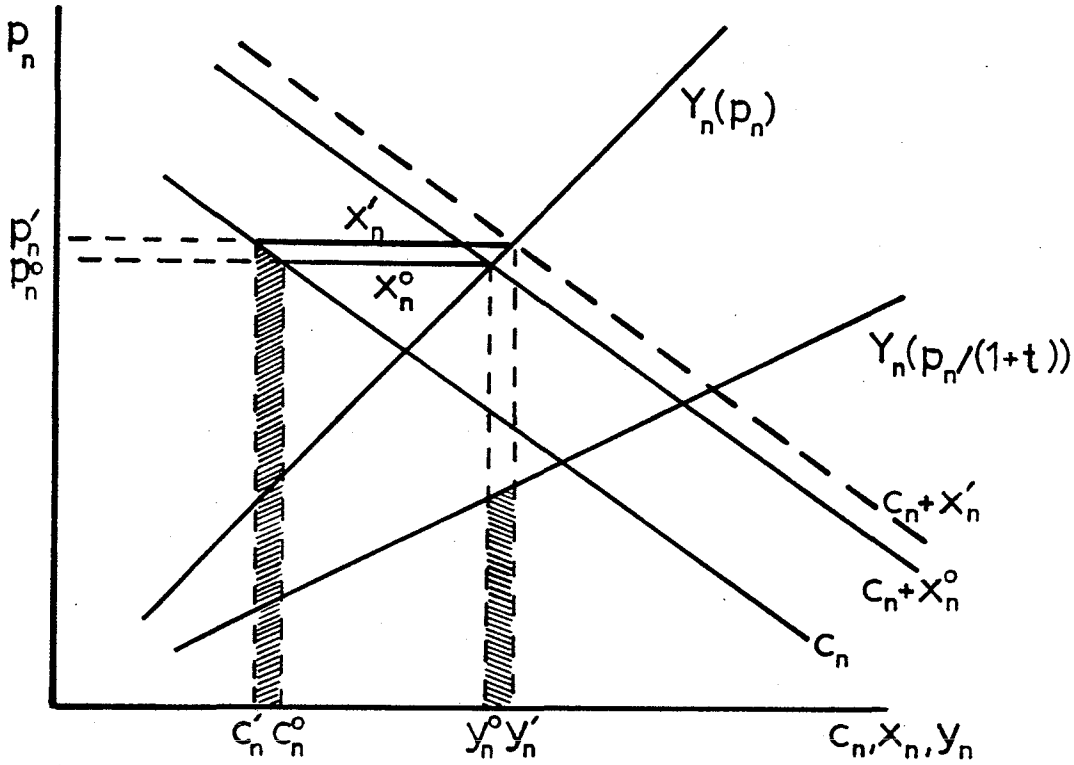


FIGURE 1