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EXCHANGE RATE BEHAVIOR WITH CURRENCY INCONVERTIBILITY

Jorge Braga de Macedo September 1979

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EXCHANGE RATE BEHAVIOR WITH CURRENCY INCONVERTIBILITY

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*Lecturer, Department of Economics and Economic Growth Center, Yale University. This paper constitutes Essay II of my Ph.D. dissertation. I am grateful to the members of my committee, in particular Pentti Kouri, for encouragement with respect to the research described in this paper. I also benefitted from the advise of Paul Krugman and the discussion in a seminar at Princeton. I am further indebted to Mike Porter for financial assistance at an early stage. Errors are my own.

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1 Effects of Various Disturbances

Introduction and Summary

This paper explores the consequences of currency inconvertibility for exchange rate and balance of payments policies. It can be said that, in the same way that the theory of trade restrictions has not been as fully worked out as the theory of "free" international trade, the theory of exchange rate behavior with inconvertibility has not been given the same attention as the theory of "freely" flexible exchange rates. The two problems are indeed quite similar since the responsibility to convert the national currency into foreign exchange is a restriction on international trade in much the same way as a direct trade restriction. Furthermore, currency inconvertibility also has a restrictive impact on trade in assets, which is not the case for direct trade restrictions. In fact, the meaning of convertibility in the IMF Articles of Agreement is confined to "restrictions on the making of payments and transfers for current international transactions" (Art. VIII, Section 2) and to date, only 46 of the 134 member countries have accepted to refrain from imposing such restrictions and still prevail themselves of the transitional status allowed by Article XIV. Acceptance

See Annual Report 1978, p. 109 and Annual Report on Exchange Restrictions, p. 422-426. The countries include Western Europe, the Middle East and most of Latin America.

of the Article VIII obligations, however, does not imply that the country cannot directly restrict trade since the restrictions are only lifted for authorized current account transactions.

The development of private international financial intermediation has led to an unprecedented growth of trade in assets taking place not only amongst industrialized countries but also spreading to semi-industrialized and even some less developed countries. This combination has provided incentives for individuals using an inconvertible currency in domestic transactions to attempt at building up foreign exchange balances, so that they can diversify their portfolios across assets denominated in different currencies. In response, many governments have attempted to recapture foreign exchange by offering special advantages to certain types of transactions, namely the ones where evasion of exchange controls would be easier, like tourist services and migrant's remittances. Nevertheless, in many countries with inconvertible currencies, "black" markets for foreign exchange have developed to the point that the relative price between domestic and foreign money that is determined in these markets can provide relevant information about the balance of payments problem of the country in question.

The importance of illegal transactions in international trade and payments is of course not new. The Italian criminologist Beccaria wrote a condition for profitable smuggling at about the same time as Hume wrote about the specie flow mechanism. Even though Beccaria's concern was the comparison between lost government revenue from the tariff and increased cost of supervision, it can be adapted to an inconvertible currency system by saying that smuggling

See Mikesell (1947) for an early discussion of the relation between trade and exchange restrictions.

The work is quoted and discussed in Bhagwati-Hansen (1973), footnote 1.

is profitable when the black market premium is lower than the tariff.Despite these ancient analytic origins and the apparent spread of the phenomenon, smuggling and related features of international trade was only recently analyzed from a welfare viewpoint in the framework of the conventional 2x2x2 trade model. Empirical work on the subject by Cooper pointed out the positive relationship between the amount of smuggling and the level of the tariff, whereas the literature on the detection of smuggling has pointed out methods to use partner country data to detect underinvoicing of exports or of imports, capital flight, etc, together with analyzing some of the consequences on the balance of payments.

The literature on black markets emerged in the late 40's in the context of the effects of price controls. Using the conventional graphics of demand and supply, Boulding pointed to the possibility that the black market price could be below the equilibrium price, namely when penalties are mostly directed to buyers.

An analysis of the black market for foreign exchange linking the early contributions to the illegal transactions literature referred to above was recently carried out by Sheikh (1976) whereas black market rates have been used by Blejer (1978) in monetarist models of exchange rate determination and by Giddy (1978) in Box-Jenkins efficiency tests.

There has, therefore, been no attempt to explicitly apply the "modern" theory of exchange rate determination emphasizing the importance of asset

¹ See Cooper (1974)

²See Morgenstern (1950), Bhagwati (1964) and (1967) and Bhagwati-Krueker-Wibulswaski (1974).

See Bhagwati (1967)

See Boulding (1947). Another useful reference is Michaely (1954).

markets to inconvertible currencies. Even the monumental work on trade and exchange liberalization directed by Bhagwati (1978) and Krueger (1978) implicitly take the traditional view of exchange rate determination by the equality of flow supply and demand for foreign exchange either in the official or in the black market. In this paper a stock view of the determination of the black market exchange rate is linked to a flow view of the official rate under various official exchange rate regimes.

The paper is organized into three Sections and two Appendices. After a brief review of the black market literature in Section 1, the analysis proceeds from a simple dynamic partial equilibrium model of the two markets in Section 2 to a general equilibrium model of the determination of the two rates and a brief conclusion in Section 3. Appendix 1 derives the stability condition of the partial equilibrium model in terms of the supply and demand of exports and imports and Appendix 2 extends the results of the several equilibrium model to an economy with non-traded goods.

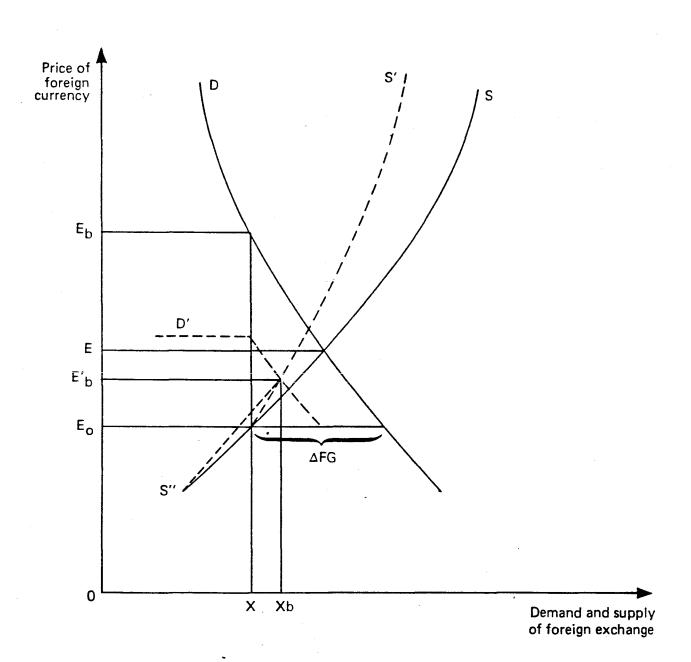
⁵Some of the recent literature on dual exchange rates has emphasized asset markets but ignores the linkages between the two markets. See Flood (1978). The linkage was considered by Fleming (1968) and (1974), but taking current accounts as given. Other useful references are Lanyi (1975) and Swohoda (1974).

- I. Black markets for foreign exchange: the traditional and monetarist approaches.
- 1. The graphic analysis of the black market for foreign exchange is essentially based on the "textbook" or BRM view of the foreign exchange market first advanced by Bickerdicke (1920), Robinson (1937), Metzler (1949), and Machlup (1949) where the exchange rate equates flow supply and flow demand for foreign exchange and such schedules are derived from the demand and supply of exports and imports respectively together with the Boulding (1947) and Michaely (1954) analysis of black markets.

In Figure 1 S and D represent respectively unrestricted flow supply and demand of funds to the foreign exchange market and their intersection E determines the unrestricted equilibrium price of foreign currency in terms of domestic currency. If the official price is set at E_0 < E the authorities would have to sell foreign exchange at the rate ΔFG per unit of time. If they do not, exporters would resell OX in the black market at price E_b^{max} if the threat of penalties is ignored. Penalties on sellers would shift S to S' and on buyers would shift D to D' thus establishing a rate E_b' such that $E_0^{<}E_b'^{<}E$. The quantity bought and sold in the black market is XX_b , more than in the earlier situation because of the new supply of funds but less than in the unrestricted case.

It is of course unlikely that the shift from official supply to black market supply would occur only for the foreign exchange supplied at a price higher than ${\bf E}_0$. Boulding thus discussed the "encroachment" of the black market or the official market which would lead to the supply schedule like S"S'.

FIGURE 1
BOULDING'S BLACK MARKET

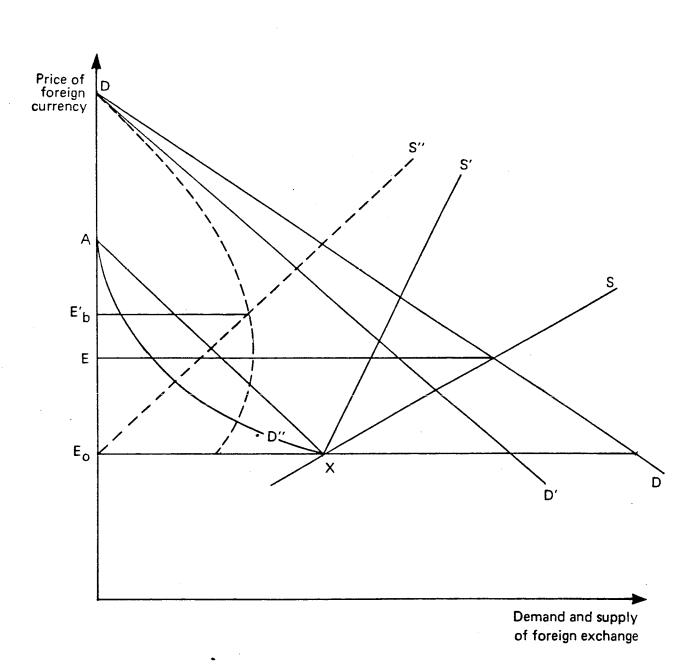


This analysis and the conclusion that $E_b^{\prime} < E$ --was challenged by Michaely who based the supply and demand of black market foreign exchange on the decreasing availability of official supply as the black market premium increases, which, for neutral rationing of official demand, will imply that the demand for foreign exchange is positive in a range and therefore that the black market price is higher than the equilibrium price. In Figure 2the supply of foreign exchange to the black market E_0s'' is the horizontal difference between a downward sloping availability curve AX and the Boulding supply XS'. The demand curve DD" is the difference between the rationed demand curved AX and the Boulding demand DD'. It is upward sloping initially because of the dominance of the effect of the decreased supply. Keeping essentially the same framework, Sheikh (1976) has shown that with perfect resaling for official foreign exchange $E_h^{\,\prime}$ = $E_h^{\,\prime}$ which was only a limiting case in the analysis of Michaely. Even though Sheikh's analysis is carried out for the foreign exchange market, by incorporating a demand for capital flows independent of the exchange rate it reveals the limitation of the flow approach to exchange rate determination more than it adds to the early contribution.

2. Another strand in the black market literature is basically empirical and of monetarist persuasion.

¹⁰ther references can be found in Michaely (1954). In minor points, namely the rationing of official funds, Sheikh (1976) follows Brofenbrenner (1947). Giddy (1976) links an analysis of Culbertson (1975) based on resale of foreign exchange with no effect of demand to the supply side of the no resale analysis of Sheikh (1976).

FIGURE 2
MICHAELY'S BLACK MARKET



The basic reference is the model of Blejer (1978a) who has adapted his work on the monetary approach to the Mexican balance of payments (1977a) to derive a PPP-monetarist model explaining the black market rate. 1

Denoting proportional rates of change by ^ we can write his money market equilibrium condition as

(1)
$$\hat{a} + (1 - \gamma) \hat{R} + \gamma \hat{D} = \hat{P} + \hat{m}_d$$

where $\gamma = D/(D+R)$

D = domestic monetary base

R = foreign monetary base

a = money multiplier

P = domestic price level

m_d = real demand for money

Other references are Culbertson (1975) who simply used annual data on black market rates in India, Phillipines and Turkey for PPP calculations, Giddy (1978) who attempted to show the "efficiency" of black markets for foreign exchange in Colombia, Brazil and Israel by testing the randomness of the black market premiums and changes in the premium using weekly data and Blejer (1978b) who used annual data on a black market based real exchange rate for Brazil, Chile and Colombia to estimate money demand functions in these countries.

Domestic inflation is a weighted average of changes in traded and nontraded goods prices, the latter bering the world price level plus the change in the official rate

(2)
$$\hat{P} = \hat{\beta}\hat{P}^* + \hat{\beta}\hat{e} + (1-\beta)\hat{P}^{NT}$$

where P* = the world price level

 \tilde{e} = the official rate of exchange

P^{NT} = the price of non-traded goods

He defines changes in monetary disequilibrium δ as the percentage gap between ex-ante changes in supply (including the multiplier) and changes in nominal demand for money or

(3)
$$\hat{\delta} = \hat{\gamma}D + \hat{a} - (\hat{M}_d + \hat{P})$$

The reasoning being that only D -- and presumably a--can be controlled by the authorities in a small open economy under fixed rates.

Changes in the price of non-traded goods are given by the assumption that relative prices have an elasticity with respect to nominal monetary disequilibrium, 1 or

(4)
$$\hat{P}_{NT} - \hat{P}_{T} = \lambda \hat{\delta}$$

 $¹_{
m In}$ Blejer (1977.), relative prices levels depended on δ which led to lagged terms in (5) and was replaced by a formulation like (4) in (1978a)

Given the assumptions domestic inflation is a weighted average of world inflation and real monetary disequilibrium, the weight on the latter being zero if the share of expenditure on non-traded goods is zero

(5)
$$\hat{P} = \theta \hat{P}_T + (1-\theta)(\hat{\delta}+\hat{P})$$

where
$$\theta = \frac{1}{1+\lambda(1-\beta)}$$

The black market rate is modelled by postulating that the supply is an increasing function of the premium and demand function of expected inflations at home and abroad as proxies for the nominal interest rate. The expected rate change in the real black market rate is a negative function of the log of the actual value.

(6)
$$\log S_B = c_1 + a \log p$$

(7)
$$\log D_{R} = c_{2} + b \operatorname{dlog} r^{E}$$

(8)
$$d\log r^{E} = -\log r$$

where $p=e/\tilde{e}$ is the black market premium e being the black market rate $r=\frac{eP^{\star}}{p} \ \ \text{is the real exchange rate}$

and E denotes expectation

Substituting for the expected change in the real rate and making flow supply equal to flow demand we have

(9)
$$\hat{e} = \frac{1}{a+b} [a\hat{e} + b(\hat{P}-\hat{P}^*)]$$

but from (5) we can eliminate P and we have

IUnder rational expectations the real exchange rate will be given by
rt = exp(exp(-t)) where r(0)=1. This implies a maximum devaluation after the initial
moment, appreciating gradually to 1.

(10)
$$\hat{e} = \frac{a+b\theta}{a+b} \hat{e} + \frac{b(1-\theta)}{a+b} (\hat{\delta} + \hat{P} - \hat{P}^*)$$

Finally Blejer assumes a PPP reaction function for $\tilde{\mathbf{e}}$ and finds

$$(11) \hat{e} = \psi \hat{x}$$

where
$$\hat{x} = \gamma \hat{D} + \hat{a} - (\hat{m}_d + \hat{P}^*)$$

and
$$\psi = \frac{(\alpha a + b)(1 - \theta)}{(a + b)(1 - \theta \alpha)}$$

That is to say that the black market rate only depends on the "gap", measuring nominal money demand in foreign prices. If credit creation is larger the black exchange rate depreciates. Using real money demand fitted a regression on permanent income and an adaptive expected domestic inflation variable, Blejer constructs his independent variable x, runs the regression

(12)
$$e_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1}$$

and reports that $\alpha_1 + \alpha_2 \ge 1$ for the three countries analyzed.

The \underline{ad} \underline{hoc} nature of the whole exercise and the weak nature of the test are apparent. The implication of an equation like (11) is, assuming complete sterilization of the changes in the official rate,

$$(13) \quad \hat{\mathbf{e}} = \psi \frac{\tilde{\mathbf{e}} \mathbf{B}_{\mathbf{t}}}{\mathbf{M}} + \frac{\psi}{\alpha} \quad \hat{\tilde{\mathbf{e}}}$$

where B, is the balance of payments.

There is no reason, however, to expect that the black market rate is related to the changes in the foreign exchange reserves of the central bank, F^G as a proportion of the money supply. If there were no capital movements B_t would be the current account. In a 100 per cent money regime $e^{G} = M$ and if relative PPP holds for the official rate $\alpha = 1$ and $\psi = 1$ therefore

establishing that the black market premium increases as central bank reserves decrease. Now given the assumptions about the black market in (6-8) and reversing the causation, we can see that changes in

expected real exchange rate determine changes in the stock of foreign assets of the central bank

$$(14) \quad \hat{F}^G = h \quad \hat{r}^E$$

where h = a/b

but this raises the question of whether the appropriate exchange rate to use in the computation of r should be the black market rate. l

In short, Blejer's adaptations of the PPP-monetarist model suffer not only from the general inadequacies of the approach but also from an <u>ad-hoc</u> formulation of the black market which leads to a relationship that presumes the absence of controls and intervention since it implicitly identifies the foreign monetary base with the stock of foreign assets held by the private sector, which is the relevant determinant of the black market rate or of the black market premium, depending on the relative size of the black market.

We now turn to a more promising approach to exchange rate determination than the one underlying Blejer's construct.

Using annual data over a sufficiently long range it is quite plausible that the current account should respond to the official real rate. But there is no monetary assumption whatsoever behind such an equation.

II. Exchange rates and the current accounts under inconvertibility.

1. We will first review briefly the theory of exchange rate determination which views the decisions of domestic and foreign residents about the composition of their portfolios as the primary determinants of exchange rates in the short run, given expectations about the fundamental determinants of the long run equilibrium value of the exchange rate.

The portfolio approach to exchange rate determination stems from the general equilibrium approach to monetary theory advanced by Tobin (1969) and explored in Branson (1976). Kouri (1975) has shown that the essential features of the portfolio approach are captured by a model where residents only hold two assets, domestic and foreign money, whereas foreigners do not hold domestic currency. This assumption is retained in Calvo and Rodriguez (1976) who call it "currency substitution" and it allows an immediate statement of the relationship between the exchange rate and the current account as demonstrated in Kouri (1976b), Rodriguez (1978) and Dornbusch-Fischer (1978).

If, following Kouri (1978), we assume that there is no inflation, no real growth and no systematic intervention on the part of the central bank and if asset demand functions are homogeneous in wealth the short run equilibrium condition in the foreign exchange market is

(1)
$$eF = a(\pi)W$$

where $\pi = (e/e)^E$ and a'>0

W = financial wealth in domestic currency.

The stock of foreign assets held by the private sector changes with the current account $\ensuremath{\mathtt{B}}$

(2)
$$f = B(e, y)$$

where B is the current account surplus,

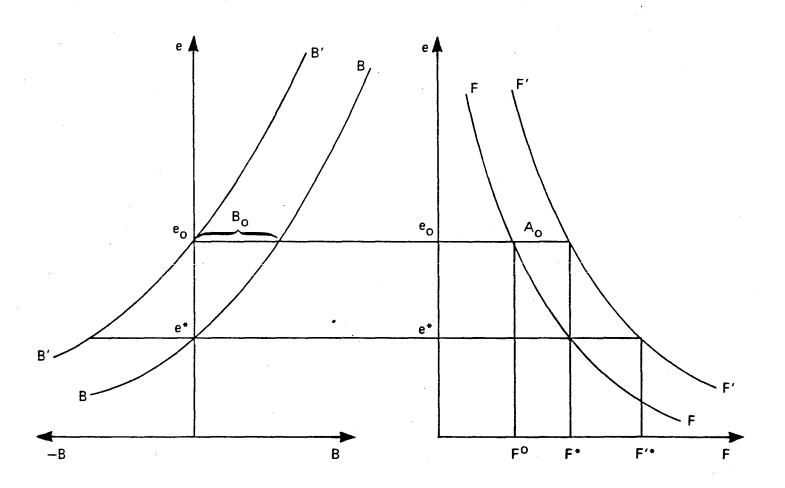
Y are other determinants of the current account

and $B_e > 0$ is the Marshall-Lerner condition, which implies a unique e* such that B=0.

The system is illustrated in Figure 3 for a given initial stock of foreign assets F_0 . On the right hand panel the temporary equilibrium e_0 is illustrated on the intersection of the e=aW/F curve, given π and W, and the initial stock F_0 . The left hand panel depicts the current account as a function of the exchange rate. At rate e_0 there is a surplus B_0 which leads to an increase in the stock of foreign assets and an appreciation of e until long run equilibrium is reached at e*, with $B(e^*)=0$ and $F=F^*$.

The effect of an increase in the expected rate of depreciation will be to shift up the FF schedule to F'F', depreciate the currency back to e_0 and equilibrium will be restored at e* with stock of foreign assets F'*. An increase in the domestic demand for imports, on the other hand shifts BB up to B'B' so that the current account is in deficit and equilibrium is restored at exchange rate e_0 and stock of foreign assets F_0 from the

INTERACTION BETWEEN THE CURRENT AND CAPITAL ACCOUNTS: THE KOURI DIAGRAM



initial e*, F*.

If there is inflation in the domestic and foreign economies at rates π_p and π_{p^*} respectively,the long run value of the exchange rate will be $\pi_p^{-\pi}_{p^*}$. By defining the real value of the stock of foreign assets held by domestic residents as $\tilde{F} = (F/P^*)$ and the inflation adjusted current account $\bar{B}=B/p^*-\pi_{p^*}\bar{F}$ which is an increasing function of the real exchange rate $\bar{e}=ep^*/p$ we can apply the earlier analysis to the system

$$\overline{eF} = a(\overline{\pi})W$$

$$\frac{.}{F} = \overline{B}(\overline{e})$$

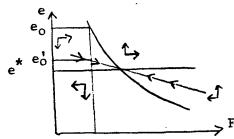
where $\bar{\pi}$ is the expected rate of depreciation of the real exchange rate, and W is real wealth.

Even though this generalization is useful for empirical purposes it does not treat inflation adequately and in any case, does not handle the case where there are new savings to be allocated from real growth in the two economies. In the analysis that follows, therefore, we continue to assume no inflation and no real growth.

$$eF = a(\dot{e}/e)W$$

$$F = B(e)$$

and steady-state equilibrium, such that $\dot{e}/e = \dot{f} = 0$, is a saddlepoint as illustrated below. With initial stock F_0 the rate consistent with rational expectations e^{0} is below the stationary expectations rate e^{0} .



¹ For the effects of other disturbances and the modifications introduced by assuming that expectations are rational rather than stationary, see Kouri (1976a). The system can then be written as

2. The foregoing analysis of the interaction of the current and capital accounts can be modified to understand a regime of currency inconvertibility. The regime involves the prohibition of capital account transactions in the official market for foreign exchange, where the monetary authorities use a mixture of intervention and exchange rate adjustment in order to balance the reported current account. Exporters have an incentive to under-invoice and the "subsidy" implied by the difference between the premium and the reported export subsidy increases supply of exports together with the supply of foreign exchange to the "free" market. Residents do not consume exports and the effects of the subsidies on consumption are ignored in this partial equilibrium framework with no cross-price effects between goods.

Importers will purchase funds from the black market to acquire unauthorized goods and services insofar as the tariff or equivalent is less than the black market premium, given the foreign price. The absence of import substitutes is also assumed.

Equilibrium in the black market is exactly the same as in the Kouri analysis just described. The difference is now that the underinvoicing of exports and the acquisition of unauthorized imports provides a link between the official and the black market but there remain two markets, one with a flow equilibrium and the other with a stock equilibrium.

Under plausible conditions 1 the domestic currency value of reported exports, X_r , will increase with a devaluation of the official rate, or an increase in the export subsidy, whereas it will decrease with a depreciation of the black market rate. Conversely, the foreign currency value of reported

¹Set out in Appendix 1, using the popular Bickerdicke-Robinson-Metzler framework.

imports, M_r^* , will decrease with a devaluation of the official rate or an increase in the import tariff or quota, whereas it will increase with a depreciation of the black market rate. Ignoring the tariff and the subsidy, the reported current account measured in foreign currency, B_r , can thus be written as

(3)
$$B_r = X_r (e, \tilde{e})/\tilde{e} - M_r^* (e, \tilde{e})$$

where
$$\frac{\partial X_r}{\partial e} < 0$$
; $\frac{\partial X_r}{\partial \tilde{e}} < 0$; $\frac{\partial M_r^*}{\partial \tilde{e}} > 0$; $\frac{\partial M_r^*}{\partial \tilde{e}} < 0$

The effect of a depreciation of the black market rate on the reported current account is clearly negative, since

(4)
$$\frac{\partial B_{\mathbf{r}}}{\partial e} = -\frac{1}{e} \left(\frac{X_{\mathbf{r}}}{\tilde{e}} \eta_{\mathbf{r}}^{\mathbf{x}} + M_{\mathbf{r}}^{\dagger} \eta_{\mathbf{r}}^{\mathbf{M}} \right)$$

and
$$\eta_r^x = -\hat{x}_r/\hat{e}; \eta_r^M = \hat{M}_r^*/\hat{e}$$
 as assumed.

The effect of a devaluation of the official rate on the reported current account is, however, ambiguous. In fact

(5)
$$\frac{\partial B_r}{\partial z} = \frac{1}{z} \left[\frac{X_r}{z} (\tilde{\eta}_r^X - 1) + \tilde{\eta}_r^M M_r^* \right]$$

and
$$\tilde{\eta}_{r}^{x} = \hat{X}_{r}/\tilde{e}; \ \tilde{\eta}_{r}^{M} = -\hat{M}_{r}^{*}/\hat{e} \text{ as above.}$$

The expression in square brackets is the Marshall-Lerner condition, so that, for initial equilibrium, a positive effect of the devaluation will require that the exchange rate elasticity of the reported exports in domestic currency and the reported imports in foreign currency be greater than one.

The analysis for the unreported current account, measured in foreign currency $\boldsymbol{B}_{_{11}}$, is along the same lines. Thus

(6)
$$B_u = X_u (e, \tilde{e})/e - M_u^* (e, \tilde{e})$$

where
$$\frac{\partial X_{u}}{\partial e} > 0$$
; $\frac{\partial X_{u}}{\partial \tilde{e}} < 0$; $\frac{\partial M_{u}^{*}}{\partial e} < 0$; $\frac{\partial M_{u}^{*}}{\partial \tilde{e}} > 0$

(7)
$$\frac{\partial B_{u}}{\partial \tilde{e}} = -\frac{1}{\tilde{e}} \left(\begin{pmatrix} x_{u} & \tilde{\eta}_{u}^{x} + M_{U}^{*} & \tilde{\eta}_{u}^{m} \end{pmatrix} \right)$$

(8)
$$\frac{\partial B_{u}}{\partial e} = \frac{1}{e} \left[\frac{X_{u}}{e} (\eta_{u}^{x} - 1) + M_{u}^{*} \eta_{u}^{m} \right]$$

The condition for $\partial B_{\mathbf{u}}/\partial \mathbf{e} > 0$ being with initial equilibrium

$$\eta_u^X + \eta_u^m > 1$$

Assuming that the Marshall-Lerner condition does hold in both cases, we can write our system as

(9) eF = aW

(10)
$$B_u \stackrel{+}{(e, e)} = F$$

(11)
$$B_r(\bar{e}, \dot{e}) = \mathring{F}^G$$

Equations (9), (10) and (11) provide a description of the interaction between the two markets. If the official rate is freely floating to balance the reported current account the description is complete. If the official rate is pegged, the consequences of a decrease in central bank reserves have to be spelled out by incorporating the money supply process. We assume for the moment that it is constant and therefore that domestic credit compensates for a decline in the foreign money base. Between these two polar cases there are the mixed regimes. We will concentrate first on regimes where the monetary authorities set the official rate based on central bank reserves, or alternatively where the rate of crawl depends on the current account. 1

The nature of the interaction between the two markets is apparent in Figure 4 where an initial situation of equilibrium is assumed. In the right panel we have, as before, temporary equilibrium at e_0 with stock of foreign assets F_0 . In the left panel, the reported current account is shown as an increasing function of the official rate, whereas the unreported current account is a decreasing function of the official rate. Defining the black market rate in such a way as to incorporate a constant risk premium (or for that matter a given export subsidy or import tariff), the equilibrium of the two markets obtains at rate $e_0 = \tilde{e}_0$. Now consider that the desired stock increases, say because a political crisis brings about an expected depreciation of the domestic currency. The FF schedule shifts up to F'F', the black market rate in-

In Section 3 we analyze the implications of an exogenous rate of crawl, where the official exchange rate devalues in steady-state.

creases to e' and this deteriorates the reported current account to $B_{r}^{\dagger}B_{r}^{\dagger}$ whereas the unreported current account increases to $B_{u}^{\dagger}B_{u}^{\dagger}$ increasing the stock of foreign assets to F_{1}^{\dagger} and pushing down to e to e". The official rate, however, has remained at e_{0} and the deficit on the reported current must be financed either by the sale of foreign assets by the central bank ΔF_{0}^{G} , or by an increase in the export subsidy or the tariff, or by devaluation of the official rate Δ \tilde{e} . With no exogenous increase in the risk premium, the latter three cases would decrease the profit from using the black market and funds would shift out until the two rates would again be equal to $e' = \tilde{e}'$. The first case could imply an exhaustion of reserves, with a large initial stock, ignoring expectations, 1 a continuing premium would pull resources to the black market and could therefore bring equilibrium back to $e_{0} = \tilde{e}_{0}$.

Consider now the steady-state properties of this model more explicitly in Figure 5. On the right panel we have portfolio balance as before. On the left panel we draw in e, $\tilde{\mathbf{e}}$ space the loci along with the reported and unreported current account are zero. To the left of the $\mathbf{B_r} = 0$ locus the official current account is in deficit and to the right it is in surplus. To the left of the $\mathbf{B_u} = 0$, the unofficial current account is in surplus and to the right it is in deficit.

 $^{^{1}}$ A similar model with expectations was used to analyze foreign exchange crises of this type by Krugman (1078).

FIGURE 4
TEMPORARY EQUILIBRIUM WITH INCONVERTIBILITY

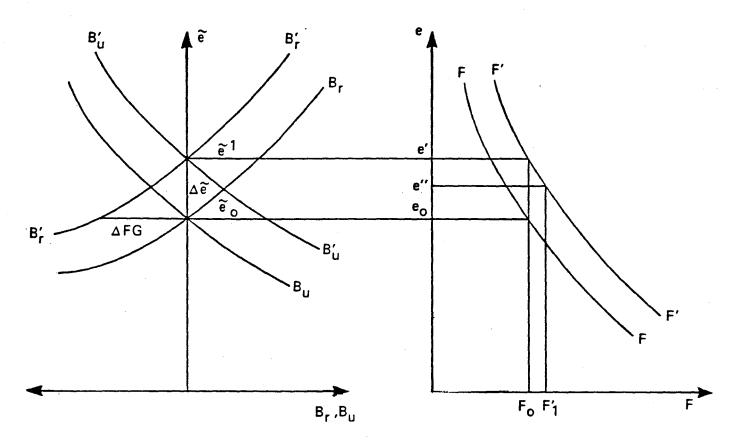
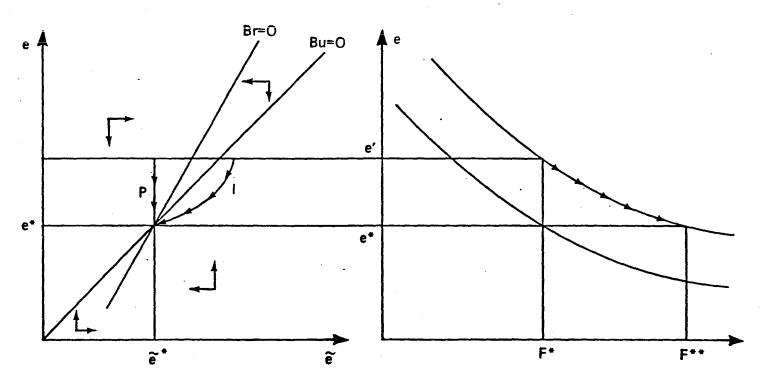


FIGURE 5
STEADY STATE EQUILIBRIUM WITH INCONVERTIBILITY



The slopes of the two loci can be written as

$$B_r = 0$$
 $\frac{de}{d\tilde{e}} = -\frac{\partial B_r/\partial \tilde{e}}{\partial B_r/\partial e}$

$$B_u = 0$$
 $\frac{de}{d\tilde{e}} = -\frac{\partial B_u/\partial \tilde{e}}{\partial B_u/\partial e}$

If the reported current account responds more to the official rate and the unreported current account responds more to the black market rate than they respond to the other rate, then the slope of the $B_r = 0$ locus, which has the "own" effect on the numerator will be larger.

Suppose that the official rate is devalued. This leads to an unreported deficit and a decrease in the stock of foreign assets held by the public which will push up the black market rate. Conversely for an unanticipated revaluation of the official rate (see vertical arrows in the left panel of figure 5). On the other hand, a decrease in the stock of foreign assets of the central bank motivated by an increase in the black market rate leads to a devaluation of the official rate (horizontal arrows).

If the slope of the ${\rm B_r}$ schedule is lower than the slope of the ${\rm B_u}$ schedule, then there will be situations where the unofficial current account is in deficit and the official rate is increasing, which increases the deficit, whereas the reported current account surplus also increases. Then the two rates depreciate without limit.

The steady-state adjustments to an increase in the demand for foreign assets after the initial derectation to e' is to put the official current account in deficit and the unofficial current account in surplus. As the stock of private foreign assets increases the free market rate appreciates and returns to its original value. With our intervention assumption the decline in the stock of foreign assets of the central bank would lead to a depreciation of the official rate and therefore to a path like I rather than P. The new equilibrium stock of foreign assets would be F'* and the two rates would remain e* and e*.

Therefore while the initial stock of foreign assets determines the free rate at each instant, steady-state equilibrium is determined by trade in goods, namely the combination of the two rates that keeps official and unofficial current accounts in balance. That the capital account matters in the short run and the current account in the long run is a conclusion of models of this type under stationary expectations. How long is the long run furthermore depends on the relative magnitudes of the current and capital accounts.

See Kouri (1978) p. 27ss. If the stock of foreign assets is half of the current account, 90% of the deviation from long run equilibrium is eliminated in 1.2 years.

3. We continue to focus on exchange rate regimes where there is a mixture of central bank intervention and exchange rate adjustment, like a crawling peg based on the state of the current account, which is just a continuous version of the following reaction function

(12)
$$\tilde{e} = g(F^{G})$$
 $g' < 0$

since it implies by differentiation that

(13)
$$\frac{e}{e} = -\gamma \frac{B_r}{F^G}$$

where
$$\gamma = -F^{G}g'/g$$

The rate of crawl is a fraction γ of the ratio of the current account to the stock of reserves of the central bank. In this case, Kouri's "acceleration hypothesis," according to which the domestic currency appreciates when the current account is in surplus and depreciates when it is in deficit, is the result of central bank policy.

In the black market, though, the "acceleration hypothesis" is the result of participant's behavior. In fact, differentiating (9) holding a and W constant and substituting (11) we get

$$(14) \quad \frac{e}{e} = -\frac{B_u}{F}$$

The direction of the arrows in the left panel of figure 5 follows by differentiating (13) with respect to e and (14) with respect to e. Thus an increase in the black market (official) rate pushes the official (black market) rate up

$$\frac{\partial \dot{\hat{e}}}{\partial e} = -\gamma \frac{\partial B}{\partial e} > 0$$

$$\frac{\partial e}{\partial \tilde{e}} = -\frac{\partial u}{\partial \tilde{e}} > 0$$

The system described by (13) and (14) has a linear approximation around the steady-state values e^* and \tilde{e}^* such that $B_r = B_u = 0$. Using (4), (5), (7) and (8) the system can be written as

(15)
$$\begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{e}} \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{\mathbf{u}}^{*}/\mathbf{F} & 0 \\ \mathbf{u} \end{bmatrix} \begin{bmatrix} -(\eta_{\mathbf{u}}^{\mathsf{X}} + \eta_{\mathbf{u}}^{\mathsf{M}} - 1) & p(\tilde{\eta}_{\mathbf{u}}^{\mathsf{X}} + \tilde{\eta}_{\mathbf{u}}^{\mathsf{M}}) \\ \mathbf{u} & \mathbf{u} \end{bmatrix} \begin{bmatrix} \mathbf{e} - \mathbf{e}^{*} \\ \mathbf{u} \end{bmatrix} \begin{bmatrix} \mathbf{e} - \mathbf{e}^{*} \\ \mathbf{e} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{e}} \end{bmatrix} \begin{bmatrix} \mathbf{e} \\ \mathbf{e} \end{bmatrix} \begin{bmatrix}$$

Given that the Marshall-Lerner conditions hold, the trace condition is obviously met. For the determinant to be positive, however, we need a stronger condition, namely for the strength of the "own" effects to be such that the product of their difference over one is larger than the product of the "cross" effects.

Note that this condition carries over to a model where wealth is composed of foreign money and domestic money divided as

(16)
$$eF = b(\pi)H$$

where b = a/1 - a

and H = (1-a)W

Consider now, even though it is not necessary for the analysis, that we are in a pure gold standard system, in that there is no domestic base money. Their (11) above gives the increase in the money supply, and, assuming that capital gains and losses of the central bank due to changes in the official rate are not a source of new money, we can write

(17)
$$\dot{H} = \tilde{e}B_{r}$$

By differentiating the portfolio balance condition and substituting from (10) and (16) we find the equation of motion for the black market rate:

(18)
$$e = \frac{b\tilde{\epsilon}}{F}^{B}r - \frac{eB_{u}}{F}$$

From (12) we have the equation of motion for the official rate:

The slope of the $\dot{\tilde{\bf e}}$ = 0 locus is thus the same as before, whereas the slope of the $\ddot{\bf e}$ = 0 locus becomes

$$\frac{de}{d\tilde{e}} \mid_{\bar{e}} = -\frac{\tilde{e}\partial B_r/\partial \tilde{e} - M \partial B_u/\partial \tilde{e}}{\tilde{e}\partial B_r/\partial e - M \partial B_u/\partial e} > 0$$

But the condition for the slope of the $\stackrel{\circ}{e}$ = 0 locus to be larger than the slope of the e = 0 locus, as in the left panel of Figure 5 above 1, is the same, namely the dominance of the "own" effects

(19)
$$\frac{\partial B_{\mathbf{u}}}{\partial \mathbf{e}} \frac{\partial B_{\mathbf{r}}}{\partial \tilde{\mathbf{e}}} > \frac{\partial B_{\mathbf{u}}}{\partial \tilde{\mathbf{e}}} \frac{\partial B_{\mathbf{r}}}{\partial \mathbf{e}}$$

The linear approximation around equilibrium confirms that the trace condition is met and that the determinant will be positive if (19) holds.

4. In terms of the underlying elasticities of supply of exports and demand for imports, in the small country case where demand for exports and supply for imports are infinitely elastic and initial equilibrium obtains, the condition on the determinant in (15) simplifies considerably. In fact the determinant will be positive provided that the sum of the premium elasticities of the supply of exports and demand for imports be smaller than

Since the money supply is continuously changing, the right panel is less useful for the analysis.

the sum of the premium elasticities of their respective reported shares. As one would expect, if the two market shares change sufficiently, the system will be dynamically stable.

The result hinges of course on the particular form of setting the official exchange rate. If $\tilde{\mathbf{e}}$ were pegged, the analysis would not be informative because the monetary consequences of the loss in reserves cannot be tracked down in a partial equilibrium framework. Similarly, for an exogenous rate of crawl ϕ , which would make the present system unstable.

Before proceeding to a more general analysis, it is worth emphasizing the main feature, of the inconvertible currency system described above. It is that, even though the combination of the Marshall-Lerner condition and of the acceleration hypothesis underlying the government's behavior is not sufficient for stability, as it would be in a unified regime with stationary expectations, it is sufficient that the response of the official market shares to the incentive provided by the black market premium be larger than the responses of supply and demand. If the latter are zero, the stability requirements are the exact analog of a unified regime.

¹See Appendix 1.

III. Currency Inconvertibility, Money and Relative Prices

1. In the preceding section we presented a condition for the stability of a black market system which carried over to a simple gold standard general equilibrium model.

We now bring, in the spirit of Kouri (1975), the wealth effects on consumption into the picture, together with a more satisfactory money supply process. This will allow us to analyze the case where the official exchange rate is changed by an exogenous crawling peg, a limiting case of which is the fixed rate system.

Consider an economy where two traded goods are produced and consumed 1.

One of them is traded through the official foreign exchange market and the other is traded through the black market. We are thus aggregating imports and exports channeled through each one of the markets. The foreign price of the two goods is given and we can therefore take e as being the price of the good traded through the official market and e as being the price of the good traded through the black market. The black market premium is thus the relative price of the two goods.

Assuming full employment of the factors of production, we can write the supply function of the two goods as depending only on that relative price

 $^{^{1}{\}mbox{The implications of the existence of nontraded goods are analyzed in Appendix 2.}$

1)
$$X = X \begin{pmatrix} + \\ p \end{pmatrix}$$

2)
$$\tilde{X} = \tilde{X} (\bar{p})$$

where X is the output of the good traded through the black market

X is the output of the good traded through the official market Domestic consumption depends on prices, income and wealth

where Y = XP + XP

These functions are homogeneous of degree zero in prices and income. The properties hold for/the excess supply functions, which are just the reported and unreported current accounts: 1

$$\frac{dC}{dP} = \frac{\partial C}{\partial P} + \frac{\partial C}{\partial Y} X + \frac{\partial C}{\partial Y} (dX_p + dX)$$

The term in parenthesis vanishes by the assumption that we are on the transformation curve and therefore the marginal rate of substitution in production is equal to the price ratio. Now since the Slutsky decomposition of the price effect is

$$\frac{\partial C}{\partial P} = \frac{\partial C}{\partial P} \mid_{\overline{u}} - \frac{\partial C}{\partial Y} X$$

we have the total effect being negative, like the compensated effect.

¹The effect of a change in relative prices in the demand functions can be given a sign because the income effect is cancelled out. In fact differentiating (3) with respect to P, we have

5)
$$X - C = B_u (p, W/\tilde{e}) = F$$

6)
$$X - C = B_r (p, W/e) = F^G$$

Wealth is composed of two assets, domestic money M and foreign money F.

The asset demand functions depend on the relative returns, on goods prices and on wealth.

$$-+++$$
 (7) $M^{d} = M (\pi, e, \bar{e}, W)$

(8)
$$eF^{d} = F (\pi, e, \tilde{e}, W)$$

(9)
$$W = M + eF$$

Relative returns are captured by the expected change in the black market exchange rate.

If the share of expenditure on the official traded good is α , a price index for a Cobb-Douglas utility function will be $\mathbf{e}^{\alpha} e^{\mathbf{1}-\alpha}$ so that the real return on domestic money is $-\alpha \hat{\mathbf{e}} - (1-\alpha)\hat{\mathbf{e}}$. The real return on foreign money is the change in $e/\tilde{\mathbf{e}}^{\alpha}e^{\mathbf{1}-\alpha}$ or $-\alpha \hat{\mathbf{p}}$. The difference is therefore $\hat{\mathbf{e}}$.

We now define the assets in terms of the official rate and assume continuous portfolio balance:

9)
$$\tilde{W} = \tilde{M} + pF$$

where $\tilde{W} = W/e$
 $\tilde{M} = M/e$

10) $\tilde{M} = h (\pi, p, W)$
 $+ t$

11) $pF = f(\pi, p, W)$

We eliminate (11) by the wealth constraint and write the protfolio balance condition as

12)
$$pF = f(\pi, p, M + pF)$$

We now specify the money stock as

13)
$$M = C_T + \tilde{e}F^G - K$$

where $\boldsymbol{C}_{\boldsymbol{T}}$ is total domestic credit K is the central banks net worth.

Even though changes in credit, in particular to the public sector, are exogeneously given to the central bank, we can assume that there is a part that is induced by the increases in net worth of the central bank due to devaluations and revaluation of gold. This is particularly the case if the central bank is not independent of the government. We thus postulate that credit has an antonomous and an induced component or

14)
$$C_T = C_0 + C (K)$$
 $C' > 0$

The change in K in turn can be written as

$$\dot{\mathbf{K}} = \mathbf{\tilde{e}} \phi_{\mathbf{G}}^{\mathbf{P}} \mathbf{G}^{\mathbf{G}} + \mathbf{\tilde{e}} \mathbf{F}^{\mathbf{G}}$$

where G is the gold stock, P_G is the price of gold and $\phi_G = P_G/P_G$ is the percentage difference between the market price of gold in foreign currency and the official price.

Thus the change in the money stock expressed in foreign currency is

15)
$$\dot{M} = \delta + B_r - \phi (1 - C')M$$

where $\delta = C'[\phi(K - C) + \phi_G^P_G^G]$ is the induced credit creation;

$$K = K/e$$

$$C = C/e$$

and $\varphi = \dot{\tilde{e}}/e$ is the exogenous rate of crawl.

2. The model consists therefore of the portfolio balance condition (11), which given expectations and asset supplies determines the temporary equilibrium value of the black market premium², and of the domestic money

the signs of the partials are discussed below.

Note that $\tilde{F}^G = \tilde{F}_e + P_G^G$ where F_e are the foreign exchange reserves

2 In fact from (12) we get as a reduced form + + - $p = p (\pi, \tilde{M}, F)$

supply equation given by (15) and of the unreported current account equation given by (5); which together determine the dynamic adjustment to steady-state equilibrium where the premium, and the stock of foreign assets are constant and money grows at rate ϕ .

Since (11) holds at all times we substitute for $\mathfrak p$ and keep as state variables M and F. The model can then be written in compact form as

(16a)
$$\tilde{M} = \delta + B_r (\pi, M, F) - \phi(1 - C') \tilde{M}$$

(16b)
$$\dot{F} = B_u (\pi, M, F)$$

The signs of the partials in (16) hinge on the assumption that

(17)
$$\psi = 1 - \alpha n - \varepsilon > 0$$
where $\alpha = pF/W$

 η is the wealth elasticity of the demand for foreign assets ϵ is the premium elasticity of the demand for foreign assets

Even if $\eta=1$, as is usually assumed in portfolio models, this condition will be true if the stock of foreign assets is not too large a proportion of wealth, as might be the case in an inconvertible currency regime¹, and the premium elasticity is fairly small as well. Condition

Empirical evidence on these proportion for major currencies can be found in Kouri-Macedo (1978) and Macedo (1979)

(17) simply says that the valuation effect of a change in the premium has to dominate the relative price and the weighted wealth effect combined.

The effects of changes in asset supplies on the unreported current account hinge on further restrictions. Thus

(18)
$$\frac{\partial B_{u}}{\partial M} > 0 \text{ if } \omega_{p} > \omega_{W} \frac{1-\epsilon}{n} = \omega_{W} a$$

where $\omega_i=i\partial B_u/\partial i$ i = p, W are the positive semi-elasticities of the unreported current account with respect to the premium and wealth and

(19)
$$\frac{\partial Bu}{\partial F}$$
 < 0 if $\omega_P > \omega_W \frac{\alpha \varepsilon}{1 - \alpha \eta} = \omega_W b$

Note that, given (17)-which implies that a > b -- (19) is true if (18) is true. For an increase in the stock of foreign assets to deteriorate the current account we thus need that the wealth effect be weaker than the price effect on goods demands. In that case ω_p will be large and ω_W will be small, for given sizes of the price and wealth elasticities of asset demands on the small size of which hinges the basic condition (17). Defining as $\tilde{\omega}$ the semi-elasticities for the reported current account, the slope of the two loci along which $B_r = 0$ and $B_u = 0$ are respectively

$$\frac{d\tilde{M}}{dF}|_{B_{\mathbf{r}}} = 0 = \frac{p}{b} \frac{\tilde{\omega}_{P} + \tilde{\omega}_{W} b}{\tilde{\omega}_{P} + \tilde{\omega}_{W} a}$$

$$\frac{\tilde{dM}}{dF}\Big|_{B_{11}} = 0 = \frac{p}{b} \frac{\omega_P - \omega_W b}{\omega_P - \omega_W a}$$

The condition for the slope of the B_u = 0 locus to be larger than the slope of the B_r = 0 locus reduces to

(20) (a - b)
$$(\omega_p \tilde{\omega}_W + \tilde{\omega}_p \omega_W) > 0$$

and we know that a > b from condition (17). Under the conditions stated the system is indeed stable. Solving the crawling peg system around equilibrium we get

(21)
$$\begin{pmatrix} \tilde{M} \\ \tilde{M} \\ \vdots \end{pmatrix} = \frac{1}{4} \begin{bmatrix} -\left[\alpha\eta \left(\tilde{\omega}_{p} + \tilde{\omega}_{W}a\right) + \phi PF \psi\right]/P & (1 - \alpha\eta) \left(\tilde{\omega}_{p} + \tilde{\omega}_{W}b\right) \\ \alpha\eta \left(\omega_{p} - \omega_{W}a\right)/P & -\left(1 - \eta\alpha\right) \left(\omega_{p} - \omega_{W}b\right) \end{bmatrix} F - F*$$

The central role of condition (17) is again apparent. The trace condition is met, under assumption (19). The condition on the determinant being

slope of the $B_r = 0$ locus. In fact:

$$\frac{d\tilde{M}}{dF} \Big|_{\tilde{M}}^{2} = 0 = \frac{p(1 - \eta\alpha) (\tilde{\omega}_{p} + \tilde{\omega}_{W} b)}{\alpha\eta(\tilde{\omega}_{p} + \tilde{\omega}_{W} a) + \phi p F (1 - \alpha\eta - \epsilon)}$$

This shows, incidentally, the conditions under which the pegged official exchange rate case is stable. Reverting to our basic model we confirm that a positive rate of crawl lower the slope of the M = 0 locus relative to the

positive is weakened relative to (20) because of the positive value of ϕ .

3. We depict the two loci where M=0 and $\dot{f}=0$ in Figure 6, where the arrows indicate the directions of motion. Above the $\dot{M}=0$ locus the real money supply is decreasing and the reported current account is in deficit for a given induced credit creation. Above the $\dot{f}=0$ locus the stock of foreign assets is increasing and the unreported current account is in surplus. Steady state equilibrium obtains at A. If the official exchange rate is pegged, the reported current account could have a deficit equal to the credit creation induced by gold sales, since then the money supply would not be changing. However, at A, $B_r=0$ because $B_u=0$ and we are in a two good economy. In a crawling peg system the money supply valued at the official rate will converge to

$$\tilde{M}^* = \frac{\delta}{(1-c') \phi}$$

so that the induced credit creation being non zero in the steady-state is associated with the continuing devaluation of the official rate, the rate of which determines the growth in nominal money and the black market rate, when the reported and unreported current accounts are zero.

$$\psi^{-1}$$
 [$\alpha\eta$ (1 - $\alpha\eta$) (a - b) ($\omega_p\tilde{\omega}_W + \tilde{\omega}_p\omega_W$) + $\phi PF\psi$ (1 - $\alpha\eta$) ($\omega_p - \omega_W b$)

The trace condition, however, becomes stronger, namely

$$\phi F \psi > \frac{\alpha \eta}{p} (\tilde{\omega} + \tilde{\omega}_W^a) + (1 - \alpha \eta) (\tilde{\omega} - \tilde{\omega}_W^b)$$

In fact when a < b -- and therefore ψ < 0 -- the determinant in (21) reduces to the positive expression:

The long run effects of an increase in the expected rate of depreciation of the black market rate are also depicted in Figure 6. At point A_0 there is now a deficit in the reported current account and a surplus on the unreported current account, so that the $\dot{F}=0$ schedule shifts to the right and the $\dot{M}=0$ schedule shifts down. At point A_1 the real money supply has decreased and the stock of foreign assets has increased. A devaluation of the official rate, by lowering the real stock of money would have the same qualitative effects.

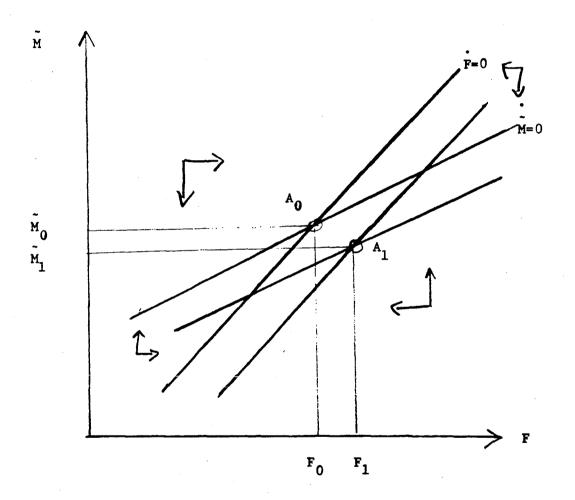
The effects of various disturbances on the three endogenous variables can be ascertained by total differentiation of the system, to yield

Given condition (17) the determinant of the Jacobian in positive. Note that the system can easily be reinterpreted for a pegged official rate by making $\phi = 0$ and introducing in the right hand side a column identical to the first column of the Jacobian, without the α 's, or

$$[1 - \varepsilon - \eta - \widetilde{\omega} - \widetilde{\omega}_{W} \omega - \omega_{W}]^{*}$$

Figure 6

Steady-State Effect of an Expected Depreciation of the Black
Market Rate



The results are collected in Table 1.1

4. The analysis in Figure 6 can be checked with reference to a unified regime where the premium is always one and the two current accounts can be aggregated, together with the stocks of foreign assets. We are basically back to a simple general equilibrium version of the partial equilibrium model reviewed in Section I.1, rewritten as

$$F = a(\pi)M/e$$

24)
$$\dot{M} = D + B(M/e + F)$$

We represent in Figure 7 the steady state effects of an increase in π by a downward rotation of the upward slopping portfolio balance condition in (23). The locus where $\hat{\mathbf{M}}=0$ and hence $\mathbf{B}=0$ is downward slopping with

$$\frac{1-\eta}{1-\alpha\eta-\varepsilon} \qquad \frac{\omega_W(1-\tilde{\omega}) + \tilde{\omega}_W(1-\omega)}{\omega_W^{\tilde{\omega}} + \tilde{\omega}_W^{\tilde{\omega}}}$$

 $^{^{1}}$ In the devaluation case the effect on domestic money is ambiguous. It will be negative if

Another check is provided in Appendix 2 where the analysis is extended to an economy with a non traded good.

Table 1

Effect of Various Disturbances

Effect on	P	M	F
π	+	-	+
δ	+	.	+
ē	+	-	+

slope-1. If the former contradictory efforts of π on the two current accounts offset each other the schedule keeps unchanged. Then we see a decrease in real balances from \tilde{M}_0 to \tilde{M}_1 and an increase in foreign assets from F_0 to F_1 as before. If the effect of an increase in π is to move the current account into surplus, as one would expect, then the schedule shifts to the right and the increase in F is larger (to F_2) and the increase in real balances smaller (to \tilde{M}_2), than it would otherwise had been. In the perverse case where the schedule shifts to the left (not shown) it is the increase in real balances which is larger.

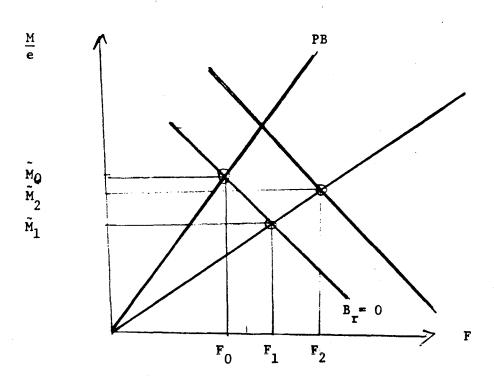
As we found in the partial equilibrium analysis in Section II, the inconvertible currency regime had an analog with a unified floating regime, when the supply and demand responses were small relative to the linkage between the two markets. Similarly, if the effects of the black market premium or the current account are weak, the analysis can ignore the unreported current account and the difference between the stock of foreign assets of the central bank and of the private sector. These are, of course, testable implications of the analysis. If true, they would show that the portfolio approach to exchange rate determination in a regime of pure convertibility is able to capture the essence of the problem even in a regime of inconvertibility where the official foreign exchange market is controlled.

The main message of the portfolio approach to exchange rate determination under currency convertibility is the acceleration hypothesis. We hope to have shown that when the exchange rate policy of the central bank involves continuous interventions and controls in the foreign exchanges this message remains.

See some empirical evidence for major countries in Kouri-Macedo (1979).

Figure 7

Steady State Effect of an Expected Depreciation of the Exchange Rate in a Regime of Unified Float



Appendix 1

Derivation of the Stability Conditions in Terms of of Demand and Supply for Imports and Exports

The argument in the text is based on the responsiveness of the reported and unreported trade flows to the official and black market rates. Introducing the markets for exports and imports gives, however, further insights and, in fact, shows that, under plausible assumptions, the determinant of (15) in the text is positive and therefore that the dynamic partial equilibrium system is stable.

Equilibrium in the market for the export good is characterized by

(1)
$$S^{X}(P^{X}, p) = D^{X}(P_{X}^{*}) = \overline{X}$$

where $S^{\mathbf{X}}$ is the supply of our exports

DX is the demand for our exports

PX is the domestic currency price of exports

 P_{X}^{*} is the foreign currency price of exports and $p = e/\tilde{e}$ is the black market premium and

$$(2) P^{X} = \tilde{e}P_{X}^{*}$$

where & is the official exchange rate.

Equilibrium in the market for the import goods, in turn, is, in similar notation, characterized by:

(3)
$$S^{M}(P_{M}^{*}) = D^{M}(P^{M}p) = \overline{M}$$

and

$$(4) P^{M} = \tilde{e}P_{M}^{\star}$$

The difference of equations (1) to (4) from the usual Bickerdicke-Robinson-Metzler specification is simply that the black market premium is an argument in the export supply and import demand functions. More specifically the assumption is that

(5)
$$\frac{\partial S^{X}}{\partial p} > 0 \text{ and } \frac{\partial D^{M}}{\partial p} < 0$$

Aside from the supply and demand decision, the black market premium has an effect in the decision about the share of exports and imports that will go through the black market. Institutional constraints clearly prevent a "bang-bang" solution by which all exports would go though the black market and all imports would go through the official market when p > 1. One way to incorporate these rigidities is to make the share of the official market a function of the black market premium. Thus,

(6)
$$X_r^* = \alpha$$
 (p) X^*

(7)
$$M_{T}^{*} = \beta$$
 (p) M^{*}

where
$$X^* = \overline{X}P_{x}^*$$

and
$$M^* = \overline{MP}_m^*$$

The assumption is that

(8)
$$\frac{\partial \alpha}{\partial P}$$
 < 0 and $\frac{\partial \beta}{\partial P}$ > 0

More institutional features could be incorporated without great gain in insight. Thus the existence of a subsidy on reported exports, S, and of a tariff on reported imports, T, would make the profits from under-invoicing exports Π_{X} and imports Π_{M} like

(9)
$$\Pi_{x} = (e - \bar{e}s) [1 - \alpha(p,s)] P_{x}^{*} \bar{X} (P_{x}^{*}, p, s)$$

(10)
$$\Pi_{M} = (e - \bar{e}\tau) [1 - \beta(p, \tau)] P_{M}^{\star} \bar{M} (P_{M}^{\star}, p, \tau)$$

where
$$s = 1 + S$$

 $\tau = 1 + T$

which would imply that profits would disappear when

$$(11) p = s = \tau$$

a result in the spirit of Beccaria (1764-65) but which can also be found in Winston's (1969) analysis of the overinvoicing of imports in Pakistan.

Going back to the simipler model, we define the following positive elasticities, using "hat" notation

$$\epsilon_{x} = -\hat{D}^{x}/\hat{P}_{x}^{*}$$

$$\epsilon_{y} = -\hat{D}^{M}/\hat{P}^{M}$$

$$\eta_{x} = \hat{S}^{x}/\hat{P}_{x}$$

$$\epsilon_{p} = -\hat{D}^{M}/\hat{P}$$

$$\eta_{p} = \hat{S}^{x}/\hat{P}$$

$$\eta_{m} = \hat{S}^{M}/\hat{P}_{m}^{*}$$

$$\omega_{p} = -\hat{\alpha}/\hat{P}$$

$$\theta_{p} = \hat{\beta}/\hat{P}$$

$$\tilde{\theta}_{p} = \frac{\beta}{1-\beta} \theta_{p}$$

The logarithmic differentiation of (1) and (3) then yields

(12)
$$\hat{P}_{x}^{*} = -\frac{\eta_{x}\hat{\epsilon} + \eta_{p}\hat{p}}{\eta_{x} + \epsilon_{x}}$$

(13)
$$\hat{P}_{M}^{\star} = -\frac{\eta_{M}\hat{\epsilon} + \epsilon_{p}\hat{p}}{\eta_{M} + \epsilon_{M}}$$

We now have, from (6) expressed in domestic currency

(14)
$$x_r = \tilde{\epsilon} \alpha \bar{x} P_{\bar{x}}^*$$

which, upon differentiation yields

(15)
$$\hat{X}_r = \hat{\epsilon} - \omega_p \hat{P} + (1 - \epsilon_X) \hat{P}_X^*$$

Valuing unreported exports at the black market rate we have, similarly

(16)
$$\hat{X}_{u} = \hat{e} + \tilde{\omega}_{p}\hat{p} + (1 - \epsilon_{X})\hat{P}_{X}^{*}$$

Doing the same operations from (7) and substituting from (12) and (13) respectively, we obtain the decomposition of the elasticities mentioned in the text as

(17)
$$\eta_{\mathbf{r}}^{\mathbf{x}} = \frac{\varepsilon_{\mathbf{X}}(\omega_{\mathbf{P}} - \eta_{\mathbf{P}}) + W_{\mathbf{P}}\eta_{\mathbf{X}} + \eta_{\mathbf{P}}}{\varepsilon_{\mathbf{x}} + \eta_{\mathbf{x}}}$$

(18)
$$\eta_{\mathbf{r}}^{M} = \frac{\eta_{M} (\theta_{\mathbf{P}} - \varepsilon_{\mathbf{P}}) + \theta_{\mathbf{P}} \varepsilon_{M} - \varepsilon_{\mathbf{P}}}{\eta_{M} + \varepsilon_{\mathbf{P}}}$$

(19)
$$\tilde{\eta}_{\mathbf{r}}^{X} = \frac{\varepsilon_{X}(\omega_{p} - \eta_{p} + 1 + \eta_{X}) + W_{p}\eta_{X} + \eta_{p}}{\varepsilon_{x} + \eta_{x}}$$

(20)
$$\tilde{\eta}_{\mathbf{r}}^{\mathbf{M}} = \frac{\eta_{\mathbf{M}}(\theta_{\mathbf{p}} - \varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{M}}) + (1 + \theta_{\mathbf{p}}) \varepsilon_{\mathbf{M}} - \varepsilon_{\mathbf{p}}}{\varepsilon_{\mathbf{M}} - \varepsilon_{\mathbf{p}}}$$

(21)
$$\eta_{u}^{x} = \frac{\varepsilon_{x} \left(1 + \tilde{\omega}_{p} + \eta_{p}\right) + \left(1 + \tilde{\omega}_{p}\right)\eta_{x} - \eta_{p}}{\varepsilon_{x} + \eta_{x}}$$

(22)
$$\eta_{\mathbf{u}}^{\mathbf{M}} = \frac{\eta_{\mathbf{M}} (\theta_{\mathbf{p}} + \varepsilon_{\mathbf{p}}) + \theta_{\mathbf{p}} \varepsilon_{\mathbf{M}} + \varepsilon_{\mathbf{p}}}{\eta_{\mathbf{M}} + \varepsilon_{\mathbf{M}}}$$

(23)
$$\tilde{\eta}_{u}^{X} = \frac{\varepsilon_{X}(\tilde{\omega}_{P} + \eta_{P} - \eta_{X}) + (1 + \tilde{\omega}_{P}) \eta_{X} - \eta_{P}}{\varepsilon_{X} + \eta_{X}}$$

(24)
$$\tilde{\eta}_{u}^{M} = \frac{\eta_{M} (\tilde{\theta}_{p} + \varepsilon_{p} - \varepsilon_{M}) + (1 + \tilde{\theta}_{p}) \varepsilon_{M} + \varepsilon_{p}}{\varepsilon_{M} + \eta_{M}}$$

With the exception of (22), the signs of all these elasticities are not unambiguously positive, as they should be for the sign assumption in the text to be vindicated.

In order to concentrate on the differences with the usual decomposition of the BRM elasticities, consider, however, the limiting use of a small country which faces an infinitely elastic demand for its exports and infinitely elastic supply of its imports. Then the only expression which remain are the factors of $\varepsilon_{\mathbf{x}}$ and $\eta_{\mathbf{x}}$.

Then assuming initial equilibrium, the sign presumption in the text become

(25)
$$\omega_{P} + \theta_{P} - \eta_{P} - \varepsilon_{P} > 0$$
 for $\frac{\partial B_{r}}{\partial e} < 0$

(26)
$$\eta_X + \varepsilon_X + \omega_P + \theta_P - \eta_P - \varepsilon_P > 0$$
 for $\frac{\partial B_r}{\partial \tilde{e}} > 0$

(27)
$$\tilde{\omega}_{P} + \tilde{\theta}_{P} + \eta_{P} + \varepsilon_{P} > 0$$
 for $\frac{\partial B_{u}}{\partial e} > 0$

(28)
$$\tilde{\omega}_{P} + \tilde{\theta}_{P} + \eta_{P} + \varepsilon_{P} - \eta_{X} - \varepsilon_{X} > 0$$
 for $\frac{\partial B_{u}}{\partial \tilde{\epsilon}} < 0$

The source of the ambiguities is that there are two affects of an increase in the premiums on the reported current account. One is through the change in the share, the other is through the change in the quantity. The share elasticities are larger than the quantity elasticities. If this were not the case, the improvement in, say, the quantity exported brought about by a devaluation of the official rate would be offset by the effect of the decrease in the premium or

$$(29) \qquad \eta_{X} + \omega_{P} < \eta_{P}$$

The deterioration of the reported current account from an increase in the black market rate presumes, further, that the share elasticity is larger than the supply elasticity or

(30)
$$\omega_p > \eta_p$$

For the unreported current account to deteriorate when the official exchange rate is devalued, however, the condition is that the quantity elasticities are overtaken by the direct effects of the decreased premium which are then in the same direction since both the share and the quantity effect tend to deteriorate the unreported current account. When the former are absent as in the effect of the black market rate on the unreported current account, there is no ambiguity, as equation (27) reveals.

The stability condition of the model becomes quite simple with a bit of additional notation.

(31)
$$\tilde{\sigma} = \tilde{\omega}_{P} + \tilde{\theta}_{P} + \eta_{P} + \varepsilon_{P}$$

(32)
$$\sigma = \omega_p + \theta_p - \eta_p - \varepsilon_p$$

Thus the sum of the premium elasticities of the share of the unreported current account is added to the sum of the premium elasticities of supply and demand to form σ and these are subtracted to form σ .

Furthermore call ϕ the sum of the price elasticities

(33)
$$\phi = \eta_X + \epsilon_M$$

Then the determinant of the system described in the text becomes

(34)
$$\Delta = \begin{bmatrix} -\sigma & \sigma - \phi \\ \sigma & -\sigma - \phi \end{bmatrix}$$

which, going back to our earlier notation and expressing ω_p and θ_p in terms of ω_p and θ_p becomes:

(35)
$$\Delta = (\eta_X + \varepsilon_X) \left(\frac{\omega_P}{1-\alpha} + \frac{\theta_P}{1-\beta} \right)$$

which is always positive.

Appendix 2

A General Equilibrium Model With Non-Traded Goods

The model of Section III can be extended to include a non-traded good. The importance of non-traded goods goes back (at least:) to the Australian "dependent economy" model exposited in Pearce (1961), was developed in Diaz (1963) and related to the monetary approach to the balance of payments in Dornbusch (1973) and Krueger (1974). Kouri (1975) introduced non-traded goods in the asset market view of the exchange rate.

This model has three goods and thus two relative prices. Work with three good models has been done by Jones (1974), Takayama (1974) and Dornbusch (1975) and (1979).

If the price of the cross traded good is $P^{\mbox{\scriptsize NT}}$, define

$$q = P^{NT}/\tilde{e}$$

and the model in Section III becomes

1) $E^{NT}(\bar{q}, p, \tilde{\omega}) = 0$

- equilibrium in the non-traded goods market
- 2) $pF = a(\pi, q, p)M$
- portfolio balance (where the wealth elasticity is one for simplicity)
- 3) $\tilde{M} = \delta + B_r(\bar{q}, p, \tilde{\omega}) \phi(1-c')\tilde{M}$ money supply process and reported current account
- 4) $\dot{\mathbf{r}} = \mathbf{B}_{ij} (\bar{\mathbf{q}}, \dot{\mathbf{p}}, \tilde{\omega})$

unreported current account

Equations (1) and (2) define the temporary equilibrium. Denoting by ϵ_1, ϵ_2 and ϵ_W the positive semi-elasticities of the excess demand for non-traded goods with respect to q, p and W and by η_1, η_2 and η_3 the elasticities of the a function with respect to q, p and π , total differentiation of equations 1 and 2 yields

5)
$$\begin{bmatrix} -\epsilon_1 & \epsilon_2 + \epsilon f \\ -\eta_1 & 1 - \eta_2 \end{bmatrix} \begin{bmatrix} \hat{q} \\ \hat{p} \end{bmatrix} = \begin{bmatrix} \hat{c}(1-f)\tilde{M} - \epsilon f\hat{F} \\ \hat{M} - \hat{F} + \eta_3\hat{\pi} \end{bmatrix}$$

The determinant $\Delta = \eta_1(\varepsilon_2 + \varepsilon f) - \varepsilon_1 (1 - \eta_2)$ will be negative if the relative price effects on the asset demand functions, captured by η_1 and η_2 are small, the effect of the "own" price on the excess demand for non-traded goods is large relative to the effect of the prices of traded goods (ε_1 large and ε_2 small) and, the relative share of foreign assets in wealth (f) is small. Under that assumption we can write the reduced forms for p and \P as

6)
$$q = q (\tilde{M}, F, \pi)$$

7)
$$p = p(M, \overline{F}, \frac{+}{\pi})$$

Given a system of asset demands as in Section III, but with a unitary wealth elasticity, the a function is simply the ratio of the demand for foreign assets fW and the demand for money (1-f)W.

We give in Figure 8 a graphical illustration of temporary equilibrium.

The slope of the locus where the non-traded goods marker is in equilibrium is, from (1):

$$\frac{\hat{q}}{\hat{p}}\Big|_{NT} = \frac{\epsilon_2 + \epsilon f}{\epsilon_1}$$

and is represented as the NT schedule in Figure 8. When the black market premium increases, demand for the unofficial traded good decreases and only an increase in the relative price of the non-traded good will keep the market in equilibrium. The schedule is shifted up by an increase in asset supplies.

The slope of the portfolio balance schedule is, from (2):

$$\frac{\hat{q}}{\hat{p}} \mid_{PB} = \frac{1 - \eta_2}{\eta_1}$$

which will be positive as well, unless the demand elasticity is greater than the share of domestic money in wealth. We have assumed, however, that the valuation effect of an increase in p dominates this direct demand effect so that when the premium increases and demand for foreign assets

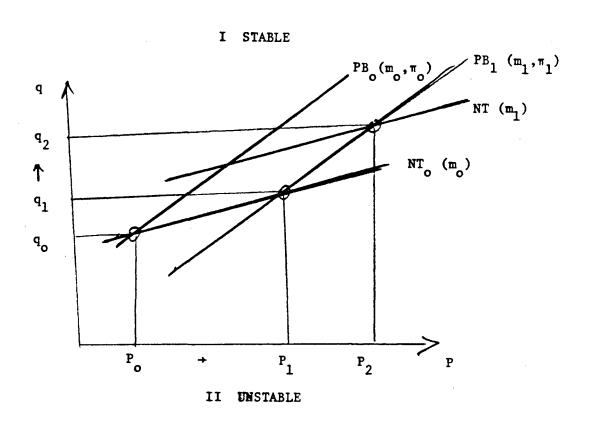
¹ Since $\eta_2^a > 1$ implies $\eta_2^f > 1 - f$.

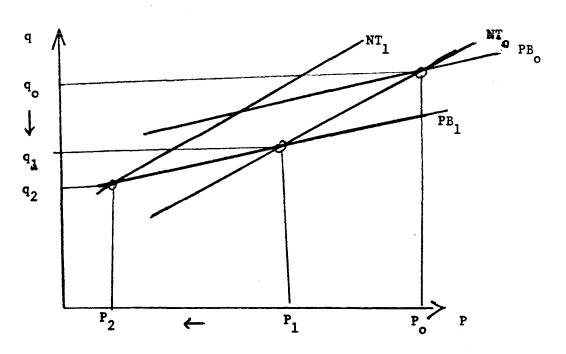
increases, the increase in the relative price of non-traded goods is able to preserve portfolio balance, given asset supplies and expectations. The portfolio balance schedule is shifted up by an increase in the stock of foreign assets and down by an increase in the expected rate of depreciation or in the money stock.

The slope of the portfolio balance locus would be vertical if relative prices would not enter the asset demand functions. As we pointed out, if they do enter, the elasticities should be smaller than in the non-traded goods sector. In figure 8 we show first the effects of an increase in the expected rate of an increase in the expected rate of depreciation as a downward shift of PB which depreciates the black market rate from P_0 to P_1 and increases the price of the non traded good from q_0 to q_1 , since the NT schedule does not move. If the slope was steeper an increase in π would bring about an appreciation and a decrease in the price of the non-traded good. The effect of an increase in the money stock reveals that the latter case is unstable. When \tilde{M} increases PB_0 shifts to PB_1 and NT_0 to NT_1 . Thus bringing q_0 to q_2 and p_0 to p_2 . If the NT schedule were steeper the effect would be a reduction in q which would increase excess demand further and an appreciation of p which would decrease the demand for \tilde{M} .

Substituting (6) and (7) in the total differential of the B_r and B_u equations, and defining as before the positive semi-elasticities $\omega_1, \omega_2, \ \tilde{\omega}, \ \omega_1, \ \omega_2$ and ω for B_r and B_u respectively, adding the positive elasticities q_1, q_2 and q_3 for M, F and π and similarly for the p function, we can write

Figure 8 Temporary Equilibrium Effect of an Increase in $\widetilde{\mathbf{M}}$





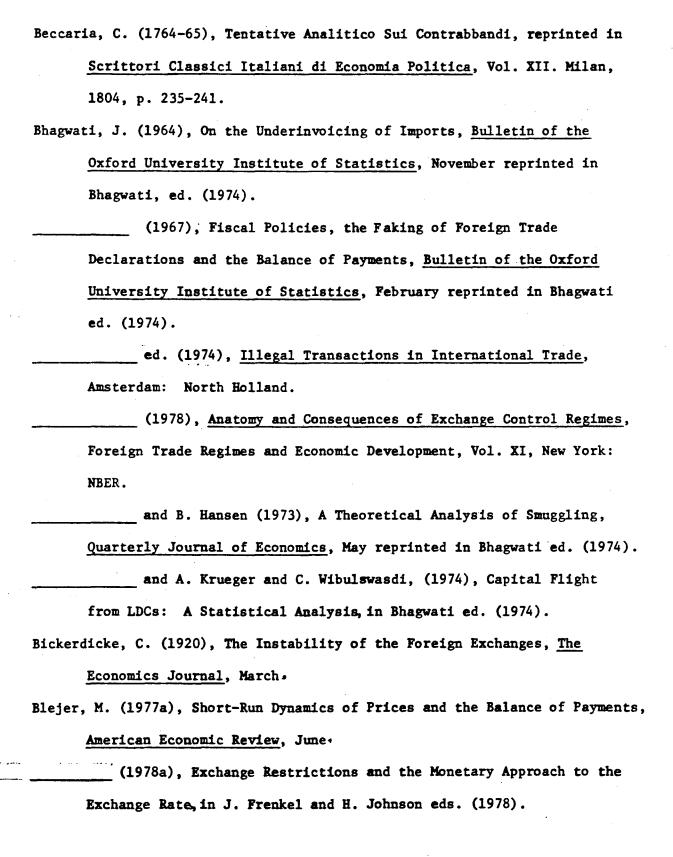
(8)
$$dB_{r} = -\tilde{M} \left[\tilde{\omega}_{1} q_{1} + \tilde{\omega}_{2} p_{2} + \tilde{\omega} (1 + f(p_{2} - 1)) \right] + \hat{F} \left[-\tilde{\omega}_{1} q_{2} + \tilde{\omega}_{2} p_{2} - \omega f \right] - \hat{\pi} (\tilde{\omega}_{1} q_{3} + \tilde{\omega}_{2} p_{3})$$

and

(9)
$$dB_{u} = \hat{H}[\omega_{1}^{q}_{1} - \omega_{2}^{p}_{2} + \omega(1 + f (p_{2} - 1))]$$
$$-\hat{F}(\omega_{1}^{q}_{2} + \omega_{2}^{p}_{2} + \omega f) + \hat{\pi} (-\omega_{1}^{q}_{3} + \omega_{2}^{p}_{3})$$

If the terms in brackets are positive the reduced form of B $_{\rm r}$ and B $_{\rm u}$ is exactly the same as in the model of section 3.

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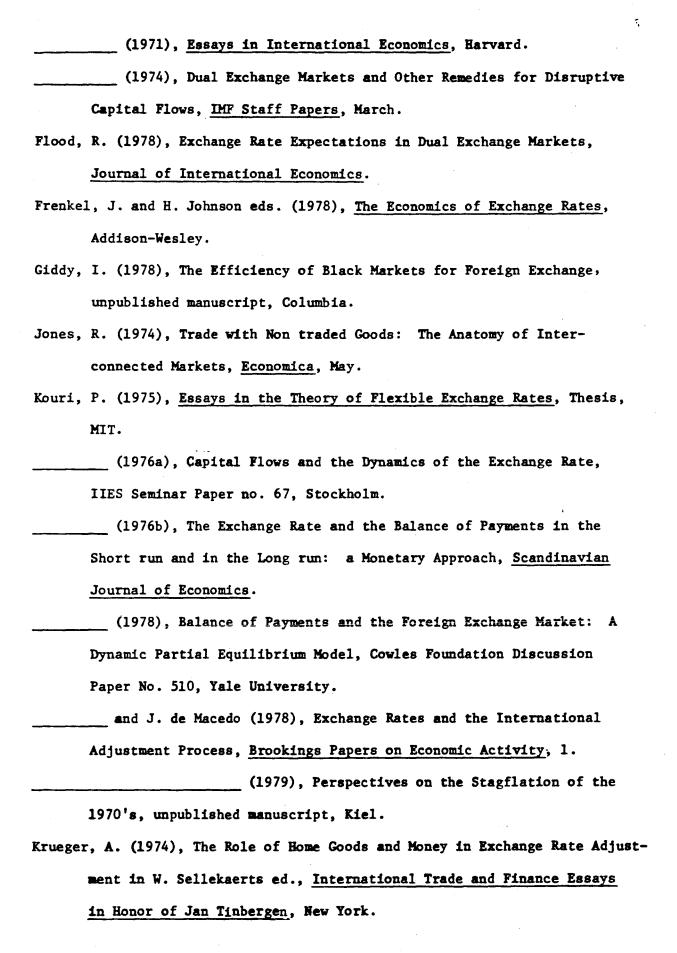
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