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## Quality, Distance and Trade: a Strategic Approach

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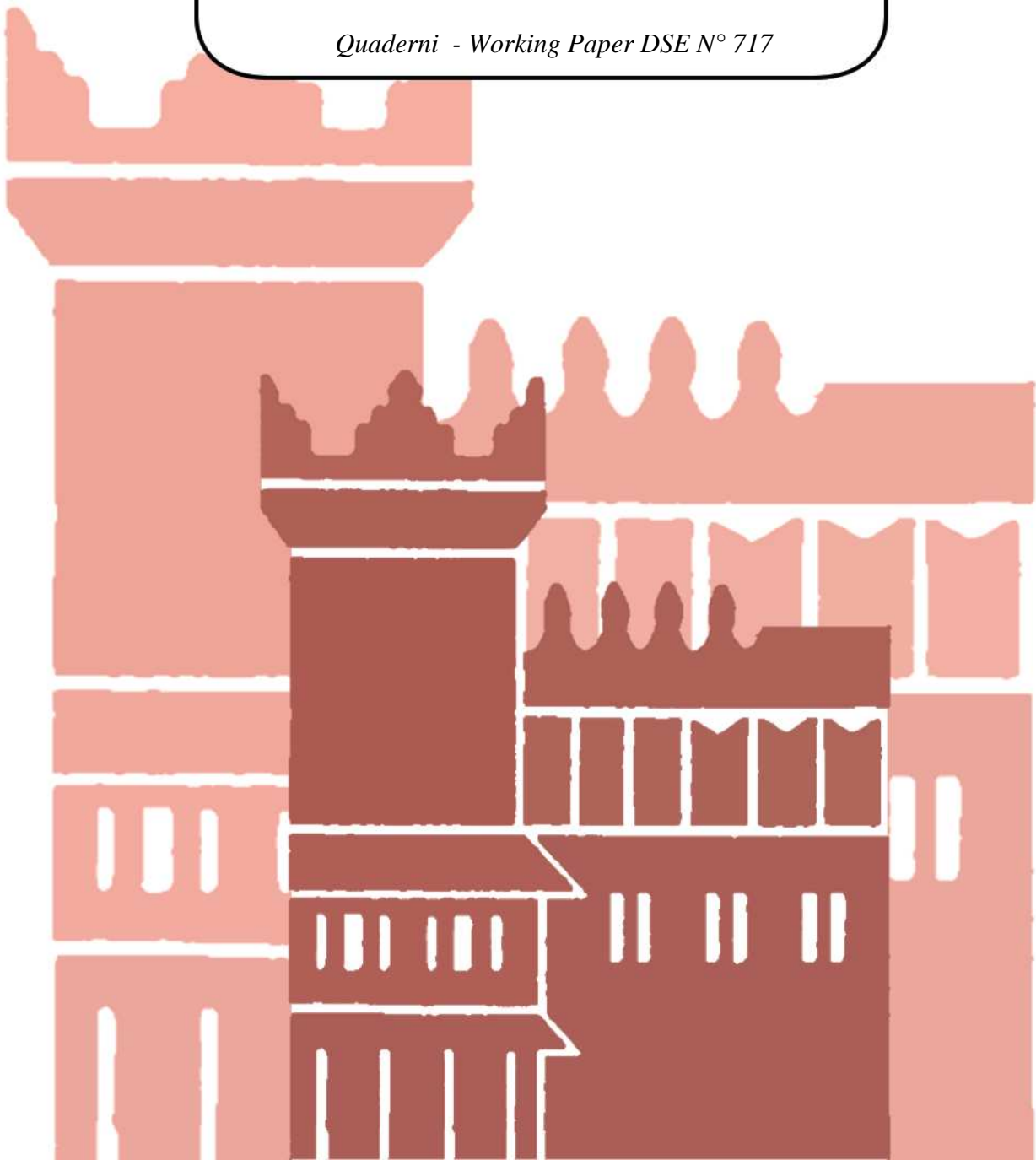


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DEPARTMENT OF ECONOMICS

**Quality, Distance and Trade:  
a Strategic Approach**

Emanuele Bacchiega  
Antonio Minniti  
Arsen Palestini

*Quaderni - Working Paper DSE N° 717*



# Quality, Distance and Trade: a Strategic Approach\*

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## Abstract

This paper contributes to the vast and growing literature on trade and quality by providing a parsimonious explanation of the observed increase in unit values (and thus quality) of shipped goods with the distance of the country of destination. This mechanism is based on the influence of distance on firms' strategic behavior when the quality level of goods is a choice variable for firms, and complements the ones already proposed in the literature. Our approach differs from the extant literature in that it does not rely on technology or preference/income differentials to identify the determinants and drivers of trade flows. Moreover, it allows to clearly disentangle between the price setting and quality choice of firms. We find that distance has an unambiguously positive effect on the average quality of traded goods, as well as a negative one on the likelihood of "trade zeros". Our results suit the acquired empirical evidence on distance and quality and contribute, therefore, to the research on the determinants of trade performances of firms and countries. Also, our model suggests some useful insights on the relation between distance and free-on-board prices.

**Keywords:** Product Quality, Destination Market, Strategic Interaction.

**JEL Codes:** L13, F12

## 1 Introduction

In the discussion of the determinants of trade flows, the focus of trade economists has gradually shifted from features such as comparative advantage, increasing returns to scale and consumer preferences to factors operating at the very firm level, see Bernard *et al.* (2007), for a discussion on this. In particular, firm heterogeneity has been emphasized as a fundamental element to understand the drivers of trade flows. In this respect, the literature

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recognizes two main dimensions along which firms may be heterogeneous (see, for instance, Hallak and Sivadasan 2009). The first relates to productivity (see, among others, Melitz 2003; Chaney 2008; Melitz and Ottaviano 2008). The other dimension of firm heterogeneity is connected with the quality level of the output (see, e.g., Baldwin and Harrigan forthcoming; Johnson 2010; Kugler and Verhoogen forthcoming).<sup>1</sup> The present paper is concerned about this second kind of heterogeneity. In particular, we explore the relationship between the quality level of traded goods and the distance of trading partners. Indeed, recent analyses have unveiled empirical regularities concerning the relationship between the quality of exported goods and both the distance and the income of the country of destination. More specifically, they show that unit values (f.o.b. prices) of exported goods increase with the distance of the trading partner, which suggests that firms upgrade the quality level of the goods they export to more distant markets compared to closer ones.<sup>2</sup> This evidence is robust both at the product and firm levels, see, for example, Baldwin and Harrigan (forthcoming); Bastos and Silva (2010); Helble and Okubo (2008); Manova and Zhang (2009).

We propose an explanation for this stylized fact that is based on the strategic behavior of firms. For this purpose, we modify the classical model of vertical product differentiation with oligopolistic competition (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982) to account for distance between trading partners. We use this model to investigate the effect of distance on the quality level of traded goods, when quality is a strategic variable for firms. In order to make our analysis sharper, we abstract from any supply-side (productivity) differences between trading partners and we normalize the consumer's income, so as to focus on the pure role of strategic interaction among firms. This approach implicitly assumes that firms are not negligible with respect to the market. In our framework, this is consistent with the observation that exporting firms are on average bigger than non-exporters (Bernard *et al.* 2003, using U.S. data, find that they ship on average 5.6 times more).

Our work directly relates to the flourishing literature analyzing the effects of distance on the quality level of traded goods, as measured by unit values. Many works tackle this issue by using monopolistic competition (Krugman and Helpman, 1985), comparative advantage (Eaton and Kortum, 2002) or monopolistic competition with heterogeneous firms (Melitz, 2003).<sup>3</sup> Recently, useful insights on the behavior of firms exposed to trade competition have been drawn by addressing to the Industrial Organization literature. This strand of literature usually combines consumer and/or firm heterogeneity with partial equilibrium analysis (see, e.g., Verhoogen 2008 and Khandelwal forthcoming). In particular, Verhoogen (2008) works out a model of North-South trade where the exporting firms of the poor country ship to the rich country commodities of higher quality relative to those produced for the domestic market, in order "to appeal to richer northern consumers" (Verhoogen, 2008, p. 489). In general, this literature delivers a mapping between *ex-ante* characteristics of trading part-

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<sup>1</sup> The role of product quality in international trade has been receiving a growing theoretical and empirical attention since the seminal contributions by Linder (1961) and Alchian and Allen (1964).

<sup>2</sup> It is common, in the literature, to measure the quality level of a commodity by its unit value (see e.g. Greenaway *et al.* 1995 and Wooldridge 2002). Noticeable exceptions are Hallak and Schott (2008) and Khandelwal (forthcoming), that disentangle unit values into quality and quality-related price components.

<sup>3</sup> See Baldwin and Harrigan (forthcoming) for an excellent recapitulation of these approaches and an exposition of their empirical implications.

ners -productivity, factor endowments, capability, consumer preferences or income- and the features of trade. More productive firms supply higher-quality products, earn larger profits and are more likely exporters, while richer countries tend to consume and import commodities of higher quality. Our paper is also related to two other strands of literature. The first investigates trade with quality-differentiated products (see, e.g., Eaton and Kierzkowski 1984; Shaked and Sutton 1984; Flam and Helpman 1987; Motta *et al.* 1997; Frascatore 2001; Cabrales and Motta 2001; Schott 2004; Hallak 2006; Choi *et al.* 2006; Sutton 2007). A second stream analyzes the optimal trade policy under trade with quality-differentiated products, and includes papers such as (Herguera *et al.*, 2002; Zhou *et al.*, 2002; Saggi and Sara, 2008).

The two distinguishing features of our approach are (i) the focus on strategic interaction with quality choice as a strategic instrument and (ii) the elimination of any supply-side differences among trading partners.

Our modeling choice proves to be useful in two respects. First, it identifies a new force shaping trade flows, which is based on the effect exerted by distance on the firms' strategic behavior both in price and non-price competition. Second, it allows to clearly distinguish between the price-setting and the quality choice (product design) activities, so that empirical implications for prices and qualities are drawn separately. As a last remark, we would like to stress that we are fully convinced that supply- and demand-side drivers play a crucial role in shaping trade flows, nonetheless we believe that our work sheds light on a novel, complementary mechanism that has been neglected so far.

Our modeling strategy is as follows. We consider an industry/sector with two firms producing at zero costs variants of a vertically differentiated commodity. One firm is located "away" from consumers so that a transport cost has to be paid to consume the good sold by that firm. We set up and solve a two-stage game where firms first simultaneously and at no cost select the quality level of the good they produce and then simultaneously set prices. We consider the location where the consumers reside as the "destination market", and focus on the prices set as well as on the quality levels of the goods available there. We show that trade costs and strategic behavior determine the role of "high-" versus "low-quality" producer for domestic and foreign firms, and that they influence the average quality level of goods in the "destination" country for given "roles" assigned to firms. In particular, the main outcomes of our model are the following. (i) when trade costs are "low" either firm may be the high- or low-quality producer, while when trade costs are "high", the firm located "away" from consumers can only be the low-quality producer, and that (ii) trade costs always increase the average quality level of the traded good. Furthermore, we show that (iii) both trade costs and the extent of feasible quality differentiation are crucial to determine the viability of trade. In particular, when trade costs are too high, or the quality spectrum is too narrow no trade is possible.

From the empirical standpoint, our results are in accordance both with the acquired evidence on the positive effect of distance on the quality level of traded goods, and with a higher likelihood of "zeros" in trade patterns as distance (measured by trade costs) increases. By contrast, the behavior of f.o.b. prices generated by our basic model is not consistent, at first sight, with empirical evidence. Indeed our analysis predicts that these prices should decrease with distance, whereas factual evidence maintains the opposite. In the section devoted to

the discussion of our results, however, we argue that extending our model to account for the consumers' income in the destination market may help to reconcile our outcomes with the data.

The paper is organized as follows. Section 2 presents the model and Section 3 solves it. Section 4 discusses our results with reference both to the empirical and to the theoretical literature. Section 5 proposes some extensions and, finally, Section 6 provides a short conclusion. All the proofs are relegated in the Appendix.

## 2 The Model

**Preliminaries** Our model is a partial-equilibrium one, analyzing a single industry/sector which is small compared to the rest of the economy. Assume that two firms operate in one country, one of them is physically settled in that country (domestic firm,  $D$ ) and the other one is producing its commodity far away (foreign firm,  $F$ ), so that a transport cost has to be paid in order to consume its good. Firms produce variants of a vertically differentiated good, we investigate the subgame-perfect Nash equilibria in a game where firms simultaneously select the quality level of their goods and subsequently set prices.

**Consumers** The country is inhabited by a continuum of consumers differing in their willingness to pay for quality. Consumers are uniformly distributed with unit density over the interval  $[0, 1]$  according to their appreciation for quality,  $\theta$ . Consumers purchase either one or zero units of the good. The utility derived by consumer  $\theta$  when purchasing one unit of variant  $j$  is  $U(\theta) = \theta u_j$ , with  $u_j \in ]0, \bar{u}]$  being the (commonly perceived) quality level of variant  $j$ . Similarly, let  $\rho_j$  denote the total price paid by consumers to buy one unit of good  $j$ . We define  $\rho_j$  as  $p_j + t$  with  $t \geq 0$  representing the unit transport cost paid by consumers, clearly  $t = 0$  if the purchased variant is produced locally, otherwise  $t > 0$ .<sup>4</sup> It is reasonable to assume that transport costs increase with the distance of the trading partner, thus, in the following, we will refer to  $t$  simply as to the “distance” for the foreign-firm to the destination market. Assume that the utility of no consumption is zero. Following Mussa and Rosen (1978) the surplus of consumer  $\theta$  is

$$U(\theta) = \begin{cases} \theta u_j - \rho_j & \text{when buying one unit of good } j ; \\ 0 & \text{when abstaining from consumption.} \end{cases}$$

Demands are obtained through the standard marginal consumer approach. Consumers choose the version of the good providing them with the largest surplus as long as this is positive, else they do not buy. Let  $h$  identify the firm selling the high-quality variant, and, similarly, let  $l$  label the firm supplying the low-quality one.<sup>5</sup> Standard computations return the value of  $\theta$  identifying the consumer indifferent between purchasing one unit of the high- and low-quality

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<sup>4</sup> With this specification the burden of transport costs is on consumers, so that the present is a shopping model. Our results, however, do not change if firms pay per unit transport costs in order to deliver the good to customers (shipping model). Calculations are available upon request.

<sup>5</sup>By anticipating an equilibrium argument, we assume that qualities are different at an equilibrium with two active firms in order to avoid price competition with an homogeneous good. See Section 3 and the Appendices for a detailed proof.

good and that indifferent between purchasing one unit of the low-quality good and abstaining from consumption. Their expressions are reported in the following.

$$\theta_1 = \frac{\rho_h - \rho_l}{u_h - u_l}; \quad \theta_0 = \frac{\rho_l}{u_l}. \quad (1)$$

Once obtained the marginal consumers expressions, the demand system under duopoly is easily derived.

$$\mathcal{D}_h = (1 - \theta_1); \quad \mathcal{D}_l = (\theta_1 - \theta_0). \quad (2)$$

Notice that  $\theta_1$  and  $\theta_0$  define the demands' bounds only if they lie within the interval  $[0, 1]$  and  $\theta_1 > \theta_0$ . Yet they may not do for some combinations of prices, qualities and transport costs. In this case the demand for the imported good vanishes.<sup>6</sup> Imagine for example high levels of  $t$  (when transport costs are prohibitively high the foreign good is not traded), or a domestic low-quality good with a quality level "close enough" to the imported high-quality one (all consumers prefer to patronize a -slightly- lower quality domestic producer but save on transport costs). In this case the demand addressing to the domestic firm needs to be re-defined accordingly. Furthermore, notice that  $\theta_1$  is not defined for  $u_h = u_l$ . Since in our model both prices and qualities are endogenously determined, this behavior of demands has to be carefully examined when characterizing the possible Nash equilibria of the game. We refer the reader to the Sections devoted to the equilibrium analysis for an accurate discussion on this point. As a minimal requirement for our analysis we make the following

**Assumption 1.**  $u_l^F - t > 0$ .

Assumption 1 states that the transport cost is such that the consumer with the highest willingness to pay for quality (located in 1 along the distribution of types) is willing to buy the low-quality good when produced abroad and priced at marginal cost (zero). Stated differently, this assumption implies that the domestic market is not a natural monopoly.

**Firms** Since the aim of our paper is to delve into the effects of distance of strategic behavior in determining the direction and the characteristics of trade flows, we abstract from any supply-side issue by (a) normalizing firms' production costs to zero and (b) assuming that product design in terms of vertical differentiation is costless as in Choi and Shin (1992).<sup>7</sup> This choice allows us to eliminate from quality setting any effects that are not linked to trade costs to the destination market, and thus to focus on the pure effect of distance on the quality of the shipped good. Firm's profits are, thus

$$\pi_j^i = \mathcal{D}_j^i p_j^i, \quad (3)$$

where  $j \in \{h, l\}$  is the quality level of firm  $i = D, F$ .

**Timing** The game we analyze has two stages. At the first firms simultaneously select the quality levels of their variants, at the second one they simultaneously set prices.

<sup>6</sup>The fact that some demands may be driven down to zero following parameter changes is not new in the literature on vertical differentiation, both with and without trade, see for example Wauthy (1996) and Frascatore (2001).

<sup>7</sup> Many papers dealing with quality and trade study the role of productivity/factor endowments asymmetries in shaping trade flows. See, in addition the papers cited in Section 1 and the references therein contained.

Our model then takes the form of a game  $\Gamma$  where the players are the two firms  $i = D, F$ , their strategies are price-quality vectors  $(p_i, u_i)$  and their payoffs are their profits  $\pi_i(\cdot)$ .

### 3 Equilibrium

The game is solved by backward induction, by addressing to the price stage first.

#### 3.1 Price stage

In this section we fully develop the pricing stage of the standard duopoly case only, namely that when both expressions in (1) lie within the  $[0, 1]$  interval. We will cope with the situations when they do not in the Section devoted to Equilibrium Analysis. The solution to this case involves standard calculations in the class of models of vertical product differentiation (see e.g. Gabszewicz and Thisse (1979)), so it will be quickly dealt with. The pricing decisions of firms depend upon their roles as high- or low- quality suppliers which, in turn, are determined at the first stage. Thus, we consider the two cases that correspond to the two different branches of game  $\Gamma$ , in the first (i) the high-quality producer is the domestic firm, in the second (ii) the foreign one.

##### (i) Domestic high-quality producer

In this case the  $D$ -firm has selected the high-quality version of the good at the first stage. Therefore profits accruing to firms are:

$$\pi_h^D = \mathcal{D}_h^D p_h^D, \quad \pi_l^F = \mathcal{D}_l^F p_l^F. \quad (4)$$

Simultaneous maximization of (4) w.r.t.  $p_h^D$  and  $p_l^F$  respectively yields the following expressions for the optimal first-stage prices.<sup>8</sup>

$$\hat{p}_h^D(u_h^D, u_l^F) = \frac{u_h^D[2(u_h^D - u_l^F) + t]}{4u_h^D - u_l^F}, \quad \hat{p}_l^F(u_h^D, u_l^F) = \frac{u_l^F(u_h^D - u_l^F) - t(2u_h^D - u_l^F)}{4u_h^D - u_l^F}. \quad (5)$$

By plugging (5) back into (4) we obtain firms' profits at the second stage:

$$\hat{\pi}_h^D(u_h^D, u_l^F) = \frac{u_h^{D2}[2(u_h^D - u_l^F) + t]^2}{(4u_h^D - u_l^F)^2(u_h^D - u_l^F)}, \quad \hat{\pi}_l^F(u_h^D, u_l^F) = \frac{u_h^D[(u_h^D - u_l^F)u_l^F - t(2u_h^D - u_l^F)]^2}{u_l^F(u_h^D - u_l^F)(4u_h^D - u_l^F)^2} \quad (6)$$

As noted above, the duopoly demand system  $\mathcal{D}_h^D, \mathcal{D}_l^F$  involves non-negative prices and quantities for firm  $F$  if and only if the difference between firms' quality levels is large enough. The precise condition is reported in the following.

**Remark 1.** *The foreign firm's price and demand are non negative under duopoly pricing if and only if  $u_h^D \geq \frac{u_l^F(u_l^F - t)}{u_l^F - 2t} > u_l^F$ .*

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<sup>8</sup>Easy calculations show that second order conditions are met as long as  $u_h^D > u_l^F$ , which is true by assumption in case (i).



If qualities are too similar, the low-quality firm (which is disadvantaged because of transport costs) cannot enjoy a positive market share under duopolistic competition. In other terms, if  $u_h^D < \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$  entry in the market is “blockaded”. We will expand on this point in section 4.

## (ii) Foreign high-quality producer

This case mirrors the previous, in that the foreign firm is now offering the high quality variant of the good. Accordingly, firms’ profits now are

$$\pi_h^F = \mathcal{D}_h^F p_h^F, \quad \pi_l^D = \mathcal{D}_l^D p_l^D \quad (7)$$

As in the previous case simultaneous maximization of profits in (7) gives the optimal prices in this case, which are reported hereafter.<sup>9</sup>

$$\hat{p}_h^F(u_h^F, u_l^D) = \frac{2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)}{4u_h^F - u_l^D}, \quad \hat{p}_l^D(u_h^F, u_l^D) = \frac{(u_h^F - u_l^D + t)u_l^D}{4u_h^F - u_l^D}. \quad (8)$$

Substitution of (8) back into (7) yields the following expressions for firms’ profits.

$$\pi_h^F(u_h^F, u_l^D) = \frac{[2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)]^2}{(u_h^F - u_l^D)(4u_h^F - u_l^D)^2}, \quad \pi_l^D(u_h^F, u_l^D) = \frac{u_h^F u_l^D (u_h^F - u_l^D + t)}{(u_h^F - u_l^D)(4u_h^F - u_l^D)^2}. \quad (9)$$

Inspection of (8) reveals that the price of the domestic low-quality producer is always positive, whereas the foreign high-quality producer’s optimal price (and consequently demand) is positive if, and only if  $u_h^F$  is large enough. More precisely

**Remark 2.** *The foreign firm’s price and demand are non negative under duopoly pricing if and only if  $u_h^F \geq \frac{1}{2} \left( t + u_l^F + \sqrt{t^2 + (u_l^F)^2} \right) > u_l^D$ .*

Similarly to case (i) if qualities are too similar a profit-dissipating price competition prevails and the disadvantaged firm is the foreign one, which in this case produces the high-quality good. Again, we refer to Section 4 for a discussion on this point.

In all the ensuing analysis we will focus on the situation where both firms enjoy strictly positive market shares, so as to rule out duopoly equilibria where one firm has a zero market share even if the price set is nil. With a slight abuse of terminology, we will refer to these “subgame-perfect Nash equilibria with two active firms” simply as “subgame-perfect Nash equilibria”.

## 3.2 Quality choice

Let us move now to the core of our paper, namely quality choice.<sup>10</sup> We will tackle separately cases (i) and (ii), proving the -possible- existence of subgame-perfect Nash equilibria in the two

<sup>9</sup>Again second order conditions are satisfied if  $u_h^F > u_l^D$ .

<sup>10</sup>One natural reference for this analysis is the article by Choi and Shin (1992). Their main result is that with costless quality choice and firms located in the same market (no transport costs) the firm producing the high-quality selects the upper bound in the quality space,  $\bar{u}$  in our notation, while its low-quality rival chooses a quality which is  $\frac{4}{7}\bar{u}$ . Choi and Shin (1992) develop a sequential-move game, while ours is simultaneous-move. By setting  $t = 0$  our model boils down to the simultaneous-move version of Choi and Shin (1992).

cases (Propositions 1 and 3) and performing comparative statics analysis on the equilibrium quality levels (Propositions 2 and 4). We will thus report our main economic results in Theorems 1 and 2. We start by case (i). In the ensuing analysis, for the sake of readability, we will omit the first-stage optimal prices when describing firms' strategies, in order to focus on quality levels instead.<sup>11</sup> We provide the economic intuition of our results at the end of this section, and we refer to Section 4 for a more articulated discussion on them.

### (i) Domestic high-quality producer

The main result in this case is the following.

**Proposition 1.** *In game  $\Gamma$  there exists a unique cutoff value for  $\bar{u}$ ,  $\tilde{u} > \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$  and a unique cutoff value for  $t, \bar{t} \in ]0, u_l^F[$  such that for all  $\bar{u} > \tilde{u}$  and  $t \in [0, \bar{t}[$  there is one and only one SPNE where the high-quality producer is domestic and the low-quality one is foreign. At this equilibrium  $u_h^{D*} = \bar{u}$  and  $u_l^{F*}(u_h^{D*}) = \underline{u}^F \in ]0, \bar{u}[$ .*

*Proof.* See Appendix A. □

We can proceed further by delving into the behavior of the equilibrium strategy  $\underline{u}^F$  as a function of  $t$ . Our findings are reported in what follows.

**Proposition 2.** *For every  $t < \frac{u^F}{2}$ ,*

- (i)  $\underline{u}^F \in ]\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}[$ ;
- (ii)  $\frac{\partial \underline{u}^F}{\partial t} > 0$  in the whole interval  $]\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}[$ .

*Proof.* See Appendix B. □

### (ii) Foreign high-quality producer

In case (ii) the main result is summarized in the following.

**Proposition 3.** *In game  $\Gamma$ , (a) when  $t < \tilde{t} \approx \frac{3\bar{u}}{125}$  and  $\bar{u} > u_l^D + t + \sqrt{(u_l^D)^2 + t^2} \equiv \hat{u}$  there exists one and only one SPNE where the high-quality producer is foreign and the low-quality one is local, in this case  $u_h^{F*} = \bar{u}$  and  $u_l^{D*} \equiv \underline{u}^D \in ]0, \bar{u}[$ ; (b) when  $t \geq \tilde{t}$  there exists no SPNE with a domestic low-quality producer and a foreign high-quality producer.*

*Proof.* See Appendix C. □

As in case (i) we analyze the characteristics of the optimal quality level for the domestic firm  $\underline{u}^D$ . The next proposition summarizes our results.

**Proposition 4.** *For every  $t \in [0, \bar{t}]$*

- (i)  $\underline{u}^D \in [\frac{4}{7}\bar{u}, \frac{4}{5}\bar{u}]$ ;
- (ii)  $\frac{\partial \underline{u}^D}{\partial t} > 0$ .

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<sup>11</sup>According to our choice firm  $i$ 's strategy  $(\hat{p}_j^i, u_j^i)$  will be reported as  $(u_j^i)$ .

*Proof.* See Appendix D. □

Propositions 1 and 3 characterize the subgame-perfect Nash equilibria of the two branches of game  $\Gamma$ . They define parameter regions in the  $(t, \bar{u})$  space where the game  $\Gamma$  has pure strategy SPNE with both firms selling positive quantities at positive prices.

In order to finalize equilibrium analysis, however, we need to analyze the relative size of  $\tilde{t}$  and  $\bar{t}$ . The next Lemma tackles this point.

**Lemma 1.**  $\tilde{t} < \bar{t}$  as long as  $\frac{27}{25}u_i^F < \bar{u} < \frac{164}{25}u_i^F$

*Proof.* Follows from direct comparison. □

Since Propositions 2 and 4 state that  $\underline{u}^F \in [\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}]$ , and  $\underline{u}^D \in [\frac{4}{7}\bar{u}, \frac{4}{5}\bar{u}]$  the condition in Lemma 1 is satisfied at equilibrium. We complete then our equilibrium analysis of this game by putting our results together in what follows.

**Theorem 1.** Let  $\bar{u} > \max\{\hat{u}, \tilde{u}\}$  and  $t < \bar{t}$ .

- (i) If  $0 \leq t < \tilde{t}$  then game  $\Gamma$  has two subgame-perfect Nash equilibria. At the first one the high-quality (res. low-quality) producer is the domestic (res. foreign) firm, at the second one the high-quality (res. low-quality) producer is the foreign (res. domestic) firm.
- (ii) If  $\tilde{t} \leq t < \bar{t}$  then game  $\Gamma$  has one and only one subgame-perfect Nash equilibrium at which the high-quality producer is the domestic firm and the low-quality producer is the foreign one.

Part (i) of Theorem 1 states that when distance is small all quality configurations may arise at equilibrium. This is in accordance with intuition: when distance is low firms compete “almost” in the same country, thus there is no reason for one of them to be more likely the high- (or low-) quality producer. By contrast, when distance is “large” (ii), our analysis predicts that one configuration only arises at equilibrium. Specifically the high-quality version of the good is locally produced, while the low-quality one is imported. Transport costs induced by distance impose a lower bound *above* marginal cost (zero in our case) to the price paid by consumers for the imported good, the domestic producer can exploit this asymmetry and “blockade high-quality entry” on the market by credibly committing to begin a price war with a -almost- homogeneous good. In this case distance acts as a credible aggressive-behavior commitment device for the local producer. Theorem 1 also stresses the role of the quality spectrum’s span. Indeed, when the feasible quality spectrum is “too narrow”  $\bar{u} < \max\{\hat{u}, \tilde{u}\}$  some of the above equilibria do not exist. In particular, when  $\bar{u} < \tilde{u}$  there is no equilibrium with a domestic high-quality producer and a foreign low-quality one, and when  $\bar{u} < \hat{u}$  there exists no equilibrium with a foreign high-quality producer and domestic low-quality one.<sup>12</sup>

The effect of distance is not restricted to assigning the *role* (high- versus low-quality producer) of the domestic and foreign firm. Indeed it also determines the *quality level* of the low-quality variant (see Propositions 2 and 4). The following theorem summarizes the results of Propositions 2 and 4.

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<sup>12</sup> These claims follow from the proofs of Propositions 1 and 3, Appendix E provides a heuristic argument for these results.

**Theorem 2.** *Let  $\bar{u} > \max\{\hat{u}, \tilde{u}\}$  and  $t < \bar{t}$ . The quality level of the low-quality good is always increasing with distance.*

Theorem 2 delivers a clear-cut result. Distance increases the quality level of the low-quality good, and the increase in quality is larger if the distance, as long as it is small enough ( $t < \bar{t}$ ) in order to make trade itself viable.

Although the result is the same in the case the low-quality producer is domestic or foreign, the economic intuition underlying the two cases is different. Consider first the case of a foreign low-quality producer. Trade costs harm the low-quality foreign firm since they increase the total price paid by consumers for the variant. Then the  $F$ -firm reaction is twofold. On the one hand it reduces the optimal price it charges on consumers, so as to dampen the effect of distance on consumers' choice (it is easily ascertained that  $\frac{\partial p^F(\cdot)}{\partial t} < 0$ ). On the other hand the  $F$ -firm attempts to recover the competitive edge eroded by (increased) distance by increasing the utility consumers derive from low-quality consumption through an increase in quality. On top of this, notice that the quality increase makes the domestic and foreign products more similar, harshening further price competition. Consider now the case where the low-quality producer is domestic. An increase in the transport costs causes an increase in the total price consumers have to pay for the high-quality variant, making it less attractive and dampening price competition. Consequently, the domestic low-quality producer can increase the quality level of its good (and extract more surplus from consumers), and yet avoid to engage in a fiercer price war due to more homogeneous goods.

We may combine the results of Theorems 1 and 2 in the following.

**Corollary 1.** *The expected quality level of the traded good always increases with distance.*

The term “expected” can obviously be dropped for  $t > \bar{t}$ , while it comes from equilibrium multiplicity when  $t \in [0, \bar{t}]$ , since each firm has a positive probability to be the quality leader.

## 4 Discussion

In this section we discuss our results with reference to the existing empirical literature in order to put our analysis into perspective.

**Quality and distance.** The main message conveyed by our paper (Corollary 1) is that distance increases the average quality level of goods available for consumption in the country of destination. Corollary 1 is in direct reference with the literature highlighting the increase of the quality level of traded goods the greater the distance of their destination market (Baldwin and Harrigan, forthcoming; Bastos and Silva, 2010; Johnson, 2010; Helble and Okubo, 2008; Manova and Zhang, 2009). When distance is high enough -case (ii) of Theorem 1- the interpretation is straightforward, since the quality level of the imported (low-quality) good is increasing in distance. When distance is “large” (Theorem 1(i)) the interpretation is for the “expected quality level of import”. Indeed, an increase in the distance results in an increase of the quality level of the imported good only when the low-quality producer is foreign, while

it has no effect on the import quality level when the low-quality firm is local.<sup>13</sup> As explained above, however, equilibrium multiplicity makes both configurations *ex ante* possible, thus we conclude that distance causes the *expected* quality level of import to increase also when trade partners are “close”. Equilibrium multiplicity may be eliminated by applying, for example, the risk-dominance selection criterion (Motta *et al.*, 1997). In this case the only surviving equilibrium would be that with a domestic high-quality producer and a foreign low-quality producer, which would reinforce our results. Our result provides an argument for this empirical evidence which hinges on the interaction of distance and non-price strategic competition and suggests that distance itself does not only influence the intensity of trade flows (as suggested by “gravity” models, see e.g. Feenstra *et al.* (2001)), but qualitatively shapes them.

Corollary 1 may also be interpreted with reference to the “Alchian-Allen Conjecture” (AAC), that predicts that per unit trade costs increase the relative price of low-quality goods in their country of destination relative to their country of origin and thus reduce their consumption share at destination, resulting in a higher average quality consumption there (Alchian and Allen, 1964, p. 64); for a recent treatment of the AAC see Hummels and Skiba (2004). The AAC compares consumption shares, but our analysis suggests that per unit trade costs should cause an increase in the average quality of consumption also because *the average quality of the goods itself increases due to trade*, not only because the relative consumption of goods of *given* quality modifies. In particular, the quality level of the imported low-quality goods is higher the farther away are trading partners, and the farther is the high-quality foreign producer, the higher the quality of the domestic production. These remarks suggest that the AAC should have a broader scope than it is currently acknowledged, since it should encompass both consumption shares and quality levels.

**F.o.b. prices and distance.** It is worth to keep in mind that the empirical analyses use f.o.b. prices as proxies of quality, and the conclusion that quality increases with distance is drawn by the observation that f.o.b. prices increase with distance. In the version of the model we have presented above, by contrast, f.o.b. prices *decrease* with distance. This is due to two different effects. On the one hand it is easy to ascertain from (5) and (8) that, *ceteris paribus*, the price of any imported good is always decreasing in  $t$ : the higher is the distance to the trading partner, the higher is the “discount” on the f.o.b. price that the exporting firm should set to compensate for the distortion due to transport costs (see e.g. Melitz and Ottaviano 2008; Kneller and Yu 2008). On the other hand, an increase in  $t$  triggers an increase in the quality level of the low-quality good. This makes variants more homogeneous and thus harshens price war, with a decrease in the general price level and, in particular, in the price of the imported good. Both these effects entail a decrease in f.o.b. prices. Yet our model may accommodate with the behavior of unit values that is empirically accepted once the income of consumers is explicitly taken into account. In fact, our choice to set equal to 1 the upper bound of the support of the willingness to pay for quality  $\theta$  amounts to normalizing the income of the richest consumer also, since a consumer’s higher willingness to pay for quality is commonly assumed to reflect a higher income of that consumer. A side-effect of this normalization is

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<sup>13</sup>Recall that the high-quality firm always selects the highest feasible quality level because of costless product design.

that income disappears in all the equations of the model. If the upper bound of the income distribution were not normalized, but set equal to, say,  $\bar{\theta}$ , then equilibrium prices (and profits) would positively depend on this parameter.<sup>14</sup> In our case, letting  $\theta \sim U[0, \bar{\theta}]$  would yield as second stage prices

$$\hat{p}_h^D = \frac{u_h^D [2\bar{\theta}(u_h^D - u_l^F) + t]}{4u_h^D - u_l^F} \quad (10)$$

$$\hat{p}_l^F = \frac{\bar{\theta} u_l^F (u_h^D - u_l^F) - t(2u_h^D - u_l^F)}{4u_h^D - u_l^F} \quad (11)$$

in case (i), and similar expressions in case (ii). It is straightforward to observe from these expressions that, all else equal, prices increase with  $\bar{\theta}$  (a higher income results in a higher willingness to pay for quality and thus firms may charge higher prices). Now, recall that Baldwin and Harrigan (forthcoming) show that the presence of “zeros” is negatively related to the country of destination’s GDP and positively related to distance. Stated differently, the higher the trading partner’s income, the more likely trade is non-zero and the farther away is the partner, the less likely trade is. Thus, the more distant a country, the more likely this country should be of “high” income if trade flows are non-zero. This suggests the existence of a positive correlation between distance and income of the trading partner when observing a non-zero trade flow. In our model with non-normalized income, this assumption results in a positive relation between  $\bar{\theta}$  and  $t$ , namely  $\bar{\theta}(t)$  such that  $\bar{\theta}'(t) > 0$ . On the basis of our previous observations, this introduces a force pushing up prices the farther away the trading partner. The extended model explicitly incorporating income is analytically solvable, however, its outcome is little informative because of the complexity of the equilibrium values. Yet, some useful insights can be drawn by making use of numerical examples. These exercises confirm the trade-off we have outlined above, which is depicted in Figure 1. The “flat” surface represents  $p_l^F(\bar{u}, \underline{u}^F(\bar{\theta}, t))$  -the price of the exported good in case (i)- for  $\bar{u} = 3$ . The f.o.b. price decreases along the distance axis  $t$  whereas it increases with income of the consumers in the destination market,  $\bar{\theta}$ . The functions on the surface represent two possible relations between  $\bar{\theta}$  and  $t$ . The function  $\bar{\theta}^1(t)$  (dashed curve) is characterized by a “slow” increase with distance, while in  $\bar{\theta}^2(t)$  (solid curve) income steeply increases with it. If the relation between the distance to the export market and the income of the trading partner is as in  $\bar{\theta}^1(t)$  then the f.o.b. price decreases with distance, while if it is as in  $\bar{\theta}^2(t)$  the f.o.b. price increases with the distance of the destination market. The overall effect on f.o.b. prices depends on the trade-off between the negative direct effect distance exerts on f.o.b. prices and its positive effect through income.

**Zeros and distance** The last point has introduced another issue which is empirically relevant, namely that of trade zeros. Following the definition of Baldwin and Harrigan (forthcoming), page 16, a zero is “a trade flow which could have occurred but did not”. Our model is in accordance with the factual evidence reported above on the positive relation between the likelihood of zeros and the distance to the destination market. In fact Theorem 1 states that a duopoly equilibrium with two active firms exists only when trade costs are low enough ( $t < \bar{t}$ ). When  $t$

<sup>14</sup> This is a standard feature of vertical differentiation models, see Gabszewicz and Thisse (1979) or Wauthy (1996).

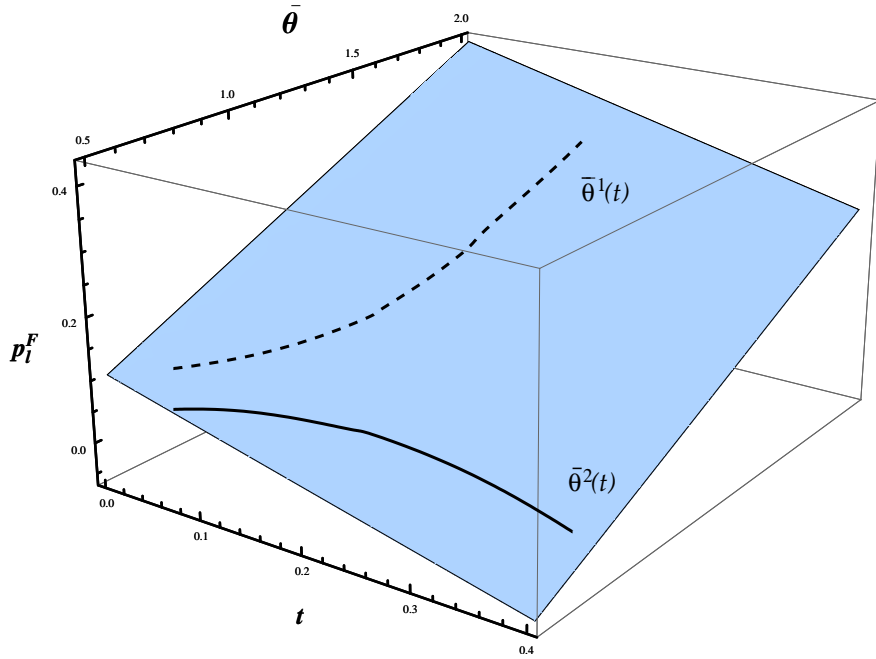


Figure 1: F.o.b. prices, income and distance.

is large there is no duopoly equilibrium with two active firms and thus no trade. Furthermore, as noted above, for given qualities, the higher is  $t$  the less likely the condition ensuring that second-stage prices are positive (see Remarks 1 and 2.) Although comparative statics cannot be performed on the value of  $\bar{t}$ , numerical exercises suggest that this cutoff value is increasing with  $\bar{u}$ , the upper bound of the feasible quality spectrum. This is in accordance with common sense: the wider is the range over which qualities can be selected, the easier is to “relax price competition through product differentiation”,<sup>15</sup> and thus the larger the distance compatible with viable trade. Stated differently, the wider the technical capacity to differentiate products within the industry, the more distant can be the trading partners. Finally, notice that the capacity to differentiate, as summarized by  $\bar{u}$  also influences the equilibrium configuration: it is easy to ascertain that  $\bar{t}$  is increasing in  $\bar{u}$ : for given  $t$ , the wider the feasible quality range, the larger is the parameter region where the foreign firm may be the quality leader.

## 5 Extensions

The present model is built on the easiest case of two single-product firms, and focuses on the trade flow from a “foreign” firm to the destination market where consumers reside and another producer is settled. This section discusses some possible extensions.

**Two-way trade flows and asymmetric trade costs.** A straightforward extension consists in modeling consumers in the “foreign” location as well, and to allow firms to produce

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<sup>15</sup>Gabszewicz and Thisse (1979); Shaked and Sutton (1982).

one variant of the good for each market (that is to say let firms design a “domestic” version of the good and an “export” one). In this way, the model would be a full-fledged intra-industry trade model. When distance is not prohibitively high ( $t < \bar{t}$ ), two-way trade flows will emerge at equilibrium. If arbitrage is excluded, the model’s outcomes should be similar to those of Theorem 1, with each firm being a high- or low-quality producer on each market for  $t \in [0, \tilde{t}]$ , but playing the role of high-quality producer for the domestic market and low-quality exporter to the other one for  $t \in [\tilde{t}, \bar{t}]$ . Similarly, by reverting to the interpretation of  $t$  as a generic trade cost, including distance as well as any other obstacle to trade, asymmetric unit trade costs could be taken into account, say  $t^F$  from the Domestic to the Foreign country and  $t^D$  from country  $F$  to  $D$ . In this case we would obtain parameter regions in the  $t^F$  and  $t^D$  segments similar to these in Propositions 1 and 3. By combining them one would obtain the equivalent of Theorem 1. In all cases the quality of the low-quality good should increase with the distance of the destination market, as in Theorem 2. Thus, a result in line with Corollary 1 should hold as well. A similar outcome should obtain if transport costs were the same for any trade direction but consumers had different incomes. Similarly, if there were more than two countries, and each firm were still restricted to ship one variant only to each market, one should expect that the quality of the good shipped by each firm to each destination should increase with its distance, as long as trade costs allow for viable trade.

While the extension to intra-industry trade with firms producing a single variant for each market is straightforward, more problematic would be accounting for firms producing many variants for each market (see, e.g. Bernard *et al.* 2010; Eckel and Neary 2010). Indeed, in this case, domestic firms may use non-price competition in the form of product proliferation to limit, if not blockade at all, entry of foreign firms (see Gabszewicz and Thisse 1980 or De Fraja 1996 for example). The domestic incumbent may try to fill in the lower-quality product niches in order to leave no room in the product space for potential foreign entrants, at the cost of a partial self-cannibalization. Clearly, the effectiveness of this instrument may be reduced by the presence of costs of product development, but the incentive for firms to use their product range as an anticompetitive device may substantially modify the analysis.

**Many firms** Another possible extension contemplates to keep the focus on the  $D$  country and assume that many single-product firms may operate there, some being settled in  $D$  and some others exporting their product from  $F$ . In this case the equilibrium quality specialization pattern of firms (which firm is the top quality producer, which is the next, and so on) would be much more complicated than that found in this paper. It is reasonable to conjecture, however, that in such a model there would be (i) a cutoff value for distance above which no trade would be observed and (ii) another threshold above which the top quality producer(s) is local and the quality followers are foreign. Also, in this case, a (negative) relation between distance and the probability of finding foreign producers among the “quality leaders” should emerge. Finally, it is reasonable to think that the effect of distance on the quality level of goods should keep unchanged.

**Production costs** Our analysis abstracts from any supply-side feature. Fixed production costs do not modify firm’s pricing and quality design decisions, thus would leave our results



qualitatively unchanged. Similarly, introducing symmetric marginal production costs should not modify the main outcomes of our paper. By contrast, if production costs are quality-dependent, a large difference between the production costs of the high-quality variant and that of the low-quality one may prevent the foreign firm to become the high-quality producer, so that part (ii) only of Theorem 1 would still hold at equilibrium. A result in line with Theorem 2 should hold in this case.

On the other hand, a costly product design would have as a first effect that the high-quality producer would possibly not select the high-end of the quality spectrum. Furthermore, if quality adjustment costs increase “too rapidly” in quality, then it may be not profitable to become the high-quality producer, thus our results would be invalidated. By contrast, if these costs are not “too convex”, then the mechanism we have highlighted in this paper should be still present and should be included among the forces governing equilibrium outcomes.

## 6 Conclusion

This paper contributes to the vast and growing literature on trade and quality by identifying a mechanism explaining the observed increase in unit values (and thus quality) with the distance of the country of shipping. This mechanism, which complements the ones that are usually reckoned as shaping quality-differentiated trade flows, acts through the effect of distance on the strategic behavior of firms *when quality is a choice variable for oligopolistic firms*. In fact, the literature usually seeks the determinants of firms’ specialization, trade viability and trade flows characteristics among ex ante productivity (broadly speaking) and/or preference differences between firms and countries. In contrast, by removing any ex ante asymmetry (except location) between trading partners, we have delved into the interaction of distance and strategic behavior in determining both the role (high- versus low-quality producer) of domestic and foreign firms and the effects of trade cost on quality-differentiated trade flows. In particular, we have analyzed the relation between trade, quality and distance by means of a parsimonious trade model with endogenous quality choice and oligopolistic competition. We have modeled the interaction between firms as a two-stage game (quality design then price setting) with simultaneous moves at each stage. We have identified the conditions under which trade is viable and characterized the subgame-perfect Nash equilibrium of such a game. Our main result is that distance unambiguously increases the average quality level of the traded good. This is in accordance with recent empirical findings on the relationship between the quality of traded goods and the distance of the country of destination. Also, by accounting for income variability, our model provides insights on the behavior of f.o.b. prices with respect to distance of the trading partner.

## A Proof of Proposition 1

We prove Proposition 1 through a series of lemmata.

**Lemma 2.** *Let  $u_h^D \geq u_l^F$ , then there exists a unique  $\tilde{u} > \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$  such that for all  $\bar{u} > \tilde{u}$  the unique maximizer of  $\hat{\pi}(u_h^D)$  is  $u_h^{D*} = \bar{u}$ .*

*Proof.* (i) Consider the range  $u_h^D \in ]\frac{u_l^F(u_l^F - t)}{u_l^F - 2t}, \bar{u}]$  first. In this case the market structure is duopolistic and second stage profits are as in (4). The partial derivative of  $\hat{\pi}(u_h^D)$  w.r.t.  $u_h^D$  is

$$\frac{\partial \hat{\pi}_h^D}{\partial u_h^D} = \frac{u_h^D(t + 2u_h^D - 2u_l^F)f(u_h^D)}{(u_h^D - u_l^F)^2(4u_h^D - u_l^F)^3} \quad (12)$$

where

$$f(u_h^D) = \left[ 8(u_h^D)^3 - (u_h^D)^2(4t + 14u_l^F) - u_h^D u_l^F t - 10(u_l^F)^2 - 4(u_l^F)^3 + 2(u_l^F)^2 t \right] \quad (13)$$

The denominator and the first two terms of the numerator are positive. Our task is therefore to prove that the polynomial  $f(u_h^D)$  is positive. Notice that  $f(\frac{u_l^F(u_l^F - t)}{u_l^F - 2t}) > 0$ . Consider now the first-order derivative of (13)

$$f'(u_h^D) = 24(u_h^D)^2 - 4(2t + 7u_l^F)u_h^D - (t - 10u_l^F)u_l^F,$$

it can be proven that for all  $u_h^D \geq \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$  this expression is positive, and thus so is  $f(u_h^D)$ . This implies that (12) is positive for all  $u_h^D \in ]\frac{u_l^F(u_l^F - t)}{u_l^F - 2t}, \bar{u}]$  and ultimately that the profit-maximizing quality is  $\bar{u}$ .

(ii) Consider now the interval  $[u_l^F, \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}]$ . In this case both the price and the demand of the foreign-quality firm are zero under the pricing rules (5), thus (2) no longer defines the demand system. The price charged by the domestic quality firm is then its best reply at the second stage against the foreign-quality firm setting a zero price, namely  $\tilde{p}_h^D(u_h^D, u_l^F) = \frac{1}{2}(u_h^D - u_l^F + t)$ , and its profits are  $\tilde{\pi}_h^D(u_h^D, u_l^F) = \tilde{p}_h^D(\cdot)(1 - \frac{\tilde{p}_h^D(\cdot) - t}{u_h^D - u_l^F})$ . It is then a matter of simple calculations to ascertain that  $\tilde{\pi}_h^D$  is maximized for  $u_h^D = \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$ .

(iii) Since  $\hat{\pi}_h^D(\cdot)$  is increasing in  $\bar{u}$  there exists a threshold  $\tilde{u} > \frac{u_l^F(u_l^F - t)}{u_l^F - 2t}$  such that for all  $\bar{u} > \tilde{u}$ .

$$\tilde{\pi}_h^D(\frac{u_l^F(u_l^F - t)}{u_l^F - 2t}) < \hat{\pi}_h^D(\bar{u})$$

(iv) Notice that if  $u_h^D = u_l^F$  the good is homogeneous and thus equations in (2) no longer represent demands. In this case the price of the domestic high-quality good is  $t - \varepsilon$  while that of the foreign low-quality one is zero. At these prices the demand of the foreign good is zero, while the one of the domestic firm is  $1 - \frac{t - \varepsilon}{u_l^F}$ . Accordingly, its profits write  $(1 - \frac{t - \varepsilon}{u_l^F})(t - \varepsilon)$ . It is easily ascertained that this profit falls short of  $\hat{\pi}_h^D(\bar{u})$  as long as  $u_h^D > \tilde{u}$ . □

Move now to the low-quality firm. We state the following

**Lemma 3.** Let  $u_h^D \geq u_l^F$ , then there exists a unique  $\bar{t} \in ]0, u_l^F[$  such that for all  $t < \bar{t}$  the unique maximizer of  $\hat{\pi}_l^F(\cdot)$ ,  $u_l^{F*}(u_h^D) \in ]0, u_h^D[$ .

*Proof.* (i) Assume that  $u_l^F < u_h^D$  first and consider the derivative of  $\hat{\pi}_l^F$  w.r.t.  $u_l^F$ .

$$\frac{\partial \hat{\pi}_l^F}{\partial u_l^F} = - \frac{u_h^D \left[ u_l^{F^2} - u_l^F(t + u_h^D) + 2tu_h^D \right] g(u_l^F)}{u_l^{F^2}(u_h^D - u_l^F)^2(4u_h^D - u_l^F)^3}, \quad (14)$$

where

$$g(u_l^F) \equiv \left[ (u_l^F)^3(7u_h^D - 2t) + (u_l^F)^2 \left[ 9tu_h^D - 11(u_h^D)^2 \right] + u_l^F \left( 4(u_h^D)^3 - 18t(u_h^D)^2 \right) + 8tu_h^D^3 \right].$$

First of all notice that over the relevant interval  $[0, u_h^D[$  the derivative (14) is continuous in  $u_l^F$ . Second, notice that concavity of (14) with respect to  $u_l^F$  requires that  $t < \bar{t}$ , with  $0 < \bar{t} < u_l^F$ .<sup>16</sup> We proceed by finding the zeros of (14). This function has five roots, namely the two zeros of the first term at the numerator, and three zeros of  $g(u_l^F)$ . The roots of the first term are real but they can be disregarded as candidate maximizers because, although lying in the interval  $[0, u_h^D]$ , they do not fulfill local second order conditions and in correspondence of these values  $\hat{\pi}_l^F(\cdot) = 0$ .<sup>17</sup> Consider now the remaining factor, the polynomial function  $g(u_l^F)$ , which is a one-parameter family of cubics depending on  $t > 0$ . First note that  $g(0) = 16t(u_h^D)^3 > 0$ . Since  $\lim_{u_l^F \rightarrow -\infty} g(\cdot) = -\infty$  this implies that  $g(u_l^F)$  and consequently (14) admit a negative real root. In the following we prove that  $g(\cdot)$  admits two further real roots for every value of  $t > 0$ , but only one of them belongs to the relevant interval  $[0, u_h^D[$ , whereas the other one is necessarily larger than  $u_h^D$  and thus not acceptable. To see this, notice that  $g(u_h^D) = -9t(u_h^D)^3 < 0$  and  $\lim_{u_l^F \rightarrow \infty} g(\cdot) = \infty$ . By continuity the function  $g(\cdot)$  (and thus (14)) must cross the real axis at a value larger than  $u_h^D$ . As noted above this root is not acceptable. Consequently there exists another value  $u_l^{F*} \in ]0, u_h^D[$  such that  $g(u_l^{F*}) = \frac{\partial \hat{\pi}_l^F(u_l^{F*}, u_h^D)}{\partial u_l^F} = 0$ . Since the solutions to the first term of (14) are internal to  $[0, u_h^D]$  and correspond to local minima, the solution  $u_l^{F*}(u_h^D)$  lies between them and is a local maximum for all  $t \in [0, \bar{t}]$ .

- (ii) Assume now that  $u_l^F = u_h^D$ . In this case (2) is no longer the demand system since price competition with a homogeneous good triggers a Bertrand war. It is straightforward to ascertain that in this case the optimal price of the high-quality firm is  $t - \varepsilon$  and that the foreign one is 0. Consequently no consumer patronizes the foreign producer and thus its profit is nil.

□

**Lemma 4.** There exists a unique cutoff  $\tilde{u} > u_l$  such that for all the  $\bar{u} > \tilde{u}$ , the pair  $(\bar{u}, \underline{u}^F)$ , where  $\underline{u}^F \equiv u_l^{F*}(\bar{u})$ , is a couple of mutual best replies at the quality-choice stage of game  $\Gamma$  for the domestic and foreign firms respectively, thus they are the quality levels chosen at a subgame perfect Nash equilibrium of this game.

<sup>16</sup>The cumbersome expression of  $\bar{t}$  is available upon request.

<sup>17</sup>Calculations are available upon request.

*Proof.* The proof of this lemma requires that no firm has a profitable deviation from the candidate equilibrium strategies  $\bar{u}$  for the  $D$  firm and  $\underline{u}^F$  for the  $F$ -firm<sup>18</sup>. Note that the robustness of these strategies has to be checked against unilateral deviations in the whole strategy space, not only in that defined by case (i). In other terms, to demonstrate Lemma 3 we need to show that no firm wants to leapfrog its rival (see, for example, Motta *et al.* 1997).

- (i) Consider firm  $D$  first. We need to prove that this firm has no profitable deviations from  $(\bar{u})$  when its rival plays  $(\underline{u}^F)$ . Clearly the only strategy (sub-)space where deviations have to be looked for is  $[0, \underline{u}^F]$ . In this case the actual high-quality producer is the foreign firm, while the domestic one plays the role of the low-quality producer, thus we need to re-define firm- $D$ 's profits to take into account this fact. Let  $u_{\mathcal{L}}^D \in [0, \underline{u}^F]$  and  $p_{\mathcal{L}}^D$  be the quality level and the price the  $D$ -firm deviates to, and, accordingly, let its deviation demand be<sup>19</sup>

$$\mathcal{D}_{\mathcal{L}}^D = \frac{\hat{p}_I^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D}, \quad (15)$$

thus the deviation profits are

$$\pi_{\mathcal{L}}^D = p_{\mathcal{L}}^D \left[ \frac{\hat{p}_I^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D} \right]. \quad (16)$$

Simple calculations show that there exists a unique price that maximizes (16), namely  $\hat{p}_{\mathcal{L}}^D = \frac{u_{\mathcal{L}}^D [2t\bar{u} + \bar{u}u^F - (u^F)^2]}{2u^F(4\bar{u} - u^F)}$ , which can be plugged back into (16) to obtain the expression for the deviation profits as a function of quality levels only:

$$\hat{\pi}_{\mathcal{L}}^D(u_{\mathcal{L}}^D, \bar{u}, \underline{u}^F) = \frac{[2t\bar{u} + (\bar{u} - \underline{u}^F)\underline{u}^F]^2 u_{\mathcal{L}}^D}{4u^F(4\bar{u} - \underline{u}^F)^2(\underline{u}^F - u_{\mathcal{L}}^D)}. \quad (17)$$

It can be proved that the demand  $\mathcal{D}_{\mathcal{L}}^D$  is always increasing in  $u_{\mathcal{L}}^D$ , however its upper bound stops growing as  $u_{\mathcal{L}}^D$  hits  $\hat{u}_{\mathcal{L}}^D = \frac{2\bar{u}u^F[3u^F - \bar{u}]}{u^F[7\bar{u} - \underline{u}^F] - 2t\bar{u}} < \underline{u}^F$ . For all  $u_{\mathcal{L}}^D \in [\hat{u}_{\mathcal{L}}^D, \underline{u}^F]$  the deviating firm's profit is  $(1 - \frac{\hat{p}_{\mathcal{L}}^D}{u_{\mathcal{L}}^D})p_{\mathcal{L}}^D$  which is always increasing in  $u_{\mathcal{L}}^D$ , thus the profit-maximizing quality level is  $\underline{u}^F$ .<sup>20</sup> The corresponding profit is:

$$\hat{\pi}_{\mathcal{L}}^D(u_{\mathcal{L}}^{D*}, \bar{u}, \underline{u}^F) = \pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F) = \frac{[\underline{u}^F(7\bar{u} - \underline{u}^F) - 2t\bar{u}][\underline{u}^F(\bar{u} - \underline{u}^F) + 2t\bar{u}]}{4u^F(4\bar{u} - \underline{u}^F)^2}. \quad (18)$$

Direct comparison of  $\hat{\pi}_h^D(\bar{u}, \underline{u}^F)$  and  $\pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F)$  returns that

$$\hat{\pi}_h^D(\bar{u}, \underline{u}^F) > \pi_{\mathcal{L}}^{D*}(\bar{u}, \underline{u}^F). \quad (19)$$

In principle there is another possible deviation available to the domestic firm, namely

<sup>18</sup>Recall that we summarize firms' strategies by reporting optimal qualities only,  $(\hat{p}_j^i, u_j^{i*})$  is represented by  $(u_j^{i*}(\bar{u}))$ .

<sup>19</sup>The demand is as in (16) as long as  $p_{\mathcal{L}}^D$  and  $u_{\mathcal{L}}^D$  are such that  $\frac{\hat{p}_I^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} < 1$ . If  $\frac{\hat{p}_I^F(\cdot) + t - p_{\mathcal{L}}^D}{\underline{u}^F - u_{\mathcal{L}}^D} \geq 1$  then the demand is  $1 - \frac{p_{\mathcal{L}}^D}{u_{\mathcal{L}}^D}$ .

<sup>20</sup>Notice that a Bertrand war is avoided here because the lower bound to the foreign good price is equal to  $t$ , the transport cost. Furthermore, at the optimal deviation quality the condition defined by Remark 1 is not met, thus the demand for the foreign good goes down to zero, and the market is solely served by the deviating firm.

to set  $u_L^D = \underline{u}^F$  and  $p_L^D = \hat{p}_l^F(\bar{u}, \underline{u}^F) + t - \varepsilon$ . It can be proven, however, that the profit earned in this case never exceeds  $\hat{\pi}_h^D(\bar{u}, \underline{u}^F)$  as long as  $\bar{u} > \tilde{u}$ . We conclude thus that there exists no profitable deviation from  $\bar{u}$  for firm  $D$  when firm  $F$  selects  $\underline{u}^F$ .

- (ii) Firm  $F$  has no profitable deviation for  $u_l^F < \bar{u}$ , since  $\underline{u}^F$  is the unique profit maximizer for all  $u_l^F < u_h^D$ , and by construction it cannot deviate to a quality higher than  $\bar{u}$ , the maximum level of the quality spectrum. Finally, a deviation to  $\bar{u}$  is not profitable because it would entail a price war over a homogeneous good.

□

Proposition 1 is obtained by combining the results of Lemmata 2-4.

## B Proof of Proposition 2

Although we are able to compute explicitly the equilibrium value for  $u_l^F$ , this turns out to be exceedingly cumbersome and thus little informative.<sup>21</sup> We shall therefore perform comparative statics analysis by means of indirect methods.

As a general remark notice that  $\bar{t} < \frac{u_l^F}{2}$ .

- (i) Consider the function  $g(u_l^F)$  defined in Lemma 3:

$$g(u_l^F) = (u_l^F)^3(7\bar{u} - 2t) + (u_l^F)^2[9t\bar{u} - 11(\bar{u})^2] + u_l^F(4(\bar{u})^3 - 18t(\bar{u})^2) + 8t\bar{u}^3,$$

within the interval  $[\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}]$ . At the left-end boundary of the interval the function's value is  $g(\frac{4}{7}\bar{u}) = \frac{96t\bar{u}^3}{343} > 0$ , while at the right-end its value is

$$g\left(\frac{29}{49}\bar{u}\right) = \frac{(9973t - 4060\bar{u})\bar{u}^3}{117649}.$$

Notice, however that the restriction  $t < \frac{u_l^F}{2}$  implies  $t < \frac{29\bar{u}}{98}$ , thus the value of  $g(\frac{29}{49}\bar{u})$  can be evaluated as

$$\frac{(9973t - 4060\bar{u})\bar{u}^3}{117649} < -\frac{108663}{11529602}\bar{u}^4 < 0.$$

By continuity, the relevant root of  $g(u_l^F)$ , that is the optimal strategy for the foreign firm, belongs to the interval  $(\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u})$ .

- (ii) Consider  $g(u_l^F, t)$  as a two-variable function of  $u_l^F$  and  $t$ , restricted to the domain  $[\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}] \times [0, \frac{29}{98}\bar{u}]$ . By the Implicit Function Theorem in this domain there exists a  $C^1$  function  $\underline{u}^F(t)$  such that:

$$\underline{u}^{F'}(t) = -\frac{\frac{\partial g}{\partial t}}{\frac{\partial g}{\partial \underline{u}^F}} = \frac{2(\underline{u}^F)^3 - 9\bar{u}(\underline{u}^F)^2 + 18\bar{u}^2\underline{u}^F - 8\bar{u}^3}{3(7\bar{u} - 2t)(\underline{u}^F)^2 + 2(9t\bar{u} - 11\bar{u}^2)\underline{u}^F + 4\bar{u}^3 - 18t\bar{u}^2}. \quad (20)$$

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<sup>21</sup>The value of  $\underline{u}^F$  is available upon request from the authors.

In order to evaluate the sign of (20) consider first the function at the numerator

$$N(\underline{u}^F) = 2(\underline{u}^F)^3 - 9\bar{u}(\underline{u}^F)^2 + 18\bar{u}^2\underline{u}^F - 8\bar{u}^3.$$

Notice that  $N(\frac{4}{7}\bar{u}) = -\frac{96\bar{u}^3}{343}$ ,  $N(\frac{29}{49}\bar{u}) = -\frac{3084\bar{u}^3}{15625}$ , and since

$$N'(\underline{u}^F) = 6[(\underline{u}^F)^2 - 3\bar{u}\underline{u}^F + 6\bar{u}^2] > 0,$$

we conclude that  $N(\underline{u}^F)$  is monotonically increasing and negative in the whole domain  $[\frac{4}{7}\bar{u}, \frac{29}{49}\bar{u}]$ .

Move now to the function at the denominator

$$D(\underline{u}^F, t) = 3(7\bar{u} - 2t)(\underline{u}^F)^2 + 2(9t\bar{u} - 11\bar{u}^2)\underline{u}^F + 4\bar{u}^3 - 18t\bar{u}^2.$$

Since  $\frac{\partial D}{\partial t} < 0$  for every admissible  $\underline{u}^F$ , the gradient of  $D(\cdot)$  never vanishes in the rectangle under scrutiny, hence we evaluate the function at its boundary to establish whether it may change its sign.

Firstly the evaluation of  $D(\cdot)$  at the boundaries for the choice variable yields:

$$\begin{aligned} D\left(\frac{4}{7}\bar{u}, t\right) &= -\frac{6}{49}(79t + 14\bar{u})\bar{u}^2 < 0, \\ D\left(\frac{29}{49}\bar{u}, t\right) &= -\frac{3}{2500}(7982t + 1413\bar{u})\bar{u}^2 < 0, \end{aligned}$$

for all values of  $t$ .

Secondly the evaluation of  $D(\cdot)$  at the boundaries for the unit transport cost returns:

$$D(\underline{u}^F, 0) = \bar{u}(21(\underline{u}^F)^2 - 22\underline{u}^F + 4\bar{u}^2) < -\frac{417}{343}\bar{u}^3 < 0,$$

and

$$\begin{aligned} D\left(\underline{u}^F, \frac{29}{98}\bar{u}\right) &= -\frac{\bar{u}}{49}(65\bar{u}^2 + 817\bar{u}\underline{u}^F - 942(\underline{u}^F)^2) < \\ &< -\frac{\bar{u}}{49}\left(65\bar{u}^2 + \frac{13237}{29}(\underline{u}^F)^2\right) < 0. \end{aligned}$$

Since  $D(\cdot)$  is negative on the boundary of the rectangle and has no stationary points inside it, its negativity over the whole rectangle is ensured. Then, we conclude that the sign of (20) is positive.

## C Proof of Proposition 3

Similarly to the proof of Proposition 1, we proceed by demonstrating a series of Lemmata.

**Lemma 5.** *Let  $u_l^D \leq u_h^F$  and  $\bar{u} > u_l^D + t + \sqrt{(u_l^D)^2 + t^2} \equiv \hat{u}$ , then the unique maximizer of  $\pi_h^F$  is  $\bar{u}$ .*

*Proof.* First of all recall (see Remark 2) that if  $\bar{u} \leq \hat{u}$  there is no quality level along all the feasible quality spectrum for the foreign firm compatible with a positive demand. Remark 2 implies also that if  $\bar{u} < \hat{u}$  the foreign firm cannot select optimally any quality level below  $\hat{u}$  because this would entail a zero demand. Focus therefore on the interval  $u_h^F \in ]\hat{u}, \bar{u}]$  and consider now the partial derivative of the foreign firm's profits w.r.t.  $u_h^F$ :

$$\frac{\partial \pi_h^F}{\partial u_h^F} = \frac{h(u_h^F)[2u_h^F(u_h^F - u_l^D) - t(2u_h^F - u_l^D)]}{(u_h^F - u_l^D)^2(4u_h^F - u_l^D)^3}, \quad (21)$$

where

$$h(u_h^F) = 8(u_h^F)^2(t + u_h^F) + 5(u_l^D)^2(t + 2u_h^F) - 2u_h^F u_l^D(5t + 7u_h^F) - 4(u_l^D)^3. \quad (22)$$

Notice that the denominator of (21) is positive for all  $u_l^D < u_h^F$  and that the term within square brackets is positive when  $u_h^F > \hat{u}$ . Move now to the polynomial (22), it can be ascertained that  $h(u_l^D) > 0$ . Consider now the partial derivative  $h'(u_l^D)$ . Standard computations show that  $h'(\cdot) > 0 \forall u_h^F > u_l^D$ . Thus maximization of  $\hat{\pi}_l^F(\cdot)$  requires the foreign firm to hit the upper bound of the quality spectrum.  $\square$

**Lemma 6.** *Let  $u_l^D \leq u_h^F$ , then (i) if  $t < \tilde{t}$  there exists a unique maximizer of  $\hat{\pi}_l^D, u_l^{D*}(u_h^F) \in ]0, u_h^F[$ ; (ii) if  $t \geq \tilde{t}$  the unique maximizer of  $\hat{\pi}_l^D$  is  $u_h^F$ .*

*Proof.* (i) Assume  $u_l^D < u_h^F$  and  $t < \frac{u_h^F}{11}$ . Consider the partial derivative

$$\frac{\partial \hat{\pi}_l^D(\cdot)}{\partial u_l^D} = \frac{u_h^F(t + u_h^F - u_l^D)m(u_l^D)}{(u_h^F - u_l^D)^2(4u_h^F - u_l^D)^3}, \quad (23)$$

where  $m(u_l^D) \equiv (u_l^D)^2(7u_h^F - 2t) + u_h^F u_l^D(t - 11u_h^F) + 4(u_h^F)^2(t + u_h^F)$ .

The denominator and the first term of the numerator are positive as long as  $u_l^D > u_h^F$ , thus we focus on the polynomial  $m(\cdot)$ . This function has two real roots within the interval  $]0, u_h^F[$  as long as  $t < \frac{u_h^F}{11}$ , furthermore only one of these roots fulfills the second-order partial derivative of  $\hat{\pi}_l^D(\cdot)$ . Label this solution  $u_h^{F*}(u_l^D)$ .

- (ii) Keep assuming that  $u_l^D < u_h^F$  but move now to the case  $t \geq \frac{u_h^F}{11}$ . In this case (23) has no real roots within  $]0, u_h^F[$  and it is always increasing over this interval. Thus the domestic firm finds it profitable to increase its product's quality as much as possible, in principle up to  $u_h^F$ . This would eventually violate the condition reported in Remark 2. However it can be easily ascertained that as the quality of the domestic firm raises to  $u_h^F$  its demand increases while the optimal price decreases. The demand stops growing as the condition in Remark 2 is met. Any further increase in  $u_l^D$  would entail no increase in the demand but a reduction in the optimal price, resulting in a profit loss. These observations allow us to conclude that in this case the profit-maximizing quality level is  $u_l^D = \frac{2u_h^F(u_h^F - t)}{2u_h^F - t} < u_h^F$ .
- (iii) Finally consider the situation  $u_l^D = u_h^F$ . In this case the good is homogeneous and a price war arises, again (2) is no longer the demand system. Optimal prices are thus  $t - \varepsilon$ , with  $\varepsilon$  positive and arbitrarily small, for the domestic firm and 0 for the foreign one.

At these prices the foreign producer has no demand, while that of the domestic firm is  $1 - \frac{t-\varepsilon}{u_h^F}$ , so that its profit is  $\tilde{\pi}_l^D \equiv (1 - \frac{t-\varepsilon}{u_h^F})(t - \varepsilon)$ . It can be proved that, while  $\tilde{\pi}_l^D > \hat{\pi}_l^D(\frac{2u_h^F(u_h^F-t)}{2u_h^F-t})$ ,  $\tilde{\pi}_l^D \leq \hat{\pi}_l^D(u_l^{D*}) \Leftrightarrow t \in [0, \tilde{t}]$ , where  $\tilde{t} = \frac{u_h^F}{229}[137 - \frac{1143^{2/3}}{\sqrt[3]{1131 - 458\sqrt{6}}} - 6\sqrt[3]{3(1131 - 458\sqrt{6})}] \approx \frac{3u_h^F}{125}$ .  $\square$

**Lemma 7.** (i) Let  $t < \tilde{t}$  and  $\bar{u} > \hat{u}$ , then the pair  $(\bar{u}, \underline{u}^D)$ , where  $\underline{u}^D \equiv u_l^{D*}(\bar{u})$  is a couple of mutual best replies at the quality-choice stage of game  $\Gamma$  for the domestic and foreign firms respectively, thus they are the quality levels chosen at a subgame perfect Nash equilibrium of this game. (ii) Let  $t > \tilde{t}$  or  $\bar{u} < \hat{u}$ , then there is no subgame perfect Nash equilibrium of game  $\Gamma$  with the domestic firm producing the low-quality good and the foreign firm supplying the high-quality one.

*Proof.* (i) It is necessary to show that there are no profitable unilateral deviations from  $(\bar{u}, \underline{u}^D)$  in all the strategy space. Since the proof parallels that of Lemma 4 it will just be sketched. If the foreign high-quality producer leapfrogs downwards its rival it finds it profitable to increase the quality of its variant up to  $\underline{u}^D$ . In this case the good would be homogeneous, thus the optimal price of the deviating firm would be  $\hat{p}_l^D(\bar{u}, \underline{u}^D) - t - \varepsilon$ . Accordingly all the demand would be served by the foreign firm. It is a matter of calculations to prove, however, that although positive, the deviation payoff is lower than that of the strategy under scrutiny, thus the deviation is not profitable.

Consider now the domestic high-quality producer. From Lemma 6 we know that as long as  $t < \tilde{t}$  the unique maximizer of the domestic firm's profits is  $\underline{u}^D \in [0, \bar{u}]$ . Deviations strictly above  $\bar{u}$  are not admissible by construction.

(ii) From Lemma 6(ii)-(iii) it follows that if  $t > \tilde{t}$  the domestic firm wants to deviate from the prescribed strategy  $\underline{u}^D$  and set  $u_l^D = \bar{u}$ . Furthermore if  $\bar{u} < \hat{u}$  there is no quality level for the foreign firm compatible with a positive demand under duopoly pricing. This firm may therefore undercut its rival and become the low-quality producer earning a positive profit (see Lemma 7 (i)). In both cases  $(\bar{u}, \underline{u}^D)$  cannot be an equilibrium.  $\square$

The results of Lemmata 5 to 7 prove Proposition 3.

## D Proof of Proposition 4

The optimal quality level of the domestic firm is:

$$\underline{u}^D(t) = \frac{\bar{u}(11\bar{u} - t - \sqrt{9\bar{u}^2 - 102\bar{u}t + 33t^2})}{14\bar{u} - 4t}$$

It is easily ascertained that  $\underline{u}^D(0) = \frac{4}{7}\bar{u}$  and  $\underline{u}^D(\frac{\bar{u}}{11}) = \frac{4}{5}\bar{u}$ . Moreover

$$\frac{\partial \underline{u}^D}{\partial t} = \frac{\bar{u}^2 [15\bar{u}\sqrt{(3\bar{u}-t)(\bar{u}-11t)} + \sqrt{3}\bar{u}(113\bar{u}-43t)]}{2\bar{u}(7\bar{u}-2t)^2\sqrt{(3\bar{u}-t)(\bar{u}-11t)}} > 0 \forall t < \frac{\bar{u}}{11}.$$



## E Non existence of equilibria for “small” $\bar{u}$

This Appendix provides a heuristic argument to ascertain that when  $\bar{u} < \max\{\hat{u}, \tilde{u}\}$  equilibria with two active firms (that is to say with firms charging non negative prices and serving non negative demands) fail to exist.

Consider case (i) first and assume that  $t < \tilde{t}$  but  $\bar{u} < \tilde{u}$ . From Lemma 2 (iii) we know that the domestic firm’s profit maximizing quality for a given  $u_l^F \leq u_h^D$  is  $\frac{u_l^F(u_l^F - t)}{(u_l^F - 2t)}$ . The price and demand of the foreign producer are zero (see Remark 1), thus its profit is zero too. Two cases may arise. First, if  $\bar{u} < \frac{u_l^F(u_l^F - t)}{(u_l^F - 2t)}$  then there exists no possible quality leapfrogging for the foreign firm, so that  $\frac{u_l^F(u_l^F - t)}{(u_l^F - 2t)}$  for the  $D$ -firm and any  $u_l^F \leq u_h^D$  is a SPNE. Second if  $\bar{u} > \frac{u_l^F(u_l^F - t)}{(u_l^F - 2t)}$  then there may be no profitable leapfrogging deviation (thus the quality pair above is again a SPNE), or there may be one (but this contradicts the assumption that the high-quality producer is domestic).

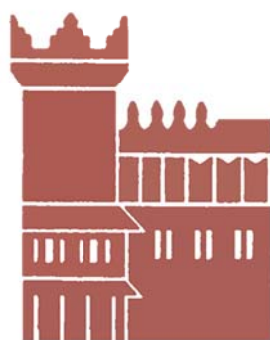
Consider case (ii) and assume now that  $t < \tilde{t}$  but  $\bar{u} < \hat{u}$ . From Remark 2 we know that the high-quality foreign firm maximizes its profit by setting  $u_h^F = \bar{u}$ , but the demand it serves is zero, thus its profit is zero as well. Again, if there is a profitable deviation this must be downwards, below the quality level of the domestic producer. This, however, contradicts the assumption of foreign high-quality producer.

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