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# Vertical Integration and Operational Flexibility\*

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## Abstract

The main aim of the paper is to highlight the relation between flexibility and vertical integration. To this purpose, we go through the selection of the optimal degree of vertical disintegration of a flexible firm which operates in a dynamic uncertain environment. The enterprise we model enjoys flexibility since it can switch from a certain amount of disintegration to vertical integration and viceversa. This means that the firm never loses vertical control, i.e., the ability to produce all inputs even when it buys them in the market. This sort of flexibility makes for results which are somehow contrary to the Industrial Organization recent literature and closer to the Operations Research results. In this sense we provide a bridge between the two approaches and rescue Industrial Organization from counterintuitive conclusions.

**Keywords:** vertical integration, outsourcing, entry, flexibility

**JEL Classification:** L24; G 31; C61

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# 1 Introduction

A crucial question in both Industrial Organization and Operations Research concerns the extension of the control of a vertical production process and its flexibility. This topic has come recently to the limelight (Acemoglu, Aghion, Griffith and Zilibotti, 2005; Acemoglu, Johnson and Mitton, 2005; Rossini, 2005, 2007) following the current wave of international outsourcing (Antràs and Helpman, 2004; Grossman and Helpman, 2002, 2005) with many firms increasing vertical disintegration on a crossborder basis by buying a growing chunk of inputs from independent foreign enterprises.

Even though most of current outsourcing is due to lower labour costs in the countries where portions of the vertical process of production is offshored, vertical disintegration is often chosen for reasons not directly linked to labour costs. As Grossman and Helpman (2002) pointed out, vertical disintegration may allow finer specialization in input production due to scale economies in R&D and production. This effect may be amplified by trade deepening, owing to better opportunities to concentrate on fewer stages of production. Nonetheless, vertical disintegration may also be adopted as a cushion against external shocks. By sharing the vertical control of production with other, preferably foreign, enterprises, each firm may enjoy a more flexible and less risky production organization, especially whenever different stages of the vertical production process require bearing relevant sunk costs. On the contrary, high quality firms may decide to increase their degree of vertical integration to make sure that quality standards are more likely met.

All these considerations are bound to explain *i*) why vertical disintegration shows variable and opposite trends across time and industries, *ii*) why we observe outsourcing among similar countries and within the same country, *iii*) why there are many examples of insourcing<sup>1</sup>, i.e., increased vertical integration, for instance, as a result of vertical mergers<sup>2</sup>.

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<sup>1</sup>In June 7, 2005 the president of the Confederation of Indian Industry (CII) Sunil Bharti Mittal told Democratic presidential candidate Hillary Clinton about Indian companies outsourcing to US firms, investing in the US and creating jobs ( IANS, 2005). Moreover, in 2007 Glaxo-Smith-Kline has decided to bring back to Europe labs involved in advanced R&D located in China. Moreover, some increased insourcing has been noticed recently in some Japanese high tech firms and in some high quality - high tech US firms. These are just few examples of reverse outsourcing which can be found in newspapers and in specialized publications.

<sup>2</sup>In the food industry, in the automotive industry in emerging countries, such as Indian Tata recently buying steel factories.

So far the decision as to vertically disintegrate and/or integrate has been analyzed as a once and for all choice, i.e., without contemplating any opportunity for its reversal. Yet, a question arises as to whether a firm may be better off adopting a radical vertical disintegration with the loss of control of entire phases of the production process (Helpman and Grossman, 2005) or it may be preferable to retain the ability to produce the inputs bought from independent enterprises. For instance, this may be obtained by producing in-house just a quota of the input requirement. As a matter of fact, for each level of external input procurement, there are many ways to vertically disintegrate. When a firm decides to offshore an input production to a foreign independent firm, the associated risk (due to exchange rates, foreign rules, production conditions in a remote country, quality standards, delivery time, shipment costs, etc.) may be quite high and worth some prudential behavior. An industrial real (not financial) way to decrease, or diversify away, this risk may require either keeping a share of the process of production in-house as a buffer or simply preserving the ability (know how and some facilities) to make it and, perhaps, reverse the offshoring decision by bringing back sections of the vertical chain of production whenever circumstances dictate so.

All these considerations confirm that the decision to outsource is neither a simple binary choice, nor a matter of sheer input costs of production. Yet, it appears to be quite related to the extent of flexibility needed by a firm so as to safely face uncertainty.

Industrial Organization (Alvarez and Stenbacka, 2007) and Operations Research (Van Mieghem, 1999) offer different answers as to the optimal timing and extent of outsourcing vis à vis internal production. The Operations Research interpretation<sup>3</sup> maintains that firms subcontract more as market uncertainty (risk) increases, closely paralleling the behavior of financial options, whose worth increases with uncertainty.

In Industrial Organization literature, once the decision as to the vertical arrangement of a firm is taken there is no possibility of reversing it: the amount of flexibility that outsourcing is meant to provide cannot change once a certain vertical organization has been decided. Moreover, in Alvarez and

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<sup>3</sup>Van Mieghem (1999) contribution belongs to Operations Research literature which highlights the flexibility that subcontracting offers to production and capacity planning. Van Mieghem (1999) paper extends and generalizes previous studies (Li, 1992 and McCardle, 1997) which dealt only with the strategic unidimensional problem of optimal production with outsourcing, without considering capacity selection, i.e. the optimal size of investment.

Stenbacka (2007), at higher levels of uncertainty the adoption of outsourcing is being postponed. This result is actually puzzling, contrary to both Van Mieghem (1999) and common sense, since, in an uncertain dynamic framework, outsourcing is a way to do "production and capacity smoothing" making for the presumption that higher uncertainty should accelerate the adoption of outsourcing. But in current Industrial Organization literature the firm has an option to do outsourcing, yet not to do "production (and profit) smoothing" by switching among different levels of vertical integration to minimize profit variability.

Recent literature (Acemoglu, Aghion, Griffith and Zilibotti, 2005) has cast some light on the relationship between vertical integration and size finding a direct link. A further confirmation on a different data set<sup>4</sup> can be found in Table 1 below, where we present indices of vertical integration (*VIX*) computed at firm level for years 2000 through 2004.

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<sup>4</sup>Data used come from Osiris, a database set up by the Bureau Van Dijk.

**Table 1:** Vertical integration indices<sup>5</sup>

		<i>small</i> <sup>6</sup>	<i>medium</i> <sup>7</sup>	<i>large</i> <sup>8</sup>
2000	<i>VIX</i>	.28	.34	.38
	<i>obs</i>	1075	836	1938
	<i>SD</i>	.36	.35	.37
	<i>cv</i>	1.28	1.01	.97
2001	<i>VIX</i>	.28	.33	.40
	<i>obs</i>	1219	839	1908
	<i>SD</i>	.36	.35	.38
	<i>cv</i>	1.29	1.06	.95
2002	<i>VIX</i>	.28	.34	.41
	<i>obs</i>	1256	815	1780
	<i>SD</i>	.37	.35	.37
	<i>cv</i>	1.32	1.03	.90
2003	<i>VIX</i>	.27	.34	.40
	<i>obs</i>	1323	787	1704
	<i>SD</i>	.36	.36	.37
	<i>cv</i>	1.33	1.06	.93
2004	<i>VIX</i>	.27	.34	.37
	<i>obs</i>	1321	716	1637
	<i>SD</i>	.36	.36	.38
	<i>cv</i>	1.34	1.03	1.02

As it can be seen, vertical integration increases as firms get larger. The variability of the index of vertical integration goes down with size even though it remains quite high, somehow blurring the revelation power of data. Nonetheless, our empirical evidence is consistent with what found on a different empirical basis by Acemoglu, Aghion, Griffith and Zilibotti (2005),

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<sup>5</sup> *obs* stands for number of observations, *SD* stands for standard deviation, *cv* stands for the coefficient of variation which is the ratio of *SD* over the sample mean of the *VIX* index.

<sup>6</sup> Less than 300 employees.

<sup>7</sup> Between 300 and 1000 employees.

<sup>8</sup> More than 1000 employees.

even though the result on variability does not have, to our knowledge, any comparability in the literature.

On the trace of recent mentioned literature and evidence presented in Table 1, we try to explain the amount of flexibility acquired by firms and the observed relationship between size and vertical integration.

We wish to explore the choice of the extent of flexibility that can be secured by the mix between outsourcing and insourcing in a dynamic uncertain framework when the scale of production changes and input price volatility varies.

We shall analyze the choice of the vertical arrangement together with the entry decision when the firm is able to revise its vertical commitment if market conditions require it. This will partially bridge the gap between Operations Research and Industrial Organization and propose a unified interpretation of flexibility in terms of vertical disintegration and/or integration.

In the next section we present the model. In the third part we go through the entry process. In the fourth we analyze the choice of capacity. In the fifth we go through some comparative statics. The epilogue contains a concluding summary.

## 2 The model

We consider a flexible vertical integration arrangement (i.e. *FVI*) in an industry in which the market price of the final good is certain and given to the firm.<sup>9</sup>

To perform its task the company buys a unit of a fundamental input for each unit of output (*perfect vertical complementarity*). The firm can either produce entirely a perfectly divisible intermediate good in-house at the marginal cost  $d_t$  or buy a share  $\alpha \in (0, 1]$  of the input at the market price  $c_t$ . The enterprise may costlessly switch from making the input, when  $\hat{c}_t \equiv \alpha c_t + (1 - \alpha)d_t$  rises above  $d_t$ , to buying it, if  $\hat{c}_t$  falls below  $d_t$ . Therefore, once decided the portion of input to procure from an independent enterprise, the instantaneous profit is:

$$\begin{aligned} \pi_t &= \max[(p - d_t), (p - \hat{c}_t)]X \\ &\equiv [p - d_t + \max(d_t - \hat{c}_t, 0)]X \end{aligned} \tag{1}$$

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<sup>9</sup>The output price may be constant due to regulation.

where  $p$  is the output market price. The profit function (1) draws on a simplified linear technology with only one input. The quantity of output is  $X$  units per year. When  $\alpha \leq 1$ , with technology *FVI* the firm manufactures the output by using a linear combination of produced and procured input, while keeping the flexibility of going back to total vertical integration every time  $c$  becomes too high.<sup>10</sup>

The market price,  $c_t$ , of the input needed for the production of the final good is uncertain. On the contrary, the marginal cost of internal production is constant, i.e.,  $d_t = d$ . Finally, for the sake of simplicity, we assume that  $p - d > 0$ . Even though outsourcing may induce cost advantages, as  $c$  goes down, the firm retains its know-how to manufacture the input in a viable and profitable way.

Dynamic uncertainty in the market price of the input,  $c_t$ , boils down to a geometric Brownian motion:

$$dc_t = \gamma c_t dt + \sigma c dz_t \quad (2)$$

with  $dz_t$  as the increment of a Wiener process (or Brownian motion), uncorrelated over time. The drift parameter is lower than the riskless interest rate, i.e.,  $\gamma \leq r$ .<sup>11</sup> The process  $dz_t$  satisfies the conditions that  $E(dz_t) = 0$  and  $E(dz_t^2) = dt$ . Therefore,  $E(dc_t)/c_t = \gamma dt$  and  $E(dc_t/c_t)^2 = \sigma^2 dt$ , i.e., starting from the initial value  $c_0$ , the random position of the cost  $c_t$  at time  $t > 0$  has a normal distribution with mean  $c_0 e^{\gamma t}$  and variance  $c_0^2 (e^{\sigma^2 t} - 1)$ , which increases as we look further and further into the future. Notice that the process “has no memory” (i.e., it is Markovian), and hence *i*) at any point in time  $t$ , the observed  $c_t$  is the best predictor of future profits, *ii*)  $c_t$  may next move upwards or downwards with equal probability.

## 2.1 The value of the *FVI* technology

With a *FVI* technology we have to distinguish between two opposite cases. If  $\hat{c}_t > d$  the firm is *Effectively Vertically Integrated (EVI)*. It does possess the facilities and produces its own input, while keeping the option of buying

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<sup>10</sup>We note that if  $\alpha = 1$ , with a technology *FVI* the firm can switch between two extremes: total vertical integration and total vertical disintegration.

<sup>11</sup>Alternatively, we could use an interest rate that includes an appropriate adjustment for risk and take the expectation with respect to a distribution of  $c$  adjusted for risk neutrality (see Cox and Ross, 1976; Harrison and Kreps, 1979; Harrison, 1985).



it. On the contrary, if  $\hat{c}_t < d$  the firm is only *Virtually Vertically Integrated* (*VVI*). It buys a share  $\alpha$  of the input while producing a portion  $1 - \alpha$  and keeps the ability (option) to manufacture the whole input requirement if  $\hat{c}_t$  goes up. Since for  $\alpha \geq 0$ , the condition  $\hat{c}_t > d$  implies  $c_t > d$ <sup>12</sup>, the value of the firm is given by the solution of the following free boundary dynamic programming problems (Dixit, 1989; Dixit and Pindyck 1994; Moretto, 1996):

$$\Gamma V^{EVI}(c_t; \alpha) = -(p - d)X, \quad \text{for } c_t > d \quad (3)$$

and

$$\Gamma V^{VVI}(c_t; \alpha) = -(p - \alpha c_t - (1 - \alpha)d)X, \quad \text{for } c_t < d, \quad (4)$$

where  $\Gamma$  indicates the differential operator:  $\Gamma = -r + \gamma c \frac{\partial}{\partial c} + \frac{1}{2} \sigma^2 c^2 \frac{\partial^2}{\partial c^2}$ . The solution of the differential equations (3) and (4) requires the following boundary conditions:

$$\lim_{c \rightarrow \infty} \left\{ V^{EVI}(c_t; \alpha) - \frac{p - d}{r} X \right\} = 0 \quad (5)$$

and

$$\lim_{c \rightarrow 0} \left\{ V^{VVI}(c_t; \alpha) - \left( \frac{p - (1 - \alpha)d}{r} - \frac{\alpha c_t}{r - \gamma} \right) X \right\} = 0, \quad (6)$$

where  $\frac{p-d}{r}X$  indicates the present value of operating the firm forever while “making” the input and  $\left( \frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} \right) X$  is the present value of operating the firm forever while “buying” a share  $\alpha$  of the input in the market. Then, from the assumptions and the linearity of the differential equations (3) and (4), using (6) and (5), we get:

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<sup>12</sup>It is easy to show that

$$\begin{aligned} \alpha c_t + (1 - \alpha)d &> d, \\ \alpha(c_t - d) &> 0, \\ c_t &> d. \end{aligned}$$

1. the firm's value under  $EVI$  :<sup>13</sup>

$$V^{EVI}(c_t; \alpha) = \frac{(p-d)}{r}X + \hat{A}c_t^{\beta_2}$$

2. and under  $VVI$ :

$$V^{VVI}(c_t; \alpha) = \left( \frac{p - (1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} \right) X + \hat{B}c_t^{\beta_1}.$$

Putting together the two equations  $V^{EVI}(c_t; \alpha)$  and  $V^{VVI}(c_t; \alpha)$  we get the net discounted flows of profits that takes into account the value of changing the vertical arrangement:

$$V(c_t; \alpha) = \begin{cases} EVI & \left\{ \begin{array}{l} \frac{p-d}{r}X + \hat{A}c_t^{\beta_2} \\ \left( \frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} \right) X + \hat{B}c_t^{\beta_1} \end{array} \right. & \begin{array}{l} \text{if } c_t > d \\ \text{if } c_t < d. \end{array} \end{cases} \quad (7)$$

Having indicated with  $\frac{p-d}{r}X$  and  $\left( \frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} \right) X$  the present value of operating the firm forever, the additional terms  $\hat{A}c_t^{\beta_2}$  and  $\hat{B}c_t^{\beta_1}$  indicate respectively the value of the option to go from  $EVI$  to  $VVI$  and the value of the option to move the other way round. Therefore, the constants  $\hat{A}$  and  $\hat{B}$  must be positive. As it may be seen, under  $FVI$  the opportunity to “make” the input with unit profits  $p-d > 0$  rules out any closure option. Notice that  $V(c_t)$  is a convex, decreasing function, with  $V(0) = \frac{p-(1-\alpha)d}{r}X$  and  $\lim_{c \rightarrow \infty} V(c_t) = \frac{p-d}{r}X < \frac{p-(1-\alpha)d}{r}X$ .

To evaluate the constants we have to meet the value matching and the smooth pasting conditions at  $c_t = d$ . That is<sup>14</sup>:

$$V^{EVI}(d; \alpha) = V^{VVI}(d; \alpha),$$

and

$$V_c^{EVI}(d; \alpha) = V_c^{VVI}(d; \alpha),$$

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<sup>13</sup>Where  $\beta_2 < 0$  and  $\beta_1 > 1$  are respectively, the negative and positive roots of the characteristic equation  $Q(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r = 0$  (See Dixit and Pindyck, 1994, pp.187-189).

<sup>14</sup>Where  $V_c = \frac{\partial V}{\partial c}$ .

whose solutions give:

$$\begin{cases} \hat{B} = \alpha B \equiv \frac{\alpha}{\beta_1 - \beta_2} (r - \gamma\beta_2) d^{1-\beta_1} \frac{1}{r(r-\gamma)} \\ \hat{A} = \alpha A \equiv \frac{\alpha}{\beta_1 - \beta_2} (r - \gamma\beta_1) d^{1-\beta_2} \frac{1}{r(r-\gamma)}. \end{cases} \quad (8)$$

The constant  $\hat{A}$  represents the value of the option to go from *EVI* to vertically disintegrated mode.  $\hat{B}$  is the value of the option to go from *VVI* to vertically integrated mode. First: it can be seen that  $\hat{A}$  and  $\hat{B}$  are always nonnegative (Dixit and Pyndyck, 1994, p.189). Second: both constants are linear in  $\alpha$ . If  $\alpha = 0$ , the firm is always vertically integrated,  $\hat{A} = 0$  and also  $\hat{B} = 0$ . If  $\alpha = 1$  the input is bought entirely from an independent firm. Nonetheless the firm keeps the option to switch to internal production.

## 2.2 Optimal $\alpha$ with an *EVI* technology

During the last decades we have extensively observed firms decreasing over time their degree of vertical integration<sup>15</sup>. On the contrary, during the first industrial revolution in Europe, at the end of the eighteenth century, and, at the beginning of the nineteenth century, an opposite trend occurred, i.e., external provision of inputs manufactured by artisans would be internalized in large firms, thanks to the introduction of new production techniques calling for more integrated processes (Grossman and Hart, 1986; Wallerstein, 1980). Nowadays, lower transaction costs, due to technology changes in transport and communication and more efficient domestic and international markets, seem to stimulate vertical disintegration, even though, as emphasized in the introduction, we observe also instances of insourcing.

In order to better interpret observed facts, we start, as in Alvarez and Stenbacka (2007), considering the case where a firm is manufacturing in-house its own input, while holding the option to switch to a mixed technology if, at a future date,  $c_t$  becomes lower than  $d$ , i.e., the firm operates as *EVI* with the option to become *VVI*.

With  $d < c_t$ , the firm's problem is to choose the optimal  $\alpha$  once the option to switch towards *VVI* is being exercised. The enterprise must select  $\alpha$  that maximizes (7) minus the cost of setting up a dedicated production organization based on outsourcing:

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<sup>15</sup>Empirical analysis on firm data over the last two decades is provided for Italy by Rossini and Ricciardi (2005).

$$\alpha^* = \arg \max [NPV^{EVI}(c_t; \alpha)]$$

where:

$$NPV^{EVI}(c_t; \alpha) \equiv V^{EVI}(c_t; \alpha) - I(\alpha)$$

and  $I(\alpha)$  is the direct cost of developing the mixed technology, covering search for subcontractors, monitoring input quality and contract enforcement. We model  $I(\alpha)$  as Cobb-Douglas with increasing costs-to-scale and we translate it into a cost function which is quadratic in  $\alpha$  multiplied by a unit organization cost  $K$ , i.e.:

$$I(\alpha) = \frac{K}{2}\alpha^2. \quad (9)$$

Then, without loss of generality, if  $\alpha = 0$ , the organizational cost of producing a quantity of output  $X$  is normalized to zero, i.e.,  $I(0) = 0$ . On the contrary, if  $\alpha = 1$ , the cost of using complete outsourcing to produce  $X$  is  $I(1) = \frac{K}{2}$ .<sup>16</sup>

We can now establish the following result:

**Proposition 1** *The optimal proportion of outsourced input is given by:*

$$\alpha^* = \begin{cases} 1 & \text{if } c_t \leq \tilde{c} \\ \frac{A}{K}c_t^{\beta_2} & \text{if } c_t > \tilde{c} \end{cases} \quad (10)$$

and the state-contingent net present value of the EVI technology associated with the option to switch to a mixed vertical mode is:

$$NPV^{EVI}(c_t, \alpha^*(c_t)) = \begin{cases} \frac{p-d}{r}X + Ac_t^{\beta_2} - \frac{K}{2} & \text{if } d < c_t \leq \tilde{c} \\ \frac{p-d}{r}X + \frac{1}{2}\frac{A^2}{K}c_t^{2\beta_2} & \text{if } c_t > \tilde{c}, \end{cases} \quad (11)$$

where  $\tilde{c} \equiv \left(\frac{K}{A}\right)^{1/\beta_2}$ .

**Proof.** See Appendix A ■

The above proposition shows that, if  $c_t$  is low it is better to choose complete outsourcing, while, as  $c_t$  increases  $\alpha$  goes down and tends to zero for high values of  $c_t$ .

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<sup>16</sup>Results will not be altered if the investment cost is  $K\alpha^\delta$  (with  $\delta > 1$ ) and the organization cost is  $I(0) = k > 0$ , even when  $\alpha = 0$ .

### 3 The choice of the optimal entry

Since we know the net present value of the project,  $NPV^{EVI}(c_t, \alpha^*(c_t))$ , we can find the value of the option to invest in the project,  $F(c_t)$ , as well as the optimal timing rule. The value of the option to invest is given by the solution of the following free boundary dynamic programming problem:

$$\Gamma F(c_t) = 0, \quad \text{for } c_t > c^* \quad (12)$$

where  $c^*$  is the threshold at which it is efficient to activate the *EVI* technology. In the case in which the option to switch to a mixed vertical arrangement is never exercised it will not be convenient for the firm to acquire the *EVI* technology. Then, the solution of the differential equation (12) requires the following boundary condition:  $\lim_{c \rightarrow \infty} F(c_t) = 0$ .

By the linearity of the differential equation (12) and using the boundary condition, the value of the option to enter becomes:<sup>17</sup>

$$F(c_t) = Cc_t^{\beta_2}. \quad (13)$$

To evaluate the constant  $C$  and the optimal entry trigger  $c^*$ , we have to meet the corresponding matching value and smooth pasting conditions:

$$F(c^*) = NPV^{EVI}(c^*, \alpha^*(c^*)),$$

$$F'(c^*) = NPV_c^{EVI}(c^*, \alpha^*(c^*)),$$

where the second equality<sup>18</sup> follows from  $NPV_\alpha^{EVI}(c^*, \alpha^*(c^*)) = 0$ .

Furthermore, a necessary condition to make an investor enter the market with an *EVI* technology is that the optimal entry trigger  $c^*$  be larger than  $d$ . In words, at the entry the input price must be higher than the cost of producing it internally. Otherwise it would be optimal to switch immediately to *VVI*. Figure 1 below plots  $F(c_t)$  and  $NPV^{EVI}(c_t, \alpha^*(c_t))$  where the optimal trigger  $c^*$  is determined, from the smooth-pasting condition, at the point of tangency  $T$  of the two curves.

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<sup>17</sup>The general solution of the above differential equation is:

$$F(c_t) = Cc_t^{\beta_2} + Dc_t^{\beta_1},$$

since the investor decides the commitment only if  $c_t$  goes down, we can set  $D = 0$ .

<sup>18</sup>It should be:  $F'(c^*) = NPV_c^{EVI}(c^*, \alpha^*(c^*)) + NPV_\alpha^{EVI}(c^*, \alpha^*(c^*))$ .

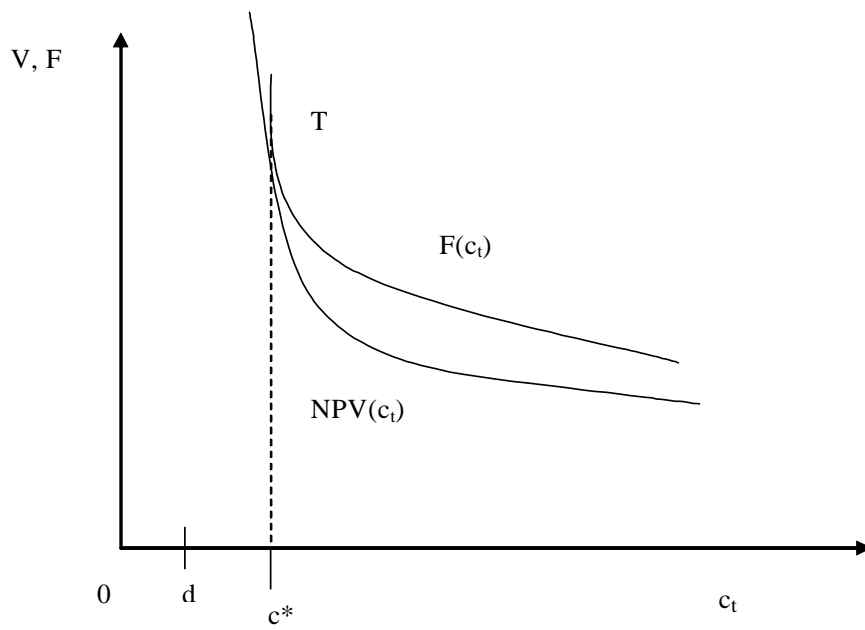


Figure 1:

**Figure 1:** Value of the Investment Opportunity  $F(c_t)$  and  $NPV^{EVI}(c_t, \alpha^*(c_t))$

Then, we can establish the following proposition regarding the dynamics of the optimal entry timing and the choice of the production mode:

**Proposition 2** 1) If  $K \in \left(2\frac{p-d}{r}X, \frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)}\right)$  then the optimal entry trigger for the *EVI* technology is:

$$c^* = \left(\frac{\sqrt{2\frac{p-d}{r}XK}}{A}\right)^{1/\beta_2} \in [\tilde{c}, \infty), \quad (14)$$

where  $A = \frac{(r-\gamma\beta_1)d^{1-\beta_2}}{(\beta_1-\beta_2)r(r-\gamma)}$ , and the optimal  $\alpha$  is:

$$\alpha^*(c^*) = \sqrt{\frac{2\frac{p-d}{r}X}{K}} \in (0, 1). \quad (15)$$

2) The value of the option to invest in the *EVI* technology which accounts for the optimal level of outsourcing can be written as:

$$F(c_t) = \begin{cases} A\sqrt{2\frac{p-d}{rK}X}c_t^{\beta_2} & \text{for } c_t > c^* (> \tilde{c}) \\ \frac{p-d}{r}X + \frac{1}{2}\frac{A^2}{K}c_t^{2\beta_2} & \text{for } d < c_t \leq c^* \end{cases}. \quad (16)$$

**Proof.** See Appendix B ■

The condition  $K \in \left(2\frac{p-d}{r}X, \frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)}\right)$  specifies the circumstances in which it is profitable to enter with the *EVI* technology, i.e., producing the input, while keeping alive the option to switch to a mixed vertical arrangement. However, if  $K \notin \left(2\frac{p-d}{r}X, \frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)}\right)$  the firm never invests in the *EVI* technology. In particular, if  $K < 2\frac{p-d}{r}X$ , by (15), we get  $\alpha^* > 1$ , which means that the firm invests only in *VVI*. On the other hand, if  $K > \frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)}$  the entry cost is so high that the firm stays out.

Unlike in Alvarez and Stenbacka<sup>19</sup> (2007), if the firm enters with an *EVI* technology, it is never optimal to choose complete outsourcing, i.e.,  $\alpha^* < 1$

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<sup>19</sup>See: Theorem 2.2.

and the firm keeps on manufacturing in-house a small fraction of the input as a sort of prudential behavior. Condition (14) can also be written as:

$$Ac^{*\beta_2} = \sqrt{2\frac{p-d}{r}XK}, \quad (17)$$

which says that the trigger, letting the firm enter, can be obtained by equating the value of the option to switch from *EVI* to *VVI* to the constant value  $\sqrt{2\frac{p-d}{r}XK}$ , which depends on, among other things, the cost of the in-house input  $d$  and the sunk cost  $K$ . A larger sunk cost  $K$  generates, *ceteris paribus*, a reduction of the optimal  $c^*$ , making the firm delay entry. An increase of the cost of producing in-house the input boosts the optimal  $c^*$  letting the firm anticipate entry.

## 4 The choice of capacity

So far we have considered the optimal entry-timing and the optimal mix of outsourcing with capacity fixed at  $X$ . Empirical evidence presented in the introduction and coming from recent literature (Acemoglu, Aghion, Griffith and Zilibotti, 2005) invites some investigation on the trade-off between entry costs, outsourcing and capacity. To this purpose, we generalize the model allowing the firm to adjust size continuously.

We suppose that capacity can be indexed by a continuum  $X \in [\underline{X}, \bar{X}]$  and that the investment costs to set up an outsourcing network go up with  $\alpha$  and the size of the firm  $X$ . In particular, extending the cost function (9), we assume that the unit organization cost  $K$  depends on size. That is:

$$I(\alpha, X; w) = \frac{1}{2}K(X; w)\alpha^2, \quad (18)$$

where  $w$  represents the price of the (fixed) factors needed to set up the subcontractors network, to write contracts, to monitor input quality. The organization cost function  $K = K(X; w)$  shows the usual properties:  $K_X(X; w) > 0$ ,  $K_{XX}(X; w) > 0$ ,  $K_w(X; w) > 0$  and  $K_{Xw}(X; w) = 0$ .<sup>20</sup> By the definition

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<sup>20</sup>By the Shephard's Lemma, if the firm's conditional factor demand is positively sloped with respect to  $X$ , we get  $K_{Xw}(X; w) \geq 0$ . However, we may alternatively assume that demand for the factors required to build up the subcontractors network is independent of capacity.



of  $I$ , if  $\alpha = 0$ , the organizational cost drops to zero regardless of the size of the firm.<sup>21</sup>

Here, we consider a firm with a perfectly divisible plant of maximum size  $\bar{X}$ . The firm has to decide which plant to build, taking into account entry costs  $I$ . By (16) the optimal dimension requires choosing  $X$  for which the constant  $C(X; w) \equiv A\sqrt{2\frac{p-d}{rK(X;w)}}X$  is the largest. This is equivalent to maximizing the ratio  $\frac{X}{K}$  subject to  $K = K(X; w)$ . Then, we get the following first order condition (FOC)<sup>22</sup>:

$$K(X^*, w) - XK_X(X^*, w) = 0. \quad (19)$$

From (19), a necessary condition for an optimal solution is a cost elasticity  $\varepsilon_{KX} \equiv \frac{X^*K_X(X^*;w)}{K(X^*;w)} = 1$ , i.e., the average cost  $AC(X; w) \equiv \frac{K(X;w)}{X}$  is constant around the optimum (constant returns to scale). Then, the optimal size is always, conditionally on  $w$ , efficient, i.e.,  $X^*(w) = \arg \min AC(X; w)$ .

If there are economies or diseconomies of scale, the optimum comes from a binary comparison between the smallest and the largest size. In the first case, the optimal policy requires waiting to invest in the largest plant in the spectrum of output capacity, i.e.,  $X^* = \bar{X}$ . In the second case, investment occurs soon in the smallest plant in the spectrum, i.e.,  $X^* = \underline{X}$ .

Finally, within the range where the SOC holds, an increase of  $w$  implies an increase of the firm's optimal size. That is, by (19) we get:

$$\frac{\partial X^*(w)}{\partial w} = -\frac{K_w(X^*; w) - X^*K_{Xw}(X^*; w)}{SOC} > 0. \quad (20)$$

## 5 Comparative Statics

The analysis performed so far could have been carried out using traditional economic analysis tools. However, our model allows for a deeper study of the effect of both the uncertainty and the project size on the entry policy as well as on the optimal outsourcing mix.

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<sup>21</sup>The results would not change if we introduced a technology cost such that  $I(0, X; w) = k(X) > 0$ .

<sup>22</sup>The second order condition (SOC) is always satisfied because of the convexity of  $K(X; w)$  with respect to  $X$ .

## 5.1 The effect of uncertainty

It is possible to show that an increase in the risk concerning the input market price ( $\sigma$ ) always entails an increase in the optimal entry trigger  $c^*$ , i.e.,  $\frac{\partial c^*}{\partial \sigma} > 0$  and the firm outsources earlier. On the contrary, by (15), we easily see that  $\alpha^*$  is not going to change as uncertainty soars.

If we analyze the condition  $K \in \left(2\frac{p-d}{r}X, \frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)}\right)$ , we see that on the left side it is simply determined by parameters. On the right side it depends on the degree of uncertainty.<sup>23</sup> The interval becomes wider as uncertainty grows making the adoption of *EVI* more likely.

Finally, once the firm has resolved to invest in the *EVI* technology, we may investigate how uncertainty affects the probability of outsourcing the input manufacturing. According to (7), the probability of investing in the technology *EVI* is represented by the likelihood that the input price touches the critical  $d$  from above starting from an initial  $c_t > d$ . This may be written as (Dixit, 1993, p.54):

$$\Pr(c_t) = \begin{cases} 1 & \text{if } \frac{2\gamma}{\sigma^2} \leq 1 \\ \left(\frac{d}{c_t}\right)^{\frac{2\gamma}{\sigma^2}-1} & \text{if } \frac{2\gamma}{\sigma^2} > 1 \end{cases}$$

Starting at  $c_t$  in the interior of the range  $[d, \infty)$ , after a “sufficient” long interval of time the process will for sure hit the barrier  $d$  if the trend is positive, but low with respect to uncertainty, or if it is negative. However, if  $\gamma$  is positive and sufficiently high with respect to volatility, the process may drift away and never hit  $d$ . Furthermore, higher volatility increases the probability of hitting the barrier  $d$  making more attractive, *ceteris paribus*, outsourcing. Indeed, the derivative of  $\Pr(c_t)$  with respect to  $\sigma$  is unambiguously positive

$$\frac{d\Pr}{d\sigma} = -\frac{4\sigma\gamma}{(\sigma^2)^2} \ln\left(\frac{d}{c_t}\right) \left(\frac{d}{c_t}\right)^{\frac{2\gamma}{\sigma^2}-1} > 0$$

All these results can be summarized in the following proposition:

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<sup>23</sup>Defining  $\frac{(r-\gamma\beta_1)d}{(\beta_1-\beta_2)r(r-\gamma)} \equiv R$ , it can easily compute:

$$\frac{\partial R}{\partial \sigma} = \frac{8d\sigma^3}{[8r\sigma^2 + (\sigma^2 + (\sigma^2 - 2\gamma)^2)^{\frac{3}{2}}]} \geq 0.$$

**Proposition 3** 1) *The optimal threshold to enter with the technology that allows to switch, in the future, to a partial outsourcing production is a strictly increasing function of input price volatility, i.e.  $\frac{\partial c^*}{\partial \sigma} > 0$ . This means that higher uncertainty makes for earlier entry with the EVI technology.*

2) *The optimal share of outsourcing at entry does not depend on input price volatility, i.e.,  $\frac{\partial \alpha^*}{\partial \sigma} = 0$ .*

3) *However, once the enterprise has adopted the EVI technology, an increase of volatility boosts the probability of switching to the VVI, i.e., to outsourcing.*

**Proof.** See Appendix C ■

The above results run counter the Industrial Organization findings (Alvarez and Stenbacka, 2007). Entry is anticipated due to the opportunity to switch to outsourcing. Once a firm has entered with an EVI technology the likelihood that it will resort to outsourcing increases with uncertainty. However, the extent of outsourcing established at the time of entry does not depend on input price volatility. This is due to the fact that the firm we consider is flexible and can change the decision to vertically integrate or disintegrate. In this sense our results provide a generalization of previous ones since our firm is flexible also in the choice of "being flexible".

## 5.2 The effect of the firm's size

Here we wish to explore the effects of size on entry and outsourcing decisions. By (20), an increase of the organization cost  $K(X;w)$  translates into an increase of the firm's optimal size  $X^*$ . Then, we may write the following:

**Proposition 4** 1) *An increase in the entry cost which translates into a larger size  $X^*$  produces an entry delay, i.e.,*

$$\frac{\partial c^*}{\partial w} < 0.$$

2) *An increase in the entry cost making for a larger size of the firm produces a decrease in the degree of vertical disintegration [as evidence in the introduction suggests], i.e.,*

$$\frac{\partial \alpha^*}{\partial w} < 0.$$

**Proof.** See Appendix 4 ■

## 6 Conclusions

We have analyzed the decision to outsource input production totally or partially in a dynamic uncertain environment. The enterprise analyzed must decide entry, vertical mode and capacity. The firm is flexible and it can revise the vertical organization decision if market requires to do so. This flexibility makes for results which are at odds with received results of Industrial Organization, stating, among other things, that uncertainty is going to postpone the adoption of outsourcing. In our framework outsourcing provides a sort of cushion against risk and becomes more appealing in risky conditions. Therefore, outsourcing is anticipated as uncertainty soars, as common sense suggests. Nonetheless, flexibility makes for the level of vertical integration independent of uncertainty at entry, since the firm possesses an option to vary the level of vertical integration once it is in. Even this second result is at odds with received literature and is due to the extent of flexibility the firm is thought to possess. Finally, as evidence from the introduction and recent literature suggests, when size increases, the complexity of the outsourcing network may overcome that of internal organization leading to higher vertical integration for large firms.

## A Proof of Proposition 1

Since  $\hat{A} = \alpha A$ , the optimal vertical arrangement is given by:

$$\begin{aligned}\alpha^* &= \arg \max [NPV^{EVI}(c_t, \alpha)] \\ &= \arg \max \left[ \frac{p-d}{r}X + \alpha A c_t^{\beta_2} - \frac{K}{2}\alpha^2 \right].\end{aligned}\tag{21}$$

Then, the FOC is:

$$A c_t^{\beta_2} - K\alpha = 0\tag{22}$$

while the SOC is always satisfied. From (22) it is immediate to show that:

$$\alpha^* = \begin{cases} 1 & \text{if } c_t \leq \tilde{c} \\ \frac{A}{K}c_t^{\beta_2} & \text{if } c_t > \tilde{c} \end{cases}\tag{23}$$

where  $\tilde{c} \equiv \left(\frac{K}{A}\right)^{1/\beta_2}$ . Finally, by substituting  $\alpha^*$  in (7) and (9), we get (11) in the text.

## B Proof of Proposition 2

The operating constraints to find  $C$  and  $c^*$  are:

$$F(c^*) = NPV^{EVI}(c^*, \alpha^*(c^*))\tag{24}$$

and

$$F'(c^*) = NPV_c^{EVI}(c^*, \alpha^*(c^*)).\tag{25}$$

We distinguish two cases according to the value taken by the optimal trigger  $c^*$ :

- If  $c^* \leq \tilde{c}$  (and  $d < c^*$ ), if the firm invests, it will choose  $\alpha = 1$  and the two conditions (24) and (25) become:

$$C c^{*\beta_2} = \frac{p-d}{r}X + A c^{*\beta_2} - \frac{K}{2}$$

and

$$\beta_2 C c^{*\beta_2-1} = A \beta_2 c^{*\beta_2-1}.$$

However,  $Cc^{*\beta_2}$  cannot simultaneously satisfy value matching and smooth pasting conditions with  $\frac{p-d}{r}X + Ac^{*\beta_2} - \frac{K}{2}$ . In other words, to make  $c^* \leq \tilde{c}$ , we need  $\frac{p-d}{r}X \geq \frac{K}{2}$ , which implies  $\alpha = 1$ . Then, it is not possible to obtain  $c^*$ .

- If  $c^* > \tilde{c}$  (and  $d < \tilde{c}$ ), by (23) the firm will choose  $\alpha < 1$ . Then the equations (24) and (25) become:

$$Cc^{*\beta_2} = \frac{p-d}{r}X + \frac{1}{2} \frac{A^2}{K} c^{*2\beta_2}$$

and

$$\beta_2 Cc^{*\beta_2-1} = \frac{A^2}{K} \beta_2 c^{*2\beta_2-1}.$$

By some substitutions we get:

$$\begin{aligned} \frac{A^2}{K} c^{*2\beta_2} &= \frac{p-d}{r}X + \frac{A^2}{K} c^{*2\beta_2} - \frac{K}{2} \left( \frac{A}{K} \right)^2 c^{*2\beta_2} \\ &= 2 \frac{p-d}{r}X. \end{aligned}$$

Then, we have that:

$$Ac^{*\beta_2} = \sqrt{2 \frac{p-d}{r} X K} \quad (26)$$

$$C = A \sqrt{2 \frac{p-d}{r K} X} > 0. \quad (27)$$

Finally, substituting (26) into (23) we get the value of  $\alpha^*$  in the text.

Let us now consider the conditions which guarantee the existence of the optimal trigger  $c^*$ . Recalling that  $A\tilde{c}^{\beta_2} = K$ , by (26), the first condition  $c^* > \tilde{c}$  is satisfied iff:

$$\sqrt{2 \frac{p-d}{r} X K} < K.$$

Therefore, since  $K > 0$ , we may write:

$$\frac{p-d}{r}X < \frac{K}{2}.$$

From (8) we get  $Ad^{\beta_2} = \frac{1}{\beta_1 - \beta_2}(r - \gamma\beta_1)d\frac{1}{r(r-\gamma)}$ , and the second condition  $d < \tilde{c}$  is satisfied iff:

$$K < \left( \frac{1}{\beta_1 - \beta_2}(r - \gamma\beta_1)d\frac{1}{r(r-\gamma)} \right).$$

Putting all results together, we get:

$$2\frac{p-d}{r}X < K < \left( \frac{1}{\beta_1 - \beta_2}(r - \gamma\beta_1)d\frac{1}{r(r-\gamma)} \right).$$

## C Proof of Proposition 3

Let us consider first the effect of uncertainty on  $c^*$ . From (26) and the implicit function theorem we get:

$$\frac{\partial c^*}{\partial \sigma} = -\frac{\frac{\partial A}{\partial \sigma}c^{*\beta_2} + A\frac{\partial \beta_2}{\partial \sigma}(\ln c^*)c^{*\beta_2}}{A\beta_2c^{*\beta_2-1}}$$

where  $A = \frac{1}{\beta_1 - \beta_2}(r - \gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)}$ . Taking the derivative of  $A$  with respect to  $\sigma$  we obtain:

$$\begin{aligned} \frac{\partial A}{\partial \sigma} &= -\frac{\frac{\partial \beta_1}{\partial \sigma} - \frac{\partial \beta_2}{\partial \sigma}}{(\beta_1 - \beta_2)^2}(r - \gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)} - \\ &\quad -\frac{1}{\beta_1 - \beta_2} \left[ \gamma\frac{\partial \beta_1}{\partial \sigma}d^{1-\beta_2} + (r - \gamma\beta_1)\frac{\partial \beta_2}{\partial \sigma}(d^{1-\beta_2}) \ln d \right] \frac{1}{r(r-\gamma)} \\ &= \frac{1}{r(r-\gamma)} \frac{1}{(\beta_1 - \beta_2)}(r - \gamma\beta_1)d^{1-\beta_2} \\ &\quad \left[ -\frac{\frac{\partial \beta_1}{\partial \sigma} - \frac{\partial \beta_2}{\partial \sigma}}{(\beta_1 - \beta_2)} - \frac{\gamma}{(r - \gamma\beta_1)} \frac{\partial \beta_1}{\partial \sigma} - \frac{\partial \beta_2}{\partial \sigma} \ln d \right] \\ &= A \left[ -\frac{1}{(\beta_1 - \beta_2)} \frac{\partial \beta_1}{\partial \sigma} + \left( \frac{1}{(\beta_1 - \beta_2)} - \ln d \right) \frac{\partial \beta_2}{\partial \sigma} - \frac{\gamma}{(r - \gamma\beta_1)} \frac{\partial \beta_1}{\partial \sigma} \right]. \end{aligned}$$

Since  $\frac{\partial \beta_1}{\partial \sigma} < 0$  and  $\frac{\partial \beta_2}{\partial \sigma} > 0$ ,  $d < 1$  is a sufficient condition to get  $\frac{\partial A}{\partial \sigma} > 0$ , even though we know that (as shown in Dixit and Pindyck, 1994, pp. 189-190 Figure 6.1) the result holds also for much higher values of  $d$ . Therefore,  $\frac{\partial A}{\partial \sigma} > 0$ ,  $\frac{\partial \beta_2}{\partial \sigma} > 0$  and  $c^* > d$  let us conclude that  $\frac{\partial c^*}{\partial \sigma} > 0$ .

Consider now the effect of uncertainty on  $\alpha^*$ . Since the firm enters when  $c_t = c^*$ , from (23) and (26) it is immediate to show that  $\frac{\partial \alpha^*}{\partial \sigma} = 0$ . To verify this, we take the derivative of  $\alpha^* = \frac{1}{K} A c^{*\beta_2}$  with respect to  $\sigma$ :

$$\frac{\partial \alpha^*}{\partial \sigma} = \frac{1}{K} \left[ \frac{\partial A}{\partial \sigma} c^{*\beta_2} + A \left( \frac{\partial \beta_2}{\partial \sigma} (\ln c^*) + \beta_2 \frac{1}{c^*} \frac{\partial c^*}{\partial \sigma} \right) c^{*\beta_2} \right].$$

This expression can be simplified by substituting  $\frac{1}{c^*} \frac{\partial c^*}{\partial \sigma}$ , i.e.:

$$\frac{1}{c^*} \frac{\partial c^*}{\partial \sigma} = - \frac{\frac{\partial A}{\partial \sigma} c^{*\beta_2} + A \frac{\partial \beta_2}{\partial \sigma} (\ln c^*) c^{*\beta_2}}{A \beta_2 c^{*\beta_2}} = - \frac{\frac{\partial A}{\partial \sigma}}{A \beta_2} - \frac{\frac{\partial \beta_2}{\partial \sigma} (\ln c^*)}{\beta_2}.$$

Then:

$$\begin{aligned} \frac{\partial \alpha^*}{\partial \sigma} &= \frac{1}{K} \left[ \frac{\partial A}{\partial \sigma} c^{*\beta_2} + A \left( \frac{\partial \beta_2}{\partial \sigma} (\ln c^*) - \frac{\frac{\partial A}{\partial \sigma}}{A} - \frac{\partial \beta_2}{\partial \sigma} (\ln c^*) \right) c^{*\beta_2} \right] \\ &= \frac{1}{K} \left[ \frac{\partial A}{\partial \sigma} c^{*\beta_2} + A \left( -\frac{\frac{\partial A}{\partial \sigma}}{A} \right) c^{*\beta_2} \right] \\ &= \frac{1}{K} \left[ \frac{\partial A}{\partial \sigma} c^{*\beta_2} - \frac{\partial A}{\partial \sigma} c^{*\beta_2} \right] = 0. \end{aligned}$$

## D Proof of Proposition 4

Let us consider first the effect of  $w$  on  $c^*$ . From (14) we get:

$$\begin{aligned} \frac{\partial c^*}{\partial w} &= \frac{1}{\beta_2} \left( \frac{\sqrt{2^{\frac{p-d}{r}} X^*(w) K(X^*(w); w)}}{A} \right)^{1/\beta_2-1} \times \\ &\quad \frac{1}{A} \left( 2^{\frac{p-d}{r}} X^*(w) K(X^*(w); w) \right)^{-1/2} \frac{p-d}{r} \frac{\partial X^*(w) K(X^*(w); w)}{\partial w} \end{aligned}$$

Therefore the sign of  $\frac{\partial c^*}{\partial w}$  is driven by the sign of  $-\frac{\partial X^*(w) K(X^*(w); w)}{\partial w}$ , i.e.,

$$\begin{aligned} \frac{\partial c^*}{\partial w} &\propto - \frac{\partial X^*(w) K(X^*(w); w)}{\partial w} \\ &= - \left[ \frac{\partial X^*(w)}{\partial w} K(X^*(w); w) + \right. \\ &\quad \left. + X^*(w) \left( K_X(X^*(w); w) \frac{\partial X^*(w)}{\partial w} + K_w(X^*(w); w) \right) \right] < 0. \end{aligned}$$



Let us consider now the effect of  $w$  on  $\alpha^*$ . From (15) we get:

$$\frac{\partial \alpha^*}{\partial w} = \left( \frac{2^{\frac{p-d}{r}} X^*(w)}{K(X^*(w), w)} \right)^{-1/2} \frac{p-d}{r} \frac{\partial \frac{X^*(w)}{K(X^*(w), w)}}{\partial w}.$$

Again the sign of  $\frac{\partial \alpha^*}{\partial w}$  is driven by the sign of  $\frac{\partial \frac{X^*(w)}{K(X^*(w), w)}}{\partial w}$ , i.e.,

$$\begin{aligned} \frac{\partial \alpha^*}{\partial w} &\propto \frac{\partial \frac{X^*(w)}{K(X^*(w), w)}}{\partial w} \equiv \frac{\partial \frac{1}{K_X(X^*(w), w)}}{\partial w} \\ &= \frac{-(K_{XX}(X^*(w), w)) \frac{\partial X^*(w)}{\partial w} - K_{Xw}(X^*(w), w)}{K_X(X^*(w), w)^2} < 0 \end{aligned}$$

where the first equality follows from the FOC.

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