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## Volatility Matters: Taylor Rules and Capital Accumulation

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ABSTRACT. The operational performance of a set of simple monetary policy rules *à la* Taylor in a model with capital accumulation and nominal and real rigidities is discussed with a special emphasis on the volatility of output, nominal rate and inflation rate. Within an enriched modelling framework it is shown that output targeting plays a more crucial role than what has been assessed in the current literature for models without capital accumulation. In fact, with a small value of the output targeting coefficient, monetary authority is not completely successful in stabilizing the volatility of output, nominal rate and inflation rate only by acting on inflation targeting. Moreover, a too strong concerns towards inflation relatively to output translates into a lower ability to control inflation volatility, together with a strong policy reaction with respect to an exogenous shock hitting the economy. Impulse response analysis shows that the risk of an excessive concerns towards inflation might end up in counterproductive results on output, after a positive technological shock. The model also shows a better internal propagation mechanism than what has been previously showed. Finally, the results show that it is no longer possible to miss capital accumulation in modelling monetary policy analyses and calls for further generalizations of the existing modelling framework.

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## 1. INTRODUCTION

This paper shows the crucial role played by the output targeting parameter in a monetary policy function *à la* Taylor in implementing successful stabilization plans. In fact, if a Taylor-type rule presents an inflation targeting parameter too high, this could induce a recession and could prevent the economy from taking advantage of a positive shocks. Moreover, a relatively high inflation targeting coefficient (with respect to output) might also induce output instability. The dynamic is simple: the fear towards the increase of expected inflation can force central bank in setting interest rates so high that a recession might occur even if the economy is hit by a positive technological shock. The main message from this paper is that if monetary authority wants to minimize variable fluctuations, it has to actively work on both inflation and output targets. The key element in the explanation of this results lies in the high level of persistency associated to the economy under analysis, generated by the presence of real and nominal rigidities.

From previous research we know that in order to simultaneously achieve welfare maximization, and determinacy, monetary policy parameters have to be within a well specified region. However, the question is: what the choice of these parameters does imply in terms of volatility in a model with capital accumulation and real rigidities ? This is the question to be answered in the following pages.

In recent years, the economic profession has taken a big effort in studying the operational performance of monetary policy rules. The main goal of this literature is to provide conclusions about the size of monetary policy parameters, by using a criterion function derived or directly imposed on the model. The conclusions found by this literature is that a simple Taylor rule where nominal rate reacts to contemporaneous inflation and output maximizes the welfare of the representative agent when the inflation targeting parameter is higher than the output targeting parameter.

In this paper the impact on volatility and persistency in a model with capital accumulation and real and nominal rigidities is studied with respect to the magnitude of parameters of monetary policy rules. The focus here is on the evaluation of simple Taylor rules with respect to their impact on the volatility of output, nominal rate and inflation rate. The modelling strategy considers an explicit role of nominal rigidities, modeled via quadratic cost of price adjustment *à la* Rotemberg (1982), with capital accumulation. Real Rigidities are inserted via cost of capital installation along the lines proposed by Lubik (2000). This paper lies within the recent developments of Dynamic Stochastic General Equilibrium monetary models, mixing the Real Business Cycles (RBC, henceforth) modelling framework together with the microfoundations of nominal rigidities, forming the standard apparatus of Keynesian models. This literature started with the work by Rotemberg and Woodford (1999) who built a very simple dynamic stochastic general equilibrium model without capital accumulation with price rigidities. Other remarkable examples of variations on the same themes are the collection of papers in the book edited by Taylor (1999). This literature suffers of an important drawback: there is no capital accumulation. Monetary policy is effective if it is able to affect real rate, which, in a model without investment choice and truly intertemporal allocation problem, remains an exogenous shock. The role of capital accumulation in monetary models has been studied by Kim (2000), Casares and McCallum (2001) and Neiss and Pappa (2002).

None of these papers, however, examined the operational performance of interest rate rules along a specific criterion (volatility or welfare).

The rest of the paper is organized as follows. The next section is devoted to the description of the modelling setup, with a complete discussion about the assumptions underlying the consumer's and firm's behavior. A following section is devoted to the discussion of monetary policy rules analyzed in the paper, together with a description of the fiscal policy reaction function. The calibration analysis, the solution method and other computational details are the subject of section 4. Section 5 reports volatility results together with a brief discussion on the effects of changing some parameters characterizing the degree of competition in the economy. Section 6 discussed the impact of monetary policy parameter's choice on the internal dynamic of the model showing the pattern of impulse response function for output, nominal rate and inflation rate for different monetary policy parameters' choice. Concluding remarks in section 7 closes up the paper.

## 2. THE MODEL

The model is populated by a large number of consumers and firms. Each representative agent in both final goods and capital markets acts as a price taker. He (She) consumes a large variety of final goods, each produced by a different firm, accumulates capital and rents it to firms. Each agent can consume and invest a composite good formed by the total amount of varieties  $j \in [0, 1]$  produced by each single firm.

**2.1. The consumer's side.** The economy is populated by many identical households indexed on the real line  $i \in [0, 1]$ . Each households optimizes over an infinite horizon the following utility function:

$$U_t = E_t \sum_{t=0}^{\infty} u(C_{it}, L_{it}) \quad (1)$$

where  $C_{it}$  and  $L_{it}$  indicate the amount of consumption and labor effort supplied on labor market by each  $i$ -th represented agent, respectively. The instantaneous utility function  $u(C_{it}, L_{it})$  is assumed to be:

$$u(C_{it}, L_{it}) = \frac{[C_{it}^{(1-\gamma)} (1 - L_{it})^\gamma]^{(1-\frac{1}{\sigma})}}{(1 - \frac{1}{\sigma})} \quad (2)$$

The utility function (1)-(2) is optimized with respect to the following intertemporal budget constraint for each agent  $i$ :

$$\begin{aligned} \frac{B_{it}}{P_t} + \frac{M_{it} - M_{it-1}}{P_t} + C_{it} (1 + \xi_t f(V_{it})) + I_{it} = Z_{it} K_{it} + W_{it} L_{it} + \\ + R_{t-1} \frac{B_{it-1}}{P_t} + \int_0^1 \eta_i(j) \Omega_t(j) dj - T_{it} \end{aligned} \quad (3)$$

Additionally, I assume that capital stock  $K_{it}$  owned by each agent  $i$  evolves according to the following equation:

$$K_{it} = \Phi \left( \frac{I_{it-1}}{K_{it-1}} \right) K_{it-1} + (1 - \delta) K_{it-1} \quad (4)$$

where  $I_{it}$  is the amount of investment at time  $t$ , and  $\delta$  is the depreciation rate on the capital stock. From (4), investment is productive next period and the stock of capital is predetermined.

From equation (3), each agent  $i$  enters a time  $t$  with a predetermined stock of capital  $K_{it}$ , nominal bond holdings  $B_{it}$ , and nominal money holdings  $M_{it}$ . Moreover, the representative agent receives its wage income times unit of labor worked,  $W_{it}L_{it}$ , the rental income from investment in physical capital  $Z_{it}K_{it}$ , where  $Z_{it}$  is the rental rate on capital, and the return (gross) on the investment in government bonds  $R_{t-1}\frac{B_{it-1}}{P_t}$ . In (3),  $R_t$  indicates the gross nominal interest rate. Moreover, each agent  $i$  participates in the profit of each firm  $j$  through a constant share  $\eta_i(j)$  on the profit of firm  $j$ ,  $\Omega_{jt}$ . I assume that profit share  $\eta_i(j)$  is constant over time and is out of the control of each single agent. This is done in order to simplify the analytical solution of the model and to save a first order condition. In what follows, I assume also the existence of a set of complete markets: this is a simplifying assumption to allow a simple relationship between the discount factor of households and that of firms.

From (2) and (3), money is introduced with a liquidity cost function inserted in the representative agent's budget constraint, as discussed, among others, by Sims (1994).

The approach taken here assumes that each agent incurs into a cost  $\xi_t f(V_{it})$  in order to make a transaction. Therefore, to consume an amount  $C_{it}$  each agent should plan to spend an amount equal to  $C_{it}(1 + \xi_t f(V_{it}))$  to complete the transaction. I assume that  $f(V_{it})$  is assumed to be linear:  $f(V_{it}) = V_{it}$ , where  $V_{it}$  is money velocity, defined as  $V_{it} = \frac{P_t C_{it}}{M_{it}}$ . As showed by Sims (1994), the functional form of  $f(V_{it})$  is crucial to determine the existence and the stability of an equilibrium for real money balances. The assumption of convex transaction costs ( $f''(V_{it}) \geq 0$ ) rules out indeterminacy under 'active' monetary policy and the existence of a barter equilibrium for this economy<sup>1</sup>.

In equation (4) I inserted real rigidities through the cost of capital installments captured by function  $\Phi\left(\frac{I_{it-1}}{K_{it-1}}\right)$ , along the same lines of Lubik (2000). Function  $\Phi(\cdot)$  is assumed to have the following properties  $\Phi'(\cdot) > 0$ ,  $\Phi''(\cdot) > 0$ . According to the formulation adopted in (4), one unit of investment  $I_{it}$  adds only  $\Phi\left(\frac{I_{it}}{K_{it}}\right)K_{it}$  units of capital to the next period's capital stock. Here, the  $q$ -theory is replaced by a  $\mu$ -theory of the households, where  $\mu$  is the Lagrange multiplier associated to investment constraint, measured in terms of marginal utility.

Money velocity shock  $\xi_t$  is modelled via an AR(1) process:

$$\log \xi_t = \rho_\xi \log(\xi_{t-1}) + (1 - \rho_\xi) \log(\xi) + \varepsilon_{\xi t} \quad (5)$$

where  $\varepsilon_{\xi t}$  is a i.i.d. random variable normally distributed  $N\left(0, \sigma_\xi^2\right)$ . Such type of shocks can be alternatively inserted as preference shock, when real money balances enter directly into the utility function.

The consumption bundle  $C_{it}$  is an aggregate of all the  $j \in [0, 1]$  varieties of goods

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<sup>1</sup>For the discussion about the influence of functional form of transaction cost function on the determinacy of the equilibrium see Sims (1994).

produced in this economy, described by:

$$C_{it} = \left[ \int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad (6)$$

where  $\theta$  is the elasticity of substitution between different varieties of goods produced by each firm  $j$ , with  $\theta > 1$ . Equation (6) is CES (Constant Elasticity of Substitution) aggregator, as in Dixit and Stiglitz (1977). To determine the optimal allocation across varieties of final goods, each consumer maximizes equation (6) subjected to the constraint that spending on all goods varieties  $\int_0^1 p_t(j) c_t^i(j)$  must not exceed aggregate expenditure  $P_t C_{it}$ . The solution to the allocation program leads to the following constant-elasticity inverse demand function:

$$\frac{c_t^i(j)}{C_{it}} = \left[ \frac{p_t(j)}{P_t} \right]^{-\theta} \quad (7)$$

where  $p_t(j)$  is the price of variety  $j$  and  $P_t$  is the general price index defined as:

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \quad (8)$$

The final goods aggregator (6) together with (7) and (8), define the so called *intra-temporal* optimization problem of the representative agent. When we aggregate  $C_{it}$  and  $c_t^i(j)$  across all agents  $i$ , we get the aggregate demand for final goods and for variety  $j$ , given, respectively, by:  $Y_t = \int_0^1 C_{it} di$  and  $Y_{jt} = \int_0^1 c_t^i(j) di$ , for all  $j \in [0, 1]$ .

**2.2. Intertemporal First-Order Conditions.** The optimal intertemporal allocation is obtained via the maximization of the utility (1)-(2) subjected to the intertemporal budget constraint (3) and the accumulation constraint (4). The first order conditions with respect to  $C_{it}$ ,  $L_{it}$ ,  $M_{it}$ ,  $B_{it}$ ,  $K_{it}$ ,  $I_{it}$  are, respectively:

$$(1 - \gamma) C_{it}^{(1-\gamma)(1-\frac{1}{\sigma})-1} (1 - L_{it})^{\gamma(1-\frac{1}{\sigma})} = \lambda_t (1 + 2\xi_t V_{it}) \quad (9)$$

$$\gamma C_{it}^{(1-\gamma)(1-\frac{1}{\sigma})} (1 - L_{it})^{\gamma(1-\frac{1}{\sigma})-1} = \lambda_t W_t \quad (10)$$

$$\beta E_t \frac{\lambda_{t+1}}{P_{t+1}} = (1 - \xi_t V_{it}^2) \frac{\lambda_t}{P_t} \quad (11)$$

$$\beta R_t E_t \frac{\lambda_{t+1}}{P_{t+1}} = \frac{\lambda_t}{P_t} \quad (12)$$

$$\beta E_t \mu_{t+1} \left[ 1 - \delta + \Phi \left( \frac{I_{it}}{K_{it}} \right) - \Phi' \left( \frac{I_{it}}{K_{it}} \right) \frac{I_{it}}{K_{it}} \right] + \lambda_t Z_t = \mu_t \quad (13)$$

$$\beta E_t \mu_{t+1} \Phi' \left( \frac{I_{it}}{K_{it}} \right) = \lambda_t \quad (14)$$

In (9)-(14)  $\lambda_t$ ,  $\mu_t$  are, respectively, the Lagrange multiplier attached to the representative agent's budget constraint (3) and to capital accumulation constraint (4):  $\lambda_t$  represents marginal utility of consumption, while  $\mu_t$  indicates marginal utility of capital.

Equation (9) equates the marginal utility of consumption to  $\lambda_t$ , while from equation (10) the disutility of labor effort equates the utility value of real wage. Equation (11)

is the First Order condition on money holdings and (12) represents the Euler equation obtained as optimal bond allocation. Finally, equations (13) - (14) indicate the optimal allocation choice between capital and investment. In particular, from (13), we have that the optimal allocation of a marginal unit of capital is governed by its marginal productivity (equal to the rental rate)  $\lambda_t Z_t$ , the discounted contribution to next period's capital stock  $(1 - \delta) \beta E_t \mu_{t+1}$ , and the contributions to the reduction of the adjustment cost overall  $-\Phi' \left( \frac{I_{it}}{K_{it}} \right) \frac{I_{it}}{K_{it}}$ .

The set of FOCs (9)-(14) is completed by the constraints (3), (4), by the stochastic process (5) and by two Transversality Conditions on bonds and capital.

From (11)-(12) we derive money demand as:

$$\frac{M_{it}}{P_t C_{it}} = \xi_t^{\frac{1}{2}} \left( \frac{R_t - 1}{R_t} \right)^{\frac{1}{2}} \quad (15)$$

The elasticity of money demand with respect to nominal interest rate is approximately equal to  $-\frac{1}{2R}$ , which, for a nominal interest rate equal to 6%, (the approximate historical value of the Federal Funds Rate for US, 1959:1-2001:4), produces a value of 0.47, slightly bigger number than what we observe in the data (around 0.2).

**2.3. Firms.** I assume the existence of a large number of firm indexed by  $j \in [0, 1]$ , each producing a single good variety. With respect to the varieties of final goods supplied by other competitors, each firm acts as a price taker. The production function for the firm  $j$  producing  $Y_{jt}$  units of output of the variety  $j$  is:

$$Y_{jt} = A_t K_{jt}^\alpha (g_j^t L_{jt})^{(1-\alpha)} - \Phi_t \quad (16)$$

where  $K_{jt}$  and  $L_{jt}$ , respectively, indicate the amount of capital stock and labor employed in the production process. Moreover,  $A_t$  is a technological shock common to all  $j$ -firms.

In (16) I assume the presence of a fixed cost of production represented by  $\Phi_t$ , common to all firms. Intuitively, in each period  $t$  an amount equal to  $\Phi_t$  must be employed for administration purposes. This cost can be interpreted as a pure cost necessary to start up with the business. The determination of the steady state level of  $\Phi_t$ ,  $\Phi$ , is obtained by imposing a zero-profit condition on the representative firm in the long run.

A sufficient condition for profit maximization is to require  $\Phi_t$  to be non-negative.

For  $A_t$  and  $\Phi_t$  we assume the following representation (in logs):

$$\log(A_t) = \rho_A \log(A_{t-1}) + (1 - \rho_A) \log(A) + \varepsilon_{At} \quad (17)$$

$$\log(\Phi_t) = \rho_\Phi \log(\Phi_{t-1}) + (1 - \rho_\Phi) \log(\Phi) \quad (18)$$

where  $A$  and  $\Phi$ , respectively, represent the steady state values of  $A_t$  and  $\Phi_t$ , respectively; moreover,  $\varepsilon_{At}$  is an i.i.d. normally distributed random variable with  $N(0, \sigma_A^2)$ .

To introduce nominal rigidities, each firm who wants to change her product prices has to pay a penalty in terms of output equal to  $Y_t$ . The price adjustment cost function is assumed to be:

$$AC_t^P(j) = \frac{\phi_P}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 Y_t \quad (19)$$

where  $\phi_p$  is the adjustment cost parameter and  $\pi$  is the steady state gross inflation rate. From (19) the cost of price adjustment is measured in terms of the final output and increases in a proportional way with the overall size of the economy  $Y_t$ .

**2.4. Profit Maximization.** The presence of price adjustment cost given by (19) makes the profit maximization problem of the firm dynamic. The optimal choice of input and prices is done through the maximization of the future stream of profit evaluated with the stochastic pricing kernel  $\rho_t$  for contingent claims, assumed to be the firm's discount factor. Thus, each firm maximizes:

$$\max_{\{K_t(j), L_t(j)\}} E_0 \left[ \sum_{t=0}^{\infty} \rho_t \Omega_t(j) \right] \quad (20)$$

subject to:

$$\Omega_t(j) = P_t(j) Y_{jt} - W_t L_{jt} - P_t R_t K_{jt} - P_t AC_t^P(j) \quad (21)$$

given the demand for differentiated products given by (7), after having aggregated over all  $i \in [0, 1]$  agents in the economy, and the price adjustment cost function given by (19).

The first order conditions with respect to  $K_{jt}$  and  $L_{jt}$  are, respectively:

$$\frac{R_t}{P_t} = \alpha \left( \frac{Y_{jt} + \Phi_t G^t}{K_{jt}} \right) \left( \frac{P_t(j)}{P_t} \right) \left( 1 - \frac{1}{\varepsilon_t(j)} \right) \quad (22)$$

$$\frac{W_t}{P_t} = (1 - \alpha) \left( \frac{Y_{jt} + \Phi_t G^t}{L_{jt}} \right) \left( \frac{P_t(j)}{P_t} \right) \left( 1 - \frac{1}{\varepsilon_t(j)} \right) \quad (23)$$

where  $\varepsilon_t(j)$  is the output demand elasticity, including the price adjustment cost, whose analytic expression is given by:

$$\frac{1}{\varepsilon_t(j)} = \frac{1}{\theta} \left\{ 1 - \phi_p \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi \right) \frac{P_t}{P_{t-1}(j)} \frac{Y_t}{Y_{jt}} + E_t \left[ \frac{\rho_{t+1}}{\rho_t} \phi_p \left( \frac{P_{t+1}(j)}{P_t(j)} - \pi \right) \frac{P_{t+1}}{P_t(j)} \frac{P_{t+1}(j)}{P_t(j)} \frac{Y_{t+1}}{Y_{jt}} \right] \right\} \quad (24)$$

Equation (24) measures the gross markup of price over marginal costs. Without cost of price adjustments, i.e. when  $\phi_p = 0$ , the markup would be  $\frac{\theta}{\theta-1}$ , constant over marginal cost. If  $\theta \rightarrow \infty$  and  $\phi_p = 0$ , then the markup becomes constant and equal to 1. With the intertemporal dimension considered in this model, both technology and demand shocks affect the cyclical properties of the markup, which becomes one of the shocks' transmission channels.

### 3. POLICY RULES

**3.1. Monetary Policy Rules.** The type of monetary policy rules analyzed here belong to a general class of interest-rate pegging rules (in log linear form) as<sup>2</sup>:

$$\tilde{R}_t = \sum_{l=1}^{T_i} \phi_{Rl} \tilde{R}_{t-l} + \sum_{l=0}^{T_\pi} \phi_{\pi l} \tilde{\pi}_{t-l} + \sum_{l=0}^{T_y} \phi_{yl} \tilde{Y}_{t-l} + \eta_t^M \quad (25)$$

<sup>2</sup>The variables of the model are all expressed in log-linear form. Thus,  $\tilde{z}_t$  stays for the deviation of  $\log Z_t$  from its steady state level  $\log Z$ .



where  $\tilde{R}_t$  is a measure of the nominal (gross) interest rate in period  $t$  used by monetary authority as instrument (the Federal Funds Rate in US),  $\tilde{\pi}_t$  is the (gross) inflation rate. Moreover,  $\tilde{Y}_t$  indicates the actual output and  $\eta_t^M$  is an exogenous monetary policy shock. According to (25), nominal interest rate controlled by Central Bank depends from the past history of output, inflation and nominal interest rate. In the present paper I focus only on rules without interest smoothing obtained by setting  $\phi_{Rl} = 0$  for all  $l$  in (25).

In (25)  $\eta_t^M$  indicates a monetary policy shock, for which I consider a general specification under the form of an AR(1) process as follows:

$$\log(\eta_t^M) = \rho_M \log(\eta_{t-1}^M) + (1 - \rho_M) \log(\eta^M) + \varepsilon_t^M \quad (26)$$

with  $\varepsilon_t^M \sim N(0, \sigma_M^2)$ , i.i.d. process. This assumption is made in order to guarantee a better persistency to policy effects, as remarked by Furher and Moore (1995).

The simplest Taylor Rule I am going to examine assumed as a benchmark is:

**Rule 1:**

$$\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t + \tilde{\eta}_t^M \quad (27)$$

Coefficients  $\phi_\pi$ ,  $\phi_y$  represent the strength to which monetary authority decides to react to shocks to inflation rate and output. Taylor (1993) sets  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$  and finds that this rule mimics quite well the pattern of nominal interest rate in the period 1974-1993 for the US economy. By setting  $\phi_y = 0.0$  in (27) we obtain a pure inflation targeting rule.

A variant of rule (27) considers nominal interest rate as reacting to expected inflation, in place of the contemporaneous inflation rate. This is motivated by two reasons: first, it is reasonable to think that monetary authority has some informative advantage in discovering the signals of incoming inflation. An expected inflation targeting rule (**Rule 2**) is:

$$\tilde{R}_t = \phi_\pi E_t \tilde{\pi}_{t+1} + \phi_y \tilde{Y}_t + \tilde{\eta}_t^M \quad (28)$$

As for rule (27), by setting  $\phi_y = 0.0$  in (28) we obtain a pure expected inflation targeting rule.

**3.2. Fiscal Policy.** Consider the following government's budget constraint, in real terms:

$$\frac{B_t}{P_t} + \frac{M_t}{P_t} = (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + G_t - T_t \quad (29)$$

In (29)  $T_t$  is the level of real lump sum taxes, while  $B_t$  and  $M_t$  indicates the stock of public debt and money issued by the government and held by investors. Bonds, money and taxes are aggregated as:  $B_t = \int_0^1 B_t^i di$ ,  $M_t = \int_0^1 M_t^i di$ ,  $T_t = \int_0^1 T_t^i di$ .

The interactions between fiscal and monetary policy are crucial in the determination of the price level. Leeper (1991), Sims (1994) have shown that to achieve full price stability we need to include a specific fiscal policy rule, given by:

$$T_t = \psi_0 + \psi_1 \frac{B_{t-1}}{P_t} \quad (30)$$

from (30) taxes react to real debt with a coefficient  $\psi_1$  whose magnitude is set in order to guarantee an upper bound in the growth rate of real debt,  $\psi_0$  is a constant term<sup>3</sup>. According to Leeper (1991), a ‘passive’ fiscal policy is when  $(\beta^{-1} - 1) < \psi_1 < (\beta^{-1} - 1)$ ; otherwise, a fiscal policy is said to be ‘active’.

Public expenditure  $G_t$  is modelled via an AR(1) as:

$$\log(G_t) = \rho_G \log(G_{t-1}) + (1 - \rho_G) \log(G) + \varepsilon_t^G \quad (31)$$

where  $\varepsilon_t^T$  is i.i.d. variable normally distributed, such that  $\varepsilon_t^G \sim N(0, \sigma_G^2)$ .

In this setting, fiscal policy is not neutral, even without distortionary taxation or finite lives. The transmission channel of the shocks included in the model is offered by the budget constraint of private sector. This can be done only via the implementation of a fiscal policy rule like (30), or a similar one. In fact, consequences on fiscal policy from monetary authority are not negligible, even if seigniorage revenues are small.

#### 4. STEADY STATE AND CALIBRATION

**4.1. Steady State.** In the system reduction process, I impose a condition of symmetry on the variables indexed by  $i$  and  $j$ , by setting  $X_t(j) = X_t$  and  $x_t^i(j) = x_t$ , for all  $i, j \in [0, 1]$ . In this way, all firms and consumers choose the identical set of behavioral relationship. This assumption avoids to keep track of the entire price distribution among various firms.

As it is customary in the RBC literature, all variables of the present model are transformed by assuming the existence of a Balanced Growth Path (BGP, henceforth). The relationship between the discount factors of the transformed economy and that of the non-transformed economy is given by:  $\beta_x = \beta g_y^{(1-1/\sigma)(1-\gamma)}$ , where  $g_y$  is the output growth rate on a BGP. Moreover, after combining the representative agent’s budget constraint and the Government budget constraint, we get the following expression for the Social Resource constraint:

$$C_t(1 + \xi V) + I_t + G_t = Y_t \left[ 1 - \frac{\phi_P}{2} (\pi_t - \pi)^2 \right]$$

I also assume that each agent has access to a set of complete markets for contingent claims. This allows to get a unique discount factor for consumers and firms on the basis of the following condition:

$$\frac{\rho_{t+1}}{\rho_t} = \beta_x \frac{\lambda_{t+1}}{\lambda_t}$$

This condition can be simply rationalized by assuming the existence of a representative agent who freely engage a set of exchanges of the firm’s share at zero transaction costs. An additional first order condition will produce the same type of equation.

Moreover, combining equations (13) -(14) we get:

$$\beta_x E_t \mu_{t+1} \left[ 1 - \delta + \Phi \left( \frac{I_t}{K_t} \right) - \Phi' \left( \frac{I_t}{K_t} \right) \frac{I_t}{K_t} + \Phi' \left( \frac{I_t}{K_t} \right) Z_t \right] = g_y \mu_t \quad (32)$$

<sup>3</sup>It is no difficult to show that a fiscal policy rule of the type designed above encompasses many other fiscal policy rules, like f.e. balanced budget rules. See Cochrane (1998) for further details.

If we impose the steady state condition on equation (32) we get:

$$1 - \delta + Z = \frac{g_y}{\beta_x} \quad (33)$$

which highlights the relationship between the rental rate  $Z$ , and the real rate of return  $g_y/\beta_x$ . From the above relationships, we observe that the capital transaction costs do not affect the steady state of the model.

**4.2. Calibration.** The model is calibrated on quarterly observation based on the US economy (sample (1959:1 - 2001:4)). The main parameters of the model are contained in Table 1 below:

Parameter	$\beta_x$	$\alpha$	$\gamma$	$\sigma$	$g_y$	$\psi_1$	$\delta$	$\xi$	$\theta$	$\eta_k$
Value	0.997	0.33	0.64	0.1	1.004	free	0.025	0.106	10	0.1

**Table 1**

From steady state relationship existing between nominal and real interest rate derived from (12), we have  $\frac{R}{\pi} = \beta_x^{-1}$ . The discount rate of the transformed economy has been set equal to  $\beta_x = 0.997$ . This value implies that the discount factor of the non-transformed economy is 1.0071. The growth rate of real GDP in the sample period is 1.76%, equivalent to  $g_y = 1.0044$  per quarter. The nominal interest rate is set accordingly to the mean value of the Federal Funds Rate which, over the specified sample period, is given by 6.47% per year, (1.61% in quarterly terms). Given such informations, we find that from (12) the inflation rate is 1.009499, equivalent to 3.8% in annual terms, very close to empirical observations.

Therefore, the real interest rate is given by  $g_y/\beta_x = 1.0066$ , equivalent to 2.64% in annual terms. If  $\delta = 0.025$ , as in the largest part of RBC studies, the rental rate of capital from (33) is  $Z = 0.0316$ , which is a value very close to that obtained in other studies.

The share of consumption over GDP,  $S_c$  as been set equal to 0.57, as in Rotemberg and Woodford (1999). From our dataset, the value of velocity  $V$ , turns out to be  $V = 0.33$ .

If we assume that the total transaction costs are 2% of GDP, we have that:  $\xi S_c V = 0.02$ , from which we get  $\xi = 0.106$ .

The share of capital in production is  $\alpha = 0.33$ , as it is customary in RBC literature.

The elasticity of intertemporal substitution  $\sigma$  is set to 0.1, implying a degree of risk aversion equal to 10: this is a compromise value between  $\sigma = 0.17$  adopted by Rotemberg and Woodford (1999), and  $\sigma = 0.085$ , adopted by Kim (2000), in an estimated model, similar to the present one.

From data, the ratio of market  $L$  to non-market  $(1 - L)$  activities is  $L/(1 - L) = 0.289017$ , from which we find  $L = 0.2243$ .

From (9)-(10) we calibrate the share of leisure in the utility  $\gamma$ , given by  $\gamma = 0.79$ . The capital/output ratio implied by the model is given by  $K/Y = \alpha/Z = 10.44$ , very close to the values reported in the literature (see, for example, Christiano, 1991).

As in Schmitt-Grohé and Uribe (2004), I fix the share of public expenditure to GDP equal to  $S_g = 0.2$ . From the social resource constraint, we obtain a share of investment over GDP equal to  $S_I = 0.21$ .

Parameter  $A$  is calibrated in order to match the steady state value of output from US data, given<sup>4</sup> by 20.12. The elasticity of substitution across differentiated goods  $\theta$  is set equal to 10, implying a *mark up* of 1.1, as in Rotemberg and Woodford (1999).

A difficult parameter to pin down is  $\phi_p$ . Ireland (1997) and Kim (2000) have values for this parameter derived from empirical estimates. In what follows  $\phi_p$  is basically left as a free parameter and it is set equal to 4, a value close to the number obtained by Ireland (1997), where  $\phi_p = 3.45$ . It should be clear, however, that this is just a working hypothesis.

The investment adjustment cost function is assumed to be zero in the steady state so that:  $\Phi(I/K) = I/K$ ,  $\Phi'(I/K) = 1$  where  $I/K$  is the steady state investment/capital ratio set equal to 0.025. Because of this characteristics, the investment adjustment cost affects the model only in the neighborhood of the steady state. The elasticity of the marginal adjustment cost function defined as  $\eta_k^{-1} = -(\Phi'/\Phi'') \div I/K$ , entirely determines the speed of adjustment together with investment volatility. To generate an empirical plausible investment volatility  $\eta_k = 0.1$ , as showed by Lubik (2000). Moreover, I set  $\psi_1 = 0.0055$ , which configures fiscal policy as to be 'passive', or Ricardian.

Let vector  $\tilde{F}_t$ , be defined as  $\tilde{F}_t = [\tilde{A}, \tilde{\xi}_t, \tilde{\eta}_t^i, \tilde{\eta}_t^T]'$ , so that:

$$\tilde{F}_t = \Omega \tilde{F}_{t-1} + \Theta_t \quad (34)$$

where  $\Theta_t$  indicates the vector of the disturbances, defined as follows:  $\varepsilon_t = \text{diag}(\Theta_t) = [\varepsilon_t^A, \varepsilon_t^\xi, \varepsilon_t^i, \varepsilon_t^T]'$ . In (34), the diagonal matrix  $\Omega$  group the AR(1) coefficients of the disturbance equations.

The set of autoregressive coefficients is:

$$\Omega = \begin{bmatrix} \rho_A & 0 & 0 & 0 \\ 0 & \rho_\xi & 0 & 0 \\ 0 & 0 & \rho_M & 0 \\ 0 & 0 & 0 & \rho_G \end{bmatrix} = \begin{bmatrix} 0.98 & 0 & 0 & 0 \\ 0 & 0.96 & 0 & 0 \\ 0 & 0 & 0.98 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix} \quad (35)$$

The autocorrelation parameter for the fixed-cost deterministic process is taken from Kim (2000) with  $\rho_\Phi = 0.92$ .

The variance-covariance matrix of the shocks  $\Sigma_\varepsilon$  is given by:

$$\Sigma_\varepsilon = \begin{bmatrix} \sigma_A^2 & 0 & 0 & 0 \\ 0 & \sigma_\xi^2 & 0 & 0 \\ 0 & 0 & \sigma_R^2 & 0 \\ 0 & 0 & 0 & \sigma_G^2 \end{bmatrix} = \begin{bmatrix} .000345 & 0 & 0 & 0 \\ 0 & .001962 & 0 & 0 \\ 0 & 0 & .0011 & 0 \\ 0 & 0 & 0 & .0004 \end{bmatrix} \quad (36)$$

Among the stochastic terms, the parameters  $(\rho_A, \sigma_A^2)$  have been set equal to (0.98, .000345), while the public expenditure parameters  $(\rho_G, \sigma_G^2)$ , are set to be (0.9, .0004), as in Chari et al. (2000). For what concerns the parameters of the stochastic process leading  $\xi_t$ , I constructed the time series of values for  $\xi$ , by assuming that in each year the transaction

<sup>4</sup>Data are transformed as suggested by Kim (2000).

costs are 2% per year. Then, I estimated equation (5) over the sample 1959:1-2001:4, by using Instrumental Variables<sup>5</sup> (Instruments chosen: four lagged values of  $\xi$ ). The results of such estimation are (standard errors in brackets):  $\rho_\xi = .99$  (.0016), D.W.: 1.059. The variance of residuals is  $\sigma_\xi^2 = .00196$ .

To get a value for monetary policy shocks, I estimated a simple monetary policy reaction function given by:

$$R_t = \phi_R R_{t-1} + \sum_{s=0}^4 \phi_{\pi_s} \pi_{t-s} + \sum_{s=0}^2 \phi_{y_s} Y_{t-s} + \eta_t^M \quad (37)$$

The sample chosen for the estimation goes from 1979:3 -2001:4, in order to make the results closest to the analysis of Clarida et al. (2000). Equation (37) has been estimated by Generalized Method of Moments proposed by Hansen (1982). A selective report of coefficients is collected in Table 2 ( $t$ -statistics in brackets):

$\phi_{\pi_0}$	$\phi_{\pi_1}$	$\phi_{\pi_2}$	$\phi_{y_0}$	$\phi_{y_1}$
1.6	1.45	0.91	0.8	0.2
(2.88)	(2.7)	(2.62)	(2.5)	(2.1)

**Table 2:** Estimated Coefficients of Equation (37)

The  $\overline{R}^2$  of the regression is 0.78, and the overidentifying restrictions are passed with a 1% of significance. The results reported in Table 2 are in line with what has been reported by Clarida et al. (2000). In particular, the coefficients of inflation and output targeting are slightly bigger than what has been reported by the existing studies.

Finally, by using a LM test, I tested for level of autocorrelation of order 1 of  $\eta_t^M$ , the test statistics of 26.714325, which, compared to  $\chi^2(1)$ , forces us to reject the null of absence of autocorrelation of order 1. Furthermore, I estimated regression (26) to get the value of the coefficient  $\rho_M$  and the volatility of residuals. The results of such regression are:  $\rho_M = 0.77$  ( $t$ -statistics: 7.4), with  $\sigma_R^2 = .0011$ .

**4.3. Solution Method.** To solve out the model, I follow a generalization of the approach taken by Blanchard and Kahn (1980) based on a QZ decomposition method proposed by Sims (2000). According to this method, the model can be written in the following form:

$$\Gamma_0 \tilde{z}_t = \Gamma_1 \tilde{z}_{t-1} + \Gamma_2 \varepsilon_t + \Gamma_3 (\tilde{z}_t - E_{t-1}[\tilde{z}_t]) \quad (38)$$

where  $\tilde{z}_t$  is a vector formed by all the variables of the model expressed as in percentage deviation from their steady state<sup>6</sup>, while  $\varepsilon_t$  is a vector formed by the exogenous shocks of the system (technology, transaction cost, monetary and fiscal policy). Matrices  $\Gamma_0$ ,  $\Gamma_1$ ,  $\Gamma_2$ ,  $\Gamma_3$  are nonlinear function of the deep parameters of the model. If there is a unique equilibrium, the solution of the model can be displayed as follows:

$$\tilde{z}_t = \Pi_1 \tilde{z}_{t-1} + \Pi_2 \varepsilon_t \quad (39)$$

<sup>5</sup>The overidentifying restrictions are passed with 1% of significance.

<sup>6</sup>Note that  $\tilde{z}_t$  is defined as  $\tilde{z}_t = \log Z_t - \log Z$ .

where matrices  $\Pi_1, \Pi_2$  are functions of the parameters of the model. By letting  $g_z$  as the growth rate of variable  $Z$ , the solution (39) can be also rewritten as:

$$\begin{aligned} \log Z_t = & [(I - \Pi_1) \log z + \Pi_1 \log g_z] + [(I - \Pi_1) \log g_z] t + \\ & + \Pi_1 \log Z_{t-1} + \Pi_2 \varepsilon_t \end{aligned} \quad (40)$$

with  $Z$ , non-transformed variable,  $z$  transformed variable, as discussed in Kim (2000). Now the system is equivalent to a VAR (Vector Autoregression) with a constant and a time trend. This approach is more stable and can easily handle non-singularities in the system of equation involved. Second moments matrix  $\widehat{\Sigma}_\varepsilon$  were computed through a simulation of the dynamic Lyapunov equations defined as follows:

$$\widehat{\Sigma}_\varepsilon = \Pi_1 \Sigma_0 \Pi_1' + \Pi_2 \Sigma_\varepsilon \Pi_2' \quad (41)$$

where  $\Sigma_0$  is the initializing matrix is defined as in (36), while  $\Pi_1$  and  $\Pi_2$  are from (39).

## 5. RESULTS

**5.1. Volatility.** In this section I am going to study the impact on volatility determined by the choice of monetary policy function parameters. The range of parameter chosen for the simulation is inspired by the choices made by other contributors, in order to make the results presented here more comparable with the existing literature. The combination  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$  is from Taylor (1993), and the value 0.06 for  $\phi_y$  has been found by Rotemberg and Woodford (1999) to be the welfare maximizing value when coupled with  $\phi_\pi = 1.22$ .

Tables 3-8 report the pattern of standard deviation for output, nominal interest rate and inflation rate, under different parameter configurations, together with the US data computed over the sample 1959:1-2001:4.

		$\phi_y$					
		0.0	0.06	0.5	3	5	10
$\phi_\pi = 1.5$	$\sigma_y$	19.58	6.844	.452	.1361	.06833	.034
	$\sigma_R$	3.159	1.1031	.0718	.03	.022	.02
	$\sigma_\pi$	2.099	0.9782	.1292	.2657	.208	.208
$\phi_\pi = 3$	$\sigma_y$	4.64	3.66	.84	.082	.22	.06
	$\sigma_R$	.745	.587	.13	.022	.04	.023
	$\sigma_\pi$	.251	.26	.165	.063	.39	.20
$\phi_\pi = 5$	$\sigma_y$	2.29	2.05	.96	.12	.068	.136
	$\sigma_R$	.36	.32	.15	.02	.022	.033
	$\sigma_\pi$	.079	.091	.11	.068	.062	.27
$\phi_\pi = 10$	$\sigma_y$	1.16	1.1	.77	.24	.13	.062
	$\sigma_R$	.18	.175	.12	.043	.029	.023
	$\sigma_\pi$	.02	.027	.049	.073	.068	.0619
US	$\sigma_y$	0.77					
	$\sigma_R$	.078					
	$\sigma_\pi$	.66					

**Table 3:** Taylor Rule 1

The numbers reported in Tables 3-8 were obtained by simulating the model via the stochastic shocks included in the model: technology shock, monetary policy, transaction cost shocks and public expenditure shocks<sup>7</sup>.

From Table 3, we observe that an inflation targeting rule with  $\phi_\pi = 1.5$ ,  $\phi_y = 10$  delivers the lowest degree of output volatility. The volatility of inflation drops further down if we set  $\phi_\pi = 10$ ,  $\phi_y = 10$ , as in Rotemberg and Woodford (1999). However, a pure inflation targeting rule (with  $\phi_y = 0.0$ ) produces results in terms of volatility which are always dominated by any other possible combinations of policy parameters. Thus, according to the results reported in Table 3, it is never optimal to set  $\phi_y = 0.0$ .

A second issue concerns the role of coefficient  $\phi_\pi$ . By raising  $\phi_\pi$  we find a substantial reduction of inflation volatility only if high values of  $\phi_\pi$  are accompanied by high values of  $\phi_y$  as well: to be successful in terms of stabilization policies, an aggressive inflation targeting policy should never miss the impact on the real side of the economy. The presence of an AS curve relating expected inflation with current inflation and output plays a key role in explaining this result: when  $\phi_y$  is too low and  $\phi_\pi$  is too high, after an inflationary shock the response towards inflation turns out to be very strong, implying a reduction in output. As a consequence, the reduction in output, implies an increase of the expected inflation. So, only a mixed response both towards inflation and output allows to correct the consequences of an inflationary shock without causing a recession and an increase of inflation.

The large importance of output targeting coefficient  $\phi_y$  can be also seen by considering the asymmetric response of volatility of output with respect to change in  $\phi_\pi$ , given a specific value for  $\phi_y$ . When  $\phi_y = 0.06$ , we observe that by raising  $\phi_\pi$  helps to reduce output volatility, nominal rate and inflation rate. However, when  $\phi_y = 0.5$  and  $\phi_\pi = 1.5$  we observe a sharp reduction in the volatility of  $Y$ ,  $R$ , and  $\pi$ , but by raising  $\phi_\pi$  we find an increased volatility of output until we set values of  $\phi_\pi$  bigger than 5.

To visualize these results, Figure 1 plots the standard deviation of  $Y$ ,  $R$ ,  $\pi$  for different values of  $\phi_\pi$ , taking as fixed <sup>8</sup>  $\phi_y = 0.5$ . In particular, the dark line shows the pattern of volatility of  $Y$ ,  $R$  and  $\pi$ , respectively, for different values<sup>9</sup> of  $\phi_\pi \in [0, 10]$ , given  $\phi_y = 0.06$ , while the dashed line is for  $\phi_y = 0.5$ . Figure 1 reports exactly the patterns that Table 1 has showed in numerical terms: the parameter combination  $\phi_\pi = 1.5$ ,  $\phi_y = 0.06$  delivers the highest level of volatility within the range and a better stabilization results can be obtained by setting  $\phi_y = 0.5$  for all  $\phi_\pi \in [0, 10]$ . This is a striking results, because a combination of monetary policy parameters like  $\phi_\pi = 1.2$ ,  $\phi_y = 0.06$  turns out to be

<sup>7</sup>Following the suggestions by Cogley and Nason (1995), in order to avoid spurious business cycle dynamics, I did not pass the time series results via Hodrik-Prescott filter. Thus, the numbers reported in Tables 3-8 are the results (averaged) over 200 simulations. For a critique to the common practice of H-P filter, see Cogley and Nason (1995).

<sup>8</sup>In Figures 1-2 the fiscal policy parameter  $\psi_1$  has been set equal to 0.0055. The present paper focuses on monetary policy function, given the fiscal policy reaction function parameters. It is clear, however, that an high value of the fiscal policy parameter and an high value of the monetary policy parameter correspond to the *passive-active* combinations in Leeper's (1991) terminology.

<sup>9</sup>Each point of the Figure is obtained by varying  $\phi_\pi$  withing the grid  $[0,10]$  with increments 0.01.

welfare maximizing in a simple model *à la* Rotemberg and Woodford (1999). These results show that the inclusion of capital accumulation changes dramatically the role of monetary policy in stabilization plans, by making the role of output much more important.

In Table 4 I report selected results for autocorrelations up to second order. By increasing  $\phi_y$ , given  $\phi_\pi = 1.5$ , the autocorrelation of the inflation rate increases, as well as that of output and nominal rate. However, if monetary authority adopts an aggressive inflation rate policy, the autocorrelation function is lower, no matters how big is the value for  $\phi_y$ .

	autocorrelation order	$\phi_y$					
		0.06			0.5		
		$\rho_y$	$\rho_R$	$\rho_\pi$	$\rho_y$	$\rho_R$	$\rho_\pi$
$\phi_\pi = 1.5$	1	.25	.24	.24	.57	.57	.33
	2	.036	.023	.027	.41	.4	.13
$\phi_\pi = 3$	1	.18	.19	.24	.3	.31	.26
	2	-.02	-.02	.03	.097	.11	.047
US	1	.98	.88	.87			
	2	.96	.8	.81			

**Table 4:** Autocorrelations for model with Rule 1

If we compare the results obtained for the autocorrelations with US data, we find that the model lacks of a substantial internal propagation mechanism. In fact, autocorrelation of order 1 from data is .98, while from the model we find only .25 (if  $\phi_\pi = 1.5$ ,  $\phi_y = .06$ ) and .57 when (if  $\phi_\pi = 1.5$ ,  $\phi_y = .5$ ).

An interesting fact to be noted is that by increasing the output targeting parameter variables autocorrelation increases with respect to the case with  $\phi_y = .06$ . Moreover, when  $\phi_\pi = 3$ , we observe a considerable drop in the output autocorrelation after the first period<sup>10</sup>.

The results obtained in Table 4 differ substantially from what has been collected by Casares and McCallum (2001), who find a level of autocorrelation implied by the model much closer to the empirical values.

Let us move on to study the empirical effects of parameters choice for a model including an expected inflation targeting rule, like Rule 2. Table 5 reports the volatility results of the inflation targeting rule, for the same range of parameters already employed in the analysis of the simple Taylor rule in Tables 3 and 4.

<sup>10</sup>The trend showed by the autocorrelations in Table 3 is stable also for all the other combinations of  $\phi_\pi = 5, 10$  and  $\phi_\pi = 3, 5, 10$ , not reported here, but available upon request.



		$\phi_y$					
		0.0	0.06	0.5	3	5	10
$\phi_\pi = 1.5$	$\sigma_y$	6.063	3.45	.36	.055	.033	.0164
	$\sigma_R$	5.68	4.3	.69	1.042	1.031	1.031
	$\sigma_\pi$	0.73	.613	.1021	.033	.029	.0274
$\phi_\pi = 3$	$\sigma_y$	1.71	1.36	.43	.063	.036	.018
	$\sigma_R$	1.29	1.14	.709	1.078	1.056	1.04
	$\sigma_\pi$	.11	.11	.076	.0341	.030	.027
$\phi_\pi = 5$	$\sigma_y$	.964	.86	.43	.076	.043	.021
	$\sigma_R$	.971	.94	.78	1.12	1.093	1.069
	$\sigma_\pi$	.046	.048	.051	.034	.030	.028
$\phi_\pi = 10$	$\sigma_y$	.544	.52	.38	.114	.067	.0324
	$\sigma_R$	.9351	.93	.88	1.16	1.15	1.09
	$\sigma_\pi$	.018	.019	.026	.033	.0311	.0289
US	$\sigma_y$	0.77					
	$\sigma_R$	.078					
	$\sigma_\pi$	.66					

**Table 5:** Expected Inflation Rule

When  $\phi_\pi = 1.5$ ,  $\phi_y = 0.06$ , the model<sup>11</sup> delivers a volatility of inflation equal to 3.45 (in the data is 0.77), and a volatility of nominal interest rate equal to 4.3 (0.078 from data). On the other hand, if we raise the value of  $\phi_y$  up to 0.5, we find that the combination  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$  implies a volatility of inflation equal to 0.36, and a volatility of nominal interest rate equal to 0.69.

From Table 5 we observe a pattern of output, nominal rate and inflation volatility for some aspects similar to what we have seen in Table 3, without the expectation terms included.

However, the lowest degree of output and inflation volatility can be obtained with  $\phi_\pi = 1.5$ ,  $\phi_y = 10$ . Therefore, output targeting is more important with a monetary rule including an expected inflation target. This result is not in accord with what other authors have found in terms of welfare maximization and opens up for different policy recommendations. It should be stressed that given a value for  $\phi_y$ , it is *not* true that raising  $\phi_\pi$  helps to stabilize output, nominal rate and inflation rate. Even in this case we note the same pattern already considered for the previous model: when  $\phi_y = 0.5$  is kept fixed, by varying  $\phi_\pi$  we obtain an increase of output volatility until when  $\phi_\pi = 5, 10$ .

The impact on volatility induced by changes in the inflation target coefficient is represented also in Figure 2, where the evolution of standard deviation of output, nominal rate and inflation rate is plotted for increments of  $\phi_\pi$ . Figure 2 replicates the results reported in Table 6: the dark line is for  $\phi_\pi = 1.5$ ,  $\phi_y = 0.06$ , while the dashed line is for  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$ . The dashed line always dominates the dark line, even if by raising  $\phi_\pi$  the volatility of all variables is immediately under control as long as  $\phi_\pi > 1$ .

<sup>11</sup>Recall that  $\phi_y = 0.06$  is the welfare-maximizing value chosen by Rotemberg and Woodford (1999).

Autocorrelations up to second order are reported in Table 6 for the model simulated with Taylor Rule 2.

	autocorrelation order	$\phi_y$					
		0.06			0.5		
		$\rho_y$	$\rho_R$	$\rho_\pi$	$\rho_y$	$\rho_R$	$\rho_\pi$
$\phi_\pi = 1.5$	1	.43	.98	.47	.64	.98	.48
	2	.22	.96	.28	.47	.97	.27
$\phi_\pi = 3$	1	.32	.97	.53	.46	.98	.44
	2	.11	.93	.37	.26	.96	.23
US	1	.98	.88	.87			
	2	.96	.8	.81			

**Table 6:** Autocorrelations for model with Rule 3

From Table 6 we observe that the model with Rule 3 shows an higher autocorrelation pattern for output, inflation and nominal rate, if compared with the results obtained for the model simulated with Rule 1. In particular, the autocorrelation of the inflation rate is .47 for  $\phi_\pi = 1.5$ ,  $\phi_y = 0.06$  and reaches 0.53 for  $\phi_\pi = 3$ ,  $\phi_y = 0.06$ , a value considerably bigger than what has been obtained by other papers in this literature.

**Summing up.** From the results collected in Tables 3-6, we found that an aggressive inflation targeting policy induces an excess of volatility. This effect can be mitigated by setting an higher output targeting coefficient.

The model here studied considers a multiplicity of shocks: three out of four are demand shocks (transaction cost, public expenditure and monetary policy shock), while the technology shock is a typical supply-side shock. In case of demand shock, it is clear that by raising feedback parameters in monetary policy reaction function tends to eliminate the impact of such shocks.

However, supply shocks require a more moderate response from monetary authority. In the simulation here considered, all the shocks are given equal importance, so the results come from the need to control demand shocks. On the other hand, the results do not change dramatically if the predominance were assigned to supply shocks, as it will be discussed in the analysis of impulse-response functions.

**5.2. Sensitivity analysis.** It has been argued<sup>12</sup> that the role of distortions in the economy is crucial in determining the optimality of a given rule. To have a metric of the degree of distortions existing in the economy, I follow the approach taken by Lubik (2001) who defines a variable - called the ‘degree of distortions’ - formed by the ratio of the elasticity of substitution among final goods  $\theta$  and the cost of price adjustment parameter  $\phi_p$ . Higher is the ratio  $\theta/\phi_p$ , less distorted is the economy, because an high level of goods substitutability (lower markup) is associated to a low value for the price adjustment cost parameter.

<sup>12</sup>See Rotemberg and Woodford (1999).

To explore these issues, I reported in Tables 7 and 8 standard deviations of output ( $\sigma_y$ ), nominal rate ( $\sigma_R$ ) and inflation rate ( $\sigma_\pi$ ) for two different values of the distortion index. The first column in normal characters indicates a ratio  $\theta/\phi_p = 0.15$ , obtained with  $\theta = 1.5$ ,  $\phi_p = 10$ . The numbers in italics are obtained for an economy with  $\theta = 11$ ,  $\phi_p = 0.001$ . Of course, the former case corresponds to an heavily distorted economy (where the markup is equal to 3), while the figures in italics are for an economy very close to perfect competition (with a markup equal to 1.05).

Volatility		$\phi_y$							
		0.0		0.06		0.5		3	
$\phi_\pi = 1.5$	$\sigma_y$	11.08	<i>19.59</i>	3.36	<i>6.88</i>	.37	<i>.45</i>	.07	<i>.1</i>
	$\sigma_R$	4.81	<i>3.22</i>	1.56	<i>1.13</i>	.27	<i>.07</i>	.046	<i>.027</i>
	$\sigma_\pi$	3.25	<i>2.14</i>	0.96	<i>1.00</i>	.13	<i>.13</i>	.09	<i>.2</i>
$\phi_\pi = 3$	$\sigma_y$	.89	<i>4.68</i>	.73	<i>3.69</i>	.27	<i>.85</i>	.13	<i>.083</i>
	$\sigma_R$	.69	<i>.76</i>	.54	<i>.60</i>	.23	<i>.14</i>	.06	<i>.023</i>
	$\sigma_\pi$	.25	<i>.25</i>	.19	<i>.27</i>	.08	<i>.16</i>	.12	<i>.065</i>

**Table 7:** Sensitivity Analysis, Rule 1.

In general, a more distorted economy shows a lower degree of volatility when a policy rule like Rule 1 is introduced in the model. Thus, we may conclude that a more distorted economy calls for a more interventionist policy rule. In fact, if  $\phi_\pi = 1.5$  and  $\phi_y = 0.5$  we observe a lower degree of volatility for all the variables, if compared with the analogous parameter combination for the less distorted economy ( $\theta = 10$ ).

Let us look now at the same type of results when monetary policy rule is given by Rule 2, with expected inflation targeting.

Volatility		$\phi_y$							
		0.0		0.06		0.5		3	
$\phi_\pi = 1.5$	$\sigma_y$	4.62	<i>6.05</i>	1.87	<i>2.6</i>	.31	<i>.38</i>	.054	<i>.056</i>
	$\sigma_R$	5.49	<i>5.7</i>	2.43	<i>2.68</i>	.63	<i>.69</i>	1.04	<i>1.047</i>
	$\sigma_\pi$	.47	<i>.76</i>	.21	<i>.39</i>	.047	<i>.10</i>	.022	<i>.034</i>
$\phi_\pi = 3$	$\sigma_y$	1.018	<i>1.75</i>	.78	<i>1.4</i>	.27	<i>.44</i>	.0543	<i>.63</i>
	$\sigma_R$	1.15	<i>1.31</i>	1.02	<i>1.15</i>	.68	<i>.71</i>	1.07	<i>1.078</i>
	$\sigma_\pi$	.077	<i>.12</i>	.065	<i>.11</i>	.03	<i>.08</i>	.021	<i>.035</i>

**Table 8:** Sensitivity Analysis, Rule 3.

In Table 8 with an heavily distorted economy, we observe a set of results similar to those reported in Table 4. An interesting fact is given by the lower degree of volatility associated to a more distorted economy including a monetary policy rule with an output targeting coefficient higher than the optimal value considered in the literature. In fact, higher is the number of distortions, bigger is the emphasis on the real-side stabilization goals that a policy rule should have, to contribute also to stabilize the inflation rate.

## 6. MODEL DYNAMICS

How parameters of monetary policy reaction function affects the response of the variables of the economy and the adjustment towards the initial steady state, after an exogenous technological shock ?

To answer this question, we can examine the impulse-response pattern for output, nominal rate and inflation under different parameter configurations of the monetary policy reaction function. In what follows, I am going to concentrate on the impact effect derived from inflation targeting parameter, given its importance in the recent monetary policy literature.

In Figure 2, column, I reported the plot of impulse response functions for output, nominal rate and inflation rate for Rule 1: the dark line is drawn for  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$ ; the dashed line is for  $\phi_\pi = 5$ ,  $\phi_y = 0.5$ , and the dashed/dotted line is for  $\phi_\pi = 10$ ,  $\phi_y = 0.5$ , while in the right column I reported similar responses for the model including Rule 2.

Let us focus now on the left column. The impact of a positive technological shock is expansionary: output raises, labor effort increases, consumption increases, and investment in physical capital increases. In particular, an expansionary technological shock is deflationary. The deflationary effect is due to the fact that the steady state level of output is inefficient (actual output is below potential output) because of the presence of monopolistic competition and transaction costs (positive in steady state).

If monetary authority reacts too aggressively to inflation, by setting  $\phi_\pi = 5$ ,  $\phi_y = 0.5$  (dashed line), from Figure 3 (left column) we observe that the effect is still expansionary and deflationary, but output, nominal rate and inflation rate all show larger swings before turning back to their steady state position. For example, output shows an expansionary peak which is even stronger than in the case with  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$  (dark line), but it passes through a recession before reaching the steady state level.

When  $\phi_\pi = 10$ ,  $\phi_y = 0.5$  the results are, in some sense reverted, because the expansionary technological shock determines a recession instead than an expansionary effect: nominal interest rate shows large swings for any observed movement of the inflation rate. This determines a recession in response to a positive technological shock. In this case, monetary authority reacts so strongly that it is not possible to take advantage of the positive technological expansion.

Let us compare these results with the case under expected inflation targeting. In the right column of Figure 2 are plotted the impulse response function for the model including Rule 2 in (28). The dark line is plotted for  $\phi_\pi = 1.5$ ,  $\phi_y = 0.5$ ; the dashed line is for  $\phi_\pi = 5$ ,  $\phi_y = 0.5$ , and the dashed/dotted line is drawn for  $\phi_\pi = 10$ ,  $\phi_y = 0.5$ . The qualitative responses of variables are not dissimilar to what has been observed with Rule 1. On the other hand, if  $\phi_\pi = 5$ ,  $\phi_y = 0.5$  the expansionary effect due to the technology shock is washed out by the strong reaction of the monetary authority. The same is true when  $\phi_\pi = 10$ ,  $\phi_y = 0.5$ .

Thus, the adoption of the expected inflation target implies a reduced degree of volatility of the variables, but an higher sensitivity of the variable response to changes in the parameters of monetary policy rules.

## 7. CONCLUSION

In this paper I have studied the operational performance of a set of simple monetary policy rules *à la* Taylor in a model with capital accumulation and nominal and real rigidities. In fact, with a small value of output targeting coefficient, monetary authority is not completely successful in stabilizing the volatility of output, nominal rate and inflation only by acting on the inflation targeting coefficient. Moreover, a too strong concerns towards inflation rate relatively to output reduces the ability of monetary authority to control inflation volatility and determines a stronger reaction with respect to an exogenous shock hitting the economy.

The model is also able to provide better statistics for inflation persistence than what has been shown in the other literature.

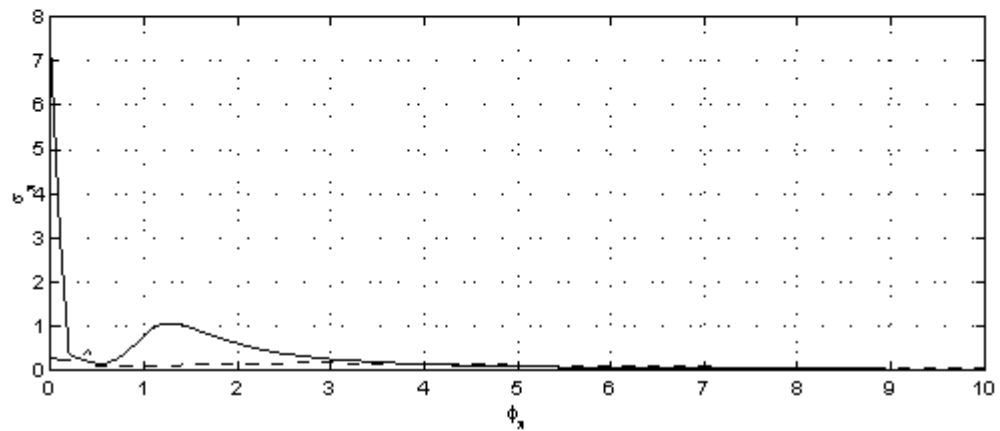
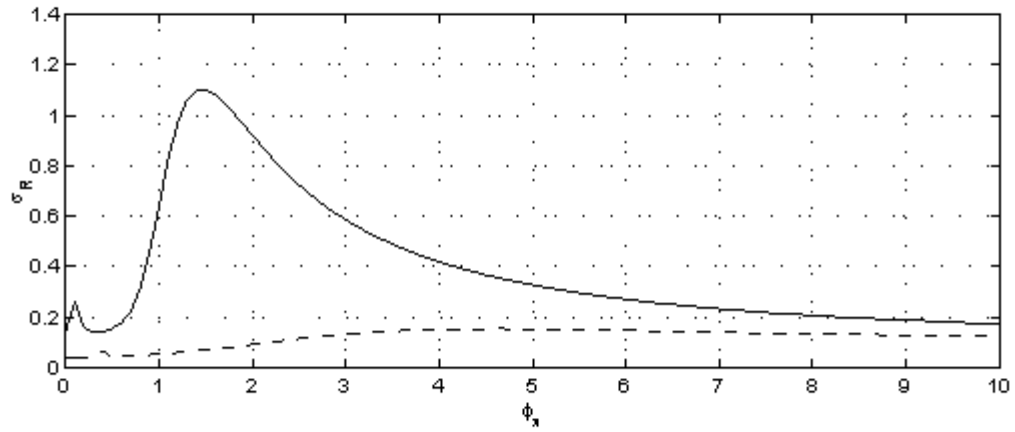
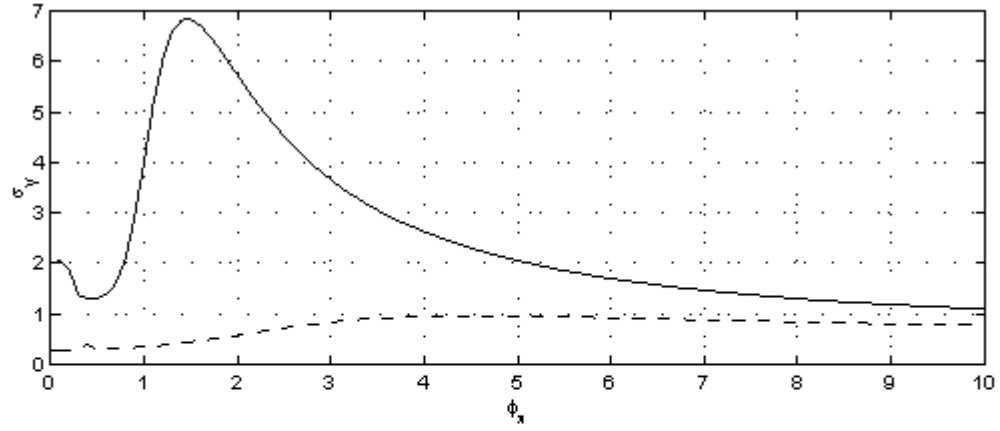
These results contrasts with the conclusion obtained in models with a naive modelling of rigidities and intertemporal trade-offs that an optimal monetary policy function should posit a disproportional weight on the inflation rate rather than on output.

The work here analyzed calls for further generalizations with respect to the way of modelling nominal rigidities (especially with respect to wage rigidities) and for the analysis of distortionary taxation.

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