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Endogeneous outsourcing and vertical integration with process R&D *

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Abstract

According to received literature, vertical integration may enjoy a social superiority due to the ability to internalize the externality that goes back from the pricing policy of the downstream firm to the profits of the upstream firm. We challenge this result introducing process R&D in a broad set of scenarios with vertically symmetric and asymmetric R&D commitments. In some of these contexts a reversed sequence of socially desirable vertical arrangements arises, making outsourcing superior. In other circumstances disintegration is privately superior but socially inefficient. Finally, vertically asymmetric costs of R&D are considered to allow for a wider range of applications.

JEL Classification: L13, L22, O31.

Keywords: Vertical Integration, Process R&D.

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1 Introduction

Vertical integration is the object of two main streams of literature: Industrial Organization (Grossman Hart, 1986; Perry, 1989) and Economics of Institutions (Coase, 1937; Williamson, 1971, 1985, 1986). In this second strand, transaction cost economics has played a prominent role and has become a paradigm that other disciplines eventually borrowed. In most investigations of the internal organisation of the firm, vertical integration appears as a reaction either to excessive costs of transacting inputs or to some of the externalities that make markets socially inferior to the governance system provided by a corporate organization embodied in a firm.

A large set of externalities is also directly or indirectly associated to vertical integration and its opposite, vertical segmentation. Among them, the one represented by double marginalization seems to be one of the most crucial. Whenever a downstream (D) firm increases its market price, this move affects negatively the profit of the upstream (U) firm (or firms). This firm sells its output which is used as an input for D production. The integrated firm embodying both production stages (U and D) is able to internalize this externality and, therefore, produces a larger output which is sold at a lower price (Spengler, 1951; Tirole, 1989).

Other externalities occur in vertical relationships. Some of them make unsustainable the coexistence of firms with idiosyncratic levels of vertical integration in the same sector. This class of externalities is due to the captive relationship in which the U firm is trapped when selling to a single D firm. When the number of firms operating in the D section of the market increases, for instance because of trade integration, U firms have broader choice and their captive condition can be relaxed. As the number of firms ”breaking their chains” increases, it becomes easier for U firms to find D buyers who will compete among them to get their inputs. Then U firms will be less liable to be cornered. The externality emerges as the number of D firms increases since the captive position becomes less and less burdensome and the alternative arrangement, i.e. vertical integration looses its attractiveness as a way to escape from the hold up problem (McLaren, 1999, 2000; Grossman and Helpman, 2002; Spencer and Jones, 1991). Despite of the attractiveness of the theoretical interpretation of this externality, we find also models where integrated and nonintegrated firms coexist because of contractual specificities of firms producing close substitutes or simply because of product differentiation (Jansen, 2003; Lambertini and Rossini, 2003; Pepall and Norman,
Empirical analyses concerning the strategic effects of vertical integration and vertical disintegration are quite few and do not address the question of the coexistence of different vertical arrangements in the same industry, yet they rather go through the welfare effects of integration and disintegration (Slade, 1998 a,b).

As a matter of fact, the decision to either vertically integrate or resort to outsourcing is highly dependent upon the ability and the incentives of firms to undertake productive activities in an efficient way in one of the two alternative organizational arrangements. However, the production activities of a firm very often include R&D commitments that may innovate either the production process or the product (Teece, 1976; Armour and Teece, 1980). These activities may change the incentive to integrate or to adopt outsourcing when they give rise to vertical spillover, or take place among firms with heterogeneous objectives (Rossini, 2003). The existence of externalities in R&D is widely analyzed in the literature (d’Aspremont and Jacquemin, 1988) but it has never been related to vertical relationships. Our aim is to analyse vertical process innovating R&D activities with spillovers and we shall concentrate on their effects on the incentives to integrate. In some circumstances, technological spillovers may neutralize or even be larger than the traditional externality that exists along the vertical chain. To this purpose, we shall walk along many scenarios in which the vertical R&D spillover takes place. In particular, we shall investigate the effects of spillovers flowing two ways from U to D and from D to U. The case of asymmetric externalities will also be considered so as to contribute to a more general picture. As we shall see, the introduction of process R&D with vertical spillover changes the received result of the superior social performance of vertical integration when there are imperfectly competitive firms.

The paper is organized around the model of process vertical R&D with spillovers that is introduced in the next section. In section 3, we provide some comparisons when symmetric spillovers occur. In section 4, we go through the case of asymmetric spillovers. In section 5, we consider asymmetric costs of R&D. Conclusions are in section 6.
2 The model with symmetric R&D spillovers

In this section we model the two alternative vertical arrangements, one with integration and the other with decentralization or disintegration. We go through the case of a monopoly since it is the only one analytically tractable. However, the externality that exists in the vertical chain when integration is assumed away remains in all cases of non perfect competition. Therefore, most of these results may be extended to oligopolistic markets.

2.1 Vertical decentralization

We consider a homogeneous monopoly facing a linear demand curve for a final, or downstream (D) good sold to consumers

\[ p_D = a - q_D \]  

(1)

Production is organized along a vertical process whereby the final output is assembled by a D monopolist who needs an intermediate good for its production. The intermediate good is produced by an upstream (U) firm that requires no produced input. The cost structure of the D firm is given by the following function

\[ c_D = c - x_D - \beta x_U \]  

(2)

where \( c_D \), the marginal cost of production of the D firm, is composed by a simple marginal cost \( c \) minus an amount \( x_D \) that reduces the marginal cost thanks to process R&D undertaken by the D firm. This activity is modeled, according to received literature, using a convex technology (d’Aspremont and Jacquemin, 1988) represented by an R&D total cost function with increasing marginal cost

\[ k_D = \gamma \frac{x_D^2}{2} \]  

(3)

where \( \gamma \) is a technological parameter related to the marginal cost of R&D. The spillover parameter \( \beta \in [0, 1] \) in (2) multiplies the amount of cost reduction \( x_U \) undertaken by the firm in the U stage and it is the vehicle for the external beneficial effect of the R&D undertaken by the U firm upon the marginal cost of the D firm. \( k_D \) represents the total cost of R&D. The objective of the D firm is to maximise

\[ \pi_D(q_D, x_D) = q_D(p_D - c_D - g) - \gamma \frac{x_D^2}{2} \]  

(4)
where $g$ is the price paid by the D firm for one unit of the intermediate good needed to produce one unit of the final good (perfect vertical complementarity assumption). The D monopolist maximises profit with respect to the quantity sold and the amount of cost reduction to be obtained via process innovating R&D.

Now we go to the U firm that chooses its optimal price and cost reduction by maximising:

$$\pi_U(g, x_U) = q_U(g - c_U) - \gamma \frac{x_U^2}{2}$$  \hspace{1cm} (5)

where the cost reduction follows a similar rule as in (3) making for the following marginal cost:\(^1\)

$$c_U = c - x_U - \beta x_D.$$  \hspace{1cm} (6)

As it appears, we assume that the vertical spillover is symmetric and goes both ways. Taking FOCs for the U and D firms and doing substitutions we get:\(^2\)

$$x^*_U = \frac{A(1 + \beta)}{4\gamma - 3 - \beta(4 + \beta)}$$  \hspace{1cm} (7)

$$g^* = \frac{(a - c)(2 + 3\beta - 2\gamma) + c(1 + \beta(1 + \beta) - 2\gamma)}{4\gamma - 3 - \beta(4 + \beta)}$$  \hspace{1cm} (8)

$$\pi^*_U = \frac{A^2\gamma}{2(4\gamma - 3 - \beta(4 + \beta))}$$  \hspace{1cm} (9)

$$\pi^*_D = \frac{A^2\gamma(2\gamma - 1)}{2(4\gamma - 3 - \beta(4 + \beta))^2}$$  \hspace{1cm} (10)

$$p^*_D = \frac{(3a + 2c)\gamma - a(1 + \beta)(3 + \beta)}{4\gamma - 3 - \beta(4 + \beta)}$$  \hspace{1cm} (11)

$$x^*_D = \frac{A}{4\gamma - 3 - \beta(4 + \beta)}.$$  \hspace{1cm} (12)

\(^1\)We make the simplifying assumption that the marginal costs are symmetric along the vertical chain (i.e., both firms bear the same marginal cost). The entire analysis of the paper has also been conducted with asymmetric marginal costs, reaching qualitatively analogous conclusions.

\(^2\)We adopt the short hand $A \equiv a - 2c.$
Second order conditions (SOCs) and nonnegativity constraints on market variables and marginal costs after R&D are assured by the condition:\(^3\)

\[
\gamma \geq \frac{a(1 + 2\beta) + c(1 + \beta^2)}{4c} = \gamma_d. \tag{13}
\]

### 2.2 Vertical integration

In the vertically integrated monopoly the input is produced by the U division and internally transferred at its opportunity cost, equal to the marginal cost. Demand, production and R&D technologies are the same as above and, therefore, the objective of the integrated (I) monopoly is:

\[
\pi_I(q_{DI}, x_{DI}, x_{UI}) = q_{DI}(p_{DI} - c_{DI} - c_{UI}) - \gamma \left(\frac{x_{DI}^2 + x_{UI}^2}{2}\right). \tag{14}
\]

By taking systemic FOCs we can get reduced form controls:

\[
x^*_{DI} = \frac{A(1 + \beta)}{2[\gamma - (1 + \beta)^2]} = x^*_UI \tag{15}
\]

\[
q^*_{DI} = \frac{A\gamma}{2[\gamma - (1 + \beta)^2]} \tag{16}
\]

\(^3\)The SOCs that must be simultaneously met are:

\[
\gamma \geq \frac{1}{2b} = \gamma_a
\]

\[
\gamma \geq \frac{3 + \beta(4 + \beta)}{4b} = \gamma_c
\]

\[
\gamma \geq \frac{1 + \beta}{2b} = \gamma_b.
\]

However, the binding condition involves \(\gamma_c\) since \(\gamma_c \geq \gamma_b \geq \gamma_a\). Moreover, we have to assure that R&D does not make for negative marginal costs, i.e.:

\[
c - x_U - \beta x_D \geq 0
\]

\[
c - x_D - \beta x_U \geq 0.
\]

These two conditions require that

\[
\gamma \geq \frac{a(1 + 2\beta) + c(1 + \beta^2)}{4c} = \gamma_d.
\]

Hence the final binding condition is \(\gamma \geq \gamma_d\), since \(\gamma_d \geq \gamma_c\).
\[ \pi_I^* = \frac{A^2 \gamma}{4[\gamma - (1 + \beta)^2]} \]  
\[ p_{DI}^* = \frac{(a + 2c)\gamma - 2a(1 + \beta)^2}{2[\gamma - (1 + \beta)^2]} \]

SOCs and nonnegativity constraints boil down to:

\[ \gamma \geq \frac{a(1 + \beta)^2}{2c} = \gamma_i. \]  

Straightforward comparison reveals that \( \gamma_i \geq \gamma_d \). Then, the admissible parameter region for the integrated firm to invest in R&D is a subset of that associated with the disintegrated firm, investing also for lower levels of \( \gamma \in [\gamma_d, \gamma_i] \), where the integrated firm does not.

3 Integration vs non integration with symmetric spillovers

We are now ready to undertake some intriguing comparisons between the two market arrangements.

First, we confine the analysis to a parameter space where both market settings invest in R&D in the two production stages (U and D). Then we shall consider also the parameter space in which only the decentralized market arrangement invests in R&D while the integrated firm does not.

\[ \text{SOCs require} \quad \gamma \geq \frac{(1 + \beta)^2}{2} = \gamma_e. \]

Nonnegativity requires

\[ \gamma \geq (1 + \beta)^2 = \gamma_f \geq \gamma_e. \]

Moreover, nonnegativity of marginal costs require:

\[ \gamma \geq \frac{a(1 + \beta)^2}{2c} = \gamma_i \geq \gamma_f. \]
3.1 Both arrangements invest in R&D

Compare the profits generated by vertical integration ($\pi^*_I$) against those of the disintegrated arrangement ($\pi^*_U + \pi^*_D$). Their difference is:

$$\pi^*_I - \pi^*_U - \pi^*_D = \frac{\gamma A^2}{4} \left[ \frac{4 \beta^2 (1 + \beta) - 1 + \beta^4 + 4 \gamma - 6 \beta^2 \gamma - 4 \gamma^2}{(3 + 4 \beta^2 + 4 \gamma^2)[(1 + \beta)^2 - \gamma]} \right]$$

(20)

From the analysis of (20), we may draw the following:

**Proposition 1** In the parameter region where both arrangements invest:

- profits are always higher for the integrated firm, and $p^*_D I \leq p^*_D$;
- the overall R&D investment carried out by the integrated firm is larger, i.e., $x^*_D I + x^*_U U \geq x^*_D + x^*_U$;
- the traditional result of the social superior performance of the integrated firm replicates since social welfare with integration ($SW_I$) is larger than with a non integrated arrangement ($SW$).

**Proof.** To begin with, examine **profits**. Consider first the denominator of the expression in square brackets, on the r.h.s. of (20). This is always negative in the feasible set of $\gamma$. Then analyse the numerator. This is positive for $\gamma \in [\gamma_1, \gamma_2]$, where $\gamma_1 = \frac{2 - 3 \beta^2 - \beta}{4 + 16 \beta + 13 \beta^2}$ and $\gamma_2 = \frac{2 - 3 \beta^2 + \beta}{4 + 16 \beta + 13 \beta^2}$ are the two roots of the numerator. However, since $\gamma_2 \leq \gamma_i$, it is established that, in the common feasible set of parameters, the profits from integration are always higher than the sum those accruing to the non integrated firms.

As for **prices**, just calculate the difference

$$p^*_D - p^*_D I = \frac{A \gamma (2 \gamma - 1 + \beta^2)}{2(3 + 4 \beta + \beta^2 - 4 \gamma)(1 + 2 \beta + \beta^2 - \gamma)}$$

(21)

which is non negative for $\gamma \geq \gamma_i$.

Eventually we compare the **R&D investments** undertaken in the two vertical arrangements. Just compare the levels of R&D in the U stage:

$$x^*_D I (= x^*_U I) - x^*_D = \frac{A(1 + \beta)}{2 \gamma - 2(1 + \beta)^2} - \frac{A^2 \gamma (2 \gamma - 1)}{2(4 \gamma - 3 - \beta(4 + \beta))^2}$$
which is nonnegative. The same happens for the D stage:

\[ x^*_D = x^*_U = \frac{A(1 + \beta)}{2\gamma - 2(1 + \beta)^2} - \frac{A^2\gamma}{2(4\gamma - 3 - \beta(4 + \beta))} \]

Welfare comparisons are straightforward and are already implicit in higher profits and lower prices of the integrated arrangement. It is just worth noting that the potentially counterproductive effect associated with the larger R&D effort carried out by the integrated structure is more than offset by the resulting reduction observed both in production costs and in the market price.

3.2 Only the nonintegrated firms invest in R&D

There is a region of the parameter space for \( \gamma \in [\gamma_d, \gamma_i] \) where only the disintegrated firms invest in R&D. In this region the integrated firm does not invest in process R&D and therefore maximization of its monopoly profits lead to the following equilibrium quantity, profit and price

\[ q^*_m = \frac{A}{2}; \quad \pi^*_m = \frac{A^2}{4}; \quad p^*_m = \frac{(a + 2c)}{2} \]

where the subscript \( m \) stands for the vertically integrated monopoly that does not invest.

We are then able to write the following the following:

**Proposition 2** For all \( \gamma \in [\gamma_d, \gamma_i] \) where the non integrated monopoly undertakes process R&D along both stages of the vertical production process, while the integrated monopoly does not. In a subset of this area, \( \gamma \in [\gamma_d, \gamma_{2n}] \subset [\gamma_d, \gamma_i] \), non integration is more efficient from a private (profit level) point of view than its integrated counterpart not investing in R&D in that same parameter space. The private superiority extends to social welfare since the price of the integrated firms is lower, when the market size is not too large. Otherwise, the social superiority occurs in a subset of the above interval.

**Proof.** The comparison of profits requires evaluating:

\[ \pi^*_m - \pi^*_U - \pi^*_D = \frac{A^2}{4} \left\{ 1 - \frac{2\gamma[6\gamma - (2 + \beta)^2]}{[3 + \beta(4 + \beta) - 4\gamma]^2} \right\} \] (23)
which is negative for $\gamma \in [\gamma_{1n}, \gamma_{2n}]$.

However, we have that $\gamma_{1n} \leq \gamma_d \leq \gamma_{2n} \leq \gamma_i$ whenever

$$2 \leq \frac{8 + 3\beta (4 + \beta) + M}{2(1 + \beta)^2} \leq \frac{\alpha}{c} \leq \frac{7 + 2\beta (6 + \beta) + M}{1 + 2\beta},$$

with the lower limit ranging between 4.94 and 6.65 and the upper limit between 11.9 and 12.3 and $M = \sqrt{(2 + \beta (4 + \beta))(14 + 5\beta (4 + \beta))}$. Then, if $\gamma \in [\gamma_d, \gamma_{2n}] \subset [\gamma_d, \gamma_i]$, the integrated firm makes less profits than the disintegrated one only in a subset of the feasible set where the market size is relatively large. While for $\frac{\alpha}{c} \leq \frac{8 + 3\beta (4 + \beta) + M}{2(1 + \beta)^2}$, we have that $[\gamma_d, \gamma_i] \subset [\gamma_d, \gamma_{2n}]$ and the private superior performance of the disintegrated firm goes through the entire feasible set. Notice that this happens for a smaller market size.

We then compare the price of the integrated firm that does not invest in R&D and the price of the disintegrated firm investing in R&D. The difference is

$$p_m^* - p_D^* = \frac{A(2\gamma - 3 - \beta (4 + \beta))}{2(3 + \beta (4 + \beta) - 4\gamma)}$$

which is positive for $\gamma \in [\gamma_{aa}, \gamma_{bb}]$, where $\gamma_{aa} = \frac{3 + \beta (4 + \beta)}{4}$ and $\gamma_{bb} = \frac{3 + \beta (4 + \beta)}{2}$. If we compare the limits of the interval in which profits are higher for the non-integrated firm with that for $\gamma \in [\gamma_{aa}, \gamma_{bb}]$ we find that $[\gamma_{aa}, \gamma_{bb}] \subset [\gamma_{1n}, \gamma_{2n}]$ regardless of the market size. Moreover we find that $\gamma_d \geq \gamma_{aa}$ for all market sizes and $\gamma_d \leq \gamma_{bb}$ if $\frac{\alpha}{c} \leq \frac{5 + 8\beta + \beta^2}{1 + 2\beta}$ that ranges between 4.6 and 5.00. This means that the social superiority of the nonintegrated firm happens to be associated to smaller market sizes.

This result may be considered as a partial counterpart of the empirical investigation found in Slade (1998a,b) where the pricing policy of competing heterogeneous vertical arrangements are compared in markets where vertically integrated and disintegrated firms coexist and compete.

---

5The two roots of (23) are $\gamma_{1n}$ and $\gamma_{2n}$ respectively equal to

$$8 + 12\beta + 3\beta^2 - \sqrt{28 + 96\beta + 104\beta^2 + 40\beta^3 + 5\beta^4}$$

$$4$$

and

$$8 + 12\beta + 3\beta^2 + \sqrt{28 + 96\beta + 104\beta^2 + 40\beta^3 + 5\beta^4}$$

$$4$$
4 Asymmetric spillovers

Here are two cases in which the spillover flows vertically from one stage to the other.

4.1 From D to U

We first investigate the case of the externality in R&D going from D to U. This happens when the final producer’s process R&D stimulates R&D in U, for instance, asking for a rationalisation of U production processes to reduce costs. This is the most common direction of the spillover. The next one, from U to D, is more rare in actual vertical relationships.

We first analyse the nonintegrated case. The cost function for the D firm is

\[ c_D = c - x_D \]  

while the profit function is

\[ \pi_D(q_D, x_D) = q_D(p_D - c + x_D - g) - \gamma x_D^2 \]  

The cost function of the U firm is (6) and the profit function is (5). By going through the optimisation program of the two firms we get the optimal controls:

\[ q_D^* = \frac{A}{4\gamma - 3 - 2\beta} \]  

\[ x_U^* = \frac{A}{4\gamma - 3 - 2\beta} = x_D^* \]  

This result is due to the fact that the externality is not internalised.

The price charged by the U firm is:

\[ g^* = \frac{2a(1 + \beta + \gamma) - c}{3 + 2\beta - 4\gamma} \]  

The profits are respectively:

\[ \pi_U^* = \frac{A^2\gamma}{(8\gamma - 6 - 4\beta)} \] ;  

\[ \pi_D^* = \frac{A^2\gamma(2\gamma - 1)}{2(3 + 2\beta - 4\gamma)^2} \]  

11
The market price of the final good is

$$p^*_D = \frac{a(3 + 2\beta) - (3a + 2c)\gamma}{3 + 2\beta - 4\gamma}. \quad (31)$$

Now we analyse the integrated arrangement with the same cost and R&D structure. The profit function is:

$$\pi_I(q_I, x_{ID}, x_{IU}) = q_I(p_I - c_D - c_U) - \gamma \frac{x_{ID}^2}{2} - \gamma \frac{x_{IU}^2}{2}. \quad (32)$$

From the FOCs we get:

$$q^*_I = \frac{A\gamma}{2\gamma - 2\beta - \beta^2} \quad (33)$$

$$x^*_I = \frac{A(1 + \beta)}{2\gamma - 2\beta - \beta^2} \quad (34)$$

$$x^*_I = \frac{A}{2\gamma - 2\beta - \beta^2} \quad (35)$$

which show that internalisation of the externality makes for a different level of R&D in the two stages of the production process: the commitment at the D stage is larger than at the U stage.\(^7\) The reduced form profits and the market price are:

$$\pi^*_I = \frac{A^2\gamma}{2(2\gamma - 2\beta - \beta^2)} \quad (36)$$

$$p^*_I = \frac{a(2 + \beta(2 + \beta)) - (a + 2c)\gamma}{2 + \beta(2 + \beta) - 2\gamma}. \quad (37)$$

We are then able to write:

**Proposition 3** When the R&D spillover flows from D to U, the profits of the integrated firm are larger than the aggregate profits of the non integrated arrangement. For all $\gamma \in [\gamma_{A1}, \gamma_{A3}]$, the non integrated arrangement makes higher profits. However, the integrated part is always able to secure lower prices.

---

\(^6\)SOCs and nonnegativity constraints require $\gamma \geq \frac{3 + 2\beta}{4} = \gamma_{A2}$.

\(^7\)SOCs and nonnegativity constraints require $\gamma \geq \frac{1 + (1 + \beta)^2}{2} = \gamma_{A1} \geq \gamma_{A2}$.
Proof. We carry out the comparisons among the two arrangements. First consider profit:

\[ \pi_I^* - \pi_U^* - \pi_D = -\frac{A^2\gamma\left[-2\beta^3 + (1 - 2b\gamma)^2 + \beta^2(6\gamma - 4)\right]}{2(3 + 2\beta - 4\gamma)^2(2 + 2\beta + \beta^2 - 2\gamma)}. \]  

(38)

whose sign is nonnegative for \( \gamma \geq \frac{2 + \sqrt{4 + \beta(8 + 9\beta)}}{4\beta} = \gamma_{A3} \) and for \( \gamma \leq \frac{2 - \beta(3\beta + \sqrt{4 + \beta(8 + 9\beta)})}{4\beta} = \gamma_{A4} \). However, by a further comparison we see that \( \gamma_{A3} \geq \gamma_{A1} \) if \( 1 \leq \frac{2 - 3\beta^2 + \beta\sqrt{4 + \beta(8 + 9\beta)}}{4 + 4\beta + 2\beta^2} \). Then, in the interval \( \gamma \in [\gamma_{A1}, \gamma_{A3}] \) the nonintegrated firms make a larger profit than their integrated counterpart.

When the prices are considered the comparison is between (37) and (31). It appears that

\[ p_D^* - p_I^* = -\frac{A\gamma(2\gamma + \beta^2 - 1)}{(3 + 2\beta - 4\gamma)(2 + \beta(2 + \beta) - 2\gamma)} \]

is nonnegative in the feasible space of parameters. 

Here again, as in the previous section we can explore what happens in the region of the parameters space in which only the nonintegrated arrangement invests in process R&D while the integrated counterpart does not. The investigation of this case can be summarized in the following

**Corollary 4** The feasible set of the VI firm is a subset of the feasible set of the NI firm. Accordingly, there exists an interval of the parameter space where the VI does not undertake any R&D while the decentralised arrangement does. If \( \gamma \) is sufficiently low, the nonintegrated firm is both privately and socially more efficient. Conversely, if \( \gamma \) is sufficiently large, the non integrated firm is privately more efficient yet it charges a higher price than the integrated counterpart.

Proof. The optimal values of controls for the nonintegrated arrangement are (27), (28), (29) and (31), while profits are (30).

The marginal costs borne by the integrated (but non investing) firm are

\[ c_U = c_D = c. \]

The controls and the optimal profits of the VI firm that does not invest are:

\[ q_{DInoInv}^* = \frac{A}{2b}; \quad \pi_{InoInv}^* = \frac{A^2}{4b}; \quad p_{DInoInv}^* = \frac{a + 2c}{2}. \]  

(39)
Comparison of profits leads to:
\[
\pi^*_{\text{inov}} - \pi^*_U - \pi^*_D = \frac{A^2(9 + 4\beta^2 - 16\gamma + 4\gamma^2 - 12\beta(\gamma - 1))}{4(3 + 2\beta - 4\gamma)^2}
\]
which is nonnegative if
\[
\gamma \geq \frac{4 + 3\beta + \sqrt{(1 + \beta)(7 + 5\beta)}}{2} = \gamma_{A5}.
\]
However \(\gamma_{A5} \geq \gamma_{A6} = \frac{3 + 2\beta}{4}\) that defines the minimum level of feasible \(\gamma\) for the nonintegrated investing arrangement. Then for \(\gamma \in [\gamma_{A6}, \gamma_{A5}]\) the nonintegrated profits are larger than the profits of the integrated firm, that does not invest. The comparison of prices leads to
\[
p^*_D - p^*_{D\text{inov}} = \frac{A(2\beta - 3 - 2\beta)}{6 + 4\beta - 8\gamma}
\]
It appears that the above difference is positive for \(\gamma \in [\gamma_{A6}, \gamma_{A7}]\) and negative for \(\gamma \in [\gamma_{A7}, \gamma_{A5}]\), where \(\gamma_{A7} = \frac{3 + 2\beta}{2}\).

### 4.2 From U to D

Here we go through the opposite case in which the externality flows from the U stage down to the final D stage. This kind of spillover may be thought to be more common since progress in the production process of the input tends to stimulate D firms to improve the cost profile of their assembling structure.

The cost structure is now given by (2) and:
\[
c_U = c - x_U.
\]
Optimal controls and profits for the nonintegrated case are:\(^8\)
\[
\begin{align*}
x^*_U &= \frac{A(1 + \beta)}{(4\gamma - 3 - \beta(2 + \beta))} \\
x^*_D &= \frac{A}{(4\gamma - 3 - \beta(2 + \beta))}
\end{align*}
\]
\(^8\)SOCs and nonnegativity constraints boil down to the condition \(\gamma \geq \frac{3 + \beta(2 + \beta)}{4} = \gamma_{B1}\).
\( q_D^* = \frac{A \gamma}{(4 \gamma - 3 - \beta(2 + \beta))} \) (43)

\( \pi_D^* = \frac{A^2 \gamma(2 \gamma - 1)}{2(4 \gamma - 3 - \beta(2 + \beta))^2} \) (44)

\( \pi_U^* = -\frac{A^2 \gamma}{2(4 \gamma - 3 - \beta(2 + \beta))} \) (45)

\( p_D^* = -\frac{a(3 + \beta(2 + \beta)) - (3a + 2c)\gamma}{4 \gamma - 3 - \beta(2 + \beta)} \), (46)

while in the integrated we have:

\( x_{UI}^* = \frac{A(1 + \beta)}{(2 \gamma - 2 - \beta(2 + \beta))} \) (47)

\( x_{DI}^* = \frac{A}{(2 \gamma - 2 - \beta(2 + \beta))} \) (48)

\( q_{DI}^* = \frac{A \gamma}{(2 \gamma - 2 - \beta(2 + \beta))} \) (49)

\( p_{DI}^* = \frac{2c - a(1 + \beta(2 + \beta))}{2 \gamma - 2 - \beta(2 + \beta)} \) (50)

\( \pi_I^* = \frac{A^2 \gamma}{2(2 \gamma - 2 - \beta(2 + \beta))} \) (51)

If we compare the VI and the decentralised case we get:

**Proposition 5** When the spillover flows from U to D, the VI arrangement is always superior to the decentralised case from both the private and the social standpoint.

**Proof.** Just compare (51) with the sum of (44) and (45). It appears that in the common feasibility parameter space (i.e. for \( \gamma \geq \frac{1 + (1 + \beta)^2}{2} = \gamma_B^2 \)) the profits of the integrated arrangements are larger. Then compare (50) with (46). We see that the final D price in the non-integrated arrangement is higher.

Moreover, in the parameters space in which just the nonintegrated firm invests, we get:
Corollary 6 If the spillover goes from U to D there exists an interval of the parameter space where only the nonintegrated arrangement invests in process R&D. The lower part of this space is associated with a nonintegrated firm that is making larger profits while setting lower prices than the integrated counterpart, i.e., a second best optimum obtains. An adjacent interval for higher levels of \( \gamma \) has the nonintegrated firms making a larger aggregate profit but setting a higher market price.

Proof. Controls and profit of the VI firm that does not invest are in (39). If we now calculate

\[
\pi^*_{\text{InoInv}} - \pi^*_D - \pi^*_U;
\]

using (44) and (45)) we find that (52) is nonnegative for

\[
\gamma \geq \frac{8 + 3\beta(2 + \beta) + \sqrt{(2 + \beta(2 + \beta))(14 + 5\beta(2 + \beta))}}{4} = \gamma_{B2}.
\]

Since \( \gamma_{B2} \geq \gamma_{B3} = \frac{3 + \beta(2 + \beta)}{4} \) below which the investing nonintegrated firms are no longer active, we have that, for \( \gamma \in [\gamma_{B3}, \gamma_{B2}] \) the nonintegrated investing firms make larger aggregate profits than the integrated non investing firm.

If we compare market prices, we see that

\[
p^*_{D\text{InoInv}} - p^*_D
\]

which, from (46), is nonnegative for \( \gamma \in [\gamma_{B4}, \gamma_{B2}] \), while it is negative for \( \gamma \in [\gamma_{B3}, \gamma_{B4}] \), where \( \gamma_{B4} \) is the value of \( \gamma \) below which (53) is nonnegative.

5 Asymmetric costs of R&D

Here, we adopt a more realistic framework with asymmetric costs of R&D along the vertical chain of production. Realism is suggested by observation of industries. For instance, the amount of R&D needed to decrease the cost of production of a chip is much higher than the amount needed to reduce the cost of assembling a PC that uses that chip as a component. However, in other industries it may be the other way around, as for instance in the automotive industry, where the R&D for a steering wheel is much lower than that required for the production of a car or a truck. This may also be the case
in other mature sectors such as the textile/clothing industry. As a matter of fact, asymmetric R&D costs lend themselves to a more realistic analysis of vertical R&D.

Here we just assume that there are two distinct $\gamma$, a $\gamma_U$ and a $\gamma_D$. Then, we repeat what done in the previous sections. Unfortunately, unlike above, we are not able to derive analytical results. We can only pin down a remark that concerns the private incentives for firms:

**Remark 1** When $\gamma_U$ is large with respect to $\gamma_D$, the integrated arrangement is privately more efficient. When the opposite obtains, the decentralised arrangement is more profitable. Both arrangements possess ranges of social second best efficiency.

**Proof.** The difference between the profits of the two arrangements is

$$
\pi_I^* - \pi_D^* - \pi_U^* = \frac{A^2 \gamma_D \gamma_U^2 (\beta^4 \gamma_D - (1 - 2 \gamma_D)^2 \gamma_U + 2 \beta^4 (\gamma_D + \gamma_U) + \beta^2 (\gamma_D + 3 \gamma_U - 6 \gamma_D \gamma_U))}{2(1 + \beta) \gamma_U + \gamma_D (1 + 2 \beta + \beta^2 - 4 \gamma_U)^2 ((1 + \beta)^2 \gamma_U + \gamma_D ((1 + \beta)^2 - 2 \gamma_U))}
$$

The sign of the above equation is nonnegative for $\gamma_D \in [\gamma_D1(\gamma_U), \gamma_D2(\gamma_U)]$. This statement cannot be proved analytically but only numerically, yet for the whole range of the feasible set of parameters. Since $\gamma_U \geq \gamma_D2(\gamma_U)$ we have that for all $\gamma_U \geq \gamma_D$ the integrated firm makes larger profits. When the opposite obtains, the disintegrated arrangement is privately more efficient. The analysis of prices carries over the same way and provides subranges of social second best efficiency in both arrangements.

**6 Conclusions**

The maintained wisdom that, in presence of non perfectly competitive patterns of behaviour, vertical integration is a second best, has been challenged in the preceding sections by introducing process R&D with spillovers along the vertical chain of production in a simple homogeneous monopoly environment. Some results are worth emphasizing since they provide a basis for a fresh explanation of outsourcing or vertical separation in a framework that does not take into account any externality coming from firms changing their
vertical organization (as it occurs in recent contributions by Grossman and Helpman (2002), McLaren (2000) and Jansen (2003)).

As we consider vertical R&D with spillovers, it appears that the non integrated firms invest in R&D over a wider set of parameters. When both the integrated firm and its decentralised counterparts invest in process R&D, the integrated firm is both privately and socially superior. This result no longer holds when we consider two distinct marginal costs of R&D in the U and D stage of production. When R&D activity is more expensive in the D stage than in the U stage, the decentralised arrangement is privately superior, in the region of parameters containing the interval of social second best. This result, that is consistent with some received investigations (Slade, 1998a,b) may explain the evolution of outsourcing policies as the relative costs of U and D R&D vary along the vertical chain. Moreover and more importantly, we find that, when the integrated firm refrains from committing to process R&D, while the decentralized firms do it, outsourcing may provide larger aggregate profits, and, in a narrower parameter range, also socially superior results even if the costs of R&D are symmetric along the vertical chain. These results obtain in the case of both symmetric and asymmetric R&D spillovers.

We believe these conclusions may be relevant to explain the waves of vertical integration just on the basis of (i) the changes in the state of the art of process R&D, and (ii) the amount of spillovers in the downstream and upstream sections of the vertical chain of production. Further research may go through the policy implication of these conclusions.

References


