

Lambertini, Luca; Orsini, Raimondello

**Working Paper**

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Quaderni - Working Paper DSE, No. 471

**Provided in Cooperation with:**

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*Suggested Citation:* Lambertini, Luca; Orsini, Raimondello (2003) : Network Externality and the Coordination Problem: A Generalization of Rohlfs's Model, Quaderni - Working Paper DSE, No. 471, Alma Mater Studiorum - Università di Bologna, Dipartimento di Scienze Economiche (DSE), Bologna, <https://doi.org/10.6092/unibo/amsacta/4823>

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# Network Externality and the Coordination Problem: A Generalization of Rohlfs's Model

Luca Lambertini<sup>§</sup> - Raimondello Orsini  
Dipartimento di Scienze Economiche  
Università degli Studi di Bologna  
Strada Maggiore 45, 40125 Bologna, Italy  
fax +39-051-2092664  
e-mail [lamberti@spbo.unibo.it](mailto:lamberti@spbo.unibo.it)  
e-mail [orsini@spbo.unibo.it](mailto:orsini@spbo.unibo.it)  
§ Corresponding author

April 4, 2003

## Abstract

We show that Rohlfs's (1974) model is a special case of a spatial monopoly model *à la* Hotelling (1929) with quadratic transportation costs, where location is exogenous and the good yields no intrinsic utility. By relaxing these assumptions, we prove that the coordination problem typically thought to affect markets for network goods may not arise in general. Endogenising location makes it easier for the monopolist to extract consumer surplus but also to cover the entire market.

**JEL Classification:** D62, L12

**Keywords:** monopoly, network externality, critical mass

# 1 Introduction

The recent literature in the theory of industrial organization has devoted a large attention to the characteristics of markets for goods whose consumption involves network effects, such that the utility that a consumer derives from the purchase of a good or service is increasing in the number of other consumers doing the same (see Economides and Encaoua, 1996; Shy, 2000). The software and telecommunications industries and, more generally, the markets for information goods, are examples of sectors where such externalities operate.

Intuitively, joining the network is more attractive to the generic consumer the larger is the size of the network. This may give rise to a coordination problem, since the market performance of a service/product depends upon the achievement of a *critical mass* of adopters/consumers. The most widely used illustration of this issue dates back to Rohlfs (1974), assuming that the utility associated with consumption is fully determined by the network effect. This can be the case, e.g., of telecommunication networks, which is the example used by Rohlfs himself. Thereafter, the coordination problem related to the issue of the critical mass has been generally associated to the presence of network effects. Yet, this is not true in general, since there exist many goods which exhibit network externalities but carry also an intrinsic utility justifying by itself consumption.<sup>1</sup> These considerations suggest that the issue of a critical mass is crucial only for a subset of all the goods yielding network effects.<sup>2</sup>

In the remainder of the paper, we proceed as follows. As a first step, we summarise Rohlfs's model. In section 3, we show that it can be obtained as a special case of a Hotelling-like monopoly model with quadratic transportation costs, with the monopolist being located at one endpoint of the support of consumer preferences. Then, we endogenise location keeping unchanged the assumption that the only source of satisfaction is the external effect. In this way, we show that the monopolist will choose location so as to meet the preference of the average consumer. Conversely, by introducing an intrinsic utility under the assumption that location is fixed at one extreme, we prove

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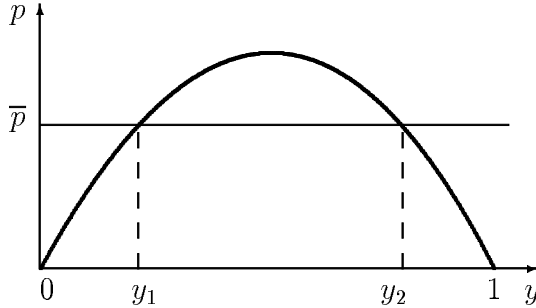
<sup>1</sup>For instance, this is the case of personal computers, CD players, TV sets, etc.

<sup>2</sup>An alternative way out of this issue consists in introducing vertical differentiation, in which case the demand function is linear and everywhere downward sloping, so that the optimum of the firm does not require solving a coordination problem. See Lambertini and Orsini (2001).

that the coordination problem may in fact disappear. Finally, putting both views together, we also prove that the coexistence of endogenous location and intrinsic utility makes it easier for the monopolist to serve all consumers in the market, at equilibrium.

## 2 Preliminaries

Here, we briefly summarise the basic model as in Rohlfs (1974) (see also Shy, 1998, pp. 256-9). A unit mass of consumers is uniformly distributed over  $[0, 1]$ , in decreasing order w.r.t. their willingness to enter the network. The value of joining the network is increasing in the network size. Therefore, the overall willingness to pay for the good or service of a consumer at  $m \in [0, 1]$  is  $w = y(1 - m)$ , where  $y$  is the size of the network, i.e., the market demand for that good. The consumer's net surplus is  $U = y(1 - m) - p$ , where  $p$  is the market price. In order to determine demand  $y$ , the firm identifies the marginal consumer in  $m = y$  and sets  $p = y(1 - y)$ . Now observe that, being the overall willingness to pay of the marginal consumer,  $\bar{w} = y(1 - y)$ , concave in  $y$ , for any given price  $p < 1/4$  there are two economically admissible sizes of the network, as it appears from figure 1.



**Figure 1:** Equilibrium network sizes

This model is usually considered as blackboxing an underlying dynamic process whereby consumers enter sequentially the network, starting from the left boundary of the unit interval. The adjustment is driven by the assumption that  $y$  increases whenever  $p < y(1 - y)$ , and conversely. Hence, as soon as  $y_1$  consumers buy at price  $\bar{p}$ , the market immediately jumps into  $\{y_2, \bar{p}\}$ . This amounts to saying that  $\{y_1, \bar{p}\}$  is unstable. Nevertheless, its

economic interest lies in the fact that  $y_1$  is the so-called *critical mass*, ensuring that marketing the product is going to be successful.

### 3 Generalisation

First, observe that, given  $m = y$ , the consumer surplus can be rewritten as  $U = y - y^2 - p$ . In this form, the term  $y - y^2$  can be interpreted as the difference between the network effect and a quadratic cost associated to the distance  $y$  to the left boundary of the unit interval (where the first consumer entering the network is located). Accordingly, the utility function employed here can be considered as a special case of the following Hotelling-like utility function (Hotelling, 1929) with quadratic disutility of transportation and linear network effects:

$$U = s - p - vd^2 + \alpha y \quad (1)$$

where  $s$  is the intrinsic satisfaction from consumption (independent of the network size),  $d$  is the distance between the generic consumer and the firm, and  $v$  and  $\alpha$  are positive parameters. In Rohlfs (1974),  $s = 0$  and  $\alpha = v = 1$ .

Assume that (i) consumers are uniformly distributed along  $[0, 1]$ , with density one; (ii) production takes place at constant returns to scale, and the marginal cost is normalised to zero; (iii) the monopolist locates in zero. If so, the marginal consumer is at  $d = y$ . Therefore, the optimal price driving to zero the surplus of the marginal consumer is:

$$p = s + y(\alpha - vy) \quad (2)$$

and the profit function is:

$$\pi = [s + y(\alpha - vy)]y, \quad (3)$$

to be maximised w.r.t.  $y$ . The related first order condition is:

$$\frac{\partial \pi}{\partial y} = s + y(2\alpha - 3vy) = 0. \quad (4)$$

If the good does not produce any intrinsic utility,  $s = 0$  and the optimal size of the network is  $y^* = 2\alpha/(3v)$ , with equilibrium price and profits equal to  $p^* = 2\alpha^2/(9v)$  and  $\pi^* = 4\alpha^3/(27v^2)$ , respectively. These magnitudes coincide with the results of Rohlfs's model if  $\alpha = v = 1$ . Moreover, they qualify as the

internal solution for all  $\alpha \in [0, 3v/2)$ . In this parameter interval,  $y^* \in [0, 1)$ . Otherwise, if  $\alpha \geq 3v/2$ , the monopolist serves all consumers at the price that drives to zero the surplus of the individual located at the opposite end of the unit interval, so that  $p^* = \alpha - v = \pi^*$ .<sup>3</sup>

At this stage, it is worth noting that, here, the concavity of the firm's optimum problem obtains on the basis of two assumptions: (a) preferences are convex; (ii) production costs are linear. The same would obtain by considering (c) linear preferences (i.e., linear transportation costs) and (d) convex production costs. However, while in the former case the convexity of preferences implies a concave demand function, in the latter case the linearity of preferences would imply a linear demand function, i.e., no coordination issue.

Now, leaving aside these considerations, we can depart from the basic model along two different directions (which may also be taken simultaneously):

- Given the spatial interpretation of the setup, one can endogenously examine the issue of the optimal monopoly location.
- What happens in equilibrium if the good under consideration jointly produces network effects and intrinsic satisfaction, i.e.,  $s > 0$ ?

### 3.1 Optimal location

This issue can be quickly dealt with, as the monopolist cannot do any better than locating at  $1/2$ . This can be easily shown by the following procedure.

Let the net utility of a generic consumer be defined as in (1). Define the location of the monopolist as  $x \in [0, 1/2]$ . The second half of the segment can obviously be disregarded in view of the symmetry of the model. From Bonanno (1987), we know that, when consumer utility includes an intrinsic satisfaction but not a network effect, (i) the monopolist finds it optimal to locate at  $1/2$  in order to maximise the extraction of surplus; and (ii) the monopolist cannot do any worse than locating at either endpoint of the space of consumer preferences.

Now, suppose  $s = 0$ , so that the net utility of a generic consumer is  $U = \alpha y - p - vd^2$ . The monopoly problem can be characterised in the following terms. In general, under partial market coverage, there exist two marginal

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<sup>3</sup>Note that  $\alpha - v \leq 4\alpha^3 / (27v^2)$  for all  $\alpha \in (1, 3v/2]$ .

consumers located respectively at  $m \in [0, x)$  and  $2x - m \in (x, 1]$ , who are indifferent between joining the network or not. Accordingly, the size of the network is  $y = 2(x - m)$ , and imposing

$$2\alpha(x - m) - p - v(x - m)^2 = 0 \quad (5)$$

we obtain the monopoly price for a generic pair  $\{m, x\}$  under partial market coverage:

$$p = (x - m)[2\alpha - v(x - m)]. \quad (6)$$

The monopoly profit function is  $\pi = py$ , to be maximised w.r.t.  $m$  and  $x$ :

$$\frac{\partial \pi}{\partial m} = 2(x - m)[3v(x - m) - 4\alpha] = 0 \quad (7)$$

$$\frac{\partial \pi}{\partial x} = -2(x - m)[3v(x - m) - 4\alpha] = 0 \quad (8)$$

which of course implies that the system (7-8) cannot determine the optimal values of both choice variables. However, solving (7), we have:<sup>4</sup>

$$m^M = x - \frac{4\alpha}{3v} \quad (9)$$

which entails:

$$y^M = \frac{8\alpha}{3v}; p^M = \frac{8\alpha^2}{9v}; \pi^M = \frac{64\alpha^3}{27v^2}. \quad (10)$$

Any location  $x^M \in (0, 1/2]$  is admissible, as long as  $m^M \in [0, x)$ . I.e., the monopolist cannot do any better than choosing  $x^M = 1/2$ .

Now, observe that  $y^M = 4y^*$ ,  $p^M = 4p^*$ ; therefore, by moving away from zero, the monopolist obtains a profit which is sixteen times as large as the one he would obtain by locating at one endpoint of the unit interval. This is due to the combined effects of (i) enlarging demand (at a given price) by moving away from zero; and (ii) enlarging the consumers' marginal willingness to pay measured by the network effect  $\alpha y$ , which enhances the demand expansion and also allows for a price increase.

The above solution defines the monopoly optimum with partial market coverage for all  $\alpha \in (0, 3v/8)$ . If instead  $\alpha \geq 3v/8$ , under full market coverage the monopoly price extracts all the surplus from the consumer located at one:

$$U = \alpha - p - v(1 - x)^2 = 0 \Rightarrow p = \alpha - v(1 - x)^2 = \pi \quad (11)$$

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<sup>4</sup>There also exists the solution  $m = x$ , which can be disregarded as it implies  $y = 0$ .

The derivative

$$\frac{\partial \pi}{\partial x} = 2v(1 - x) \quad (12)$$

is always positive for  $x \in [0, 1/2]$ . Hence, the monopolist locates at  $x^M = 1/2$  and sets the price driving to zero the marginal consumers located at zero and one:

$$p^M = \alpha - \frac{v}{4} = \pi^M. \quad (13)$$

The foregoing discussion proves that the following holds:

**Lemma 1** *If consumers are rich enough to yield full market coverage, then the unique optimal monopoly location is  $x^M = 1/2$ . Otherwise, the monopolist may choose any location  $x^M \in (0, 1/2]$  such that equilibrium demand is evenly distributed around  $x^M$ .*

Once we have clarified that the consumer utility function considered in Rohlfs's model contains both a linear externality and a convex transportation cost, it is straightforward to conclude that, indeed, the monopolist could not do any worse than locating at either endpoint of the support of consumer preferences. Having said that, as long as partial market coverage obtains in equilibrium, even with endogenous location, we still observe a concave demand function and the associated issue of the critical mass, as in figure 1 above.

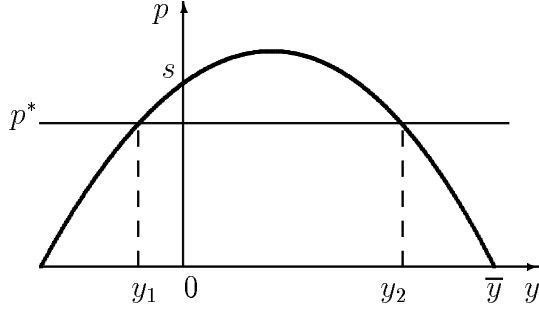
### 3.2 Intrinsic satisfaction

Now suppose  $x = 0$  but  $s > 0$ . The monopolist must select a single marginal consumer at  $m = y \in (0, 1]$ . The inverse demand is given by:

$$p = s + y(\alpha - vy) \quad (14)$$

and the corresponding profit function is  $\pi = [s + y(\alpha - vy)]y$ . To begin with, observe that the presence of  $s > 0$  preserves the concavity of the demand function, but implies that, when  $y = 0$ ,  $p = s$ . This entails that, if the monopolist sets a price  $p^* \in [0, s)$ , there exists no issue of a critical mass. This situation is represented in figure 2.





**Figure 2:** The demand function with intrinsic utility

From (14), we know that  $p = s$  in  $y = 0$ . Moreover,  $p = 0$  in:

$$y = \frac{\alpha \pm \sqrt{\alpha^2 + 4vs}}{2v} \quad (15)$$

with the smaller root being negative for all  $s > 0$ , and the larger root  $\bar{y} \in [0, 1)$  for all  $v > \alpha + s$ . In the opposite case, where  $v < \alpha + s$ , also the farthest consumer located at one is able to afford a positive mill price.

Profit maximisation requires:

$$\frac{\partial \pi}{\partial y} = s + y(2\alpha - 3vy) = 0 \Rightarrow y^* = \frac{\alpha + \sqrt{\alpha^2 + 3sv}}{3v} \quad (16)$$

which yields:

$$p^* = \frac{6sv + \alpha(\alpha + \sqrt{\alpha^2 + 3sv})}{9v}; \quad \pi^* = \frac{\alpha(2\alpha^2 + 9sv) + 2\sqrt{(\alpha^2 + 3sv)^3}}{27v^2}. \quad (17)$$

Partial market coverage prevails for all

$$s \in (0, 3v - 2\alpha) \text{ and } \alpha \in (0, 3v/2); \quad (18)$$

otherwise  $y^* = 1$ . Using the expression of  $p^*$  in (17), we can easily verify that:

$$p^* \in [0, s) \text{ for all } s > \hat{s} \equiv \frac{\alpha^2}{v}. \quad (19)$$

Therefore, we can claim:

**Lemma 2** *If the intrinsic utility is large enough, the coordination problem associated to the issue of a critical mass disappears.*

This is due to the fact that there are consumers (typically, those located next to zero) who buy irrespective of the network size. In so doing, they in fact create the network that, through a bandwagon effect, draws other customers into the network. The critical threshold of  $s$  is increasing in  $\alpha$  and decreasing in  $v$ . The reason for this is to be found in the comparative statics properties of the equilibrium price:

$$\frac{\partial p^*}{\partial \alpha} > 0, \frac{\partial p^*}{\partial v} < 0 \Rightarrow \frac{\partial \hat{s}}{\partial \alpha} > 0, \frac{\partial \hat{s}}{\partial v} < 0. \quad (20)$$

### 3.3 Two eggs in one basket

Here,  $s > 0$  and location is endogenously chosen. Under partial market coverage, the location of the marginal consumer at  $m \in [0, x)$  can be written as  $m = (2x - y)/2$ , and the corresponding demand function is  $p = s + y(4\alpha - vy)/4$ . The first order condition is:

$$\frac{\partial \pi}{\partial y} = s + 2\alpha y - \frac{3vy^2}{4} = 0 \Rightarrow y^M = \frac{2(2\alpha + \sqrt{4\alpha^2 + 3sv})}{3v} \quad (21)$$

with  $y^M \in (0, 1)$  for all

$$s \in (0, (3v - 8\alpha)/4) \text{ and } \alpha \in (0, 3v/8); \quad (22)$$

while  $y^M = 1$  otherwise. Note that the above interval is smaller than the corresponding interval defined in (18), since choosing endogenously a location along the segment reduces transportation costs and facilitates the attainment of full coverage. Once again, location is indeterminate as long as the market is not fully covered. Equilibrium price and profits are:

$$\begin{aligned} p^M &= \frac{2[3sv + \alpha(2\alpha + \sqrt{4\alpha^2 + 3sv})]}{9v}; \\ \pi^M &= \frac{4\left[8\alpha^3 + 9\alpha sv + \sqrt{(4\alpha^2 + 3sv)^3}\right]}{27v^2}. \end{aligned} \quad (23)$$

which, of course, are higher than those obtained in the previous case. Concerning the coordination problem, we can check that:

$$p^* \in [0, s) \text{ for all } s > \frac{4\alpha^2}{v}, \quad (24)$$

that is: when location is endogenous, the existence of a critical mass is more easily observed, due to the increase in the equilibrium price as compared to the case where  $x = 0$ , all else equal.

If the market is fully covered, the monopoly price extracts all the surplus from the consumer located at one:

$$U = s + \alpha - p - v(1 - x)^2 = 0 \Rightarrow p = s + \alpha - v(1 - x)^2 = \pi \quad (25)$$

Accordingly, the maximum profit is attained at  $x^M = 1/2$ , with  $p^M = s + \alpha - v/4 = \pi^M$ .

Summing up, the general model produces the following results, which are the combination of those individually highlighted in Lemmata 1-2:

**Proposition 3** *If the monopolist is free to choose location optimally and consumption yields both an intrinsic satisfaction and a network externality, then:*

- *the monopolist locates in correspondence of the average consumer;*
- *the average consumer may want to buy irrespective of the network effect;*
- *the coordination problem vanishes if intrinsic satisfaction is sufficiently high;*
- *the condition for full market coverage is milder than otherwise.*

Note that, once the market is fully covered and the monopolist is at  $1/2$ , the social welfare is maximised, although of course the surplus distribution may be different from the one that would obtain under social planning.

## 4 Concluding remarks

We have shown that Rohlfs (1974) is a special case of a spatial monopoly model à la Hotelling (1929) with quadratic transportation costs. By endogenising the location of the firm, and introducing an intrinsic utility from consumption, we have proved that the coordination problem typically thought to affect markets for network goods may not arise in general. Endogenising location makes it easier for the monopolist to extract consumer surplus but also to cover the entire market.

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