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**Working Paper**

## Reimbursing Preventive Care

Quaderni - Working Paper DSE, No. 403

**Provided in Cooperation with:**

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*Suggested Citation:* Barigozzi, Francesca (2001) : Reimbursing Preventive Care, Quaderni - Working Paper DSE, No. 403, Alma Mater Studiorum - Università di Bologna, Dipartimento di Scienze Economiche (DSE), Bologna,  
<https://doi.org/10.6092/unibo/amsacta/4898>

This Version is available at:

<https://hdl.handle.net/10419/159244>

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# Reimbursing Preventive Care\*

Francesca Barigozzi<sup>†</sup>

First version: December 1998.

This version: January 2001

## Abstract

The paper focus on secondary prevention (diagnostic screening, medical examinations, checks-up...) which refers to the early detection of disease. In particular secondary prevention is analyzed as an instrument of self-insurance: if illness occurs, the negative health shock decreases. Both the case in which secondary prevention and treatment are complementary goods, and that in which they are substitutes, are analyzed. Optimal reimbursement for prevention and treatment is derived when insurance uses a linear mechanism. Results show that, starting from a situation with no insurance, a linear contract always encourages treatment consumption whereas it may either encourage or discourage secondary prevention consumption. Prevention consumption is discouraged when the two goods are complementary and encouraged when the two goods are substitutes. In the former case, one of the two goods is taxed and the other is subsidized, while in the latter case the two goods are either both subsidized or both taxed.

**JEL classification:** D81; I11.

**Keywords:** Health insurance, secondary prevention, self-insurance, linear taxation.

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\*I am grateful to Giacomo Calzolari, Eric Malin, Gwenael Piaser, Bertrand Villeneuve and especially to Helmuth Cremer for helpful comments and discussions. I thank Lorenzo Montanaro and Alessandra Tomasi for their medical viewpoint on the subject of secondary prevention. Paper presented at the Spring School in International and Industrial Economics, Aix en Provence, May 1999 and at the European Economic Association Annual Congress, Santiago de Compostela, September 1999.

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# 1 Introduction

The object of preventive medicine is to protect, promote, and maintain health and to prevent disease, disability and premature death. Thus preventive medicine includes activities or medical services that might reduce the future incidence or severity of illness. In particular, primary prevention reduces the *probability of illness* while secondary prevention reduces the *vulnerability to illness*.<sup>1</sup>

Primary prevention concerns the avoidance of undesirable outcomes. Behavior such as doing physical exercise, dieting or reducing cigarette consumption, have an impact on illness probability and are primary prevention measures. Such actions, which influence an individual's life-style, are generally not observable by insurers and represent a case of ex-ante moral hazard. However, not all primary prevention measures are not observable: vaccinations, for example, can be, and are in fact, controlled by the Health Authorities (compulsory vaccination policies).

Ehrlich and Becker (1972) define a reduction in the *probability* of a loss as self-protection. Self-protection has been used to analyze primary prevention in many Health Insurance models, the standard issue being the trade-off between incentives and risk-spreading<sup>2</sup>. Recent works have also looked at vaccines and the problem of disease eradication and incentives towards inoculation<sup>3</sup>.

Secondary prevention refers to the early detection of disease. Medical examinations, checks-up and diagnostic screening (scanning, mammography) help to reduce the severity of illness, and as such are to be considered secondary prevention measures. In fact, the detection of an illness when it is still asymptomatic means reducing its incidence and making recovery easier.<sup>4</sup>

Once again, according to Ehrlich and Becker (1972), a reduction in the *size* of a potential loss is called self-insurance. Contrary to the best part of primary prevention (all behavior characterizing the consumer's life-style), secondary prevention consumption is in fact observable. This means that secondary prevention measures are contractible: in other words, they can constitute the subject-matter of an insurance contract. Nevertheless, with regard to the insurability of preventive measures, insurance theory would argue that these measures are predictable. In fact they are employed before the realization of the risk. On the other hand, if preventive services are to be encouraged, insurance coverage represents the easier way to increase utilization.

As I mentioned before, both vaccination and secondary prevention measures are observable. It is interesting to note that while some vaccinations are compulsory, there is no compulsory diagnostic screening. This is probably due to the fact that vaccines lead to important

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<sup>1</sup>Tertiary prevention has been defined as "all the actions reducing disability associated with a chronic illness". For example: educating diabetic patients about foot care in order to prevent complications.

<sup>2</sup>This is the case in the work of Pauly (1974), for example, and (with a different context) in that of Bond and Crocker (1991).

<sup>3</sup>Among others Geoffard and Philipson (1997).

<sup>4</sup>"To illustrate the magnitude of the differences in health outcomes possible with secondary prevention, in randomized controlled trials that included women 50 and older, mammography screening for breast cancer is estimated to reduce breast cancer mortality by 20 to 30 percent." From Kenkel (2000, page 1683).

positive externalities, and so their subsidization is generally accepted: whereas health authorities are more skeptical about the cost-containment value of other form of preventive care. Economists are still debating this question, and recent studies seems suggest that most preventive measures fail the benefit-cost test<sup>5</sup>.

The following table summarizes the characteristics of primary and secondary preventive measures:

Primary Prevention	Secondary Prevention
<i>Self-protection</i> - Observable and contractible: vaccines - Not observable: life-style	<i>Self-insurance</i> Observable and contractible

Table 1: Primary and secondary prevention

As opposed to primary prevention, secondary prevention has not received much attention in the field of Health Economics, and the problem of how to reimburse secondary prevention has yet to be explored. It is my intention to address this particular issue in this paper.

The question of preventive care reimbursement is currently of great interest. This firstly because the way insurance companies reimburse preventive care directly influences important individual health-related decisions. Secondly administrators of public health insurance systems will be interested in the impact preventive care coverage has on public sector budgets. Thirdly there has been a recent increase in interest in disease prevention throughout the developed world.<sup>6</sup> In general there has been an evolution in insurance reimbursement plans. For example, in the U.S., whereas traditional health insurance reimburses prevention less generously than it does treatment costs (thereby artificially reducing the demand for preventive services), managed care programs are characterized by the considerably higher coverage of prevention. Moreover, employers are increasingly turning to prevention as an alternative to treating patients for their health problems: many firms offer annual physical exams, employee fitness tests, together with weight and stress control programs.

I previously mentioned the paper which first introduced the concepts of self-protection and self-insurance in Insurance Theory: Ehrlich and Becker (1972). A recent work by Eeckoudt *et al.* (1998) has now applied these concepts to preventive medicine. In this paper the authors analyze individuals' decisions on preventive care consumption in the case of both primary and secondary prevention. Some comparative statics complete their study. However, their model, takes no account of insurance problems.

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<sup>5</sup>Garber and Phelps (1987) argue that most preventive care, while improving health, does not actually save money in the long run. In particular, they show that preventive care can, and often does, provide health improvements at relatively low cost per life-year saved, but prevention does not actually save money very often. Russell (1986), in his review of the evidence that prevention can add to medical expenditure rather than reducing it, has shown that prevention is not necessarily a system-wide cost-containment strategy. At present, prenatal-care seem to be the only form of secondary prevention which clearly passes the cost-benefit analysis.

<sup>6</sup>In 1977 the U.S. federal government set up the Office of Disease Prevention and Health Promotion.

This present paper looks in detail at the problem of insuring secondary prevention, and analyzes both the case in which treatment and prevention are complementary goods and that in which they are substitutes. In solving the insurance problem I calculate the optimal reimbursement for prevention and treatment when insurance uses a linear mechanism. Results show that, starting from a no-insurance situation, a linear contract always encourages treatment consumption, whereas it may either encourage or discourage secondary prevention consumption. Prevention consumption is discouraged when the two goods are complementary, and encouraged when the two goods are substitutes. In the former case, one of the two goods is taxed and the other is subsidized, while in the latter case the two goods are either both subsidized or both taxed. Taxation and/or subsidization are determined according to the “comparative advantage” of the prices of the two goods in influencing compensated demands for treatment and prevention.

The remainder of the paper is organized as follows. The model is laid out in section 2. Then section 3 describes the consumer’s problem. Section 4 solves the insurance program and provides the interpretation of the results. Finally, section 5 concludes.

## 2 The model

A patient’s utility depends on his health, the benefits of the health care received, and that income available for spending on others goods after the cost of prevention and treatment has been deducted. Initial income is  $W$ . The patient is ill with probability  $p$ . When ill, the patient is subject to a negative health shock with a monetary equivalent of  $H_0$ . Health can be (partially) recovered according to a strictly concave function  $H(e, x)$  representing the monetary equivalent of the benefits of prevention and health care, where  $e$  denotes secondary prevention consumption, and  $x$  the quantity of treatment<sup>7</sup>.  $H$  is increasing in  $e$  and  $x$ , and ranges from 0 to  $H_0$ . As usual, the marginal productivity of  $e$  and  $x$  is decreasing, and the lower limits of the two variables are set at zero.

Preventive care consumption increases the benefit of treatment, in particular, for any  $e'' \geq e'$ ,  $H(e'', x) > H(e', x)$ . This represents the self-insurance property of secondary preventive care: prognosis is a function of the promptness of detection, and promptness of detection is made possible by secondary prevention. The cross derivative of the  $H(\cdot)$  function will be discussed in the next section.

For the sake of simplicity, I assume that treatment and prevention are produced by a linear technology which is subject to constant returns to scale. Their costs are constant and set at one. The consumer purchases on the market the chosen quantity of treatment and prevention, as a consequence of which I implicitly assume that the physician is acting as a perfect agent for his patient.

Using the strictly concave function  $U(\cdot)$  to represent the risk-averse consumer’s prefer-

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<sup>7</sup>In Ma and McGuire (1997), the monetary equivalent of the benefits of health care also depends on two variables: the first is the quantity of treatment, and the second is the physician’s input into the production of health benefits (effort). In this paper the authors study the strategic interaction between insurance, consumer and physician.

ences, the consumer's expected utility without any insurance is:

$$pU[W - e - x - H_0 + H(e, x)] + (1 - p)U(W - e)$$

To assure an interior solution for  $e$  I assume that:

$$pU'[W - x - H_0 + H(0, x)] H_e(0, x) > pU'[W - x - H_0 + H(0, x)] + (1 - p)U'(W).$$

The special feature of this model is that preventive care consumption is decided ex-ante, while treatment consumption is decided ex-post (after the realization of the risk). As a consequence, preventive care is consumed in both states of nature, while treatment is only consumed in the case of illness. As the reader will see, this implies that the Slutsky equations are non-standard.

## 2.1 The relationship between secondary prevention and treatment

The cross derivative  $H_{ex}$  represents the technological relationship between preventive care and treatment. Both complementary and substitute cases are interesting aspects of the real world, and therefore I will analyze both of them.<sup>8</sup>

### • Complementary goods ( $H_{ex}(e, x) > 0$ )

In this case the marginal benefit of treatment increases with the purchase of preventive care: for any  $e'' \geq e'$   $H_x(e'', x) > H_x(e', x)$ <sup>9</sup>. In other words, treatment becomes more effective if preventive measures have been taken. Prevention and treatment may be complementary in the case, for example, of prenatal and cervical screening. Take firstly cervical screening (Pap smear) for cervical cancer and pre-cancerous cervical conditions. This test also checks for the presence of human papillomavirus (HPV) on the cervix, and research has shown a positive correlation between HPV and cervical cancer. Thus when HPV is detected, many experts prefer to treat their patients with a cone biopsy, even if only a small percentage of such patients will actually develop cervical cancer.<sup>10</sup> With regard to prenatal screening, let us look at the case of amniocentesis. Amniocentesis can detect a genetic abnormality called XYY syndrome, which is associated with mild mental retardation and in some cases, to aggressive behavior. Therefore, when XYY syndrome is detected in an unborn child, this child can receive psychological treatment, in the form of special education and training, from a very early age.<sup>11</sup>

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<sup>8</sup>In general, the sign of the cross derivative depends both on  $e$  and  $x$ , that is, on the preventive measure in question, the detected illness and the type of therapy involved. To simplify the analysis, I will treat the two cases of complementary and substitutes goods separately.

<sup>9</sup>In fact  $H_{ex}(e, x) > 0 \Leftrightarrow \int_{e'}^{e''} H_{ex}(t, x) dt > 0$ ,  $\forall e'' > e'$  and  $x$ .

<sup>10</sup>Given that cervical dysplasia develops slowly, other physicians prefer to put off diagnosis for a few months while waiting to repeat the Pap test.

<sup>11</sup>There are many cases in which diagnostic screening leads to further testing. Given that a new test also implies another physician's opinion and advice, in a broad sense such cases may be seen as further examples of complementarity between secondary prevention and treatment. The alpha-fetoprotein (AFP) test is a screening method used to check for the possibility of Down syndrome or spina bifida in unborn children. About 5% of tested women result positive, although, over 95% of babies who test positive, have neither Down syndrome nor spina bifida. Abnormal results are usually followed up by the performance of further tests,

• **Substitutes** ( $H_{ex}(e, x) < 0$ )

In this case the marginal benefit of treatment decreases with the purchase of preventive care: for any  $e'' \geq e'$   $H_x(e'', x) < H_x(e', x)$ . In other words, treatment is less effective if preventive measures have been taken. Prevention is, nevertheless, still beneficial to consumer's health, hence we are always considering  $H(e'', x) > H(e', x)$ . It is the *marginal* benefit of treatment that decreases when preventive care has been purchased.<sup>12</sup> Cases of the substitutability of secondary prevention and treatment are more common. Dental screening is clearly one of these cases. In fact, routine check-ups mean that most dental caries are detected early on (while pain may not be present until the advanced stages of tooth decay). When dental caries are discovered in the early stages, less treatment is needed, complication are prevented, and the tooth can be preserved. Let us now consider mammography, a breast scan used to detect cancers before they are clinically palpable. Mammography enables us to detect ductal carcinoma in situ (DCIS, a non-spreading form of breast cancer) which is generally treated by needle-localized biopsy. If DCIS is not detected and treated, it develops into ductal carcinoma. This type of cancer is normally treated by mastectomy, the removal of all the axillary lymph nodes, together with pharmacological therapy.<sup>13</sup>

Being averse to risk, consumers demand health insurance. In the next paragraph I am going to illustrate the optimal insurance plan from the consumer's point of view.

## 2.2 First-best

First-best insurance is characterized by two monetary transfers contingent upon disease. The social planner maximizes consumer's expected utility according to the following program:

$$\begin{cases} \underset{T_H, T_0, x, e}{Max} & pU[W - e - x + T_H - H_0 + H(e, x)] + (1 - p)U(W - e + T_0) \\ s.t. : & pT_H + (1 - p)T_0 = 0 \end{cases} \quad (P1)$$

where consumers receive  $T_H$  in the case of illness and  $T_0$  ( $< 0$ ) when healthy. Later on,  $C_0$  will denote consumption when the consumer is healthy, and  $C_H$  will denote consumption when he is ill. From FOCs with respect to  $T_H$  and  $T_0$  one finds the full insurance condition ( $C_0 = C_H$ ). Moreover FOCs with respect to  $e$  and  $x$  yield:

$$\frac{H_e}{H_x} = \frac{1}{p} \quad (1)$$

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including amniocentesis. A second example again involves genetic testing. Research into the parental origin of Down Syndrome has shown that a parent with a Robertsonian translocation t(14;21) (a genetic abnormality which can be only detected by chromosome mapping) is highly likely to produce a baby affected by Down syndrome. Thus when such a Robertsonian translocation is detected, pregnancy is generally followed by more genetic testing.

<sup>12</sup> Generally speaking, where prevention and treatment are substitutes, the case in which it is  $H(e'', x) < H(e', x)$  may arise for sufficiently large  $x$ . However, empirical evidence leads us to exclude this case as being unlikely.

<sup>13</sup> Before the advent of breast screening, DCIS was rarely diagnosed, as the illness was only discovered at an advanced, far more serious stage.

The previous expression shows that the marginal rate of substitution between prevention and treatment is not equal to their price ratio. This is because  $e$  is a certain form of consumption, while there is only a probability  $p$  that  $x$  will be consumed. As a result,  $H_x < H_e$ : at the optimum the marginal benefit of treatment is lower than the marginal benefit of prevention.

In the following pages I am going to focus on reimbursement insurance: in other words, I will be analyzing a linear mechanism. This linear mechanism corresponds to an insurance plan that is frequently used in reality. However, as I pointed out in the introduction, we can reasonably assume that insurance can ex-post verify both secondary prevention and treatment consumption<sup>14</sup>. Thus more complex, non-linear mechanisms could be analyzed as well, although these mechanisms are not often employed, and the analysis would be considerably more complicated. In fact, I am going to leave non-linear contracts for future study.

### 3 The consumer's problem

I will analyze the linear mechanism characterized by a monetary transfer  $P$  and by two parameters representing *consumption* prices of prevention and treatment,  $\alpha$  and  $\beta$  respectively.  $P$  corresponds to the actuarially fair premium ( $P = (1 - \alpha)e + p(1 - \beta)x$ ). When  $0 < \alpha < 1$  and  $0 < \beta < 1$ ,  $\alpha$  and  $\beta$  represent, respectively, the fraction of the cost of preventive care and that of the cost of treatment, to be borne by the consumer:  $\alpha$  and  $\beta$  are the coinsurance parameters. In this case, prevention and treatment are both subsidized. This is a standard insurance contract in which the premium  $P$  corresponds to a negative lump-sum transfer. On the contrary, when  $\alpha, \beta > 1$ , prevention and treatment are both taxed. In this case  $P$  corresponds to a positive lump-sum transfer. Also, the case may arise whereby one good is taxed while the other is subsidized. Obviously, in the latter case, the premium  $P$  can be either a positive or a negative monetary transfer.

Given the insurance contract  $(P, \alpha, \beta)$ , the consumer maximizes his expected utility with respect to prevention and treatment:

$$\underset{e, x}{Max} \quad EU = pU[W - P - \alpha e - \beta x - H_0 + H(e, x)] + (1 - p)U(W - P - \alpha e) \quad (P2)$$

From FOC with respect to  $e$  one finds:

$$pU'(C_H)H_e(e, x) = \alpha E[U'(C)] \quad (2)$$

where  $E[U'(C)] = pU'(C_H) + (1 - p)U'(C_0)$  and  $H_e(e, x) > \alpha$ . (2) shows that the marginal benefit of preventive care, which exists only when there is illness, is equal to its marginal cost, which, on the contrary, exists in both situations. From FOC with respect to  $x$  one finds:

$$H_x(e, x) = \beta \quad (3)$$

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<sup>14</sup>They are both ex-post verifiable through a doctor's certification or through a hospital's/doctor's bill.



(3) shows that the patient chooses  $x$  to make the marginal benefit of treatment equal to its out-of-pocket cost per unit  $\beta$ .<sup>15</sup>

It is easy to see that the second derivatives of expected utility ( $EU_{ee}$  and  $EU_{xx}$ ) are negative. Moreover, computing the Hessian matrix determinant (see appendix A.1), one can establish the following.

**Remark 1** *The concavity of  $H(\cdot)$  is a sufficient condition for the strict concavity of the expected utility.*

Obviously, the sign of  $EU_{ex}$  depends on the sign of  $H_{ex}$ . One can easily see that, when  $e$  and  $x$  are complements,  $\frac{dx}{de} > 0$ , on the other hand, when  $e$  and  $x$  are substitutes,  $\frac{dx}{de} < 0$ . This means that when  $H_{ex} > 0$ , the more preventive care has been consumed, the more the consumer demands treatment. While when  $H_{ex} < 0$ , the more preventive care has been consumed, the less the consumer demands treatment.

### 3.1 Slutsky equations and comparative statics

As I said in section 2, the consumer program analyzed in this model is not a standard one. In fact, prevention is consumed in both situations, while treatment is consumed only if illness occurs (remember that the two budget constraints are, respectively:  $W = P + \alpha e + \beta x + H_0 + C_H - H(e, x)$  and  $W = P + \alpha e + C_0$ ). This implies that when the derivative with respect to  $\beta$  is considered, the Slutsky equation has to be redefined. As will be shown in the next section, Slutsky equations play an important role in the solution of the insurance program. In the following  $y^C$ ,  $y = e, x$  will indicate compensated demand.

**Lemma 1** *Slutsky equations are:*

$$\frac{\partial y^C}{\partial \alpha} = \frac{\partial y}{\partial \alpha} + e \frac{\partial y}{\partial W} \quad (4)$$

$$\frac{\partial y^C}{\partial \beta} = \frac{\partial y}{\partial \beta} + x \frac{pU'(C_H)}{E[U'(C)]} \frac{\partial y}{\partial W} \quad (5)$$

*And the Slutsky matrix is not symmetric  $\left(\frac{\partial x^C}{\partial \alpha} \neq \frac{\partial e^C}{\partial \beta}\right)$ .*

**Proof.** See the appendix. ■

As (5) shows, when compensated demand derivatives are considered with respect to  $\beta$ , a new multiplication term appears in the Slutsky equation. Note that  $\beta$  is the price of the good which is consumed only when illness occurs. It is well known that, as prices vary, compensated demand represents the level of demand that would arise if a consumer's wealth were simultaneously adjusted to keep his utility level constant. From equation (5) wealth

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<sup>15</sup>In Ma and McGuire (1997) treatment demand was not dependent on the insurance premium. Here, even if both the negative health shock and the benefit of treatment and prevention are expressed in monetary terms, insurance premium  $P$  affects consumer's demand for treatment through the demand for preventive care.

compensation has to be multiplied by the term  $\frac{pU'(C_H)}{E[U'(C)]}$ : wealth compensation is less than one. From (4) and (5) it is evident that the Slutsky equations are no longer symmetrical.

The following Lemma describes conditions for prevention and treatment as Hicksian complements or substitutes. As one would expect, there is a one-to-one relationship between the cross derivative of  $H(\cdot)$  and the sign of the compensated demand derivatives.

**Lemma 2** *When  $H_{ex} > 0$ ,  $e$  and  $x$  are Hicksian complements  $\left(\frac{\partial e^C}{\partial \beta}, \frac{\partial x^C}{\partial \alpha} < 0\right)$ .*

*When  $H_{ex} < 0$ ,  $e$  and  $x$  are Hicksian substitutes  $\left(\frac{\partial e^C}{\partial \beta}, \frac{\partial x^C}{\partial \alpha} > 0\right)$ .*

*When  $H_{ex} = 0$ ,  $\alpha$  does not affect compensated demand for treatment and  $\beta$  does not affect compensated demand for prevention  $\left(\frac{\partial e^C}{\partial \beta}, \frac{\partial x^C}{\partial \alpha} = 0\right)$ .*

**Proof.** See the appendix. ■

Finally, notice that when  $e$  and  $x$  are substitutes, if the consumption of one of the two goods increases with wealth, then the other decreases. This implies that, when  $H_{ex} < 0$ , one of the two goods must be an inferior good. This unexpected property of the two goods is a simple consequence of considering treatment and prevention consumption as separate choices (see remark 2 in the appendix A.3).<sup>16</sup>

## 4 Insurance program

In this section the optimal plan  $(P, \alpha, \beta)$  will be characterized in detail. The insurer here may be taken to be a private firm in a competitive market, or a monopolistic public insurer.

The insurance program is:

$$\left\{ \begin{array}{l} \underset{\alpha, \beta, P}{Max} \quad pU[W - P - \alpha e^* - \beta x^* - H_0 + H(e^*, x^*)] + (1 - p)U(W - P - \alpha e^*) \\ s.t. : \quad P = (1 - \alpha)e^* + p(1 - \beta)x^* \quad (\lambda) \\ e^*, x^* = \arg \max_{e, x} \{pU[W - P - \alpha e - \beta x - H_0 + H(e, x)] + (1 - p)U(W - P - \alpha e)\} \end{array} \right. \quad (P3)$$

where  $\lambda$  is the Lagrange multiplier associated with the budget constraint. I do not impose any constraint on  $\alpha$  and  $\beta$ ; as a consequence, prevention and treatment are allowed to be either subsidized ( $0 \leq \alpha < 1$ ,  $0 \leq \beta < 1$ ) or taxed ( $\alpha, \beta > 1$ ).

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<sup>16</sup>It is well accepted that the rich consume more primary and secondary prevention than the poor, even among fully insured people. Thus, prevention is commonly considered a normal good. “One possible explanation may be the existence of a socio-economical bias: for example, a psychological cost of access to medical services that prevents poor people from implementing suitable preventive measures” (Henriet and Rochet (1998), page 20). On the contrary, *when we consider treatment as a separate good from prevention*, both the case where treatment is a normal good and the case where it is an inferior good may be plausible. In particular, we may think that the treatment demand curve is increasing for low revenue values and decreasing for high values. Thus, when studying the case of substitutes, we will implicitly assume that prevention is the normal good and treatment is the inferior one (we will consider the “decreasing with revenue” part of the treatment demand curve).

Using the Envelop Theorem, deriving the expression for  $\lambda$  from FOC with respect to  $P$  and substituting it in FOCs with respect to  $\alpha$  and  $\beta$ , one finds:

$$(1 - \alpha) \left( \frac{\partial e}{\partial \alpha} - e \frac{\partial e}{\partial P} \right) + p(1 - \beta) \left( \frac{\partial x}{\partial \alpha} - e \frac{\partial x}{\partial P} \right) = 0 \quad (6)$$

$$E[U'(C)] \frac{px - (1 - \alpha) \frac{\partial e}{\partial \beta} - p(1 - \beta) \frac{\partial x}{\partial \beta}}{1 - (1 - \alpha) \frac{\partial e}{\partial P} - p(1 - \beta) \frac{\partial x}{\partial P}} = xpU'(C_H) \quad (7)$$

These equations respectively characterize the optimal values of  $\alpha$  and  $\beta$ <sup>17</sup>. Whatever the state of nature realized,  $e$  is always paid entirely by the consumer:  $e$  is a “certain” consumption. Hence, consumption marginal utility does not appear in (6). The consumer’s decisions are modified at the margin by  $\alpha$  (see FOC (2)) but the consumer’s budget constraint does not change. Then, using the instrument  $\alpha$ , the insurer imposes either a tax or a subsidy on preventive services: there is no insurance against prevention expenses. When  $\alpha$  increases, insurance has to balance the *total* effect of this variation on the consumption of  $e$  and  $x$ . This total effect is the sum of the direct effect on consumption due to a rise of  $\alpha$  and the indirect effect on consumption due to the premium fall, as (6) shows.<sup>18</sup>

The l.h.s. of equation (7) represents the marginal benefit, while the r.h.s. represents the marginal cost, of a rise in  $\beta$ . When  $\beta$  increases, the consumer’s out-of-pocket expenses increase and his premium decreases. Marginal benefit is the consequence of the reduction in the premium. On the l.h.s. of (7), the direct effect of a marginal variation of  $\beta$  in the premium is divided by the indirect effect of the premium fall on consumption. This ratio is multiplied by the average marginal utility of consumption; in fact the premium is paid in both states of nature. On the r.h.s. of (7), the negative income effect due to an increase in  $\beta$  is given by the product of treatment quantity and the marginal utility of consumption in the case of illness.

Notice that  $W - P$  is perceived by the consumer as the net exogenous revenue, then  $\frac{\partial e}{\partial P} = -\frac{\partial e}{\partial W}$  and  $\frac{\partial x}{\partial P} = -\frac{\partial x}{\partial W}$ . As a consequence, by rearranging (6) and (7) we find that:

$$(1 - \alpha) \left( \frac{\partial e}{\partial \alpha} + e \frac{\partial e}{\partial W} \right) + p(1 - \beta) \left( \frac{\partial x}{\partial \alpha} + e \frac{\partial x}{\partial W} \right) = 0 \quad (8)$$

$$(1 - \alpha) \left( \frac{\partial e}{\partial \beta} + x \frac{pU'(C_H)}{E[U'(C)]} \frac{\partial e}{\partial W} \right) + p(1 - \beta) \left( \frac{\partial x}{\partial \beta} + x \frac{pU'(C_H)}{E[U'(C)]} \frac{\partial x}{\partial W} \right) + px(1 - p) \frac{U'(C_H) - U'(C_0)}{E[U'(C)]} = 0 \quad (9)$$

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<sup>17</sup>Note that  $\alpha = \beta = 1$  cannot be a solution because, in this case,  $C_H \neq C_0$ , such that condition (7) does not hold.

<sup>18</sup>The term  $\frac{\partial e}{\partial \alpha}$  is a measure of moral-hazard. A number of empirical studies have analyzed the impact of cost-sharing on the consumption of preventive care. The Rand Health Insurance Experiment found that, in general, coinsurance reduces the use of preventive care services, although not to the point where it also reduces visits for non-preventive care services. However, the negative impact that cost-sharing has on preventive care consumption increases when low-income people are taken into consideration. (Manning *et al.* (1987))

In keeping with the previous result, another study (Cherkin *et al.* (1990)), has shown that *few* preventive care services were adversely effected by cost-sharing among the non-poor population: even though co-payment led to reductions in the rate of physical examinations and of visits by persons with cardiovascular disease, the rates of cancer screening, childhood immunization and the use of cardiovascular drugs by those needing them, were not adversely affected.

Using the Slutsky equations derived in Lemma 1, one can rewrite the previous expressions in the following way.

$$(1 - \alpha) \frac{\partial e^C}{\partial \alpha} + p(1 - \beta) \frac{\partial x^C}{\partial \alpha} = 0 \quad (10)$$

$$(1 - \alpha) \frac{\partial e^C}{\partial \beta} + p(1 - \beta) \frac{\partial x^C}{\partial \beta} + px(1 - p) \frac{U'(C_H) - U'(C_0)}{E[U'(C)]} = 0 \quad (11)$$

From (10) and from Lemma 2 it follows that, when  $H_{ex} > 0$  it must be either  $\alpha < 1$  and  $\beta > 1$  or the opposite. While when  $H_{ex} < 0$  it must be either  $(\alpha < 1, \beta < 1)$  or  $(\alpha > 1, \beta > 1)$ . Moreover, when  $H_{ex} = 0$  the substitution effects are zero, then (10) and (11) show that  $\alpha = 1$  and  $\beta < 1$ . These results are included in the following proposition:

**Proposition 3** *If  $e$  and  $x$  are complements, one good is subsidized while the other is taxed. If  $e$  and  $x$  are substitutes, the two goods are either both subsidized or both taxed. However, if the two goods are neither complements nor substitutes, then treatment is subsidized and the price of prevention is unaffected.*

Proposition 3 declares that when  $H_{ex} = 0$ , the optimal linear contract subsidizes treatment and leaves prevention price unaffected. This means that if prevention consumption does not affect treatment demand, then secondary prevention is not included in the insurance contract. This result anticipates the interpretation of the optimal insurance policy when  $H_{ex} \neq 0$ .

To define the optimal policy one must solve the system of equations (10) and (11). Deriving the expression for  $\alpha$  from (10), substituting it in (11) and rearranging things, we find that the sign of  $\frac{\partial x^C}{\partial \beta} - \frac{\partial x^C}{\partial \alpha} \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}}$  is of crucial importance in calculating whether  $\beta$  is higher or lower than 1. The result of the insurance program (P3) can be summarized in table 2.

	$H_{ex} > 0$	$H_{ex} < 0$
$\frac{\partial x^C}{\partial \beta} > \frac{\partial x^C}{\partial \alpha} \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}}$	(a) $\alpha > 1, \beta < 1$ $(\alpha \rightarrow e^C, \beta \rightarrow x^C)$	(c) $\alpha > 1, \beta > 1$ $(\alpha \rightarrow x^C, \beta \rightarrow e^C)$
$\frac{\partial x^C}{\partial \beta} < \frac{\partial x^C}{\partial \alpha} \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}}$	(b) $\alpha < 1, \beta > 1$ $(\alpha \rightarrow x^C, \beta \rightarrow e^C)$	(d) $\alpha < 1, \beta < 1$ $(\alpha \rightarrow e^C, \beta \rightarrow x^C)$

Table 2: the solution in the case of complements and substitutes.

The foregoing discussion proves the following:

**Proposition 4** *The optimal values of  $\alpha$  and  $\beta$  depend on the comparative advantage of the two prices in influencing compensated demand for treatment and prevention. As table 2 shows, four different cases must be distinguished here:*

*Complementary goods ( $H_{ex} > 0$ ):*

- *when prevention price has a comparative advantage in influencing compensated demand for prevention, preventive care is taxed and treatment is subsidized.*
- *when prevention price has a comparative advantage in influencing compensated demand for treatment, preventive care is subsidized and treatment is taxed.*

*Substitutes ( $H_{ex} < 0$ ):*

- *when prevention price has a comparative advantage in influencing compensated demand for prevention (treatment), preventive care and treatment are both subsidized (taxed).*

Proposition 4 show that, in both the complement and substitute cases, the question of whether it is best to tax or to subsidize the two goods depends on the relative size of the compensated demand derivatives. Why *compensated* demand derivatives one may ask? To understand this, we need to remember that the insurance contract is characterized by the *three* variables  $P, \alpha$  and  $\beta$ . The optimal contract has been defined only in term of the *two* variables  $\alpha$  and  $\beta$ . Nevertheless, when we consider a change in prevention and treatment prices, the premium  $P$  must also be modified in order to balance the budget constraint. And a change in the premium affects consumption choices because it modifies the consumer's net wealth. Compensated demand derivatives take into account the effects of a change of  $\alpha$  and  $\beta$  in the premium  $P$ , and the consequences of premium variation on consumption choices. In other words, compensated demand derivatives enable us to consider both the direct and indirect effects on consumption of a change in  $\alpha$  and  $\beta$ , where the indirect effects are the financial consequences of the premium variation.

As we will see, the compensated demand derivatives ratio can be interpreted in terms of “comparative advantage”. This situation gives us an *assignment rule*: insurance should use that instrument ( $\alpha$  or  $\beta$ ) with a comparative advantage in influencing a given target (compensated demand for prevention or for treatment).<sup>19</sup> Table 2 shows (in bracket) the assignment rule for each of the four cases. In appendix A.4 I explain in detail how this assignment problem can best be formulated. Here I simply want to underline the fact that insurance has two separate objectives: the efficient consumption of treatment and of preventive care. It also has two instruments which it can use to achieve these two objectives: treatment and preventive care consumption prices. As I said before, in order to internalize the financial effects of a variation of  $\alpha$  and  $\beta$ , we must consider compensated demands for treatment and prevention. The latter both depend on the two instruments  $\alpha$  and  $\beta$ , and the problem is how to effectively use the two instruments. As results show, this goal is achieved if each instrument is used specifically to influence the compensated demand for the good which is more “elastic”

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<sup>19</sup>This rule governing the matching of instruments to targets has become known as the Principle of Effective Market Classification (PEMC), Mundell (1962). Alternatively we can refer here to the “Principle of targeting” enunciated by Dixit (1985). According to this principle, a required distortion is attained at least welfare cost by a most direct policy. In our context such a policy corresponds to the tax instrument that acts on a specific target most directly.

with respect to the instrument it-self.<sup>20</sup>

Let us consider complementary goods, and remember from Lemma 2 the signs of the compensated demand derivatives. Table 2 shows that, when  $\alpha$  has a comparative advantage in influencing compensated demand for prevention and  $\beta$  has a comparative advantage in influencing compensated demand for treatment (case (a)), insurance assigns instrument  $\alpha$  to compensated demand for preventive care, and instrument  $\beta$  to compensated demand for treatment. On the contrary, when  $\beta$  has a comparative advantage in influencing compensated demand for prevention and  $\alpha$  has a comparative advantage in influencing compensated demand for treatment (case (b)), insurance assigns instrument  $\beta$  to compensated demand for preventive care, and the instrument  $\alpha$  to compensated demand for treatment.<sup>21</sup> Table 2 can be read in the same way for the case of substitutes.

## 4.1 Interpretation

A glance at table 2 shows that the only intuitive case is (d), where both preventive care and treatment are subsidized. In all the other cases either one of the two goods, or both of them, ought to be taxed. This looks like a paradox: given that prevention and treatment are both inputs in the health production function, how can these policies maximize the consumer's utility?

In analyzing this result, it is important to remember that the consumer is averse to risk, and demands insurance against the financial risk associated with buying medical care. Insuring treatment is the main objective of the insurer, and so treatment consumption is always encouraged. On the contrary, with regard to prevention, the insurer's reasoning is purely financial: if prevention increases the cost of insuring treatment, then prevention is to be discouraged; whereas if it reduces this cost, then prevention is to be encouraged.<sup>22</sup> Thus when the two goods are complements, prevention consumption is discouraged. In fact, in this case secondary prevention leads to a rise in the purchase of treatment.<sup>23</sup> On the contrary, when

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<sup>20</sup>To understand which good ought to be taxed and which one ought to be subsidized, it is not necessary for the insurer to measure the compensated demand derivatives. As is shown in appendix A.4, the insurer simply has to observe the sign of the compensated demand derivatives: this sign is determined by the cross derivative of the health production function (see Lemma 2).

<sup>21</sup>Notice that  $-\frac{\frac{\partial x^C}{\partial \beta}}{\frac{\partial x^C}{\partial \alpha}} = \frac{d\alpha}{d\beta} |_{x^C=C}$  and  $-\frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}} = \frac{d\alpha}{d\beta} |_{e^C=C}$  are the locus of the price pairs  $(\alpha, \beta)$  such that compensated demands for prevention and for treatment are constant. The relative slope of  $\frac{d\alpha}{d\beta} |_{x^C=C}$  and  $\frac{d\alpha}{d\beta} |_{e^C=C}$  guarantees the stability of the allocation: if an exogenous shock moves prices from their optimal values,  $\alpha$  and  $\beta$  can return to their initial position. In fact the assignment rule states that a *stable* adjustment will be achieved by assigning each instrument to the target with respect to which it has a comparative advantage.

<sup>22</sup>Eeckhoudt *et al.* (2000) also arrived at a similar conclusion when investigating the optimal subsidization of primary prevention. Their paper came to my attention after my own had been written. With regard to their comparative statics of the consumer's problem, as usually happens when dealing with the case of self-protection, results are somewhat ambiguous. Their analysis of the insurance problem is, on the other hand, much simpler because the health production function does not depend on prevention.

<sup>23</sup>Let us consider the financial consequences when preventive care and treatment are complementary and are both subsidized. It is clear that in this situation, the consumption of both goods would increase so much

the two goods are substitutes, prevention consumption is encouraged. In fact, in this case secondary prevention consumption leads to a reduction in the purchase of treatment. The result when  $H_{ex} = 0$  is in perfect keeping with this interpretation: if secondary prevention does not affect treatment demand, then the price of prevention is not distorted and treatment is subsidized.

Take, for example, the case of complementary goods. In solution (a) preventive care is taxed ( $\alpha > 1$ ) and treatment is subsidized ( $\beta < 1$ ). Prevention and treatment are complementary, and therefore starting from a situation in which  $\alpha = 1$ , the increase in prevention price makes demand for both goods fall. In the same way, if we start from a situation in which  $\beta = 1$ , the reduction in treatment price makes demand for both prevention and treatment rise. As the relative size of the compensated demand derivatives shows, in case (a) prevention price has a comparative advantage in influencing compensated demand for prevention. As a consequence, the net effect of prices variation is a fall in prevention demand and a rise in treatment demand. In other words, prevention consumption is discouraged while treatment consumption is encouraged. Let us now look at solution (b), whereby preventive care is subsidized ( $\alpha < 1$ ) and treatment is taxed ( $\beta > 1$ ). Prevention and treatment are complementary, and therefore if we start from a situation in which  $\alpha = 1$ , the reduction in prevention price leads to a rise in demand for both goods. In the same way, starting from a situation in which  $\beta = 1$ , the increase in treatment price leads to a fall in demand for both prevention and treatment. As the relative size of the compensated demand derivatives shows, in case (b) prevention price has a comparative advantage in influencing compensated demand for treatment. Thus the net effect of the price variation will once again be, a fall in prevention demand and a rise in treatment demand. This means that prevention consumption is indirectly discouraged by treatment taxation, while treatment consumption is indirectly encouraged by prevention subsidization. The above reasoning is summarized in table 3. The same reasoning can also be applied to the case where the two goods are substitutes (see table 4).

<div style="border: 1px solid black; padding: 10px;"> <p>(a)</p> <p><math>\alpha &gt; 1 \Rightarrow e \searrow, x \searrow</math></p> <p><math>\beta &lt; 1 \Rightarrow e \nearrow, x \nearrow</math></p> <p>but <math>\alpha \rightarrow e^C, \beta \rightarrow x^C</math></p> <p>then: <math>e \searrow</math> and <math>x \nearrow</math></p> </div>	<div style="border: 1px solid black; padding: 10px;"> <p>(b)</p> <p><math>\alpha &lt; 1 \Rightarrow e \nearrow, x \nearrow</math></p> <p><math>\beta &gt; 1 \Rightarrow e \searrow, x \searrow</math></p> <p>but <math>\alpha \rightarrow x^C, \beta \rightarrow e^C</math></p> <p>then: <math>e \searrow</math> and <math>x \nearrow</math></p> </div>
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Table 3: intuition of the result when  $H_{ex} > 0$ .

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that the consumer's expenses (and insurance reimbursement) would burst out.

<div style="border: 1px solid black; padding: 10px;"> (c)  <math>\alpha &gt; 1 \Rightarrow e \searrow, x \nearrow</math>   <math>\beta &gt; 1 \Rightarrow e \nearrow, x \searrow</math>   but <math>\alpha \rightarrow x^C, \beta \rightarrow e^C</math>   then: <math>e \nearrow</math> and <math>x \nearrow</math> </div>	<div style="border: 1px solid black; padding: 10px;"> (d)  <math>\alpha &lt; 1 \Rightarrow e \nearrow, x \searrow</math>   <math>\beta &lt; 1 \Rightarrow e \searrow, x \nearrow</math>   but <math>\alpha \rightarrow e^C, \beta \rightarrow x^C</math>   then: <math>e \nearrow</math> and <math>x \nearrow</math> </div>
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Table 4: intuition of the result when  $H_{ex} < 0$ .

Given the above considerations, we can now formulate the following proposition.

**Proposition 5** *Starting from a no-insurance situation, a linear contract always encourages treatment consumption, whereas it may either encourage or discourage secondary prevention consumption. That is, prevention consumption is discouraged when the two goods are complements, and encouraged when the two goods are substitutes.*

## 4.2 Constrained insurance program

In this section I assume that institutional and/or informational constraints do not allow the insurer to tax either prevention or treatment. This implies that consumption prices must be such that  $0 \leq \alpha \leq 1$  and  $0 \leq \beta \leq 1$ . As a result, the insurer's Lagrangian objective function becomes:

$$L = pU[W - P - \alpha e^* - \beta x^* - H_0 + H(e^*, x^*)] + (1-p)U(W - P - \alpha e^*) - \lambda[P - (1-\alpha)e^* - p(1-\beta)x^*] + \mu_1(1-\alpha) + \mu_2(1-\beta) \quad (12)$$

where  $\mu_1, \mu_2 \geq 0$  are the Kun Tacker multipliers of the insurance program verifying  $\mu_1(1-\alpha) = 0$  and  $\mu_2(1-\beta) = 0$ . Consider the first order conditions of (12) with respect to  $\alpha, \beta$  and  $P$ . Using the Envelop Theorem, and substituting the expression for  $\lambda$  and rearranging, the result is the following system of equations:

$$\left\{ \begin{array}{l} (1-\alpha) \left[ \frac{\partial e^C}{\partial \alpha} + \frac{\mu_1}{E[U'(C)]} \frac{\partial e}{\partial W} \right] + p(1-\beta) \left[ \frac{\partial x^C}{\partial \alpha} + \frac{\mu_1}{E[U'(C)]} \frac{\partial x}{\partial W} \right] + \frac{\mu_1}{E[U'(C)]} = 0 \\ (1-\alpha) \left[ \frac{\partial e^C}{\partial \beta} + \frac{\mu_2}{E[U'(C)]} \frac{\partial e}{\partial W} \right] + p(1-\beta) \left[ \frac{\partial x^C}{\partial \beta} + \frac{\mu_2}{E[U'(C)]} \frac{\partial x}{\partial W} \right] + \frac{\mu_2}{E[U'(C)]} + px(1-p) \frac{U'(C_H) - U'(C_O)}{E[U'(C)]} = 0 \end{array} \right. \quad (13)$$

I am now going to analyze the inequality constraints on  $\alpha$  and  $\beta$ .

Case (1): neither of the two inequality constraints is binding. Then  $1-\alpha > 0$  and  $1-\beta > 0$  both stand, such that  $\mu_1 = \mu_2 = 0$ . As a result, the equations in (13) are equivalent to (10) and (11). Table 2 shows that  $\alpha, \beta < 1$  is only possible when treatment and prevention are substitutes, and when  $\alpha$  has a comparative advantage in influencing compensated demand for prevention.



Case (2): only the inequality constraint on  $\alpha$  is binding. Then  $1 - \alpha = 0$  and  $1 - \beta > 0$ , such that  $\mu_1 \geq 0$  and  $\mu_2 = 0$ . The system of equations (13) becomes:

$$\begin{cases} p(1 - \beta) \left[ \frac{\partial x^C}{\partial \alpha} + \frac{\mu_1}{E[U'(C)]} \frac{\partial x}{\partial W} \right] + \frac{\mu_1}{E[U'(C)]} = 0 \\ p(1 - \beta) \frac{\partial x^C}{\partial \beta} + px(1 - p) \frac{U'(C_H) - U'(C_o)}{E[U'(C)]} = 0 \end{cases} \quad (14)$$

Notice that, because of the second equation in (14),  $\beta = 1$  is not a possible solution. The first equation in (14) clearly show that a solution only exists if  $\frac{\partial x^C}{\partial \alpha} + \frac{\mu_1}{E[U'(C)]} \frac{\partial x}{\partial W} < 0$ . The previous condition is compatible with both the case where the goods are substitutes and that in which they are complementary.

Case (3): only the inequality constraint on  $\beta$  is binding. Then  $1 - \alpha > 0$  and  $1 - \beta = 0$ , such that  $\mu_1 = 0$  and  $\mu_2 \geq 0$ . The system of equations (13) clearly becomes impossible. Thus, we can exclude case (3).

Case (4): the inequality constraints are both binding. This means that  $1 - \alpha = 1 - \beta = 0$ , such that  $\mu_1$  and  $\mu_2$  are both non negative. It is easy to see that  $\alpha = \beta = 1$  is not a solution to (13). Thus, we can also exclude case (4).

The previous results can be summarized in the following proposition.

**Proposition 6** *With regard to the constrained insurance program:*

- $(\alpha < 1, \beta < 1)$  is a possible solution when treatment and prevention are substitutes, provided  $\alpha$  has a comparative advantage in influencing compensated demand for prevention.
- $(\alpha = 1, \beta < 1)$  is a possible solution when  $\frac{\partial x^C}{\partial \alpha} + \frac{\mu_1}{E[U'(C)]} \frac{\partial x}{\partial W} < 0$ .
- $(\alpha < 1, \beta = 1)$  and  $(\alpha = 1, \beta = 1)$  are not possible solutions.

Proposition 6 shows that solutions whereby treatment is not subsidized are to be excluded. This is in keeping with the result of the non-constrained problem. In fact, I showed that treatment consumption is always encouraged, either directly (when  $\beta < 1$ ) or indirectly (when  $\beta > 1$ ) (table 2 and proposition 5). Here, such “indirect subsidization” of treatment consumption is not possible. Hence, the only feasible solutions are those whereby  $\beta < 1$ . This proves once again that the primary objective of the insurer is to encourage treatment consumption. In the case of prevention, the insurer’s policy reflects a purely financial line of reasoning: prevention is subsidized only if it enables the insurer to save on treatment.

## 5 Conclusion

As Kenkel (2000, page 1686) has observed: “... the economist’s notion of an optimal level of prevention where the marginal benefit equals the marginal costs remains somewhat foreign and even controversial.” This model has tried to fill the gap by analyzing secondary prevention as an instrument of self-insurance (Ehrlich and Becker (1972)): if illness occurs, the negative health shock decreases. This approach represents an interesting alternative to the cost-effectiveness (CE) analysis normally used in Health Economics to evaluate public health measures and to investigate private goods and services such as prevention. The limitations of

CE analysis as an analytic framework are discussed in Garber (2000)<sup>24</sup>, the main unresolved issue being the difficulty in using the result of CE analysis to formulate health policy at the societal or group level. The analysis developed in this paper, however, enables us to formulate a clear policy rule.

In the present model, consumer's utility depends on the monetary equivalent of the benefits of preventive care and treatment. Both the case of complementary goods and that of substitutes are analyzed here, and optimal reimbursement is defined when insurance adopts a linear mechanism. Starting from a no-insurance situation, the most important result is that a linear contract always encourages treatment consumption, whereas it may encourage or discourage secondary prevention. Prevention consumption is discouraged when the two goods are complementary, but encouraged when the two goods are substitutes.<sup>25</sup>

This normative approach shows that when deciding whether to reimburse prevention costs or not, the insurer should reason in purely financial terms: if prevention reduces the cost of insuring treatment (i.e. in the case of substitutes), then prevention has to be encouraged, while if it increases this cost (i.e. in the case of complements) then prevention has to be discouraged. The optimal policy when prevention does not affect treatment demand (i.e. when the cross derivative of the health production function is zero) is in keeping with this interpretation: in such a case, the price of secondary prevention must not be distorted.

It has been noticed that policy proposals often seem to be made according to the assumption that prevention needs to be encouraged. However, so far health economics research has shed more light on which policy tools may succeed in achieving greater prevention, than on whether encouraging greater prevention is in fact a suitable policy goal (Kenkel (2000)). With regard to this question, our model first of all shows that the distinction between complementary and substitute goods is a crucial one. Secondly it shows that, when treatment and secondary prevention are complements, the insurer has no incentive towards encouraging (or paying for) preventive care, even though the latter reduces the severity of illness. This is indeed consistent with the observation that, in reality, reimbursement insurance does not very often reimburse preventive services.

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<sup>24</sup>Many controversies exist concerning CE methods. For example, the lack of specific standards for the application of CE analysis and the methodological controversies over the measurement of both the future cost of health care and the value of the health outcome.

<sup>25</sup>With regard to the ease with which this policy may be implemented, it should be said that in some cases it may be difficult for the insurer to distinguish between consumption of preventive services and treatment. As a result, and with the physician's help, the insured party may be able to misrepresent his consumption in order to avoid taxation.

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## A Appendix

### A.1 The concavity of the consumer's problem

From the first order conditions of program (2) one finds:

$$\begin{aligned}
 EU_{ee} &= pU''(C_H)(-\alpha + H_e)^2 + \alpha^2(1-p)U''(C_0) + pU'(C_H)H_{ee} < 0 \\
 EU_{xx} &= pU''(C_H)(-\beta + H_x)^2 + pU'(C_H)H_{xx} < 0 \\
 EU_{ex} &= pU''(C_H)(-\beta + H_x)(-\alpha + H_e) + pU'(C_H)H_{ex} = pU'(C_H)H_{ex}
 \end{aligned}$$

Second order conditions are seen to exist if the Hessian matrix is negative semi-definite. Let  $D$  denote the determinant of the Hessian matrix  $\begin{bmatrix} EU_{ee} & EU_{ex} \\ EU_{xe} & EU_{xx} \end{bmatrix}$ . It must be:

$$\begin{aligned}
 D &= p^2U'(C_H)U''(C_H)(-\alpha + H_e)^2 H_{xx} + \alpha^2p(1-p)U'(C_H)U''(C_0)H_{xx} \\
 &\quad + p^2[U'(C_H)]^2(H_{ee}H_{xx} - H_{ex}^2) \geq 0
 \end{aligned} \tag{15}$$

From (15) it is easy to see that a sufficient condition for the concavity of expected utility is:  $H_{ee}H_{xx} - H_{ex}^2 \geq 0$ . This means that, not surprisingly, the concavity of  $H(\cdot)$  is a *sufficient* condition for the concavity of the whole objective function. While a *necessary* condition for the concavity of the expected utility is  $H_{xx} \neq 0$ .

## A.2 The expenditure minimization problem and the Slutsky equations (proof of Lemma 1)

The consumer's problem is characterized by two budget constraints:

$$W = P + \alpha e + \beta x + H_0 - H(e, x) + C_H$$

$$W = P + \alpha e + C_0$$

thus the expenditure minimization problem can be written as follows:

$$\underset{e, x, D}{Min} \quad \Lambda = D + \gamma \left\{ \bar{U} - pU[D - P - \alpha e - \beta x - H_0 + H(e, x)] - (1 - p)U(D - P - \alpha e) \right\}$$

where  $\gamma$  is the Lagrange multiplier associated with the utility constraint. From this program we can deduce the expenditure function  $D(\alpha, \beta, \bar{U})$ , where  $D(\alpha, \beta, \bar{U}) = W$  when  $\bar{U} = EU[e^*(\alpha, \beta, W), x^*(\alpha, \beta, W)]$ , and the Hicksian or compensated demands  $y^C(\alpha, \beta, \bar{U})$ ,  $y = e, x$ .

It is commonly agreed that, one can relate Hicksian and Walrasian demand as follows:

$$y^C(\alpha, \beta, \bar{U}) = y[\alpha, \beta, D(\alpha, \beta, \bar{U})]$$

By differentiating the previous expression with respect to prices, one gets:

$$\frac{\partial y^C}{\partial \alpha} = \frac{\partial y}{\partial \alpha} + \frac{\partial y}{\partial W} \frac{\partial D}{\partial \alpha} \quad (16)$$

$$\frac{\partial y^C}{\partial \beta} = \frac{\partial y}{\partial \beta} + \frac{\partial y}{\partial W} \frac{\partial D}{\partial \beta} \quad (17)$$

According to the Envelope Theorem, differentiation of  $\Lambda$  yields the following expressions:

$$\frac{\partial \Lambda}{\partial D} = 1 - \gamma E(U'(C)) = 0, \text{ as a consequence } \gamma = \frac{1}{E(U'(C))}$$

$$\frac{\partial D}{\partial \alpha} = \frac{\partial \Lambda}{\partial \alpha} = e\gamma E(U'(C)) \text{ and then, substituting } \gamma, \frac{\partial D}{\partial \alpha} = e^C = e.$$

$$\frac{\partial D}{\partial \beta} = \frac{\partial \Lambda}{\partial \beta} = x\gamma pU'(C_H) \text{ and then, substituting } \gamma, \frac{\partial D}{\partial \beta} = x^C \frac{pU'(C_H)}{E(U'(C))} = x \frac{pU'(C_H)}{E(U'(C))}.$$

From the latter expression, it is clear that in the case of  $x$  compensated demand *is not* the derivative of the expenditure function with respect to  $\beta$ . Substituting  $\frac{\partial D}{\partial \alpha}$  and  $\frac{\partial D}{\partial \beta}$  in (16) and (17) one obtains the Slutsky equations (4) and (5) in section 3.1.

## A.3 Comparative statics (proof of Lemma 2)

$$\frac{\partial x}{\partial W} \text{ and } \frac{\partial e}{\partial W} \text{ can be calculated as follows: } \begin{bmatrix} EU_{ee} & EU_{ex} \\ EU_{xe} & EU_{xx} \end{bmatrix} \begin{bmatrix} \frac{\partial e}{\partial W} \\ \frac{\partial x}{\partial W} \end{bmatrix} = - \begin{bmatrix} EU_{eW} \\ EU_{xW} \end{bmatrix}$$

where:

$$EU_{eW} = -\alpha E[U''(C)] + pU''(C_H)H_e$$

$$EU_{xW} = 0$$

Solving for  $\frac{\partial x}{\partial W}$  and  $\frac{\partial e}{\partial W}$  using Cramer's rule one finds:

$$\frac{\partial e}{\partial W} = -\frac{\begin{bmatrix} EU_{eW} & EU_{ex} \\ EU_{xW} & EU_{xx} \end{bmatrix}}{D} \quad \text{and} \quad \frac{\partial x}{\partial W} = -\frac{\begin{bmatrix} EU_{ee} & EU_{eW} \\ EU_{xe} & EU_{xW} \end{bmatrix}}{D}$$

where  $D > 0$  because  $H(\cdot)$  is concave.

As a consequence:

$$\frac{\partial e}{\partial W} = -\frac{1}{D}pU'(C_H)H_{xx}[-\alpha E[U''(C)] + pU''(C_H)H_e] \quad (18)$$

$$\frac{\partial x}{\partial W} = \frac{1}{D}pU'(C_H)H_{ex}[-\alpha E[U''(C)] + pU''(C_H)H_e] \quad (19)$$

From (18) and (19) the following remark holds:

**Remark 2** *If  $e$  and  $x$  are complements, then the two goods are either both normal goods or both inferior goods. If  $e$  and  $x$  are substitutes, one good is a normal good while the other is an inferior good. On the other hand, if the two goods are neither complements nor substitutes, treatment demand is unaffected by changes in revenue.*

In both the complementary and substitute case, we will assume that  $-\alpha E[U''(C)] + pU''(C_H)H_e > 0$ .<sup>26</sup> This condition implies that, when  $H_{ex} > 0$ , prevention and treatment are normal goods. While, when  $H_{ex} < 0$ , prevention is a normal good and treatment is an inferior good.

In order to calculate  $\frac{\partial e}{\partial \alpha}$ ,  $\frac{\partial x}{\partial \alpha}$ ,  $\frac{\partial e}{\partial \beta}$  and  $\frac{\partial x}{\partial \beta}$ , let us first consider:

$$\begin{aligned} EU_{e\alpha} &= -E[U'(C)] + \alpha eE[U''(C)] - epU''(C_H)H_e \\ EU_{x\alpha} &= 0 \\ EU_{e\beta} &= -xpU''(C_H)(-\alpha + H_e) \\ EU_{x\beta} &= -xpU''(C_H)(-\beta + H_x) - pU'(C_H) \end{aligned}$$

Applying Cramer's rule once again we find that:

$$\frac{\partial e}{\partial \alpha} = \frac{1}{D}pU'(C_H)H_{xx}E[U'(C)] - e\left[-\frac{1}{D}pU'(C_H)H_{xx}[-\alpha E[U''(C)] + pU''(C_H)H_e]\right] \quad (20)$$

$$\frac{\partial x}{\partial \alpha} = -\frac{1}{D}pU'(C_H)H_{ex}E[U'(C)] - e\left[\frac{1}{D}pU'(C_H)H_{ex}[-\alpha E[U''(C)] + pU''(C_H)H_e]\right] \quad (21)$$

$$\frac{\partial e}{\partial \beta} = -\frac{1}{D}p^2[U'(C_H)]^2 H_{ex} - x\left[-\frac{1}{D}p^2U'(C_H)U''(C_H)H_{xx}(-\alpha + H_e)\right] \quad (22)$$

$$\begin{aligned} \frac{\partial x}{\partial \beta} &= \frac{1}{D}p^2U'(C_H)U''(C_H)(-\alpha + H_e)^2 + \frac{1}{D}\alpha^2p(1-p)U'(C_H)U''(C_0) + \\ &\quad + \frac{1}{D}p^2[U'(C_H)]^2 H_{ee} - x\left[\frac{1}{D}p^2U'(C_H)U''(C_H)H_{ex}(-\alpha + H_e)\right] \end{aligned} \quad (23)$$

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<sup>26</sup>Considering FOC (2) one can see that when revenue increases, the marginal cost of prevention decreases. However, as a result of the function  $H(\cdot)$ , the marginal cost of prevention decreases as well. As a consequence, in order for  $e$  to be a normal good, one also needs to assume that an increase in wealth makes the marginal cost of  $e$  fall more than its marginal benefit does. This condition is necessary because function  $H(\cdot)$  represents the *monetary* equivalent of benefit to health care.

The Slutsky equation (4) implies that the partial derivatives with respect to  $\alpha$  can be broken down into substitution and revenue effects in the usual way. This is clear from (20) and (21), where the revenue effects correspond to expressions (18) and (19) respectively.

(20) shows that the substitution effect is negative ( $\frac{\partial x^C}{\partial \alpha} < 0$ ). Moreover, as we expected, the sign of the substitution term in (21) is determined by the cross derivative of  $H(\cdot)$ . In particular,  $H_{ex} > 0$  implies  $\frac{\partial x^C}{\partial \alpha} < 0$ :  $x$  is a Hicksian complement for  $e$ . While  $H_{ex} < 0$  implies  $\frac{\partial x^C}{\partial \alpha} > 0$ :  $x$  is a Hicksian substitute for  $e$ .

With regard to the partial derivatives with respect to  $\beta$ , on the contrary, (5) implies that the standard break-down into substitution and revenue effects does not hold (the revenue effects in (22) and (23) do not correspond to the expressions (18) and (19)).

(23) shows that the substitution effect is negative ( $\frac{\partial x^C}{\partial \beta} < 0$ ). As for (21), the sign of the substitution term in (22) is determined by the cross derivative of  $H(\cdot)$ , but the reader can see that  $\frac{\partial e^C}{\partial \beta} \neq \frac{\partial x^C}{\partial \alpha}$ : the Slutsky matrix is not symmetric. As before,  $H_{ex} > 0$  implies  $\frac{\partial e^C}{\partial \beta} < 0$ , that is,  $e$  is a Hicksian complement for  $x$ . While  $H_{ex} < 0$  implies  $\frac{\partial e^C}{\partial \beta} > 0$ , that is,  $e$  is a Hicksian substitute for  $x$ . Finally, when  $H_{ex} = 0$ , (21) and (22) show that the substitution effects are zero ( $\frac{\partial x^C}{\partial \alpha} = \frac{\partial e^C}{\partial \beta} = 0$ ). The previous results are summarized in Lemma 2.

#### A.4 The “assignment problem”<sup>27</sup>

Consider the two policy instruments  $\alpha$  and  $\beta$  and the two target variables:

$$\begin{aligned} e^C &= e^C(\alpha, \beta) \\ x^C &= x^C(\alpha, \beta) \end{aligned}$$

where  $e^C$  and  $x^C$  are compensated demand for prevention and for treatment as a function of consumption prices. The two equations express the endogenous variable in terms of the exogenous variables. It is assumed that the insurer knows the signs of the partial derivatives  $\frac{\partial e^C}{\partial \alpha}$ ,  $\frac{\partial e^C}{\partial \beta}$ ,  $\frac{\partial x^C}{\partial \alpha}$  and  $\frac{\partial x^C}{\partial \beta}$ . Let us assume that the insurer has certain given targets,  $\bar{e}^C$  and  $\bar{x}^C$ , that satisfy:

$$\begin{aligned} \bar{e}^C &= e^C(\bar{\alpha}, \bar{\beta}) \\ \bar{x}^C &= x^C(\bar{\alpha}, \bar{\beta}) \end{aligned} \tag{24}$$

By choosing the above values for the policy instruments, he will ensure that the target objectives be met. Suppose we formulate a general policy adjustment rule as follows:

$$\dot{\alpha} = a_{11}(e^C - \bar{e}^C) + a_{12}(x^C - \bar{x}^C) \tag{25}$$

$$\dot{\beta} = a_{21}(e^C - \bar{e}^C) + a_{22}(x^C - \bar{x}^C) \tag{26}$$

As they are, equations (25) and (26) assert that *both* policy variables  $\alpha$  and  $\beta$  are adjusted in accordance with the differences between *both* target variables and their respective goals. What we want to do is to assign *one* target to *one* instrument in such a way that the target

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<sup>27</sup>From Turnousky (1997).

variables will ultimately converge towards their chosen values, irrespective of the speed with which the instrument are adjusted. Such a policy assignment is obtained by setting either  $a_{11}$  and  $a_{22} = 0$ , or  $a_{12}$  and  $a_{21} = 0$ . Linearizing (25) and (26) about the equilibrium defined in (24) yields:

$$\begin{aligned}\dot{\alpha} &= a_{11} \left[ \frac{\partial e^C}{\partial \alpha} (\alpha - \bar{\alpha}) + \frac{\partial e^C}{\partial \beta} (\beta - \bar{\beta}) \right] + a_{12} \left[ \frac{\partial x^C}{\partial \alpha} (\alpha - \bar{\alpha}) + \frac{\partial x^C}{\partial \beta} (\beta - \bar{\beta}) \right] \\ \dot{\beta} &= a_{21} \left[ \frac{\partial e^C}{\partial \alpha} (\alpha - \bar{\alpha}) + \frac{\partial e^C}{\partial \beta} (\beta - \bar{\beta}) \right] + a_{22} \left[ \frac{\partial x^C}{\partial \alpha} (\alpha - \bar{\alpha}) + \frac{\partial x^C}{\partial \beta} (\beta - \bar{\beta}) \right]\end{aligned}$$

where the partial derivatives are evaluated at the equilibrium  $(\bar{\alpha}, \bar{\beta})$ . Letting  $\alpha_o = \alpha - \bar{\alpha}$  and  $\beta_o = \beta - \bar{\beta}$ , the adjusted system can be written in deviation form as

$$\begin{pmatrix} \dot{\alpha}_o \\ \dot{\beta}_o \end{pmatrix} = \begin{pmatrix} a_{11} \frac{\partial e^C}{\partial \alpha} + a_{12} \frac{\partial x^C}{\partial \alpha} & a_{11} \frac{\partial e^C}{\partial \beta} + a_{12} \frac{\partial x^C}{\partial \beta} \\ a_{21} \frac{\partial e^C}{\partial \alpha} + a_{22} \frac{\partial x^C}{\partial \alpha} & a_{21} \frac{\partial e^C}{\partial \beta} + a_{22} \frac{\partial x^C}{\partial \beta} \end{pmatrix} \begin{pmatrix} \alpha_o \\ \beta_o \end{pmatrix} \quad (27)$$

This equation describes the adjustment of the system in the neighborhood of equilibrium, when both policy variables are simultaneously adjusted to both targets. The necessary and sufficient condition for the stability of the linearized system is that:

- (i)  $\text{tr} \Delta < 0$
- (ii)  $\det \Delta > 0$

where  $\text{tr}$  denotes trace and  $\Delta$  denotes the matrix of coefficients in (27). Thus (27) is stable if, and only if,

$$a_{11} \frac{\partial e^C}{\partial \alpha} + a_{12} \frac{\partial x^C}{\partial \alpha} + a_{21} \frac{\partial e^C}{\partial \beta} + a_{22} \frac{\partial x^C}{\partial \beta} < 0 \quad (28)$$

$$(a_{11}a_{22} - a_{12}a_{21}) \left( \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} \right) > 0 \quad (29)$$

The question now is: can we set either  $a_{11} = a_{22} = 0$  or  $a_{12} = a_{21} = 0$  and have both (28) and (29) met?

Let us start with the case of complementary goods. When  $H_{ex} > 0$ , we know from Lemma 2 that  $\frac{\partial e^C}{\partial \alpha}$ ,  $\frac{\partial x^C}{\partial \alpha}$ ,  $\frac{\partial x^C}{\partial \beta}$  and  $\frac{\partial e^C}{\partial \beta}$  are all negative. Consider the first assignment. If  $a_{12} = a_{21} = 0$ , (28) will be satisfied for every arbitrary positive value of  $a_{11}$  and  $a_{22}$ . Moreover, for such arbitrary positive values, (29) will be met provided  $\left( \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} \right) > 0$ . In this case, the adjustment of the system will be stable for all magnitudes of the adjustment speeds  $a_{11} > 0$  and  $a_{22} > 0$ . Alternatively, if  $a_{11} = a_{22} = 0$ , (28) will be satisfied for every arbitrary positive value of  $a_{12}$  and  $a_{21}$ . Moreover, for these arbitrary positive values, (29) will be met provided  $\left( \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} \right) < 0$ . In this case, the adjustment of the system will be stable for all magnitudes of the adjustment speeds  $a_{12} > 0$  and  $a_{21} > 0$ .

To summarize then, when  $H_{ex} > 0$ , the stable assignments are as follows:

$$(a) \text{ if } \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} > 0 \left( \Rightarrow \frac{\frac{\partial x^C}{\partial \beta}}{\frac{\partial x^C}{\partial \alpha}} > \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}} \right)$$



$$\begin{aligned}
\dot{\alpha} &= a_{11} (e^C - \bar{e}^C) & a_{11} &> 0 \\
\dot{\beta} &= a_{22} (x^C - \bar{x}^C) & a_{22} &> 0 \\
\text{(b) if } \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} &< 0 & \left( \Rightarrow \frac{\frac{\partial x^C}{\partial \beta}}{\frac{\partial x^C}{\partial \alpha}} < \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}} \right) \\
\dot{\alpha} &= a_{12} (x^C - \bar{x}^C) & a_{12} &> 0 \\
\dot{\beta} &= a_{21} (e^C - \bar{e}^C) & a_{21} &> 0.
\end{aligned}$$

Let us now consider the case of substitutes. When  $H_{ex} < 0$ , we know from Lemma 2 that  $\frac{\partial e^C}{\partial \alpha}$  and  $\frac{\partial x^C}{\partial \beta}$  are negative while  $\frac{\partial x^C}{\partial \alpha}$  and  $\frac{\partial e^C}{\partial \beta}$  are positive. The first assignment implies  $a_{12} = a_{21} = 0$ , thus (28) will be satisfied for every arbitrary positive value of  $a_{11}$  and  $a_{22}$ . Moreover, for these arbitrary positive values, (29) will be satisfied provided  $\left( \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} \right) > 0$ . In this case, the adjustment of the system will be stable for all magnitudes of the adjustment speeds  $a_{11} > 0$  and  $a_{22} > 0$ . Alternatively, if  $a_{11} = a_{22} = 0$ , (28) will be satisfied for every arbitrary negative value of  $a_{12}$  and  $a_{21}$ . Moreover, for these arbitrary negative values, (29) will be satisfied provided  $\left( \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} \right) < 0$ . In this case, the adjustment of the system will be stable for all magnitudes of the adjustment speeds  $a_{12} < 0$  and  $a_{21} < 0$ .

Summarizing once again, when  $H_{ex} < 0$ , the stable assignments are as follows:

$$\begin{aligned}
\text{(c) if } \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} &< 0 & \left( \Rightarrow \frac{\frac{\partial x^C}{\partial \beta}}{\frac{\partial x^C}{\partial \alpha}} > \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}} \right) \\
\dot{\alpha} &= a_{12} (x^C - \bar{x}^C) & a_{12} &< 0 \\
\dot{\beta} &= a_{21} (e^C - \bar{e}^C) & a_{21} &< 0 \\
\text{(d) if } \frac{\partial e^C}{\partial \alpha} \frac{\partial x^C}{\partial \beta} - \frac{\partial e^C}{\partial \beta} \frac{\partial x^C}{\partial \alpha} &> 0 & \left( \Rightarrow \frac{\frac{\partial x^C}{\partial \beta}}{\frac{\partial x^C}{\partial \alpha}} < \frac{\frac{\partial e^C}{\partial \beta}}{\frac{\partial e^C}{\partial \alpha}} \right) \\
\dot{\alpha} &= a_{11} (e^C - \bar{e}^C) & a_{11} &> 0 \\
\dot{\beta} &= a_{22} (x^C - \bar{x}^C) & a_{22} &> 0.
\end{aligned}$$