

Bertarelli, Silvia; Bernardini Papalia, Rosa

**Working Paper**

## An Optimal Partnership Search Model: Theoretical Implications for the Europarteneriat Event

Quaderni - Working Paper DSE, No. 310

**Provided in Cooperation with:**

University of Bologna, Department of Economics

*Suggested Citation:* Bertarelli, Silvia; Bernardini Papalia, Rosa (1998) : An Optimal Partnership Search Model: Theoretical Implications for the Europarteneriat Event, Quaderni - Working Paper DSE, No. 310, Alma Mater Studiorum - Università di Bologna, Dipartimento di Scienze Economiche (DSE), Bologna,  
<https://doi.org/10.6092/unibo/amsacta/4998>

This Version is available at:

<https://hdl.handle.net/10419/159152>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



<https://creativecommons.org/licenses/by-nc/3.0/>

# An Optimal Partnership Search Model: Theoretical Implications for the Europartenariat Event

by

**Rosa Bernardini Papalia**  
ISTAT - Istituto Nazionale di Statistica  
Roma, Italy  
E-mail: robernar@istat.it

**Silvia Bertarelli**  
Dipartimento di Scienze Economiche  
Bologna, Italy  
E-mail: bertarel@spbo.unibo.it

*Last draft: april 8, 1998*

## Abstract

A theoretical search model applied to collaboration among firms with a public authority as an intermediate is the primary issue of this paper. Specifically, we consider the Europartenariat event since it represents a good opportunity to study a problem of interaction between the decision to search a collaboration and the exchange of information involved in a bargaining process. Search costs play a central role in this framework. We show the optimality of a strategy as to the number of contacts with host companies in a problem of dynamic programming. Precisely, solutions in a search model with the presence of fixed and quadratic costs are derived in terms of an optimal threshold number of contacts and of an optimal interval of the number of contacts, respectively. For the case of fixed costs, we find that the minimum number of contacts for which the firm accepts to participate is negatively related to the expected profit and to the probability of contacting one firm with a common project of collaboration. This is confirmed by a simulation. In the quadratic cost hypothesis we investigate learning phenomena and revelation of private information.

*JEL Classification: C78, D83.*

## **1. Introduction**

Firms can benefit in the exchange of information with other firms as to commercial, technological, and productive skills or in a co-operation for implementing new activities. Especially small sized enterprises have no sufficient resources and capabilities to implement such a relationship. Many problems are faced in this project. First, there is a huge difficulty to identify some firms with the characteristics required, most of all if operating in other countries. Second, the firm must incur in high costs to contact them in order to establish a collaboration.

Recently, we observe the development of projects proposed by public authorities or by associations of firms to boost collaboration among small firms. The Chamber of Commerce of Milan collects information about foreign firms interested in expanding their activity in the Italian market or in their local market but in collaboration with Italian firms. Some of these information are weekly published by an economic newspaper "Il Sole 24-Ore" to guarantee a large circulation. Similar services are offered by associations of firms. However, direct contacts are very limited in number because they need high costs to be implemented. To overcome these problems, the European Union has established the Europartenariat event. By collecting and selecting a large number of small sized firms invited to offer their collaboration, every six months the EU authorities organise a meeting where all European firms can obtain free assistance and advice in order to take direct contacts with host companies.

Further detail about this event is reported in section 2 describing objectives and organisation of the program. In section 3, we study a search process and its interaction with bargaining issues in the problem of finding a partner, in section 4 we introduce the theoretical model studying the European firm's decision to participate in the meeting. Costs connected with the decision of participation and with the revelation of private information are introduced. Learning phenomena are also investigated. The dynamic and stochastic programming problem is presented in section 5. The Bellman equations for fixed and quadratic costs are respectively presented in section 6 and 7. Specifically, we show the optimality of a threshold strategy as to the number of contacts with host companies when costs are fixed; when costs are quadratic a decision of participating is taken if the number of contacts is higher than a floor value and smaller than a ceiling value. Finally, concluding remarks are discussed in section 8.

## **2. The Europartenariat Program**

Many small and medium sized companies do not have the expertise or resources to find a collaboration abroad. Often the first step to success is the identification of partners in other countries. To respond to this need, twice a year companies throughout Europe are invited to attend a two-day event, the *Europartenariat*, during which they have the opportunity to meet potential partners from a host region selected by the Commission.

The *Europartenariat* program was launched by the European Commission in 1987 in order to stimulate the development of less-favoured regions by encouraging small and medium-sized business relationships with their counterparts all over the Community.

Host companies are chosen on the strength of their performance levels in home and overseas markets and on their co-operation proposals for *Europartenariat*. The organisers tend to ensure the participation of quality companies from the widest cross section of industries possible. An *Europartenariat* catalogue contains profiles of the participating host companies with details of their co-operation proposals. 80,000 copies of the catalogue are produced in five languages (English, French, Italian, Spanish, and German) and then distributed throughout 46 countries including EFTA, Central and Eastern European and Mediterranean countries.

EC and overseas companies have the opportunity to identify from the catalogue those local companies which they would like to meet. The schedules of appointments are arranged and distributed to companies by their National Counsellors prior to the event. Visiting companies receive support as to practical arrangements for their participation in a meeting without being charged of any fee<sup>1</sup>.

### **3. Firm's decision problem**

The *Europartenariat* event is interesting since it represents a good opportunity to study a problem of interaction between the decision to search a collaboration and the exchange of information involved in such a decision. A search process is implemented in order to acquire information about potential partners; however, the possible exchange of information means that firms will strategically interact among them.

---

<sup>1</sup> For *Europartenariat* events organised in Member States, two thirds of the cost is carried by the European Commission. The other third is paid by the host region. No fees are charged to the visiting companies.

The former observation reminds us the best toolkit to study such a problem is search theory. The latter requires a sequential bargaining approach. In this framework, if neither bargainer had an alternative trading partner, sequential bargaining would divide equally the profit being bargained over, provided agents were equally patient and the time between offers was short (Bimmore, Osborne, and Rubinstein, 1992). An important strand of literature has considered the interaction between search and bargaining theory<sup>2</sup>. The existence of different potential partners, means that the bargaining process involves that the negotiated division of the profit depends upon the bargainers' search capabilities. An agent's bargaining power is affected by search in two ways<sup>3</sup>, as McMillan and Rothschild (1994) remark. First, the ability to search while negotiating effectively reduces the cost to the agent of delaying agreement. The agent who has the lower cost of delay tends to get better of the bargain, because of his negotiating partner's impatience to settle. Second, the cost of search of a new partner determines the losses from breakdown. The agent who would suffer less in a breakdown is in the better bargaining position.

At this stage of our research we have decided to avoid the complexity of the bargaining between two firms by assuming that only one firm searches and that the "bargaining" consists of the other firm making a take-it-or-leave-it collaboration offer to any potential partner who has found it. This decision is justified by the structure of the Europartenariat meeting. The small firm's decision of collaboration will be studied in an uncertain environment, and reference is to one-sided matching models. Search models have been widely exploited to explain voluntary unemployment in job markets and to give microfoundations to counter-cyclical mark-ups in good markets<sup>4</sup>. The two-sided matching model represents a desirable generalisation we will develop in the future with the purpose

---

<sup>2</sup> Reviews of these theories are in *Handbook of Game Theory* (Vol. 1, 1992 - Vol. 2, 1994). Search theory is presented by McMillan and Rothschild (ch. 27), and sequential bargaining analysis is reviewed by Binmore, Osborne, and Rubinstein (ch. 7). An investigation of the interaction between search and bargaining is that of Wolinsky (1987) based on noncooperative bargaining concepts. Diamond (1981, 1982, 1984), Diamond and Maskin (1979, 1981), and Mortenson (1982) face the same problem with reference to the cooperative Nash bargaining model. See McMillan and Rothschild (1994) for other references.

<sup>3</sup> In equilibrium, an agreement is reached immediately when a buyer and a seller meet because the bargainers foresee these losses from protracted negotiations: there is no time for search during the negotiations. But the implicit, and credible, threat of search shapes the terms of the agreement.

<sup>4</sup>In the former group of models an equilibrium rate of unemployment is obtained and research has been devoted to studying its dynamics. The first formal models are in Phelps (1970). Lucas and Prescott (1974) analyses a search model with a stochastic sectoral shock and a one-period lag by workers in moving between sectors. Diamond (1981, 1982) and Pissarides (1985) studies continuous time general equilibrium models of search. Reviews are given in Mortenson (1986), Sargent (1987), and McKenna (1987). In the latter approach, Stiglitz (1979, 1987) explores the properties of individual firm's demand curves under different hypothesis about the search technology. Stiglitz shows that a convex search technology implies a kinked demand function. Stiglitz considers a model where consumers make decisions by using a sequential search mechanism.

to study the behaviour of any firm interested in a search process of a collaboration in a more general environment. We have already observed that in the interaction with potential partners, to a firm it is important to evaluate the bargaining power conditioned by search abilities. When an agent has high search costs the negotiated division of profits is not good for him, so there is little incentive to search a collaboration. Since search abilities are important in the interaction with potential partners we need to identify these capabilities in terms of costs for small-sized firms. As a result, we can think firms take into account costs and benefits connected with the possible partnership. First of all, small enterprises have few resources to invest in a search process; therefore it is difficult to identify potential partners, and high costs to contact them are necessary. In addition, we can find indirect costs and benefits connected with incentives to exchange private information in a perfect-competition framework..

In economic literature, properly in the field of industrial organisation, we find papers about the exchange of information among firms<sup>5</sup>. The main purpose of this research stream is the identification of conditions that ensures the convenience to make private information common knowledge to the economic system in the hypothesis of different market regimes (perfect competition, Cournot and Bertrand duopoly, etc..). For example, Vergauwen (1994) describes a technological information trading system organised as a barter system where no price can be set<sup>6</sup>. A firm can take advantage in supplying his private information, because it expects positive feed-backs by other firms (not necessarily by direct rivals). An important conclusion is that with the higher competition among firms, the less information is shared. The reason is due to a high strategic cost on information revelation, connected with a high degree of product homogeneity and a high degree of absorption of rivals' private information. Furthermore, no information revelation takes place when the number of competitors in the industry is very large. This family of models can therefore explain in more detail why small firms do not have incentives to offer private information in perfect competition-like markets. In our model, the difficulty to collect information about potential partners as well as the hypothesis of competitive markets, where small firms operate, can motivate the presence of high search costs. It is quite possible that no collaboration is implemented by firms with a consequent loss

---

<sup>5</sup>The origin of these works on information transfers, say Vergauwen (1994), are related to R&D and innovation processes. Pioneering articles are Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985). The central idea is that information is considered an important asset, which is managed by a firm in order to maximise its value and, on the other hand, to restrict rivals' information asset value.

<sup>6</sup>Vergauwen (1994) refers to a general article on this topic by Okuno-Fujiwara *et al.* (1990). Other papers on technology and information transfers cited by Vergauwen (1994) are Shrader (1991), Rogers (1982), Marjit (1990), Vives (1984), and Raith (1993).

of opportunities to improve their economic performance. In such a situation it could be necessary the intermediation of a planner in order to reduce the impact of the cost of search. In addition, with reference to search theory, we need to remember that searchers fail to take into account the benefits their search decisions convey to agents on the other side, and too little research takes place. In other words, there is an externality private agents do not internalise in their private decisions. These ideas can justify the intervention of EU authorities to organise the Europartenariat.

#### **4. The theoretical model**

In this section we consider the European firm's decision to participate in a Europartenariat meeting with reference to search theory. In this framework, if a firm directly contacts other firms with no intervention of EC authorities we can refer to a standard model (i.e. Lancaster and Chesher, 1983), where contacts arrive one at a time and randomly with the addition of very high costs of search. If we want alternatively describe an event like Europartenariat a different search model is needed. In the latter situation, we assume that a firm always prefers to participate in a meeting instead of contacting potential partners by her own, because of more limited costs connected with it. One more observation. Search theory has been constructed in the hypothesis of decentralised markets. A firm who decides to go to Europartenariat meetings does not give the authorisation to a planner to decide for him. Therefore, collaboration always remains an individual decision like the decision to accept a work in job search models.

We can draw a step-by-step parallel between our problem and the standard job search model. An individual looking for a job does not know all the opportunities offered by firms, except for the probability distribution of wage offers. He or she decides to accept or to reject an offer at time  $t$ . If he accepts, he will work forever after; if he does not accept, he will search another offer next period. In our model, there is a firm looking for a collaboration with another enterprise. The firm does not know its optimal partner and obtains information in the Europartenariat program necessary to calculate the probability distribution of the number of contacts and the probability of success in a possible business.

The firm has to choose to participate in the event or not. If it participates, without high direct costs to take contacts, it will have the opportunity of meeting selected firms among which to find a

partner in its business. In addition, actual collaboration prevents a new participation.<sup>7</sup> If it does not participate or if it participates but it does not find a partner, next period it can decide to participate in a new meeting. The scheme of actions is reported in figure 1, where P, NP, C, NC actions stand for participation, no participation, collaboration, no collaboration respectively.

In job models, worker's decision is to accept or reject an offer and, in the case of acceptance, the agreement is achieved with certainty. On the contrary, the firm's choice concerns participation in the Europartenariat and is not directly related to actual collaboration, because the event "no collaboration" is enclosed in the state of possible realisations. Therefore the decision to participate is conditioned by the collaboration event with a probability less than one.

In addition, such a probability is also conditioned by decisions of participation taken in past meetings. The conditional probability of participating to next Europartenariat event, when the "collaboration" event has already occurred, is equal to zero. Otherwise, when the event "no collaboration" occurred, the conditional probability of participation has to be estimated depending on several variables as to firms characteristics, industry performance, etc.

We want to point out that we evaluate the visiting firm's decision which is interested in a collaboration with another firm operating in a less-favoured region; therefore we will not refer to the latter as the main target of our analysis but we will consider the former, that is any European firm.

## 5. The dynamic and stochastic programming problem

### Elements:

1. State space  $S$ : the state of the system  $s \in S$ , where  $s = (w, z)$  and  $S = W \times Z$ , at the start of a given period include exogenous variables relative to current information about firms collected in the catalogue,  $x$ , and the (relative) number of possible contacts at the event,  $n$ , with vector  $z = (n, x)$  and endogenous variables as to the profit earned in the previous period  $\pi^8$ , where

$$p = \begin{cases} 0 & \text{if } P \cup NC \text{ or } NP \\ p^* & \text{if } P \cup C \end{cases}$$

---

<sup>7</sup>The evaluation of the effects of past collaboration on the participation to future meetings represents an interesting development of our model. This is an objective of future works.

<sup>8</sup> For notational convenience, we will use the following symbols to indicate events: P "participation", NP "no participation, C "collaboration", and NC "no collaboration".



with  $\pi^*$  being the single period profit which is invariant across time<sup>9</sup>. So we define the vector of endogenous variables as  $w = \pi$ .

2. A stochastic mechanism of motion of the exogenous variables (Markov process). The number of contacts  $n$  and local firms' characteristics  $x$  are martingale variables.
3. Probability of the number of contacts: the probability to find  $n$  contacts given  $N$  host companies is binomial

$$n \sim P(n) = \binom{N}{n} \tilde{p}^n (1 - \tilde{p})^{N-n}$$

where  $\tilde{p}$  is the probability to have one contact.

4. Probability of collaboration: we assume that the firm accepts the first collaboration emerging in an Europartenariat meeting, given the number  $n$  of possible contacts. This hypothesis is justified because the firm has neither costs nor advantages in further contacts with host companies in the same meeting. We define the probability of success in a collaboration given  $i$  direct contacts and  $n$  host companies at Europartenariat the firm has already decided to contact as

$$p(C) \equiv \text{prob}(C, i \leq n) = \sum_{i=0}^{n-1} (1-p)^i p = p \frac{1 - (1-p)^n}{1 - (1-p)} = 1 - (1-p)^n$$

where  $p$  is the probability of success in a collaboration with one firm. We assume  $p$  is invariant across time. We justify this hypothesis as the best estimate of the probability given information at time  $t$ . We are avoiding learning phenomena<sup>10</sup>. We remark the hypothesis that  $n$  does not affect the single period profit  $\pi^*$  but only the probability of success.

5. Total cost function: we consider the following components of  $K(n)$  a) *participation cost*:  $k$  fixed; b) *private information revelation cost*: quadratic function  $f(n) = wn^2$ ; c) *learning opportunity cost*: quadratic function  $g(n) = \gamma n^2$ . The b) component is related to the fact that a firm has no convenience to supply its private information in all situations, because it can expect negative feed-backs from other firms (not necessarily from direct rivals). An important result on information-exchange theory is that with the higher competition among firms, the less information is shared. The reason is due to a high strategic cost on information revelation, connected with a high degree of product homogeneity and a high degree of absorption of rivals' private information. Furthermore, no information revelation takes place when the number of competitors in the

---

<sup>9</sup> In terms of rational expectations,  $\pi^*$  is the best estimate of the single period profit, given information at time 0.

industry is too large. However, the opportunity to meet an increasingly number of potential collaborators can shift downward the cost curve for learning effects - *c*) component.

6. A choice correspondence:  $\Gamma(w, z)$ , with  $\Gamma: W \times Z \rightarrow Y$  is a set of feasible choices when the state which is inherited is  $(w, z)$  and  $y$  is the firm's choice to participate or not

$$y_t = \Gamma(w_t, z_t) = \begin{cases} \mathbf{p}^* & \text{if } P \cap C & \text{at } (t-1) \\ H(z_t) \cup D & \text{if } P \cap NC \text{ or } NP & \text{at } (t-1) \end{cases}$$

where

$D$ : choice set if the firm's decision at  $t$  is NP

$H(z_t)$ : choice set if the firm's decision at  $t$  is P

7. The single period return function:  $F(w_t, y_t, z_t)$ ,  $F: A \rightarrow R$ , where  $A$  is the graph of  $\Gamma$ ,  $A \equiv \{(w_t, y_t, z_t) \mid y_t \in \Gamma(w_t, z_t)\}$ , and  $F$  is given by the following expression:

$$F(w_t, y_t, z_t) = \begin{cases} (\mathbf{p} - K(n)) p(C) & \text{if } P \cap C \text{ at } t \\ -K(n)[1 - p(C)] & \text{if } P \cap NC \text{ at } t \\ 0 & \text{if } NP \text{ at } t \end{cases}$$

where  $K(n)$  is the total participation cost at the Europartenariat event. Our problem is to establish when it is convenient to participate in a meeting or not with reference to a possible collaboration. We therefore need to formulate the single period pay-off defined as a function of C and NC events in a different way, in terms of "participation" and "no participation" events. Given the expected profit conditioned to the event C/NC, we determine the unconditional expected profit as to the "participation" event. We have already defined the correspondent expected profit for the NP event. If we refer to the event P and NP, we have respectively:

$P \cap C \cup P \cap NC$  and  $\pi = \pi^*$  for the event P

NP and  $\pi = 0$  for the event NP

so the single period return function in terms of these events is:

$$F(w_t, y_t, z_t) = \begin{cases} \mathbf{p}^* p(C) - K(n) & \text{if } P \\ 0 & \text{if } NP \end{cases}$$

8. Law of motion of endogenous state variables:  $\phi: Y \times Z \rightarrow W$

$$w_{t+1} = \phi(y_t, z_t) = y_t$$

9. Discount factor  $\beta \in (0, 1)$ , where  $\beta \equiv 1/(1+r)$ ,  $0 \leq r \leq 1$ , and initial state  $s_0 = (w_0, z_0)$  where  $w_0$  entails the hypothesis of "no participation" before time 0.

---

<sup>10</sup> This is a simplifying hypothesis we would like to generalise in the future, with the purpose to study the effects in participating to several meetings. Nevertheless, we introduce a cost function (element 5) where learning benefits are

## 6. Bellman equation: fixed costs

For the construction of the Bellman equation we need to remember all hypothesis we have described in section 4. We assume that acceptance of a collaboration prevents a new participation. When the collaboration is realised, the single-period profit  $\pi^*$  is repeated for all remaining periods. If participation to the Europartenariat event is rejected, the firm will wait until next period when it decides to participate or not in a new meeting. This waiting makes sense because the small firm has no possibility to realise a collaboration outside the Europartenariat meeting, given the difficulty to gather information at low cost by other channels.

In a model with two periods, the dynamic programming problem is very simple. We consider the firm's decision backward in order to construct the Bellman equation in terms of a value function. In the second period, we identify an optimal strategy of participation as follows: if in the first period the firm has already established a relation he gets profits so there is no choice to be made; if it has not participated or there has been no relation established in the first period, next period it decides to participate in a new meeting if the expected profit from a collaboration, weighted by the probability of success in creating it, is larger than participation costs. Value function  $V$  must satisfy the following Bellman equation:

$$V_1 = \mathbf{b}V_2 + \max \left\{ 0, \left[ \frac{1 - \mathbf{b}^2}{1 - \mathbf{b}} \mathbf{p}^* - \mathbf{b}V_2 \right] p(C) - k \right\} \quad (1)$$

In a more general model with an infinite horizon, the Bellman equation is as follows

$$V(s_t) = \mathbf{b}V(s_{t+1}) + \max \left\{ 0, \left[ \frac{\mathbf{p}^*}{1 - \mathbf{b}} - \mathbf{b}V(s_{t+1}) \right] p(C) - k \right\} \quad (2)$$

where  $\beta = 1/(1+r)$ ,  $1 - \beta \cong r$ , and  $0 \leq r \leq 1$ .

Throughout this paper we will focus on the infinite horizon model, for which we refer to equation (2) in order to determine the optimal strategy<sup>11</sup>. A firm decides to participate or not in an Europartenariat event if available information suggests a high probability to succeed in a collaboration such that benefits from participating are greater than related costs. In job search models the strategy of solution is to find a reservation wage defined as the wage level that equals

---

considered.

benefits and opportunity costs. We can identify an optimal threshold number of contacts with host firms interested in the same objectives: an European firm is indifferent between “participation” and “no-participation” events. Specifically, we are interested in an optimal policy in terms of the number of contacts sufficient to prefer to participate that is invariant over time<sup>12</sup>.

Let  $\tilde{n}$  be the optimal threshold number of contacts to participate;  $\tilde{n}$  is such that the expected profit from participating is equal to the expected profit in case of no participation, that is

$$\left[ \frac{\mathbf{p}^*}{1-\mathbf{b}} - \mathbf{b}V(s_{t+1}) \right] p(C) - k = 0 \quad (3)$$

where  $p(C) = 1 - [1-p]^n$ .

While in job search models, we find a reservation wage  $w^*$  such that any offer greater than  $w^*$  ensures the worker to become employed with certainty, in our model  $\tilde{n}$  does not guarantee the success to realise a collaboration, therefore there is one more stage in the decision process to participate. So  $\tilde{n}$  defines the advantage to participate or not, based on a positive (but less than 1) probability of the “no collaboration” event.

More precisely, it is possible to show that  $V$  is the supremum function in equation (2) that exists and is a bounded continuous function.

**HYPOTHESIS 1:** We assume that  $\frac{\mathbf{p}^*}{1-\mathbf{b}} - k > A$ .

This assumption states that a firm takes advantage from participating when a collaboration is implemented with certainty.

**PROPOSITION 1:** *Value function  $V(s_t)$  (equation (2)) is an increasing function of  $n$ , whose form is depicted in Fig. 2, and a reservation number of contacts is the optimal strategy.*

**Proof:** Value function (2) is obtained by the combination of two different functions. We study them separately. The first function  $V(s_t) = A$  is constant with respect to  $n$ . The second function

$V(s_t) = A + \left( \frac{\mathbf{p}^*}{1-\mathbf{b}} - A \right) [1 - (1-p)^n] - k$  is increasing with respect to  $n$  ( $dV(s_t)/dn > 0$ ); in addition

$V(0) = A - k$  and  $\lim_{n \rightarrow \infty} V(s_t) = \frac{\mathbf{p}^*}{1-\mathbf{b}} - k > A$ . The two functions intersect once. In this point of

---

<sup>11</sup>To solve this dynamic programming problem we referred to Stokey and Lucas (1989).

<sup>12</sup> Lancaster and Chesher (1983) offer a detailed discussion about the choice of a stationary solution instead of a time-dependent one.

intersection the number of contacts is defined as  $\tilde{n}$ . For  $n < \tilde{n}$ , the first function is the maximum so a strategy of no participation is chosen. For  $n \geq \tilde{n}$ , the second function takes higher values than the first function so a firm decides to participate. In addition, since the two functions are bounded, the value function is bounded.  $\square$

We can find  $V$  as a solution to Bellman equation (2) more sharply if we exploit its particular form. Imposing the identity  $A \equiv \mathbf{b}V(\cdot)$  and given the reservation number of contacts  $\tilde{n}$  as an optimal strategy the value function is

$$V(s_i) = A + \begin{cases} 0 & \text{if } n < \tilde{n} \\ \left[ \frac{\mathbf{p}^*}{1 - \mathbf{b}} - A \right] p(C) - k & \text{if } n \geq \tilde{n} \end{cases} \quad (4)$$

Remarking that the number of contacts is not certain, and specifically it is distributed as a binomial, we determine the value of the parameter  $A$  such that  $V$  be a solution to the Bellman equation

$$A = \mathbf{b} \sum_{i=\tilde{n}}^N \frac{1}{1 - \mathbf{b}} \left\{ \left[ \frac{\mathbf{p}^*}{1 - \mathbf{b}} - A \right] p(C) - k \right\} \binom{N}{i} \tilde{p}^i (1 - \tilde{p})^{N-i} \quad (5)$$

We substitute the expression of  $p(C)$  into equation (5). We recall that we are assuming that both revelation costs and learning effects are null. Equation (5) has two unknowns. So we need one more condition to solve for  $A$  and  $\tilde{n}$ . Given that  $\tilde{n}$  is obtained by a condition of indifference between a decision of “participation” and one of “no participation”, we can exploit it as reported in equation (3). Thus,  $A$  and  $\tilde{n}$  are obtained solving the system of equations (3) and (5). By substitution, we obtain an equation with a unique unknown ( $\tilde{n}$ ). More specifically, the optimal  $\tilde{n}$  is the solution of equation

$$\frac{\mathbf{p}^*}{\mathbf{b}k} = \frac{1}{1 - (1 - p)^{\tilde{n}}} \left\{ \frac{1 - \mathbf{b}}{\mathbf{b}} + \sum_{i=\tilde{n}}^N \binom{N}{i} \tilde{p}^i (1 - \tilde{p})^{N-i} \left[ (1 - p)^{\tilde{n}} - (1 - p)^i \right] \right\} \quad (6)$$

where  $\pi^*$ ,  $\beta$ ,  $p$ ,  $\tilde{p}$ ,  $N$ ,  $k$ ,  $w$ ,  $\gamma$  are all parameters we have already introduced in section 5.

Equation (6) can be studied by simulation in order to give qualitative results about the solution. As a first step, we have calculated the right-hand-side (r.h.s.) of equation (6) based on the following values of parameters:  $N = 100$ ,  $r = 2\%$ , so  $\beta = 0.9804$ ,  $p = 0.1 - 0.5 - 0.9$  (probability of success in a collaboration with one firm),  $\tilde{p} = 0.1 - 0.5 - 0.9$  (probability to have one contact). The simulation confirms an intuitive negative correlation between the optimal threshold number of contacts  $\tilde{n}$  and the participation cost  $k$ , and between the threshold number of contacts  $\tilde{n}$  and the single-period profit  $\pi^*$ . If the fixed cost  $k$  increases, it is necessary a greater expected benefit, by meeting a higher

number of firms, to balance this increase. So the probability to find a partner increases, for a given  $\pi^*$ . If the single period profit  $\pi^*$  increases, for a given cost of participation  $k$ , the expected benefit increases, therefore to the firm it is sufficient to meet a smaller number of agents. Furthermore, as  $p$  increases  $\tilde{n}$  declines for a given value of l.h.s. of equation (6). If  $r$  increases, as the European firm is less interested in future profits, i.e.  $\beta$  lowers, the minimum number of contacts  $\tilde{n}$  to decide to participate is higher. Figure 3 shows our results for a given set of parameters  $p = 0.013$  (frequency of contacts at 1992 Europartenariat event),  $p = 0.1 - 0.5 - 0.9$ , and a discount rate  $r = 0.02$ . The shape is always the same for all cases considered. The effects of the parameters is only on levels.

## 7. Bellman equation: quadratic costs

If we consider quadratic revelation costs the expression for  $K(n)$  is the following

$$K(n) = k + (w - g)n^2$$

and the corresponding Bellman equation in an infinite horizon is

$$V(s_t) = bV(s_{t+1}) + \max\left\{0, \left[\frac{p^*}{1-b} - bV(s_{t+1})\right]p(C) - K(n)\right\} \quad (7)$$

In this framework a reservation number of contacts is not the optimal strategy anymore. We can show that the optimal decision is to participate if  $n \in [n_1, n_2]$  where  $n_1$  is the minimum number of contacts and  $n_2$  is the maximum number of contacts for which benefits from participation are greater than costs.

**PROPOSITION 2:** *Value function  $V(s_t)$  (equation (7)) is characterised by the diagram depicted in Fig. 4, and a decision of participation is optimal if the number of contact  $n \in [n_1, n_2]$ .*

**Proof:** Value function (7) is obtained by the combination of two different functions as we have already seen for the case of fixed costs. The first function  $V(s_t) = A$  is constant with respect to  $n$ . The second function  $V(s_t) = A + \left(\frac{p^*}{1-b} - A\right)[1 - (1-p)^n] - K(n)$  is a concave function:  $dV(s_t)/dn > 0$  for small levels of  $n$ , and  $dV(s_t)/dn < 0$  when  $n$  is large  $\left(\lim_{n \rightarrow \infty} V(s_t) = -\infty\right)$ . In addition  $V(0) = A - k$ . The two functions intersect twice. The two points of intersection identify two values of  $n$  we call  $n_1$  and  $n_2$ , with  $n_1 < n_2$  for notation convenience. For  $n < n_1$  and  $n > n_2$  the constant function is chosen,

so a strategy of no participation is optimal. If  $n_1 \leq n \leq n_2$  the second function takes higher values so participation is an optimal strategy.  $\checkmark$

As we have previously shown for the fixed cost case, the value function is still a bounded and continuous function when we assume quadratic costs. Imposing the optimal strategy  $n \in [n_1, n_2]$  the value function can be written as follows

$$V(s_t) = A + \begin{cases} 0 & \text{if } n < n_1 \\ \left( \frac{P^*}{1-b} - A \right) [1 - (1-p)^n] - K(n) & \text{if } n_1 \leq n \leq n_2 \\ 0 & \text{if } n > n_2 \end{cases} \quad (8)$$

Given that  $A \equiv \beta V(s_{t+1})$ , we can write a more explicit condition:

$$A \frac{1-b}{b} = \sum_{i=n_1}^{n_2} \binom{N}{i} \bar{p}^i (1-\bar{p})^{N-i} \left[ \left( \frac{P^*}{1-b} - A \right) [1 - (1-p)^n] - \sum_{i=n_1}^{n_2} \binom{N}{i} \bar{p}^i (1-\bar{p})^{N-i} K(n) \right] \quad (9)$$

We can find the values of  $n_1$  and  $n_2$  by solving a system of three equations. The first one is equation (9). Second equation and third equation are similar to condition (3) presented for the fixed cost case:

$$\left[ \frac{P^*}{1-b} - \beta V(s_{t+1}) \right] [1 - (1-p)^{n_i}] - K(n_i) = 0 \quad i = 1, 2 \quad (10)$$

Comparing the two indifference conditions we can get the following equation:

$$\frac{1 - (1-p)^{n_1}}{1 - (1-p)^{n_2}} = \frac{K(n_1)}{K(n_2)} \quad (11)$$

Given equations (9) and (11) we can determine  $n_1$  and  $n_2$  as functions of exogenous parameters  $\pi^*, \beta, p, \bar{p}, N, k, w, \gamma$ .

## 8. Concluding remarks

In this paper the European firm's decision to participate in an Europartenariat event by a search model is studied. We propose a simplified bargaining model where it is possible to search the potential partner among different alternatives. In the interaction of bargaining with a search problem, search costs play a central role. They come from (i) difficulty in collecting information; (ii) revelation and learning of private information, with reference to small firms.

We consider the behaviour of a representative visiting firm interested in a collaboration with host companies by an empirical dynamic programming model in which the optimal policy can be calculated and interpreted. The firm's choice concerns participation to the Europarteneriat related to a distribution of the number of contacts. An interesting element introduced in our model is the presence of uncertainty in the realisation of a collaboration. The associated probability of collaboration is also conditioned by decisions of participation taken in past meetings. We derive the Bellman equation in both a fixed cost hypothesis and a quadratic cost case; the optimal strategy with reference to the number of contacts are then derived in terms of an optimal threshold value and a two-sided rule respectively. For the fixed cost case, by simulation we find that the minimum number of contacts for which the firm accepts to participate is negatively related to the expected profit and to the probability of contacting one firm with a common project of collaboration.

We want to remember that in our model the solutions are obtained under two basic assumptions: first, acceptance of a collaboration prevents a new participation; second, when a firm find a partner, a take-it-or-leave-it offer is made by this potential partner. An implicit hypothesis we have when considering a take-it-or-leave-it offer is that the potential collaborator has the ability to make commitments. Otherwise the offer is not subgame perfect; if the first firm rejects the offer, it is not in the partner's interest to refuse to bargain. It would be then necessary to reformulate the problem in a more general way by mixing an explicit bargaining process with search. This will be the object of our future research. In addition, it will be interesting to evaluate the effects of past collaborations on the participation to future meetings.

## References

- Binmore K., M.J. Osborne, and A. Rubinstein (1992). 'Noncooperative Models of Bargaining' in R.J. Aumann and S. Hart (eds.), *Handbook of Game Theory*, Volume 1, Elsevier Science, Amsterdam, ch. 7.
- Blanchard, O.J. and S. Fischer (1989). *Lectures on Macroeconomics*, MIT Press, Cambridge MA.
- Christensen, B.J. and N.M. Kiefer (1991). 'The Exact Likelihood Function for an Empirical Job' *Econometric Theory*, **7**, 464-486.
- Diamond, P.A. (1981). 'Mobility Costs, Frictional Unemployment, and Efficiency', *Journal of Political Economy*, **89**, 798-812.



- Diamond, P.A. (1982). 'Wage Determination and Efficiency in Search Equilibrium', *Review of Economic Studies*, **49**, 217-227.
- Diamond, P.A. (1984). *A Search Theoretic Approach to the Micro Foundations of Macroeconomics*, MIT Press, Cambridge MA.
- Diamond, P.A. and E. Maskin (1979). 'An equilibrium analysis fo search and breach of contract I: Steady states', *Bell Journal of Economics*, **10**, 282-316.
- Diamond, P.A. and E. Maskin (1981). 'An equilibrium analysis fo search and breach of contract II: A non-steady state example', *Journal of Economic Theory*, **25**, 165-195.
- Eckstein, Z.E. and K.I. Wolpin (1990). 'Estimating a Market Equilibrium Search Model from Panel Data', *Econometrica*, **58**, 783-808.
- Gal-Or, E. (1985). 'Information Sharing in Oligopoly', *Econometrica*, **53**, 329-343.
- Howitt, P. (1988). 'Business Cycles with Costly Search and Recruiting', *Quarterly Journal of Economics*, **103**, 147-166.
- Lancaster, T. and A. Chesher (1983). 'An Econometric Analysis of Reservation Wages', *Econometrica*, **51**, 1661-1676.
- Lucas, R.E. and E. Prescott (1974). 'Equilibrium Search and Unemployment', *Journal of Economic Theory*, **7**, 188-209.
- McMillan, J. and M. Rothschild (1994). 'Search' in R.J. Aumann and S. Hart (eds.), *Handbook of Game Theory*, Volume 2, Elsevier Science, ..., ch. 27.
- Marjit, S. (1990). 'On A Non-Cooperative Theory of Technology Transfer', *Economic Letters*, **33**, 293-298.
- McKenna, C.J. (1987). 'Theories of Individual Search Behavior', in J.D. Hey and P.J. Lambert (eds.), *Surveys in the Economic of Uncertainty*, Blackwell, Oxford, 91-109.
- Mortenson, Dale (1982). 'The Matching Process as a noncooperative bargaining game', in J.J. McCall (ed.), *The Economics of Information and Uncertainty*, University of Chicago Press, Chicago.
- Mortenson, Dale (1986). 'Job Search and Labor Market Analysis', in Orley Ashenfelter and Richard Layard (eds.), *Handbook of Labor Economics, Volume 2*, North-Holland, Amsterdam, 849-920.
- Novshek, W. and H. Sonnenschein (1982). 'Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release', *Bell Journal of Economics*, **13**, 214-218.

- Okuno-Fujiwara, M. A. Postlewaite and K. Suzumura (1990). 'Strategic Information Revelation', *Review of Economic Studies*, **57**, 25-47.
- Phelps, E. (1970). *Microeconomic Foundations of Employment and Inflation Theory*, Norton, New York.
- Pissarides, C. (1985). 'Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real', *American Economic Review*, **75**, 676-690.
- Raith, M.A. (1993). 'A General Model of Information Sharing in Oligopoly', Working Paper Theoretical Economics, TE/93/260, STICERD, London School of Economics.
- Sargent, T. (1987). *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, MA.
- Shrader, S. (1991). 'Informal Technology Transfer Between Firms: Cooperation Through', *Research Policy*, **20**, 153-170.
- Stiglitz, J.E. (1979). 'Equilibrium in Product Markets with Imperfect Information', *American Economic Review*, **69**, 339-345.
- Stiglitz, J.E. (1987). 'Competition and the Number of Firms in a Market: Are Duopolies More Competitive than Atomistic Markets?', *Journal of Political Economy*, **95**, 1041-1061.
- Rogers, E.M. (1982). 'Information Exchange and Technological Innovation', in Devendra S. (ed.) *The Transfer and Utilization of Technical Knowledge*, Lexington Books, Lexington, MA.
- Stokey, N.L. and R.E. Lucas (1989). *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, MA.
- Vergauwen, P. (1994). 'Strategic Information Sharing: Exchanging Pieces of a Jigsaw Puzzle', University of Louvain, mimeo.
- Vives, X. (1984). 'Duopoly Information Equilibrium: Cournot and Bertrand', *Journal of Economic Theory*, **34**, 71-94.
- Wolinsky, A. (1987). 'Matching, Search, and Bargaining', *Journal of Economic Theory*, **32**, 311-333.