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**Working Paper**

## On the Regulation of A Vertically Differentiated Market Through Taxation/Subsidization of Product Quality

Quaderni - Working Paper DSE, No. 247

**Provided in Cooperation with:**

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*Suggested Citation:* Lambertini, Luca; Mosca, Manuela (1996) : On the Regulation of A Vertically Differentiated Market Through Taxation/Subsidization of Product Quality, Quaderni - Working Paper DSE, No. 247, Alma Mater Studiorum - Università di Bologna, Dipartimento di Scienze Economiche (DSE), Bologna, <https://doi.org/10.6092/unibo/amsacta/5063>

This Version is available at:

<https://hdl.handle.net/10419/159090>

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# On the Regulation of a Vertically Differentiated Market<sup>1</sup>

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November 10, 1998

<sup>1</sup>This paper was written while the first author was at the Institute of Economics, University of Copenhagen. A previous version was part of the first author's DPhil thesis at Linacre College, Oxford. We thank Svend Albæk, Norman Ireland, Paul Klemperer and Martin Slater for helpful discussion and suggestions on topics treated in this paper. The responsibility obviously rests with us only.

## Abstract

The aim of this paper is to investigate a vertically differentiated market served either by a multiproduct monopolist or by duopolists, in which a public authority aiming at increasing the welfare level can choose among two instruments, namely, quality taxation/subsidization, and minimum quality standard. In the monopoly case they are equivalent as to the social welfare level, in that both allow the regulator to achieve the second best level of social welfare he would attain if he were to set qualities under the monopoly pricing rule, while they are not equivalent in terms of the distribution of surplus. In the duopoly regime, we show that there exists a taxation/subsidization scheme inducing firms to produce the socially optimal qualities.

JEL classification: H21, L12, L13

Keywords: vertical product differentiation, taxation/subsidization, minimum quality standard

# 1 Introduction

The welfare analysis of taxation or subsidization in imperfectly competitive markets stems from the convergence of two originally different streams. The first deals with the definition of social welfare. The second derives from the notion of market failure and the consequent envisaging of a public intervention to correct it. The earliest analytical treatment of these issues dates back to Dupuit (1844, 1849), who claims that a monopolist, independently of her property structure, should price-discriminate among consumers (see also Ekelund, 1970), although the private entrepreneur would entirely extract consumer surplus, while the social planner would price proportionally to individual reservation prices, so as to break even (Dupuit, 1854). The issue of optimal pricing appears in the English literature with Ramsey (1927) who analyses a tariff schedule minimizing the distortion affecting demand, under the constraint that firms' profits be non-negative.<sup>1</sup> In Hotelling's (1938) view, a public firm should price at marginal cost and finance any loss through taxation. Questioning the applicability of marginal cost pricing, Allais (1947, p. 220) proposes the adoption of tariffs departing proportionally from marginal costs. Then, Boiteaux (1956) adopts an approach to optimal pricing that completely dispenses with the concept of consumer surplus.

The literature in this field has largely disregarded the interaction between firms' pricing behaviour and their ability to differentiate products, although Dupuit (1849) stresses that a monopolist supplying several varieties of the same good or service may offer suboptimal qualities to low-income consumers in order to force richer ones to pay higher prices for superior qualities. This issue has received a first characterization by Spence (1975), who highlights that a profit-maximizing monopolist will distort the output for a given quality level, and vice versa. In the same vein, Mussa and Rosen (1978) show that, under full market coverage, the extent to which a multiproduct monopolist undersupplies qualities increases as the income of consumers who purchase such qualities decreases, so that, as the number of varieties tends to infinity, only top-income consumers are being provided with the same quality as under social planning. An inductive proof of this is in Lambertini (1997a), where it is also shown that the quality distortion vanishes under partial market coverage. Several measures have been envisaged to regulate the behaviour

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<sup>1</sup>For a recent exposition of this topic, see Baumol and Bradford (1970). Later developments are discussed by Yang and Stitt (1995), *inter alia*.

of a vertically differentiated multiproduct monopolist. Spence (1975) shows that rate of return regulation (RORR) can be attractive if quality is capital-intensive. Besanko et al. (1987, 1988) compare such a policy with minimum quality standard (MQS) and maximum price regulation (MPR). They find that, under price ceiling, the monopolist increases the distortion at the high end of the market, and conversely at the low end, and welfare increases if price regulation is slight enough. As to the MQS policy, they claim that its welfare effects are ambiguous because it might exclude some consumers from the market; moreover, those consumers for whom the standard is not binding receive the same quality they would purchase in the absence of MQS. A growing amount of effort is being devoted to investigating the design of MQSs (Ronnen, 1991; Motta and Thisse, 1993; Crampes and Hollander, 1995; Ecchia and Lambertini, 1997; Scarpa, 1998). Some general results are that (i) all qualities increase as a reaction to the MQS; (ii) the MQS may increase the profit of the low-quality firm; (iii) welfare is higher after the introduction of the MQS, provided the latter does not induce any firm to exit. MQS's main shortcoming is the asymmetry introduced in the quality stage, so that the resulting misallocation of demand prevents the regulator from attaining the first best. To the best of our knowledge, the only existing study of taxation in a vertically differentiated duopoly is due to Cremer and Thisse (1994), who consider the impact of an ad valorem tax, showing that a uniform ad valorem tax, with the same rate applying to both products, lowers both equilibrium qualities and distorts the allocation of consumers between firms. If the tax rate is sufficiently low and its proceeds are redistributed as a lump-sum transfer, it turns out to be always welfare-improving, although unable to attain the first best. The adoption of a non-uniform tax scheme may or may not appear socially desirable.

In the real world, a regulator may find it profitable to use a mix of several instruments, but our aim is to investigate the impact of regulative measures directly affecting quality levels in a multiproduct monopoly, in isolation from any other policy instruments, such as RORR, MPR or ad valorem and excise taxation. The policy we consider are related to the hedonic characteristics of the goods under consideration, and can take the form of either a taxation/subsidization scheme, or an MQS. Examples of both policies can be found in the automobile industry, with (i) tax schedules related to fiscal horsepower rather than the actual engine horsepower; and (ii) the widespread compulsory adoption of airbags and safety belts, while examples of the former are penalties introduced to limit the production of polluting components

in the chemical industry, where quality standards have also been adopted in order to preserve the environment. We show that the effectiveness of these instruments drastically depends upon market structure. In a duopoly regime with single-product firms, quality taxation/subsidization achieve the social optimum, while an MQS policy is unable to do so, due to its inherent asymmetry in affecting product qualities. In a multiproduct monopoly regime, we prove that, when the quality range is discrete while consumer distribution is continuous, the adoption of an MQS affects the entire range. This contrasts with previous contributions in this field (Besanko et al., 1987, 1988). Moreover, we show that, unlike what happens in oligopolistic settings, the two policies are equivalent in a monopolistic market, being both able to achieve the second best level of social welfare that a public authority would attain by choosing qualities under the monopoly pricing rule, the social optimum remaining out of reach.

The remainder of the paper is structured as follows. The basic model is described in section 2. Section 3 deals with social planning and the unregulated monopoly and duopoly regimes. The optimal design of regulation policies is investigated in section 4, and finally section 5 provides some concluding comments.

## 2 The Setting

We adopt the model of vertical differentiation due to Cremer and Thisse (1994), which is a two-product specification of Mussa and Rosen's (1978). The model is worked out under the hypothesis of complete information. In particular, we assume that quality levels are common knowledge, i.e., they are immediately observable by the regulator.<sup>2</sup> Two vertically differentiated goods are supplied, whose respective quality levels are denoted by  $q_H > q_L > 0$ : Both varieties are being produced through the same technology, represented by the following cost function:

$$C_i = tq_i^2x_i; \quad t > 0; \quad i = H; L; \quad (1)$$

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<sup>2</sup>This amounts to assuming that technology is common knowledge. A wide literature deals with the case where there exists an information asymmetry between producers and the public authority concerning the cost structure (and thus product quality). See Lafont and Tirole (1989a,b; 1993).

where  $x_i$  denotes firm  $i$ 's output level. Consumers are uniformly distributed over the interval  $[\underline{\mu}; \bar{\mu}]$ ; with  $\underline{\mu} = \bar{\mu} - 1$ . Their total density can be normalised to one without loss of generality. Parameter  $\mu$  represents each consumer's marginal willingness to pay for quality. The market is fully covered, and each consumer buys one unit of the variety  $i$  which maximizes the following indirect utility function:

$$U = \mu q_i - p_i \geq 0 \quad (2)$$

Given production costs (1), the maximization of consumer surplus (2) would imply that the quality preferred by a generic consumer indexed by a marginal willingness to pay  $\mu$  would be  $\mu = 2t$ , so that the range of preferred qualities is defined by  $[(\bar{\mu} - 1) = 2t; \bar{\mu} = 2t]$  (see Cremer and Thisse, 1994, p. 617). Consumers can be divided into two groups: those buying the high-quality good, and those buying the low-quality good, the indifferent consumer being at  $\beta = (p_H - p_L) / (q_H - q_L)$ . Hence the demands for the two commodities are, respectively:

$$x_H = \bar{\mu} - \beta = \bar{\mu} - \frac{(p_H - p_L)}{q_H - q_L} \quad \text{iff} \quad 0 < \bar{\mu} - \beta < 1; \quad (3)$$

$$x_H = 1 \quad \text{iff} \quad \bar{\mu} - \beta \geq 1; \quad x_H = 0 \quad \text{iff} \quad \bar{\mu} - \beta \leq 0; \\ x_L = \beta - (\bar{\mu} - 1) \quad \text{iff} \quad 0 < \beta - (\bar{\mu} - 1) < 1; \quad (4)$$

$$x_L = 1 \quad \text{iff} \quad \beta - (\bar{\mu} - 1) \geq 1; \quad x_L = 0 \quad \text{iff} \quad \beta - (\bar{\mu} - 1) \leq 0;$$

### 3 Social optimum and the unregulated equilibria

Under full market coverage, pricing above marginal cost only activates a mechanism to redistribute surplus from consumers to producers, without affecting the overall level of welfare. Hence, provided that producers are among consumers, a social planner can neglect any redistribution issue. Then, the problem of welfare maximization consists in finding the quality pair that maximizes the following social welfare function, defined as the sum of consumer and producer surplus:

$$SW = \int_{\underline{\mu}}^{\beta} (\mu q_L - tq_L^2) d\mu + \int_{\beta}^{\bar{\mu}} (\mu q_H - tq_H^2) d\mu \quad (5)$$

By solving the system of first derivatives of (5) w.r.t.  $q_H$  and  $q_L$ , it can be verified that the social planner "locates" qualities in the first and third quartiles of the interval of consumers' preferred varieties, setting  $q_H^{sp} = (4\bar{\mu} + 1)/(8t)$  and  $q_L^{sp} = (4\bar{\mu} - 3)/(8t)$ : The level of welfare at the social optimum is  $SW^{sp} = (16\bar{\mu}^2 + 16\bar{\mu} + 5)/(64t)$ : Equilibrium demands are  $x_H^{sp} = x_L^{sp} = 1/2$ :

For future reference, we briefly illustrate the unregulated duopoly and monopoly equilibria. We borrow the former from Cremer and Thisse (1994) and Lambertini (1996). Firms' objective functions are  $\pi_H = (p_H - tq_H^2)x_H$  and  $\pi_L = (p_L - tq_L^2)x_L$ , respectively. They play a non-cooperative two-stage game in qualities and prices, whose solution yields  $q_H^d = (4\bar{\mu} + 1)/(8t)$ ;  $q_L^d = (4\bar{\mu} - 5)/(8t)$ ; and  $p_H^d = 49/(64t)$ ;  $p_L^d = 25/(64t)$ : Notice that equilibrium qualities are positioned well outside the socially preferred interval, i.e., duopoly is characterized by excess differentiation at equilibrium. Demand is equally split at equilibrium, with  $x_H = x_L = 1/2$ : Social welfare under duopoly is  $SW^d = (16\bar{\mu}^2 + 16\bar{\mu} + 1)/(64t)$ : The welfare loss, as compared to the social optimum, is due to the strategic effect leading firms to seek reciprocal differentiation in order to soften price competition (Shaked and Sutton, 1982; 1983).<sup>3</sup> The constraint  $\bar{\mu} > 9/4$  must be satisfied in order for the poorest consumer located at  $\underline{\mu}$  to be able to buy in equilibrium.

Consider now an unregulated profit-seeking monopolist offering both varieties (see Lambertini, 1997a). She maximizes  $\pi^m = (p_H - tq_H^2)x_H + (p_L - tq_L^2)x_L$  w.r.t. prices and qualities. Provided that the poorest consumer must be able to buy the low-quality good, the optimal monopoly price for that variety is  $p_L^m = (\bar{\mu} - 1)q_L$ , at which the consumer whose marginal willingness to pay is  $\underline{\mu} = \bar{\mu} - 1$  is exactly indifferent between purchasing the low-quality good and not purchasing at all. The optimal price for the high-quality good obtains from the first order condition (FOC):

$$\frac{\partial \pi^m}{\partial p_H} = \frac{\bar{\mu}q_H + \bar{\mu}q_L - 2p_H - 2q_L + tq_H^2 - tq_L^2}{q_H - q_L} = 0; \quad (6)$$

yielding  $p_H^m = (\bar{\mu}q_H + \bar{\mu}q_L - 2q_L + tq_H^2 - tq_L^2)/2$ : Then, differentiating  $\pi^m$  w.r.t.

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<sup>3</sup>This is analogous to a well known result characterizing the Hotelling duopoly model with quadratic transportation costs. Cremer and Thisse (1991, p. 386) establish its formal equivalence to the vertically differentiated duopoly with quadratic variable costs of quality improvements employed here.



qualities, we obtain the following FOCs:

$$\frac{\partial \pi^m}{\partial q_H} = \frac{(\bar{\mu} - tq_H - tq_L)(\bar{\mu} - 3tq_H + tq_L)}{4} = 0; \quad (7)$$

$$\frac{\partial \pi^m}{\partial q_L} = \frac{(2 - \bar{\mu} - tq_H + 3tq_L)(\bar{\mu} - 2 - tq_H - tq_L)}{4} = 0; \quad (8)$$

whose solution yields the monopolist's optimal qualities, i.e.,  $q_H^m = (2\bar{\mu} - 1)/(4t)$  and  $q_L^m = (2\bar{\mu} - 3)/(4t)$ ; Equilibrium prices are  $p_H^m = (2\bar{\mu}^2 - 3\bar{\mu} + 2)/(4t)$  and  $p_L^m = (\bar{\mu} - 1)(2\bar{\mu} - 3)/(4t)$ ; equilibrium demands are  $x_H^m = x_L^m = 1/2$ ; profits amount to  $\pi^m = (4\bar{\mu}^2 - 8\bar{\mu} + 5)/(16t)$ ; The corresponding welfare is  $SW^m = \bar{\mu}(\bar{\mu} - 1)/(4t)$ ; The constraint ensuring the positivity of all equilibrium magnitudes is  $\bar{\mu} > 3/2$ :

The profit-maximizing monopolist undersupplies both qualities as compared to social planning, the distortion being larger in the low-quality segment of the market. Furthermore, the degree of differentiation chosen by the monopolist is twice as wide as under social planning, due to the monopolist's attempt at extracting as much consumer surplus as possible by enhancing differentiation beyond the socially preferable level. As a consequence, the monopolist's price-and-quality scheme determines different demands as well as different profits for the two varieties at equilibrium. This will have relevant bearings on the choice of the optimal policy by the regulator.

## 4 Alternative regulation policies

The policy menu we analyse includes two kinds of intervention that share the features of being aimed at affecting product choice, i.e., quality taxation/subsidization and minimum quality standard. In the previous section, we have calculated the level of social welfare in the first best configuration. It will become clear in the remainder of this section that a second best analysis is unnecessary in the case of duopoly, in that we will show that there exists a policy measure capable to attain the social optimum. On the contrary, as a benchmark for the regulation of a two-product monopolist, we need to work out the maximum level of social welfare attainable given the monopolist's pricing rule. Under the latter, maximizing social welfare as defined in (5) yields

$$q_H^{sb} = \frac{6\bar{\mu} - 3 + \sqrt{3}}{12t}; \quad q_L^{sb} = \frac{2\bar{\mu} - 3 + \sqrt{3}}{4t}; \quad (9)$$

where superscript sb stands for second best. The following inequalities hold:

$$q_i^{sb} > q_i^{sp} > q_i^m; \quad i = H; L; \quad (10)$$

$$q_H^m > q_L^m > q_H^{sb} > q_L^{sb} > q_H^{sp} > q_L^{sp}; \quad (11)$$

The fact that each second best quality is higher than its counterpart under social planning can be interpreted by observing that the socially optimal qualities are defined in correspondence of marginal cost pricing. Given the distortion in prices introduced by the profit-seeking monopolist, the compensation operated on the quality side is such that both qualities end up being higher than under social planning. It is also worth noting that the increase in the quality level is decreasing as we step up along the quality range, i.e.,  $q_L^{sb} - q_L^{sp} > q_H^{sb} - q_H^{sp}$ : This mirrors the fact that, under monopoly, the distortion increases in the opposite direction. As a result of the upgrading observed in qualities, the equilibrium demand levels are  $x_H^{sb} = 0.2113$  and  $x_L^{sb} = 0.7887$ : The second best level of social welfare is  $SW^{sb} = (6\bar{\mu}^2 - 6\bar{\mu} + 3) = (24t)$ , with  $SW^{sp} > SW^{sb} > SW^m$ : To attain  $SW^{sb}$ , the regulator avails of two alternative instruments, namely, the adoption of a taxation/subsidization of possibly both product qualities or the introduction of an MQS affecting directly the low quality and only indirectly the high quality. In the next subsection we explore the monopoly setting.

## 4.1 Monopoly

### 4.1.1 Taxation/subsidization of product quality

Here we examine the effects exerted on the monopolist's quality choice by a taxation/subsidization policy designed as follows:

$$T_i = k_i(q_i - j_i)^2 + ck_i; \quad k_i \in \mathbb{R}; \quad i = H; L; \quad (12)$$

where  $k_i$  is the unit tax/subsidy rate applied to variety  $i$ ,  $j_i$  is the starting point appropriately selected by the regulator for the taxation/subsidization schedule of variety  $i$ , and  $ck_i$  is a lump-sum transfer. The sign of  $c$  determines whether  $T_i$  is a tax or a subsidy.

The goal of the social planner is to identify the pair  $(k_H^m; k_L^m)$  that forces the monopolist to supply the second best quality levels (9). In case of taxation, the proceeds are assumed to be redistributed among consumers. The

quadratic term in  $T_i$  preserves both the manageability and the concavity of the problem.<sup>4</sup> We are going to show that the following holds:

**Proposition 1** If  $j_L \cdot (6\bar{\mu} + 15j_L \sqrt{3}) > 12t$ , there exists an optimal pair of unit rates  $k_H$  and  $k_L$  such that the second best level of social welfare is attained in a monopoly regime. If  $c$  is sufficiently high, the monopolist is being taxed, otherwise she is being subsidized.

**Proof.** The objective function of the monopolist looks now as follows:

$$\frac{1}{4}^m(T) = (p_H - tq_H^2)x_H + (p_L - tq_L^2)x_L - T_H - T_L \quad (13)$$

Assuming the monopolist has optimally set prices, we can concentrate our attention to quality choice. By differentiating  $\frac{1}{4}^m$  w.r.t.  $q_H$  and  $q_L$  and substituting both with their respective second best levels (9), we can simplify the FOCs as follows:

$$\frac{\partial \frac{1}{4}^m}{\partial q_H} = \frac{k_H(3j_L \sqrt{3} - 6\bar{\mu} + 12tj_H)}{6t} = 0; \quad (14)$$

$$\frac{\partial \frac{1}{4}^m}{\partial q_L} = \frac{k_L(3j_L \sqrt{3} - 2\bar{\mu} + 4tj_L) - t(1 + \sqrt{3})}{6t} = 0; \quad (15)$$

From (14) it appears that  $k_H^m = 0$ : As a consequence, the high-quality product is neither taxed nor subsidized. On the other hand, from (15) we obtain  $k_L^m = t(1 + \sqrt{3}) / [3(3j_L \sqrt{3} - 2\bar{\mu} + 4tj_L)]$ ; which represents the optimal rate for the low-quality good, provided that the second order condition for concavity is met. It can be shown that

$$\frac{\partial^2 \frac{1}{4}^m}{\partial q_L^2} < 0 \Leftrightarrow j_L \cdot \frac{(6\bar{\mu} + 15j_L \sqrt{3})}{12t} > 0; \quad (16)$$

Moreover, the critical threshold established for  $j_L$  in (16) is lower than  $q_L^{sb}$  for all admissible values of  $\bar{\mu}$ : Finally, we are now in a position to verify that

$$T_L > 0 \Leftrightarrow c > \frac{(1 + \sqrt{3})4tj_L - 8tj_L^2 - 2j_L \sqrt{3}}{8}; \quad (17)$$

which implies that the low-quality good is being taxed if the lump-sum transfer is sufficiently high, otherwise it's being subsidized. Equilibrium profits are  $\frac{1}{4}^m(T) = [6(3j_L \sqrt{3} - 2\bar{\mu} + 4tj_L) - t(1 + \sqrt{3})] \bar{\mu} + 19j_L \sqrt{3} - [24(3j_L \sqrt{3} - 2\bar{\mu} + 4tj_L) - t(1 + \sqrt{3})] T_L(k_L^m)$ : ■

<sup>4</sup>The concavity of the firm's maximum problem is guaranteed if the public authority introduces either (i) a linear tax or subsidy function; or (ii) a convex tax function; or (iii) a concave subsidy function, where the convexity, concavity or linearity is meant w.r.t. quality. Analogous consideration holds in a duopoly setting. For a proof, see Lambertini (1997b), where a Hotelling model with quadratic transportation cost is used.

### 4.1.2 Minimum quality standard

Consider now the situation where regulation takes place through an MQS aimed at increasing the average quality level by setting a lower bound to the monopolist's quality range. We assume that in setting the standard, labelled as  $q_L^{mqs}$ , the regulator acts as if he were playing simultaneously a noncooperative game with the monopolist. We prove the following

**Proposition 2** As a result of the adoption of an MQS, the monopolist produces the second best qualities  $q_H^{sb}$  and  $q_L^{sb}$ :

**Proof.** The FOC for welfare maximization w.r.t.  $q_L$  is

$$\frac{\partial SW}{\partial q_L} = \frac{8\bar{\mu} - 4 - 3\bar{\mu}^2 - 16tq_L + 12\bar{\mu}tq_L + 3t^2q_H^2 - 6t^2q_Hq_L - 9t^2q_L^2}{8} = 0: \quad (18)$$

As to the monopolist, the FOC for profit maximization w.r.t.  $q_H$  is

$$\frac{\partial \pi^m}{\partial q_H} = \frac{\bar{\mu}^2 - 4\bar{\mu}tq_H + 3t^2q_H^2 + 2t^2q_Hq_L - t^2q_L^2}{4} = 0: \quad (19)$$

The only root of the system (18-19) such that the concavity conditions are satisfied for both the monopolist and the regulator, and  $q_H > q_L \geq 0$ ; is given by  $q_H^{mqs} = (6\bar{\mu} - 3 + \sqrt{3})/(12t)$ ;  $q_L^{mqs} = (18\bar{\mu} - 11 + 4\sqrt{3})/(36t)$ : It can be immediately verified that these qualities correspond to  $q_H^{sb}$  and  $q_L^{sb}$ , respectively. Equilibrium profits are  $\pi^m(MQS) = \pi^m(T) + T_L(k_L^m)$ : ■

The same correspondence obviously holds as to welfare levels and the distribution of demand across products in the two settings. As a straightforward consequence, we may establish the following

**Corollary 1** As to the welfare level achieved, the regulator is indifferent between adopting a quality taxation/subsidization scheme and introducing an MQS.

This also proves that the regulator's optimal behaviour consists indeed in simulating a simultaneous and noncooperative game against the firm. In other terms, the public authority could not do any better by acting as a

Stackelberg leader, i.e., by maximizing social welfare w.r.t.  $q_L$  under the constraint represented by the monopolist's reaction function implicitly defined by (19).<sup>5</sup> Another relevant corollary of Proposition 2 is

**Corollary 2** As a result of the introduction of an MQS, quality increases both in the low segment and in the high segment of the market.

This contrast with the findings of Besanko et al. (1987, 1988), who claim that the quality increase is observed only in the low segment, where the MQS is binding (see their Proposition 1, 1987, p. 750). This strictly depends on the assumption that the quality range is continuous, eliminating thus any adjustment triggered by regulation along the remainder of the surviving product range. In a discrete setting, the immediate consequence of a standard is to induce the monopolist to "relocate" upwards all her quality spectrum, since the unregulated level initially chosen for the high-quality product would no longer be optimal in presence of the MQS. It is worth noting here that, under the alternative assumption of partial market coverage, the monopolist would exactly provide the socially optimal qualities, and the distortion would solely affect the output level, so that an MQS policy would be completely ineffective (see Lambertini, 1997a).

## 4.2 Duopoly

### 4.2.1 Taxation/subsidization of product quality

As before, we consider a taxation scheme defined by (12). Given the symmetry of the model and the assumption of full market coverage, the scheme we propose cannot affect total output being supplied by duopolists, and it is only meant to ensure an efficient distribution of firms (or their products) along the quality range. Each firm's profit function is then  $\pi_i^d = (p_i - tq_i^2)x_i - k_i(q_i - j_i)^2 - ck_i$ ;  $i = H, L$ . We prove the following:

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<sup>5</sup>The same conclusion has been drawn by Ecchia and Lambertini (1997) in a duopoly setting. The above proof could also be generalized to show that any lower bound imposed on the monopolist's strategy space would induce her to supply the same qualities the public authority would produce, in that both agents have the same FOC w.r.t.  $q_H$ . E.g., it could be easily shown that the regulator could compel the monopolist to produce  $q_H^{sp}$  and  $q_L^{sp}$  by setting  $q_L^{mqs} = q_L^{sp}$ , but this obviously would not maximize welfare under the monopolist's pricing rule.

**Proposition 3** For all  $j_H \in (2\bar{\mu} - 1, 4\bar{\mu} - 1)$  and for all  $j_L \in (4\bar{\mu} - 3, 2\bar{\mu} - 1)$ ; there exists an optimal rate  $k$  maximising social welfare. If parameter  $c$  is sufficiently high, farms are being taxed, otherwise they are being subsidized.

**Proof.** Proceeding backwards, it is immediate to verify that equilibrium prices, as a function of qualities, are the same as in the unregulated game, since FOCs at the market stage are unmodified. Moreover, by symmetry, we can set  $k_H = k_L = k^d$ : Equilibrium qualities can be found by solving the FOCs at the first stage,

$$\frac{\partial \mathcal{W}_H^d}{\partial q_H} = \frac{3t^2 q_H^2 + 2t^2 q_H q_L + t^2 q_L^2 + 4\bar{\mu} t q_H + 4t q_H + 18k^d (q_H + j_H) + \bar{\mu}(\bar{\mu} + 2) + 1}{9} = 0; \quad (20)$$

$$\frac{\partial \mathcal{W}_L^d}{\partial q_L} = \frac{t^2 q_H^2 + 2t^2 q_H q_L + 3t^2 q_L^2 + 4\bar{\mu} t q_L + 8t q_L + 18k^d (q_L + j_L) + \bar{\mu}(\bar{\mu} - 4) + 4}{9} = 0; \quad (21)$$

using the additional information that  $j_H$  and  $j_L$  must obviously be symmetric and respect the relationship:

$$j_H + j_L = \frac{\bar{\mu}}{2t} + \frac{\bar{\mu} - 1}{2t} = \frac{2\bar{\mu} - 1}{2t}; \quad (22)$$

This can be exploited by plugging  $j_L = (2\bar{\mu} - 1) - j_H$  into the low-quality farm's profit function. As a result, qualities are set at their socially optimal levels,  $q_H^d = q_H^{sp} = (4\bar{\mu} - 1) - (8t)$  and  $q_L^d = q_L^{sp} = (4\bar{\mu} - 3) - (8t)$ ; if

$$k_H = k_L = k^d = \frac{2t}{3(4\bar{\mu} - 8tj_H - 1)}; \quad (23)$$

It is easy to verify that (i) the denominator of the expression in (23) is positive if the "upper starting point" of the scheme,  $j_H$ , lies within the interval of the socially preferred varieties; and (ii) the second order conditions for the concavity of the profit functions are met if  $j_H \in (2\bar{\mu} - 1, 4\bar{\mu} - 1)$  and  $j_L \in (4\bar{\mu} - 3, 2\bar{\mu} - 1)$ .

Moreover, notice that the behaviour of the optimal rate  $k^a$  is hyperbolic, with

$$\lim_{j_H \rightarrow q_{H_i}^{SP}} k^d = 1; \quad \lim_{j_H \rightarrow \frac{2\bar{\mu}-1}{4t}} k^d = \frac{2}{3}t: \quad (24)$$

This implies that, if the "starting point" selected by the regulator is very close to the socially preferred qualities, the rate  $k^d$  needed to induce firms to supply precisely those qualities becomes infinitely high.

Equilibrium profits are  $\pi_H^d = \pi_L^d = 2ct = (3(8tj_H - 4\bar{\mu} + 1) + (8tj_H + 7 - 4\bar{\mu})) = (96t)$ : Finally, it can be verified that

$$T_i = \frac{4\bar{\mu} - 8tj_H - 1}{96t} + \frac{2ct}{3(4\bar{\mu} - 8tj_H - 1)} > 0 \iff c > e = \frac{(4\bar{\mu} - 8tj_H - 1)^2}{64t^2} < 0: \quad (25)$$

This condition states that there exists a critical threshold  $e$  above (below) which firms are being taxed (subsidized). E.g., when  $j_H = j_L = (2\bar{\mu} - 1)/(4t)$ ; i.e.,  $j_H$  and  $j_L$  coincide with the midpoint of the interval of preferred qualities, it is a straightforward exercise to show that  $T_i$  is positive for all  $c > e = (64t^2)$ : ■

#### 4.2.2 Minimum quality standard

Here, we briefly resume the analysis contained in Ecchia and Lambertini (1997). The main result is stated in the following

**Proposition 4** In a duopoly market, the MQS policy is unable to attain the first best qualities.

The derivation of the optimal MQS under a duopoly market regime yields

$$q_L^d(\text{MQS}) = \frac{20\bar{\mu} - 34 + 9\bar{P}_6}{40}: \quad (26)$$

Given  $q_L^d(\text{MQS})$  and its equilibrium price, full market coverage is possible if and only if  $\bar{\mu} \geq 2.23926$ : Observe that the introduction of the standard slightly loosens such a constraint as compared to the unregulated setting.

The new level of the high quality is the best reply of the high-quality firm to the MQS:

$$q_H^d(\text{MQS}) = \frac{20\mu^1 + 2 + 3\frac{P_L}{6}}{40} \quad (27)$$

The new equilibrium profits are  $\pi_L^d(\text{MQS}) = 0.22153$  and  $\pi_H^d(\text{MQS}) = 0.06714$ : As a result of the adoption of the MQS, the degree of differentiation decreases, and the demand for the high quality decreases while the demand for the low quality increases. Moreover, notice the drastic reduction in the high-quality firm's profits. Since the increase observed in the profit accruing to the low-quality firm is lower, total industry profits decrease considerably as compared to the unregulated equilibrium.

Social welfare amounts to  $SW^d(\text{MQS}) = [200\mu^1(\mu^1 - 1) + 18\frac{P_L}{6} - 13] = 800$ ; which is obviously higher than that observed in the unregulated setting, but lower than  $SW^{sp}$ . The increase in welfare is due to two effects: (i) the increase in both quality levels; (ii) the increase in price competition, due to a reduced degree of product differentiation. However, the effect of the MQS on consumer surplus is not identical across consumers. The MQS increases the surplus of consumers purchasing the low quality for all acceptable values of  $\mu^1$ , while it decreases the surplus of consumers patronizing the high quality if  $\mu^1$  is sufficiently high. Summing up, in this case it appears that the MQS policy, provided it is designed to maximize welfare regardless of its redistributive effects, trades off the losses suffered by the agents (firm and consumers) dealing with the high quality with the gains enjoyed by the other agents, but remains far from the ideal target represented by first best.

### 4.3 Discussion

The inefficiency arising in markets where a firm controls product quality as well as price is associated with the difference between the average and the marginal consumer's evaluation of quality, represented by parameter  $\mu$ : Spence (1976, pp. 426-7) shows that, when quality is a capital-intensive rather than labour-intensive feature, rate-of-return regulation is an appealing measure. This is the case, e.g., when product quality is the result of R&D effort, so that the cost function exhibits no interaction between quality and quantity. (Ronen, 1991; Motta and Thisse, 1993). In our model, the cost function (1) might suggest that quality requires a combined use of capital and labour in variable proportions. This prompts for the design of



alternative interventions, as those depicted above. In the monopoly setting, although both measures yield the same level of welfare, the adoption of a taxation/subsidization scheme entails a surplus transfer from the monopolist to consumers, which does not take place under the MQS regime. Since the regulator is assumed not to be interested in surplus distribution and the MQS policy is effort-saving as compared to the design of a taxation/subsidization policy, one might think that the former policy is more likely to be adopted. The perspective changes considerably when the market is a duopoly with single product firms, in which case the two policies are not equivalent, with the MQS being unable to yield the first best while taxation/subsidization allows the regulator to attain such a goal.

A crucial question naturally springs to mind, that is, whether the introduction of the MQS induces the monopolist to drop one of the two products, and reoptimize with respect to a single variety. This would have undesirable welfare consequences (for an example pertaining to oligopoly settings, see Scarpa, 1998). To address this question, we can work out the single-product profit the monopolist would gain by providing a quality corresponding to the optimal MQS with a single good,  $q^m = (2\bar{\mu} - 1)/(4t)$ , the price being  $p^m = (\bar{\mu} - 1)q^m$ . Such a profit amounts to  $\frac{1}{4} \pi_1^m(\text{MQS}) = (4\bar{\mu}^2 - 8\bar{\mu} + 3)/(16t)$ : Hence, we have

$$\frac{1}{4} \pi_2^m(\text{MQS}) - \frac{1}{4} \pi_1^m(\text{MQS}) = \frac{11 - 5\sqrt{3}}{48(3 - \sqrt{3})t} > 0; \quad (28)$$

so that the adoption of an MQS does not induce the firm to restrict the product range.

An analogous question can be asked concerning the duopoly setting. In the case where an MQS is adopted, we already know that profits are strictly positive. In the case of taxation/subsidization, firms' profits are positive if  $c < \mathbf{b} = (4\bar{\mu} - 1 - 8tj_H)(4\bar{\mu} - 7 - 8tj_H)/(64t^2)$ ; with  $\mathbf{b} > \mathbf{e} > 0$  everywhere and  $\mathbf{b} > 0$  for all  $j_H \in (2\bar{\mu} - 1)/(4t); (4\bar{\mu} - 1)/(8t)$ ; wherein second order conditions are satisfied. Hence, the adoption of such a policy does not drive firms out of business, as long as  $c$  is sufficiently low.

## 5 Concluding remarks

We have investigated the issue of regulating either a multiproduct monopolist or single-product duopolists operating under vertical differentiation, through

policies affecting her quality choice. The relative effectiveness of the policy instruments included in the menu we have considered turns out to be very sensitive to market structure. In the monopoly case, neither of the two alternative policy measures is able to attain first best. We have shown that there exists a second best quality range characterized by quality levels higher than their counterparts under social planning, and we have described how such second best configuration can be attained through the alternative adoption of either hedonic taxation/subsidization or a minimum quality standard (MQS). As to their final results, these measures are completely equivalent, while they are not in terms of the distribution of surplus. We have also shown that the introduction of a standard does not lead to a reduction in the number of available varieties. In the duopoly case these two instruments offer a very different performance. The asymmetric effects exerted by the MQS on the two products prevents the public authority to induce firms to supply the first best quality levels, which on the contrary can be obtained under the taxation/subsidization policy we have envisaged. This leads to prefer the latter intervention to the former.

The issue of designing the socially preferable intervention as a combination of the several policy instruments that can be envisaged to regulate a market for endogenously differentiated products is left open for future research.

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