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Profit Sharing Regulation, Repeated Bargaining and Shut-Down Option.

Michele Moretto* Gianpaolo Rossini†

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Abstract

We analyse the behavior of a firm where workers share profits with shareholders by using a model cast in an Aoki framework. Our firm faces two sorts of uncertainty: one relates to the market price assumed to follow a random walk in continuous time and the other relates to internal organization, i.e. the share of profits to be distributed between workers and shareholders. The firm is assumed to be flexible, since it has the possibility of shutting down by paying laid off workers a bonus, which represents a sunk cost for the firm. The distributive share is determined through a bargaining that takes place in two occasions: at the beginning of the firm's life and when its profits reach a certain threshold level. The second bargaining is then endogenized according to a rule that is imposed upon shareholders and workers by a regulator who may use profit distribution as a way to regulate the firm. Different share parameter patterns will result as the regulator calls for renegotiation when profits are increasing or decreasing. Moreover we distinguish between a case in which the regulator's rule is announced in advance from the one in which it is discretionally set.

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1 Introduction

Workers and firms representatives usually sign *nation-wide* labor-contracts determining the market wage. In addition to collective bargaining workers and firms are involved in many rounds of specific bargaining where each firm sets an extra wage as the result of a bilateral contract. The firm specific "extra wage" may be interpreted as a "profit-share" given to workers both as an internal incentive and as an insurance to avoid loss of human capital. Within each firm we can imagine a cooperative bargaining between shareholders and workers, whose result depends heavily on company performance normally measured in terms of profits.

Collective bargainings, for practical and institutional reasons, take place at fixed dates, while firm specific contracts, in which the extra wage or the extent of "profit sharing" is decided, should be renegotiated in accordance with changes in specific performance of each firm. This complies with the features distinguishing profit sharing from other participation schemes. However it is not easy, or even impossible, to proceed to a firm specific bargaining, or even to repeat it, when the firm's performance changes. This happens simply because there would not be any state in which workers and shareholders may agree on recontracting, as the extra wage is state-dependent. In some cases we may be even in a worse condition since the commitment power of the original contract may be lost if the parties are able to renegotiate the terms of the contract itself, as pointed out by Masten and Snyder (1989).¹

This leaves room for government intervention at two tiers. At the general normative level it encourages or even makes compulsory "profit sharing" or firm specific bargaining over extra-nation-wide wages. At the bilateral tier government tends to regulate profit sharing. In industrialised countries the former role is now well established. Recent studies have shown that the diffusion of profit sharing observed in a number of countries is driven by recent changes in legislation aimed at making these schemes more attractive

¹Voluntary agreements can be reached if, in the renegotiation, the obtained gain to one side does not imply losses to the other. This is the main reason why renegotiation through contracts can hardly give rise to state-dependent agreements, since they are rarely Pareto superior.

and even sometimes compulsory². The importance of incentives to promote adoption of profit sharing schemes is certainly relevant, yet the regulation aspect on which we concentrate is just as relevant. Because of the fundamental inability of workers and shareholders to precommit to threats and to rule out strategic commitment, we imagine out a *regulator* who has the authority to set the terms of recontracting.³ What is the objective of the regulator when he intervenes setting the terms of recontracting? Simply the regulator may wish to contrast a dominant position by calling for a recontracting of the profit share parameter when market conditions are buoyant or he may aim at lower employment variability when market conditions are depressed⁴. Actually the regulator may appear at first sight too worker-oriented when he shows this binary aim. Nevertheless, he is quite neutral if considered over the long run, since it is also in the interests of the firm to preserve its specific human capital and avoid disruptive industrial relationships.

Within the firm we may have several bargainings, even though for the sake of simplicity we confine our analysis to two bargainings over the firm's lifetime⁵. In particular, we allow for a second bargaining that takes place when the profit (market price) attains a certain predetermined level. This level may be set in two alternative ways. It may be announced by the regulator at the beginning of the planning period and made known in advance by both parties. Or else it may be discretionally set by the regulator when the state variable touches a predetermined level.

In either cases the result of the repeated bargaining is the setting of two share parameters, one for each bargaining period. To describe how these

²Profit sharing has been implemented under many different institutional rules, from being entirely or partially compulsory (Mexico and France) to being completely discretionary (for instance in North America and Canada it has been linked to the accumulation to retirement funds), (OECD, 1995; Vaughan-Whitehead, 1995; Biagioli, 1995).

³Recent literature examines the possibility of reinforcing contractual commitments when they are fairly ineffective. Specific investments by the parties are alternative instruments to commit themselves against breach of contract by making exit and entry quite costly, since investment has a much higher value inside the relationship than outside. See for example Aghion and Bolton (1987), Chung (1992, 1995), Spier and Whinston (1993).

⁴Despite peculiarities there is widespread evidence that profit sharing tends to lower employment variability and to improve internal industrial relationships (OECD, 1995).

⁵This does not imply that a firm takes part in bargaining only twice, but that a particular organization of the firm with a particular manager and ownership structure takes place only twice in labour bargainings.

results are obtained a full analysis of the dynamics of the profit distribution is provided. In particular we investigate the difference in profit distribution when: *i*) the initial market price is higher or lower than the price at which the second bargaining takes place; *ii*) the latter is announced in advance, or not, by the regulator.⁶

When the regulator sticks to a preannounced policy rule with upturn bargaining the profit share changes in favour of the workers. With downturn contracting the opposite happens. In addition to that, we show that the profit distribution differential reduces for both cases under analysis when the regulator acts discretionally.

We adopt a continuous time setup of optimal sequential decision under uncertainty which stresses the firm's need for investment flexibility, as has recently been modeled by the so-called option valuation approach to investment decisions (McDonald and Siegel 1985, 1986; Dixit and Pindyck 1994). According to this interpretation a firm can be thought of as an individual possessing an operative option that can be exercised at the most favorable occasion. This approach highlights the role played by the value of waiting for better information and its analogy with the option theory in financial markets.⁷ Investment decisions should be taken only when market conditions become extremely favorable and reversed only if they become extremely unfavorable.

Unfortunately the requirement for a firm to be as flexible as possible seems to be often in conflict with fair and efficient functioning of the labor market within the firm. Workers are usually against flexibility when this

⁶The use of downward prices adjustment when the firm's rate of return exceeds a target rate and/or adoption of prices caps as regulatory mechanisms for (private) natural monopoly and oligopoly are well known in the theory of incentives and regulation. While these regulatory schemes refer, in general, to the extent of profit (cost) sharing between firms and consumers, our notion of regulation refers to the firm's internal profit distribution.

⁷A firm with an opportunity to invest holds an option which is analogous to a financial call option. That is, it has the right but not the obligation to buy an asset of some value at a future time at a fixed "exercise price". On the other hand, an operating firm with an opportunity to abandon holds an option analogous to a financial put option which gives the right, but not the obligation, to sell an asset at some future time for a fixed price, represented by the sunk cost of exit. McDonald and Siegel (1985) in their pioneering work show that, if the price is driven by a geometric Brownian motion, a unit-investment project with fixed operating cost can be valued as the sum of an infinite set of European call options.

concerns firm's exit or even reduction of production levels. The firm may relax the rigidity of its internal organization by letting workers take part in profit distribution. The organization that parallels closely these features is the one envisaged by Aoki (1980, 1984), where profits are distributed according to bargaining between a workers' representative and a shareholders' representative.

Other works have focused on issues that are related to the subject dealt with in this paper. Moretto and Rossini (1995) consider the effects of flexibility, represented by a viable shut down option, upon the dividend distribution policy when there are different degrees of loss sharing between workers and shareholders. Elsewhere, Moretto and Pastorello (1995), with symmetric profit/loss sharing, study the effect of two compound options, a shut down and a reopening option, on the profit share distribution policy. Both articles, however, do not consider renegotiation of the terms of the bargaining when economic conditions change and a regulator is acting..

The paper is organized as follows. In section 2 we present some general characteristics of the environment in which the action takes place. In section 3 we outline the features of the bargaining. In section 4 we define the objective functions, the efficient sets of the two contenders in the second period and then we proceed to find the bargaining equilibrium. In section 5 we go back to the first period by distinguishing two cases according to the level of the market price *vis à vis* the price that triggers recontracting. In section 6 we provide some conclusions.

2 The model

We consider an incumbent firm which exhibits a constant-returns-to-scale technology and is endowed with a capital stock of infinite life. Each period the firm produces one unit of output. Marginal and average costs are equal to c . The labor force is, for the sake of simplicity, normalized to one. The internal organization of the firm is shaped by profit sharing between workers and shareholders according to Aoki's scheme. Then the extent of profit sharing is the result of bargaining between a shareholders' representative and a workers' representative.

The firm faces two kinds of uncertainty. One is concerned with its internal organization and relates to the risk of *internal conflict*, whilst the other is exogenous and concerns the *market price*, which we assume driven by a

geometric Brownian motion (random walk in continuous time):

$$dp_t = \alpha p_t dt + \sigma p_t dz_t, \quad \text{with } p_{t_0} = p_0 \text{ and } \sigma > 0, \quad (1)$$

where dz_t is the increment of a standard Wiener process, uncorrelated over time and satisfying the conditions that $E(dz_t) = 0$ and $E(dz_t^2) = dt$. The operating profit at time t is:

$$\pi(p_t) = p_t - c, \quad (2)$$

when the firm is working and zero if it decides not to produce. The firm can shut down by paying laid off workers a bonus equal to K , which represents the entire *sunk cost*⁸. As there is an opportunity cost of abandoning now rather than waiting for new information, the firm does not exit if today's price is just below the average variable cost. It is optimal to exit only if the price falls below a trigger level $p_L < c$ that has to be endogenously determined by considering future expected opportunities *vis à vis* the sunk exit cost.

Different types of profit sharing schemes can be figured out: cash-based or share-based, and many company performance indicators are currently used. We simply assume that payments are conditional on current profits and workers share symmetrically profits and losses.⁹ Therefore:

$$w_t = \bar{w} + \Delta w_t, \quad (3)$$

where \bar{w} is the market wage and Δw_t is a premium earning which represents the employees' share of profits accruing to the firm. Let $0 \leq \theta \leq 1$ be the share parameter indicating the proportion of profits and losses going to shareholders, then:

$$\Delta w_t(p_t; \theta) = (1 - \theta)\pi(p_t), \quad (4)$$

with $\pi(p_t) \geq c - p_L$, and p_L representing the price that triggers exit.

⁸In many industrialized countries the severance payment for laid off workers is determined (and enforced) by law. For that reason we consider it as an institutional parameter that cannot be negotiated upon. We could also think of different bonus schemes, for instance by linking the amount of K to the time spent by workers at the firm or assuming that only a part of the entire exit cost K is paid as bonus to the laid-off workers. However, the consideration of these different schemes will not change conclusions substantially.

⁹Theoretically, employees may receive negative extra-wages when the firm makes losses, in practice this is never the case, except when there are "solidarity contracts". However, for the sake of simplicity, we maintain this symmetry. For a variable degree of loss sharing between shareholders and workers, see Moretto and Rossini(1995).

3 Organization of the bargaining

At the beginning of its life (stage 0) the firm signs a contract with workers. The essential nature of the contract is the agreed share of profits going respectively to workers and shareholders. At the end of stage 0 the firm and the workers take part in a second bargaining (stage 1), whose outcome should be enforced for the rest of the firm's life.

The innovation we introduce is that the length of this contract is not predetermined, yet it is derived from both institutional exogenous settings and current performances of the firm. The length of stage 0 and how it is designed will characterize the rest of this paper and represents its core.

As stated in the introduction, the regulator may use profit sharing as a device to regulate the firm and for this purpose the two parties will be summoned to recontract according to particular rules, which we shall illustrate below. The second bargaining takes place when the market price hits a certain predetermined level, called p_1 , which may be announced by the regulator in advance, i.e. at the beginning of the firm's life, or discretionally announced only when the regulator decides that recontracting should take place. Consequently, in the latter case the two bargainings are independent of each other, while in the first case the two bargainings are interlinked, as the regulator commits to a renegotiation policy.¹⁰

The price p_1 may be called the *public trigger price* to be distinguished from the *private trigger price* p_{L1} , which is the one set optimally by the firm as its exit price. We shall be able to design a variety of scenarios according to the different levels at which the *public trigger price* is set and whether this is done discretionally or via a preannounced rule.

Formally we use as a benchmark the case where the regulator commits in advance to a policy rule. The comparison with the discretionary case is straightforward. The problem is a two-stage optimization problem amenable to dynamic programming. In particular, since dynamic programming provides a recursive solution, we start solving the two stages backwards by deriving the optimal share which is the solution of the bargaining at stage 1.

¹⁰Despite the above distinction, both parties may anticipate a future renegotiation event, if they believe that it will happen even in absence of a formal announcement by the regulator. Cukierman(1986) and Alesina(1988) give illuminating surveys of dynamic (un)consistent policies by government.

4 Workers and shareholders at stage 1

4.1 The contenders' objective functions

Assuming that the firm is a value maximizer operating in perfectly competitive markets for its product and for its assets, the expected present value at stage 1 of the stream of profits is given by:

$$S_1(p_1; \theta_1) = E_1 \left\{ \int_{t_1}^{T_1} \theta_1(p_t - c)e^{-\rho(t-t_1)} dt \mid p_{t_1} = p_1 \right\} \quad \text{for } p_1 \in [p_{L_1}, \infty). \quad (5)$$

Besides the *public trigger price* p_1 , the firm's value S_1 is also a function of the parameter representing the share of profits going to shareholders within the same stage θ_1 ; ρ is the discount rate ($> \alpha$); t_1 is the starting time of stage 1 and $T_1 = \inf(t > t_1 \mid p_t = p_{L_1})$ is the stochastic stopping time at which the firm will exit.

As long as workers completely share firm's losses and \bar{w} is constant, the level of their well being up to the shut down may be ordered according to the expected discounted sum of the premium earnings at the firm. That is:

$$L_1(p_1; \theta_1) = E_1 \left\{ \int_{t_1}^{T_1} \Delta w_t(p_t; \theta_1) e^{-\rho(t-t_1)} dt \mid p_{t_1} = p_1 \right\} \quad \text{for } p_1 \in [p_{L_1}, \infty). \quad (6)$$

Finally, the organizational equilibrium of the firm is characterized by the result of a bargaining process that takes place at the beginning of stage 1. Using the *Nash Bargaining Solution* (NBS) concept as formulated by Harsanyi (1956,1977), and extended by Aoki (1980, 1984) and Rubinstein (1987), the joint objective function of workers and shareholders to be maximized with respect to the distributive parameter θ_1 is:

$$\nabla_1 = \lg[U(L_1) - \hat{U}] + \lg[V(S_1) - \hat{V}] \quad , \quad (7)$$

subject to (5) and (6).

U and V represent the utility functions of workers and shareholders respectively. The predetermined levels \hat{U} and \hat{V} are the reservation levels of utility that the contenders can get if the bargaining fails to reach an agreement, i.e. the *threat points* of the bargaining. For this purpose we assume specificity by both parties. In other words, workers are not able to find a

proper job if the contract is not signed, while shareholders lose the opportunity of producing with high skilled workers who cannot be easily found on the labor market. Therefore for the rest of the paper the outside opportunities or threat points are assumed, for simplicity, to be equal to zero.

4.2 The efficient bargaining set

To identify the efficient bargaining set we start with the shareholders who independently decide the exit policy. For a given value of θ_1 , when the firm is in operation $S_1(p; \theta_1)$ must satisfy the no-arbitrage condition, which requires that the sum of the return on the investment, given by the dividend flow plus the capital gain $E(\frac{dS_1(p; \theta_1)}{dt})$, equals the market cost of capital $\rho S_1(p; \theta_1)$. Since p_t is driven by (1), applying Itô's lemma to S_1 , the expected capital gain is given by $E(dS_1) = [S_1' \alpha p_t + \frac{1}{2} S_1'' \sigma^2 p_t^2] dt$; then the asset market equilibrium condition leads to the following differential equation:

$$\frac{1}{2} \sigma^2 p_t^2 S_1'' + \alpha p_t S_1' - \rho S_1 = -\theta_1(p_t - c) \quad \text{for } p_t \in [p_{L_1}, \infty), \quad (8)$$

with boundary conditions:

$$S_1(\infty; \theta_1) = 0, \quad (9)$$

$$S_1(p_{L_1}; \theta_1) = -K, \quad (10)$$

$$S_1'(p_{L_1}; \theta_1) = 0. \quad (11)$$

While equation (9) states that when the market price goes to infinity the value of the firm must be bounded, the *value matching condition* (10) says that, when the firm exits its value must be equal to its liabilities represented by the bonus paid to laid-off workers. The *smooth pasting condition* (11) is imposed to rule out arbitrary exercise of the option to exit at a different moment. By the linearity with respect to S_1 and making use of (9), the general solution of (8), evaluated at p_1 , takes the form:

$$S_1(p_1; \theta_1) = A_1 p_1^{\beta_2} + \theta_1 \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right) \quad \text{for } p_1 \in [p_{L_1}, \infty), \quad (12)$$

where β_2 is the negative root of the quadratic equation: $\Psi(\beta) \equiv \frac{1}{2} \sigma^2 \beta^2 + (\alpha - \frac{1}{2} \sigma^2) \beta - \rho = 0$. The last term on the r.h.s. of (12) represents the discounted value of expected profits when the firm is active forever (Harrison 1985, pag.44):

$$E_1 \left\{ \int_{t_1}^{\infty} \theta_1 (p_t - c) e^{-\rho(t-t_1)} dt \mid p_{t_1} = p_1 \right\} \equiv \theta_1 F(p_1) = \theta_1 \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right), \quad (13)$$

whilst $A_1 p_1^{\beta_2}$ indicates the option value, in terms of avoidance of expected losses, of shutting down. The constant A_1 and the optimal trigger price p_{L_1} are jointly determined by using (10) and (11):¹¹

$$p_{L_1} = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c - \frac{\rho}{\theta_1} K \right), \quad A_1 = -\theta_1 \frac{1}{\beta_2} \frac{1}{\rho - \alpha} p_{L_1}^{1-\beta_2} > 0. \quad (14)$$

Finally, substituting (14) into (12) we can rewrite the firm's value in the simplified form:

$$S_1(p_1; \theta_1) = \theta_1 V(p_1; \theta_1), \quad (15)$$

where the value of the stream of profits before distribution:

$$\begin{aligned} V(p_1; \theta_1) &= A p_1^{\beta_2} + \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right) \\ &= - \left(\frac{1}{\beta_2} \frac{1}{\rho - \alpha} p_{L_1}^{1-\beta_2} \right) p_1^{\beta_2} + \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right), \end{aligned} \quad (16)$$

By using a similar procedure for workers it can be shown that:

$$L_1(p_1; \theta_1) = B_1 p_1^{\beta_2} + (1 - \theta_1) \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right) \quad \text{for } p_1 \in [p_{L_1}, \infty), \quad (17)$$

with a *matching value condition*, saying that at the exit trigger price the value for a worker of being employed at the firm is equal to the bonus. That is:

$$L_1(p_{L_1}; \theta_1) = K. \quad (18)$$

No *smooth pasting condition* is introduced since the exit decision is controlled by the firm and workers have no influence on it. Applying (18), the constant B_1 is equal to:

$$B_1 = -(1 - \theta_1) \left(\frac{1}{\beta_2} \frac{1}{\rho - \alpha} p_{L_1}^{1-\beta_2} \right) \left(\frac{c\theta_1 - \beta_2 \rho K}{c\theta_1 - \rho K} \right) + K p_{L_1}^{-\beta_2} > 0. \quad (19)$$

¹¹We assume $c\theta_1 > \rho K$ to guarantee that $p_{L_1} > 0$.

Consequently, the workers' well being value attributable to the firm's option to stop producing $B_1 p_1^{\beta_2}$ depends, *ceteris paribus*, on the size of the bonus K . Considering the firm's market value before distribution V and taking account of (16), (17), (18) and (19), we can compute:

$$L_1(p_1; \theta_1) = (1 - \theta_1)V(p_1; \theta_1) + G(p_1; \theta_1), \quad (20)$$

where $G(p_1; \theta_1) = \frac{1}{\theta_1} K \left(\frac{p_1}{p_{L_1}}\right)^{\beta_2} > 0$ indicates the increase of the lifetime well being accruing to the workers, induced by the asymmetry between shareholders and employees due to the bonus K .¹²

FIGURE 1 ABOUT HERE

4.3 The bargaining at stage 1

For a solution of the bargaining at stage 1 we have to maximize (7) with respect to θ_1 . We specify the workers' utility function as $U(L_1) = L_1^{1-R}$ and that of the shareholders as $V(S_1) = S_1^q$, where $0 < R < 1$ and $0 < q < 1$ are the respective degrees of relative risk aversion. Moreover, to concentrate attention on the profit distribution differential induced by the sunk exit cost K , we eliminate the other asymmetries posing, as already mentioned, the outside option or threat solution of the bargaining at $\hat{U} = \hat{V} = 0$. The results of the joint maximization can be summarized in the following proposition.

Proposition 1 (a) *If $K > 0$ and $\pi(p_t) \geq c - p_{L_1}$ the optimal relative share of shareholders and employees in firm's profits is state-dependent and given by the necessary condition:*

¹²As the bonus K introduces an asymmetry between the two contenders, the workers would like to exit earlier. Indeed, if they could set the exit trigger price independently, $p_{L_1}^w$, it would be (see fig.1):

$$p_{L_1}^w = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c + \frac{\rho}{1 - \theta_1} K \right) > p_{L_1}$$

$$\frac{S_1(p_1; \theta_1^*)}{L_1(p_1; \theta_1^*)} - \Phi(p_1; \theta_1^*, K) \frac{S_1(p_1; \theta_1^*)}{L_1(p_1; \theta_1^*)} = \frac{q}{1-R}, \quad (21)$$

where

$$\Phi(p_1; \theta_1^*, K) = \frac{\frac{dV}{d\theta_1} + \frac{dG}{d\theta_1}}{\frac{dS_1}{d\theta_1}} > 0.$$

(b) When the option to shut down is viable (i.e. $p_{L_1} > 0$) the shareholders' bargained share of profit is greater than Aoki's share θ_1^A . That is:

$$\frac{\theta_1^*}{1 - \theta_1^*} > \frac{\theta_1^A}{1 - \theta_1^A} = \frac{q}{1-R}.$$

Proof: see Appendix A and Moretto-Rossini (1995).

Part (b) of proposition 1 means that the bargaining over θ_1 leads to a profit distribution which is more favourable to shareholders than Aoki's original result, θ_1^A , represented by the ratio of the respective degrees of risk aversion.

As the threat point was set at zero for both actors, the only asymmetry between workers and shareholders is due to the exit cost. Then, if K tends to zero this asymmetry disappears and the profit share parameter is no longer state-dependent, and in particular:

Corollary 1 *If $K = 0$ (or $\sigma = 0$) and $\alpha > 0$, the profit share parameter reduces to Aoki's one. That is:*

$$\frac{\theta_1^*}{1 - \theta_1^*} = \frac{\theta_1^A}{1 - \theta_1^A} = \frac{q}{1-R}.$$

Proof: see Appendix A.

The corollary involves two implications. First, as long as exit is costless shareholders and workers would have chosen the same exit policy, and therefore also the same distribution policy of Aoki. Second, uncertainty affects profit distribution only if there are irreversibilities. On the other hand, if $\alpha > 0$ and $\sigma = 0$ the value of the option of shutting down goes to zero and

the result of corollary 1 is straightforward. Finally, a different result follows if $\alpha \leq 0$ and $\sigma = 0$: under certainty the firm knows exactly when it will quit: i.e. $p_{L_1} = c - \frac{\rho}{\theta_1} K$, and the option is still alive.

To end this section, we wish to summarize in the next proposition the comparative static property of the optimal sharing parameter θ_1^* with respect to the *public trigger price* p_1 :

Proposition 2 *When the option to shut down is viable, (i.e. $p_{L_1} > 0$), the profit share going to shareholders decreases as p_1 increases. That is:*

$$\frac{d\theta_1^*}{dp_1} < 0.$$

Proof: See Appendix B.

The effectiveness of the shut down threat is going to weaken as the *public trigger price* grows since it becomes less likely that it may be endorsed. This result is illustrated graphically in figure 2.

FIGURE 2 ABOUT HERE

5 Back to stage zero

At stage zero we may get a diverse distribution policy owing to a different condition wherein the firm operates. Assume the regulator has laid down a profit rule. By recursion in the solution of the dynamic programming problem workers and shareholders have to consider the opportunity for recontracting. This will take place if the market price touches the exogenously determined *public trigger price* p_1 . Since the profit distribution is state-dependent the difference between stage 0 and stage 1 will depend on the deviation from the price (profits) at the beginning of firm's life and the *public trigger price* (profits) set by the policy maker. The share parameter is decreasing in p_1 (proposition 2) and the exit trigger level p_{L_1} is decreasing in θ_1 (equation 14). Then the regulator may pursue the objective of decreasing employment

variability calling for recontracting when profits fall off, i.e. setting a *public trigger price* lower than the price at which bargaining at time zero has taken place. On the contrary, if the regulator aims at contrasting the formation of a dominating position in the market through fairer income distribution, he may call for recontracting when the firm's market performance is bouyant, setting a *public trigger price* higher than the price of the bargaining at time zero.

5.1 Downturn recontracting: $p_0 > p_1$

Let us start with the simpler case, depicted in figure 3, where the regulator lays down a policy rule in advance and $p_0 > p_1$.

Proposition 3 *If $p_0 > p_1$ it is never optimal for the firm to exit before recontracting.*

Proof: see Appendix C

Shareholders are better off the lower the price at which (re)contracting takes place since the shut down threat becomes more credible (proposition 2). If $p_0 > p_1$ it is always in their interest to wait for the realization of p_1 before setting the optimal *private trigger* p_{L_1} .

FIGURE 3 ABOUT HERE

In accordance with proposition 3, the firm's value at stage zero is given by:

$$\begin{aligned} S_0(p_0; \theta_0) &= E_0 \left\{ \int_0^{t_1} \theta_0(p_t - c)e^{-\rho t} dt + S_1(p_1; \theta_1^*)e^{-\rho t_1} \mid p_{t_0} = p_0 \right\} \quad (22) \\ &= A_0 p_0^{\beta_2} + \theta_0 \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right) \quad \text{for } p_0 \in [p_{L_1}, \infty), \end{aligned}$$

with the limit condition:

$$S_0(p_1; \theta_0) = S_1(p_1; \theta_1^*). \quad (23)$$

No *smooth pasting condition* is set since exit cannot take place before recontracting. Substituting (22) into (23) it is easy to show that $A_0 = A_1 + (\theta_1^* - \theta_0) \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right) p_1^{-\beta_2}$, which allows us to rewrite the objective function of shareholders in the simplified form:

$$S_0(p_0; \theta_0, \theta_1^*) = S_1(p_0; \theta_1^*) - (\theta_1^* - \theta_0) \Delta F(p_0; p_1 < p_0). \quad (24)$$

Going through the same steps for the workers, with $B_0 = B_1 + (\theta_0 - \theta_1^*) \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right) p_1^{-\beta_2}$, we obtain:

$$\begin{aligned} L_0(p_0; \theta_0) &= E_0 \left\{ \int_0^{t_1} (1 - \theta_0)(p_t - c) e^{-\rho t} dt + L_1(p_1; \theta_1^*) e^{-\rho t_1} \mid p_{t_0} = p_0 \right\} \\ &= L_1(p_0; \theta_1^*) + (\theta_1^* - \theta_0) \Delta F(p_0; p_1 < p_0), \text{ for } p_0 \in [p_{L_1}, \infty), \end{aligned} \quad (25)$$

For both actors the term ΔF is equal to:

$$\Delta F(p_0; p_1 < p_0) = \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right) - \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho} \right) \left(\frac{p_0}{p_1} \right)^{\beta_2} > 0.$$

As at t_1 the price p_t hits the lower level p_1 for the first time, it can be shown that $\left(\frac{p_0}{p_1} \right)^{\beta_2} = E_0 \{ e^{-\rho t_1} \mid p_{t_1} = p_1 < p_0 \}$, and ΔF turns out to be always positive (see Karlin and Taylor, 1975; Moretto, 1995). Then, this term represents the difference between the value of a firm that never shuts down and starts at p_0 and the value of a firm that never shuts down but starts later at t_1 discounted back to time zero.

As long as the public trigger price p_1 is exogenously set by the authority, the value of the firm at stage zero, S_0 , can be decomposed in two parts: the value of the firm at stage 1 with the distributive parameter θ_1^* but evaluated at the initial price p_0 , minus the opportunity cost (gain) of recontracting between the first and the second period, expressed by ΔF . Symmetrically, a similar expression holds for the workers but with the opposite sign. The well being of the workers at stage zero is equal to their well being at stage 1, with distributive parameter θ_1^* and evaluated at p_0 , plus the cost (gain) of recontracting ΔF .

Making use of (24) and (25), we are now ready to bargain over θ_0 . The following proposition defines the result in this stage:

Proposition 4 (a) *If $K > 0$, $p_0 > p_1$ and p_1 is announced in advance by the regulator, the optimal profit share at stage zero is given by the*

following necessary condition:

$$(\theta_0^* - \theta_1^*)\Delta F(p_0; p_1 < p_0) = (1 - \theta^A)L_1(p_0; \theta_1^*)\left(\frac{q}{1 - R} - \frac{S_1(p_0; \theta_1^*)}{L_1(p_0; \theta_1^*)}\right). \quad (26)$$

(b) Yet, when the option to shut down is viable (i.e. $p_{L_1} > 0$) the profit share in stage zero **with pre-announcement** is always lower than the profit share **without pre-announcement**, and both are lower than the one in stage 1. That is:

$$\theta_0^* < \theta_0^*(p_1 = 0) < \theta_1^*.$$

Proof: see Appendix D.

We said that recontracting is "imposed" by a regulator who has the authority to decide on its timing. In the benchmark case the regulator announces that renegotiation is going to take place when the market price decreases below a certain predefined level. When $p_0 > p_1$ it is never optimal for the firm to leave before recontracting. As a consequence, the shut down threat by shareholders is weakened and so is their contractual strength in the bargaining process at stage zero. Therefore the profit share is lower.

Discretionary intervention implies that the two stages are independent. This makes the circumstances in which the bargaining at stage zero takes place equivalent to those we get if the regulator sets $p_1 = 0$. Substituting $p_1 = 0$ into (26) yields the right-hand-side of the inequality in part (b) of the proposition. By the maximum principle of optimal control, the independency of the two stages reinforces the shareholders' contractual power leading to a higher profit share at stage zero as stated in the left-hand-side of the inequality in part (b) of the proposition.

Finally if K tends to zero, by Proposition 4, the asymmetry between the two players disappears and also the difference in profit share between the two stages. This is stated in the following corollary:

Corollary 2 *If the exit cost K decreases also the profit share differential reduces. In particular $K = 0$ yields:*

$$\theta_0^* = \theta_1^* = \theta_1^A$$

Proof: see Appendix D.

As expressed by corollary 1, when $K = 0$, θ_1 is no longer state-dependent. Then it becomes irrelevant when the regulator calls for recontracting. This

has a further implication. If we move back to stage 0, the symmetry of the market value leads both actors to choose the same exit policy and consequently the same θ_0 . This is shown by the contenders' bargaining power implicit in their utility functions.

5.2 Upturn recontracting: $p_0 < p_1$

Unlike in the previous section, the firm may now leave before recontracting takes place. That is, the firm may set a threshold price p_{L_0} , such that $p_{L_0} \leq p_0 \leq p_1$. While p_{L_0} is an absorbing barrier which causes exit, the *public trigger* p_1 can be seen now by the firm as a ceiling that prompts recontracting if it is touched before p_{L_0} (see figure 4). Under this hypothesis, making use of an indicator function, the firm's value at stage 0 becomes:

$$S_0(p_0; \theta_0) = E_0 \left\{ \int_0^{t_j} \theta_0(p_t - c) e^{-\rho t} dt + e^{-\rho t_j} \hat{S}_j \mid p_{t_0} = p_0 \right\}, \quad j = 0, 1. \quad (27)$$

For $j = 0$, we get $t_0 \equiv T_0 < t_1$, $\hat{S}_0 = -K$ and the firm exits before recontracting. If $j = 1$, we get, $\hat{S}_1 = S_1(p_1; \theta_1^*)$ and the associated value is equal to (22).

Since the price can freely move within the interval $[p_{L_0}, p_1]$, using the same procedure of stage 1, the general solution of (27) can be written as:

$$S_0(p_0; \theta_0) = A_{01} p_0^{\beta_2} + A_{02} p_0^{\beta_1} + \theta_0 \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right), \quad \text{for } p_0 \in [p_{L_0}, p_1]. \quad (28)$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the usual quadratic equation $\Psi(\beta)$.

Because of the existence of the ceiling p_1 , the general solution cannot be subject anymore to the boundary condition $S_0(\infty; \theta_0) = 0$. Whilst the last term on the r.h.s. of (28) gives the expected value of profits when neither exit nor the ceiling are binding, the other two terms represent the expected present value of the consequences of reaching the exit threshold and the price ceiling. In particular, the first term keeps the meaning of the value of the option to exit, while the second term represents the expected cost of reaching the price ceiling that prompts recontracting. The constant A_{01} (positive) and A_{02} (negative) are determined, together with p_{L_0} , by the following limit conditions:

$$S_0(p_{L_0}; \theta_0) = -K, \quad (29)$$

$$S'_0(p_{L_0}; \theta_0) = 0, \quad (30)$$

$$S_0(p_1; \theta_0) = S_1(p_1; \theta_1^*). \quad (31)$$

Now, we have a system composed of two value matching conditions and one smooth pasting condition for 3 unknowns: p_{L_0}, A_{01}, A_{02} . The last condition holds when the ceiling is binding, i.e. the price hits the level p_1 . Then recontracting takes place and the firm switches to a profit flow $\theta_1^*(p_t - c)$ which gives rise to a value $S_1(p_1; \theta_1^*)$.

Similarly the workers' well-being can be written as:

$$L_0(p_0; \theta_0) = E_0 \left\{ \int_0^{t_j} (1 - \theta_0)(p_t - c)e^{-\rho t} dt + e^{-\rho t_j} \hat{L}_j \mid p_{t_0} = p_0 \right\}, \quad j = 0, 1. \quad (32)$$

For $j = 0$, we get $t_0 \equiv T_0$ and $\hat{L}_0 = K$. If $j = 1$, $\hat{L}_1 = L_1(p_1; \theta_1^*)$ and the associated well being is given by (25). The solution takes the form:

$$L_0(p_0; \theta_0) = B_{01}p_0^{\beta_2} + B_{02}p_0^{\beta_1} + (1 - \theta_0) \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right), \quad \text{for } p_0 \in [p_{L_0}, p_1], \quad (33)$$

with boundary conditions:

$$L_0(p_{L_0}; \theta_0) = K, \quad (34)$$

$$L_0(p_1; \theta_0) = L_1(p_1; \theta_1^*). \quad (35)$$

The high degree of nonlinearity of the relationships associated with the equilibrium of both actors does not allow for derivation of closed form formulae for the 3 unknowns p_{L_0}, A_{01}, A_{02} . An insight into the investment value for both types of individuals outlined above can be obtained only by solving numerically the system.¹³

Plugging (28) and (33) into (7), the following proposition can be derived:

Proposition 5 *When $p_0 < p_1$ and the option to shut down at stage zero is viable (i.e. $p_{L_0} > 0$), the profit share in stage zero **with commitment** is always greater than the profit share **without commitment**, and both are greater than the one in stage 1. That is:*

$$\theta_0^* > \theta_0^*(p_1 = \infty) > \theta_1^*.$$

¹³Instances of numerical solutions can be found in Dixit (1989), Dixit and Pindyck (1994) and, with profit sharing, in Moretto and Pastorello (1995).

Proof: see Appendix E.

When $p_0 < p_1$ there is a credible threat that the firm may exit even before recontracting takes place. This can be seen in figure 4 where p_{L_0} is higher than p_{L_1} , the two trigger values imply exit either at T_0 or at T_1 . Shareholders are in a more favorable position since their shut down threat is now credible also at stage zero and the reinforcement of their bargaining power leads to $\theta_0^* > \theta_1^*$.

No preannounced rule, i.e. discretionary intervention, in this upturn case becomes equivalent to setting $p_1 = \infty$ by the regulator. This involves disappearance of the expected cost associated with the ceiling and a reduction in the shut down threat expressed by a lower value of the exit trigger price [$p_{L_0} > p_{L_0}(p_1 = \infty)$]. Therefore, the independency of the two stages weakens the shareholders' contractual power leading to a lower profit share at stage zero as stated by proposition 5.

Summarizing, if the regulator announces that the two actors recontract when profits are higher than the initial ones this results in a redistribution of profits to shareholders at stage zero which is higher than the distribution we would get if the regulator did not aim at any redistribution.

FIGURE 4 ABOUT HERE

Symmetrically with the result found in the previous section corollary 2 holds here as well. That is, even for the case of $p_0 < p_1$ this gap disappears as the exit becomes costless (see Appendix E). We provide in table 1 below a summary of the previous results.

Table 1

Profit share stage 0	commit.	no commit.	stage 1
upturn contracting	$\theta_0^* >$	$\theta_0^* >$	θ_1^*
downturn contracting	$\theta_0^* <$	$\theta_0^* <$	θ_1^*

5.3 A particular case: no exit allowed in stage 0

The last case we consider concerns the firm without the option to exit in stage zero even if $p_0 < p_1$. To justify the lack of the exit option we may assume that the regulator is now endowed with a further authority, which he did not possess before. Since he is aware of the possibility for the firm to exit before recontracting takes place, he obliges the firm to stay in the market. The regulator then becomes a sort of rescue board for firms which are quite likely to quit. It keeps them active since it expects that the market price may reach the more favorable level at which recontracting should start. This will be of some consequence for the bargaining and then it becomes interesting to see how income distribution among the two contenders is affected by this new policy of the regulator.

Formally, all this implies that the firm cannot determine any p_{L_0} at stage 0 and then p_t is free to fluctuate within the interval $(0, p_1)$. Referring to the solution (28) we need to substitute the boundary conditions (29) and (30) with the simple one $S_0(0; \theta_0) = 0$, which implies $A_{01} = 0$, while the *value matching condition* $S_0(p_1; \theta_0) = S_1(p_1; \theta_1^*)$ is still valid.

After some calculus we are able to write the shareholders' value function in the suitable form:

$$\begin{aligned} S_0(p_0; \theta_0) &= A_{02}p_0^{\beta_1} + \theta_0\left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho}\right) \\ &= S_1(p_0; \theta_1^*) - M(p_0; p_1) - (\theta_1^* - \theta_0)\Delta F(p_0; p_1 > p_0). \end{aligned} \quad (36)$$

Similarly, for the workers we obtain:

$$\begin{aligned} L_0(p_0; \theta_0) &= B_{02}p_0^{\beta_1} + (1 - \theta_0)\left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho}\right) \\ &= L_1(p_0; \theta_1^*) - N(p_0; p_1) + (\theta_1^* - \theta_0)\Delta F(p_0; p_1 > p_0), \end{aligned} \quad (37)$$

where:

$$\begin{aligned} M(p_0; p_1) &= A_1 p_0^{\beta_2} \left(1 - \left(\frac{p_0}{p_1}\right)^{\beta_1 - \beta_2}\right) > 0, \\ N(p_0; p_1) &= B_1 p_0^{\beta_2} \left(1 - \left(\frac{p_0}{p_1}\right)^{\beta_1 - \beta_2}\right) > 0, \end{aligned}$$

and:

$$\Delta F(p_0; p_1 > p_0) = \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho}\right) - \left(\frac{p_1}{\rho - \alpha} - \frac{c}{\rho}\right)\left(\frac{p_0}{p_1}\right)^{\beta_1}.$$

It is worth noting the resemblance of (36) and (37) with (24) and (25) of the downturn case. Similarly, we can interpret $(\frac{p_0}{p_1})^{\beta_1}$ as $E_0 \{e^{-\rho t_1} \mid p_{t_1} = p_1 > p_0\}$ where t_1 is now the random time when the process hits the upper level p_1 for the first time. Therefore, the expression $\Delta F(p_0; p_1 > p_0)$ represents the difference between the value of a firm that never shuts down starting at p_0 and the discounted value of a firm that starts later at t_1 when the price hits the upper boundary p_1 and never shuts down. However, unlike the downturn case, as well as $\Delta F(p_1; p_1 > p_0) = 0$ there may be another price level $\tilde{p} \leq p_1$ such that $\Delta F(\tilde{p}; p_1 > p_0) = 0$, so that the term ΔF changes sign twice within the operating interval $(0, p_1]$.

Let us now turn to the terms M and N . As $A_1 p_0^{\beta_2}$ and $B_1 p_0^{\beta_2}$ represent the values associated with the exit option owned by shareholders and workers respectively, and $0 < (\frac{p_0}{p_1})^{\beta_1 - \beta_2} < 1$, the terms M and N indicate the reduction of the exit option value due to the impossibility for the firm to leave in stage zero. Obviously such a reduction tends to zero as long as the gap between p_0 and p_1 disappears. Following the steps of the previous section, θ_0 can be obtained by rearranging proposition 4 in the following way:

Proposition 6 (a) *If $K > 0$, $p_0 < p_1$ and the shut down opportunity is excluded in stage zero, the optimal profit share is given by the following necessary condition:*

$$(\theta_0^* - \theta_1^*) \Delta F(p_0; p_1 > p_0) = (1 - \theta^A) (L_1(p_0; \theta_1^*) - N(p_0; p_1)) \times \quad (38)$$

$$\times \left(\frac{q}{1 - R} - \frac{S_1(p_0; \theta_1^*) - M(p_0; p_1)}{L_1(p_0; \theta_1^*) - N(p_0; p_1)} \right)$$

(b) *Yet, the profit share in stage zero may exhibit the following behavior: In the case $\tilde{p} < \hat{p}$ it can be shown that:*

$$\begin{aligned} \theta_0^* < \theta_1^* & \quad \text{if } \hat{p} < p_0 < p_1 \\ \theta_0^* > \theta_1^* & \quad \text{if } \tilde{p} < p_0 < \hat{p} \\ \theta_0^* < \theta_1^* & \quad \text{if } p_0 < \tilde{p} \end{aligned}$$

Or, in the case $\tilde{p} > \hat{p}$, we get:

$$\begin{aligned} \theta_0^* < \theta_1^* & \quad \text{if } \tilde{p} < p_0 < p_1 \\ \theta_0^* > \theta_1^* & \quad \text{if } \hat{p} < p_0 < \tilde{p} \\ \theta_0^* < \theta_1^* & \quad \text{if } p_0 < \hat{p} \end{aligned}$$

where:

$$\hat{p} = \inf \left\{ p > 0 \mid \frac{q}{1-R} - \frac{S_1(p; \theta_1^*) - M(p; p_1)}{L_1(p; \theta_1^*) - N(p; p_1)} = 0 \right\},$$

$$\tilde{p} = \inf \{ p > 0 \mid \Delta F(p; p_1 > p) = 0 \} < p_1.$$

Proof: see Appendix F.

Proposition 6 shows that the restriction on the shut down option by the firm during stage zero reduces the shareholders' bargaining power. This appears, as in the case $p_0 > p_1$, via a decrease of the shareholders' profit share with respect to the one bargained at stage 1. This means that the regulator ends up by providing workers with a further advantage. Workers are put at an advantage by the regulator owing to the absence of a shut down threat and the fact that recontracting takes place at a higher price. This affects the bargaining power of the shareholders making them worse off in stage 0.

Nevertheless there may exist an interval, $\tilde{p} < p_0 < \hat{p}$ (or $\hat{p} < p_0 < \tilde{p}$) where the relationship between profit share at stage zero and stage 1 reverses to the advantage of the shareholders, making our conclusions more complex. In this interval the objective of the regulator of playing the role of a rescue board is reached at a cost of a reversed profit distribution.

The obligation of the firm to stay in stage 0, does not always enhance the workers' power. It may happen that the lack of a threat of closure is substituted by a "threat of recontracting". The players know that the larger is the gap between p_1 and p_0 , the smaller the probability of reaching the ceiling p_1 in the near future. None of the contenders can influence the evolution of profits. As a consequence, when the regulator sets a high price ceiling p_1 two effects arise: *i*) the time of recontracting is postponed and indirectly also the time when the bonus is given to workers; *ii*) the profit share going to workers in stage 1 increases since the recontracting is associated to a high level of profits (proposition 2). By balancing these two effects workers may be better off with a lower profit share in stage 0. They then give up some of their bargaining power they receive by the firm's obligation to remain active. Yet they get a higher profit share in stage 1.

All this implies that the firm has an advantage in terms of profit distribution in recontracting when profits are low, once the value of p_1 is set. The picture changes when profits are very low, $p_0 < \tilde{p}$ (or $p_0 < \hat{p}$). Both

contenders are aware that recontracting will be quite unlikely and the probability of having operating losses during stage 0 increases. The ceiling loses its effectiveness and workers again prefer a larger profit share today than in a distant future.

A slightly different picture may emerge if we consider $1 < \beta_1 < 2$. In this case $(\frac{p_1}{p-\alpha} - \frac{c}{p})(\frac{p_0}{p_1})^{\beta_1}$ is concave in p , as it appears in figure 6 and appendix F. Equation $\Delta F = 0$ has only the root p_1 , which gives rise to two subintervals for profit distribution.

Corollary 3 *Since with $1 < \beta_1 < 2$ the term ΔF is always negative we have only \hat{p} . Below this price we shall get $\theta_0^* < \theta_1^*$, while above \hat{p} and below p_1 we have just the reverse $\theta_0^* > \theta_1^*$.*

Proof: see Appendix F.

Finally, despite the introduction of a new asymmetry expressed by the firm's inability to abandon within stage 0, if exit becomes costless also the profit share differential disappears, and the corollary 2 continues to hold here too (see Appendix F).

6 Conclusions

We have analysed the case of an Aoki's firm embedded in an uncertain environment where it can act in a flexible manner since it has the possibility of closing down by paying a bonus to laid off workers. Profit sharing is influenced by the shut down option in the hands of the shareholders since the bargaining power of workers is affected by the degree of credibility of the closure threat. However, carrying out this threat is costly for the firm since it has to pay a bonus to laid-off workers. Bargaining over the profit share, to be distributed among workers and shareholders, is repeated according to an exogenous mechanism endorsed by a regulator, whose aim is to regulate the firm via a workers oriented profit sharing. The regulator operates mostly in two ways. He may announce in advance a policy rule by maintaining that recontracting takes place when the market price hits either a high or a low price. Alternatively he may act discretionally by not announcing any intervention in advance. When the regulator commits to a preannounced policy rule we have two cases: *i*) with downturn bargaining (at a price p_1 lower

than the one at which bargaining takes place at stage 0) shareholders gain; *ii)* with upturn contracting the opposite happens.

Since profit sharing tends to reduce employment variability, the regulator obliges the contenders to recontract when market conditions are bad and the firm may be tempted to quit. In this case the bargaining leads to an income distribution favorable to the shareholders since workers are willing to give up some of their profits to keep the firm in operation. This result must be associated with some degree of specificity on the part of workers, while specificity on the part of shareholders is not excluded, but is assumed to be far lower than that of workers. The regulator intervenes also in different circumstances when market conditions are bouyant to reach a fairer income distribution and to this purpose he simply obliges the two parties to recontract . In that case recontracting takes place when the market price is fairly high and the shut down threat by the firm loses strength. Workers take advantage of that by gaining a larger share of profits.

When the regulator acts discretionally the profit distribution differential reduces for both cases under analysis making the time profile of the profit share parameter smoother.

We consider a further case where the regulator is empowered with the authority of prohibiting the closing down of the firm in the first period. The regulator becomes a sort of rescue board similar to those that were quite active in the 70's. The result is an overshooting of the regulator far beyond the target it used to pursue. By eliminating the shut down opportunity *only* in the first period the regulator not only makes jobs safer, but also increases the bargaining power of workers, who may take advantage of that getting a larger chunk of profits.

Other results are possible according to the level of the bonus workers get when they are laid off. The exogeneity of the bonus and the fact that it is paid to workers are the main limitations of this work. However they point to future directions of research through the consideration of a more complex structure of the sunk costs.

A Appendix

Substituting (12) and (17) into (5) and taking account of (15) and (20) we can derive the first order condition (FOC)

$$\frac{\frac{dL_1}{d\theta_1} S_1}{\frac{dS_1}{d\theta_1} L_1} + \frac{q}{1-R} = 0. \quad (39)$$

Moreover, by the fact that $S_1 = \theta_1 V$ and $L_1 = (1 - \theta_1)V + G$ we obtain $\frac{dS_1}{d\theta_1} = V + \theta_1 \frac{dV}{d\theta_1}$, and $\frac{dL_1}{d\theta_1} = -V + (1 - \theta_1) \frac{dV}{d\theta_1} + \frac{dG}{d\theta_1} = -\frac{dS_1}{d\theta_1} + \frac{dV}{d\theta_1} + \frac{dG}{d\theta_1}$, therefore the FOC can be rewritten as:

$$- \left[\frac{S_1}{L_1} - \frac{\frac{dV}{d\theta_1} + \frac{dG}{d\theta_1}}{\frac{dS_1}{d\theta_1}} \frac{S_1}{L_1} - \frac{q}{1-R} \right] = 0 \quad (40)$$

Since $\frac{dV}{d\theta_1} + \frac{dG}{d\theta_1} = -\frac{\beta_2 \rho K}{\theta_1(c\theta_1 - \rho K)} G > 0$, and $\frac{dS_1}{d\theta_1} = V + \theta_1 \frac{dV}{d\theta_1} = V + G > 0$ we can conclude that

$$\Phi(p_1; \theta_1, K) = \frac{\frac{dV}{d\theta_1} + \frac{dG}{d\theta_1}}{\frac{dS_1}{d\theta_1}} > 0$$

This proves the first part of proposition 1. As $K \rightarrow 0$ workers and shareholders are symmetric with respect to the bargaining process and then it becomes easy to show that: $\lim_{K \rightarrow 0} \Phi(p_1; \theta_1, K) = 0$ which reduces (40) to Aoki's weighting rule:

$$\frac{S_1}{L_1} \equiv \frac{\theta_1}{1 - \theta_1} = \frac{q}{1 - R} \quad (41)$$

Then, by concavity of the bargaining function ∇_1 at the optimum and by the positivity of Φ we get the second part of proposition 1. Equation (41) also proves the corollary, while the case of certainty is straightforward.

B Appendix

To prove proposition 2 let us write the FOC (40) as a functional¹⁴ of p :

¹⁴The term $\Phi(p; \theta_1^*, K) \frac{S_1(p; \theta_1^*)}{L_1(p; \theta_1^*)}$ in figure 5 is called ω .

$$y(p; \theta_1^*) = \frac{S_1(p; \theta_1^*)}{L_1(p; \theta_1^*)} - \Phi(p; \theta_1^*, K) \frac{S_1(p; \theta_1^*)}{L_1(p; \theta_1^*)} - \frac{q}{1-R} \quad \text{for } p \in [p_{L_1}, \infty). \quad (42)$$

The necessary condition for θ_1^* requires $y(p_1; \theta_1^*) = 0$ at $p = p_1$. Therefore using the implicit function theorem, we need to show that:

$$\frac{d\theta_1^*}{dp_1} = -\frac{\frac{dy(p_1; \theta_1^*)}{dp_1}}{\frac{dy(p_1; \theta_1^*)}{d\theta_1^*}} < 0 \quad (43)$$

The denominator of (43) represents the second order condition, which we assume to hold for the relevant values of θ_1 , then the sign of (43) is determined by the numerator. To derive (43) we start analyzing the ratio $\frac{S_1}{L_1}$. By the concavity of both S_1 and L_1 , the following limits hold: $\lim_{p \rightarrow 0} \frac{S_1}{L_1} = \frac{A_1}{B_1} (< 1)$, $\lim_{p \rightarrow p_{L_1}} \frac{S_1}{L_1} = -1$ and $\lim_{p \rightarrow \infty} \frac{S_1}{L_1} = \frac{\theta_1^*}{1-\theta_1^*}$. Moreover as $(\frac{S_1}{L_1})' = \frac{S_1'}{L_1} - \frac{L_1'}{L_1} \frac{S_1}{L_1}$, by evaluating it at p_{L_1} we obtain $(\frac{S_1}{L_1})'(p_{L_1}) = \frac{L_1'(p_{L_1})}{K} < 0$. Then $\frac{S_1}{L_1}$ appears as shown in fig.5. Let us now consider the term $\Phi(p)$. Without going into the details it is easy to show that the following limits hold: $\lim_{p \rightarrow p_{L_1}} \Phi(p) = -\infty$, and $\lim_{p \rightarrow \infty} \Phi(p) = 0$ and $\lim_{p \rightarrow \bar{p}} \Phi(p) > 0$.

We are now ready to analyze $y(p; \theta_1^*)$. Putting together $\frac{S_1}{L_1}$ and Φ we can conclude that $y(p; \theta_1^*)$ is monotonically increasing in p with the properties that $\lim_{p \rightarrow p_{L_1}} y(p; \theta_1^*) = -\infty$, $\lim_{p \rightarrow \bar{p}} y(p; \theta_1^*) = -\frac{q}{1-R}$, $\lim_{p \rightarrow p_{L_1}} y(p; \theta_1^*) = \frac{\theta_1^*}{1-\theta_1^*} - \frac{q}{1-R}$ and obviously $\lim_{p \rightarrow p_1} y(p; \theta_1^*) = 0$.

A graphic appearance of $y(p; \theta_1^*)$ is in figure 5. As long as the positive monotonicity of $y(p; \theta_1^*)$ holds also at $p = p_1$, proposition 2 is proved.

FIGURE 5 ABOUT HERE

C Appendix

If the firm exits before recontracting there should exist a p_{L_0} greater than p_1 at which the recontracting takes place. This also implies that in the interval

$[p_{L_0}, \infty)$ the price can move freely driven by (1). Therefore, as was done in stage 1, we can write:

$$\begin{aligned} S_0(p_0; \theta_0) &= E_0 \left\{ \int_0^{T_0} \theta_0(p_t - c)e^{-\rho t} dt - Ke^{-\rho T_0} \mid p_{t_0} = p_0 \right\} \\ &= A_0 p_0^{\beta_2} + \theta_0 \left(\frac{p_0}{\rho - \alpha} - \frac{c}{\rho} \right), \quad \text{for } p_0 \in [p_{L_1}, \infty), \end{aligned} \quad (44)$$

with boundary conditions:

$$S_0(p_{L_0}; \theta_0) = -K, \quad (45)$$

$$S'_0(p_{L_0}; \theta_0) = 0, \quad (46)$$

from which we get:

$$p_{L_0} = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c - \frac{\rho}{\theta_0} K \right). \quad (47)$$

Now, comparing p_{L_0} with p_{L_1} it is easy to note that $p_{L_0} > p_1 > p_{L_1}$ *if* $\theta_0 > \bar{\theta}_0 > \theta_1$, where $\bar{\theta}_0 = \inf \{ \theta_0 > 0 \mid p_{L_0} = p_1 \}$. However, if the firm has chosen p_{L_0} at stage zero and bargains over θ_0 with the workers, it would yield a profit sharing $\theta_0^* < \theta_1^*$ since the bargaining starts from a price p_0 larger than p_1 . This implies that $p_{L_0} < p_{L_1} < p_1$, which runs contrary to the necessary condition for an optimal exit before recontracting.

D Appendix

At stage zero the bargaining function can be expressed as:

$$\max_{\theta_0} \nabla_0 = \max \left(\lg \{ L_0^{1-R} \} + \lg \{ S_0^q \} \right).$$

Hence the FOC becomes:

$$\frac{d\nabla_0}{d\theta_0} = \frac{1-R}{L_0} \frac{dL_0}{d\theta_0} + \frac{q}{S_0} \frac{dS_0}{d\theta_0} = 0. \quad (48)$$

Recalling that $S_0 = S_1 - (\theta_1^* - \theta_0)\Delta F$ and $L_0 = L_1 + (\theta_1^* - \theta_0)\Delta F$, we get $\frac{dL_0}{d\theta_0} = -\frac{dS_0}{d\theta_0}$ which substituted into (48) together with the expressions for L_0 and S_0 yields:

$$(\theta_0^* - \theta_1^*)\Delta F(p_0; p_1 < p_0) = (1 - \theta_1^A)L_1(p_0; \theta_1^*)(\frac{q}{1-R} - \frac{S_1(p_0; \theta_1^*)}{L_1(p_0; \theta_1^*)}). \quad (49)$$

As $\Delta F > 0$ and $\frac{q}{1-R} - \frac{S_1(p_0; \theta_1^*)}{L_1(p_0; \theta_1^*)} < 0$ at p_0 (as shown in figure 5), we get $(\theta_1^* - \theta_0^*) > 0$.

As long as the discretionary intervention by the authority means that the two stages are independent of each other, in our setting this is equivalent to assuming $p_1 = 0$. If this is the case we get $\Delta F(p_0; p_1 = 0) > \Delta F(p_0; p_1 < p_0)$ which, by (49), leads to the inequality of part (b) of the proposition.

Finally, as K appears only on the r.h.s. of (49), taking the derivative with respect to K yields:

$$\frac{d(\theta_0^* - \theta_1^*)}{dK} = \frac{dL_1}{dK} \frac{q}{1-R} - \frac{dS_1}{dK} > 0. \quad (50)$$

In particular, when $K = 0$ the FOC becomes:

$$\frac{S_0}{L_0} \equiv \frac{\theta^A V - (\theta^A - \theta_0)\Delta F}{(1 - \theta^A)V + (\theta^A - \theta_0)\Delta F} = \frac{q}{1-R} \equiv \frac{\theta^A}{1 - \theta^A}, \quad (51)$$

which to hold must be $\theta_0^* = \theta^A$. This prove the corollary.

E Appendix

By the high degree of non linearity, both S_0 and L_0 cannot be written as functions of only exogenous variables. In fact, substituting (28) and (33) into the *matching value conditions* (31) and (35) we obtain the following simplified forms:

$$S_0(p_0; \theta_0, \theta_1^*) = S_1(p_0; \theta_1^*) - (\theta_1^* - \theta_0)\Delta F(p_0; p_1 < p_0) + A_{02}(\theta_0)\Omega(p_0; p_1), \quad (52)$$

$$L_0(p_0; \theta_0, \theta_1^*) = L_1(p_0; \theta_1^*) - (\theta_1^* - \theta_0)\Delta F(p_0; p_1 < p_0) + B_{02}(\theta_0)\Omega(p_0; p_1). \quad (53)$$

where $\Omega(p_0; p_1) = p_1^{\beta_1} \left[1 - \left(\frac{p_0}{p_1} \right)^{\beta_2} \right] < 0$. Unfortunately, as A_{02} and B_{02} are both functions of the unknown parameter θ_0 , the FOC (48) takes up the

uneasy form:

$$\frac{S_0(p_0; \theta_0, \theta_1^*)}{L_0(p_0; \theta_0, \theta_1^*)} = \frac{q}{1-R} \Pi(\theta_0), \quad \text{with } \Pi(\theta_0) = \left[1 - \frac{\left(\frac{dB_{02}}{d\theta_0} + \frac{dA_{02}}{d\theta_0} \right) \Omega}{\frac{dL_1}{d\theta_0}} \right] \quad (54)$$

Both the r.h.s. and l.h.s. of (54) depend on θ_0 , and then it is not possible to refer only to the behavior of the ratio $\frac{S_0}{L_0}$ to obtain the optimal profit share as for $p_0 > p_1$. Instead, let us rely on the continuity of value functions (52) and (53). Putting together (29), (30) and (31), and solving for p_{L_0} yields:

$$p_{L_0} = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c - \frac{\rho K}{\theta_0} \right) - \frac{\beta_2 - \beta_1}{\beta_2 - 1} \frac{\rho - \alpha}{\theta_0} A_{02} p_1^{\beta_1} \quad (55)$$

When $p_1 \rightarrow \infty$ recontracting becomes less likely. Therefore $A_{02} \rightarrow 0$, i.e. p_{L_0} is decreasing in p_1 and in particular when $p_1 = \infty$ the trigger exit price reduces to $p_{L_1}(p_1 = \infty) = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c - \frac{\rho K}{\theta_0} \right)$. Then it appears that $p_{L_0} > p_{L_0}(p_1 = \infty)$. However, when $p_1 = \infty$ the two stages are not related and the formula for the trigger price p_{L_0} is equal to the one at stage 1 except for θ . Therefore, if the firm bargains with workers at stage 0 with an initial price $p_0 < p_1$ we get $\theta_0^*(p_1 = \infty) > \theta_1^*$. Finally, as the price ceiling reduces the value of the firm, by the above inequality, we obtain $p_{L_0} > p_{L_0}(p_1 = \infty) > p_{L_1}$. By continuity of the value functions and proposition 2 we may conclude that also $\theta_0^* > \theta_0^*(p_1 = \infty) > \theta_1^*$.

Despite the complexity of the value functions (52) and (53), they can be used to prove corollary 2. Letting $K \rightarrow 0$, the FOC reduces to:

$$\frac{S_0}{L_0} \equiv \frac{\theta^A V - (\theta^A - \theta_0) \Delta F + A_{02}(\theta_0) \Omega}{(1 - \theta^A) V + (\theta^A - \theta_0) \Delta F + B_{02}(\theta_0) \Omega} = \frac{q}{1-R} \Pi(\theta_0) \equiv \frac{\theta^A}{1 - \theta^A} \Pi(\theta_0) \quad (56)$$

Yet, the symmetry between workers and shareholders, induced by $K = 0$, means that $A_{02}(\theta_0) = \theta_0 \hat{A}$ and $B_{02} = (1 - \theta_0) \hat{A}$, where \hat{A} is a constant common to both players. Substituting the above expressions into (56) and noting that $\frac{dA_{02}}{d\theta_0} = -\frac{dB_{02}}{d\theta_0}$, so that $\Pi(\theta_0) = 1$, the FOC gives $\theta_0 = \theta^A$.

F Appendix

As in appendix D, by the symmetry of both L_0 and S_0 with respect to θ_0 , the FOC becomes:

$$\frac{S_0}{L_0} = \frac{q}{1-R}. \quad (57)$$

Substituting the expressions for L_0 and S_0 in (36) and (37) and rearranging yields:

$$\begin{aligned} (\theta_0^* - \theta_1^*)\Delta F(p_0; p_1 > p_0) &= (1 - \theta^A) (L_1(p_0; \theta_1^*) - N(p_0; p_1)) \times \\ &\times \left(\frac{q}{1-R} - \frac{S_1(p_0; \theta_1^*) - M(p_0; p_1)}{L_1(p_0; \theta_1^*) - N(p_0; p_1)} \right) \end{aligned} \quad (58)$$

Unlike in Appendix B we now need to analyze both the ratio $\frac{S_1-M}{L_1-N}$ and the difference ΔF . Let us start with the former. By the concavity of both S_1 and L_1 and by the limits: $\lim_{p \rightarrow p_1} M = \lim_{p \rightarrow p_1} N = 0$ we get $\lim_{p \rightarrow p_1} \frac{S_1-M}{L_1-N} = \frac{S_1(p_1; \theta_1^*)}{L_1(p_1; \theta_1^*)}$. By referring to p_{L_1} , as long as $\lim_{p \rightarrow p_{L_1}} S_1 = -K$ and $\lim_{p \rightarrow p_{L_1}} L_1 = K$, we get $\lim_{p \rightarrow p_{L_1}} \frac{S_1-M}{L_1-N} = \frac{-K-M}{K-N}$, whilst by (36) and (37) we get $\lim_{p \rightarrow \infty} \frac{S_1-M}{L_1-N} = \frac{A_1}{B_1}$ (< 1) and $\lim_{p \rightarrow 0} \frac{S_1-M}{L_1-N} = \frac{\theta_1}{1-\theta_1}$. Therefore, recalling the behavior of the ratio $\frac{S_1}{L_1}$ as analyzed in appendix B, there may exist a value $p_{L_1} < \hat{p} < p_1$ such that $S_1(\hat{p}; \theta_1^*) - M(\hat{p}; p_1) = \frac{q}{1-R} (L_1(\hat{p}; \theta_1^*) - N(\hat{p}; p_1))$; for $p < \hat{p}$ the l.h.s. of (58) is positive whilst for $p > \hat{p}$ the l.h.s. is negative. Fig.5 describes also the ratio $\frac{S_1-M}{L_1-N}$, within the interval of interest $(0, p_1]$ for the case in cui $\frac{\theta_1}{1-\theta_1} > 1$.

Finally, it easy to show that $\Delta F(p_1; p_1 > p_0) = 0$ and $\Delta F(0; p_1 > p_0) = -\frac{c}{\rho}$. Hence, within the interval $(0, p_1]$, $\Delta F(p; p_1 > p)$ has a second root $\tilde{p} \leq p_1$. For $0 < p < \tilde{p}$ the r.h.s. of (58) is negative, whilst for $\tilde{p} < p < p_1$ the r.h.s. of (58) becomes positive. By joining the positive and negative intervals for the r.h.s. and l.h.s. of (58) as described above we get the second part of proposition 6.

When $K = 0$ we get $A_1 = \theta_1 A$, therefore $B_1 = (1 - \theta_1)A$, $M = \theta_1 \hat{M}$ and $N = (1 - \theta_1) \hat{M}$, where $\hat{M} = p_0^{\beta_2} \left(1 - \left(\frac{p_0}{p_1} \right)^{\beta_1 - \beta_2} \right)$. Substituting into (57) the FOC reduces to:

$$\frac{S_0}{L_0} \equiv \frac{\theta^A (V - \hat{M}) - (\theta^A - \theta_0) \Delta F}{(1 - \theta^A) (V - \hat{M}) + (\theta^A - \theta_0) \Delta F} = \frac{q}{1-R} \frac{\theta^A}{1 - \theta^A} \Pi(\theta_0) \quad (59)$$

which is satisfied when $\theta_0 = \theta^A$.

FIGURE 6 ABOUT HERE

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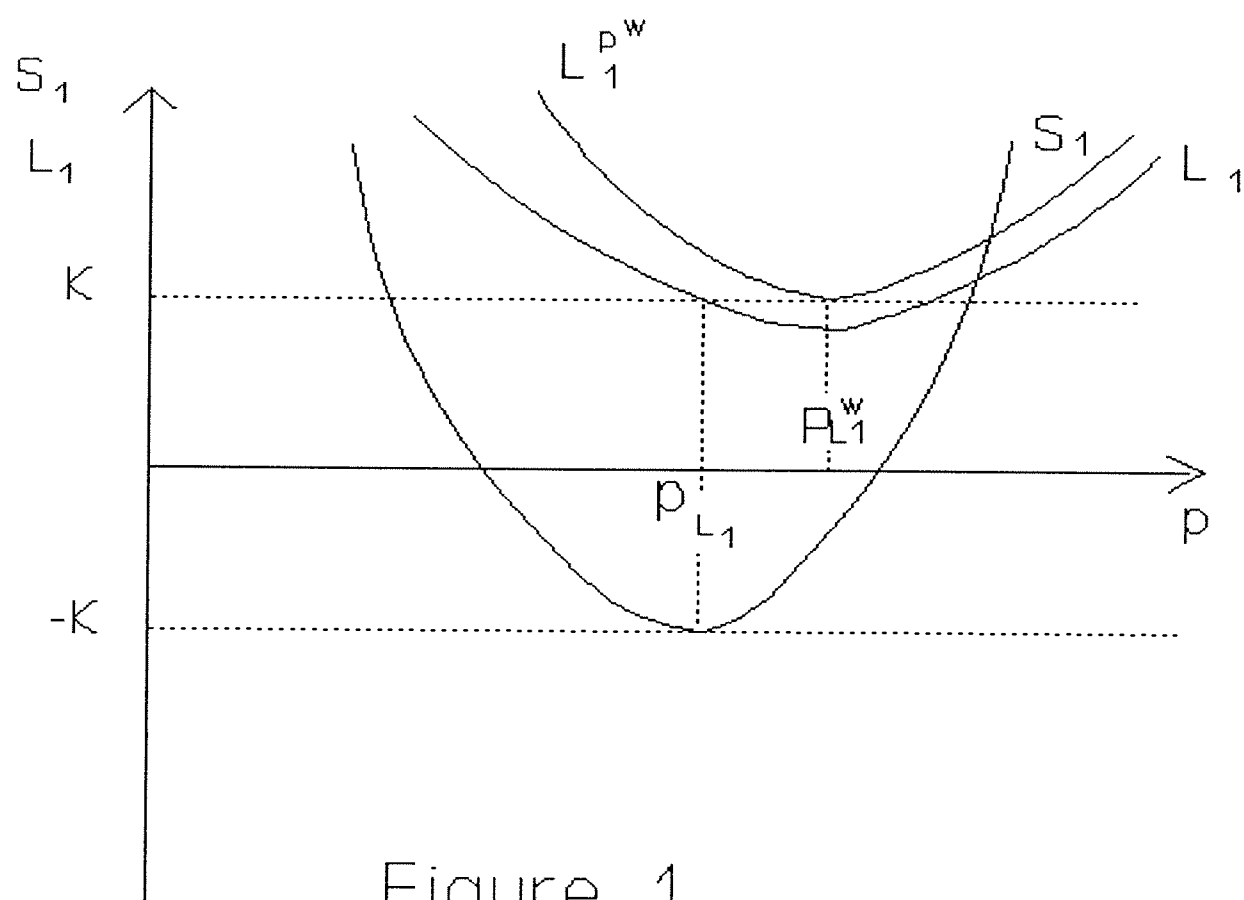


Figure 1

Figure 2

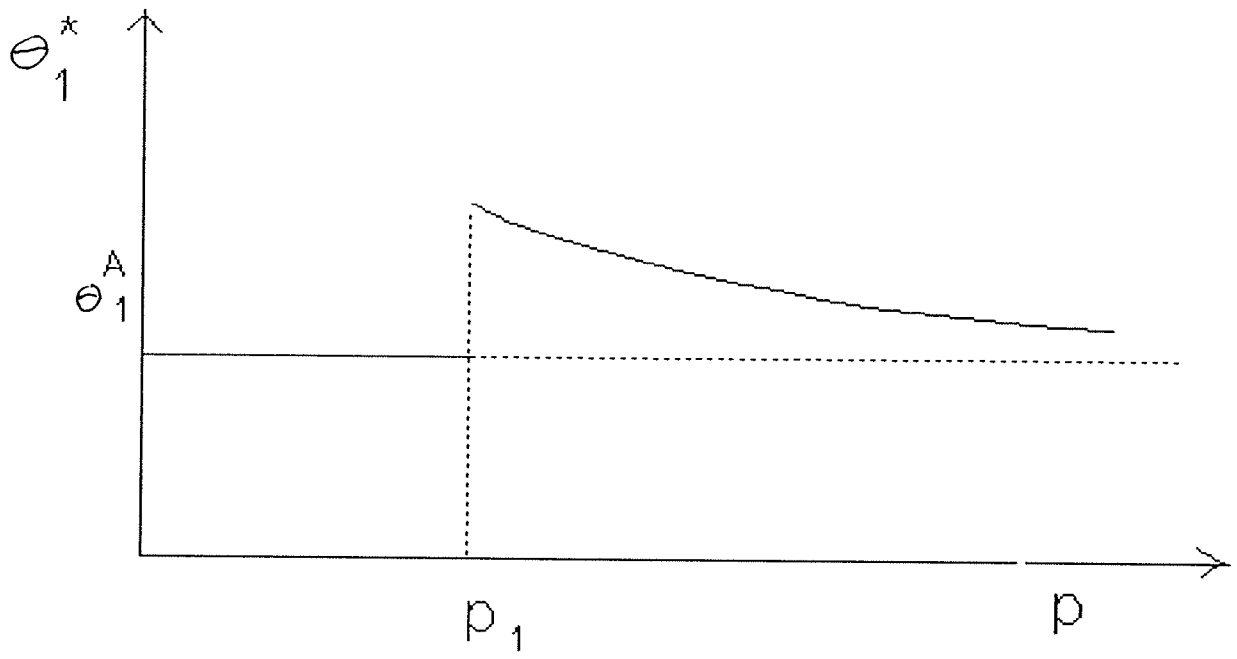


Figure 3

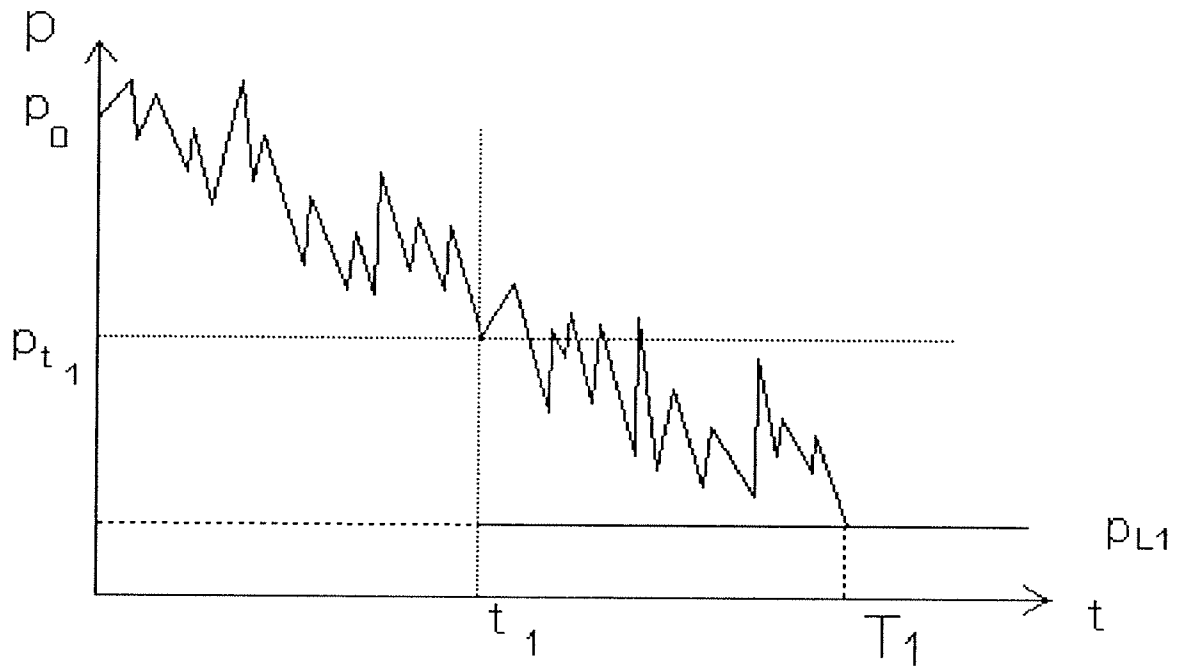


Figure 4

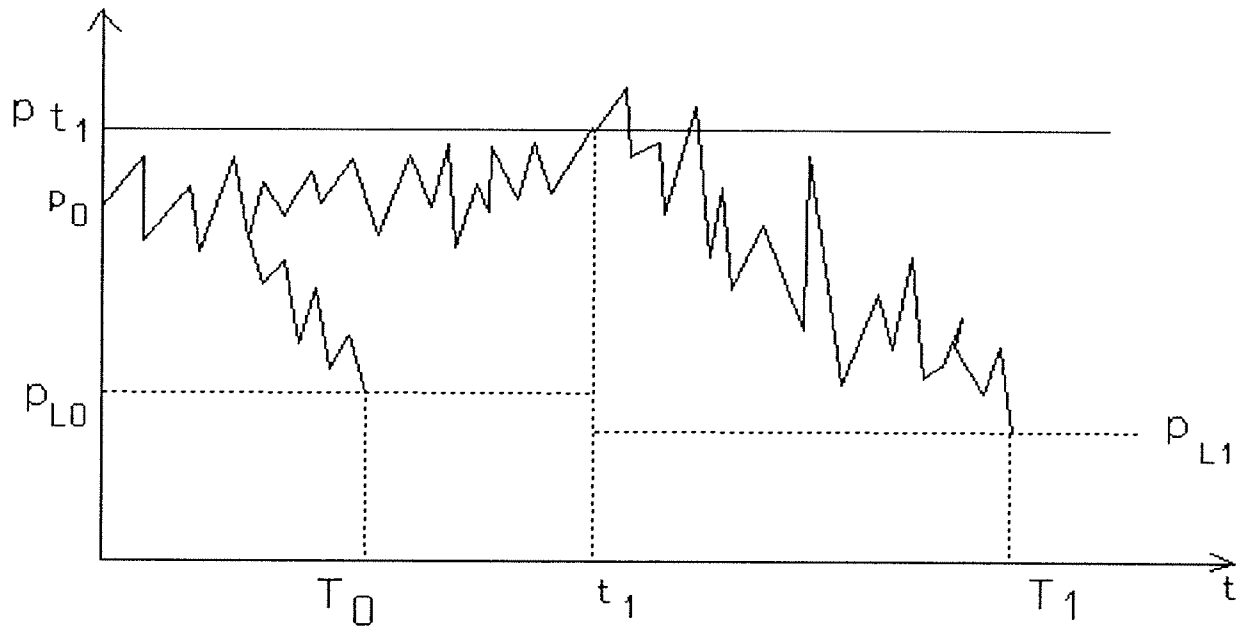
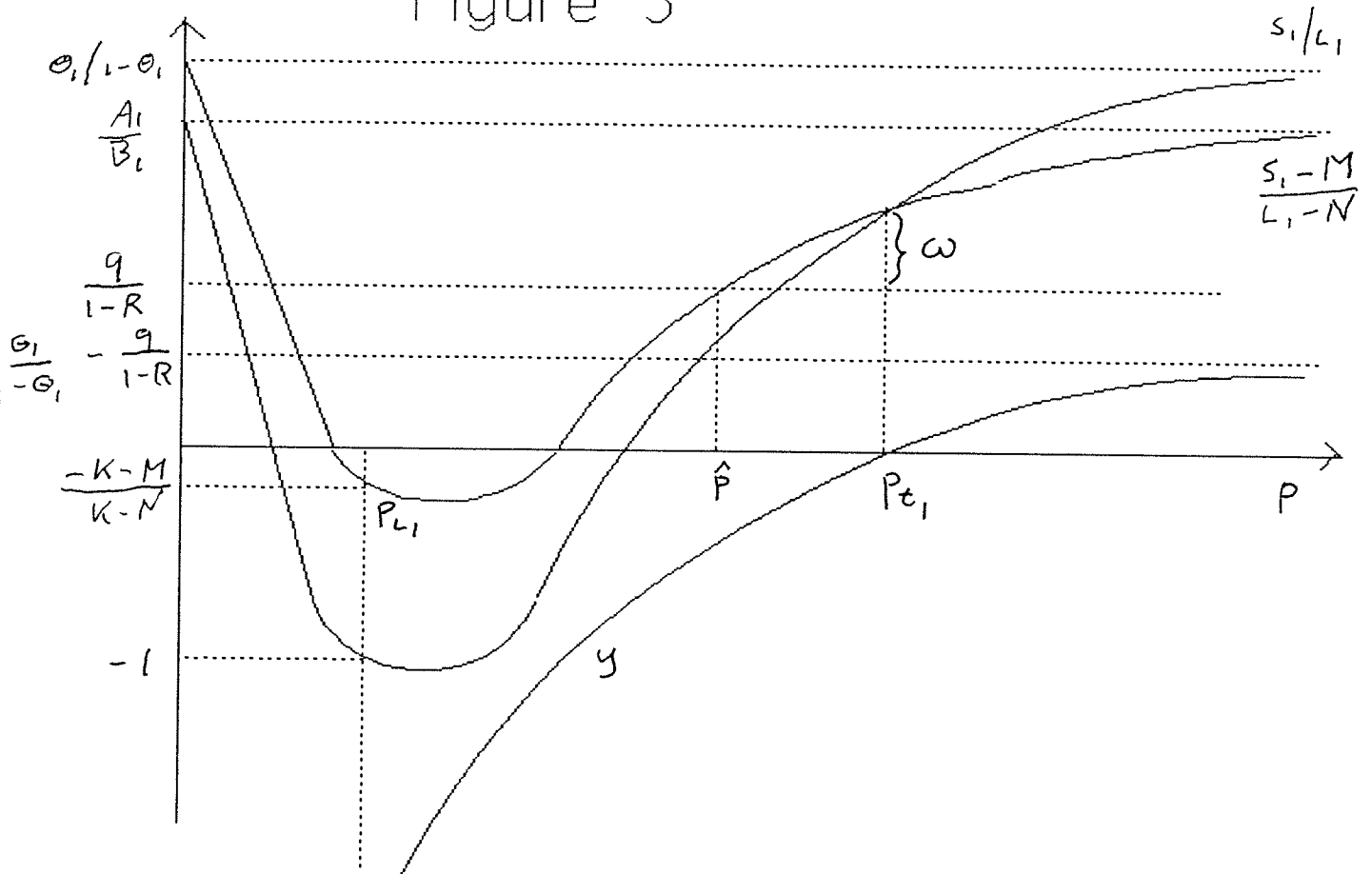


Figure 5



$$\omega = \phi \frac{s_1}{L_1} > 0$$

$$\frac{\theta_1}{1-\theta_1} > 1$$

$$\frac{A_1}{B_1} < 1$$

Figure 6

