Franzoni, Luigi Alberto

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Prosecutorial discretion and criminal deterrence

LUIGI ALBERTO FRANZONI*†
UNIVERSITY OF BOLOGNA, ITALY

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ABSTRACT. This paper develops a model of law enforcement in which
the indicted and the enforcer can negotiate the level of the penalty by means
of a settlement. The emphasis of the analysis is on the credibility of the set-
tlement offer: the enforcer cannot threaten an incredibly large conviction rate
if the negotiation fails. The introduction of the negotiation stage brings about
several novel features of the optimal enforcement policy, one of which is the
possibility that a finite penalty is optimal (globally or locally). We show that
the skimming process associated with the negotiation stage reduces the incen-
tives for the enforcer to carry out thorough investigations and increases the
rate of noncompliance.

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*Correspondence to: Luigi Alberto Franzoni, Dept. of Economics, Strada Maggiore 45, 40125
Bologna, Italy. Fax: IT-51-6402664. E-mail: franzoni@boph01.cineca.it.
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between the ex-post perspective taken by prosecutors and the ex-ante perspective usually taken by
the legislature and the economists.
1. Introduction

In many legal settings, negotiations over the level of the penalty are allowed to take place between the law enforcer and indicted individuals. The negotiation typically ends up with a guilty plea for criminal cases and a settlement for civil cases: in both cases, the offender agrees to be inflicted a certain sanction so as to avoid a regular trial. The negotiated enforcement of law is an extremely widespread practice, and targets the objective of preserving the economic resources that would be wasted for costly investigations. Its social desirability, however, is quite controversial, and has been objected by many legal scholars.¹

A strong argument in favour of pre-trial negotiations comes from the literature on law and economics, which recognizes that they reduce the conviction costs and produce an efficiency gain. The core of this argument can be found in Grossman and Katz (1983) and can be stated in the following terms: settlements, as well as plea bargains, allow guilty agents (who accept the settlement offer) to separate themselves from innocent ones (who reject the offer), with the effect of reducing the overall conviction costs. In other words, settlements act as a screening device, providing a quick and almost free conviction of the defendants who would otherwise be bound to lose their case.

On the negative side, it has been pointed out that settlements may provide indicted agents with unnecessary bargaining power, which allows them to net a discount on the due penalty. In turn, this lowers the expected sanction for offenders and induces more frequent violations. The latter argument depends strongly on the assumption that the indicted has some bargaining power at the settlement stage, and that he can compel the enforcer to agree on a settlement amount which is less than the statutory penalty for the infraction at hand.

In this paper, we provide a different explanation to the fact that settlements usually allow guilty offender to get away with a significantly reduced payment. We assume, as it is often the case, that the indicted has no bargaining power, and that the settlement offer is made on a take-it-or-leave-it base by the enforcer. The reason why settlements involve amounts lower than the statutory penalty has to be identified with the fact that the enforcer cannot commit itself to put full effort into the case after a failure to settle. This, in turn, occurs since the enforcer suffers from imperfect information at the settlement stage (he does not know whether the indicted is innocent or guilty) and the elimination of this uncertainty (by means of a thorough investigation) is costly. Hence, the enforcer may not be able to threaten a sure conviction to those who refuse to accept his offer, and the different types of agents (guilty and innocent) do not fully separate at the settlement stage. In general, if the enforcer wants his offer to be accepted with a larger probability, he will have to

¹A good introduction to the debate on the impact of pre-trial bargains on the legal system can be found in the 1991 issue of the Yale Law Journal.
content himself with a low settlement amount.\textsuperscript{2}

In our model, the terms of the enforcement process depend on the will of different institutions/individuals involved in it at different times. We assume that the enforcement agency is two-tiered; on the first tier, we have a legislature which decides the level of the penalty for convicted offenders, on the second, we have an independent investigation branch ("the enforcer"), whose aim is to discover whether the indicted is guilty or innocent with respect to a specific infraction. The two branches of the agency enter the enforcement process at different stages; the legislature decides the penalty at the outset, the enforcer decides which effort to devote to the investigation after the violation, if any, has been perpetrated. As in many real situations, the viewpoint of the enforcer is a typical \textit{ex-post} perspective, in which the attention is focused on the cost-effective conviction of guilty offenders rather than the deterrence effect on crime. The legislature, on the other hand, takes an \textit{ex-ante} perspective, and selects the penalty level that minimizes the social loss (expected harm minus net recovery) associated with the potential offense.

The main conclusion of this paper is that pre-investigation settlements play a double role in the enforcement process. On the one hand, they allow the enforcer to effectively reduce enforcement costs: if the settlement offer is accepted, the enforcer collects a settlement amount without having to prove the guilt of the defendant. On the other hand, however, the screening out of guilty defendants operated at the negotiation stage induces an overall weakening of the deterrent power of enforcement policy, because it reduces the incentives for the enforcer to carry out thorough investigations after a failure to settle. As a consequence, the introduction of the settlement stage increases both the \textit{ex-post} net recovery from possibly guilty agents and the level of crime. These two factors affect social welfare in opposing ways. Which of the two effects dominates depends on the ratio between the level of the harm associated with the crime and the amount of resources that can be extracted from the offender at the settlement stage.

An in-depth account of the credibility constraint hanging over the enforcement process allows us to identify new factors relevant to determining the optimal penalty. In particular, we will provide an alternative justification to the commonly observed lack of the maximal sanctions which are recommended by deterrence theory. While the "Principle of Maximum Deterrence" states that the optimal penalty is maximal whenever penalties can be inflicted without costs, in the real world, sanctions are rarely set to their maximum level, even when this is finite, i.e., when limited liability applies.\textsuperscript{3} Our way of reconciling theory and evidence is rather intuitive; when the level

\textsuperscript{2}The problem of the credibility of settlement offers has been forcefully raised by Nalebuff (1987) in a model involving civil suits. In Nalebuff’s model, the plaintiff has imperfect information about the true liability of the defendant and makes a settlement offer prior to the trial. The credibility problem arises because the plaintiff may be tempted to drop the case if a settlement is not reached. Nalebuff limits his analysis to the settlement stage, and does not consider the impact of the negotiation on the production of harm.

\textsuperscript{3}The optimality of a maximal penalty is derived and discussed in the early works of Becker
of harm associated with the crime is relatively small relative to the private benefit that the criminal gets from it, it may be socially desirable to let the agent engage in crime so as to reduce the uncertainty suffered by the enforcer at the settlement stage and obtain the largest benefits form the negotiation.

The explicit consideration of the *ex-ante* effects of pre-trial negotiations on the production of crime distinguishes this contribution to most of the other works on settlements. The standard view is to take the probability that a crime has been committed as given, and to study the effects of settlements on the distribution of the legal and investigative costs.\(^4\) A work related to ours is Polinsky and Rubinfeld (1988), which analyses the desirability of settlements in a simple model of harm prevention. In their model, the parties enjoy perfect information about the level of harm caused by accidents. As a consequence, trials entail pure "transaction costs." They assume that both parties held some bargaining power at the negotiation stage and that settlements reduce the expected cost of an accident for the injurer. Under these conditions, trials are socially desirable only if settlements do not provide the potential injurer with adequate incentives to care. In Franzoni (1995), a model of tax enforcement is developed in which the tax agency can precommit itself to the audit policy, and uses it to extract a settlement amount from taxpayers. Settlements represent here a form of "renegotiation" of the original enforcement policy, which proves to increase the payoff to the agency when the original policy is not optimally chosen.

The paper is organized as follows. In section 2, we introduce the model and derive the equilibrium of the game between the agent and the enforcer. The game is solved backwards from the settlement stage. This allows us to derive a theory of settlements, which is of interest in itself. In section 3, we characterize the optimal penalty and discuss the relation between our result and the Principle of Maximum Deterrence. In section 4, we discuss the desirability of the settlement stage by contrasting the outcome of our game with the outcome of a game in which negotiations cannot take place between the enforcer and the individual. In section 5, the model is extended so as to give account of a variable probability of indictment, a variable benefit from non-compliance and a variable penalty. Finally, section 6 provides a brief summary and some concluding remarks.

2. The enforcement game

The problem we are going to consider relates to the enforcement of a specific law or regulation, which requires the agent to behave in a specified manner or to refrain from taking a certain action. For example, environmental regulations might require that firms refrain from using polluting production processes, fiscal regulations that

taxpayers keep their accounts in a specific way, etc. If the agent does not abide by his obligation, (social) harm of amount $H$ is produced. Our emphasis is on crimes of economic nature, where rationality of both parties, the agent and the enforcer, can be reasonably assumed.

We assume that the agent can be of two types: honest or opportunistic. The honest type abides by his obligation out of respect for his ethical code, while the opportunistic type rationally assesses the costs and benefits of compliance, and complies only if the expected penalty is larger than his individual gain from noncompliance. The opportunistic agent maximizes a von Neumann-Morgenstern utility function and is neutral towards risk. For the honest type, the benefit from noncompliance is 0 (or negative), while for the opportunistic it is equal to $T$. The probability that the agent is honest is $p$. The reason why the opportunistic agent may decide to comply is that he may incur the risk of an investigation. This investigation can take the form of statistical testing in the case of pollution control, an audit in the case of tax infractions, and the reconstruction of the events in the case of generic crimes and frauds. If the investigation shows that the agent has defected, he has to pay a penalty in the amount $F$.

We assume that the investigation process is triggered by a random event (signal), which might be tied up with the behaviour of the agent. For the time being, however, we assume that this event is independent of the agent's choice and occurs with a constant probability $\pi$. In section 5, we will extend our model to the more realistic case in which the probability of an investigation is larger when the agent is noncompliant.

Before the investigation is actually begun, the agent and the enforcer (i.e. the administrative branch in charge of the discovery of the truth and the conviction of guilty agents) can strike a deal; the agent can plead guilty and accept to pay the settlement amount decided by the enforcer. If the settlement offer made by the enforcer is rejected, the investigation takes place. Depending on how much support the enforcer puts into the process, the investigation may reach different levels of accuracy. We assume that conviction can be obtained only in the presence of hard incriminatory evidence, i.e. evidence which proves without doubt the guilt of the defendant. Let $a$ be the probability of conviction of a guilty agent subject to the investigation (a measures the accuracy of the investigation). Since a larger investigative effort produces a greater accuracy, we can express the enforcer's problem as the choice of the appropriate level of accuracy for the case at hand. The enforcer's goal is to produce deterrence, but to recover the penalty with the lowest effort from a possibly guilty agent. In other words, the enforcer maximizes the expected net transfer from the agent. For simplicity, we assume that investigation costs are a quadratic function of the probability of conviction, $C(a) = \frac{1}{2} c a^2$. The enforcer maximizes the net recovery taking the penalty level and the noncompliance probability as given.

The penalty level is decided by the "the principal" (the legislature), whose objective is ex-ante optimality. It will choose $F$ so as to minimize the social loss from noncompliance, i.e. the difference between the expected level of harm and the expected net recovery from the agent. The principal decides the penalty at the outset,
and is aware of the way in which it affects the game between the enforcer and the agent.

The move sequence of the game is the following:
1. the principal decides the penalty level,
2. the agent decides whether to comply or not,
3. depending on an random signal, the agent may be selected for an investigation,
4. the indicted agent is offered a take-it-or-leave-it settlement offer by the enforcer,
5. the agent chooses whether to accept the offer or not,
6. if the offer is rejected, the enforcer carries out the investigation. It chooses the effort to devote to the case and the resulting conviction probability.

We solve the game by backward induction using the concept of Perfect Bayesian Equilibrium. In simple terms, a PBE is a set of strategies and beliefs such that, at any stage of the game, strategies are optimal given the beliefs, and the beliefs are obtained from equilibrium strategies and observed actions using Bayes’ rule. We solve the game backwards, considering the optimal settlement offer first.

The optimal settlement offer

In this section, we derive the optimal settlement offer for any given probability of guilt of the agent. We are therefore solving the last three stages of the game. Our objective is to show that the optimal settlement offer will depend on the conviction rate that the enforcer can credibly threaten. Given that the payment of the agent is $F$ in case of conviction and 0 in case of acquittal, the optimal settlement offer will be $q^* = a^* F$, with $a^*$ equal to the credible conviction threat. In order to derive the optimal settlement amount, the enforcer has to calculate the continuation equilibrium following each possible settlement amount $q$.

We start our analysis by considering the equilibrium of the continuation game (following the settlement offer). We derive the optimal behaviour of the defendant first. The agent only knows whether he is innocent or guilty, while the enforcer believes the agent to be guilty with probability $x$. The strategy of the agent will depend on whether he is guilty or not. Clearly, an innocent defendant will reject all settlement offers with $q > 0$. A guilty defendant will instead reject the offer only if the expected payment in case of rejection (probability of conviction $\times$ penalty) is less than $q$. Let $p(a \mid q)$ be the probability that the guilty defendant rejects an offer of amount $q$ when the expected conviction probability is $a$. We have

$$
\begin{align*}
\rho(a \mid q) &= 1 & \text{if } q > a F, \\
p(a \mid q) &= 0 & \text{if } q < a F, \\
p(a \mid q) &= [0,1] & \text{if } q = a F.
\end{align*}
$$

Consider now the enforcer’s optimal choice in case of rejection. It will choose the conviction probability $a$ (through her investigation effort) which maximizes the expected net recovery. The latter is calculated through her revised belief about the probability
of guilt of the agent,

\[ R(a) = \frac{\rho x}{\rho x + 1 - x} a F - \frac{1}{2} c a^2. \]

The enforcer knows that only the guilty defendant may be willing to accept the settlement offer. If the offer is rejected, the enforcer will attach a larger probability to the defendant being innocent rather than guilty. This in turn reduces the incentives to carry out thorough investigations. The optimal conviction probability is

\[ a(\rho | x) = \begin{cases} \frac{\rho x}{[\rho x + 1 - x]} & \text{if } \frac{\rho x}{[\rho x + 1 - x]} \frac{F}{c} \leq 1, \\ 1 & \text{otherwise.} \end{cases} \]  

(2)

The conviction probability is larger if the likelihood that the agent is guilty is larger. If the recoverable fine is far greater than the investigation costs, the enforcer can credibly threaten \( a(\rho | x) = 1 \).

Figure 1 depicts the best reply functions of the enforcer and the guilty agent.

![Figure 1](image_url)

Figure 1: Optimal rejection probability for the guilty agent and optimal conviction probability.

Let us assume that \( F < c \), so that \( a(\rho | x) \) is always less than one at the equilibrium and leave the other case to the appendix. The equilibrium following the settlement offer \( q \) is obtained combining eqs. (1) and (2), and depends on the
level of \( q \) as follows;

\[
\text{continuation equilibrium,} \quad \left\{ \begin{array}{ll}
a(q \mid x) = \frac{q}{F}, & \text{and } \rho(q \mid x) = \frac{1-x}{x} \left( \frac{1}{x} - 1 \right), \quad \text{if } q \leq x^{F^2}_c, \\
a(q \mid x) = x^{F^2}_c, & \text{and } \rho(q \mid x) = 1, \quad \text{if } q > x^{F^2}_c.
\end{array} \right.
\]

(3)

If the settlement amount is not too high, \( 0 < q \leq x^{F^2}_c \), then at the equilibrium the guilty agent rejects the offer with a probability \( \rho(q \mid x) \) which is larger if the enforcer demands a larger settlement amount; \( \frac{\partial \rho}{\partial q} > 0. \) At the equilibrium, the probability of conviction of the guilty agent in case of rejection is such as to render the agent indifferent between accepting and rejecting the offer; \( a(q \mid x) = q/F. \) As expected, investigations are less thorough if they follow the rejection of a settlement offer with a small amount. The rejection of the offer provides the enforcer with bad news about the probability of getting a conviction: the smaller the settlement offer which is turned down and the more likely it is that the defendant is innocent.

Note also that the probability that the guilty defendant accepts the settlement offer is larger if the probability of guilt, \( x, \) is larger. This because a larger probability of guilt relaxes the credibility constraint on the conviction threat by making investigations more profitable. If \( x \) goes to one (the agent is guilty with certainty), the rejection probability goes to zero. This is an interesting result, which will have important consequences for the optimal enforcement policy. One can start to see that if one of the objectives of the principal is to limit enforcement costs, then a large defection rate may have desirable consequences, since it reduces the credibility constraint on the conviction threat and makes a settlement more likely.

If the settlement amount is very large, \( q > x^{F^2}_c \), then both types reject the offer with probability one, and the probability of conviction of the guilty agent is the largest credible one; \( a(q \mid x) = x^{F^2}_c. \) In this case, the rejection does not convey any information to the enforcer and the continuation equilibrium is independent of \( q. \)

We can now turn to the problem of the optimal settlement offer. The expected net recovery following an offer of amount \( q \) is

\[
R(q \mid x) = x(1 - \rho(q \mid x)) q + x\rho(q \mid x) a(q \mid x) F - \frac{1}{2} [1 - x(1 - \rho(q \mid x))] c [a(q \mid x)]^2
\]

An increase in the settlement amount affects the net recovery in several ways: first, it increases the amount collected in case of acceptance; second, it reduces the probability

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5Note that this result is brought about by the convexity of the investigation cost function. In a linear model, with \( C(a) = ca \), the rejection probability would be independent of the settlement amount, since only one level of \( \rho \) would allow the inspector to select a conviction probability different from 0 or 1. Here, instead, different rejection rates lead to different conviction probabilities; large rejection probabilities are associated with large conviction rates and, consequently, large settlement amounts.
that the offer is accepted; third, it increases the optimal conviction rate in case of rejection (which is larger, the higher the probability that the guilty agent rejects the offer).

Using the equilibrium values just obtained, we get

\[
R(q | x) = \begin{cases} 
    x q + \frac{1}{2} (1 - x) \left[ 1 + \frac{1}{(F - \frac{c}{F} - x)} \right] c \left( \frac{x}{F - \frac{c}{F}} \right)^2, & \text{for } q \leq x \frac{F^2}{c}, \\
    x F \left( x \frac{F^2}{c} \right) - \frac{1}{2} c \left( x \frac{F^2}{c} \right)^2, & \text{for } q > x \frac{F^2}{c}.
\end{cases}
\]

For \( q > x F^2 / c \), the expected net recovery is independent of \( q \); large settlement offers are rejected out of hand.

From the previous equation, one gets

\[
R'(q | x) = \begin{cases} 
    x - (1 - x) \frac{\sqrt{(F - \frac{c}{F}) - \frac{F^2 q}{c}}}{(F - \frac{c}{F} - x)}, & \text{for } q \leq x \frac{F^2}{c}, \\
    0, & \text{for } q > x \frac{F^2}{c},
\end{cases}
\]

with \( R''(q | x) \leq 0 \) throughout. Since \( R'(0 | x) > 0 \) and \( R' \left( x \frac{F^2}{c} | x \right) < 0 \), the optimal \( q \) will belong to the interval \( \left( 0, x \frac{F^2}{c} \right) \). (For \( q = 0 \), the net recovery is obviously 0). The expected net recovery reaches its maximum when \( R'(q | x) = 0 \) and \( R''(q | x) < 0 \), that is when

\[
q(x) = \frac{F^2}{c} \left( 1 - \frac{\sqrt{(1 - x)}}{\sqrt{1 + x}} \right).
\]

The optimal settlement offer \( q(x) \) is larger if the net profitability of the investigation is larger (larger \( F \) and \( x \), and smaller \( c \)). If the agent is certainly innocent, \( x = 0 \), then the optimal amount is zero. If the agent is certainly guilty, \( x = 1 \), then the optimal amount is \( q(1) = F^2 / c \) (which is the largest credible settlement offer).

In the appendix, we derive the optimal settlement amount for the case with \( F > c \). In this case, for defection probabilities greater than the threshold \( \overline{x} \), the continuation equilibrium has \( a(q | x) = 1 \), where \( \overline{x} \) is defined as follows

\[
\overline{x} \equiv \frac{1 - \left( \frac{F - c}{F} \right)^2}{1 + \left( \frac{F - c}{F} \right)^2}.
\]

Our results are summarized by the following proposition.

**Proposition 1.** The optimal settlement offer

Let \( x \) be the probability that the individual is guilty and let \( x \) be common knowledge.
1. If $F \leq c$, the optimal settlement offer is $q^* (x) = \frac{F}{c} \left( 1 - \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}} \right)$. This offer is rejected with probability $\rho (x) = \frac{\sqrt{1-x^2} (1-x)}{\sqrt{1+x^2}}$ by a guilty agent and probability one by an innocent agent. The conviction probability for a guilty agent who rejects the offer is $a(x) = \frac{c}{F} \left( 1 - \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}} \right)$.

2. If $F > c$, we have two possible cases. If $x \leq \bar{x}$, the optimal offer and the continuation equilibrium are the same as in point 1. If $x > \bar{x}$, the optimal settlement offer is $q^* (x) = F$. This offer is rejected with probability $\rho (x) = \frac{1-x}{x} \left( \frac{1}{2} - 1 \right)$ by a guilty agent and probability one by an innocent agent. The conviction probability for a guilty agent who rejects the offer is $a(x) = 1$.

There are several interesting aspects of this result that need to be emphasized.

First, when the net recovery from the investigation is not too large ($F \leq c$ or $x \leq \bar{x}$), the optimal settlement offer is strictly increasing in the probability of guilt $x$: large defection probabilities make the investigation more remunerative and weaken the credibility constraint on the conviction threat. In turn, the guilty type is lead to reject the settlement offer with a smaller probability. The probability that a settlement is reached upon indictment is

$$\Pr(\text{sett.}) = x (1 - \rho (q)) = 1 - \sqrt{1-x^2},$$

which is strictly increasing in the probability of guilt $x$. The settlement offer is accepted with certainty only if the agent cannot be or pretend to be innocent.

Second, if net return from the investigation is large ($F > c$ and $x > \bar{x}$), the enforcer can threaten the defendant with a sure conviction ($a = 1$). In this case, the credibility constraint does not bind, and the enforcer can ask for a settlement amount equal to the full penalty. The probability of a settlement is

$$\Pr(\text{sett.}) = x (1 - \rho (q)) = \frac{Fx - c}{F - c},$$

which is also strictly increasing in $x$.

The previous considerations show that pre-conviction negotiations are more effective (i.e. more likely to lead to a settlement) when the probability of guilt of the agent is larger. In order to increase the efficiency of the settlement stage, one would therefore have to raise the noncompliance probability. This fact illustrates the typical trade-off arising in the enforcement policy, where ex-post efficiency can often be reached only at the expense of ex-ante deterrence.

**Ex-post settlement and noncompliance**

Proposition 1 fully characterizes the optimal settlement offer for the enforcer given her belief about the probability of noncompliance. We can now find the solution of the enforcement game and consider how the probability of noncompliance is affected by the expected settlement offer. Recall that the penalty is taken as given by the players (the agent and the enforcer).
Let $\beta$ be the probability that the opportunist agent is noncompliant. The probability that the indicted agent is guilty when the settlement is proposed is then $x = (1 - p) \beta$. For any expected settlement amount $q$, the opportunist will choose to comply only if the expected payment in case of indictment is greater than his direct gain from noncompliance. In other terms,

$$\beta(q) = 0 \quad \text{only if} \quad T \leq \pi (1 - \rho(q)) + \rho(q) a(q) F,$$

where $\rho(q)$ is the equilibrium probability that he rejects an offer of amount $q$ and $a(q)$ is the equilibrium probability of conviction in case of rejection. Since in any continuation equilibrium $q(q) / a(q) = F$, we have

$$\begin{cases}
\beta(q) = 0 & \text{if } q > T/\pi, \\
\beta(q) \in (0, 1) & \text{if } q = T/\pi, \\
\beta(q) = 1 & \text{if } q < T/\pi.
\end{cases}$$

We can finally derive the solution of the game. The equilibrium will be characterized by the levels of $q$ and $\beta$ that allow each player to play his/her best reply. The best reply functions are depicted in figure 2.

![Figure 2: Optimal noncompliance probability of the opportunist agent and optimal settlement amount.](image)

The nature of the equilibrium will depend on whether $q(1)$ and $F$ are greater or less than $T/\pi$ (note that $q$ is now written as a function of $\beta$). If $F < T/\pi$, the enforcer cannot possibly prevent the opportunist agent from defecting; even if he were convicted with certainty upon indictment, he would face an expected penalty less than his gain from noncompliance. In this case, the equilibrium settlement amount will depend on whether "$a \leq 1$" is binding or not. In order to ease the presentation,
we assume here that the enforcement costs are high enough for 
"a ≤ 1" not to be 

binding at the equilibrium. In other terms, we assume that

\[ c > \frac{T}{\pi} (1 - P), \]

with \( P \equiv \frac{p}{1 - p}. \) The opposite case, with \( c < \frac{T}{\pi} (1 - P), \) does not present special difficulties and is dealt with in the appendix.

The most interesting case has \( F > T/\pi. \) Here, the equilibrium can be either 

internal or on the corner depending on whether \( q(1) \) is greater or less than \( T/\pi. \) We 

have \( q(1) > T/\pi \) if an only if \( \tau < 1 - P, \) with

\[ \tau \equiv \frac{T}{\pi} F^{-\frac{\pi}{\tau}}. \]

The variable \( \tau, \) which will be extensively used hereafter, is equal to the ratio between 

the private benefit from noncompliance and the largest credible expected punishment 

(which is inflicted when the agent is guilty with probability one).

The following proposition describes the equilibrium of the game on the assumption 

that "a ≤ 1" in not binding.

**Proposition 2. The equilibrium of the enforcement game**

1. If \( \tau < 1 - P \) (large penalty), the opportunistic agent is noncompliant with 

probability \( \beta^* = \frac{1 - \frac{1}{1 - \tau}}{1 - \frac{1}{1 + (1 - \tau)^2}} \) and the enforcer asks for a settlement amount \( q^* = \frac{T}{\pi}. \)

The compliant agent rejects the settlement offer with probability one, while the noncompliant agent rejects it with probability \( \rho^* = \frac{1 - \tau}{1 - \frac{\pi}{\tau}}. \) The probability of conviction of the noncompliant agent in case of rejection is \( a^* = \frac{T}{\pi F}. \)

2. If \( \tau > 1 - P \) (small penalty), the opportunistic agent is noncompliant with 

probability \( \beta^* = 1 \) and the enforcer asks for a settlement amount \( q^* = \frac{T}{\tau} (1 - P). \)

The compliant agent rejects the settlement offer with probability one, while the noncompliant agent rejects it with probability \( \rho^* = \frac{2 \pi}{1 + F}. \) The probability of conviction of the noncompliant agent in case of rejection is \( a^* = \frac{\pi}{\tau} (1 - P). \)

Some remarks are in order. Let us focus our attention on the internal equilibrium 

first (case 1). Here, \( F \) is large enough to exert some deterrence on the opportunistic agent \( (\beta^* < 1). \) At the equilibrium, the opportunist defects with a probability \( \beta = \frac{1}{1 - \frac{1}{1 + (1 - \tau)^2}} \) which is increasing in the investigation cost \( c \) and the private benefit \( T, \)

and decreasing in the penalty \( F, \) the indictment rate \( \pi \) and the probability of ethical behaviour \( p. \) The optimal settlement amount is such that the private benefit from 

noncompliance is fully expropriated (in expected terms) from the agent; \( \pi q = T. \) The 

settlement amount is larger if the private benefit from noncompliance is larger and
if the probability of indictment is smaller. The probability of a settlement (upon indictment) is

\[ Pr(\text{sett.}) = x^* (1 - \rho^*) = \frac{\tau^2}{1 + (1 - \tau)^2}, \]

which is larger if \( F, \pi \) and \( p \) are smaller, and \( c \) and \( T \) larger. An increase in the penalty reduces the defection rate and undermines the credibility of the conviction threat. As a result, the probability of a settlement is reduced. Finally, the probability that a non-complaint agent is convicted is \( q^* / F = \frac{T}{\pi F} \), which is smaller if the penalty is larger.

Since it will become relevant for the determination of the optimal penalty, note that an increase in the penalty induces a reduction in the noncompliance level, an increase in the probability of a settlement and a reduction in the conviction rate.

When the penalty is set at low levels (case 2), the enforcement policy cannot exert any deterrence on the opportunistic agent. The solution is on the corner, \( \beta^* = 1 \), and \( q = \frac{F^2}{c} (1 - P) \). Here, an increase in the profitability of the investigation (larger \( F \), smaller \( c \) or smaller \( p \)) weakens the credibility constraint on the conviction threat and allows the enforcer to demand a larger settlement amount.

It is interesting to calculate the probability of a settlement for two limit cases;

\[ \lim_{F \to \infty} Pr(\text{sett.}) = \lim_{\tau \to 0} \left( \frac{\tau^2}{1 + (1 - \tau)^2} \right) = 0, \]

\[ \lim_{F \to 0} Pr(\text{sett.}) = (1 - p) \left( 1 - \frac{2p}{1 + \pi p} \right) = \frac{(1 - p)^2}{1 + \pi p^2}. \]

For \( F \to \infty \), the opportunistic agent defects with a probability close to 0. The optimal settlement offer has \( q \to 0 \) since thorough investigations are not credible. This offer is rejected with probability one by the agent irrespective of whether he is guilty or innocent.

For \( F \to 0 \), the problem of the enforcer is somehow reversed; he cannot threaten thorough investigations because the penalty is infinitesimal. The optimal settlement offer has \( q \to 0 \) and the opportunistic agent is noncompliant with probability one. This offer is rejected with certainty by the honest agent and is rejected with positive probability by the opportunistic agent, who mimics the honest one. As \( p \to 0 \), the opportunist has no honest agent to mimic and accepts the settlement offer with probability one.

On the basis of our results, we can now calculate the equilibrium net recovery. We have

\[ R(F) = \begin{cases} \frac{T}{\pi} \frac{\tau}{1 + (1 - \tau)^2}, & \text{for } \tau \leq 1 - P \text{ (large penalty)}, \\ \frac{F^2}{c} \frac{(1 - P)^2}{1 + P^2}, & \text{for } \tau > 1 - P \text{ (small penalty)}. \end{cases} \]
The net recovery at the internal equilibrium \( (\tau \leq 1 - P) \) is decreasing in \( F \) and \( \pi \), and is increasing in \( T \) and \( c \). This because variations in the parameters that make the investigation more profitable (for a given defection probability) reduce the credibility constraint on the conviction threat and increase the deterrent effect of the enforcement process. The defection probability is consequently smaller, and so is the net recovery. It is clear that if it were net recovery what the principal was interested in rather than social loss, a large penalty level would not be desirable, since it would reduce the net recovery to zero: \( \lim_{F \to \infty} R(F) = \lim_{\tau \to 0} R(F) = 0. \)

For \( \tau > 1 - P \), the opportunistic agent defects with probability one. Variations in the parameters do not affect the deterrent capacity (or better, the lack of it) of the enforcement process. Therefore, the effect of an increase in the profitability of the investigation is to increase the net recovery.

Note that the expected recovery from the investigation reaches its maximum level when \( \tau = 1 - P \), i.e. when \( F \) is set to the largest level compatible with \( \beta = 1 \). Here the opportunistic agent defects with probability one and faces, in case of indictment, a settlement demand \( q = \frac{F^2}{c} \tau = \frac{T}{\pi} \), which he accepts with probability \( 1 - \rho = 1 - \frac{1 - P}{1 + P} \).

The expected recovery for the enforcer is \( R = \frac{F^2}{c} \left( \frac{1 - P}{1 + P} \right)^2 \). Suppose now that \( p = 0 \), i.e. that the agent is an opportunist with probability one, and that \( \tau = 1 \). In this case, the enforcer does not suffer from imperfect information at the settlement stage since the agent is guilty for sure, and there is no credibility constraint on the conviction threat. The enforcer can then extract at no cost from the agent an amount which is equal (in expected terms) to his private benefit from noncompliance: \( \pi R = \frac{F^2}{c} = T. \)

If the penalty were increased, the agent would elect to be compliant with a positive probability, creating uncertainty on the side of the enforcer. The investigation costs, in turn, would raise by an amount more than proportional to the probability of compliance. In fact, the guilty agent would now have an incentive to mimic the innocent one and reject the settlement offer with positive probability. The overall effect would be an increase in the investigation costs and a sharp reduction in the net recovery.

This is an interesting result which shows that the largest expected net recovery is obtained by limiting the punishment power of the enforcer, so as to minimize her lack of information at the settlement stage and to soften the credibility constraint on the conviction threat.

For an assessment of the overall effects of pre-investigation negotiations and for the determination of the optimal penalty, we consider now the social loss associated with the production of crime.

3. The optimal penalty

The results of the last section make clear that the social desirability of high penalties cannot be reconciled with large levels of the net recovery. High penalties are instead better considered in relation to their deterrent effect on criminal activity.

Let us consider the problem of the principal, i.e. of the authority that decides
the level of the penalty. The principal chooses the level of the penalty taking into account the effects that it will have on the process of enforcement and the level of compliance. Following Reinganum (1993), we assume that the objective of the principal is the minimization of the (uncompensated) social loss resulting from the enforcement activity, measured as the difference between the expected harm produced by the offender and the expected amount of resources recovered by the enforcer. The level of harm associated with noncompliance is denominated in terms of social utility and is equal to $H$.\footnote{Note that if the victim of crime and the enforcement agency do not coincide (they do coincide, for example, when the crime at hand is tax evasion) and if direct compensation (in expected terms) does not take place, then the enforcement process produces a transfer of `welfare’ from the victim of crime to the enforcer. The more undesirable this transfer (because of lack of indirect compensation), the larger will be the social harm $H$ associated with crime.}

The social loss can then be expressed as

$$L (F) = (1 - p) \beta^* H - \pi R^*,$$  \hspace{1cm} (4)

where $(1 - p) \beta^*$ is the equilibrium noncompliance probability and $\pi R^*$ the equilibrium expected net recovery.

We have

$$L (F) = \begin{cases} (1 - p) \frac{\pi F^2 (1 - P)^2}{c} 1 + P^2, & \text{for } \tau > 1 - P \text{ (small penalty),} \\ \frac{1 - (1 - \tau)^2}{1 + (1 - \tau)^2} H - \frac{\tau}{1 + (1 - \tau)^2} T, & \text{for } \tau \leq 1 - P \text{ (large penalty).} \end{cases}$$  \hspace{1cm} (5)

For $\tau > 1 - P$, i.e. for $F < \hat{F} = \sqrt{\frac{Tc}{\pi(1 - P^2)}}$, the opportunistic agent is noncompliant with probability one. Here, an increase in the penalty affects only the expected net recovery from the investigation and provides the enforcer with an incentive to devote more effort to the case. The net recovery is larger and the social loss smaller if the penalty is larger.

For $\tau = 1 - P$, we have $L (\hat{F}) = (1 - p) H - \frac{1 - p}{1 + P^2} T$. Note that $L (\hat{F}) \geq 0$ if and only if the social harm $H$ associated with noncompliance is greater than $\frac{1 - p}{1 + P^2} T = \frac{1}{1 + P^2} T$.\footnote{The last inequality stems from the fact that $p = \frac{2Tc}{1 - 1/P^2}$.} For $p \to 0$, $L (\hat{F}) = H - T$, which is positive only if the level of harm is greater than the private benefit from noncompliance.

For $\tau \leq 1 - P$, i.e. for $F > \hat{F}$, an increase in $\tau$ (i.e. either a reduction in $F$ and $\pi$ or an increase in $c$ and $T$) has two effects; it raises the probability that harm is produced and increases the amount of resources recovered through the enforcement system. These effects clearly pull in opposite directions for the social loss. We have

$$\frac{\partial L}{\partial F} < 0 \iff H > \frac{2 - \tau^2}{4 (1 - \tau)} T.$$  \hspace{1cm} (6)
An increase in the penalty level reduces the social loss (at the internal equilibrium) if and only if $H$ is sufficiently large. Note that the coefficient of $T$ in the previous inequality is strictly increasing in $\tau$. For $\tau \to 0$ (infinite penalty), we have $I' < 0$ if and only if $H > 1/2T$. For $\tau \to 1 - P$, we have $I' < 0$ if and only if $H > \left(\frac{1}{2} + \frac{(1-P)^2}{4P}\right)T$. If the probability of ethical behavior is negligible, $p \to 0$, then the latter condition cannot be satisfied since $\lim_{p \to 0} \left(\frac{1}{2} + \frac{(1-P)^2}{4P}\right) = \infty$. As expected, $\lim_{\tau \to \infty} L(F) = \lim_{\tau \to 0} L(F) = 0$. Therefore, $\hat{F}$ is a local minimum of $L(F)$ if $H \leq \left(\frac{1}{2} + \frac{1-P^2}{4P}\right)T$, and is a global minimum if $H < \frac{1}{1+P} T$.

Figure 3 plots the loss function for the case with no ethical behaviour ($p = 0$).

![Figure 3: Social loss as a function of the penalty level.](image)

Let us focus on figure 3 and consider how a variation in the penalty affects social loss when $p = 0$. For small levels of the penalty, $F < \hat{F} = \sqrt{cT/\pi}$, the agent is noncompliant with probability one. The enforcer enjoys full information at the settlement stage and his offer is accepted with certainty. An increase in the penalty decreases the social loss since it increases the settlement amount without affecting the probability of a settlement. At $F = \hat{F}$, the settlement amount is equal to $T/\pi$ and the agent is fully expropriated from his private benefit from noncompliance. The settlement offer is accepted with certainty and no investigation costs are incurred in by the enforcer. For penalty levels greater than $\hat{F}$, the credibility constraint on the conviction threat starts to bite. When $F$ is greater than $\hat{F}$, but less than a threshold level $\overline{F}$, social loss is strictly increasing in the penalty; an increase in $F$ reduces the noncompliance probability and tightens the credibility constraint. The guilty agent finds it more profitable to mimic the innocent one and the settlement offer is more likely to be rejected. Due to the larger probability of a (costly) investigation, the net recovery is smaller and the social loss is larger. For $F > \overline{F}$, the increase in the
rejection probability associated with an increase in the penalty is outweighed by the increase in the probability of harm, and social loss is decreasing with \( F \). If the penalty is infinite, no harm is committed, and the net recovery and the social loss are nil.

The special case just considered, with \( p = 0 \), displays two interesting properties. The first is that when the penalty is set to \( \hat{F} \), social loss is equal to \( H - T \), i.e. social harm minus private benefit from noncompliance. In this case, a settlement is reached with certainty and the enforcer can extract from the noncompliant agent his private benefit in full at zero cost. The second is that social loss is strictly increasing in the penalty for some values of \( F \). Here, an increase in the penalty level is detrimental, since it worsen the credibility constraint on the conviction threat and increases the probability of a (costly) investigation. These facts have straight consequences in terms of the optimal penalty. First, obviously, if the level of harm associated with noncompliance is less than the private benefit to the agent, then social loss is minimized at \( F = \hat{F} \). Second, if the penalty is constrained by an exogenous upper bound \( \bar{F} \in (\hat{F}, \tilde{F}) \), which may for instance derive from the limited liability of the agent, then the optimal penalty is \( \hat{F} \). When \( F = \hat{F} \), the enforcer “sells the shop” to the agent, who buys the “right to be noncompliant” at an (expected) price equal to his private benefit from it. The settlement effectively serves the purpose of legalising the harmful act, taxing individuals by an amount equal to their private benefit from the harmful activity.\(^8\) The interesting fact is that the enforcement system can be used to extract the private benefits from individuals at no costs only if two conditions are satisfied: i) the penalty is small, and ii) all individuals are interested in buying the right to be noncompliant (i.e. there are no ethical agents).

When \( p \neq 0 \), the previous results have to be qualified. The presence of unconditionally compliant agents reduces the likelihood of a cost-free conclusion of the enforcement process, since they will always refuse to settle. In turn, this leads noncompliant agents to mimic the innocent ones and reject the settlement offer with positive probability. This means that there are no ways for the enforcer to avoid the investigation costs and to recover \( T \) without effort. At \( F = \hat{F} \), the probability that a settlement is agreed upon indictment reaches its largest level and is equal to \( \frac{(1-p)^2}{1+p} \). The probability that a costly investigation is carried out and the social loss will therefore be larger if \( p \) is larger. Note also that in proximity of \( F = \hat{F}^+ \), we have \( U > 0 \) if and only if \( H < \left( \frac{1}{2} + \frac{(1-p)^2}{4p} \right) T \). This means that when \( p \neq 0 \), \( F = \hat{F} \) is local minimum only if \( H \) is sufficiently small. Actually, \( H \) has to be smaller if \( p \) is larger.

The previous arguments suggest that the optimal penalty will depend on the ratio between \( H \) and \( T \).\(^9\)

\(^8\)Note that this argument in favour of legalisation is slightly different from the standard utilitarian one (which says that an harmful act should not be deterred if the utility that the agent can get from it is larger that the level of harm it produces), since our loss function does not include the violator’s utility. See Lewin and Trumbull (1990) for a philosophical defence of this approach.

\(^9\)In the appendix, we will show that similar threshold levels can be defined also for the case in
Proposition 3. The optimal penalty

There are two threshold values $h_0 = \frac{1}{1+P}$ and $h_1 = \left(\frac{1}{2} + \frac{1-P^3}{4P}\right)$ such that:
1. If the ratio between the level of harm and the private benefit from noncompliance is less than $h_0$, then the optimal penalty is finite, $F^* = \hat{F}$.
2. If the ratio between the level of harm and the private benefit from noncompliance lies between $h_0$ and $h_1$, then an infinite penalty is globally optimal and $F = \hat{F}$ is a local optimum;
3. If the ratio between the level of harm and the private benefit from noncompliance is greater than $h_1$, then the social loss is monotonically decreasing with $F$ and the optimal penalty is infinite.

It may be useful to recall that $\hat{F} = \sqrt{\frac{P}{1+P}}$ is the largest penalty compatible with full noncompliance of the opportunistic agent.

When $H/T$ is small, the principal is better off by inducing the opportunistic agent to be noncompliant with probability one and allowing the enforcer to extract from him (in expected terms) his private benefit $T$. This amount can in turn be used to compensate the victim of crime. Note that only opportunistic agents engage in crime and pay the settlement amount, and that the enforcement systems effectively screens between the two types of agents (those who value noncompliance and those who do not). This is actually its major advantage with respect to a policy which directly legalises the harmful behaviour.

When the ratio between social harm and private benefit is larger than $h_0$, but less than the threshold $h_1$, then the optimal penalty is infinite. However, if limited liability applies and $F$ is bounded above, it may still be optimal for the principal to set $F = \hat{F}$ and maximize the probability of a settlement with the opportunistic agent. Note that the threshold value $h_1 = \left(\frac{1}{2} + \frac{1-P^3}{4P}\right)$ is decreasing with the probability of ethical behaviour. As $p$ tends to 0, $h_1$ tends to infinity, and $\hat{F}$ is a local optimum for any (finite) level of $H/T$.

Finally, if the ratio between harm and private benefit is very large, $H/T > h_1$, then deterrence becomes more important, and the penalty has to be set to its largest feasible level.

It is interesting to contrast these results with those deriving from the standard enforcement model, in which no negotiations can take place between the enforcer and the agent (see section 4). In the standard model, the Principle of Maximum Deterrence applies and the optimal penalty is the largest feasible one, as far as social harm is less than the net amount of resources that can be recovered from the agent. Note that the latter amount consists of the penalty inflicted in case of conviction less enforcement costs. When negotiations between the enforcer and the agent take place and the penalty is small, instead, the recovery can take place at no cost, and the condition for the optimality of an infinite penalty becomes more stringent. Also, if limited liability puts an upper bound on $F$, then the optimal penalty may not be the

which the constraint “$a \leq 1$” is binding.
maximal one.

Even in the case with no negotiations, before setting up an enforcement apparatus, the principal has to ask to herself whether enforcement is at all worthwhile, and whether it would be better to legalise the offence at hand. Clearly, this is definitively a feasible alternative when all individuals enjoy a private benefit from noncompliance, but is certainly more problematic when some individuals benefit from committing the offense and others do not. One can hardly imagine an auction for the right to commit criminal acts or engage in harmful behaviour. In fact, the easiest way to extract the private benefit from the agents who gain from the harmful behaviour is to refrain from setting a large penalty and allow for pre-trial negotiations. People who net a large benefit from noncompliance will then engage in crime and pay it out at the settlement stage. Agents who do not attach any value to noncompliance will instead comply with the law and go through the normal enforcement procedure in case they are (wrongfully) indicted. Clearly, this outcome is desirable only if the private benefit from noncompliance to the agents is sufficiently greater than the social harm attached to it and the legal system provides effective guarantees to innocent defendants.

In order to asses the overall effect of the settlement stage on the enforcement policy, in the next section we investigate the features of the compliance game assuming that negotiations between the enforcer and the agent cannot take place.

4. Negotiations versus straight investigations

One of the controversial issues in the theory of law enforcement and in general public debates is whether negotiations between the enforcer and ‘suspected’ offenders should be allowed to take place in the course of the enforcement process. The reason for this is that the prospect of a negotiated penalty may provide agents with additional incentives to engage in criminal activity. The increased noncompliance rate may then partially or completely offset the benefits accruing from the reduction in enforcement costs, which is provided by the introduction of the settlement stage.

In this section, we tackle this issue by comparing the equilibrium configuration of the game in which negotiations take place between the enforcer and the indicted agent with the equilibrium of a game in which negotiations are ruled out. We assume that the penalty level is given. For sake of simplicity, we limit our analysis to the case in which the probability of ethical behaviour is negligible, $p = 0$, and the enforcement costs are sufficiently large to rule out equilibria with perfect investigations, $c > T/\pi$.\footnote{When $c < T/\pi$, at the equilibrium we have $\beta = 1$ and $a = 1$ for $F \in [c, T/\pi]$. For $F$ belonging to this interval, the introduction of the settlement stage has no effect on the noncompliance probability (which is constantly one) and brings enforcement costs down to zero.}

Let us consider the case in which pre-investigation negotiations are not feasible. The enforcement game has a very simple structure; the agent chooses whether to comply or not, and the enforcer what level of effort to devote to the investigation. This model captures the typical situation in which noncompliance occurs only if the
probability of conviction is sufficiently large, while the probability of conviction is large only if there is a positive probability that the agent is guilty.

The best reply functions of the players can be easily derived. The agent chooses to comply only if the prospective punishment is larger than his private benefit from noncompliance:

\[
\begin{align*}
\beta_i(a) &= 1 & \text{if } & a_1 < \frac{T}{\pi F}, \\
\beta_i(a) &\in [0, 1] & \text{if } & a_1 = \frac{T}{\pi F}, \\
\beta_i(a) &= 0 & \text{if } & a_1 > \frac{T}{\pi F}.
\end{align*}
\]  

(7)

The net recovery for the enforcer is

\[
R_1(a_1 | \beta_i) = a_1 \beta_i F - \frac{1}{2} c a_1.
\]

For any given \( \beta_i \), the best reply for the enforcer is

\[
a_1(\beta_i) = \begin{cases} 
\beta_i F \tau & \text{if } \beta_i F \tau \leq 1 \\
1 & \text{otherwise.}
\end{cases}
\]  

(8)

The optimal accuracy of the investigation is larger if the probability of unearthing an infraction is larger and if the investigation costs are smaller.

The equilibrium of the game between the enforcer and the agent is derived from the interception of the two best reply functions, eqs. (7) and (8), and is described below.

\[
\begin{align*}
\beta_i^* &= \tau, \quad \text{and} \quad a_i^* = \frac{T}{\pi F}, \quad \text{for } \tau \leq 1, \\
\beta_i^* &= 1, \quad \text{and} \quad a_i^* = \frac{F}{c}, \quad \text{for } \tau > 1.
\end{align*}
\]

For small investigation costs, \( \tau = \frac{cT}{\pi F} \leq 1 \), the conviction rate is \( a_i^* = \frac{T}{\pi F} \) and the agent is noncompliant with probability \( \beta_i^* = \tau \). The defection probability is larger if \( c \) and \( T \) are larger, and \( F \) and \( \pi \) smaller. For large investigation costs, \( \tau > 1 \), the enforcer is not able to deter the agent; the optimal conviction rate is \( a_i^* = \frac{F}{c} \) and the agent is noncompliant with probability one.

It can be easily seen that the conviction probability in the game without negotiations, \( a_i^* \), is the same as that obtained in the game with negotiations, \( a_i^{N*} \). This is the unique conviction probability that fully expropriates the agent’s benefit from noncompliance. As far as the probability of noncompliance is concerned, one can easily see that, for any penalty level, it is larger when negotiations are feasible;

\[
\beta_{i}^{N} - \beta_{i}^{*} = \frac{1 - (1 - \tau)^2}{1 + (1 - \tau)^2} - \tau = \frac{(1 - \tau)^2}{1 + (1 - \tau)^2} > 0.
\]
When negotiations between the enforcer and the agent can take place, a fraction of guilty agents settles the penalty; this reduces the profitability of the investigation and lowers the investigative effort. As a consequence, the deterrent effect of the investigations is decreased and noncompliance occurs with a larger probability.

At the equilibrium, the expected net recovery for the enforcer is

$$R^*_i = \begin{cases} 
\frac{a_i^* \beta_i^*}{2} F - \frac{1}{2} c (a_i^*)^2 = \tau \frac{1}{2} T, & \text{for } \tau \leq 1, \\
\frac{a_i^* F}{c} - \frac{1}{2} (a_i^*)^2 = \frac{F^2}{2 c}, & \text{for } \tau > 1.
\end{cases}$$

(9)

At the internal equilibrium, $\tau \leq 1$, the expected net recovery is decreasing in $F$ and $\pi$, and is increasing in $T$ and $c$. Changes in these variables affect the probability of noncompliance and, consequently, the probability that the penalty is recovered. At the corner equilibrium, $\tau > 1$, the agent defects with probability one; a larger $F$ and a smaller $c$ increase the profitability of the investigation and increase the net recovery.

The social loss in the game with no-negotiations is

$$L^*_i = \begin{cases} 
\beta_i^* (H - T) + \frac{1}{2} c (a_i^*)^2, & \text{for } \tau \leq 1, \\
H - a_i^* F + \frac{1}{2} c (a_i^*)^2, & \text{for } \tau > 1,
\end{cases}$$

(10)

that is

$$L^*_i = \begin{cases} 
\tau \left( \frac{1}{2} T \right), & \text{for } \tau \leq 1, \\
H - \frac{1}{2} \pi \frac{F^2}{c}, & \text{for } \tau > 1.
\end{cases}$$

By looking at the social loss expression, we can see that here an infinite penalty is optimal only if the level of harm, $H$, exceeds the net recovery, $T/2$. For small levels of harm, $H < T/2$, the net recovery is larger than the level of harm and a finite penalty is optimal ($F^* = \sqrt{cT/\pi}$). At the optimum, the agent is noncompliant with probability one and faces an expected penalty equal to his private benefit. It is important to note that in order to collect this amount, the principal has to expend resources on the investigation process, since only a conviction can result in the application of the penalty. This marks a major difference between the present model and that in the previous section, since in the latter the full extraction of the private benefit could be obtained with no cost (by a pure threat) when the penalty was optimally set. Note, finally, that when the optimal penalty is infinite ($H > 1/2T$), the Principle of Maximum Deterrence applies: the principal is better off setting $F$ to its largest feasible level. This because variations in $F$ do not affect the likelihood of a cost-free conviction (settlement) of the offender.

Finally, we can compare the social loss associated with the game without negotiations ($L^*_i$) with the social loss associated with the negotiation game ($L^*_N$). From eqs.
(5) and (10), we have

\[
I_N^* - I_i^* = \begin{cases} 
(\beta_N^* - \beta_i^*) (H - T) - \pi \Pr{\text{sett.}} \frac{1}{2} c (a^*)^2, & \text{for } \tau \leq 1, \\
-\pi \frac{1}{2} c (a^*)^2, & \text{for } \tau > 1.
\end{cases}
\]

The introduction of the settlement stage increases the probability that the agent is noncompliant and reduces the expected investigation costs by an amount proportional to the probability that the settlement is accepted. The difference in the social loss is then composed of two parts: i) the increase in the expected level of harm net of the amount that can be recovered from the noncompliant agent, and ii) the reduction in the enforcement costs. Clearly, when the penalty is so small that \( \beta \) is constantly equal to 1, the introduction of the negotiation stage does not affect the expected level of harm, and the social loss in the negotiations regime is smaller by an amount equal to the reduction in enforcement costs. When the equilibria of the games with and without negotiations are internal, \( \tau < 1 \), which of the effects is the dominant one depends on the size of \( H \) compared with \( T \). In particular, we have

\[
I_N^* - I_i^* \leq 0 \iff \frac{(1 - \tau) \tau^2}{1 + (1 - \tau)^2} H - \frac{1}{2} \frac{(1 - (1 - \tau)^2)}{1 + (1 - \tau)^2} T \leq 0
\]

\[
\iff H < \frac{1 - \frac{1}{2} \tau}{1 - \tau} T.
\]

(11)

The next proposition summarizes our findings.

**Proposition 4. The optimality of the negotiation**

The introduction of the settlement stage has two effects on the enforcement system: i) it increases the probability that the agent is noncompliant, ii) it reduces the expected investigation costs by an amount which is proportional to the probability that a settlement is agreed upon.

Given a penalty level large enough to exert some deterrence on the agent (\( \tau < 1 \)), the introduction of the settlement stage is desirable if and only if the ratio between social harm and private benefit from noncompliance is less than a cut-off value \( \bar{h} (\tau) \equiv \frac{1 - \frac{1}{2} \tau}{1 - \tau} \).

Note that the threshold \( \bar{h} \) is increasing in \( \tau \), and is therefore decreasing in \( \pi \) and \( F \), and increasing in \( T \) and \( c \). For \( \tau \to 1 \) (small penalty), the condition for the desirability of the negotiation stage collapses to \( H/T < \infty \), while for \( \tau \to 0 \) (infinite penalty), this condition collapses to \( H/T < 1 \).\(^{11}\)

\(^{11}\)By comparison of eqs. (6) and (11), one may note that if the social loss (with negotiations) is increasing in the penalty, then the removal of the negotiation stage is definitely detrimental (while the reverse is not necessarily true).
Given $H$ and $T$, with $H > T$, we have

$$L^*_N - L^*_T \leq 0 \iff \tau \geq \frac{H - T}{H - \frac{1}{2}T}.$$  

The last result can be rephrased as follows.

**Corollary 5.** Given the level of the social harm $H$ and the private benefit from noncompliance $T$, with $H > T$, the introduction of the settlement stage reduces the social loss if and only if the penalty level is sufficiently small.

When the penalty level is large, the agent is noncompliant with a small probability and the possibility to reach a settlement is hampered by the credibility constraint hanging over the conviction threat. This reduces the benefits from the negotiation. On the contrary, when the penalty is small, the agent is noncompliant with a large probability: he will find it hard to pretend that he is innocent and will accept the settlement offer with a large probability. The reduction in the enforcement cost will then outweigh the increase in the expected level of harm.

One may wonder how a positive probability of ethical behaviour would affect our results. It can be easily seen that if the equilibrium is internal both with and without negotiations, then equation (11) still holds. Hence, the introduction of the negotiation stage is still desirable only if the penalty is sufficiently small. If the equilibria are on the corner, then the introduction of the negotiation stage does not affect the expected level of harm and yields a net reduction in the social loss.

5. **Extensions**

In this section we look at some ways of extending the model with pre-investigation negotiations and show how some of its limitations can be overcome.

**Selective indictment**

The analysis of the previous sections has been carried out under the assumption that the probability of indictment was the same irrespective of whether the crime had been committed or not. We intend now to show how our results can be generalized to the case in which the probability of an investigation depends on the compliance choice of the agent.

Let us assume that by not complying the agent increases the probability of the signal that triggers the investigation. Let $\pi_n$ and $\pi_c$ be the probabilities that this signal occurs when the agent is, respectively, noncompliant and compliant, and let $\pi_n > \pi_c$. One can calculate the probability that the agent is noncompliant when the signal is observed. If $x$ is the (prior) probability of noncompliance, we have

$$\hat{x} = \Pr(\text{noncompliance} \mid s) = \phi(x) = \frac{\pi_n x}{\pi_n x + \pi_c (1 - x)}.$$
Since the indictment process is biased against the noncompliant agent, we have $\dot{x} > x$ for all $x \neq 0, 1$.

Once an agent has been indicted, the case can be settled before the investigation is started. The problem is the same as that analysed in section 2; the only difference is that the belief of the enforcer about the probability of noncompliance is now $\dot{x}$ instead of $x$. The optimal settlement offer is hence that, which is described in section 2. Since $\dot{x}$ is generally greater than $x$, the optimal settlement amount will be larger: the selection bias of the indictment stage makes the conviction threat more credible.

From an *ex-ante* perspective, the fact that noncompliance increases the chances of being indicted decreases the incentive to defect. An opportunistic agent will comply only if

$$\pi_n q (\dot{x}) \leq T.$$  

The noncompliant agent faces now a larger probability of indictment and a tougher enforcer. At the equilibrium (which we assume for simplicity to be internal), the expected payment must equate the private benefit from defection, so that $q (\dot{x}) = T/\pi_n$. We have therefore $\dot{x} = q^{-1} (T/\pi_n)$ and $\dot{x} = \phi (x)$, so that $x = \phi^{-1} (q^{-1} (T/\pi_n))$. Given $\pi_n$, the equilibrium probability of noncompliance is smaller if the indictment process is more selective, i.e. if the probability of indictment of the compliant type is smaller.

The social loss associated with the enforcement process can be easily derived. Recall that $L$ is the difference between expected harm and expected net recovery. Once investigations are started, net recovery is as in section 2, with $\dot{x}$ in place of $x$. The expected probability of an investigation is $\pi (x) = \pi_c (1 - x) + \pi_n x$. Thus, the social loss becomes

$$L (x) = x H - \pi (x) R (\dot{x}).$$

An improvement in the selectivity of the indictment process leads to a reduced level of expected harm (since defection is more easily detected), a smaller probability that the investigation is undertaken (since defection takes place with a smaller probability), and to a larger net recovery if the investigation is carried out (since the probability of noncompliance given indictment is larger, and the credibility problem is relaxed).

Since $\dot{x} = x$ at $x = 0, 1$, the results on the optimality of a finite penalty for $p = 0$ are confirmed. In general, for $p > 0$, the results obtained with a constant probability of indictment have to be adjusted so as to take into account the degree of selectivity in the indictment process. Their qualitative features, however, remain unchanged.

**Variable private benefit from noncompliance**

In many situations the private benefit from noncompliance is not known with certainty by the principal, since it may vary from individual to individual. We model this lack of information by assuming that the principal and the enforcer share a common prior on $T$, represented by a cumulative probability distribution $G (T)$ with support $[0, T]$ and $G' > 0$ throughout. Since honest agents do not attach any (positive) value to noncompliance, their private gain is $T = 0$. 
We analyze the nature of the equilibrium of the game starting from the settlement stage. Let us assume that the probability of noncompliance \( x (T) \) for each type-\( T \) and the settlement amount \( q \) are given. The agent has to decide whether to accept the offer, and the enforcer what level of effort to devote to the investigation in case of rejection. As in section 2.1, a noncompliant agent will choose to reject the offer if the expected conviction probability is sufficiently low,

\[
\rho (T) = 1 \quad \text{if} \quad a < \frac{q}{F}.
\]

Note that the probability of rejection is independent of \( T \). For simplicity, we can focus on a symmetric equilibrium, in which all noncompliant agents choose the same \( \rho \). Naturally, compliant agents reject the offer with probability one.

The enforcer will choose the conviction probability that maximizes the expected net recovery. In order to do so, it has to calculate the probability that the agent is guilty, which is

\[
X = \int_0^T x (T) \, dG (T).
\]

If we concentrate on the internal solution, we get

\[
a (\rho \mid X) = \frac{X \rho}{X \rho + 1 - X} \frac{F}{c},
\]

which is the same as the expression obtained in section 2. It is then clear that the equilibrium of the settlement stage coincides with the equilibrium derived in that section, to which the reader may refer for details of the analysis. The best reply of the enforcer, \( q (X) \), is also described there. In order to derive the equilibrium of the whole game, we have to calculate the agent's optimal compliance decision. Given an expected settlement amount \( q \), the agent will defect if his gain from noncompliance is sufficiently high;

\[
x (T) = 1 \quad \text{only if} \quad T \geq \pi q.
\]

The overall probability of defection is therefore equal to the probability that \( T > \pi q \),

\[
X = 1 - G (\pi q).
\]

Note that \( X \) is a decreasing function of \( q \).

The equilibrium of the game is unique and is characterized by

\[
X^* = 1 - G (\pi q (X^*)).
\]

At the equilibrium, the agent defects only if his benefit from noncompliance is larger than the equilibrium expected settlement amount \( \pi q \). At the settlement stage, a noncompliant agent rejects the offer with probability \( \rho = \rho (X) \), and the conviction rate is \( a = a (X) \) (see the proposition on the optimal settlement offer in section 2). Clearly, the probability of a settlement is larger if the probability of noncompliance is larger.
This indicates that an increase in the penalty level still has two effects: on one hand, it reduces the expected level of harm; on the other, it increases the probability that the settlement offer is rejected. Also in this case, it may be optimal for the principal to lower the penalty in order to induce a larger probability of noncompliance and increase the probability of a settlement.

**Variable benefit and penalty**

In this section, we introduce the possibility that the penalty depends on the private benefit from noncompliance. The idea is that external circumstances allow the individual to engage in criminal activities of various kinds, and that the penalty is larger if the crime is more serious. The penalty is now represented by a function \( f \) of the private gain \( T, F = f (T) \), with \( f (0) = 0 \) and \( f' > 0 \). It can be easily shown that the nature of the equilibrium essentially depends on the shape of the penalty function. Suppose that the indictment and conviction probabilities are given. In this case, an agent who can benefit from noncompliance by an amount \( T \) will engage in crime only if

\[
\pi a f (T) < T.
\]

If the penalty function is convex, only agents with a small \( T \) will defect; if the penalty function is concave, only agents with a large \( T \) will defect. Therefore, depending on the shape of the penalty function, opposite criminal patterns may emerge. In the following, we assume that, because of reasons associated with limited liability, the penalty function is concave and that when defection takes place, it is perpetrated by individuals who get a large private benefit from it.

The purpose of this section is not to derive the explicit solution of the enforcement game, but rather to illustrate the way in which the enforcement process screens among different agents. Let us consider the settlement stage first, and let the settlement amount \( q \) and the probability of noncompliance of each type, \( x (T), \) be given. Let \( \rho (T) \) be the probability that type-\( T \) rejects the settlement offer if he has not complied. We have

\[
\rho (T \mid q) = 1 \quad \text{only if} \quad a f (T) \leq q.
\]

Let \( T_1 (a) \) be the cut-off level of \( T \) for the rejection of the offer: \( f (T_1) = q / a \). Noncompliant agents with \( T \geq T_1 (a) \) accept the settlement offer, while noncompliant agents with \( T < T_1 (a) \) reject it.

The enforcer selects the conviction rate so as to maximize the expected net recovery. An internal maximum is characterized by

\[
a (\rho) = \frac{\int_0^T [f (T) / c] \rho (T \mid q) x (T) dG (T)}{\int_0^T \rho (T \mid q) x (T) dG (T) + \int_0^T (1 - x (T)) dG (T)}.
\]

The conviction rate is larger if the average penalty that can be recovered from those who reject the settlement offer is larger. Given the settlement amount \( q \), the internal
continuation equilibrium will be characterized by

\[ a(q) = \frac{\int_{T_0}^{T_1(a)} \left[ \frac{f(T)}{c} \right] x(T) \, dG(T)}{\int_{T_0}^{T_1(a)} x(T) \, dG(T) + \int_{T_0}^{T_1(a)} (1 - x(T)) \, dG(T)}. \]

At the equilibrium, the probability that the offer is rejected and the probability of conviction are larger if \( q \) is larger.

Given the nature of the continuation equilibrium, the optimal settlement amount for the enforcer will depend positively on the expected penalty that can be recovered from the agent. Given the expected settlement amount and the related conviction probability, the agent will choose to defect if \( T > \pi_a f(T) \). For each possible level of \( a \), there is a cut-off level \( T_0 \) such that agents with \( T > T_0 \) defect, and agents with \( T < T_0 \) comply. The overall probability of defection is larger if the settlement amount is larger. In particular, the expected penalty of the noncompliant agent is larger if \( q \) and \( a \) are larger. Given these premises, the equilibrium of the game is simply obtained from the intersection of the best reply functions of the enforcer and the agents.

At the equilibrium, the type space will be partitioned in three parts by the cut-off levels \( T_0 \) and \( T_1 \). Depending on his private gain from noncompliance, the agent will adopt one of the following strategies:

1. if his private benefit is small, \( T < T_0 \), he will elect to comply and reject the settlement offer in case of indictment;

2. if his private benefit is of intermediate size, \( T_0 < T < T_1 \), he will elect not to comply and will reject the settlement offer in case of indictment;

3. if his private benefit is large, \( T > T_1 \), he will elect not to comply and will accept the settlement offer in case of indictment.

Note that the settlement acts as a screening device; from the pool of indicted agents, it screens out those who have defected and deserve the largest penalties.

Figure 4 illustrates a typical partition of the type space.

Suppose for a moment that the depth of the investigation and the conviction probability upon indictment were given (or credibly chosen in advance). Then, the effect of the introduction of the settlement stage could be easily pinned down; the settlement would act as a simple screening device separating large offenders from small ones. Offenders who voluntarily accept to pay the settlement amount are excluded from the prosecution process, and the enforcement costs born by the agency are reduced. This is the major insight of the early literature on plea bargaining (Grossman and Katz 1983) and the principal/agent approach to law enforcement (Reinganum and Wilde 1985, Kaplow and Shavell 1994). In the view of this paper, instead, the introduction of a settlement stage affects the enforcement process in a deeper way. By excluding the agents who deserve the largest penalties from the conviction process and by cutting down the expected net recovery from the investigation, the settlement
Figure 4: Equilibrium behaviour of the agent as a function of his benefit from non-compliance.

stage reduces the incentives for the enforcer to put full effort to the conviction of guilty offenders. As a result, the conviction probability is lower and the noncompliance level is boosted (in reference to figure 4, \( T_0 \) is shifted to the left).

From an ex-ante perspective, the desirability of a settlement stage will therefore depend on the relative weight of two countervailing factors: increased loss from noncompliance and reduced enforcement costs, as we have seen in section 4. The introduction of a negotiation stage is then desirable only when the social harm associated with noncompliance is small, or when the private benefit is large and evenly distributed among the population (the latter fact makes settlement more likely).

6. Final remarks

The model developed in this paper sheds some new light on the implications of negotiated law enforcement. The tenet of this paper is that the information conveyed at the settlement stage can profoundly affect the incentives for the enforcer to search for incriminatory evidence and reduces the deterrence capacity of the enforcement system. At the same time, settlements represent an effective tool to reduce the enforcement costs after the infraction has been committed. Our results show that there is a fundamental divergence between the ex-post goal of efficiency (i.e. the need to attain a cost-effective conviction process) and the ex-ante goal of deterrence (i.e. the need to ensure credible sanctions for noncompliant agents). Ex-post efficiency is maximal when the enforcer can use settlements to screen among agents, inducing the guilty to accept the offer and the innocent to reject it. This is possible, however, only when the enforcer can make credible threats, i.e. when he is convinced that the agent is guilty with a large probability. As a result, the enforcement expenditure is smaller when the probability of noncompliance is larger.

The solution of the efficiency-deterrence tradeoff will generally depend on the weight attached to these factors. In general, one may expect that settlements will be more valuable when the enforcement system is overburdened and the crime rate is
large.

7. Appendix

A. Optimal settlement amount for $F > c$.

We have to consider two cases.

1. For $xF < c$, then $a (\rho \mid q) < 1$ for all $x$, and the equilibrium is described by eq. (3).

2. For $xF \geq c$, then $a (\rho \mid q) < 1$ if $\rho < \frac{1-x}{x} \frac{c}{F-c}$, and $a (\rho \mid q) = 1$ if $\rho \geq \frac{1-x}{x} \frac{c}{F-c}$.

The continuation equilibrium is:

$$
\begin{align*}
\begin{cases}
a (q \mid x) = \frac{q}{F} & \text{and} & \rho (q \mid x) = \frac{1-x}{x} \frac{1}{(\frac{F-c}{F-c})}, & \text{if } 0 < q < F, \\
a (q \mid x) = 1, & \text{and} & \rho (q \mid x) \in \left(1-x, \frac{1}{(\frac{F-c}{F-c})}\right], & \text{if } q = F, \\
a (q \mid x) = 1, & \text{and} & \rho (q \mid x) = 1, & \text{if } q > F.
\end{cases}
\end{align*}
$$

For $q = F$, the game admits of a multiplicity of continuation equilibria, characterized by different probabilities of rejection by the guilty type. Note that for $q < F$, the continuation equilibrium is internal and is the same as in the case with $xF < c$. The same applies to the continuation net recovery. Here, however, the boundary is met at $q = F$, rather than $q = xF^2/c$.

The continuation net recovery is

$$
\begin{align*}
\begin{cases}
R (q \mid x) = xq + \frac{1}{2} (1-x) \left(1 + \frac{1}{(\frac{F-c}{F-c})}\right) c \left(\frac{q}{F}\right)^2, & \text{for } q < F, \\
R (q \mid x) = \left[xF - \frac{1}{2} c, xF + \frac{1}{2} (1-x) \left(1 + \frac{1}{(\frac{F-c}{F-c})}\right) c\right], & \text{for } q = F, \\
R (q \mid x) = xF - \frac{1}{2} c, & \text{for } q > F.
\end{cases}
\end{align*}
$$

Let $\hat{q} (x) \equiv \frac{F^2}{c} \left(1 - \sqrt{\frac{F-c}{1+c}}\right)$ (the optimal settlement amount for the case with $F < c$). Since the expression of the first line is increasing in $q$ for $q \in \left(0, \hat{q} (x)\right)$, and decreasing in $q$ for $q \in \left(\hat{q} (x), \frac{F^2}{c} \right)$, the optimal settlement amount is $q = F$ if $F \leq \hat{q} (x)$, and $q = \hat{q} (x)$ if $F > \hat{q} (x)$. In particular, we have $F \leq \hat{q} (x)$, if and only if

$$
x \geq \bar{x} \equiv \frac{1 - \left(\frac{F-c}{F-c}\right)^2}{1 + \left(\frac{F-c}{F-c}\right)^2}.
$$

It can be easily established that $\bar{x} > c/F$.

To sum up, we have proved what follows.

For $x < \bar{x}$, the continuation equilibrium is not affected by the constraint “$a \leq 1$,” and the optimal settlement amount is $\hat{q} (x)$.

For $x > \bar{x}$, the constraint “$a \leq 1$” is binding, and the optimal settlement amount is $F$. 

B. Equilibrium of the game when “a ≤ 1” is binding \( \left( \frac{1}{1-p} < \frac{T}{\pi} \right) \).

Let us recall that the optimal settlement offer of the enforcer is \( q(\beta) = \frac{p^2}{c} \left( 1 - \frac{\sqrt{1-(1-p)\beta}}{\sqrt{1+(1-p)\beta}} \right) \) for \( \beta < \frac{1}{1-p} \), and \( q(\beta) = F \) for \( \beta \geq \frac{1}{1-p} \). We consider the case in which \( q(1) = F \) when \( F = T/\pi \), i.e. \( \frac{T}{\pi} (1 - P) > 1 \). The best reply of the agent is

\[
\begin{align*}
\beta(q) &= 0 \quad \text{if} \quad q > T/\pi, \\
\beta(q) &= (0, 1) \quad \text{if} \quad q = T/\pi, \\
\beta(q) &= 1 \quad \text{if} \quad q < T/\pi.
\end{align*}
\]

The equilibrium of the game is therefore as follows.

1. If \( F \leq \frac{1}{1-p} c \), then the opportunistic agent is noncompliant with probability \( \beta^* = 1 \) and the enforcer asks for a settlement amount \( q^* = \frac{p^2}{c} (1 - P) \). The compliant agent rejects the settlement offer with probability one, while the noncompliant agent rejects it with probability \( \rho^* = \frac{p}{1-p} \). The probability of conviction of the noncompliant agent in case of rejection is \( a^* = \frac{T}{\pi} (1 - P) \).

2. If \( \frac{1}{1-p} c < F \leq \frac{T}{\pi} \), then the opportunistic agent defects with probability \( \beta^* = 1 \) and the inspector makes a settlement offer with \( q^* = F \). This offer is rejected with probability one by the compliant agent and with probability \( \rho^* = \frac{p}{1-p} \) by the noncompliant agent. The probability of conviction of the noncompliant agent in case of rejection is \( a^* = 1 \).

3. If \( F > \frac{T}{\pi} \), the opportunistic agent is noncompliant with probability \( \beta^* = \frac{1 - (1-P)^2}{1-P} \) and the enforcer asks for a settlement amount \( q^* = \frac{T}{\pi} \). The compliant agent rejects the settlement offer with probability one, while the noncompliant agent rejects it with probability \( \rho^* = \frac{1-P}{1-P} \). The probability of conviction of the noncompliant agent in case of rejection is \( a^* = \frac{T}{\pi} \).

Note that the constraint “a ≤ 1” is binding only for \( \frac{1}{1-p} c < F \leq \frac{T}{\pi} \). Here, the opportunistic agent is noncompliant with probability one, and the enforcer can credibly threaten a perfect investigation (a = 1) and demand a settlement amount equal to the penalty, \( q = F \). The noncompliant agent rejects the settlement offer with probability \( \rho = \frac{p}{1-p} \), which is larger if the investigation is more profitable.

The net revenue and the social loss function associated to the equilibrium of the game are the same as those derived in section 2 expect for \( \frac{1}{1-p} c < F \leq \frac{T}{\pi} \), where we have

\[
R = (1-p) F - \frac{1}{2} c p F \frac{F}{F-c},
\]

and

\[
L(F) = (1-p) (H - \pi F) + \pi c \frac{p F}{F-c}.
\]

Note that the settlement amount is larger and the enforcement costs are smaller if \( F \) is larger. When there are no ethical agents, \( p = 0 \), the enforcer can extract from the agent the penalty with no costs. At \( F = T/\pi \), we have \( L = H - T \).
Figure 5 plots the social loss as a function of the penalty for the case with \( p = 0 \).

![Diagram](image)

**Figure 5:** Social loss as a function of the penalty for the case in which \( a \leq 1 \) is binding.

Recall that the constraint \( a \leq 1 \) is binding for \( F \in [c, T/\pi] \). In this region, the agent defects and accepts the settlement offer with amount \( q = F \) with probability one. An increase in the penalty increases the settlement amount without affecting the credibility constraint, and reduces the social loss. For \( F = T/\pi \), the settlement amount equals the private benefit from noncompliance; the game admits of a multiplicity of equilibria characterized by a defection probability \( \beta \in [c\pi/T, 1] \) and a rejection probability for the noncompliant type equal to \( \rho \in [0, \frac{T-c\pi}{\pi-1/T-\pi}] \). The larger the defection probability, the more likely it is that the settlement is accepted. For \((T/\pi)^{-}\), we have \( \beta = 1 \) and \( L = H - T \). For \( F \) greater than \( T/\pi \), the constraint \( a \leq 1 \) is not binding, and the social loss is as in figure 3.

We can now derive the optimal penalty. Recall that

\[
L(F) = \begin{cases} 
(1 - p) H - \pi \frac{F^2}{c} \frac{(1 - p)^2}{1 + p^2}, & \text{for } F \leq \frac{c}{1 - p}, \\
(1 - p) (H - \pi F) + \pi \frac{1}{2} c p \frac{F}{F - c}, & \text{for } \frac{c}{1 - p} < F < \frac{T}{\pi}, \\
\frac{1 - (a - 1)^2}{1 + (a - 1)^2} H - \frac{\tau}{1 + (a - 1)^2} T, & \text{for } \frac{T}{\pi} < F.
\end{cases}
\]

For \( \pi F < T \), we have \( L'(F) < 0 \), while for \( \pi F > T \), we have \( L'(F) < 0 \) if and only if condition (6) is satisfied. For \( \pi F = T \), the game admits a multiplicity of equilibria; the opportunistic agent is indifferent on whether to defect with probability one and reject the settlement offer (of amount \( \pi F = T \)) with probability \( \rho = \frac{p}{1 - p} \frac{c}{F - c} \), or to defect with probability \( \beta = \frac{1 - (a - 1)^2}{a - 1 + (a - 1)^2} \) and reject with probability \( \rho = \frac{1 - \pi T}{\pi - 1/2} \). We
have

\[ L\left(\frac{T}{\pi^-}\right) = (1 - p) (H - T) + \frac{1}{2} cp_\pi \frac{T}{\pi} \frac{\pi}{T/\pi - c} \]

and

\[ L\left(\frac{T}{\pi^+}\right) = \frac{cT [H (2T - c\pi) - T^2]}{T^2 + (T - c\pi)^2}. \]

With a good deal of algebra, one can establish that

\[
\begin{cases}
L\left(\frac{T}{\pi^-}\right) \leq L\left(\frac{T}{\pi^+}\right) \iff H \leq \frac{T}{2} \frac{2T - c\pi}{T - c\pi}, \\
L\left(\frac{T}{\pi^-}\right) \geq 0 \iff H \geq \frac{T}{T - c\pi} \left(T - \frac{2 - p}{2 - 2p} c\pi\right), \\
L\left(\frac{T}{\pi^+}\right) \leq 0 \Rightarrow L'\left(\frac{T}{\pi^+}\right) \geq 0, \\
[L'\left(\frac{T}{\pi^+}\right) \geq 0 \lor L\left(\frac{T}{\pi^-}\right) \leq 0] \Rightarrow L\left(\frac{T}{\pi^-}\right) \leq L\left(\frac{T}{\pi^+}\right).
\end{cases}
\]

Again, \(\lim_{F \to \infty} L(F) = 0\). From the previous results one can infer that \(F = T/\pi\) is a global minimum if and only if \(H/T \leq h_0 \equiv \frac{1}{T - c\pi} \left(T - \frac{2 - p}{2 - 2p} c\pi\right)\) and a local minimum if and only if \(h_0 < H/T \leq h_1 \equiv \frac{1}{2} \frac{2T - c\pi}{T - c\pi}\).
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