

Cesari, Riccardo

**Working Paper**

## A generalized measure of competition

Quaderni - Working Paper DSE, No. 204

**Provided in Cooperation with:**

University of Bologna, Department of Economics

*Suggested Citation:* Cesari, Riccardo (1994) : A generalized measure of competition, Quaderni - Working Paper DSE, No. 204, Alma Mater Studiorum - Università di Bologna, Dipartimento di Scienze Economiche (DSE), Bologna,  
<https://doi.org/10.6092/unibo/amsacta/5122>

This Version is available at:

<https://hdl.handle.net/10419/159047>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



<https://creativecommons.org/licenses/by-nc/3.0/>

**A GENERALIZED MEASURE  
OF COMPETITION**

**RICCARDO CESARI**

**N° 204**

# A generalized measure of competition

Riccardo Cesari\*

April '94

JEL classification number: L13

## *Abstract*

Using the conditional probabilities of the passage of customers from one firm to another, a new, implementable measure of market competition is obtained which generalizes the classical Hirschman-Herfindahl index.

---

\* "This paper was prepared while the author was visiting professor at the Dipartimento di Scienze Economiche of the University of Bologna"

## 1. Introduction.

The degree of competition is usually considered a structural aspect of the market, i.e. a feature changing sufficiently slowly in time to represent a basic, unobservable characteristic. In this case, when other observable characteristics (barriers, concentration, etc.) are correlated with the degree of competition, the former can be used to measure the latter.

A leading example is the Hirschman-Herfindahl index of concentration:

$$HHI = \sum_{i=1}^N y_i^2$$

$y_i^2$  being the square of the market share of the  $i$ -th firm in the market <sup>(1)</sup>. Simple theories of monopolistic pricing show that HHI is positively related to non-competitive profits [see Cowling and Waterson (1976)] so that it can provide a measure of non-competitive behavior or, at least, a threshold to suspect it (see the 1982 Merger Guidelines of the U.S. Department of Justice).

Competition, however, is essentially a dynamic process so that it may be interesting to try to grasp its 'intensity' in a simple but properly dynamic context. This approach was

---

<sup>1</sup> HHI is equivalent to  $(cv^2+1)/N$  where  $cv$  is the coefficient of variation of market shares. It, therefore, ranges between  $1/N$  (equidistribution) and 1 (a single firm in the market).

taken in a seminal paper by Stigler (1964) and, surprisingly, it conduced again to HHI <sup>(2)</sup>.

In this paper, using a simple, two period model, we shall show that a more general result and a new implementable measure of competition can be obtained for which HHI represents a special (and implausible) case.

## 2. The model.

Let us consider, for clarity sake, the simplest case of a two-period market with two firms <sup>(3)</sup>,  $F_1$  and  $F_2$  selling a differentiated good at time 1 and at time 2 ('years').

If  $p(\hat{F}_i, F_j)$  is the joint probability of being customer of  $F_j$  at year 1 and of  $F_i$  at year 2 (year 2 is indicated by a hat), the 2x2 probability table is the following:

[ HERE TABLE 1 ]

Note that the marginal probability of being customer of Firm 1 at year 2 is the sum of the joint probability of being already one of its clients at year 1 and stay with it plus the joint probability of being a client of the competitor at

---

<sup>2</sup> In Stigler's paper the index is correlated with the likelihood of effective collusion.

<sup>3</sup> The extension to the general case of N firms is trivial.

time 1 and then change <sup>(4)</sup>:

$$p(\hat{F}_1) = p(F_1, \hat{F}_1) + p(F_2, \hat{F}_1) \quad (1)$$

Using the conditional probabilities:

$$\begin{aligned} p(\hat{F}_1) &= p(F_1)p(\hat{F}_1|F_1) + p(F_2)p(\hat{F}_1|F_2) \\ &\equiv p(F_1) f_{11} + p(F_2) f_{12} \end{aligned} \quad (2)$$

and for Firm 2:

$$\begin{aligned} p(\hat{F}_2) &= p(F_1)p(\hat{F}_2|F_1) + p(F_2)p(\hat{F}_2|F_2) \\ &= p(F_1) f_{21} + p(F_2) f_{22} \end{aligned} \quad (3)$$

In matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (4)$$

with the obvious conditions:

$$\sum_i y_i = 1, \quad \sum_j x_j = 1, \quad \sum_i f_{ij} = 1 \quad \forall j, \quad 0 \leq f_{ij} \leq 1 \quad (5)$$

where  $y_i$  and  $x_j$  are respectively year 2 and year 1 market shares and, following Stigler (1964), they are assumed as proxies of the marginal probabilities:  $p(\hat{F}_i) = y_i$ ,  $p(F_j) = x_j$ .

The parameters  $f_{ij}$  are the transition probabilities from Firm  $j$  to Firm  $i$ . In particular, the diagonal of the transition matrix contains the probabilities to stay with the

---

<sup>4</sup> Note that the entry in year 2 of new customers (a new column in table 1) or new firms (a new row in table 1) poses no special problem to the model.

same firm both in year 1 and year 2. They measure, therefore, the fidelity level of the customer relationship and in this sense they are correlated with market power. If market power can be excluded from the demand side, a high fidelity level means that flows among firms are small, customers are 'captured' by their suppliers and the latter benefits by a monopoly degree. On average, if the diagonal probabilities decrease it means that the market competition has increased, raising the flows of customers among firms.

This suggests a simple measure of non-competition defined by a weighted average of the diagonal conditional probabilities. Using the  $y_i$  weights we have the following fidelity index (with N firms):

$$FI = \sum_{i=1}^N f_{ii}y_i = \sum_{i=1}^N p(\hat{F}_i | F_i) p(\hat{F}_i) \quad (6)$$

Note that the value  $1/N$  is obtained when the conditional probabilities are all equals: every year the consumer chooses his/her supplier as the result of a coin toss so that there is perfect mobility of customers, no market power is accumulated by firms and the market is, in a dynamic, probabilistic sense, perfectly competitive.

At the other extreme, FI can correctly identify a situation in which a number of producers, possibly many with equal market shares (low HHI), determine a non-competitive, monopolistic configuration of the market, in which the probability of stay with the same supplier is near to 1 (high FI).

Moreover, note that in the case of independence we have  $p(\hat{F}_i|F_i) = p(\hat{F}_i)$ , meaning that becoming customer of Firm  $i$  at time 2 is independent of being already one of its customers at time 1. It is clearly an implausible assumption and in this case the fidelity index reduces to HHI:

$$HHI = \sum_{i=1}^N y_i^2 = \sum_{i=1}^N p(\hat{F}_i)p(\hat{F}_i) \quad (7)$$

### 3. Implementation.

In order to implement model (4) we have to estimate the conditional probabilities in the transition matrix, under the constraints given in (5).

Fortunately, the problem of estimating conditional probabilities from marginals (the 'ecological regression' problem) has been object of many contributions in the social sciences, starting from Miller (1952), Duncan and Davis (1953) and Goodman (1953, 1959), as well as in econometrics, from Telser (1963) to Theil and Rey (1966) and Lee and Judge (1972).

Let us assume that the extra-information needed to estimate the transition probabilities comes from  $H$  different geographical markets <sup>(5)</sup> where the 2 firms compete, under the maintained hypothesis that the transition probabilities do not vary in the considered areas.

---

<sup>5</sup> Another possibility, considered in the econometric literature, is information from the time dimension. The analysis is similar.



Introducing an error term  $\varepsilon$ , the first line (Firm 1) in (4) becomes, in the H areas:

$$\begin{aligned}
 Y_{1,1} &= x_{1,1} f_{11} + x_{2,1} f_{12} + \varepsilon_{1,1} \\
 \dots\dots\dots \\
 Y_{1,h} &= x_{1,h} f_{11} + x_{2,h} f_{12} + \varepsilon_{1,h} \\
 \dots\dots\dots \\
 Y_{1,H} &= x_{1,H} f_{11} + x_{2,H} f_{12} + \varepsilon_{1,H}
 \end{aligned} \tag{8}$$

i.e. in matrix form

$$Y_1 = X b_1 + \varepsilon_1$$

with  $\varepsilon_1 \sim D(0, \sigma_{11}I_H)$ , homoskedastic error term

and the complete system (in the general case of N firms)

becomes:

$$\begin{aligned}
 \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ Y_j \\ \cdot \\ \cdot \\ Y_N \end{bmatrix}_{HN \times 1} &= \begin{bmatrix} X & 0 & \dots & 0 \\ 0 & X & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & X & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 0 & X \end{bmatrix} \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ f_j \\ \cdot \\ \cdot \\ f_N \end{bmatrix}_{N^2 \times 1} + \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \varepsilon_j \\ \cdot \\ \cdot \\ \varepsilon_N \end{bmatrix}_{HN \times 1}
 \end{aligned} \tag{9}$$

i.e.

$$Y = Xf + \varepsilon \quad \varepsilon \sim D(0, \Sigma \otimes I_H), \quad \Sigma = [\sigma_{ij}], \quad \sigma_{ij} = E(\varepsilon_{ih} \varepsilon_{jh}) \tag{10}$$

with the constraints  $\sum_{i=1}^N f_{ij} = 1 \quad \forall j \quad 0 \leq f_{ij} \leq 1$

Note that  $\Sigma$  is singular because the errors are linearly dependent:

$$\sum_{j=1}^N y_{j,h}=1, \sum_{j=1}^N x_{j,h}=1 \quad \text{so that} \quad \sum_{j=1}^N e_{j,h}=0 \quad \forall h \quad (11)$$

The system (10) is a special case of Zellner's SUR equations with identical regressors and cross-equation parameter restrictions. An estimation method is suggested by Judge et al. (1985, p.499) and Theil and Rey (1966). Following this method, a first application to the Italian banking markets has been developed by the author obtaining different levels and behaviors of the fidelity and Herfindahl indexes. In particular, differently with respect to previous alternative measure, the fidelity index resulted to be non correlated with HHI.

## References

- Cowling, K. and M. Waterson, 1976, Price-costs Margins and Market Structure, *Economic Journal*, 43, 267-74.
- Duncan, O. D. and B. Davis, 1953, An Alternative to Ecological Correlation, *American Sociological Review*, 18, 665-66.
- Goodman, L. A., 1953, Ecological Regressions and Behavior of Individuals, *American Sociological Review*, 18, 663-64.
- Goodman, L. A., 1959, Some Alternatives to Ecological Correlation, *American Journal of Sociology*, 64, 610-25.
- Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lütkepohl and T. C. Lee, 1985, *The Theory and Practice of Econometrics*, (Wiley, New York).
- Lee, T. C. and G. G. Judge, 1972, Estimation of Transition Probabilities in a Nonstationary Finite Markov Chain, *Metroeconomica*, 24, 180-201.
- Miller, G. A., 1952, Finite Markov Process in Psychology, *Psychometrika*, 17, 149-67.
- Stigler, G., 1964, A Theory of Oligopoly, *Journal of Political Economy*, 72, 44-61.
- Telser, L. G., 1963, Least Squares Estimates of Transition Probabilities, in: V.A., *Measurement in Economics. Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, (Stanford University Press, Stanford) ch. 11.

Theil, H. and G. Rey, 1966, A Quadratic Programming Approach to the Estimation of Transition Probabilities, Management Science, 12, 714-21.

TABLE 1

		Y E A R 1		
		Firm 1	Firm 2	
Y E A R  2	Firm 1	$p(\hat{F}_1, F_1)$	$p(\hat{F}_1, F_2)$	$p(\hat{F}_1)$
	Firm 2	$p(\hat{F}_2, F_1)$	$p(\hat{F}_2, F_2)$	$p(\hat{F}_2)$
		$p(F_1)$	$p(F_2)$	1