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## Article

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Journal of Industrial Engineering International

## Provided in Cooperation with:

Islamic Azad University (IAU), Tehran

*Suggested Citation:* Seyedhoseini, S.M.; Rashid, Reza; Zangeneh, Erfan; Kamalpour, Iman (2015) : Application of queuing theory in inventory systems with substitution flexibility, Journal of Industrial Engineering International, ISSN 2251-712X, Springer, Heidelberg, Vol. 11, pp. 37-44, <https://doi.org/10.1007/s40092-015-0099-5>

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# Application of queuing theory in inventory systems with substitution flexibility

S. M. Seyedhoseini · Reza Rashid · Iman Kamalpour · Erfan Zangeneh

Received: 26 August 2014 / Accepted: 5 January 2015 / Published online: 23 January 2015  
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**Abstract** Considering the competition in today's business environment, tactical planning of a supply chain becomes more complex than before. In many multi-product inventory systems, substitution flexibility can improve profits. This paper aims to prepare a comprehensive substitution inventory model, where an inventory system with two substitute products with ignorable lead time has been considered, and effects of simultaneous ordering have been examined. In this paper, demands of customers for both of the products have been regarded as stochastic parameters, and queuing theory has been used to construct a mathematical model. The model has been coded by C++, and it has been analyzed due to a real example, where the results indicate efficiency of proposed model.

**Keywords** Inventory management · Substitution flexibility · Simultaneous ordering · Stochastic demand · Queuing theory

## Introduction

One of the challenges in supply chain management is to find optimal policy for inventory system, the main objective of inventory management is to balance conflicting goals like optimization of stock costs and shortage costs (Arda and Hennet 2006). Using flexible inventories is one of the ways to reduce inventory costs. Flexibility could be

considered in different ways, for example, through using product substitution, postponement (Tibben-Lembke and Bassok 2005) and lateral transshipments (Herer et al. 2006).

In substitution systems, flexible stock (mostly more expensive) will be used only when regular (cheaper) item stockout (Deflem and van Nieuwenhuysse 2011). For instance, if inventory of regular product cannot satisfy its demand, a higher quality item can be used as a substitute inventory (Liu and Lee 2007).

In summary, it is clear that despite many contributions in inventory management, there is little consideration due to substitution inventory models. In this paper, a stochastic stock control model has been proposed for two substitute products when lead time is ignorable, where the main contributions are summarized as follows.

- Demands have been considered as stochastic parameters.
- To prepare a comprehensive model, bi-level Markov process has been used.
- All the steady-state equations have been solved in terms of one state.

This model can be applied probably for inventory systems where demand is uncertain and two way substitutions can be used. For example, some items of dairy inventories have mostly stochastic demand, and some of them can use substitution with each other.

The reminder of this paper is organized as follows. In “Literature review”, a brief literature review has been presented. In “Model development and analysis”, first we represent a mathematical model for an inventory system with substitute products. The model is validated and some numerical examples are tested in “Solving approach”. We conclude our study in “Numerical results”.

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## Literature review

There are lots of researchers that worked on inventory models. For instance, AriaNezhad et al. (2013) proposed a two echelon system for perishable items in supply chains, and used a case study to analyze their model.

Generally, in a supply chain, most of the parameters are not deterministic, for this reason, some researchers used queuing theory to construct a mathematical model. In this area, there are some who prepared a model for stochastic demand and some who prepared a model for stochastic lead time. Parlar (1996) presented an inventory model which was combined with queuing theory to consider demand and lead time stochastic parameters. Hosseini et al. (2013) considered stochastic lead time, and developed a multi-objective pricing-inventory model for a retailer, where their main objective was to maximize retailer's profit and service level. Seyedhoseini et al. (2014) considered poison demand for customers in a cross-docking problem, and prepare a stochastic model.

For better modeling of stochastic environment, some researchers used queuing theory, for example, Ha (1997) considered poison demand and exponential production times for a single-item make-to-stock production system. He proposed an  $M/M/1/S$  queuing system for modeling the system. Arda and Hennet (2006) analyzed inventory control of a multi-supplier strategy in a two-level supply chain. They considered random arrival for customers and random delivery time for suppliers, and represented their system as a queuing network. Isotupa (2006), considered a lost sales ( $s, Q$ ) inventory system with two customer groups, and illustrated the model by Markov processes.

Babai et al. (2010) considered demand and lead time stochastic and analyzed a single-echelon single-item inventory system by means of queuing theory. Considering effectiveness of queuing theory in inventory problems, we also used queuing theory to develop our model. Toktas-Palut and Ülengin (2011) coordinated the inventory policies in a two-stage decentralized supply chain, where each supplier has been considered as an  $M/M/1$  queue and the manufacture has been assumed as a  $GI/M/1$  queue.

Alimardani et al. (2013) applied continuous review ( $S-I, S$ ) policy for inventory control and supposed a bi-product three-echelon supply chain which is modeled as an ( $M/M/I$ ) queue model for each type of products offered through the developed network. In addition, to show the performance of the proposed bi-product supply chain, they also considered a network including two ( $M/M/I$ ) queue for each type of products.

Some researchers studied substitution flexibility. For example, Bayindir et al. (2007) consider a one-way substitution system with two products which uses  $S-I, S$  policy. They use a two-dimensional Markov process to develop

the model, where the objective of their research was to find the optimal order up to levels. Liu and Lee (2007) proposed three different policies to use one-way substitution, and developed an inventory system with backlogs.

Olssen (2010) considered a continuous review inventory system where one-way lateral transshipment is allowed. Nagarajan and Rajagopalan (2008) dealt with a two-product problem, where substitution has been assumed for both of the products. Bahri and Tarokh (2012) presented a coordinated seller-buyer supply chain model in two stages, which is called Joint Economic Lot Sizing (JELS) in the literature. They assumed that the delivery lead time is stochastic and follows an exponential distribution and delivery activities consist of a single raw material. Tan and Karabati (2013) proposed an inventory management that incorporates the effects of stockout-based dynamic substitutions.

Deflem and van Nieuwenhuysse (2011) presented an approach to analyze two-item periodic inventory system with one-way substitution flexibility, where the objective function was to minimizing the expected purchasing costs, holding costs, shortage costs and adjustment costs. Ye (2014) dealt with the problem of inventory management and simultaneously horizontal (equivalently inter-brand) and vertical (equivalently intra-brand) substitution.

Considering great literature of stochastic inventory we prepare a table in which research on substitute flexibility and stochastic inventory have been analyzed.

As demonstrated in Table 1, there are some papers which dealt with the problem of ( $R, Q$ ) inventory in stochastic environment; however, only some of them considered substitution flexibility. Even among those researchers who worked on substitution flexibility, assumptions of two way of substitution and stochastic ( $R, Q$ ) inventory are new.

## Model development and analysis

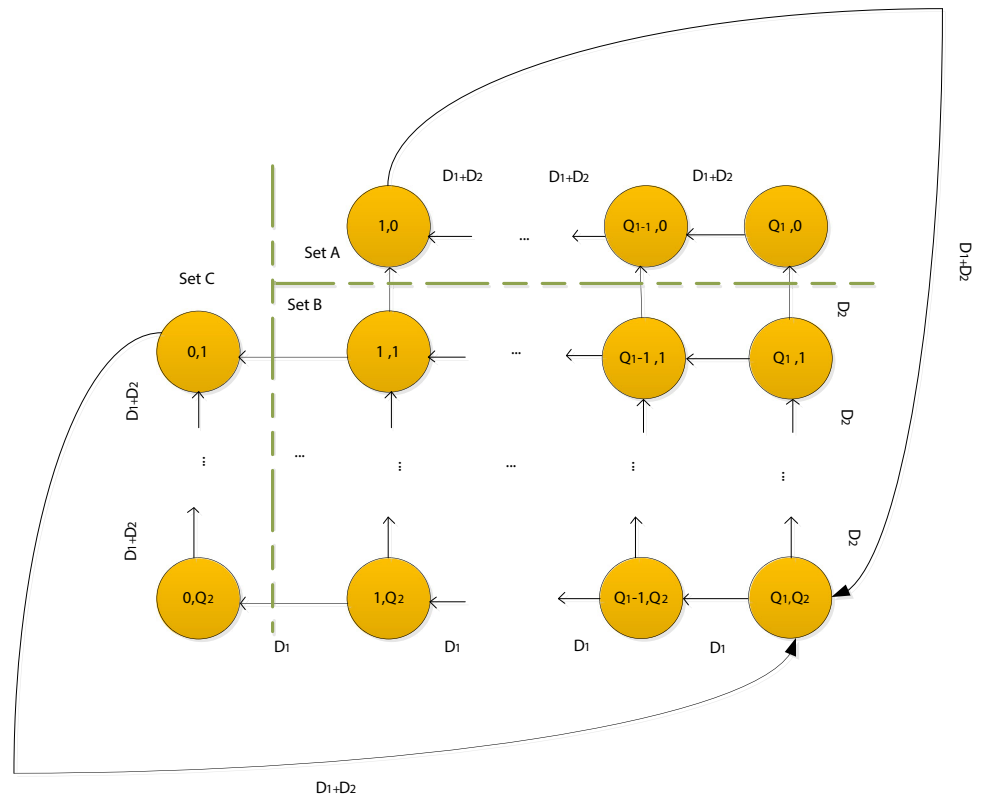
In this paper, an inventory system with two substitute products has been considered where shortage is not allowed, and replacement causes costs for both of the products. For simplification of the system, it has been assumed that lead time is an ignorable value. In this research, demand of each product has been considered poison distributor, and Markov process has been used to model the system, where the states have been represented by ( $I, J$ ).  $I$  and  $J$  represent inventory levels for each type of products. For this queue, Fig. 1 demonstrates the transition diagram when both of the products are ordered together.

As it has been demonstrated in Fig. 1, states have been put into three sets, set  $A$  and set  $C$  represent the states which have only one type of inventories and the inventory level will be reduced if a demand for each kind of products

**Table 1** Classification of papers in stochastic inventory and substitution flexibility

Authors	Shortage		Lead time	Demand	Replenishment policy	Substitute flexibility	
	Lost sales	Back order				One way	Two way
Parlar (1996)		✓	Stochastic	Stochastic	( $R, Q$ )		
Isotupa (2006)		✓	Stochastic	Stochastic	( $R, Q$ )		
(Arda and Hennet 2006)		✓	Stochastic	Stochastic	( $S-I, S$ )		
Boute et al. (2007)		✓	Stochastic	Stochastic	( $T, S$ )		
Olssen (2010)	✓	✓	Deterministic	Stochastic	( $R, Q$ ), ( $S-I, S$ )		
Hill et al. (2007)	✓		Deterministic	Stochastic	( $S-I, S$ )		
Hannet and Arda (2008)		✓	Stochastic	Stochastic	( $S-I, S$ )		
Teimoury et al. (2010)		✓	Stochastic	Stochastic	( $R, Q$ )		
Toktas-Palut and Ülengin (2011)		✓	Stochastic	Stochastic	( $S-I, S$ )		
Babai et al. (2010)		✓	Stochastic	Stochastic	( $S-I, S$ )		
Tili et al. (2012)	✓		Deterministic	Stochastic	( $T, s, S$ )		
Bahri and Tarokh (2012)		✓	Stochastic	Deterministic	( $R, Q$ )		
Alimardani et al. (2013)		✓	Stochastic	Stochastic	( $S-I, S$ )		
Guerrero et al. (2013)	No shortage		Deterministic	Stochastic	( $T, s, S$ )		
Yu and Dong (2014)	No shortage		Deterministic	Stochastic	( $R, Q$ )		
Baek and Moon (2014)		✓	Stochastic	Stochastic	( $R, Q$ )		
Deflem and van Nieuwenhuyse (2011)	Combination of both models		Deterministic	Stochastic	( $T, S$ )	✓	
Ye (2014)	✓		Deterministic	Stochastic	( $R, Q$ )	✓	
Ahiska and kurtul (2014)	Combination of both models		Deterministic	Stochastic	( $T, S$ )	✓	
Salameh et al. (2014)	✓		Deterministic	Deterministic	( $R, Q$ )		✓
Krommyda et al. (2015)	✓		Deterministic	Deterministic	( $R, Q$ )		✓
Our model	✓		Stochastic	Deterministic	( $R, Q$ )		✓

**Fig. 1** Transition diagram of proposed model



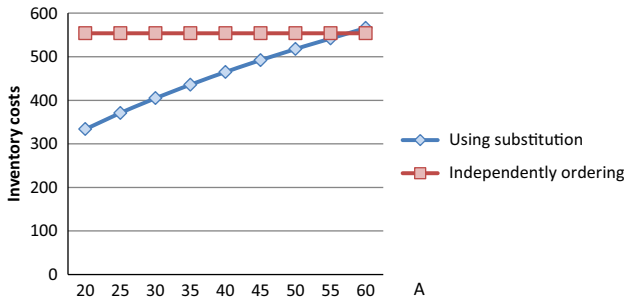


Fig. 2 Sensitivity of the models due to A

inters to the system. Set B represents the states which have inventories for both kinds of products, and no substitution happens.

To prepare a comprehensive model, four lemmas have been proposed to calculate steady-state probabilities in terms of probability of state  $(Q_1, Q_2)$ . Henceforth, let  $\pi_{i,j}$  denote steady-state probability of state  $(I, J)$ ,  $D_1$  denote demand rate for type-one product, and  $D_2$  denote demand rate for type-two product.

**Lemma 1** In a network when flows arrive only through one state like set B, and flows only move to right or top, arrival flows to node  $(I, J)$  would be sum of the flows which come from node  $(Q_1, Q_2)$  through all the directions. If  $IF_{i,j}$  denotes interval flow for state  $(I, J)$ , and  $IF_{i,j}^k$  demonstrates arrival flow which comes through path  $k$ , and there are  $N$  paths between state  $(Q_1, Q_2)$  and  $(I, J)$ , so we have:

$$IF_{i,j} = \sum_{k=1}^N IF_{i,j}^k \tag{1}$$

**Proof** Arrival flow for this network only goes through node  $(Q_1, Q_2)$ . Consequently, each flow goes to state  $(I, J)$  comes from state  $(Q_1, Q_2)$ . Considering flows only move to right or top, each path between node  $(Q_1, Q_2)$  and node  $(I, J)$ , which has  $I-1$  lateral movement and  $J-1$  vertical movement consists a flow. Also flows only go through these paths, so  $IF_{i,j}$  is sum of flows that go through these paths and could be calculated by Eq. 1.

**Lemma 2** If Fig. 2, demonstrates a queuing network, where each node has vertical output rate with value of  $D$  and lateral output rate with value of  $\mu$ .  $IF_{i,j}$  could be calculated by Eq. 2.

$$IF_{i,j} = \left( \left( \frac{D_1^{Q_1-i} D_2^{Q_2-j}}{(D_1 + D_2)^{Q_2+Q_1-i-j}} \right) \binom{Q_2 + Q_1 - i - j}{Q_2 - j} \right) IF_{Q_1, Q_2} \tag{2}$$

**Proof**  $IF_{k,m}$  has a vertical movement with probability of  $\frac{D_2}{D_1+D_2}$  and a lateral movement with probability of  $\frac{D_1}{D_1+D_2}$ . Consequently if flow has a vertical movement, it will be

multiplied by coefficient  $\frac{D_2}{D_1+D_2}$ , and if it has a lateral movement, it will be multiplied by coefficient  $\frac{D_1}{D_1+D_2}$ . Also between node  $(Q_1, Q_2)$  and node  $(I, J)$  there are  $Q_2 - j$  vertical movements and  $Q_1 - i$  lateral movements, consequently, in each path, input flow for  $(I, J)$  is:

$$\left( \frac{D^{i-1} \mu^{j-1}}{(\mu + D)^{j+i-2}} \right) IF_{1,1} \tag{3}$$

Also number of paths between node  $(Q_1, Q_2)$  and node  $(I, J)$  is:

$$\binom{Q_2 + Q_1 - i - j}{Q_2 - j} \tag{4}$$

Considering lemma 1 and Eqs. 3 and 4, below expression can be obtained.

$$IF_{i,j} = \sum_{k=1}^N IF_{i,j}^k = \left( \frac{D_1^{Q_1-i} D_2^{Q_2-j}}{(D_1 + D_2)^{Q_2+Q_1-i-j}} \right) IF_{Q_1, Q_2} \sum_{k=1}^N 1 = \left( \left( \frac{D_1^{Q_1-i} D_2^{Q_2-j}}{(D_1 + D_2)^{Q_2+Q_1-i-j}} \right) \times \binom{Q_2 + Q_1 - i - j}{Q_2 - j} \right) IF_{Q_1, Q_2} \tag{5}$$

In this Markov process, output rate is equal to  $D_1 + D_2$ , and below equation is true for steady-state probabilities.

$$(D_1 + D_2)\pi_{i,j} = IF_{i,j} \tag{6}$$

Consequently,  $IF_{Q_1, Q_2}$  is equal to  $\frac{IF_{Q_1, Q_2}}{D_1+D_2}$  and steady states for set B can be calculated by Eq. 7.

$$\pi_{i,j} = \left( \left( \frac{D_1^{Q_1-i} D_2^{Q_2-j}}{(D_1 + D_2)^{Q_2+Q_1-i-j}} \right) \binom{Q_2 + Q_1 - i - j}{Q_2 - j} \right) \pi_{Q_1, Q_2} \tag{7}$$

To calculate steady-state probabilities of set A, lemma 3 has been proposed.

**Lemma 3** Steady-state probabilities for set A can be calculated as follows

$$\pi_{i,0} = \sum_{s=0}^{Q_1-i} \pi_{Q_1-s,1} \cdot \frac{D_1}{D_1 + D_2} = \sum_{s=0}^{Q_1-i} \left( \binom{Q_1 + Q_2 - s - 1}{Q_2 - 1} \right) \cdot \pi_{Q_1, Q_2} \cdot \frac{D_1^{Q_1-s} \cdot D_2^{Q_2-1}}{(D_1 + D_2)^{Q_1+Q_2-s-1}} \cdot \frac{D_1}{D_1 + D_2} \tag{8}$$

**Proof** All of the flows which arrive to set A come from states which have  $J = I$ , and also states of set A have only lateral rate with value of  $D_1 + D_2$ . For state  $(I, 0)$ , Eq. 9 is true.

$$\mu(\pi_{i,0}) = D(\pi_{i,1} + \pi_{i+1,1} + \dots + \pi_{Q_1,1}) \tag{9}$$

Considering Eq. 9 and lemma 2, lemma 3 is proved.

Similar to set A, steady-state probabilities of set C can be obtained as follows:

$$\begin{aligned} \pi_{0,j} &= \sum_{s=0}^{Q_2-j} \pi_{0,Q_2-s} \cdot \frac{D_2}{D_1 + D_2} \\ &= \sum_{s=0}^{Q_2-j} \binom{Q_1 + Q_2 - s - 1}{Q_2 - s} \cdot \pi_{Q_1, Q_2} \\ &\quad \cdot \frac{D_1^{Q_1-1} \cdot D_2^{Q_2-s}}{(D_1 + D_2)^{Q_1+Q_2-s-1}} \cdot \frac{D_2}{D_1 + D_2} \end{aligned} \tag{10}$$

Lemma 4 Amount of substitution for product 1 and product 2 is represented by Eqs. 10 and 11.

$$\sum_{j=1}^{Q_2} \pi_{1,j} \cdot j \cdot \left( \frac{D_1}{D_1 + D_2} \right)^2 \tag{11}$$

$$\sum_{i=1}^{Q_1} \pi_{i,1} \cdot i \cdot \left( \frac{D_2}{D_1 + D_2} \right)^2 \tag{12}$$

Proof State of  $\pi_{1,j}$  changes to  $\pi_{0,j}$  by probability of  $\frac{D_1}{D_1+D_2}$ , and then  $\frac{D_1 \cdot j}{D_1+D_2}$  substitution inventories must be used to satisfy demand of product 1. Consequently, Eq. 11 calculates substitution amount for product 1 and similar to product 1, Eq. 12 calculates substitution amount for product 2.

This section presents a mathematical model to solve the problem described above.

**Parameters**

- $h_1$  Holding costs for product 1
- $h_2$  Holding costs for product 1
- $A_1$  Ordering cost for product 1 for independently ordering
- $A_2$  Ordering costs for product 2 for independently ordering
- $A$  Ordering costs of simultaneous ordering
- $D'_1$  Average demand for inventory of type one in a specific decision period
- $D'_2$  Average demand for inventory of type two in a specific decision period
- $c_1$  Replacement costs for product 1
- $c_2$  Replacement costs for product 2

**Variables**

- $\pi_{i,j}$  Steady-state probability of state  $(I, J)$
- $Q_1$  Quantity of ordering for product one
- $Q_2$  Quantity of ordering for product two

Objective:

$$\begin{aligned} Z &= \sum_{j=0}^{Q_2} \sum_{i=0}^{Q_1} (h_1 \cdot i) \cdot \pi_{i,j} + \sum_{j=0}^{Q_2} \sum_{i=0}^{Q_1} (h_2 \cdot j) \cdot \pi_{i,j} + A \\ &\quad \cdot \left( \frac{D'_1 + D'_2}{Q_1 + Q_2} \right) + c_1 \cdot \sum_{j=1}^{Q_2} \pi_{1,j} \cdot j \cdot \left( \frac{D_1}{D_1 + D_2} \right)^2 \end{aligned} \tag{13}$$

**Table 2** Example

$c_2$	$c_1$	$D_2$	$D_1$	$D'_2$	$D'_1$	$A$	$A_2$	$A_1$	$h_2$	$h_1$
20	20	10	5	200	100	30	30	30	8	10

**Table 3** Comparison between using substitution and independently ordering

	Using substitution		Independently ordering	
	Product 1	Product 2	Product 1	Product 2
Ordering quantity	10	37	24	38
Costs of inventory system		405		554
Substitution costs		33		0

The objective function is composed of five sections. The first two sections are for calculating holding costs, third section calculates ordering costs. Forth section and fifth sections are to calculate substitution costs.

In this system, when each product is ordered independently, substitution will not used, and optimal ordering quantity for product 1 and 2 can be calculated by  $\sqrt{\frac{2 \cdot A_1 \cdot (D'_1)}{h_1}}$  and  $\sqrt{\frac{2 \cdot A_2 \cdot (D'_2)}{h_2}}$ .

**Solving approach**

When there are two types of products with substitution flexibility, and  $Q_1$  and  $Q_2$  represent ordering quantity of these products, then if  $Q_2$  becomes zero, optimal  $Q_1$  will be the biggest amount that it can be. If only product 1 be ordered  $CO_1$  represents costs of the system and if only product 2 be ordered  $CO_2$  represents costs of the system.

$$CO_1 = A \cdot \left( \frac{D'_1 + D'_2}{Q_1} \right) + h_1 \cdot \bar{I}_1 + D'_2 \cdot c_2 \tag{14}$$

$$CO_2 = A \cdot \left( \frac{D'_1 + D'_2}{Q_1} \right) + h_2 \cdot \bar{I}_2 + D'_1 \cdot c_1 \tag{15}$$

where  $\bar{I}_1$  and  $\bar{I}_2$  are equal to  $\frac{Q_1}{2}$  and  $\frac{Q_2}{2}$ . And optimal  $Q_1$  and  $Q_2$  for  $CO_1$  and  $CO_2$  are  $\sqrt{\frac{2 \cdot A \cdot (D'_1 + D'_2)}{h_1}}$  and  $\sqrt{\frac{2 \cdot A \cdot (D'_1 + D'_2)}{h_2}}$ . Consequently, Eqs. 16 and 17 can be used to limit feasible space of the problem.

$$Q_1 \leq \sqrt{\frac{2 \cdot A \cdot (D'_1 + D'_2)}{h_1}} \tag{16}$$

$$Q_2 \leq \sqrt{\frac{2 \cdot A \cdot (D'_1 + D'_2)}{h_2}} \tag{17}$$

### Numerical results

In this section, proposed model has been coded by C++ software and an example has been produced in Table 2.

For this example, Table 3 compares using substitution with independently ordering, where superiority of substitution has been demonstrated.

Decision for this system, can be influenced by different parameters. For that matter, this section has analyzed the model due to different parameters. For the previous example, Fig. 2 illustrates sensitivity of the model due to A. Decision of using substitution or not depends on value of

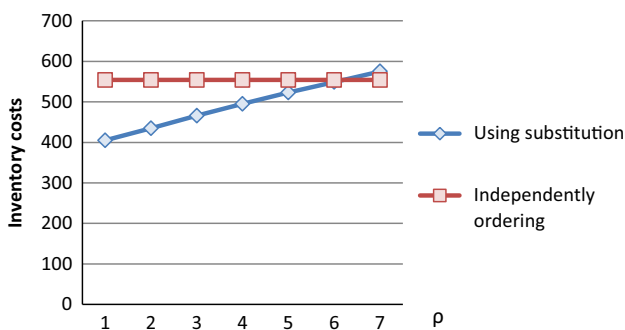


Fig. 3 Sensitivity of the models due to substitution costs

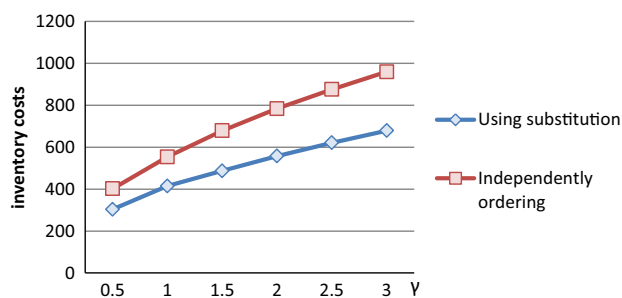


Fig. 4 Sensitivity of the models due to holding costs

A. For this example, if A be less than 57 it would be affordable to use substitution, otherwise it can cause more costs than independently ordering. Consequently, it can be inferred that decision of independently ordering or simultaneous ordering (using substitution) are dependent on parameter of A.

To analyze sensitivity of substitution costs, it has been assumed that rho be a coefficient for both of products substitution costs and systems costs have been analyzed in Fig. 3. Substitution costs effect decision of using substitution flexibility. In this example, if substitution costs be bigger than 120, it is not affordable to use substitution flexibility.

To analyze sensitivity of holding costs, it has been assumed that gamma be a coefficient for both of products holding costs and systems costs have been analyzed in Fig. 4, where sensitivity of independent ordering due to holding costs is more than when substitution flexibility is used.

### Real example

The proposed model can be appropriately used for Dairy supply chains. KALE Company produces Dairy products, and this company has two factories located in Amol and Karaj cities, and also 25 cities consisting majority of KALE customers.

In this supply chain, 25 retailers exist, where 18 retailers are for KALE and the other 7 retailers are acting in a decentralized supply chain. For most of these retailers, transportation is more than 3 h and there are only five retailers near KALE factories, which have less transportation time than 3 h. For this reason, we only considered one retailer which has less transportation time than the other retailers. Ahamd Abad retailer is located at Tehran city and it acts in a centralized supply chain with KALE.

KALE has some characteristics that make it suitable for our model. First its production rate is high, and it has a

Table 4 Comparison between using substitution and independently ordering for cream cheese and Amol cheese

	Using substitution		Independently ordering		Current policy in company	
	Product 1	Product 2	Product 1	Product 2	Product 1	Product 2
Ordering quantity	1,520	3,640	3,330	3,560	2,100	21,000
Costs of inventory system	1,513		2,633		2,124	

Table 5 Comparison between using substitution and independently ordering for low-fat Tetra milk and low-fat Manshori milk

	Using substitution		Independently ordering		Current policy in company	
	Product 1	Product 2	Product 1	Product 2	Product 1	Product 2
Ordering quantity	1,330	2,640	2,630	2,780	1,850	1,850
Costs of inventory system	1,851		3,210		2,520	

**Table 6** Comparison between using substitution or independently ordering IML yogurt and Ps yogurt

	Using substitution		Independently ordering		Current policy in company	
	Product 1	Product 2	Product 1	Product 2	Product 1	Product 2
Ordering quantity	2,160	1,210	2,300	2,120	1,450	1,450
Costs of inventory system	1,100		1,950		1,760	

small transportation time for Ahmad Abad retailer, so we could consider lead time as an ignorable parameter. On the other hand, demand of Ahmad Abad is high. For this reason, we considered poison distribution for demand of each retailer.

Although KALE Company produces different kinds of Dairy products, we only considered three kinds of products that can use substitution. For this company, cream cheese and Amol cheese, low-fat Tetra milk and low-fat Manshori milk, and IML yogurt and Ps yogurt are substitute products. This company has same ordering quantity for each of these kinds of products. For this case, analyses of these three substitution products have been represented in Tables 4, 5 and 6, where costs are in 1,000,000 Rials.

As demonstrated in these tables, current policy is to order in same quantity for each kind of substitute products. Using this policy, their costs are lesser than when they use independent ordering. But using substitution flexibility they can improve inventory costs significantly, where major reason is decrease in ordering costs. Using substitution, overall ordering quantity has been increased, where 1794 items have been increased for cheese, 270 items increased for milk, and 470 items for yogurt; however, 611,000,000 Rials for cheese, 669,000,000 Rials for milk and 660,000,000 Rials for yogurt would diminish.

## Conclusion

In this research, an inventory system with two substitute products with ignorable lead time and stochastic demand has been considered, and by means of queuing theory a mathematical model has been proposed. For this system, steady-state equations have been solved and all of the steady-state probabilities have been calculated in terms of  $\pi_{Q_1, Q_2}$ .

In this paper, the model has been analyzed due to different parameters and their behaviors have been discovered. Using substitution requires simultaneous ordering and it can be compared with the situation when different items use independent ordering. It is clear that, sometimes decision of using substitution and simultaneous ordering is not affordable and it depends on values of problem parameters, so we analyzed this too.

We also prepared a real example, which is for Kale Company. Inventory costs of this Company for three kinds of substitution products have been analyzed, and effectiveness of the model has been revealed.

For future studies, this research can be extended by considering substitution stock control system for more than two products with different substitution relations, this may increase complexity of the problem but the model would become more realistic. Another extension of this research is possible by considering rate of corruption for perishable inventories.

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