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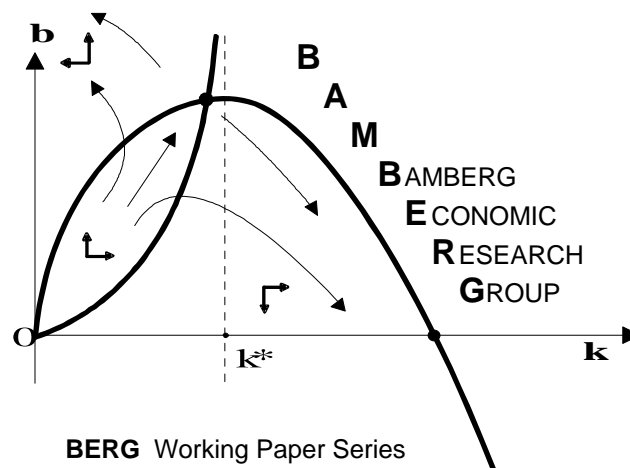
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# Risk Aversion and Prudence in Contests

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# Risk Aversion and Prudence in Contests

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## Abstract

I examine the impact of risk preferences on efforts and winning probabilities in generalised Tullock contests between two players. The theoretical analysis yields two main results. First, I specify a sufficient condition on the agents' comparative prudence under which a higher common level of risk aversion leads to lower aggregate effort in symmetric contests. Second, I show that for a certain range of parameters in asymmetric contests, higher risk-aversion will be a disadvantage if the agent is comparatively prudent.

**Keywords:** Tullock Contest; Risk Aversion; Prudence

**JEL classification:** C72, D72

## 1 Introduction

Contests are situations in which participants compete for some exogenous rent (prize) by spending non-refundable effort which increases their likelihood of winning. Examples from business, politics, sports, and many other areas of life abound (see e.g. Konrad, 2009). As some element of luck or uncertainty is inherent in contests, the question how individual attitudes towards risk affect the contestants' behaviour suggests itself.

In this paper, I study the influence of individual risk preferences on the equilibrium behaviour in the model of a general two-person Tullock contest under perfect information. The contestants simultaneously choose their effort levels (investments) in order to maximise their expected utility.

First, I compare aggregate efforts across two *symmetric* contests in one of which the common level of risk aversion is higher than in the second. I show that an increase in the common level of risk aversion will reduce aggregate efforts if it comes along with a sufficiently high measure of comparative prudence. This generalises the result that, in a symmetric contest, effort will be lower under risk aversion than under risk neutrality if the risk averse individuals are prudent (Treich, 2010).

Second, I analyse individual efforts and odds within an *asymmetric* contest between two agents one of which is more risk averse than the second. For a wide range of parameters, I show that the more risk averse agent will exert less effort and have a smaller chance of winning if she is sufficiently prudent compared to the second player.

To gain some intuition for these results, notice for a start that the impact of risk aversion on the behaviour in contests is generally ambiguous (see e.g. Konrad and Schlesinger, 1997). Since participation in the contest comes along with an uncertain payment, it may be regarded as a lottery. The general ambiguity then arises because risk aversion induces two opposing effects (see e.g. Skaperdas and Gan, 1995). On the one hand, there is the so called *gambling effect*: The more risk averse the agents are, the fewer lottery tickets they buy since this reduces their safe payment. On the other hand, there is the so called effect of *self-protection*: Buying more lottery tickets, the players can reduce their probability of losing. Therefore, the more risk averse they are, the more they invest. However, a prudent agent is downside risk averse, i.e. the agent prefers some lottery at a higher wealth level over the same lottery at a lower wealth level (Menezes et al., 1980). Thus, as higher investment lowers the wealth level and increases downside risk, the gambling effect will dominate the self-protection effect if the agent is comparatively prudent.

The remainder of this paper is organised as follows: In Section 2, I review the related literature. Section 3 introduces the model. In Section 4, I consider symmetric contests, and in Section 5 contests with asymmetric risk preferences. Section 6 concludes.

## 2 Related Literature

Relatively few papers explicitly address the role of risk preferences in contests. Most of them assume homogeneous players and focus on aggregate effort (rent dissipation). In general, the influence of risk aversion on aggregate effort in symmetric contests with a finite number of players and general contest success functions is ambiguous (Konrad and Schlesinger, 1997). Hillman and Katz (1984) consider a Tullock contest with linear technologies and a large number of homogeneous contestants with a common degree of constant relative risk aversion (CRRA). They show that rent dissipation decreases as the common degree of CRRA increases. Millner and Pratt (1991) present a qualified extension of this result to symmetric Tullock contests with linear technologies and two homogeneous participants: If the agents are risk averse, rent dissipation will be lower than in the case of risk neutral players if and only if the agents are also prudent. Treich (2010) extends the result of Millner and Pratt (1991) to symmetric contests with general contest success functions and any finite number of players showing that aggregate rent seeking efforts of risk averse agents will be lower than in the risk neutral case if the agents are also prudent. As Cornes and Hartley (2012) demonstrate, this result will also hold in asymmetric Tullock contests with linear technologies if no single agent has an equilibrium probability of winning greater than one half. Moreover, they show that in a large symmetric Tullock contest with general technologies rent dissipation is the smaller the higher the common level of risk aversion thereby generalising the original result in Hillman and Katz (1984).

While all these papers examine how changes in common risk preferences influence aggregate behaviour and rent dissipation *across* contests, I also ask how differing risk preferences affect individual behaviour and winning probabilities *within* a given contest. This question has not yet been addressed in the literature for general risk preferences but only for the specific cases of constant absolute risk aversion (CARA, Skaperdas and Gan, 1995, Cornes and Hartley, 2003) and CRRA (Bozhinov, 2006) both of which imply prudence. The authors show that, for such preferences, the less risk averse (and thus less prudent) of any two agents exerts more effort and, therefore, has the better chance of winning.

Hence, the contribution of my paper is twofold. First, I specify a sufficient condition on relative prudence under which a higher common level of risk aversion leads to lower aggregate effort in symmetric contests, thereby generalising the result of Treich (2010). Second, I find that comparative prudence plays an important role for risk aversion to decrease relative individual effort and thus winning probabilities within a certain contest, thereby extending the analysis of asymmetric risk preferences from the CARA and CRRA cases to general utility functions.

Despite the ambiguous theoretical predictions, most of the experimental studies on the impact of risk preferences in contests find that risk aversion significantly

reduces mean individual effort; see e.g. Millner and Pratt (1991), Anderson and Freeborn (2010), or Sheremeta (2011). As there usually is a positive correlation between prudence and risk aversion (Noussair et al., 2014, Ebert and Wiesen, 2014), my theoretical analysis suggests that these results may be driven indirectly by prudence.

### 3 The Model

Consider a contest between two agents which are identical except for possibly differing risk preferences. Each agent  $i \in \{1, 2\}$  has an initial wealth endowment  $I$  and can spend some resources  $x_i \geq 0$  in order to improve her probability  $p_i$  of winning some exogenously given rent  $R > 0$ . The winning probability is determined by the following contest success function (CSF): If  $x_1 = x_2 = 0$  then  $p_i := 1/2$ , else

$$p_i := \frac{f(x_i)}{f(x_1) + f(x_2)}, \quad (1)$$

where  $f : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$  is a twice continuously differentiable function of  $x_i$  satisfying  $f_i'' \leq 0 < f'$  and  $f_i(0) = 0$ .

The agents are weakly risk averse. The utility  $u_i(z)$  agent  $i \in \{1, 2\}$  derives from a certain wealth level  $z$  can be expressed by means of some three times continuously differentiable function  $u_i : \mathbb{R} \rightarrow \mathbb{R}$  with  $u_i'' \leq 0 < u_i'$ .

The contest is organised as a simultaneous move game with complete information. The players choose their effort levels  $x_i$  in order to maximise their expected utility  $Eu_i$  from wealth  $z_i$ , which will equal  $W_i := I - x_i + R$  if agent  $i$  wins the contest and  $L_i := I - x_i$  otherwise. Hence, for  $i, j \in \{1, 2\}, i \neq j$ ,

$$\begin{aligned} Eu_i &= p_i u_i(W_i) + (1 - p_i) u_i(L_i) \\ &= \frac{f(x_i)}{f(x_1) + f(x_2)} u_i(I - x_i + R) + \frac{f(x_j)}{f(x_1) + f(x_2)} u_i(I - x_i). \end{aligned}$$

Cornes and Hartley (2012, Theorem 3.1) show that under these assumptions a Nash equilibrium in pure strategies always exists. Moreover, they derive some regularity condition on the curvature of the utility functions  $u_i$  under which the Nash equilibrium is unique (Cornes and Hartley, 2012, Theorem 4.2). Yamazaki (2009) shows that, under the assumptions made, the Nash equilibrium in pure strategies will be unique if the Arrow-Pratt measure of absolute risk aversion  $RA(u_i, z) = -\frac{u_i''(z)}{u_i'(z)}$  is non-increasing in the wealth level  $z$  for all agents  $i$ . All these results hold even for the more general case of a contest between an arbitrary number  $n \in \mathbb{N}$  of participants and includes the possibility that some of them might be inactive in equilibrium, i.e. exert zero effort. However, in any equilibrium of a two-player contest, both players will obviously exert positive effort. The corresponding effort levels will hence be fully characterised by the two first order

conditions for maximum expected utilities:

$$\frac{\partial p_i}{\partial x_i} = \frac{p_i u'_i(W_i) + (1 - p_i) u'_i(L_i)}{u_i(W_i) - u_i(L_i)}. \quad (2)$$

#### 4 Symmetric Contests

In this subsection, I suppose that contestants are homogeneous, i.e.  $u_i = u$  for  $i \in \{1, 2\}$ . The contest then has a unique symmetric Nash equilibrium in pure strategies (Cornes and Hartley, 2012, Theorem 4.1). In order to analyse how a change in the common risk preferences influences (aggregate) effort, I compare the symmetric equilibria of two symmetric contests  $A$  and  $B$  which are identical but with respect to the participants' risk preferences: In contest  $A$ , both agents  $i \in \{1, 2\}$  have identical preferences described by  $u_i = u_A$  and in contest  $B$ , both have identical preferences described by  $u_i = u_B$ .

In general, there is ambiguity whether a rising level of risk aversion *ceteris paribus* decreases the contestant's effort due to the gambling-effect or increases it due to the effect of self-protection (Skaperdas and Gan, 1995, Konrad and Schlesinger, 1997). However, Treich (2010, Proposition 2) shows that if all agents are risk neutral in contest  $A$  but risk averse in contest  $B$ , efforts will be lower in contest  $B$  if the agents in contest  $B$  are prudent, i.e. if  $u_B''' > 0$ .<sup>1</sup> I will generalise this result to the case in which agents are more risk averse in contest  $B$  than  $A$  with respect to their Arrow-Pratt measure of absolute risk aversion, i.e.  $RA(u_B, z) > RA(u_A, z)$  for all possible wealth levels  $z$ . In this case, there exists a function  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  with  $\phi'' < 0 < \phi'$  and  $u_B(z) = \phi(u_A(z))$  for all possible wealth levels  $z$ , i.e.  $u_B$  is a concave transformation of  $u_A$  (Mas-Colell et al., 1995, Proposition 6.C.2). I will call such a transformation *prudent*, if  $\phi''' > 0$ . Note that a prudent transformation preserves the prudence property of risk preferences: If  $u_B(z) = \phi(u_A(z))$  and  $\phi''' > 0$  then  $u_B''' \geq 0$  will imply

$$u_B''' = \phi'''(u_A)(u_A')^3 + 3\phi''(u_A)u_A'u_A'' + \phi'(u_A)u_A''' > 0.$$

**Proposition 1** *Suppose that contests  $A$  and  $B$  differ only in the participants' common risk preferences such that agents are more risk averse in contest  $B$ , i.e.  $u_B = \phi(u_A)$  with  $\phi'' < 0 < \phi'$ . If  $u_B$  is a prudent transformation of  $u_A$ , i.e. if  $\phi''' > 0$ , then efforts will be lower in contest  $B$ .*

The proof can be found in the Appendix. Proposition 1 includes the special case of players in contest  $A$  being risk neutral as discussed by Treich (2010, Proposition

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<sup>1</sup>In the case of two players, prudence is even necessary for efforts to be lower (Treich, 2010, Corollary 1).



2) and Cornes and Hartley (2012, Proposition 6.1), respectively.<sup>2</sup> In this case,  $u_A'' \equiv 0 \equiv u_A'''$  and  $\phi''' > 0$  then implies  $u_B''' > 0$ , i.e. the prudence of players in contest  $B$ . Note, however, that Proposition 1 will hold even for the players in contest  $A$  not being prudent. In this case, the condition  $\phi''' > 0$  neither implies the prudence of players in contest  $B$ . Just as little, the condition  $\phi''' > 0$  generally implies that agents in contest  $B$  are more prudent (less imprudent) than in contest  $A$  with respect to the coefficient of absolute prudence  $PA(u, z) = -\frac{u'''(z)}{u''(z)}$ . In fact, the condition is weaker in the sense that it only assures that players in contest  $B$  are *comparatively* prudent, i.e. sufficiently prudent (not too imprudent) compared to players in contest  $A$ . These considerations show that rather a relative measure of prudence than prudence per se drives the result.

## 5 Contests with Asymmetric Risk Preferences

In order to analyse how differences in the agents' risk preferences influence individual efforts and odds within a certain contest, I assume agent 1 to be more risk averse than agent 2 with respect to the Arrow-Pratt measure of absolute risk aversion, i.e.  $RA(u_1, z) > RA(u_2, z)$  for all possible wealth levels  $z$ . As above, there exists a function  $\phi : \mathbb{R} \rightarrow \mathbb{R}$  with  $\phi'' < 0 < \phi'$  and  $u_1(z) = \phi(u_2(z))$  for all possible wealth levels  $z$ , i.e.  $u_1$  is a concave transformation of  $u_2$ .

If players exhibit CARA, the less risk averse will exert higher effort and therefore have a better chance of winning (Skaperdas and Gan, 1995, Cornes and Hartley, 2003). The following example shows, however, that this result does not hold in general.<sup>3</sup>

**Example 1** Assume  $u_1(z) = -(1 - z)^2$ ,  $u_2(z) = z$ ,  $f(x) = x$ , and  $I + R < 1$ , i.e. a lottery contest between some risk averse agent 1 and some risk neutral agent 2. Straightforward calculations show that the game has a Nash equilibrium in pure strategies in which both agents choose the same effort level  $x_1 = x_2 = R/4$  and, hence, have the same winning probability  $p_1 = p_2 = 1/2$ .

The example is instructive in explaining why an agent's risk aversion alone will not necessarily induce a behaviour that is different from the one of a risk neutral opponent. While the latter only cares about the mean of the lottery associated with the contest, quadratic risk preferences imply that the former only cares

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<sup>2</sup>For contests with an arbitrary number  $n$  of participants, Cornes and Hartley (2012, Corollary 6.1) show that the effort of the more risk averse players in contest  $B$  will be smaller than the effort of the less risk averse players in contest  $A$  if  $n$  is sufficiently large. Under the additional condition that  $u_B$  is a prudent transformation of  $u_A$ , Proposition 1 extends their finding to the case of  $n = 2$ .

<sup>3</sup>In a model of optimal prevention, Eeckhoudt and Gollier (2005, Proposition 1) find that, when the optimal probability of loss of the risk neutral agent is  $1/2$ , adding risk aversion but not prudence has no effect on the optimal level of effort. The example illustrates that this result may hold even in (asymmetric) situations of strategic interaction.

about the mean and variance which equals  $p(1-p)R^2$ . Hence, for  $p = 1/2$  a marginal change in the winning probability induced by a marginal investment in effort has no effect on the variance of the lottery, i.e. the marginal calculus is identical for both agents.

The crucial difference between this example and the CARA-case is the fact that CARA-preferences imply that the respective agent is not only risk averse but also prudent. Prudent agents are downside risk averse and, therefore, prefer some lottery at a higher wealth level over the same lottery at a lower wealth level. Hence, even in a situation like in the example above in which the marginal investment has no impact on the variance, a prudent contestant has, ceteris paribus, an incentive to invest less than a risk neutral agent in order to increase the wealth level  $I - x$  at which the lottery takes place.

These considerations provide some intuition for why the comparative level of prudence plays an important role for the agents' relative efforts. The following Lemma states that, unlike in the example above, the asymmetric contest will never have a symmetric equilibrium if  $u_1$  is a prudent transformation of  $u_2$ .

**Lemma 1** *Suppose that the contestants differ only in their risk preferences such that agent 1 is more risk averse than agent 2, i.e.  $u_1 = \phi(u_2)$  with  $\phi'' < 0 < \phi'$ . If  $u_1$  is a prudent transformation of  $u_2$ , i.e. if  $\phi''' > 0$ , then equilibrium efforts and odds will differ, i.e.  $x_1 \neq x_2$  and  $p_1 \neq p_2$ .*

The proof can be found in the Appendix. Proposition 1 and the CARA-case suggest that the more risk averse agent will exert less effort and have a smaller chance of winning if her preferences are a prudent transformation of the second player's preferences. The proof that this conjecture holds for a wide range of parameters  $(R, I, f, u, \phi)$  specifying the contest is based on the following idea. Start from a contest  $(R_0, I_0, f_0, u_0, \phi_0)$  for which the parameters satisfy the conditions of Lemma 1 and the more risk averse agent indeed exerts less effort in equilibrium, i.e.  $x_1(R_0, I_0, f_0, u_0, \phi_0) < x_2(R_0, I_0, f_0, u_0, \phi_0)$ , like e.g. for some case with CARA-preferences. Then, any continuous change to the parameters  $(R, I, f, u, \phi)$  within the class satisfying the conditions of Lemma 1, which leads to some continuous shift of the equilibrium,<sup>4</sup> preserves  $x_1(R, I, f, u, \phi) < x_2(R, I, f, u, \phi)$  by Lemma 1 and the intermediate value theorem.

Formalising this idea in Proposition 2 requires some further notation. Denote by  $\mathbb{C}^k(X)$  the set of all  $k$ -times continuously differentiable real functions on some compact  $X \subset \mathbb{R}_0^+$  and define

$$\begin{aligned} A &:= \{f \in \mathbb{C}^2(X) \mid f(0) = 0, f'(x) > 0 \geq f''(x) \text{ for all } x \in X\}, \\ B &:= \{u \in \mathbb{C}^3(X) \mid u'(x) > 0 \geq u''(x) \text{ for all } x \in X\}, \\ C &:= \{\phi \in \mathbb{C}^3(X) \mid \phi'(x) > 0 > \phi''(x), \phi'''(x) > 0 \text{ for all } x \in X\}. \end{aligned}$$

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<sup>4</sup>By the implicit function theorem, this is the case e.g. whenever the parameter-functions  $f$ ,  $u$ , and  $\phi$  can be parameterised by finitely many real parameters and the considered change in these parameters is sufficiently small.

Note that  $X$ ,  $A$ ,  $B$ , and  $C$  are connected topological spaces and so is the product space  $P = X \times X \times A \times B \times C$ . Let  $M$  be a connected subset of  $P$  for which the contest has a unique equilibrium in pure strategies.<sup>5</sup> For each element of  $M$ , denote this equilibrium by  $(x_1^*(R, I, f, u, \phi), x_2^*(R, I, f, u, \phi))$ .

**Proposition 2** *Let  $S$  be a connected subset of  $M$  on which*

$$\Delta x(R, I, f, u, \phi) := x_2^*(R, I, f, u, \phi) - x_1^*(R, I, f, u, \phi)$$

*is continuous. If there is some  $(R_0, I_0, f_0, u_0, \phi_0) \in S$  with  $\Delta x(R_0, I_0, f_0, u_0, \phi_0) > 0$ , then  $\Delta x(R, I, f, u, \phi) > 0$  for all  $(R, I, f, u, \phi) \in S$ .*

**Proof.** Applying the intermediate value theorem to  $\Delta x$  on  $S$ , the statement follows immediately from Lemma 1. □

## 6 Concluding Remarks

In this paper, I have examined the impact of risk preferences on efforts and winning probabilities in generalised Tullock contests between two players. I have specified a sufficient condition on the agents' comparative prudence under which a higher common level of risk aversion leads to lower aggregate effort in symmetric contests. Moreover, I have shown that in asymmetric contests, higher risk-aversion will be a disadvantage for an abstract range of parameters if the agent is comparatively prudent. Future work may further specify this range of parameters by characterising the (maximum) subset  $S$  on which Proposition 2 applies.

Lemma 1 implies that the effort levels or winning probabilities of two agents, one of which is more risk averse and sufficiently prudent, may coincide only if they differ also in some other characteristic besides risk preferences. Put differently, whenever Proposition 2 applies, keeping up with the less risk averse requires, ceteris paribus, an advantage in a second dimension, e.g. higher ability (March and Sahm, 2017).

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<sup>5</sup>Yamazaki (2009) as well as Cornes and Hartley (2012, Theorem 4.2) provide sufficient conditions for equilibria in pure strategies to be unique.

## Appendix: Proofs

### Proof of Proposition 1.

Remember that the symmetric contest  $C \in \{A, B\}$  has a unique symmetric equilibrium  $(x_C, x_C)$  satisfying condition (2). Define

$$p_x := \frac{\partial p_i(x, x)}{\partial x_i} = \frac{1}{4} \frac{f'(x)}{f(x)}$$

for  $i \in \{1, 2\}$  and

$$F_C(x) := p_x[u_C(W) - u_C(L)] - \frac{1}{2}[u'_C(W) + u'_C(L)]$$

for all  $x > 0$ . By definition,  $F_C$  is continuous in  $x$ , has a unique zero at  $x_C$ , and satisfies  $\lim_{x \rightarrow 0} F_C(x) > 0$  as well as  $\lim_{x \rightarrow \infty} F_C(x) < 0$  due to the properties of  $f$ . The intermediate value theorem then implies that  $F_C(x) > 0$  if and only if  $x < x_C$ . In particular,  $x_B < x_A \Leftrightarrow F_A(x_B) > 0$ . In what follows, I will show the validity of the latter inequality.

With  $u_B = \phi(u_A)$ , the identity  $F_B(x_B) = 0$  implies

$$p_{x_B} = \frac{1}{2} \frac{\phi'(u_A(W_B))u'_A(W_B) + \phi'(u_A(L_B))u'_A(L_B)}{\phi(u_A(W_B)) - \phi(u_A(L_B))}.$$

Using this equation,  $F_A(x_B) > 0$  is equivalent to

$$\frac{[\phi'(u(W))u'(W) + \phi'(u(L))u'(L)][u(W) - u(L)]}{[\phi(u(W)) - \phi(u(L))][u'(W) + u'(L)]} > 1 \quad (3)$$

with  $u = u_A$ ,  $W = W_B = I - x_B + R$ , and  $L = L_B = I - x_B$ . To show the validity of inequality (3), first note that

$$\frac{\phi'(u(W))u'(W) + \phi'(u(L))u'(L)}{u'(W) + u'(L)} \geq \frac{1}{2}[\phi'(u(W)) + \phi'(u(L))]$$

$$\Leftrightarrow [\phi'(u(W)) - \phi'(u(L))][u'(W) - u'(L)] \geq 0,$$

which is true due to the concavity of  $\phi$  and  $u$  since  $u$  is increasing and  $W > L$ . Hence, it is sufficient to show that

$$\begin{aligned} & \frac{1}{2}[\phi'(u(W)) + \phi'(u(L))]\frac{u(W) - u(L)}{\phi(u(W)) - \phi(u(L))} > 1 \\ \Leftrightarrow & \frac{1}{2}[\phi'(u(W)) + \phi'(u(L))] - \frac{\phi(u(W)) - \phi(u(L))}{u(W) - u(L)} > 0. \end{aligned}$$

However, this last inequality follows from Eeckhoudt and Gollier (2005, Lemma 1) and the strict convexity of  $\phi'$ .

□

### Proof of Lemma 1.

The proof is by contradiction. I write  $u_2 = u$  and  $u_1 = \phi(u)$  as well as  $W_i = I - x_i + R$  and  $L_i = I - x_i$  for short. Note that

$$\frac{\partial p_i}{\partial x_i} = \frac{f'(x_i)f(x_j)}{[f(x_i) + f(x_j)]^2}$$

for  $i, j \in \{1, 2\}$  with  $i \neq j$ . Dividing the first order condition (2) for agent 1 by the one for agent 2 yields

$$\frac{f'(x_1)f(x_2)}{f'(x_2)f(x_1)} = \frac{[p_1\phi'(u(W_1))u'(W_1) + p_2\phi'(u(L_1))u'(L_1)][u(W_2) - u(L_2)]}{[\phi(u(W_1)) - \phi(u(L_1))][p_2u'(W_2) + p_1u'(L_2)]} \quad (4)$$

Now suppose that there is an equilibrium with  $x_1 = x_2 = x$  and, hence,  $W_1 = W_2 = W$ ,  $L_1 = L_2 = L$ , as well as  $p_1 = p_2 = 1/2$ . Then, obviously the left hand side of equation (4) equals 1. However, the right hand side of equation (4) is equal to the term on the left hand side of inequality (3) and, hence, larger than 1 as show in the proof of Proposition 1.

□

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