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# ESTIMATING THE EFFECTS OF THE MINIMUM WAGE IN A DEVELOPING COUNTRY: A DENSITY DISCONTINUITY DESIGN APPROACH

HUGO JALES<sup>1</sup>

This paper proposes a framework to identify the effects of the minimum wage on the joint distribution of sector and wage in a developing country. I show how the discontinuity of the wage distribution around the minimum wage identifies the extent of non-compliance with the minimum wage policy, and how the conditional probability of sector given wage recovers the relationship between latent sector and wages. I apply the method in the “PNAD”, a nationwide representative Brazilian cross-sectional dataset for the years 2001 to 2009. The results indicate that the size of the informal sector is increased by around 39% compared to what would prevail in the absence of the minimum wage, an effect attributable to (i) unemployment effects of the minimum wage on the formal sector, (ii) movements of workers from the formal to the informal sector as a response to the policy.

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Despite its widespread use, controversy persists regarding the economic impact of the minimum wage. A simple one-sector competitive market model predicts that a minimum wage will generate unemployment when the minimum exceeds the market-clearing wage. However, if the employer has market power, then a minimum wage can lead to an increase in wages and employment. In an economy with a large informal sector, where some employers do not comply with the minimum wage policy, the minimum wage might not generate unemployment effects even in the absence of market power on the part of the employer. This will hold as long as the workers can freely migrate from one sector to the other and the informal sector is sufficiently large to accommodate such movements.

These conflicting theoretical predictions provide a strong motivation for empirical studies on the effects of minimum wage policies. In this paper, I develop a Dual-economy model based on Doyle (2006) to assess the impacts of the minimum wage on (a) unemployment, (b) average wages, (c) wage inequality, (d)

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sector mobility, (e) the size of the informal sector and (f) labor tax revenues. I model the joint distribution of wages and sectors (latent and observed), as opposed to the marginal distribution of wages, as in Doyle (2006). A model for the joint distribution of sector and wages allows me to infer the size of the formal sector that would prevail in the absence of the minimum wage and compute the proportion of workers who move to the informal sector in response to the policy. I provide the conditions for identifying the Dual-economy model parameters and the latent joint distribution of sector and wages, that is, the distribution that would prevail in the absence of the minimum wage policy. My identification strategy relies on the discontinuity in wage density at the minimum wage and the differences in the response to the minimum wage between the formal and informal sectors.

This paper’s contributions to the literature are the following: (i) I document key empirical facts concerning the relationship between formal and informal wage distributions that have been overlooked in previous research, namely, the similarity between these distributions conditional on values above the minimum wage. (ii) I provide a novel identification strategy that combines a non-parametric density discontinuity design with a parametric model for the conditional probability of sector given the wage. In particular, I show that under reasonable conditions, the parameters that describe the effects of the minimum wage and the underlying latent joint distribution of sector and wages are identified using only cross-sectional data. (iii) I estimate a sector mobility parameter, the probability that a worker in the formal sector moves to the informal sector in response to the minimum wage. (iv) I estimate the effect of the minimum wage on the joint distribution of sector and wages and (v) I estimate the effect of the minimum wage on labor tax revenues. To the best of my knowledge, this is the first paper that attempts to identify both the latent share of the formal sector and the effects of the minimum wage on labor tax revenues.

The identification problem studied in this paper includes minimum wage policies as a special case. In general, any policy that introduces a boundary type of restriction can potentially be analyzed under this framework. Examples of such policies are price floors, price ceilings, and age restrictions (such as the restrictions for alcohol and tobacco consumption). This paper provides a set of assumptions and data requirements under which it is possible to identify the effects of such policies on these outcomes. This approach is closely related to a growing literature that exploits “bunching” and discontinuities in density functions to identify structural parameters and policy effects (Saez (2010), Kleven and Waseem (2014)). See Kleven (2016) and Jales and Yu (2016) for recent reviews.

The model is estimated using the years 2001 to 2009 from “*Pesquisa Nacional por Amostra de Domicílios*” (PNAD), a dataset comprising repeated cross sections of an annual household survey that is representative of the Brazilian population. I find that the probability of a formal worker switching to the informal sector as a result of the policy is small – approximately 12%. The combined effect of unemployment and transitions to the informal sector generated by the introduction of the minimum wage leads to an 9% decrease in the size of the formal sector relative to the counterfactual state defined by the absence of the minimum wage. This associated growth in the size of the informal sector as a result of the policy is 39% - an effect attributable to the fact that the latent formal sector is four times larger than the informal sector of the economy. Unemployment effects of the minimum wage are, as expected, highly correlated with the real value of the minimum wage. Moreover, the minimum wage strongly affects average wages (promoting an increase of approximately 16%), wage inequality, and labor tax revenues (-6%).

## 1. BACKGROUND

In Brazil, all workers are required to carry a government document called a “*Carteira de Trabalho*”, or worker’s card. This document, introduced in 1932, serves as proof of the worker’s legal employment status. If a worker is formally employed in the Brazilian labor market, then her contract is signed by the employer on a page of the worker’s card. This labor contract implies that the worker’s employment is in compliance with labor taxes and labor regulations such as the minimum wage. Formal employment gives the worker access to benefits that include unemployment insurance and severance payments.

Not all labor contracts are signed by the employer and included in the worker’s card. When an employer and a worker agree to a labor contract but decide not to formally sign it and include it in the worker’s card, the worker’s employment is called informal. Reasons for the existence of informal contracts include the evasion of labor regulations, such as the payment of labor taxes, compliance with the minimum wage, job safety standards, and restrictions on hours worked per week.<sup>1</sup> This definition of informality is tightly related to compliance with the minimum wage. However, these concepts are not equivalent. A worker with a wage below the minimum wage level is surely an informal worker. However, a worker whose wage is above the minimum wage may be formal or informal

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<sup>1</sup>Firms face a trade-off between the costs of complying with the regulation and the probability/magnitude of punishment. The firms’ decision to hire formal versus informal workers was investigated in Almeida and Carneiro (2012), Mattos and Ogura (2009), Galiani and Weinschelbaum (2012), among many others.

depending on whether his contract is signed by the employer. Pooling data from years 2001 to 2009, the proportion of private sector workers between the ages of 19 and 59 who are employed in the formal sector is .74. In other words, more than one quarter of private sector workers do not have a signed contract included in their worker's card.

The history of the minimum wage in Brazil began during the Getulio Vargas government, on May 1st, 1940. Initially, the minimum wage varied across regions to accommodate differences in price levels across the country. Subsequently, in 1984, regional minimum wages were unified into a single wage at the national level. The periodicity of changes in the minimum wage has been annual since the economy stabilized in 1994 (Lemos, 2009). In theory, all jobs are covered, meaning that the (same) minimum wage level should apply to every worker.<sup>2</sup> In practice, coverage only extends to workers with a contract written on the worker's card (Lemos, 2009).

A unified minimum wage set at the federal level with full coverage complicates the task of finding an appropriate control group. This is because cross-border differences-in-differences analysis, such as that in Card and Krueger (1994), is ruled out as a practical option, as the same level prevails in all states. Another feature of the minimum wage changes in Brazil is that since 2005, they have been linked to inflation and GDP growth, which poses further challenges to the use of time-series variation to estimate the effects of the minimum wage. Under these conditions, it is more difficult to disentangle the effects of the minimum wage from other sources of changes in the wage distribution that are linked to changes in economic activity.

Despite these challenges, it is nevertheless possible to identify the effect of the minimum wage using only cross-sectional data on sector and wages. This paper describes a set of *a priori* restrictions – on the joint distribution of sector and wage, and on the effects of the policy – that allows for identification of the effects of the minimum wage using only cross-sectional data on sector and wages. This research design is well suited to analyze markets characterized by the absence of cross-sectional or time series variation on the policy level that can be used for identification, such as the case of the minimum wage in the Brazilian labor market.

The points of departure for this paper are the works of Meyer and Wise (1983) and Doyle (2006). These papers show how to identify the effects of the minimum wage on the distribution of wages. I extend their model to a two-sector,

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<sup>2</sup>The constitution allows for states to set occupation specific wage floors, as long as they are above the federal level. On appendix F, I provide a detailed discussion about the institutional background and its implications for my analysis.

or dual-economy, setting with sector mobility. This allows wages in both sectors to be affected, but in different ways, by the minimum wage. It also allows me to capture the effects of the minimum wage on the size of the formal sector and other related outcomes, such as labor tax revenues.

The dual-economy extension I develop presents new challenges for identification. This is because the techniques presented in Doyle (2006) are not alone sufficient to recover the sector-specific parameters of the model in this general version. The reason is that applying Doyle’s strategy to the aggregate economy only recovers a weighted average of the parameters, which will be uninformative for most of the outcomes of interest. Applying his method to each sector separately is not feasible, as workers have moved from one sector to the other as a result of the policy. Thus, one of the contributions of this paper is demonstrating how to identify the effects of the minimum wage in this dual-economy setting.

In the next sections, I briefly describe the model of Doyle (2006) to highlight the similarities and differences between his paper and the approach followed here.

## 2. MODEL

The effect of the minimum wage on a worker’s wage is the difference between her wage under the policy and the wage she would receive in its absence. The fundamental problem of causal evaluation is that this difference is conceptually well defined but never observed in the data. This is true because we can at most observe the wages for each worker in one of the two possible states of the world. However, it is nevertheless helpful to consider these objects. Thus, let worker  $i$  be characterized by an observed wage  $W_i(1)$  and a corresponding latent wage  $W_i(0)$ , which is defined as the wage that the worker would receive in the absence of the minimum wage. I will denote the minimum wage level by  $m$ . I will denote by  $F_0(w)$  ( $f_0(w)$ ) the CDF (pdf) of latent wages. Similarly, denote by  $F(w)$  ( $f(w)$ ) the CDF (pdf) of observed wages. To keep the notation simple, assume that these workers come from a population with similar observable characteristics, and hence, we do not need to control for them. In the absence of the minimum wage, every worker  $i$  in this population obtains a draw  $W_i(0)$  from the distribution  $F_0$ , which I will refer to as the underlying wage distribution or the distribution of “market” wages. Although workers are intrinsically homogeneous *ex-ante*, meaning that they draw their wages from the same distribution, they will have different wages *ex-post*.

In the presence of the minimum wage policy, the worker will receive a draw  $W_i(1)$  from the distribution  $F$ , which I will refer to as the distribution of

observed wages.

To make the problem of identifying the effects of the minimum wage tractable, I follow Doyle (2006) by imposing a set of *a priori* restrictions on the distribution of the latent variables and on the effects of the policy. As we will see, these restrictions allow me to identify the effects of the policy without relying on exogenous policy variations or time-series data.

### 2.1. Doyle's Approach

This section describes the assumptions and estimation strategy used by Doyle (2006). Assume that the econometrician observes a random sample of observed wages  $\{W_i(1)\}$  of size  $N$  from a population of interest. Let the following hold:

ASSUMPTION MW1 *The density of latent wages,  $f_0(m)$ , is non-zero and continuous at  $m$ .*

ASSUMPTION MW2 *There are no spillovers from the minimum wage. This means that  $W(1) = W(0)$  when  $W(0) > m$ .*

ASSUMPTION MW3 *If  $W(0) < m$ , then with probability  $\pi_m$ , the worker receives the minimum wage ( $W(1) = m$ ). With probability  $\pi_d$ , ( $W(1) = W(0)$ ), the worker's wage is the same as the latent wage (non-compliance). With the complementary probability  $\pi_u = 1 - \pi_m - \pi_d$ , the worker becomes unemployed ( $W(1) = \cdot$ ).<sup>3</sup>*

The probabilities  $(\pi_m, \pi_d, \pi_u)$  represent the likelihood of receiving the minimum wage, non-compliance and unemployment. The goal of the exercise is to recover the parameters of the underlying latent distribution of market wages  $(\mu, \sigma)$  and the parameters  $(\pi_d, \pi_m, \pi_u)$  that govern how the minimum wage affects the economy. The key contribution of Doyle is to show that those parameters are identified using data on observed wages  $(W_i(1))$ . Perhaps surprisingly, one need not have any variation in the policy to recover its effects.<sup>4</sup>

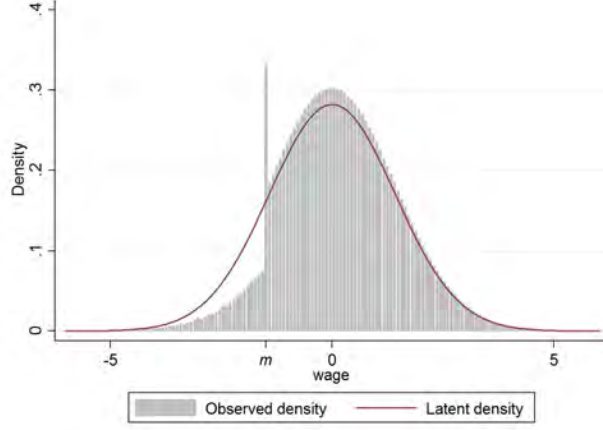
As discussed in Doyle (2006), assumption MW1 exploits the fact that the distribution of worker productivity is likely to be smooth, but the observed density of wages has a jump around the minimum wage. This jump may provide just enough information to trace back the effects of the policy.

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<sup>3</sup>Strictly speaking, the appropriate expression should be “non-employment”. I will refer to this effect as the “unemployment” effect of the minimum wage. Throughout the paper I will use non-employment and unemployment exchangeably, given that the model can not distinguish these effects.

<sup>4</sup>Meyer and Wise (1983) showed that these parameters were identified under the assumption of log-normality of latent wages.

FIGURE 1.— Doyle's Model



Under assumptions MW2 and MW3, there is a relationship between the latent and observed distribution of wages. This relationship is given by:  $f(w) = \mathbb{I}\{w < m\} \frac{\pi_d f_0(w)}{c} + \delta(w - m) \frac{\pi_m F_0(w)}{c} + \mathbb{I}\{w > m\} \frac{f_0(w)}{c}$ , where  $c = 1 - \pi_u F_0(m)$  and  $\delta$  is the Dirac delta function. Figure 1 displays a graphical example of the relationship between the observed density, as approximated by a histogram, and the latent density.

Taking the ratio of the density of observed wages just below and above the minimum wage, that is, considering the lateral limits of the density at  $m$ , we have  $\frac{\lim_{w \rightarrow m-} f(w)}{\lim_{w \rightarrow m+} f(w)} = \frac{\lim_{w \rightarrow m-} \frac{\pi_d f_0(w)}{c}}{\lim_{w \rightarrow m+} \frac{f_0(w)}{c}} = \pi_d$ , where the last equality is obtained using assumption MW1. To recover the remaining parameters, it is easy to see that by integrating the density of observed wages up to the minimum wage, we have  $Pr[W(1) < m] = \frac{\pi_d F_0(m)}{c}$ . Then, we have  $\frac{Pr[W(1)=m]}{Pr[W(1) < m]} = \frac{\pi_m}{\pi_d}$ . Because the left-hand side of this equation is identified from the data and the right-hand side is a function of only one unknown, this implies that  $\pi_m$  is identified. This also implies that  $\pi_u = 1 - \pi_d - \pi_m$  is identified.

To identify  $F_0(m)$  one can use the fact that  $F_0(m) = \frac{Pr[W(1) < m]}{Pr[W(1) < m] + \pi_d Pr[W(1) > m]}$ , which follows from:  $\frac{Pr[W(1) < m]}{Pr[W(1) < m] + \pi_d Pr[W(1) > m]} = \frac{\frac{\pi_d F_0(m)}{c}}{\frac{\pi_d F_0(m)}{c} + \pi_d (1 - F_0(m)) / c} = \frac{F_0(m)}{F_0(m) + 1 - F_0(m)} = F_0(m)$ .

This implies that the latent distribution of wages can be recovered under assumptions MW1, MW2, and MW3. The discontinuity in the observed distribution around the minimum wage identifies the probability of non-compliance with the policy  $\pi_d$ . This in turn allows us to recover  $\pi_m$ ,  $F_0(m)$  and, conse-



quently, the entire latent wage distribution.<sup>5</sup>

As shown by Meyer and Wise (1983), the estimates of the disemployment effects of the policy are obtained from the measure of the “missing mass” in the wage distribution. It is easy to show that the unemployment probability is equal to  $\pi_u = \frac{F_0(m)-F(m)}{F_0(m)} \frac{1}{1-F(m)}$ . The first part of the right hand side of this equation is the fraction of missing workers in the bottom part of the observed wage distribution. In the next section, I will show that, in a dual economy model, the observed density of wages in the informal sector may present the opposite feature: Due to the inflow of workers from the informal sector, the informal sector wage density may have an “excess of mass”.

## 2.2. Minimum Wage Effects in a Dual Economy

The Brazilian economy, similar to those of many other developing countries, is characterized by a large informal sector. In Brazil, an informal worker is defined as a worker whose worker’s card does not include a signed labor contract. Informality is thought to arise in developing countries as a result of restrictive and costly labor laws. Note that once the worker’s card is signed, the collection of labor taxes should follow and compliance with minimum wage and other labor standards has to be assured. A natural question that arises in this context is the following: What is the role of the minimum wage in a economy with such a large informal sector? A large fraction of contracts outside the “umbrella” of the labor laws may be a consequence of the minimum wage, meaning that many workers (intentionally or not) have moved to the informal sector as a consequence of the minimum wage policy. However, in principle, it could also be the case that the observed proportion of workers in the informal sector is completely unrelated to the level of the minimum wage. Informality may instead depend on labor taxes and other forms of labor regulation (hours worked, job safety standards and so forth) that have to be met regardless of where the worker is located in the wage distribution. These two explanations have markedly different policy implications but are in principle equally plausible explanations for the observed size of the informal sector. One of the goals of this paper is to assess the relative importance of these explanations.

To do so, I generalize Doyle’s (2006) model to the case of a dual economy. I model the joint distribution of wages and sectors (latent and observed), as opposed to the marginal distribution of wages. This allows me to infer the size of the formal sector that would prevail in the absence of the minimum wage

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<sup>5</sup>The relationship between the observed and latent densities can be inverted once we know the rescaling factor  $c$  and  $\pi_d$ . To see this, note:  $f_0(w) = \frac{f(w) \cdot c}{\pi_d}$  if  $w < m$ , and  $f_0(w) = f(w) \cdot c$  if  $w > m$ .

and compute the proportion of workers who move to the informal sector in response to the policy.

Let worker  $i$  be characterized by a pair of wage ( $W_i(1)$ ) and sector ( $S_i(1)$ ), which is equal to one if she is employed in the formal sector and zero otherwise. Compliance with minimum wage legislation is perfect in the formal sector but not in the informal sector. This effectively means that the workers in the formal sector are not allowed to have wages below the minimum wage once the policy is introduced. If they remain employed in the presence of the policy, they must either move to an informal contract or comply with the policy by receiving a wage equal to  $m$ . In addition, for each worker, define a pair ( $W_i(0), S_i(0)$ ) that denotes the counterfactual - or latent - wage and sector in the absence of the minimum wage. Finally, define  $F_0(w)$  ( $f_0(w)$ ) as the c.d.f (p.d.f) of  $W(0)$  and  $F(w)$  ( $f(w)$ ) as the c.d.f (p.d.f) of observed wages ( $W_i(1)$  or, in short notation,  $W_i$ ). I will assume that the *latent* wage and sector distribution have the following characteristics:

**ASSUMPTION 1 (Continuity)** *The density of latent wages and its first derivative exist, are non zero, and are continuous at  $m$ .*

Because this is a model of the joint distribution of sector and wages, we need to define another object,  $Pr[S(0) = 1|W(0) = w]$ :

**ASSUMPTION 2 (Conditional probability of (latent) sector given the wage)** *The conditional distribution of latent sector given the latent wage belongs to a parametric family  $\{\Lambda(w, \beta) : \beta \in B \subset \mathbb{R}^k\}$ . That is,  $Pr[S(0) = 1|W(0) = w] = \Lambda(w, \beta_0)$  for some  $\beta_0 \in B$ . Moreover,  $Pr[\Lambda(W(0), \beta_0) \neq \Lambda(W(0), \beta)|W(0) > m] > 0$  for all  $\beta \neq \beta_0$ .*

With the conditional probability of latent sector (given the wage) and the marginal distribution of latent wages, we have completely specified the joint distribution of these variables.<sup>6</sup> The restrictive part of this assumption is that the conditional probability of the latent sector given latent wages can be described by a parametric model. The first part of the above assumption states that there is a parameter  $\beta_0$  for which the probability of the latent sector given the latent wage  $w$  is exactly equal to  $\Lambda(w, \beta_0)$ . The second part of the assumption ensures that there is only one parameter for which this condition

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<sup>6</sup>This joint distribution could come, for example, from a Roy-type model of sector choice, in which workers would choose the sector that yields the highest utility. Another model would be one in which workers are assigned to firms that, based on labor taxes and probability of punishment, decide whether they will employ formal or informal workers.

holds. Both assumptions are standard in models with binary outcomes. For concreteness, assume that the parametric model is a logit.

One relevant special case of assumption 2 occurs when  $Pr[S(0) = 1|W(0) = w] = Pr[S(0) = 1]$  for all  $w$ . In this case, the conditional probability of latent sector given the latent wage does not depend on  $w$ . As a result, the model for the conditional probability of latent sector given the wage is trivially given by a constant.<sup>7</sup>

**ASSUMPTION 3 (No spillovers)** *Workers whose latent wages would be above the minimum wage are not affected by the policy. That is,  $W(1) = W(0)$  and  $S(1) = S(0)$  when  $W(0) > m$ .*

This assumption is potentially strong. The absence of spillovers is, however, implied by a simple Roy-model with a binding minimum wage. In the empirical application, I show that my estimates of the effects of the minimum wage on the size of the formal sector are robust to deviations from assumption. The estimates remain of a similar magnitude when I assume limited spillovers instead.<sup>8</sup>

To complete the model we need to define the minimum wage effects in the lower-tail of the wage distribution. As discussed by Meyer and Wise, workers in sectors operating in competitive markets whose wages would be below the minimum might become unemployed as a result of the minimum wage. If there is some bargaining involved in the wage determination, or if the employers hold market power, some workers will “bunch” at the minimum as a result of the policy. Finally, because compliance with the minimum is imperfect in some markets, workers might migrate from the formal to the informal sector to avoid unemployment. In terms of the model, this leads to the following assumption:

**ASSUMPTION 4 (Minimum wage effects)** *For wages below the minimum wage ( $W(0) < m$ ), we have the following: If  $S(0) = 0$ , then  $S(1) = S(0)$ . Moreover, with probability  $\pi_d^{(0)}$ , the wage continues to be observed ( $W(1) = W(0)$ ). With the complementary probability  $\pi_m^{(0)} = 1 - \pi_d^{(0)}$ , the worker earns the minimum wage ( $W(1) = m$ ).<sup>9</sup> If  $S(0) = 1$ , then with probability  $\pi_d^{(1)}$ , the wage continues*

<sup>7</sup>This seems to be approximately true in the case of the Brazilian labor market. Thus, I will go back to this discussion in Section 5.

<sup>8</sup>This assumption can be relaxed if one is willing to assume a parametric family for the latent wage distribution. Moreover, bounds can be computed for the parameters when positive spillovers are assumed to exist but the researcher has no prior information on their magnitude. See the Appendix D for further discussion of this issue.

<sup>9</sup>The reason for allowing  $\pi_m^{(0)}$  to be greater than zero is to account for the empirical fact that the informal sector wage distribution also presents bunching at the minimum wage.

to be observed ( $W(1) = W(0)$ ), meaning that the worker successfully transits from the formal sector to the informal sector.<sup>10</sup> In this case, the observed sector will be  $S(1) = 0$ , being different from the latent sector. With probability  $\pi_m^{(1)}$ , the worker earns the minimum wage ( $W(1) = m, S(1) = 1$ ). With the complementary probability ( $\pi_u^{(1)} = 1 - \pi_d^{(1)} - \pi_m^{(1)}$ ), the worker becomes unemployed ( $W(1) = \cdot, S(1) = \cdot$ ).<sup>11</sup>

This assumption can be summarized by the following relationships: If  $W(0) < m$  and  $S(0) = 1$ , then:

$$(W(1), S(1)) = \begin{cases} (W(0), 0) & \text{with probability } \pi_d^{(1)} \\ (m, 1) & \text{with probability } \pi_m^{(1)} \\ (\cdot, \cdot) & \text{with probability } \pi_u^{(1)} \end{cases}$$

If  $W(0) < m$  and  $S(0) = 0$ , then:

$$(W(1), S(1)) = \begin{cases} (W(0), 0) & \text{with probability } \pi_d^{(0)} \\ (m, 0) & \text{with probability } \pi_m^{(0)} \end{cases}$$

Assumption 4 is a sector-specific version of assumption MW3 imposed by Meyer and Wise (1983) and also by Doyle (2006). This assumption imposes several restrictions on the way that the minimum wage affects the bottom part of the joint distribution of sector and wages. In particular, it implies that: (i)  $\pi$  is not indexed by  $i$ , that is, the probabilities that characterize the effects of the minimum wage are the same for all workers; (ii)  $\nabla_w \pi = 0$ , that is, the probabilities of non-compliance, unemployment and earning the minimum wage are constants, as opposed to functions of workers' latent wage; and (iii) wages of sub-minimum wage informal workers are not affected by the inflow of formal workers to that sector.

Some of these restrictions can be relaxed. However, some of them are required for my identification strategy to work, so it is helpful to discuss these conditions

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<sup>10</sup>The assumption that the wage remains exactly the same when the worker moves to the informal sector, that is ( $W(1) = W(0)$ ), substantially simplifies the exposition. The same results hold when this assumption is replaced with one in which the worker draws a new wage from  $f_0(w|S(0) = 1, W(0) < m)$ .

<sup>11</sup>To ease the exposition, I have assumed that  $\pi_m^{(1)}$  and  $\pi_u^{(1)}$  do not vary as a function of the latent wage. In this case in which they vary over the latent wages, the parameter recovered by assuming that they are constants is the expectation of the distribution of  $\pi_m^{(1)}$  and  $\pi_u^{(1)}$  over the distribution of wages below the minimum. This result holds only as long as  $\pi_d^{(1)}$  remains constant as a function of the wage.

in greater detail. Condition (i) is imposed just to simplify the exposition. When the probabilities that characterize the effects of the minimum wage present unobserved heterogeneity across workers, the model will implicitly “integrate out” this heterogeneity. In that case, the estimates based on my identification strategy can be interpreted as the average  $\pi$  over the mass of affected workers, which is the natural extension of the parameter of interest when this parameter presents heterogeneity over the population.<sup>12</sup> Condition (ii) can be replaced by an assumption that makes  $\pi$  a low order polynomial (such as a linear function) of the worker’s latent wage. The second derivative of the wage density plays a role on the identification of  $\pi$  in this scenario. The second derivative of the wage density can also be used to test if condition (ii) is violated in the data.<sup>13</sup> In the application I assume a constant  $\pi$  with respect to wages since the estimation of the second derivative of the wage density at a boundary point is, in practice, challenging. Condition (iii) is necessary unless one specifies a parametric functional form for the latent wage distribution.<sup>14,15</sup>

### 2.3. Discussion

The goal of the exercise is to recover the unknown parameters  $\pi \equiv (\pi_d^{(1)}, \pi_m^{(1)}, \pi_u^{(1)}, \pi_d^{(0)}, \pi_m^{(0)})'$  and the joint distribution of latent sector and wages, that is, the joint density that would prevail in the absence of the minimum wage. By comparing this distribution with the observed distribution, I can evaluate the impact of the minimum wage on expected wages, wage inequality, employment and other labor market outcomes. By defining the latent sector and the sector-specific parameters, a broader range of implications of the minimum wage becomes assessable, such as changes in tax revenues and movements between sectors. In Sections 2.4 and 5.5, I will discuss in detail how the minimum wage affects these outcomes.

The assumptions used in this model are similar to the assumptions used previously in this literature. I maintain all assumptions from Doyle – or Meyer and Wise, if one prefers a parametric specification for latent wages – and generalize their approach to address sector-specific responses. Assumption 4, the assump-

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<sup>12</sup>The conditions needed for identification under unobserved heterogeneity are discussed in Appendix D.

<sup>13</sup>See Appendix C for further discussion of this issue.

<sup>14</sup>Condition (iii) would be violated if, for example, the informal sector present congestion externalities, due to imperfect mobility of capital across sectors or to search frictions. In this case, the inflow of workers from the formal to the informal sector could reduce the productivity of worker’s in the informal sector.

<sup>15</sup>Jales (2015) uses this strategy and finds that the results are similar to the ones found when imposing this restriction.

tion that defines the sector-specific effects of the minimum wage, implies the assumptions used by Meyer and Wise (1983) and Doyle (2006) concerning the marginal distribution of wages. That is, the marginal distribution of wages (which is obtained after integrating out the sector-specific wage distributions) will resemble the density of wages that appears in Meyer and Wise (1983) and Doyle (2006).

Despite these similarities, there are numerous advantages of using a model for the joint distribution of sector and wages. This is especially true for developing countries, where the informal sector plays an important role in the economy. This model accommodates a variety of responses of the economy to the minimum wage policy. The model allows for the standard unemployment effect. The model allows the minimum wage to have a “supporting” effect on the lower tail of the wage distribution in such a way that the policy can affect average wages and wage inequality. The model allows wages in the informal sector to be affected by the introduction of the minimum wage – an effect captured by the parameter  $\pi_m^{(0)}$ . This model also allows workers to move to the informal sector as a response to the minimum wage – this event is captured by the parameter  $\pi_d^{(1)}$ . Combined, these unemployment and sector mobility effects allow the minimum wage to affect the relative size of the formal sector in the economy, which in turn can affect labor tax revenues.

A two-sector model helps to interpret the parameters identified in the previous work from Meyer and Wise (1983) and Doyle (2006). Meyer and Wise discuss the possible reasons that one would observe a non-zero density of wages below the minimum, such as uncovered jobs and non-compliance in covered sectors. Ultimately, however, Meyer and Wise’s model identifies the aggregate likelihood of non-compliance ( $\pi_d$ ). This parameter is the proportion of workers who, following the introduction of the minimum wage, do not ultimately respond to the policy. An application of the law of iterated expectations shows that the parameter estimated in their model is a weighted average of the sector-specific parameters, with weights given by the *latent* shares of the sectors in the economy. The parameter  $\pi_d$  does not identify whether workers earn sub-minimum wages because they would already be working in non-covered sectors regardless of the policy or because they moved there as a *response* to the policy. These two different stories are implied by different values of the sector-specific parameters. However, they can imply the exact same value for  $\pi_d$ . Moreover, any combination of the two is also equally plausible when one estimates only the aggregate or “unconditional” parameter  $\pi_d$ . Thus, the sector mobility parameter  $\pi_d^{(1)}$  and the latent size of the uncovered sector  $Pr[S(0) = 1]$  are more economically meaningful than the aggregate parameters.

### 2.4. Model Analysis

In this section, I show that this model can capture a wide range of potential effects of the minimum wage policy. To do so, I discuss the model's implications for some objects of interest, such as the sector-specific wage densities and the conditional probability of formality given the wage.

Given assumptions 2 to 4 above, there is a relationship between the latent and observed unconditional wage distributions. It is given by:  $f(w) = \mathbb{I}\{w < m\} \frac{\pi_d(w)f_0(w)}{c} + \delta(w - m) \int^m \frac{\pi_m(w)f_0(w)}{c} dw + \mathbb{I}\{w > m\} \frac{f_0(w)}{c}$ , where  $c \equiv 1 - \int^m \pi_u^{(1)} \Lambda(w) f_0(w) dw$  is a rescaling factor that ensures both densities integrate to one. This parameter has the interpretation as the ratio of employment in the presence of the policy to that in the absence of the policy.

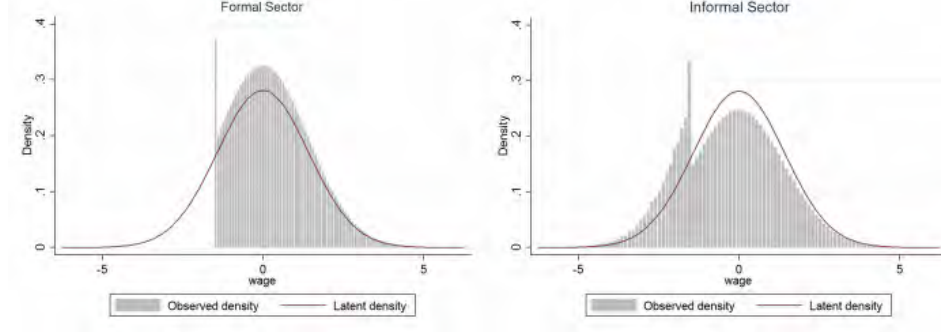
Regarding the relationship between the sector-specific parameters and the aggregate ones, we have  $\pi_d(w) \equiv \Lambda(w)\pi_d^{(1)} + (1 - \Lambda(w))\pi_d^{(0)}$ ,  $\pi_m(w) \equiv \Lambda(w)\pi_m^{(1)} + (1 - \Lambda(w))\pi_m^{(0)}$ , and  $\pi_u(w) \equiv \Lambda(w)\pi_u^{(1)}$ . The parameters  $\pi_d(w)$ ,  $\pi_m(w)$  and  $\pi_u(w)$  are weighted averages of the sector-specific parameters with weights given by the relative shares of each sector in the *latent* distribution. They describe the unconditional probability of non-compliance, “bunching” at the minimum wage level and unemployment at a given value of the wage. These are the parameters estimated in the previous approach employed by Meyer and Wise (1983) and Doyle (2006).<sup>16</sup>

Examining the sector-specific wage density, one can see that for the formal sector, we have:  $f(w|S(1) = 1) = \mathbb{I}\{w < m\} 0 + \delta(w - m) \frac{\pi_m^{(1)} F_0(m|S(0)=1)}{c^{(1)}} + \mathbb{I}\{w > m\} \frac{f_0(w|S(0)=1)}{c^{(1)}}$ . For the informal sector, we have  $f(w|S(1) = 0) = \mathbb{I}\{w < m\} \frac{\pi_d(w)f_0(w|S(0)=0)}{(1 - \Lambda(w))c^{(0)}} + \delta(w - m) \frac{\pi_m^{(0)} F_0(m|s(0)=0)}{c^{(0)}} + \mathbb{I}\{w > m\} \frac{f_0(w|S(0)=0)}{c^{(0)}}$ , where I define  $c^{(1)} \equiv 1 - F_0(w|S(0) = 1)(1 - \pi_m^{(1)})$  and  $c^{(0)} \equiv 1 + \pi_d^{(1)} \int^m \frac{\Lambda(w)}{1 - \Lambda(w)} f_0(w|S(0) = 0) dw$  so that both densities integrate to one. They have the interpretation of the ratio of employment observed in the sector to that in the absence of the policy. Figures 1 and 2 display the relationship between the latent and observed densities for the aggregate wage distribution, for the formal sector, and for the informal sector, respectively.

The dual-economy model preserves the same relationship between the latent and observed unconditional wage densities as in Meyer and Wise's model. However, the dual-economy model presents heterogeneity in the responses to the minimum wage across sectors. The formal sector wage density below the minimum wage vanishes, whereas in the informal sector, the density grows

<sup>16</sup>Note that, here, they are allowed to be functions of  $w$  as long as the latent sizes of the sectors differ across wages and the model parameters differ across sectors.

FIGURE 2.— Dual-economy Model: Latent and Observed Densities



according to the inflow of workers from the formal sector. As a result, the density in the informal sector below the minimum wage can, for some values of the model parameters, present a discontinuity at the minimum wage with the “inverse” shape relative to that observed in the aggregate wage distribution. That is, in the presence of sector-mobility, part of the “missing mass” in the bottom of the wage distribution in the formal sector reveals itself as an “excess of mass” in the bottom of the wage distribution in the informal sector.

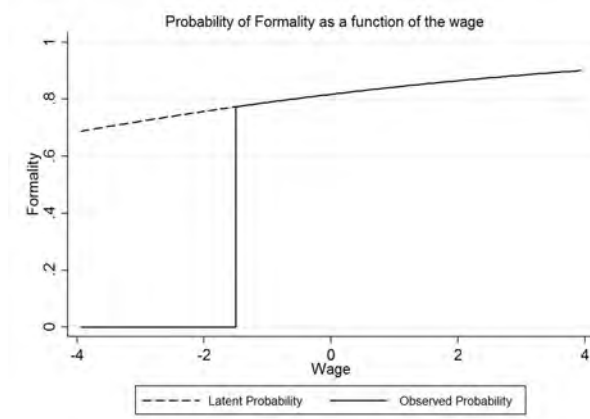
Regarding the conditional probability of working in the formal sector as a function of the wage, we have:  $Pr[S(1) = 1|W(1) = w] = \mathbb{I}\{w < m\}0 + \mathbb{I}\{w = m\} \frac{\pi_m^{(1)} \int_m^m f_0(w) \Lambda(w) dw}{\int_m^m \pi_m(w) f_0(w) dw} + \mathbb{I}\{w > m\} \Lambda(w)$ . Figure 3 graphically displays the relationship between the latent and the observed probabilities of formality with respect to the wage. The model offers a sharp prediction concerning the effect of the minimum wage on the conditional probability of the sector given the wage. It states that for values above the minimum wage, this probability is equal to the latent probability ( $Pr[S(0) = 1|W(0) > m] = Pr[S(1) = 1|W(1) > m]$ ). It states that the probability of working in the formal sector given the wage will be zero for values below the minimum wage. At the minimum wage level, it should be a particular constant ( $\frac{\pi_m^{(1)} \int_m^m \Lambda(w) f_0(w) dw}{\int_m^m \pi_m(w) f_0(w) dw}$ ), which is likely different from this function’s left and right limits. This result follows from the fact that workers are not able to maintain wages below  $m$  in the formal sector and the assumption of the absence of spillovers above the minimum wage level.

### 3. IDENTIFICATION

It is not possible to directly use the techniques developed in Doyle (2006) in each sector separately, as I have introduced movements between them. To identify the model, a different approach must be used. Below, I state the main



FIGURE 3.— Dual-economy Model: Latent and Observed Conditional Probabilities



identification results of this paper, which concern the identification of (a) the latent *joint* distribution of sector and wages; that is, the distribution that would prevail in the absence of the minimum wage; (b) the vector of parameters  $\pi$  that governs how the minimum wage affects the economy; and (c) the effects of the minimum wage on functionals of the distribution of sector and wages. In what follows, assume that the econometrician observes a random sample of the pair  $\{(W_i(1), S_i(1))\}$  of size  $N$  from a population of interest. I also assume the following easily verifiable technical conditions:  $Pr[W(1) < m] > 0$ , and  $\Lambda'(m; \beta_0) \neq 0$ .

**LEMMA 1 (*Identification of sector-specific parameters*)** Under Assumptions 1, 2, 3, and 4,  $\pi$  is identified. Proof: See Appendix A.

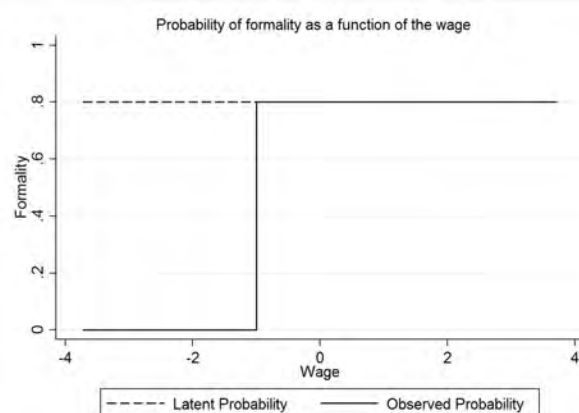
**LEMMA 2 (*Identification of latent distributions*)** Under Assumptions 1, 2, 3 and 4, the latent joint distribution of sector and wages is identified. Proof: See Appendix A.

**COROLLARY 1 (*Identification of the minimum wage treatment effects*)**

Under Assumptions 1, 2, 3 and 4, the effects of the minimum wage on functionals of the joint distribution of sector and wages are identified. Examples of such functionals are the effects of the minimum wage on average wages, on the standard deviation of wages, on quantiles of the wage distribution, on the size of the formal and informal sectors, on the average wages conditional on sectors, and on labor tax revenues.

The key points that permit the identification are as follows: The shape of the relationship between sector and wages for values above the minimum wage is preserved in the presence of the policy. This allows us to obtain estimates of the latent share of the formal sector for values below the minimum wage level by extrapolating the curve we observe in the upper part of the wage distribution.<sup>17</sup> The identification of the latent wage density builds on the approach in Doyle (2006) in the sense that the probability of non-compliance with the policy is identified using the ratio of the density of wages above and below the minimum wage level. To complete the identification, the sector-specific parameters are identified using the derivative of the wage density.<sup>18</sup>

FIGURE 4.— Conditional Probabilities under Independence



#### 4. ESTIMATION

In this section, I discuss how to estimate the model parameters and latent distributions using non-parametric kernel methods. The non-parametric esti-

<sup>17</sup>The relationship between latent sector and wages can only be observed for values above  $m$ . If this function is specified non-parametrically, the latent share of formal workers for values below  $m$  would essentially be unidentified. However, by relying on the parametric functional form, I can extrapolate the relationship observed above  $m$  to predict the latent share of workers that would prevail below the minimum wage in the absence of the policy. This is achieved by estimating the parameters of the function  $\Lambda(w)$  using wages above  $m$  and then using the estimated parameters for the prediction for *all* wages, both above and below  $m$ .

<sup>18</sup>If latent sector and wages are independent, one need not resort to the derivative of the wage density at  $m$ . In this case, identification of the sector-specific parameters can be achieved by examining the distribution of wages given the sector. See Appendix A for a detailed discussion of this issue.

mation strategy used here is local linear density estimators.

As in Doyle (2006), the model can also be estimated without assuming that the latent wage distribution belongs to a known parametric family. A crucial step in obtaining non-parametric estimates of the objects of interest, such as the model parameters and the counterfactual distributions, involves the estimation of a ratio of one sided limits of the density at the minimum wage. The estimation of these quantities can be performed using non-parametric methods. Note that because the density is discontinuous around the minimum wage, only observations below the minimum are informative for  $\lim_{w \rightarrow m^-} f(w)$  (and similarly for the density above the minimum). This implies that the estimators of these quantities will behave as if the minimum wage were a boundary point of the density, which has implications in terms of bias and variance (McCrary (2008)).

Therefore, it is advisable to use methods ensuring that the performance of the density estimator is satisfactory on points that are close to the support boundaries. I use local linear density estimators, which have the same order of bias at the boundary as at interior points of the distribution. In Appendix B, I formally describe how to non-parametrically perform the density estimation.

For the remaining terms that need to be estimated, I will use the plug-in approach and replace the unknown objects in the identification section with their consistent estimators. Thus  $\hat{\pi}_d(m) \equiv \frac{\hat{f}(m^-)}{\hat{f}(m^+)}$ , where  $\hat{f}(m^-)$  is the estimator of the density just below the minimum wage value using the local linear density estimator. In addition, for the estimator of  $\pi'_d(m)$ , we can define  $\hat{\pi}'_d(m) \equiv \left( \frac{\hat{f}'(m^-)}{\hat{f}'(m^+)} - \frac{\hat{f}(m^-)}{\hat{f}(m^+)} \right) \frac{\hat{f}'(m^+)}{\hat{f}(m^+)}$ .

To complete the process of recovering the structural parameters  $\pi$ , one requires estimates of  $\Lambda(m)$  and  $\Lambda'(m)$ . These objects are the latent share of the formal sector and the change in it at  $m$ . For that purpose, one needs to estimate  $\beta_0$ . Given Assumptions 2 and 3, the estimator can be defined as:  $\hat{\beta} \equiv \arg \min_{\beta} \sum_{i=1}^N (S_i - \Lambda(W_i; \beta))^2 \mathbb{I}\{W_i > m\}$ .

Then, given that we estimated  $\beta_0$ , we can plug it into the function  $\Lambda(\cdot)$  to obtain an estimate of  $\Lambda(m)$  and  $\Lambda'(m)$ . They will be given by  $\hat{\Lambda}(m) = \Lambda(m; \hat{\beta})$  and  $\hat{\Lambda}'(m) = \Lambda'(m; \hat{\beta})$ . Using the estimate  $\hat{\Lambda}(m)$  of the latent share of the formal sector, we can define the plug-in estimator for the parameters  $\pi_d^{(1)}$  and  $\pi_d^{(0)}$ :  $\hat{\pi}_d^{(0)} \equiv \hat{\pi}_d(m) - \frac{\hat{\Lambda}(m)}{\hat{\Lambda}'(m)} \hat{\pi}'_d(m)$ , and  $\hat{\pi}_d^{(1)} \equiv [\hat{\pi}_d(m) - (1 - \hat{\Lambda}(m)) \cdot \hat{\pi}_d^{(0)}] \cdot \hat{\Lambda}(m)^{-1}$ . To complete the estimation, we first need to estimate  $c$  before we recover the latent wage density:  $\hat{c} \equiv [\int^m \frac{\hat{f}(u)}{\hat{\pi}_d(u)} du + 1 - \hat{F}(m)]^{-1}$ . Then, the estimates of the latent wage distribution can be defined, for  $w \neq m$ , as:  $\hat{f}_0(w) =$

TABLE I  
DESCRIPTIVE STATISTICS

| Variable      | Obs    | Mean   | Std. Dev. | Min | Max    |
|---------------|--------|--------|-----------|-----|--------|
| Wage          | 579252 | 791.13 | 1233.28   | 1   | 350000 |
| Female        | 579252 | 0.35   | 0.48      | 0   | 1      |
| White         | 579252 | 0.56   | 0.50      | 0   | 1      |
| Education     | 579252 | 9.88   | 3.87      | 1   | 17     |
| Tenure        | 579252 | 4.24   | 5.65      | 0   | 53     |
| Age           | 579252 | 32.97  | 10.08     | 19  | 59     |
| Formal Sector | 579252 | 0.74   | 0.44      | 0   | 1      |
| Minimum Wage  | 579252 | 320.36 | 93.12     | 180 | 465    |

$$\mathbb{I}\{w < m\} \frac{\hat{f}(w)\hat{c}}{\hat{\pi}_d(w)} + \mathbb{I}\{w > m\} \frac{\hat{f}(w)}{\hat{c}}.$$

The consistency of the  $\hat{\pi}$ ,  $\hat{\beta}$  and, consequently,  $\hat{\Lambda}(w)$  and  $\hat{f}(w)$  follows directly from the identification equations and the consistency of  $\hat{f}(w)$  and  $\hat{f}'(w)$ . Closed-form expressions for the asymptotic variances can be derived. However, I will rely on resampling methods to estimate them in the empirical application.

## 5. EMPIRICAL APPLICATION: THE EFFECT OF THE MINIMUM WAGE IN BRAZIL

For my empirical application, I consider a stronger version of Assumption 2:

ASSUMPTION 5 *Independence*:  $Pr[S(0) = 1 | W(0) = w] = \Lambda \quad \forall \quad w$ .

This assumption implies that *latent* sector and wages are independent. Figure 4 displays the relationship between the latent and observed conditional probabilities of formality with respect to the wages under this assumption. This assumption is testable. Below, I provide evidence that it is not violated in the context of the Brazilian labor market.

Independence greatly simplifies the identification and estimation, as can be seen in Appendix A.2. Independence (and the absence of spillovers) allows me to identify the latent share of the formal sector by examining the observed share of the formal sector for wages that are above the minimum wage level. Moreover, it implies that the aggregate minimum wage probabilities  $(\pi_d(w), \pi_m(w), \pi_u(w))$  do not vary across wages even if the parameters differ across sectors. This is because the latent share of each sector becomes constant with respect to wages. This allows me to identify the model parameters by simply examining the discontinuity in the aggregate wage distribution at  $m$  and the sector-specific wage distributions. As a result, I will not need to rely on estimating the derivative of the wage density at the boundary point, a

TABLE II  
DESCRIPTIVE STATISTICS BY SECTOR

|                     | Formal Sector      | Informal Sector    | Difference            |
|---------------------|--------------------|--------------------|-----------------------|
| Wage                | 874.599<br>(1.716) | 556.114<br>(3.755) | 318.485***<br>(3.661) |
| Female              | 0.359<br>(0.001)   | 0.340<br>(0.001)   | 0.019***<br>(0.001)   |
| White               | 0.583<br>(0.001)   | 0.482<br>(0.001)   | 0.101***<br>(0.001)   |
| Education           | 10.246<br>(0.006)  | 8.840<br>(0.010)   | 1.406***<br>(0.011)   |
| Tenure              | 4.639<br>(0.009)   | 3.134<br>(0.013)   | 1.505***<br>(0.017)   |
| Age                 | 33.312<br>(0.015)  | 32.003<br>(0.026)  | 1.309***<br>(0.030)   |
| Minimum wage worker | 0.103<br>(0.000)   | 0.151<br>(0.001)   | -0.047***<br>(0.001)  |
| <i>N</i>            | 420,097            | 159,155            | -                     |

*Note:* Standard errors in parentheses.

problem that is both in theory and in practice much harder than estimating the density's level.

In Appendix A.2, I describe how to identify the model under this condition. The estimation strategy I use follows the same method as in the general form of the model. That is, I estimate the density of wages at the boundary using local linear density estimators and use a plug-in method for the remaining objects. Namely, once I estimate the lateral limits of the density of wages at  $m$ , I complete the estimation by replacing the objects in the identifying equations using their respective sample counterparts. In the next sections, I describe the data and discuss the results obtained when estimating this model for the Brazilian labor market.

### 5.1. *Data and Descriptive Statistics*

To evaluate the effects of the minimum wage on labor market outcomes, I used data for the period from 2001 to 2009 from the PNAD dataset. These data have been collected by the IBGE – which is a Portuguese acronym for “Brazilian Institute of Geography and Statistics” – since 1967 and contain information on income, education, labor force participation, migration, health and other socioeconomic characteristics of the Brazilian population. Workers who do not report wages, those who work in the public sector and workers who are older than 60 years of age or younger than 18 years of age were removed from the sample. The PNAD dataset includes information on the worker's labor contract status, which was used to define formality.

The variable of interest – the wage – is measured at the monthly level, which

TABLE III  
DESCRIPTIVE STATISTICS: CONDITIONAL PROBABILITIES

|                                 | $Pr[S = 1]$ | $Pr[S = 0]$ | $Pr[W = m]$ | $Pr[W < m]$ |
|---------------------------------|-------------|-------------|-------------|-------------|
| <i>Unconditional</i>            | 0.738       | 0.262       | 0.116       | 0.073       |
| <i>Conditional on sector</i>    |             |             |             |             |
| Formal                          | 1.000       | 0.000       | 0.103       | 0.006       |
| Informal                        | 0.000       | 1.000       | 0.151       | 0.265       |
| <i>Conditional on wage</i>      |             |             |             |             |
| $W < m$                         | 0.056       | 0.944       | 0.000       | 1.000       |
| $W = m$                         | 0.659       | 0.341       | 1.000       | 0.000       |
| $W > m$                         | 0.811       | 0.189       | 0.000       | 0.000       |
| <i>Conditional on gender</i>    |             |             |             |             |
| Male                            | 0.732       | 0.268       | 0.099       | 0.066       |
| Female                          | 0.748       | 0.252       | 0.147       | 0.088       |
| <i>Conditional on race</i>      |             |             |             |             |
| White                           | 0.773       | 0.227       | 0.082       | 0.047       |
| Nonwhite                        | 0.694       | 0.306       | 0.157       | 0.107       |
| <i>Conditional on education</i> |             |             |             |             |
| Less than 5 years               | 0.576       | 0.424       | 0.180       | 0.179       |
| Less than 12 years              | 0.672       | 0.328       | 0.143       | 0.108       |
| More than 12 years              | 0.797       | 0.203       | 0.041       | 0.022       |
| <i>Conditional on region</i>    |             |             |             |             |
| South                           | 0.803       | 0.197       | 0.062       | 0.040       |
| Southeast                       | 0.778       | 0.222       | 0.079       | 0.041       |
| Center-West                     | 0.706       | 0.294       | 0.129       | 0.050       |
| North                           | 0.623       | 0.377       | 0.182       | 0.106       |
| Northeast                       | 0.608       | 0.392       | 0.246       | 0.198       |

Note:  $N = 579,252$

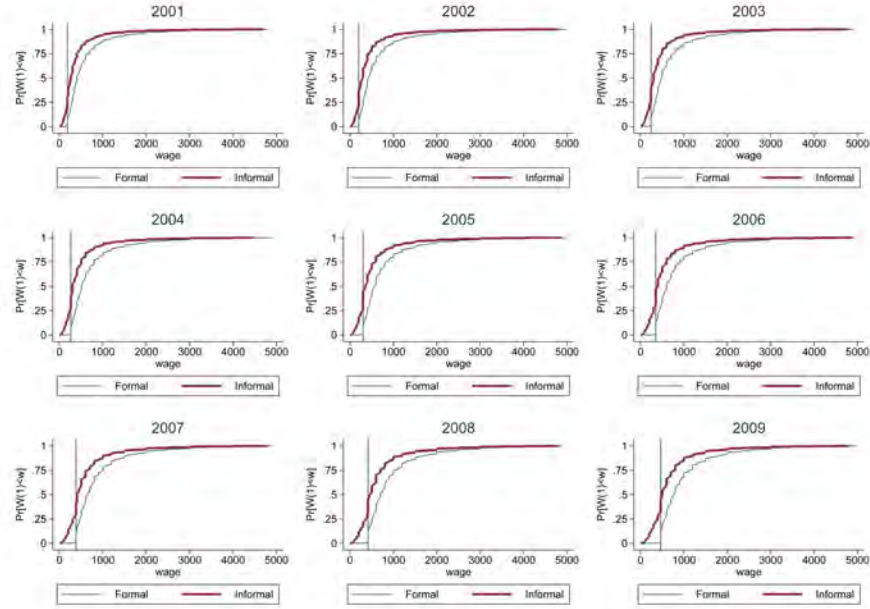
is the most natural unit in the Brazilian institutional context. A feature of the Brazilian labor market is that wages are typically specified at the monthly level, the same unit of measure as the minimum wage. The labor contract also establishes the number of hours of work per day (typically 6 or 8 hours).<sup>19</sup> I will treat the wage reported in the survey as the contracted wage, so no adjustment for hours need to be performed. As a result, wages below the minimum wage are not, in principle, a result of a “division bias”.

The empirical strategy will assume also that the wage is measured without error. This is unquestionably a strong assumption, given that the observed wage distribution presents “heaping” at round numbers. In my baseline estimates, I attempt to minimize the damage due to this particular form of measurement error by using larger values for the bandwidth. In the Appendix E.2, I estimate a modified version of the model that explicitly takes “heaping” into account, obtaining similar results to my preferred specification.

As mentioned above, all workers in Brazil carry an official document called “Carteira de Trabalho” (worker’s card). This document is signed by the em-

<sup>19</sup>At the end of the month the worker will receive a payment “pro rata” based on the actual number of days he or she worked. This payment will present some small variation across months due to reasons such as holidays, absences, overtime pay and the like.

FIGURE 5.— Empirical CDFs



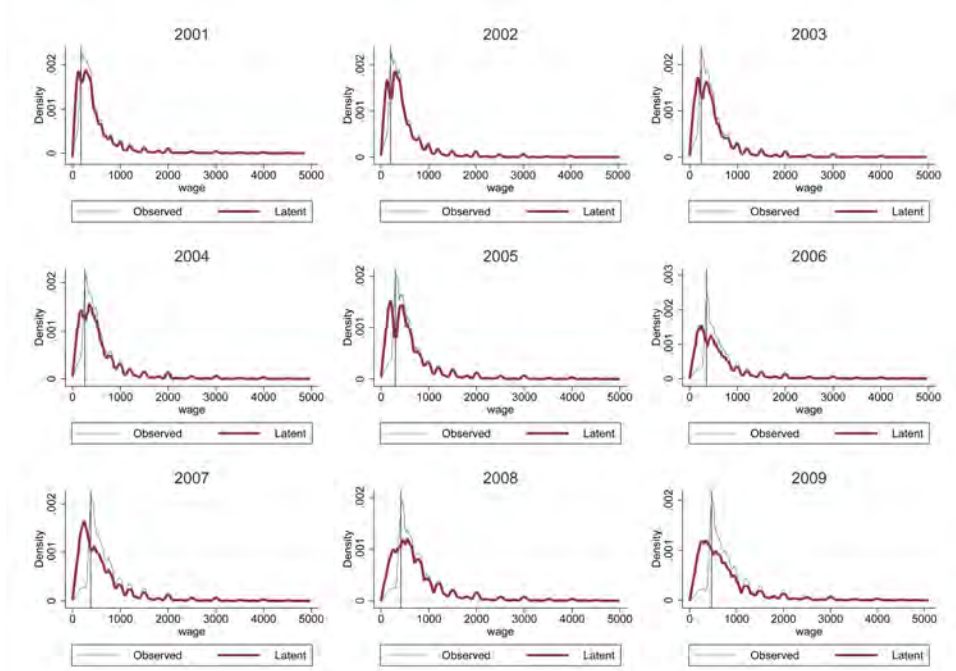
employers in the formal act of hiring. The lack of a formal signed labor contract means that the employer is not forced to collect labor taxes or to comply with the minimum wage and other types of regulation. The Brazilian economy is known to be characterized by a large informal sector. Tables I, II and III illustrate this fact and describe the main features of the data.<sup>20</sup>

Figure 5 displays the empirical CDFs of the formal and informal sectors. A few interesting facts can be noted: The empirical cumulative distribution of wages seems to have a spike at the minimum wage level in *both* sectors, and virtually no worker in the formal sector receives wages below  $m$ . Thus, informality is closely related to sub-minimum wages. However, these concepts are not equivalent, as a sizable fraction of informal workers earn wages above the minimum wage level.

Table II shows that workers in the informal sector earn on average approximately 36% less than workers in the formal sector. In addition, in terms of the observable characteristics, workers in the informal sector are more likely to be male, nonwhite, less educated and young. Considering the likelihood of earning minimum and sub-minimum wages, Table III shows the heterogeneity of these probabilities across population subgroups. For example, white workers are 48%

<sup>20</sup>All estimates are computed using survey weights.

FIGURE 6.— Kernel Density Estimates  
Unconditional Wage Distribution



less likely to earn the minimum wage than are nonwhite workers. Workers with less than 5 years of education have an approximately 18% likelihood of earning the minimum wage, whereas the corresponding likelihood is only 4% for workers with more than 12 years of education. Regarding the geographic variation, workers in the South Region have a 6% probability of earning the minimum wage, whereas workers in the Northeast have an approximately 25% probability of earning the minimum wage. A similar heterogeneity pattern appears when we consider the probability of earning sub-minimum wages.

Table III shows that formality presents considerable heterogeneity across observable characteristics. It shows that the probability of formality is close to zero for workers with wages below the minimum wage, as predicted by the dual-economy model. Also, it shows that the probability of working in the formal sector is lower for low education groups, nonwhite, and in the North and Northeast regions.



TABLE IV  
MODEL PARAMETER ESTIMATES BY YEAR

|               | 2001                | 2002                | 2003                | 2004                | 2005                | 2006                | 2007                | 2008                | 2009                |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\pi_d$       | 0.202***<br>(0.006) | 0.217***<br>(0.009) | 0.206***<br>(0.005) | 0.232***<br>(0.006) | 0.180***<br>(0.006) | 0.157***<br>(0.004) | 0.113***<br>(0.002) | 0.192***<br>(0.005) | 0.121***<br>(0.003) |
| $\pi_m$       | 0.256***<br>(0.007) | 0.356***<br>(0.015) | 0.289***<br>(0.007) | 0.293***<br>(0.008) | 0.349***<br>(0.011) | 0.262***<br>(0.006) | 0.176***<br>(0.004) | 0.304***<br>(0.007) | 0.208***<br>(0.005) |
| $F_0(m)$      | 0.253***<br>(0.005) | 0.260***<br>(0.008) | 0.311***<br>(0.004) | 0.291***<br>(0.005) | 0.328***<br>(0.007) | 0.400***<br>(0.005) | 0.446***<br>(0.004) | 0.345***<br>(0.004) | 0.434***<br>(0.005) |
| $\pi_d^{(1)}$ | 0.106***<br>(0.013) | 0.222***<br>(0.021) | 0.139***<br>(0.010) | 0.179***<br>(0.012) | 0.165***<br>(0.012) | 0.077***<br>(0.007) | 0.000<br>(0.000)    | 0.131***<br>(0.008) | 0.023***<br>(0.005) |
| $\pi_m^{(1)}$ | 0.191***<br>(0.006) | 0.231***<br>(0.011) | 0.223***<br>(0.006) | 0.226***<br>(0.006) | 0.258***<br>(0.009) | 0.213***<br>(0.005) | 0.137***<br>(0.007) | 0.264***<br>(0.006) | 0.186***<br>(0.004) |
| $\pi_d^{(0)}$ | 0.525***<br>(0.018) | 0.199***<br>(0.038) | 0.461***<br>(0.017) | 0.443***<br>(0.019) | 0.244***<br>(0.026) | 0.516***<br>(0.013) | 0.640***<br>(0.015) | 0.494***<br>(0.016) | 0.669***<br>(0.011) |
| $\pi_m^{(0)}$ | 0.475***<br>(0.018) | 0.801***<br>(0.038) | 0.539***<br>(0.017) | 0.557***<br>(0.019) | 0.756***<br>(0.026) | 0.484***<br>(0.013) | 0.360***<br>(0.015) | 0.506***<br>(0.016) | 0.331***<br>(0.011) |
| $\pi_u$       | 0.543***<br>(0.012) | 0.427***<br>(0.024) | 0.506***<br>(0.011) | 0.475***<br>(0.013) | 0.471***<br>(0.016) | 0.581***<br>(0.010) | 0.711***<br>(0.005) | 0.504***<br>(0.010) | 0.670***<br>(0.007) |
| $\Lambda$     | 0.772***<br>(0.002) | 0.781***<br>(0.002) | 0.792***<br>(0.002) | 0.798***<br>(0.002) | 0.816***<br>(0.002) | 0.819***<br>(0.002) | 0.823***<br>(0.002) | 0.832***<br>(0.002) | 0.847***<br>(0.002) |
| m             | 180                 | 200                 | 240                 | 260                 | 300                 | 350                 | 380                 | 415                 | 465                 |
| N             | 55,487              | 58,223              | 58,265              | 62,576              | 65,745              | 68,193              | 68,315              | 71,051              | 71,397              |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

## 5.2. Main Results

In this section, I will discuss the results obtained after estimating the model for the Brazilian labor market. The model is estimated (separately) for the years 2001 to 2009. As discussed in the estimation section, all objects in the model can be estimated by replacing the population object with its sample analog. The only exception to this is the density of wages at the boundary. To estimate this object, I use a local linear kernel estimator with a normal kernel and a bandwidth equal to eight times Silverman's rule of thumb.<sup>21</sup>

Figure 6 shows a plot of the observed density of wages and its latent counterpart. We can see that, as a consequence of sizable unemployment effects, the observed density above the minimum wage is higher than the latent density. Due to both bunching at the minimum and unemployment, the observed density below the minimum wage is smaller than the latent density. The estimates of the model parameters used to construct this latent density are shown in Table IV.<sup>22</sup>

In examining the point estimates and standard errors in Table IV, we see sizable estimates of the unemployment probability  $\pi_u$ .<sup>23</sup> The evidence from

<sup>21</sup>I show that the results are robust to the choice of bandwidth the Appendix E.3. Jales (2015) shows, furthermore, that the estimates of  $\pi_d$  are similar to the estimates obtained using a parametric model that *bypasses* entirely the need to specify a bandwidth.

<sup>22</sup>I discuss the implication of these estimates for wage inequality on Appendix E.5.

<sup>23</sup>The unemployment probabilities I obtained are comparable to the results obtained in

Table IV also suggests that sector mobility is limited. The estimates of the sector-mobility parameter ( $\pi_d^{(1)}$ ) are approximately 12%, with a maximum of 22%. I discuss in greater detail the implications of this result in Section 5.5.

The period of 2001 to 2009 is characterized by an increase in the real value of the minimum wage. We should expect the estimates of the mass of affected workers,  $F_0(m)$ , to reflect this feature of the data. Regarding Table IV, we observe a close relationship between the minimum wage level and estimates of  $F_0(m)$ . The correlation coefficient of the estimate of  $F_0(m)$  with the (real value of the) minimum wage is approximately 0.90.

The minimum wage disemployment effects I estimate compare the employment levels in the presence of the policy with their latent counterparts. As a result, it is not trivial how to compare these estimates with most of the literature that estimates the effects of marginal changes in the policy.<sup>24</sup> To put the disemployment estimates into perspective, I perform a back of the envelope calculation to estimate the implied elasticity of employment with respect to the minimum wage level. I do so by regressing the logarithm of the estimate of  $c$ , the minimum wage employment effect on the logarithm of the minimum wage level. This exercise exploits the correlation (over time) of the minimum wage level with the estimates of the employment effects.

This exercise gives me an elasticity of approximately -.22, statistically different from zero at conventional significance levels (with a somewhat wide confidence interval from -0.06 to -.4). This elasticity is close to the bulk of the literature that estimated minimum wage effects using panel data methods and natural experiments. For the typical minimum wage increases I observe in the data – of around 12% –, the elasticity estimates would imply a decrease of approximately 2.5% on aggregate employment, *ceteris paribus*. Although the unemployment *probability* estimates seem large, they imply reasonable employment *effects* when applied to the set of workers affected by marginal changes in the policy.

### 5.3. The Geographic Heterogeneity of Minimum Wage Effects

The Brazilian economy is characterized by considerable geographic variation in the size of the formal sector, as shown in Table III. The size of the formal sector in the Southeast region is approximately 0.78, whereas in the Northeast region the size of the formal sector is approximately 0.61. Table V shows the model

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papers that tried to estimate the same parameters (Meyer and Wise (1983), Doyle (2006)).

<sup>24</sup>The unemployment effects estimated from the “missing mass” in the wage distribution should be closer to the long-run than to the short-run effect of the minimum wage. This could be one explanation for the magnitude of my estimates of the disemployment effects of the policy (Meer and West (2015), Baker, Benjamin, Stanger (1999), Sorkin (2015)).

TABLE V  
THE GEOGRAPHIC HETEROGENEITY OF MINIMUM WAGE EFFECTS

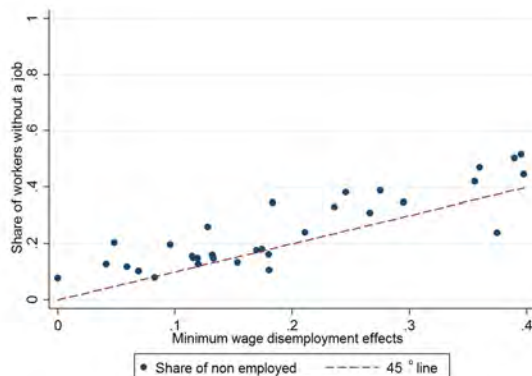
|           | $\pi_d$             | $\pi_m$             | $F_0(m)$            | $\pi_d^{(1)}$       | $\pi_m^{(1)}$       | $\pi_d^{(0)}$       | $\pi_m^{(0)}$       | $\pi_u$             | $\Lambda$           | $m$ | $N$   |
|-----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|-----|-------|
| Southeast | 0.142***<br>(0.007) | 0.204***<br>(0.009) | 0.267***<br>(0.006) | 0.044***<br>(0.013) | 0.146***<br>(0.007) | 0.552***<br>(0.025) | 0.448***<br>(0.025) | 0.654***<br>(0.015) | 0.808***<br>(0.002) | 260 | 22158 |
| Northeast | 0.328***<br>(0.011) | 0.345***<br>(0.012) | 0.532***<br>(0.007) | 0.264***<br>(0.020) | 0.306***<br>(0.012) | 0.531***<br>(0.021) | 0.469***<br>(0.021) | 0.327***<br>(0.021) | 0.760***<br>(0.004) | 260 | 14982 |

Note: Data from 2004 PNAD. \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

parameter estimates separately for the Southeast and the Northeast region. In the Northeast region the minimum wage “bites” at a much higher point in the wage distribution when compared to the Southeast. The latent size of the formal sector in the Southeast is 0.81. In the Northeast region, the latent size is 0.76. These regions also differ in their responses to the (same) minimum wage policy. In the Southeast we observe a high probability of unemployment (0.65). We also observe a low estimate of the sector mobility parameter (0.04). In the Northeast, we observe a lower probability of unemployment (0.33), higher probability of non-compliance (0.33), and higher probability of moving to the informal sector (.26).<sup>25</sup>

Regarding the informal sector parameters,  $\pi_d^{(0)}$  and  $\pi_m^{(0)}$ , I do not reject the null hypothesis that the coefficients are the same across regions. This suggest that the differences we observe in the joint distribution of sector and wages across these regions come from differences in their latent distributions and differences in the *formal* sector’s response to the minimum wage. A decomposition exercise based on the estimates from Table V show that approximately 72% of the differences in the observed size of the formal sector between the Northeast and the Southeast are a result of the minimum wage. The remaining 28% of the differences in the size of the formal sector across these regions are due to other economic factors that cause the Southeast to have a larger size of the formal sector throughout the wage distribution above and beyond their differences in the minimum wage effects. This exercise indicates that the minimum wage affects a substantially larger proportion of workers in the Northeast economy, thereby inducing a larger inflow of workers to the informal sector in that region.

FIGURE 7.— Disemployment estimates and share of disemployed workers



#### 5.4. Investigating the Disemployment Effects

Even though the minimum wage imposes the same floor for all individuals in the population, the same constraint is not equally binding for all groups of the population.<sup>26</sup> As a result, the effect of the minimum wage may be larger for young adults, when compared to their older counterparts. Similarly, we should expect larger effects for low education groups, when compared to workers with high education.

This variation can be used to informally validate the estimates of the model parameters: If larger minimum wage effects are found in sub-groups for which we do not expect the minimum wage to be binding, then one would worry that the estimates are capturing something else unrelated to the effects of the policy. However, if we find that the estimated effects tend to be larger for the sub-groups we *ex-ante* expect them to be, then we could be cautiously more confident that the estimates indeed are capturing the effects of the policy.

Thus, to verify whether the minimum wage effects obtained in my empirical exercise are reasonable, I perform the following exercise: I pool the data for all years, normalizing the wages around the minimum wage level, and estimate the model for narrowly defined groups of age, education, race and gender.<sup>27</sup> Then

<sup>25</sup>The region where the *latent* size of the formal sector is higher also presented a higher likelihood of sector mobility. This result may also suggest that formal and informal sectors operate in most cases in distinct labor markets, in the sense that they are located in different geographic regions or different industries. This could be one explanation for the small estimates of the likelihood of sector mobility found in the aggregate economy.

<sup>26</sup>The variation on the minimum wage “bite” induced by the *same* level of the minimum wage was ingeniously used to identify the effect of the policy in the US context in Card (1992).

<sup>27</sup>The groups are defined by the binary indicators of gender (male versus female), race

TABLE VI  
MINIMUM WAGE EFFECTS ON LABOR TAX REVENUES

|                                       | 2001                | 2002                | 2003                | 2004                | 2005                | 2006                | 2007                | 2008                | 2009                |
|---------------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| R                                     | 0.966***<br>(0.001) | 0.969***<br>(0.002) | 0.955***<br>(0.001) | 0.956***<br>(0.001) | 0.954***<br>(0.003) | 0.922***<br>(0.002) | 0.893***<br>(0.002) | 0.939***<br>(0.002) | 0.897***<br>(0.002) |
| Pr[S(1)=1]/Pr[S(0)=1]                 | 0.929***<br>(0.001) | 0.908***<br>(0.001) | 0.909***<br>(0.001) | 0.904***<br>(0.001) | 0.898***<br>(0.001) | 0.897***<br>(0.001) | 0.915***<br>(0.001) | 0.909***<br>(0.001) | 0.916***<br>(0.001) |
| c                                     | 0.863***<br>(0.006) | 0.889***<br>(0.008) | 0.843***<br>(0.005) | 0.862***<br>(0.006) | 0.845***<br>(0.010) | 0.767***<br>(0.007) | 0.683***<br>(0.006) | 0.826***<br>(0.006) | 0.709***<br>(0.006) |
| E[W(1)   S(1) = 1]/E[W(0)   S(0) = 1] | 1.205***<br>(0.007) | 1.201***<br>(0.009) | 1.246***<br>(0.006) | 1.227***<br>(0.007) | 1.257***<br>(0.011) | 1.340***<br>(0.008) | 1.429***<br>(0.009) | 1.251***<br>(0.006) | 1.380***<br>(0.009) |
| N                                     | 55,487              | 58,223              | 58,265              | 62,576              | 65,745              | 68,193              | 68,315              | 71,051              | 71,397              |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

I plot the total employment effects I obtained for these groups with the actual fraction of workers that are non-employed. If the estimates of the employment effects based on the “missing mass” on the wage distribution are capturing a real disemployment effect of the policy, higher missing mass for certain groups should be positively correlated with a higher fraction of workers disemployed in these subgroups.

Figure 7 displays the results of this exercise. The results suggest a positive correlation between the estimated disemployment effects of the policy and actual presence of disemployment in these sub-populations.<sup>28</sup>

There are a few groups, however, in which the estimated effects fall below the 45-degree line. The 45-degree line is the natural bound for the disemployment effect. For these groups, the estimated disemployment effects are *too large* to be justified from the fraction of disemployed workers.<sup>29</sup> Incorporating this type of moment inequality condition to improve the estimates can be a fruitful avenue of future research.

### 5.5. Tax Revenues and the Size of the Informal Sector

A comparison of Tables I and IV shows that the minimum wage reduces the share of the formal sector in the economy. This occurs through two different but related channels: First, the minimum wage reduces the size of the formal sector as long as the unemployment effects are greater than zero, as has been found in Brazil. Second, the minimum wage increases the size of the informal sector

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(white versus non-white), age (above and below 35 years old), and the education groups (less than high school, high school, some college, and college educated workers), which give me a total of 32 cells.

<sup>28</sup>The regression coefficient of the total employment effect and the observed rate of disemployment gives a point estimate of 0.99, statistically significant at conventional levels, and an R-squared of 0.75.

<sup>29</sup>In the Appendix E.1 I extent this analysis by showing the correlation between median wages and the minimum wage “bite”  $F_0(m)$  and also the correlation between  $F_0(m)$  and the estimated effects of the minimum wage on the size of the formal sector.

through sector movements that are driven by the policy itself. Overall, the share of the formal sector in the Brazilian economy is reduced by approximately 9% as a result of the minimum wage policy.<sup>30</sup>

For this reason, the minimum wage indirectly affects the government budget. The minimum wage affects the shape of wage distribution, the relative size of the formal sector and the likelihood of employment. Each of these effects has the potential to alter tax revenues.

The goal of this section is to derive an estimate of these effects. I consider the effects on revenues from the INSS tax, which is the Brazilian labor tax. The INSS is collected to fund the social insurance system in Brazil, and the rate is 20% for companies included in the regular system of taxation and 12% for small companies that opt for the “simplified” system. To estimate the effects, I will rely on the following assumption:

**ASSUMPTION 6** *No Tax Revenues in the Informal Sector*

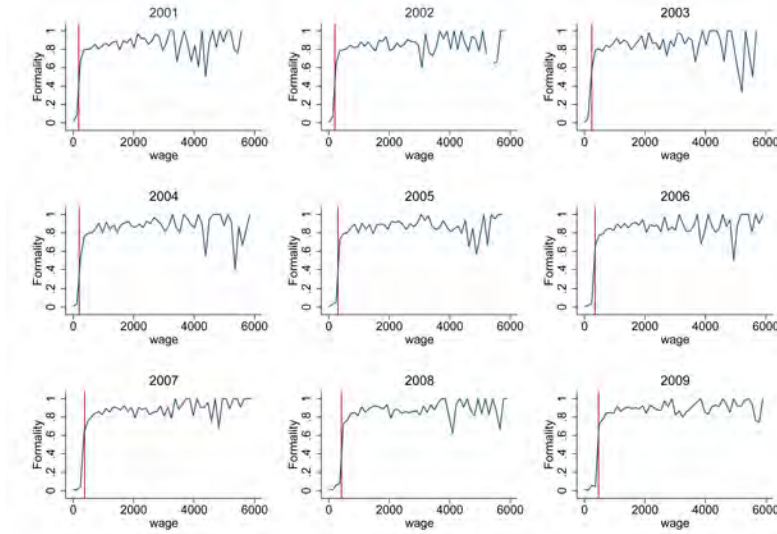
Let  $T(1)$  represent the expected value of tax revenues in the formal sector under the imposition of a minimum wage and  $T(0)$  in its absence. By definition, we have  $T(1) \equiv E[\tau(W_i(1))W_i(1)S_i(1)]$ , and  $T(0) \equiv E[\tau(W_i(0))W_i(0)S_i(0)]$ . The object of interest is the ratio between these two quantities. After some algebra, we have  $R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1)=1]}{Pr[S(0)=1]} \cdot c \cdot \frac{E[\tau(W(1))W(1)|S(1)=1]}{E[\tau(W(0))W(0)|S(0)=1]}$ . This expression is further simplified in the Brazilian case, where labor taxes are a constant fraction of wages. In this case,  $R$  is given by:  $R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1)=1]}{Pr[S(0)=1]} \cdot c \cdot \frac{E[W(1)|S(1)=1]}{E[W(0)|S(0)=1]}$ . Thus, the effects on tax revenues can be decomposed into three components: compression of the formal sector, employment effects, and change in expected wages in the formal sector. This equation shows that the tax effect of the minimum wage will depend on the relative magnitude of these effects.

I compute the tax effects of the minimum wage using a plug-in approach for the components of the equation above based on the model parameter estimates from Table IV. Table VI displays the estimated effects. The minimum wage policy seems to generate sizable unemployment effects and to reduce the size of the formal sector. These effects are large enough to compensate for the increase in expected wages. Therefore, the minimum wage reduces the mass of

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<sup>30</sup>My estimates imply that the mass of workers at and below the minimum wage level is inconsistent with absence of disemployment effects under smooth non-compliance probabilities and a continuous latent distribution of wages. The “missing” mass of workers at or below the minimum wage level is attributed in the model to unemployment effects of the policy. Similarly, high sector-mobility probabilities ( $\pi_d^{(1)}$ ) are inconsistent with my estimate of the latent share of the formal sector and the density of low-wages in the informal sector. That is, we do not observe enough small wages in the informal sector to justify larger sector mobility parameter estimates.

FIGURE 8.— Formality vs. Wages



*Note:* Conditional probabilities estimates based on a local-constant estimator using an Epanechnikov kernel and the standard “rule of thumb” bandwidth.

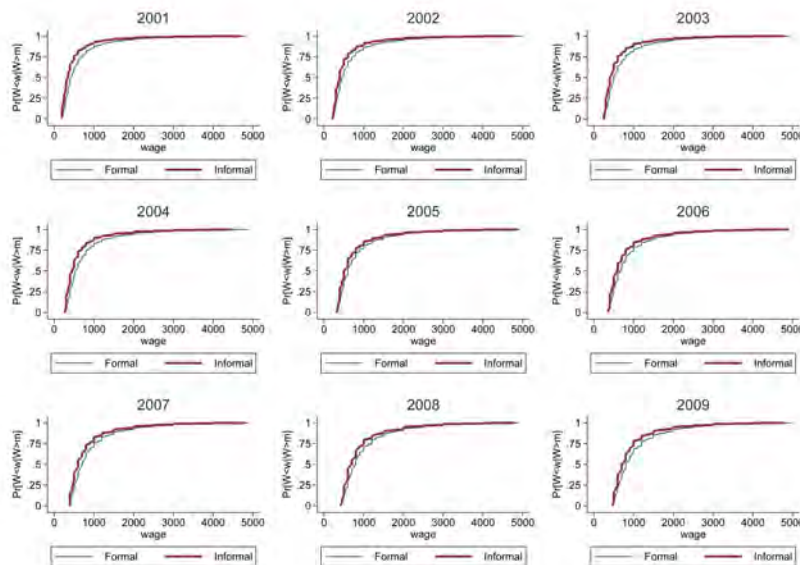
wages in the formal sector, with a corresponding decline in labor tax revenues. The estimates range from 3% in 2001 to 11% in 2007.

### 5.6. *Testing the Underlying Assumptions and Robustness Checks*

This research design allows me to graphically inspect whether some of my identifying assumptions are plausible.<sup>31</sup> First, I will indirectly assess the adequacy of Assumption 5, the independence between latent sector and wage. This assumption has different implications which can be, up to a certain extent, verified. One implication pertains to the proportion of workers in each sector as a function of the wage. If the assumption holds, this proportion should not vary with the wage for wage values that are above the minimum, as shown in Figure 4. A naive regression of formality on wages should mechanically detect a negative relationship because no worker in the formal sector can earn below the minimum wage. However, after restricting our attention to wage values above the minimum, the relationship should disappear. Another related way to test the assumption is to examine the CDF conditional on wages above the minimum. If the model is correct, differences in the observed wage distributions across sectors at values above the minimum will only be due to rescaling induced by unemployment and sector mobility. Thus, by conditioning on val-

<sup>31</sup>I discuss the theory behind the tests performed in this section in Appendix C.

FIGURE 9.— Empirical CDF by Sector above the Minimum Wage



ues above the minimum, the effects of rescaling and sector movements should disappear, and the densities should be approximately the same.

Figures 8 and 9 provide visual evidence of the accuracy of this assumption within the Brazilian context. Above the minimum wage level, the proportion of workers in the formal sector of the economy does not seem to systematically vary with the wage. This evidence supports the assumption that the underlying latent density of wages should be the same between sectors. The plots of the empirical CDFs in Figure 9 across formal and informal sectors point in the same direction: Workers in the formal and informal sectors apparently draw from similar distributions for wages above the minimum wage. This suggests that the differences between the overall distribution of wages occur mostly as a result of the different ways in which the sectors respond to the minimum wage. Note, however, that the assumption required for identification is that the entire wage distribution be the same across sectors. The presence of the minimum wage prevents me from testing this condition for values below  $m$ . Thus, it is still possible that the latent wage distributions are indeed similar conditional on wages above the minimum wage, while this is not the case for values below it. This last part of the identifying assumption is untestable. The evidence that the wage distributions are similar for values above the minimum wage seems to indicate that they may also be so for values below  $m$  in the absence of the policy. However, this conclusion is subject to debate.



TABLE VII  
FORMALITY VS. WAGES - LINEAR REGRESSION ESTIMATES

|            | 2001              | 2002              | 2003              | 2004              | 2005              | 2006              | 2007              | 2008              | 2009             |
|------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|------------------|
| $W > 0$    | 1.93***<br>(0.14) | 2.15***<br>(0.14) | 2.77***<br>(0.13) | 2.43***<br>(0.34) | 2.73***<br>(0.15) | 2.63***<br>(0.62) | 2.76***<br>(0.15) | 2.51***<br>(0.48) | 0.72<br>(0.59)   |
| $W > m$    | 0.87***<br>(0.08) | 0.71***<br>(0.08) | 0.99***<br>(0.07) | 0.79***<br>(0.12) | 0.54***<br>(0.08) | 0.61***<br>(0.15) | 0.73***<br>(0.08) | 0.51***<br>(0.11) | 0.09<br>(0.16)   |
| $W > 1.5m$ | 0.40***<br>(0.06) | 0.26***<br>(0.07) | 0.42***<br>(0.06) | 0.38***<br>(0.07) | 0.12*<br>(0.07)   | 0.15**<br>(0.06)  | 0.17**<br>(0.07)  | 0.02<br>(0.06)    | -0.03<br>(0.08)  |
| $W > 2m$   | 0.15**<br>(0.06)  | 0.07<br>(0.06)    | 0.34***<br>(0.06) | 0.08<br>(0.05)    | -0.07<br>(0.07)   | 0.07<br>(0.06)    | 0.08<br>(0.07)    | -0.05<br>(0.07)   | -0.08*<br>(0.04) |

*Note:* Estimates of the (100 times the) elasticity of formality with respect to the wage at the minimum wage level, using different sample restrictions. Standard errors in parentheses.

Table VII shows the estimates of *one hundred times* the elasticity of formality with respect to the wage based on a linear probability model, using different restrictions on the sample. The relationship between sector distribution and wages becomes substantially weaker after one conditions the regression to only consider wages above the minimum. Regarding the coefficient while conditioning on higher values, several estimates that are not different from zero were found.<sup>32</sup>

Another maintained assumption of the model is that the latent wage density is continuous around the minimum. If the wage density is continuous, then my estimates should not reveal any effect for values other than the minimum wage.

Table VIII displays the estimates of the ratio of the left and right limits of the wage density for values different than the minimum wage pooling data from all years (normalizing the minimum wage to zero). I display the point estimates for using two values for the bandwidth: the same bandwidth as in Table IV and a bandwidth half of its size. I perform the placebo test at 20 different points, from R\$200 to R\$300 above the minimum wage. The point estimates tend to be around one, which should be the case in the absence of a discontinuity. Using the baseline bandwidth, the estimates range from 0.88 to 1.17.

However, the null hypothesis of no gap is still rejected at several points. Discontinuities of such small magnitude are likely a result of “heaping” at round numbers. If a discontinuity of similar size is present in the latent wage density, then the magnitude of the bias on the estimates of the model structural

<sup>32</sup>Jales (2015) further investigates the consequences of imposing the independence assumption. Estimating a general form of the model, he finds a positive relationship between latent sector and wage. However, as suggested by the results in Table VII, the magnitude of the estimates of this relationship are very small, close to zero. As a result, the estimated effects of the minimum wage using the general form of the model are similar to the results obtained under independence.

TABLE VIII  
PLACEBO TESTS: DISCONTINUITY ESTIMATES USING VALUES OTHER THAN THE  
MINIMUM WAGE

|          | Wage | 10       | 20       | 30       | 40       | 50       | 60       | 70       | 80       | 90       | 100      |
|----------|------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Baseline |      | 1.007*** | 0.929*** | 0.993*** | 1.111*** | 1.130*** | 1.173*** | 1.097*** | 0.920*** | 0.961*** | 0.882*** |
|          | 200  | (0.008)  | (0.008)  | (0.008)  | (0.009)  | (0.009)  | (0.009)  | (0.007)  | (0.007)  | (0.007)  | (0.007)  |
| Half     |      | 1.039*** | 0.876*** | 1.003*** | 1.191*** | 1.178*** | 1.161*** | 0.969*** | 0.654*** | 0.742*** | 0.579*** |
|          |      | (0.011)  | (0.011)  | (0.010)  | (0.013)  | (0.015)  | (0.014)  | (0.010)  | (0.008)  | (0.009)  | (0.010)  |
| Baseline |      | 1.029*** | 0.907*** | 0.894*** | 1.076*** | 1.084*** | 1.104*** | 1.046*** | 0.899*** | 1.059*** | 0.995*** |
|          | 300  | (0.008)  | (0.008)  | (0.008)  | (0.009)  | (0.009)  | (0.009)  | (0.010)  | (0.009)  | (0.011)  | (0.010)  |
| Half     |      | 0.901*** | 0.698*** | 0.737*** | 1.081*** | 1.101*** | 1.174*** | 1.031*** | 0.760*** | 1.023*** | 0.865*** |
|          |      | (0.013)  | (0.011)  | (0.010)  | (0.012)  | (0.013)  | (0.013)  | (0.012)  | (0.010)  | (0.013)  | (0.013)  |

*Note:* Standard errors in parentheses.

parameters would be of negligible economic significance. All the qualitative implications of the model parameter estimates based on the continuity assumption would remain valid.<sup>33</sup>

As a robustness check, I investigate the sensitivity of my estimates to the choice of bandwidth and the presence of spillovers. A key parameter of the model,  $\pi_d$ , is identified by the ratio of the wage density above and below the minimum wage. In the baseline specification, the estimation was performed using local linear density estimators with the bandwidth equal to eight times Silverman’s rule of thumb. In the Appendix E.3, I show that I obtain similar results when estimating the model using different choices of the bandwidth.

In the Appendix E.4 I discuss the identification of the effect of the minimum wage on the size of the formal sector under the presence of spillovers. Under Assumption 5, identification of the latent size of the formal sector can be achieved by assuming that spillovers vanish at a point higher up in the wage distribution. My spillover-robust estimates of the impact of the minimum wage on the size of the formal sector are approximately -14%. Thus, these estimates are higher than the baseline estimates from Table IV that are obtained under the assumption of absence of spillovers. This suggests that the -9% effect from the baseline estimate *underestimates* of the true effect of the minimum wage on the size of the formal sector if Assumption 3 is violated.

<sup>33</sup>I discuss in Appendix D the consequences of estimating the model incorrectly assuming continuity for the latent wage distribution. The estimators of the probabilities of non-compliance and “bunching” at the minimum wage will be inconsistent if the latent wage distribution is discontinuous at the minimum wage level. The ratio between the true structural parameters and the (probability limit of the) estimators will be given by the magnitude of the discontinuity in the *latent* density at the minimum wage level. For example, adjusting the estimates for a discontinuity of 0.92 in the latent density increases the estimate of  $\pi_d$  for the year 2001 from 0.20 to 0.22. Similarly,  $\pi_m$  increases from 0.26 to 0.28.

## 6. CONCLUSION

This paper develops a dual-economy model to analyze the effects of the minimum wage in a country with a large informal sector. I discuss the conditions under which the effects of the policy are identified using only cross-sectional data on wages and sector (defined by formality status) and the same level of the policy is applied to all workers. I show that the discontinuity of the wage density at the minimum wage level identifies the probability of non-compliance with the policy, and the latent relationship between sector and wages can be recovered using data on wages and sector above the minimum wage. I then show that the latent *joint* distribution of sector and wages can be identified based solely on data on sector and wages.

The main results are that the minimum wage significantly alters the shape of the lower part of the wage distribution, inducing an increase of approximately 16% on average wages. However, the minimum wage generates sizable unemployment effects and a decrease in the size of the formal sector of the economy.

My estimates imply a decrease of approximately 9% in the size of the formal sector. This result is due to both unemployment effects on the formal sector and movements of workers from the formal sector to the informal sector as a response to the policy. My estimates also indicate that the latent size of the formal sector is approximately four times larger than the informal sector. As a result, small movements from the formal to the informal sector still induce a sizable change in the relative size of the informal sector. I estimate that the minimum wage increases the size of the informal sector by approximately 39%. Together, these effects imply a reduction in the tax revenues collected by the government to support the social welfare system of approximately 6%.

The research design based on the sharp contrast in the effects of the minimum wage between workers on each side of the minimum wage value allows for indirect tests of the underlying identification assumptions of the model. The graphical and statistical evidence supports the maintained assumptions. The robustness checks performed produced similar results to those of my preferred estimates.

There are, however, several limitations of this strategy. A fully structural model of workers and firms behavior is not specified. Thus, this approach does not recover deep parameters of the economy such as the elasticity of labor demand. An extended version of the dual-economy model presented in this paper that fully incorporate optimizing behavior from the worker's side, such as a Roy-model of sector choice, is the object of ongoing research.

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## APPENDIX A

A.1. *Identification*

This section I prove identification of the parameters of the model and the joint distribution of latent sector and wages under the assumptions of the Dual-Economy model. In what follows, assume that the econometrician observes a random sample of the pair  $\{(W_i(1), S_i(1))\}$  of size  $N$  from a population of interest. I also assume the following easily verifiable technical conditions: the minimum wage  $m$  is set at a point with non-zero density, that is,  $f_0(m) > 0$ ,  $Pr[W(1) < m] > 0$ , and  $\Lambda'(m; \beta_0) \neq 0$ .

LEMMA 1 (**Identification of sector-specific parameters**) Under Assumptions 1, 2, 3, and 4,  $\pi$  is identified.

LEMMA 2 (**Identification of latent distributions**) Under Assumptions 1, 2, 3 and 4, the latent joint distribution of sector and wages is identified.

PROOF: Given Assumptions 3 and 4, the relationship between the observed density and the latent one can be written as:

$$(1) \quad f(w) = \begin{cases} \frac{\pi_d(w)f_0(w)}{c} & \text{if } w < m \\ \int^m \frac{\pi_m(w)f_0(w)}{f_0(w)} dw & \text{if } w = m \\ \frac{f_0(w)}{c} & \text{if } w > m. \end{cases}$$

Given Assumptions 2, 3 and 4, the latent share of the formal sector  $\Lambda(w(0))$  is identified using the information above the minimum wage. This is true because  $Pr[S(0) = 1|W(0) = w] = Pr[S(1) = 1|W(1) = w]$  when  $w > m$ . Then, we have:

$$\beta_0 = \arg \min_{\beta} \int_m^{\infty} (Pr[S(1) = 1|W(1) = u] - \Lambda(u; \beta))^2 f(u|W(1) > m) du.$$

Furthermore, we have that  $\Lambda(w; \beta_0) = Pr[S(0) = 1|W(0) = w]$  for all  $w$ .<sup>34</sup> Given Assumptions 1, 3 and 4, we have:

$$\pi_d(m) = \lim_{\epsilon \rightarrow 0^+} \frac{f(m - \epsilon)}{f(m + \epsilon)}.$$

Moreover, regarding the derivative of the wage density, we have:

$$(2) \quad f'(w) = \begin{cases} \frac{\pi_d'(w)f_0(w)}{c} + \frac{\pi_d(w)f_0'(w)}{c} & \text{if } w < m \\ \frac{f_0'(w)}{c} & \text{if } w > m. \end{cases}$$

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<sup>34</sup>Note the importance of *all*  $w$  in this sentence. This means that once we recover  $\beta_0$ , we can forecast  $Pr[S(0) = 1|W(0) = w]$  for values of  $w$  that are below the minimum wage level. It should be clear here why non-parametric estimation of the conditional probability of sector given the wage is not an option. By assuming a parametric form, I can use the parameters to predict the latent probability of sector given the wage for values at which, in the data, this probability is equal to zero due to the minimum wage policy.

Then, it can be shown that:

$$\pi'_d(m) = \lim_{\epsilon \rightarrow 0^+} \left( \frac{f'(m - \epsilon)}{f'(m + \epsilon)} - \frac{f(m - \epsilon)}{f(m + \epsilon)} \right) \cdot \frac{f'(m + \epsilon)}{f(m + \epsilon)}.$$

Because the RHS of this equation contains only objects of the observed wage distribution, this implies that  $\pi'_d(m)$  is identified. Given that the function  $\Lambda(m)$  is identified, we have:

$$\pi_d^{(0)} = \pi_d(m) - \frac{\Lambda(m)}{\Lambda'(m)} \cdot \pi'_d(m)$$

$$\pi_d^{(1)} = [\pi_d(m) - (1 - \Lambda(m)) \cdot \pi_d^{(0)}] \cdot \Lambda(m)^{-1}.$$

This can be shown using the equation below and its derivative with respect to the wage:

$$\pi_d(w) = \pi_d^{(1)} \Lambda(w) + \pi_d^{(0)} (1 - \Lambda(w)).$$

Given that all terms of the equation above are identified, we have that the function  $\pi_d(w)$  is identified. Inverting the relationship between the observed and latent wage densities, we have:

$$(3) \quad f_0(w) = \begin{cases} \frac{f(w) \cdot c}{\pi_d(w)} & \text{if } w < m \\ f(w) \cdot c & \text{if } w > m. \end{cases}$$

Which implies:

$$c = \left[ \int^m \frac{f(w)}{\pi_d(w)} dw + 1 - F(m) \right]^{-1}.$$

Because the function  $\pi_d(w)$  is already identified and  $F(m)$  is simply the fraction of workers in the observed wage distribution who earn less than or equal to the minimum wage,  $c$  is identified. This implies the identification of the entire latent wage distribution  $f_0(w)$ . Using the latent wage density and the function  $\Lambda(w)$  allows the identification of the latent densities of the formal and informal sectors and, finally, the remaining parameters  $\pi_m^{(1)}$  and  $\pi_u^{(1)}$ .

$$\begin{aligned} f(W(0) = w | S(0) = 1) &= \frac{Pr[S(0)=1|W(0)=w] \cdot f_0(w)}{Pr[S(0)=1]} &= \frac{\Lambda(w) \cdot f_0(w)}{\int \Lambda(u) f_0(u) du} \\ f(W(0) | S(0) = 0) &= \frac{Pr[S(0)=0|W(0)=w] \cdot f_0(w)}{Pr[S(0)=0]} &= \frac{(1 - \Lambda(w)) \cdot f_0(w)}{\int (1 - \Lambda(u)) f_0(u) du} \\ \pi_m^{(1)} &= \frac{Pr[W(1)=m|S(1)=1]}{1 - Pr[W(1)=m|S(1)=1]} \cdot \frac{1 - F_0(m|S(0)=1)}{F_0(m|S(0)=1)} \\ \pi_u^{(1)} &= 1 - \pi_d^{(1)} - \pi_m^{(1)} \\ \pi_m^{(0)} &= 1 - \pi_d^{(0)}. \end{aligned}$$

*Q.E.D.*

It is important to note that the identification result holds if one assumes that  $\pi_m^{(1)}$  and  $\pi_u^{(1)}$  are non-specified functions of the latent wage, as long as  $\pi_d^{(1)}$  remains constant. In this scenario, the parameters recovered above are expectations -  $E(\pi_d^{(1)})$  and  $E(\pi_u^{(1)})$  - over the distribution of workers whose latent wages are below the minimum wage. Formally, the parameters identified are  $\pi_m^{(1)} = Pr[W(1) = m | S(0) = 1, W(0) < m]$  and  $\pi_u^{(1)} = Pr[W(1) =$

$\cdot |S(0) = 1, W(0) < m]$ . Under the maintained assumptions, this probability is the same for all workers regardless of their latent wage. In the case in which workers are heterogeneous in the probability of becoming unemployed, or receiving the minimum wage, with respect to their latent wages, the model recovers the natural extension of this parameter in the presence of such heterogeneity. That is, it recovers the average effect for the population of affected workers. Interestingly, this does not imply that the latent wage distributions obtained under the assumption of constant probabilities will be inconsistent. The assumption of constant probabilities is maintained only to simplify the exposition.<sup>35</sup>

Further, it should be stressed that this proof does not require the wage distribution to peak above the minimum wage. In fact, one can identify the effects of the minimum wage regardless of where in the latent wage distribution the minimum wage happens to be set, as long as the density of wages is greater than zero at the minimum wage,  $\pi_d^{(1)}$  and  $\pi_d^{(0)}$  are constants and either one of them is greater than zero.

**COROLLARY 1 (*Identification of the minimum wage treatment effects*)** Under Assumptions 1, 2, 3 and 4, the effects of the minimum wage on functionals of the joint distribution of sector and wages are identified.

**PROOF:** The identification of treatment effect parameters follows directly from the identification of the joint distribution of observed and latent sector and wages from i.i.d data on  $\{(W_i(1), S_i(1))\}$ . *Q.E.D.*

#### A.2. Identification under Independence between Sector and Wages

In this section, I discuss the identification given the independence between (latent) sector and wages, that is,  $Pr[S(0) = 1 | W(0) = w] = \Lambda \ \forall \ w$ . I maintain Assumptions 1 (continuity), 3 (no spillovers) and 4 (minimum wage effects). Given those assumptions, the aggregate wage density will be given by:

$$(4) \quad f(w) = \begin{cases} \frac{\pi_d f_0(w)}{\pi_m \bar{F}_0^c(m)} & \text{if } w < m \\ \frac{\pi_m \bar{F}_0^c(m)}{f_0^c(w)} & \text{if } w = m \\ \frac{f_0^c(w)}{c} & \text{if } w > m. \end{cases}$$

This is exactly the one-sector version of this model, as proposed by Doyle (2006). This means that at least the aggregate parameters  $\pi_d$ ,  $\pi_m$  and  $\pi_u$  are identified as:

$$\pi_d = \lim_{\epsilon \rightarrow 0} \frac{f(m - \epsilon)}{f(m + \epsilon)}.$$

To identify  $\pi_m$ , one simply needs to verify that:

$$\pi_m = \pi_d \cdot \frac{Pr[W(1) = m]}{Pr[W(1) < m]}.$$

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<sup>35</sup>In the Appendix D I show that this will hold as long as the probability of non-compliance is correctly specified with respect to latent wages – for example, if it is constant. In the Appendix C I discuss how to test if  $\pi_d$  is a function of the wage and how to estimate the model in this case.



Given  $\pi_d$ ,  $F_0(m)$  can be identified by:<sup>36</sup>

$$F_0(m) = \frac{Pr[W(1) < m]}{\pi_d Pr[W(1) > m] + Pr[W(1) < m]}.$$

The relationship between the aggregate data parameters  $\pi_d$  and  $\pi_m$  and the sector-specific model parameters can be derived as:

$$\pi_d = \Lambda \pi_d^{(1)} + (1 - \Lambda) \pi_d^{(0)}$$

$$\pi_m = \Lambda \pi_m^{(1)} + (1 - \Lambda) \pi_m^{(0)}$$

$$\pi_u = \Lambda \pi_u^{(1)}$$

$$\pi_d^{(1)} + \pi_m^{(1)} + \pi_u^{(1)} = 1$$

$$\pi_d^{(0)} + \pi_m^{(0)} = 1.$$

Having recovered the aggregate parameters, the goal is solve for the sector-specific parameters. To do so, one first needs to identify  $\Lambda$ . Note:

$$\Lambda \equiv Pr[S(0) = 1] = Pr[S(0) = 1|W(0) > m] = Pr[S(1) = 1|W(1) > m],$$

where the first equality holds because of the independence between latent sector and wages, and the second holds due to the lack of spillovers on sector probabilities. Interestingly, the identification of the latent size of the formal sector does not rely on anything but independence, the lack of spillovers, and the assumption that  $Pr[W(1) > m|W(0) < m] = 0$ .<sup>37</sup> This means that we can correctly identify the size of the formal sector even if we mis-specify the continuity of the latent distribution of wages or the way in which that the minimum wage affects the lower tail of the wage distribution. Note that, given the aggregate data parameters and  $\Lambda$ , this is a system of five equations and five unknowns. Unfortunately, the system is rank deficient, and hence, an additional equation needs to be added to recover the sector-specific parameters. Relying on the identification of  $\Lambda$ ,  $\pi_u^{(1)}$  is identified by:

$$\pi_u^{(1)} = \frac{\pi_u}{\Lambda} = \frac{1 - \pi_d - \pi_m}{\Lambda}.$$

To recover  $\pi_m^{(1)}$ , it is necessary to consider the density of the formal sector:

$$(5) \quad f(w|S(1) = 1) = \begin{cases} 0 & \text{if } w < m \\ \frac{\pi_m^{(1)} F_0(m)}{c^{(1)}} & \text{if } w = m \\ \frac{f_0(w)}{c^{(1)}} & \text{if } w > m, \end{cases}$$

where  $c^{(1)} = 1 - F_0(m)(1 - \pi_m^{(1)})$  is a scaling factor such that the two densities integrate to one. The key feature of the formal sector that allows for the identification of  $\pi_m^{(1)}$  is that because the density is zero below the minimum wage, the scaling factor on the denominator

<sup>36</sup>See the section on the identification of Doyle's model.

<sup>37</sup> $Pr[W(1) > m|W(0) < m] = 0$  is implied by Assumption 4. When Assumption 4 does not hold, the identification strategy described above will be valid if  $Pr[W(1) > m|W(0) < m] = 0$ . An example of this situation is when the probability of non-compliance is a function of the worker's latent wage. This would invalidate Assumption 4 while preserving the condition  $Pr[W(1) > m|W(0) < m] = 0$ .

is a function of only one unknown parameter (note that  $F_0(m)$  is already identified). Finally, using:

$$Pr[W(1) = m | S(1) = 1] = \pi_m^{(1)} F_0(m) / c^{(1)},$$

it is possible to show that:

$$\pi_m^{(1)} = \frac{Pr[W(1) = m | S(1) = 1]}{1 - Pr[W(1) = m | S(1) = 1]} \cdot \frac{1 - F_0(m)}{F_0(m)}.$$

The RHS of this equation consists only of quantities that are already identified. Given that  $\pi_m^{(1)}$  is identified based on the expression above, we can now return to the system and recover all the other parameters:

$$\pi_m^{(0)} = \frac{\pi_m - \Lambda \pi_m^{(1)}}{1 - \Lambda}.$$

Thus:

$$\pi_d^{(0)} = 1 - \pi_m^{(0)}.$$

Finally:

$$\pi_d^{(1)} = 1 - \pi_m^{(1)} - \pi_u^{(1)}.$$

The latent wage density can be recovered in the same way as in the baseline model, that is, by inverting the relationship and using the fact that  $c$  and  $\pi_d$  were already identified:

$$(6) \quad f_0(w) = \begin{cases} \frac{f(w) \cdot c}{\pi_d} & \text{if } w < m \\ f(w) \cdot c & \text{if } w > m. \end{cases}$$

This implies that we have identified the latent distribution of wages  $f_0(w)$ , the latent size of the formal sector  $\Lambda$  and the parameters  $\pi$  that govern how the minimum wage affects the economy.

Note that estimation in this context is considerably easier than in the baseline model. This is the case because it is not necessary to estimate the derivative of the density of wages at  $m$  to solve for the sector-specific parameters. All objects in the identifying equations – except by the lateral limit of the density of wages at  $m$  – can be estimated by replacing the population object with its respective sample counterpart. I used this plug-in method to estimate the parameters of the model in the empirical application.

## APPENDIX B: ESTIMATION

### B.1. Local Linear Density Estimation

In this section, I describe the local linear approach to density estimation. A standard approach to non-parametrically estimate densities at boundary points is to rely on local linear fitting instead of a local constant. This ensures a better order for the bias term compared to the standard, local constant, kernel density estimator. Thus, following Jones(1993), I estimate the density, for  $w \neq m$ , as the solution to the following minimization problem:

$$\begin{bmatrix} \hat{f}(w) \\ \hat{f}'(w) \end{bmatrix} = \arg \min_{(a_0, a_1)'} \int (f_n(u) - a_0 - a_1(w - u))^2 K\left(\frac{w-u}{h}\right) (\mathbb{I}\{u > m\} \mathbb{I}\{w > m\} + \mathbb{I}\{u < m\} \mathbb{I}\{w < m\}) du,$$

where  $f_n(u) = N^{-1} \sum \delta(u - W_i)$  is the empirical density function,  $K()$  is the kernel function, and  $h$  is the bandwidth.

### APPENDIX C: TESTING

This research design allows us to perform partial tests of the validity of the model's assumptions. This section I describe how these tests can be performed and their limitations.

Assumption 1, the continuity of the latent wage distribution, can be verified by visual inspection of the histogram and the kernel density estimates using different values for the bandwidth. Formally, this condition can be tested by performing a placebo test, that is, by checking whether there are differences between the left and right limits of the density estimates at wage points other than the minimum wage.

Assumption 2 can be tested by comparing the fit of the parametric model with non-parametric estimates. If  $\Lambda(w; \beta)$  is correctly specified, for the true value of the parameters  $\beta_0$ , we have:

$$\int_m^\infty (Pr[S(0) = 1 | W(0) = u] - \Lambda(u; \beta_0))^2 f_0(u) du = 0.$$

While this equation is in terms of latent variables, we can restate it using observables by relying on Assumption 3. Thus, we have:

$$I \equiv \int_m^\infty (Pr[S(1) = 1 | W(1) = u] - \Lambda(u; \beta_0))^2 \mathbb{I}\{u > m\} f(u) du = 0,$$

where  $\beta_0 \equiv \arg \min_\beta E[(S_i(1) - \Lambda(W_i(1); \beta))^2 \mathbb{I}\{W_i(1) > m\}]$ . This condition is in terms of quantities we can observe. Correctness of the specification of the model for  $\Lambda(W(0), \beta)$  implies that  $I = 0$ . This is a standard integrated mean squared error type of condition that can be used for specification testing. The idea behind it is to compare the fit of a parametric model with the fit of a non-parametric model. This type of comparison can be used to identify the proper functional form for the sector and wage relationship. This is relevant because part of the identification relies on extrapolating this conditional mean function to values below the minimum wage. It should be noted, however, that this is, at best, a partial test of the assumption. There are some deviations

from the Assumption 2 which this condition cannot capture. To make this point clear, observe the following condition:

$$\int_{-\infty}^{\infty} (Pr[S(0) = 1|W(0) = u] - \Lambda(u; \beta_0))^2 f_0(u) du = 0.$$

This condition is equivalent to the correctness of the specification of the parametric model for the conditional probability of the latent sector given the wage. The difference between this condition and that the feasible one presented before is that it can detect when the model is incorrectly specified for values below the minimum wage. It is not possible, however, to create a feasible version of this condition. Once we move from latent to observed wages, all information on the conditional probability of latent sector given the wages is lost for values below the minimum. In sum, it is conceivable that the parametric functional form holds for values above the minimum wage but fails to hold for values below it. This part of Assumption 2 is untestable.

It is also possible to test Assumption 4. In Assumption 4, the probabilities that capture the effects of the minimum wage are defined. A restriction imposed by that assumption is that the probabilities of non-compliance ( $\pi_d^{(1)}$  and  $\pi_d^{(0)}$ ) are invariant across workers with different latent wages in the same sector.<sup>38</sup> This is a restrictive assumption, as workers whose latent wage is close to the minimum wage level could be more likely to comply with the policy than workers whose latent wage is far from the minimum. To see why this assumption is testable, one must first examine the second derivative of the observed wage density:

$$(7) \quad f''(w) = \begin{cases} \frac{\pi_n''(w)f(w)}{c} + 2\frac{\pi_n'(w)f'(w)}{f''(w)} + \frac{\pi_d(w)f''(w)}{c} & \text{if } w < m \\ \frac{f''(w)}{c} & \text{if } w > m. \end{cases}$$

If the continuity assumption on the latent wage distribution is strengthened up to the second derivative, that is, if  $\lim_{w \rightarrow m^+} f''(w) = \lim_{w \rightarrow m^-} f''(w)$ , then we have:

$$\lim_{\epsilon \rightarrow 0^+} \left( cf''(m + \epsilon) - \frac{cf''(m - \epsilon) - \pi_d''(m)f_0(m) - 2\pi_d'(m)f_0'(m)}{\pi_d(m)} \right) = 0.$$

Intuitively, we can test Assumption 4 because by examining the second derivative, we have added another equation while the number of parameters remained

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<sup>38</sup>One can see that in aggregate, the likelihood of non-compliance  $\pi_d(w)$  will be a function of latent wages due to changes in the composition of each sector as we move along different wages.

the same. This provides us an overidentification condition that allows us to test the model.

It is easier to see this in the case in which one assumes a linear probability model for  $Pr[S(0) = 1|W(0) = w]$ . In this scenario, it is possible to find a closed-form solution for the model parameters using either the first or the second derivative of the wage density. These different ways of identifying the parameters must yield the same result under Assumption 4. However they do not coincide if the model is misspecified, that is, when the probabilities of non-compliance are functions of latent wages.<sup>39</sup>

#### APPENDIX D: ROBUSTNESS

In this section, I investigate the behavior of the estimands from my identification strategy under different assumptions. I argue that (i) the model is still correctly specified if the unobserved heterogeneity affects either the model parameters or the latent wage distribution, but not both, (ii) the identification strategy is robust to certain forms of unobserved heterogeneity in minimum wage effects, (iii) under Assumption 4, the latent share of the formal sector is identified even when unemployment effects cannot be, and (iv) the aggregate parameters  $\pi_d$ ,  $\pi_m$  and  $\pi_u$  are identified even when Assumption 2 does not hold or when unemployment effects are also present in the informal sector.

To show (i), I reformulate the model and allow its parameters or distributions to be functions of potentially unobservable worker characteristics. I argue that, under some conditions, the assumptions I require for the baseline model to hold will still be valid. (ii) I reformulate the model under a random coefficients framework. I show that under certain restriction on the heterogeneity of the parameters across individuals, the estimands based on the baseline model identify the expectation of the distribution of parameters over the set of workers affected by the minimum wage. To show (iii), I argue that a lack of continuity implies the inconsistency of some, but not all, of the parameters of interest in the model. To show (iv), I argue that identification of Doyle's aggregate parameters does not rely on all four assumptions that I use to identify the parameters of the dual-economy model.

These results reveal an important feature of the baseline model. It is easy and feasible to infer the direction that the parameter estimates will go when

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<sup>39</sup>If one is willing to impose further smoothing conditions on the latent wage distribution, it is possible to identify the model by imposing flexible conditions on the relationship between the parameters and the wages. For example, if one believes that  $\pi_d$  is appropriately described by a quadratic (cubic) function, then one needs to go up to the third (fourth) derivative of the wage density to estimate the model parameters.

some of the model's assumptions are violated. Moreover, as the identification is achieved using “separable” pieces – a model for the conditional distribution of sector given the wages, continuity of latent wage distribution to identify  $\pi_d$ , and so forth – some of the results will still hold when the model is partially misspecified. Taken together, these features should increase the credibility of the results when there are some concerns with the correctness of the model specification. Some pieces of information based on this approach can be useful even in the worst case scenario in which the model is guaranteed to be inconsistent for some parameters.

#### D.1. *Role of Covariates and Unobserved Heterogeneity*

By exploring the different effects of the minimum wage across sectors and the discontinuity of the density of wages around the minimum, one can estimate how the economy responds to this policy. This approach has some similarities to the quasi-experimental Regression Discontinuity Designs (Hahn et. al. (2001), Calonico et. al. (2014), Lee and Lemieux (2010)). Because one of the main advantages of Regression Discontinuity Designs is to provide a way to avoid most of the endogeneity concerns associated with using observational data to infer causality, it is useful to discuss the extent to which these advantages are also present in this method.

Assume that there is a random variable  $Z$  – say, for example, age – that is known to affect individual labor market conditions. One example is when workers with different values of  $Z$  draw from different latent wage distributions. Another way that  $Z$  can affect a worker's labor market conditions is through the model parameters. For example, after the introduction of the minimum wage, younger workers might be more likely to move into the informal sector than older workers, which, in the model, would be represented by a higher  $\pi_d^{(1)}$ . In these cases, is it necessary to estimate the model conditional on  $Z$  for the inferences to be valid?

In the following discussion, I will always assume continuity of the  $Z$ -specific latent wage distribution, an absence of spillovers and a covariate-specific version of the assumption that describes the minimum wage effects. I will also assume the following:

ASSUMPTION 7 *Conditional probability of latent sector given the wage:*

$$Pr[S(0) = 1|W(0) = w] \equiv \int Pr[S(0) = 1|W(0) = w, Z = z]f(z|w)dz = \Lambda(w; \beta).$$

This assumption simply means that whatever the model for the conditional probability of the sector given the wage and  $Z$  is, this model can be aggregated to an unconditional one with parameters  $\beta$ .<sup>40</sup> Two sufficient conditions for the inferences based on the unconditional wage distribution to be valid in the presence of covariates are the following:

Case 1:

ASSUMPTION 8 *Equality of parameters:*  $\pi(z) = \pi \quad \forall z$ .

When the effect of  $Z$  occurs through changes in the latent joint distribution of sector and wages but not through differential responses to the minimum wage, then  $Z$  can be safely ignored when making inferences with regard to the unconditional distribution. The reason for this result is simple. The assumptions above imply that all assumptions of the model for the aggregated data remain valid.

Case 2:

ASSUMPTION 9 *Equality of latent distributions:*  $W(0)|_{Z \sim F} \quad \forall z$ .

This assumption means that  $Pr[W(0) < w|Z = z] = Pr[W(0) < w|Z = z']$  for all  $(z, z')$  and all  $w$ . By restricting the latent wage distribution to be the same for all values of  $z$ , inference based on the unconditional distribution ignoring the covariate will be valid when parameters are allowed to vary over  $Z$ . The parameters  $\pi$  recovered from the aggregate data will be weighted averages of the covariate-specific ones, with correct weights to reflect the share of each group of values of  $Z$  in the population. These, of course, are much stronger conditions than those in Case 1, as the role of covariates is severely limited when they are only allowed to determine wages through the differences in minimum wage effects.

When both the latent wage distribution and the parameters are allowed to vary over  $Z$ , the estimate of  $\pi_d$  can be interpreted as a local effect, as it recovers the likelihood of non-compliance for those with latent wages around the minimum wage.

## D.2. Random Coefficients

Assume that each worker is characterized by a pair  $(W_i, S_i)$  of observed sector and wage, a vector  $(W_i(0), S_i(0), \zeta_i)$ , and a vector  $(\pi(\zeta) \equiv$

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<sup>40</sup>In general, this model will be more complex than the covariate-specific one. A simple, sufficient, but clearly not necessary, condition to guarantee that such a model will exist is when strengthened to  $Pr[S(0) = 1|W(0) = w, Z = z] = \Lambda(w, \beta)$ , that is, covariates only enter the conditional probability of sector given the wage through their effects on wages.

$(\pi_n^{(1)}(\zeta), \pi_m^{(1)}(\zeta), \pi_u^{(1)}(\zeta), \pi_n^{(0)}(\zeta), \pi_m^{(0)}(\zeta))$ , which is now  $\zeta$ -specific. One way to interpret this is that we are treating  $\zeta$  as the worker's unobserved type. For now, I will not assume anything regarding the relationship between the worker's type and his latent wages. Of course, it still holds that  $\pi_m^{(0)}(\zeta) + \pi_n^{(0)}(\zeta) = 1$  for all  $\zeta$  and similarly for the formal sector parameters. In addition to the worker's latent sector and wages, he receives a draw for the model parameters. Here, I also allow this draw to be a function of the latent wage. Thus, for example, workers with higher latent wages can have a higher probability of receiving the minimum wage versus becoming unemployed. This extension captures the idea that (i) minimum wage effects might vary across dimensions of worker's characteristics that are unobservable to the researcher and (ii) minimum wage effects can, and likely will, vary across workers with respect to the distance of their latent productivities from the minimum wage level. The rest of the model remains the same, meaning that I will retain continuity and the absence of spillovers. I will also assume independence between latent sector and wages for simplicity in the rest of this discussion.

This extension adds a great degree of flexibility to the model. It relaxes Assumption 4 in two ways. It allows different workers with similar wages to have different minimum wage response probabilities in an unknown and unspecified way. It also allows workers in the formal sector to have different probabilities of becoming unemployed ( $\pi_u^{(1)}$ ) versus bunching at the minimum wage ( $\pi_m^{(1)}$ ) for different values of the latent wage. This can be achieved without relying on any specified functional form; that is, it is not assumed that these probabilities vary over latent wages in any parametric, continuous or known way.

To analyze the model, we now need to define some new objects. Let:

$$E(\pi_m^{(1)}(\zeta)|w) = \int \pi_m^{(1)}(u) f_{\zeta|w}(u) du.$$

This expression defines the “average probability of bunching at the minimum wage for a formal sector worker with a latent wage equal to  $w$ ” as the integral of this probability for each worker's unobserved type weighted by the proportion of each type for that wage value. We can analogously define similar objects for the other probabilities.

Now, under this new set of assumptions, the relationship between latent and observed densities will be given by:

$$f(w) = \begin{cases} \frac{E(\pi_d(\zeta)|W(0)=w)f_0(w)}{f_0(w)} & \text{if } w < m \\ \frac{\int^m E(\pi_m(\zeta)|\tilde{W}(0)=u)f_0(u)du}{f_0(w)} & \text{if } w = m \\ \frac{f_0(w)}{c} & \text{if } w > m. \end{cases}$$



Now, let us consider the behavior of the estimands defined for the baseline model used under this, more general, version.

$$\pi_d \equiv \lim_{\epsilon \rightarrow 0} \frac{f(m - \epsilon)}{f(m + \epsilon)}.$$

It is easy to see that:

$$\lim_{\epsilon \rightarrow 0} \frac{f(m - \epsilon)}{f(m + \epsilon)} = \Lambda E(\pi_d^{(1)}(\zeta) | W(0) = m, S(0) = 1) + (1 - \Lambda) E(\pi_d^{(0)}(\zeta) | W(0) = m, S(0) = 0).$$

Now, it is also easy to see that this estimand will be equal to the number that we need if  $E(\pi_d^{(1)}(\zeta) | W(0) = w, S(0) = 1) = E(\pi_d^{(1)}(\zeta) | W = w', S(0) = 1)$  and  $E(\pi_d^{(0)}(\zeta) | W(0) = w, S(0) = 0) = E(\pi_d^{(0)}(\zeta) | W(0) = w', S(0) = 0)$  for all pairs  $w$  and  $w'$ . This means that the only restriction on the relationship between the types and latent wages is that the expectation of the non-compliance probabilities,  $E(\pi_d^{(1)}(\zeta) | w)$  and  $E(\pi_d^{(0)}(\zeta) | w)$ , taken with respect to the type distribution, is not a function of the wage.<sup>41</sup>

Assuming that this condition holds, we have that our baseline estimand  $\lim_{\epsilon \rightarrow 0} \frac{f(m - \epsilon)}{f(m + \epsilon)}$  identifies the expected value of  $\pi_d$  over the population of affected individuals. That is:

$$\lim_{\epsilon \rightarrow 0} \frac{f(m - \epsilon)}{f(m + \epsilon)} = Pr[W(1) = W(0) | W(0) < m].$$

Regarding the estimand of  $\pi_m$ :

$$\pi_m \equiv \pi_d \frac{Pr[W_i = m]}{Pr[W_i < m]}.$$

It can be shown that:

$$\pi_d \frac{Pr[W_i = m]}{Pr[W_i < m]} = \frac{\int^m E(\pi_m(\zeta) | W(0) = u) f_0(u) du}{\int^m f_0(u) du} = Pr[W(1) = m | W(0) < m],$$

which means that  $\pi_m$  equals to the expected value of the probability of bunching over the population of affected workers. The intuition for this result is that

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<sup>41</sup>This does not mean that the model is unidentified if this condition fails to hold. It means that in this case, we would need to rely on the derivatives of the wage density to identify the slope of the relationship between expected minimum wage probabilities and latent wages. This can be achieved in the same way as discussed in Appendix C.

the estimand of  $\pi_m$  comes from the point mass at the minimum wage level, which is obtained by integrating the probability of bunching at the minimum wage level for all workers whose latent wages are below the minimum wage level. Thus, irrespective of what functional form exists between the latent wage and the probability of receiving the minimum wage, this form reveals itself in the data in the form of the proportion of workers at the minimum wage level. The mass of wages at the minimum wage level has already “integrated out” the unobserved heterogeneity. This allows us to consider estimating  $Pr[W(1) = m | W(0) < m]$  without completely describing the shape of  $\pi_m^{(1)}(\zeta)$  as a function of  $W(0)$ . The term  $Pr[W(1) = m | W(0) < m, S(0) = 1]$  coincides with the parameter  $\pi_m^{(1)}$  as defined in the baseline model when  $\pi_m^{(1)}$  is not a function of the latent wage. When  $\pi_m^{(1)}$  is indeed a function of the latent wage (through unobserved types, for example), we can bypass the task of modeling this function and directly identify the aggregate component  $Pr[W(1) = m | W(0) < m, S(0) = 1]$ . Similar calculations show that the same is the case with respect to the estimand of unemployment probability. That is,

$$1 - \pi_d - \pi_m = Pr[W(1) = . | W(0) < m].$$

### D.3. *Lack of Continuity*

In the following discussion, I will assume independence between latent sector and wages. Now suppose that  $\pi_d$  is not identified. This could be the case for two reasons. The first case is when latent wage distribution is not continuous. In this case, the estimate of  $\pi_d$  actually identifies  $\pi_d \kappa$ , where  $\kappa$  is the (unknown) size of the discontinuity of the latent wage around the minimum wage. It is clear that as long as  $\kappa = 1$ , the estimate of  $\pi_d$  will be consistent. The second case is when spillovers are misspecified. For example, if one incorrectly assumes that spillovers are absent, when, in fact, they are present. If spillovers reduce the density of wages just above the minimum wage,<sup>42</sup>  $\pi_d$  is misspecified because the density of sector wage observed just above the minimum wage is not the correct quantity to scale the density below to measure the extent of non-compliance.

If spillovers are assumed to be weakly positive, meaning that workers above the minimum do not suffer wage cuts following the policy – and, in addition, Assumption 4 holds – then  $\pi_d$  estimated when ignoring spillovers is an upper bound of the likelihood of non-compliance. This will also provide an upper bound for  $\pi_m$  and a corresponding lower bound for  $\pi_u$ .

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<sup>42</sup>This will be the case if one assumes that spillovers are weakly positive.

If the latent wage distribution presents a discontinuity at the minimum wage level, then the ratio of the left and right limits of the density will identify  $\pi_d \kappa$ . That is, the probability of non-compliance will be scaled by the discontinuity in the latent wage density at  $m$ . Thus, if the discontinuity is of moderate size, the implied change in the estimated of  $\pi_d$  will be of a small order.<sup>43</sup> Thus, sizable distortions will be present only if the latent wage distribution presents large discontinuities at the minimum wage level.

It should also be stressed that under independence between sector and wages, the latent share of the formal sector – which is perhaps one of the most relevant parameters of the model – is still identified regardless of misspecifying how the minimum wage affects the lower part of the wage distribution or a lack of continuity in the latent wage distribution.<sup>44</sup>

#### D.4. Aggregate Parameters

Doyle (2006) and Meyer and Wise (1983) define what I call “aggregated data” probabilities  $\pi_d$ ,  $\pi_m$  and  $\pi_u$ . I call them aggregated because they are a weighted average of the corresponding sector-specific likelihood of non-compliance, bunching and becoming unemployed. Because their goal is to compute aggregate data parameters, they do not need to have a correctly specified form for the conditional probability of the sector given the wage.

The identification of the simplified version of the model here uses Doyle’s estimate as a first step. Then, the weights of the sector-specific probabilities are estimated, and finally, one can solve for the sector-specific parameters. This is a worthwhile exercise because, as I have shown above, a broader set of counterfactuals, such as labor tax and the size of the formal sector, analyses can be performed with sector-specific parameters. Moreover,  $\pi_d^{(1)}$  is a parameter with more economic meaning than  $\pi_d$  itself.

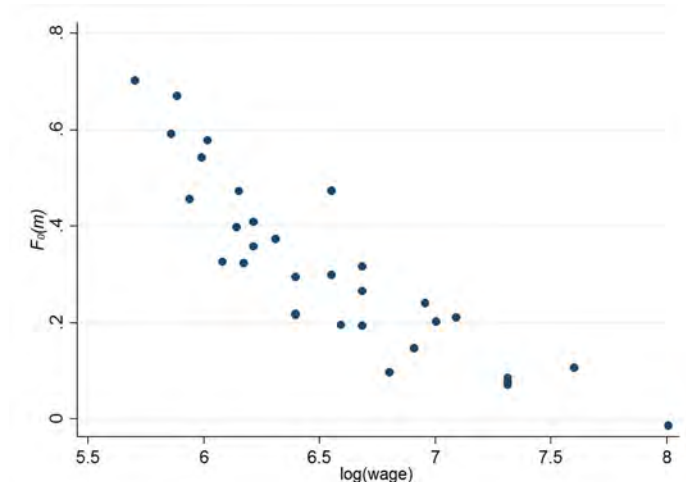
The misspecification of either sector-specific assumptions or the form of the joint distribution of sector and wages has different consequences for the aggregate parameters when compared to the sector-specific parameters. Two cases can illustrate this: If either (a) unemployment effects on the informal sector are present, which is ruled out by Assumption 4, or (b) the model for the conditional probability of the sector given the wages is incorrectly specified,

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<sup>43</sup>For example, if the left limit of the latent density of wages is 10% smaller than the right limit and  $\pi_d$  is equal to 0.2, then the estimated probability limit of the estimator of  $\pi_d$  based on this identification strategy will be 0.18, only 2 percentage points smaller than the true value.

<sup>44</sup>The only additional assumption needed for that identifying this parameter is a lack of spillovers on sector probabilities. See sections A.2 and E.4.

FIGURE 10.— Median wages and the minimum wage “bite”



then the sector-specific parameter estimators will be inconsistent whereas the aggregate parameters will not. It is straightforward to see this because neither Doyle or Meyer and Wise’s approaches rely on either (i) the validity of Assumption 2, or (ii) the implication of Assumption 4 that for the informal sector parameters it is true that  $\pi_d^{(0)} + \pi_m^{(0)} = 1$ .

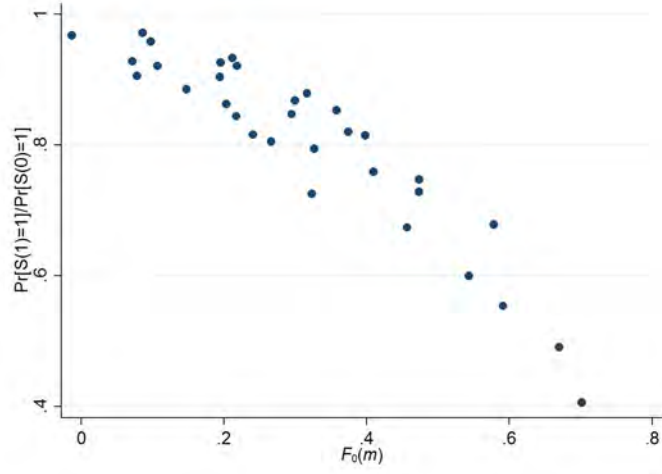
#### APPENDIX E: EMPIRICAL APPLICATION – FURTHER RESULTS

In this section, I present various empirical results that complement the main findings of the paper. First, I extend the sub-group analysis of section 5.4. I also estimate the distributional effects of the minimum wage. In addition, I perform other various robustness exercises.

##### E.1. *Analysis by sub-groups*

Figure 7 shows that the estimates of the disemployment effect of the policy seem to correlate with the actual fraction of workers that are without jobs across different sub-groups. In other words, it seems that the disemployment effects, as captured by the “missing workers” in the wage distribution, may indeed be capturing a real disemployment effect of the policy. In this section, I extend this analysis by also looking at whether the effects for groups of different characteristics follow the patterns that one would intuitively expect. In particular, I investigate two main questions: Do I obtain larger estimates of the mass of affected workers,  $F_0(m)$ , for groups of workers that we should expect this probability to be larger – that is, for low wage groups? Also, do I

FIGURE 11.— Minimum wage “bite” and the estimated effect on formality



obtain larger effects of the minimum wage on the size of the formal sector for the groups of workers for which the policy is more binding – that is, groups in which  $F_0(m)$  is larger?

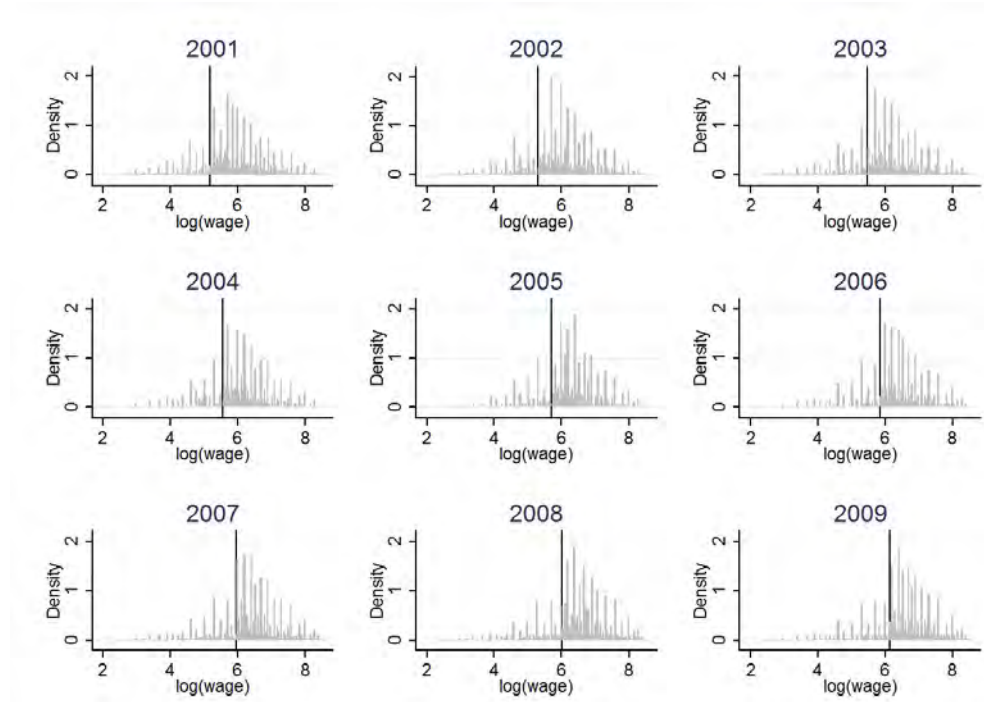
Figure 10 shows the estimated values of the minimum wage “bite” and the median of log-wages for each group. Regarding this graph, we see that groups of workers with wage distributions shifted to the right (that is, with higher medians) seem to be less likely affected by the minimum wage policy. This “sanity” check shows that the estimates of  $F_0(m)$  across different groups follow the pattern one would naturally expect: The estimated values of fraction of workers affected by the minimum wage policy,  $F_0(m)$ , decreases as we look at groups that have wage distributions with higher medians.

In Figure 11, I perform a similar exercise. I plot the estimates of the fraction of workers affected by the minimum wage and the estimated effect of the policy on the size of the formal sector. The results indicated that the estimated effect of the policy on the size of the formal sector seems to be larger for groups in which the minimum wage “bite” is more pronounced.

### E.2. *Heaping*

In my empirical exercise, I assume away measurement error in wages. The data, however, do not support this assumption: There is a tendency of wages to pile up in round numbers, multiples of 50, such as 100, 250, etc. Figure 12 displays this feature of the data. I plot the estimates of the density of (log) wages using an undersmoothed histogram. We observe several spikes in the

FIGURE 12.— Undersmoothed Histogram



Note: Histogram of the logarithm of wages, for wages not equal to  $m$ .

graphs, which coincide with the (logarithm) of round numbers.<sup>45</sup> This could be a concern since the identification of the non-compliance probability is obtained by comparing the density of wages just above versus just below the minimum wage.

When we compare the effects of the minimum wage across the different years, we expect that sometimes we would overestimate the non-compliance probability (because the minimum wage is set slightly above a heaping point), and other times we would underestimate the non-compliance probability (because the minimum wage was set slightly below a heaping point). It is reassuring to note that although the point estimates do vary from year to year, all the qualitative economic implications seem to be similar regardless of whether the minimum wage is set right below, at, or above a heaping point.

In my baseline specification, I choose not to engage in any form of correc-

<sup>45</sup>The logarithm of the minimum wage level is represented by a vertical line. I compute the density for wages not equal to the minimum wage in this graph since the height of the histogram at  $m$  is so high that it becomes hard to visualize the density of the rest of the data.

TABLE IX  
PARAMETER ESTIMATES – ROUNDING WAGES TO THE NEAREST HEAPING POINT

|               | 2001                | 2002                | 2003                | 2004                | 2005                | 2006                | 2007                | 2008                | 2009                |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\pi_d$       | 0.271***<br>(0.012) | 0.189***<br>(0.008) | 0.256***<br>(0.006) | 0.291***<br>(0.008) | 0.172***<br>(0.005) | 0.171***<br>(0.005) | 0.123***<br>(0.003) | 0.208***<br>(0.005) | 0.126***<br>(0.003) |
| $\pi_m$       | 0.344***<br>(0.015) | 0.310***<br>(0.013) | 0.359***<br>(0.009) | 0.367***<br>(0.011) | 0.335***<br>(0.010) | 0.285***<br>(0.007) | 0.193***<br>(0.004) | 0.331***<br>(0.007) | 0.217***<br>(0.004) |
| $F_{0(m)}$    | 0.201***<br>(0.007) | 0.288***<br>(0.008) | 0.266***<br>(0.004) | 0.247***<br>(0.005) | 0.338***<br>(0.006) | 0.380***<br>(0.005) | 0.424***<br>(0.004) | 0.326***<br>(0.004) | 0.424***<br>(0.004) |
| $\pi_d^{(f)}$ | 0.245***<br>(0.024) | 0.156***<br>(0.019) | 0.237***<br>(0.013) | 0.288***<br>(0.015) | 0.149***<br>(0.011) | 0.104***<br>(0.009) | 0.003<br>(0.004)    | 0.159***<br>(0.008) | 0.030***<br>(0.005) |
| $\pi_m^{(f)}$ | 0.257***<br>(0.011) | 0.201***<br>(0.009) | 0.277***<br>(0.007) | 0.283***<br>(0.009) | 0.247***<br>(0.008) | 0.232***<br>(0.006) | 0.166***<br>(0.005) | 0.286***<br>(0.006) | 0.194***<br>(0.004) |
| $\pi_d^{(h)}$ | 0.361***<br>(0.032) | 0.304***<br>(0.033) | 0.331***<br>(0.023) | 0.302***<br>(0.024) | 0.277***<br>(0.025) | 0.473***<br>(0.019) | 0.684***<br>(0.009) | 0.451***<br>(0.016) | 0.656***<br>(0.010) |
| $\pi_m^{(h)}$ | 0.639***<br>(0.032) | 0.696***<br>(0.033) | 0.669***<br>(0.023) | 0.698***<br>(0.024) | 0.723***<br>(0.025) | 0.527***<br>(0.019) | 0.316***<br>(0.009) | 0.549***<br>(0.016) | 0.344***<br>(0.010) |
| $\pi_u$       | 0.384***<br>(0.026) | 0.502***<br>(0.021) | 0.385***<br>(0.014) | 0.343***<br>(0.019) | 0.493***<br>(0.014) | 0.544***<br>(0.011) | 0.684***<br>(0.006) | 0.461***<br>(0.011) | 0.657***<br>(0.007) |
| $\lambda$     | 0.772***<br>(0.002) | 0.781***<br>(0.002) | 0.792***<br>(0.002) | 0.798***<br>(0.002) | 0.816***<br>(0.002) | 0.819***<br>(0.002) | 0.823***<br>(0.002) | 0.832***<br>(0.002) | 0.846***<br>(0.002) |
| $m$           | 180                 | 200                 | 240                 | 260                 | 300                 | 350                 | 380                 | 415                 | 465                 |
| $N$           | 55,487              | 58,223              | 58,265              | 62,576              | 65,745              | 68,193              | 68,315              | 71,051              | 71,397              |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

tion, as to keep the identification and estimation as clean as possible. Now, to investigate the robustness of my estimates to this particular form of measurement error, I perform a different exercise: I first round all wages to the nearest multiple of 25, then I estimate the model parameters.

Heaping is a threat to the identification because this form of measurement error may induce the density estimates to be artificially larger on one side when compared to the other side of the threshold. Rounding all wages to the nearest heaping point minimizes this type of concern, since now the distribution has only mass at heaping points, so the heights are comparable again at both sides of the minimum wage. That is, there is no risk to compare the height of the density at a heaping point in one side of the minimum wage with the height of the density in a non-heaping point in the other.<sup>46</sup>

Table IX displays the results obtained by this exercise. I rounded all wages to the nearest multiple of 25, naturally ensuring no wage crosses or reaches the minimum wage level in this process. The most important coefficient in this exercise is  $\pi_d$ , since this parameter estimate is obtained from the ratio of the left and the right limits of the density around the minimum wage. The results suggest that the estimates of  $\pi_d$  are not quite sensitive of whether I use the raw data or whether I deal with the heaping by rounding all wages to their

<sup>46</sup>Two other similar strategies could be performed: One is to estimate the model ignoring all data at heaping points. Another is to ignore all the data except at the heaping points. By rounding all data to the heaping points I guarantee that the sample sizes are comparable to the ones in Table IV. Also, since informal workers are slightly more likely to report a round number, I do not engage in differential sample selection when I choose to work with either one of the sub-samples.

TABLE X  
PARAMETER ESTIMATES ACROSS DIFFERENT CHOICES OF BANDWIDTH  
YEAR 2009

|               | I                   | II                  | III                 | IV                  | V                   | VI                  |
|---------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\pi_d$       | 0.143***<br>(0.005) | 0.136***<br>(0.004) | 0.133***<br>(0.004) | 0.128***<br>(0.003) | 0.121***<br>(0.003) | 0.116***<br>(0.002) |
| $\pi_m$       | 0.246***<br>(0.009) | 0.234***<br>(0.007) | 0.228***<br>(0.007) | 0.220***<br>(0.006) | 0.208***<br>(0.005) | 0.199***<br>(0.004) |
| $F_0(m)$      | 0.394***<br>(0.008) | 0.405***<br>(0.006) | 0.412***<br>(0.007) | 0.420***<br>(0.006) | 0.434***<br>(0.005) | 0.445***<br>(0.004) |
| $\pi_d^{(1)}$ | 0.059***<br>(0.009) | 0.048***<br>(0.007) | 0.042***<br>(0.007) | 0.034***<br>(0.006) | 0.023***<br>(0.005) | 0.014***<br>(0.005) |
| $\pi_m^{(1)}$ | 0.220***<br>(0.008) | 0.209***<br>(0.006) | 0.204***<br>(0.006) | 0.197***<br>(0.005) | 0.186***<br>(0.004) | 0.178***<br>(0.004) |
| $\pi_d^{(0)}$ | 0.610***<br>(0.017) | 0.628***<br>(0.014) | 0.638***<br>(0.014) | 0.651***<br>(0.013) | 0.669***<br>(0.011) | 0.683***<br>(0.010) |
| $\pi_m^{(0)}$ | 0.390***<br>(0.017) | 0.372***<br>(0.014) | 0.362***<br>(0.014) | 0.349***<br>(0.013) | 0.331***<br>(0.011) | 0.317***<br>(0.010) |
| $\pi_u$       | 0.611***<br>(0.014) | 0.630***<br>(0.010) | 0.640***<br>(0.011) | 0.652***<br>(0.009) | 0.670***<br>(0.007) | 0.685***<br>(0.006) |
| $\Lambda$     | 0.847***<br>(0.002) | 0.847***<br>(0.002) | 0.847***<br>(0.002) | 0.847***<br>(0.002) | 0.847***<br>(0.002) | 0.847***<br>(0.002) |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

nearest heaping point. The same qualitative conclusions are obtained in both exercises: the disemployment effects are large, the transition probabilities are small, and the size of the formal sector is decreased by around 10%.<sup>47</sup>

### E.3. Bandwidth Selection

Table X displays the estimates of the parameters using different choices for different values of the bandwidth, using data for the year of 2009. In column one I use half of the bandwidth I employ in my baseline specifications – that is, four times the Silverman’s rule of thumb –. In the other columns, I progressively increase the value of the (multiplier for the) bandwidth. In column VI I use 9 times the Silverman’s rule of thumb.

The bandwidth considered in all empirical specifications is eight times the Silverman’s rule of thumb at each side of the minimum wage, which corresponds to column V in Table X. We can see that increasing or decreasing the bandwidth leads to differences in the point estimates. The qualitative implications of the results, however, seem robust to the bandwidth choice. Note, also, that

<sup>47</sup>Note that point estimates of the latent size of the formal sector are invariant to the estimates of  $\pi_d$ . This is expected since the identification of  $Pr[S(0) = 1]$  comes from the conditional probability of formality for wages above the minimum.



the estimate of the latent size of the formal sector is *invariant* to the choice of the bandwidth.

I also attempt to estimate the non-compliance probability using a global approximation to the density function, as opposed to the local approximation employed using the non-parametric density estimator. The estimates of  $\pi_d$  I obtained assuming log-normality for the density of latent wages (available on request), are similar to the non-parametric estimators.<sup>48</sup>

#### E.4. *Robust Estimates of the Effects of the Minimum Wage on the Size of the Informal Sector*

This paper develops a model that allows one to estimate the effects of the minimum wage on a broad range of policy-relevant outcomes. The model captures a channel through which workers move from the formal sector to the informal sector in response to the minimum wage policy. The effects of the minimum wage on the size of the informal sector have important policy implications. For example, this parameter is key to understand the effects of the minimum wage on the government budget.

Under the assumptions of the model, this parameter, the effect of the minimum wage on the size of the informal sector, can be consistently estimated. This section discusses the extent to which those estimates are robust to deviations from these assumptions, particularly the absence of spillovers. This will be achieved using Assumption 5, the independence between latent sector and wages. The object of interest is  $\frac{Pr[S(1)=1]}{Pr[S(0)=1]}$ , that is, the ratio of the size of the formal sector in the presence of the minimum wage versus its size in the absence of the minimum wage. The numerator of this fraction can be directly estimated from the data. The counterfactual object is the latent size of the informal sector. Under independence between latent sector and wages, we have:

$$Pr[S(0) = 1] = Pr[S(0) = 1|W(0) > m] = Pr[S(1) = 1|W(1) > m].$$

This expression uses the size of the formal sector above the minimum wage as the estimate of the latent size of the formal sector in the absence of the policy. Interestingly, this result does not rely on the continuity of the latent wage distribution or the correctness of the specification of the minimum wage effects on the bottom part of the wage distribution. It relies on the independence, the

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<sup>48</sup>The results from Table X refer only to the year 2009. I also estimated the model pooling data from all years (normalizing wages around the minimum wage level). In this exercise, my largest point estimate for  $\pi_d$  was 23.1%, whereas my smallest estimate was 19.6%. These estimates are close to the baseline pooled estimate of  $\pi_d$ , which I estimated at 20% in my preferred specification. These results are available on request.

TABLE XI

SPILLOVER ROBUST ESTIMATES OF THE LATENT THE SIZE OF THE FORMAL SECTOR

| Year    | 2001                | 2002                | 2003                | 2004                | 2005                | 2006                | 2007                | 2008                | 2009                |
|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $k = 2$ | 0.867***<br>(0.002) | 0.853***<br>(0.003) | 0.865***<br>(0.003) | 0.847***<br>(0.003) | 0.860***<br>(0.003) | 0.858***<br>(0.003) | 0.879***<br>(0.002) | 0.873***<br>(0.002) | 0.883***<br>(0.002) |
| $k = 3$ | 0.853***<br>(0.003) | 0.844***<br>(0.004) | 0.849***<br>(0.004) | 0.845***<br>(0.003) | 0.866***<br>(0.004) | 0.850***<br>(0.004) | 0.872***<br>(0.004) | 0.873***<br>(0.003) | 0.878***<br>(0.003) |

*Note:* Standard errors in parentheses. Minimum wage spillovers are assumed to vanish above  $km$ . See text for details.

lack of spillovers assumption, and  $Pr[W(1) > m|W(0) < m] = 0$ . To evaluate the robustness of this estimate to departures from the absence of spillovers, one simply needs to specify a limit at which the spillovers should vanish. In the most extreme version of this assumption, the effects of the minimum wage vanish at the minimum wage level. However, one can specify that the minimum wage effects vanish at twice, or in general,  $k$ -times the minimum wage level. This lead to the following identification equation:

$$Pr[S(0) = 1] = Pr[S(0) = 1|W(0) > km] = Pr[S(1) = 1|W(1) > km],$$

show that  $k$  is a number greater than or equal to one. The first equality follows from independence between sector and wages, whereas the second follows from the absence of spillovers at points higher than  $km$ .<sup>49</sup>

Table XI reports the effects of the minimum wage on the size of the formal sector based on different assumptions concerning where spillovers should vanish. The baseline estimates are approximately 9%. The estimates robust to spillovers find an effect of around 12 to 16%. The point estimates are significantly different. The qualitative conclusions, however, remain similar. The minimum wage has a sizable impact on the size of the formal sector. This section shows that those effects should be further magnified if spillovers are indeed present. These results are based on the minimal assumptions of independence and lack of spillovers in the upper part of the wage distribution. They are robust to limited spillovers, lack of continuity in the latent wage distribution and misspecification of the minimum wage effects on the lower part of the wage distribution.

<sup>49</sup>It is interesting to note that one can allow for wage spillovers above this threshold. The only restriction that needs to be imposed for this identification to be work is the absence of spillovers on sector probabilities. That is, the conditions  $S(1) = S(0)$  if  $W(0) > km$ ,  $Pr[W(1) > km|W(0) < km] = 0$ , and  $Pr[W(1) < km|W(0) > km] = 0$  are sufficient for this identification strategy to work.

TABLE XII  
DISTRIBUTIONAL EFFECTS OF THE MINIMUM WAGE

|                                     | 2001                 | 2002                 | 2003                 | 2004                 | 2005                 | 2006                 | 2007                 | 2008                 | 2009                 |
|-------------------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| $E[\log(w)]$                        |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| Observed                            | 5.994***<br>(0.004)  | 6.054***<br>(0.003)  | 6.152***<br>(0.004)  | 6.220***<br>(0.003)  | 6.322***<br>(0.003)  | 6.400***<br>(0.003)  | 6.489***<br>(0.003)  | 6.578***<br>(0.003)  | 6.658***<br>(0.003)  |
| Latent                              | 5.793***<br>(0.009)  | 5.874***<br>(0.014)  | 5.926***<br>(0.008)  | 6.020***<br>(0.009)  | 6.090***<br>(0.012)  | 6.093***<br>(0.010)  | 6.097***<br>(0.010)  | 6.363***<br>(0.008)  | 6.310***<br>(0.008)  |
| Minimum wage effect                 | 0.201***<br>(0.008)  | 0.180***<br>(0.012)  | 0.225***<br>(0.007)  | 0.200***<br>(0.008)  | 0.231***<br>(0.011)  | 0.307***<br>(0.009)  | 0.392***<br>(0.009)  | 0.215***<br>(0.007)  | 0.348***<br>(0.007)  |
| $Sd[\log(w)]$                       |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| Observed                            | 0.769***<br>(0.004)  | 0.774***<br>(0.004)  | 0.753***<br>(0.004)  | 0.737***<br>(0.004)  | 0.719***<br>(0.004)  | 0.699***<br>(0.003)  | 0.693***<br>(0.004)  | 0.684***<br>(0.003)  | 0.661***<br>(0.003)  |
| Latent                              | 0.916***<br>(0.006)  | 0.929***<br>(0.006)  | 0.916***<br>(0.006)  | 0.885***<br>(0.006)  | 0.901***<br>(0.007)  | 0.881***<br>(0.006)  | 0.897***<br>(0.009)  | 0.846***<br>(0.006)  | 0.851***<br>(0.006)  |
| Minimum wage effect                 | -0.147***<br>(0.004) | -0.155***<br>(0.005) | -0.163***<br>(0.003) | -0.149***<br>(0.004) | -0.182***<br>(0.005) | -0.182***<br>(0.004) | -0.204***<br>(0.006) | -0.162***<br>(0.004) | -0.190***<br>(0.004) |
| $q^{80}(\log(w)) - q^{20}(\log(w))$ |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| Observed                            | 1.157***<br>(0.003)  | 1.112***<br>(0.020)  | 1.124***<br>(0.012)  | 1.015***<br>(0.016)  | 1.099***<br>(0.000)  | 1.050***<br>(0.000)  | 0.916***<br>(0.000)  | 1.062***<br>(0.000)  | 0.948***<br>(0.002)  |
| Latent                              | 1.419***<br>(0.022)  | 1.476***<br>(0.045)  | 1.267***<br>(0.019)  | 1.386***<br>(0.000)  | 1.386***<br>(0.012)  | 1.386***<br>(0.021)  | 1.447***<br>(0.015)  | 1.204***<br>(0.010)  | 1.204***<br>(0.012)  |
| Minimum wage effect                 | -0.262***<br>(0.022) | -0.364***<br>(0.052) | -0.143***<br>(0.022) | -0.372***<br>(0.016) | -0.288***<br>(0.012) | -0.336***<br>(0.021) | -0.531***<br>(0.015) | -0.142***<br>(0.010) | -0.256***<br>(0.012) |
| Gini                                |                      |                      |                      |                      |                      |                      |                      |                      |                      |
| Observed                            | 0.069***<br>(0.000)  | 0.069***<br>(0.000)  | 0.065***<br>(0.000)  | 0.063***<br>(0.000)  | 0.061***<br>(0.000)  | 0.058***<br>(0.000)  | 0.056***<br>(0.000)  | 0.055***<br>(0.000)  | 0.052***<br>(0.000)  |
| Latent                              | 0.087***<br>(0.001)  | 0.087***<br>(0.001)  | 0.085***<br>(0.001)  | 0.081***<br>(0.001)  | 0.081***<br>(0.001)  | 0.080***<br>(0.001)  | 0.080***<br>(0.001)  | 0.072***<br>(0.001)  | 0.074***<br>(0.000)  |
| Minimum wage effect                 | -0.018***<br>(0.001) | -0.018***<br>(0.001) | -0.019***<br>(0.000) | -0.017***<br>(0.001) | -0.021***<br>(0.001) | -0.022***<br>(0.000) | -0.024***<br>(0.000) | -0.017***<br>(0.000) | -0.022***<br>(0.000) |
| N                                   | 55,487               | 58,223               | 58,265               | 62,576               | 65,745               | 68,193               | 68,315               | 71,051               | 71,397               |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications.

### E.5. The Distributional Effects of the Minimum Wage

Table XII shows how the minimum wage affects the shape of the (log-) wage distribution. Here, I compute the effects of the minimum wage on the usual measures of wage inequality, such as the standard deviation of log wages and the Gini coefficient. The estimates show that the minimum wage has a positive impact on average wages (conditional on employment). The maximum difference is .39 log points in 2007, and the minimum is .18 in 2002. The minimum wage also reduces wage inequality, as measured by differences in quantiles, the standard deviation, or the Gini coefficient. These estimates indicate the trade-off faced by policy makers when choosing the minimum wage level. On the one hand, there is a gain in terms of reducing wage inequality and increasing average wages. On the other hand, workers tend to have more difficulty finding jobs.

#### E.5.1. Addressing the Disemployment Effects

By comparing the estimates of the observed and the latent wage distribution, I can evaluate the effects that the minimum wage policy has on the distribution of wages. The results of this exercise, shown in Table XII, suggest that the

minimum wage induces an increase in the expected value of (log) wages and a decrease in the inequality of (log) wages. However, the comparison performed in Table XII involves two populations that are distinct: the population of workers that would be employed in the absence of the minimum wage and the population of workers that are employed in the presence of the policy. In other words, the results on Table XII compare the inequality of the latent and observed distribution of wages *conditional on working* in each scenario.

As a result, however, this exercise ignores the effect that the minimum wage policy has on the probability of employment. Thus, perhaps a more important exercise would be to estimate the effects of the minimum wage on wage inequality taking into account the effects that the policy have on the probability of employment.<sup>50</sup>

To do that, I will make the strong assumption that in the event that the worker becomes unemployed his earnings are zero. That is, I will assume away any form of earnings in home production, self-employment, or becoming an entrepreneur. This, of course, would maximize the possible negative effects of the policy, so it should be thought as a lower bound on the positive effects of the policy (if any), or an upper bound of the negative effects of the policy.

Table XIII displays the estimated effects of the minimum wage on expected wages and on the variance of wages, inputting zero wages for those that become unemployed by the policy. The estimates in Table XIII are computed using a plug-in estimator from the closed form expression for the relationship between latent wages and the observed wages. For the case of the variances, I take advantage of the law of total variance to relate the variance of wages in the latent distribution to the variance of observed wages inputting zeros for the unemployed.

The effect of the policy on the variance of wages is ambiguous when the unemployed workers are taken into account: In one hand, the minimum wage increases the variances by pushing some workers to the lower bound of the support of the distribution, which contributes to an increase in the variance. In the other hand, the minimum wage increases the wages of low-wage workers towards the median, which has the opposite effect of reducing the variance. Depending on the model parameters, the net effect of the minimum wage on the variance of wages can be either positive or negative. For the case of the mean, we have a similar ambiguity: The minimum wage increases the wages of a fraction of the workers, however, it reduces the wages of others. The net effect of the policy on the expected value of wages will depend on the relative strength of these forces.

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<sup>50</sup>I thank a referee for suggesting this exercise.

TABLE XIII

## DISTRIBUTIONAL EFFECTS OF THE MINIMUM WAGE

Effects on the population of workers that would be employed in the absence of the policy

|                       | 2001                   | 2002                   | 2003                   | 2004                   | 2005                   | 2006                      | 2007                   | 2008                     | 2009                      |
|-----------------------|------------------------|------------------------|------------------------|------------------------|------------------------|---------------------------|------------------------|--------------------------|---------------------------|
| $E[W]$                |                        |                        |                        |                        |                        |                           |                        |                          |                           |
| Latent                | 516.621***<br>(4.504)  | 563.241***<br>(6.132)  | 583.597***<br>(4.721)  | 624.209***<br>(4.667)  | 675.151***<br>(5.392)  | 672.150***<br>(5.891)     | 681.628***<br>(6.132)  | 843.897***<br>(6.480)    | 818.473***<br>(8.414)     |
| Observed <sup>†</sup> | 496.497***<br>(4.845)  | 545.341***<br>(7.013)  | 555.477***<br>(4.883)  | 599.410***<br>(5.114)  | 643.146***<br>(6.649)  | 621.171***<br>(6.722)     | 601.051***<br>(6.311)  | 787.138***<br>(7.471)    | 728.458***<br>(9.010)     |
| Minimum wage effect   | -0.039***<br>(0.002)   | -0.032***<br>(0.003)   | -0.048***<br>(0.002)   | -0.040***<br>(0.002)   | -0.047***<br>(0.003)   | -0.076***<br>(0.003)      | -0.118***<br>(0.003)   | -0.067***<br>(0.003)     | -0.110***<br>(0.003)      |
| $Var[W]^{1/2}$        |                        |                        |                        |                        |                        |                           |                        |                          |                           |
| Latent                | 769.797***<br>(26.379) | 822.104***<br>(24.133) | 813.678***<br>(19.088) | 930.971***<br>(58.206) | 917.649***<br>(18.646) | 1,013.497***<br>(114.509) | 965.304***<br>(22.324) | 1,153.066***<br>(89.064) | 1,955.520***<br>(612.338) |
| Observed <sup>†</sup> | 769.451***<br>(25.758) | 817.923***<br>(23.610) | 813.639***<br>(18.401) | 930.502***<br>(56.911) | 913.851***<br>(18.033) | 1,016.883***<br>(109.895) | 978.276***<br>(20.960) | 1,150.864***<br>(85.297) | 1,933.712***<br>(588.493) |
| Minimum wage effect   | -0.000<br>(0.001)      | -0.005***<br>(0.001)   | -0.000<br>(0.001)      | -0.001<br>(0.001)      | -0.004***<br>(0.001)   | 0.003<br>(0.005)          | 0.013***<br>(0.002)    | -0.002<br>(0.003)        | -0.011<br>(0.013)         |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors (in parentheses) computed by 100 bootstrap replications. † Zero wages are imputed for workers disemployed by the policy. See text for the details.

Regarding Table XIII, we see that the minimum wage has a small but negative effect on the expected value of wages, meaning that the unemployment effect dominates the wage increases *under the strong assumption of zero earnings for the disemployed workers*. The estimated effect, displayed in terms of a percentage change from the baseline level, is of a small magnitude. As a result, if the assumption of zero earnings for workers disemployed by the policy is relaxed, even to a small extent, to allow for earnings derived from home production, the sign of the effect may easily become positive again.

For the case of the variance, the minimum wage seems to be close to the point in which the positive and the negative effects cancel each other out. For some years, I obtain the small but positive effect, whereas for others I obtain the opposite result. The effect is not statistically different from zero for a few years and in all cases, the effects are of a small magnitude, which even if statistically significant, should be of small economical relevance.

## E.6. Tax Effects of the Minimum Wage Under Alternative Assumptions

To provide an idea of the importance of the unemployment effects on the matter at hand, I will also compute the effects of the minimum wage on taxes based on a different model. In this version, I will force the unemployment effects to be equal to zero. By doing so, I no longer need to assume the continuity of the latent wage distribution. Formally, the model operates as follows. I will retain Assumptions 5 (independence) and 3 (no spillovers). Assumption 4 will be modified to force  $\pi_u = 0$ :

## ASSUMPTION 10 No Unemployment Effects

Under the minimum wage, a fraction  $\pi_d$  of workers will earn the same wage as in the latent wage distribution. The remaining fraction will earn the minimum wage. These fractions can be sector-specific as in the baseline model. Note that there is no need to assume continuity in this case. Under these assumptions, the observed wage density will relate to the latent density by the following equation:

$$f(w) = \begin{cases} \pi_d f_0(w) & \text{if } w < m \\ (1 - \pi_d) F_0(m) & \text{if } w = m \\ f_0(w) & \text{if } w > m, \end{cases}$$

where  $f_0(w)$  is the latent wage distribution based on this different set of assumptions. In this case, we only need to estimate  $\pi_d$ . One way to do so is by recognizing that in this case:

$$\pi_d = \frac{Pr[W < m]}{Pr[W < m] + Pr[W = m]}.$$

Therefore, a consistent estimator can be constructed by plugging in the maximum likelihood estimator of the respective quantity. With an estimate of  $\pi_d$ , the latent wage density can be easily estimated by properly reweighting the observed wage density. Then, the tax effects of the minimum wage can be computed under the “no unemployment” assumption, given by:

$$R \equiv \frac{T(1)}{T(0)} = \frac{Pr[S(1) = 1]}{Pr[S(0) = 1]} \cdot \frac{E[\tau(W(1))W(1)|S(1) = 1]}{E[\tau(W(0))W(0)|S(0) = 1]}.$$

This is exactly the same expression as before without the unemployment component  $c$ . Note, however, that the expected value of wages in the latent distribution will also change, as the estimate of the latent distribution is different under this different set of assumptions. Table XIV reports the estimates of  $R$  for the years from 2001 to 2009. The estimates under the assumption of no-unemployment indicate that the minimum wage has a sufficiently strong effect on average wages to compensate for the reduction in the share of the formal sector due to sector transition. Moreover, note that for the same data, the implied effect of the minimum wage on the average wages of those employed is, as expected, smaller when one assumes the absence of unemployment effects. This is the case because the presence of a disemployment effect induces a mechanical increase in the expected value of wages since the disemployment is concentrated in the bottom part of the wage distribution.

TABLE XIV  
LABOR TAX EFFECTS UNDER A “NO UNEMPLOYMENT” ASSUMPTION

|                                       | 2001                | 2002                | 2003                | 2004                | 2005                | 2006                | 2007                | 2008                | 2009                |
|---------------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| R                                     | 1.093***<br>(0.006) | 1.077***<br>(0.005) | 1.081***<br>(0.004) | 1.059***<br>(0.004) | 1.057***<br>(0.004) | 1.052***<br>(0.004) | 1.052***<br>(0.004) | 1.037***<br>(0.004) | 1.039***<br>(0.007) |
| Pr(S(1)=1)/Pr(S(0)=1)                 | 0.912***<br>(0.002) | 0.892***<br>(0.002) | 0.895***<br>(0.002) | 0.891***<br>(0.002) | 0.884***<br>(0.001) | 0.884***<br>(0.002) | 0.899***<br>(0.002) | 0.889***<br>(0.002) | 0.902***<br>(0.002) |
| E(W(1)   S(1) = 1)/E(W(0)   S(0) = 1) | 1.198***<br>(0.006) | 1.208***<br>(0.006) | 1.208***<br>(0.005) | 1.189***<br>(0.005) | 1.195***<br>(0.005) | 1.191***<br>(0.005) | 1.170***<br>(0.004) | 1.167***<br>(0.004) | 1.152***<br>(0.008) |
| N                                     | 71,397              | 71,051              | 68,319              | 68,196              | 65,755              | 62,587              | 58,269              | 58,241              | 55,502              |

Note: \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Standard errors computed by 100 bootstrap replications.

As a result, larger disemployment effects are associated with larger increases in the expected value of wages. Equivalently, smaller disemployment effects necessarily mean a smaller increase in the expected value of wages.

#### APPENDIX F: INSITUATIONAL BACKGROUND – FURTHER DETAILS

Although the misspecify grants the federal government exclusive rights to set the minimum wage level, some states take advantage of a provision created in the year 2000 that allows states to legislate wage floors that are “proportional to the complexity of the work” (Corseuil, Foguel, and Hecksher, (2015)).

As a result, states become allowed to set wage floors that are *above the federal minimum wage*. These wage floors are *ocupation specific* and are only valid in the absence of a federal law regulating the wage or collective bargaining that already set a floor for that occupation.

Since the year 2000, five states took advantage of the provision. These states are Rio de Janeiro (in 2001), Rio Grande do Sul (in 2001), Paraná (in 2006), São Paulo (in 2007), and Santa Catarina (in 2010). However, compliance with the state level, occupation-specific wage floor seems to be extremely low. Corseuil, Foguel, and Hecksher (2015) report that a sizable fraction of *for-mal* workers seem to be able to earn wages below the minimum wage level. Moreover, the fraction of workers that earn the federal minimum wage is frequently 10 times larger than the proportion of workers that earn the state-level wage floor in the states that implemented such policy.

For the purposes of my empirical exercise, one needs to decide whether to treat the federal or the state/occupation specific wage floor as the relevant minimum wage for the analysis. For all states that implemented the wage floor policy, the mode of the wage distribution continues to be the *federal* minimum wage level. As a result, none of the fundamental implications of assumption 4 of the dual economy model – namely (i)  $Pr[W(1) < m | S(1) = 1] = 0$ , that is, no formal sector worker earns a wage that is below the minimum wage and, as a result, (ii)  $Pr[S(1) = 1 | W(1) < m] = 0$ , the conditional probability of formality as a

function of the wage drops discontinuously to zero below the minimum wage level – seem to be reasonable when I treat the minimum wage in these states as the state legislated wage floor. The non-compliance with the federal minimum wage is somewhat high. However, it consists of *informal* workers earning wages below the minimum wage, which is in line with the assumptions of the dual economy model. In contrast, even formal workers seem to be able to ignore the state-specific wage floors.

To sum up, the studies on the effects of the state-specific wage floors suggest that the policy has been effectively ignored in the labor markets in these states (Corseuil, Foguel, and Hecksher, (2015)). The fraction of formal workers that earn wages at the federal minimum wage level or in between the federal and the state-specific wage floor are substantially larger than the fraction of workers that earn the state-specific wage floor. As a result, I choose to ignore the state-specific wage floors in my empirical exercise. The results should be interpreted as the effects of changing the federal level minimum wage holding constant the state specific (and mostly irrelevant) wage floors. Estimating the model ignoring the states that have wage floors I obtain a decrease in the size of the formal sector of 10.7%, whereas when I include these states in the sample I obtained an effect of 9%. The estimates I obtain for the probability of moving to the formal sector  $\pi_d^{(1)}$  and for the aggregate non-compliance probability  $\pi_d$  are also quite similar to the ones of my baseline estimates, with differences that are smaller than 3 percentage points. Thus, all the quantitative and qualitative implications remain the same when I focus on the states that do not have a state/occupation specific wage floor.

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