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The Impact of Growth on Unemployment in a Low vs. a High Inflation Environment

by Mewael F. Tesfaselassie and Maik H. Wolters
The Impact of Growth on Unemployment in a Low vs. a High Inflation Environment

Mewael F. Tesfaselassie* and Maik H. Wolters**

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Abstract

The standard search model of unemployment predicts, under realistic assumptions about household preferences, that disembodied technological progress leads to higher steady-state unemployment. This prediction is at odds with the 1970s experience of slow productivity growth and high unemployment in industrial countries. We show that introducing nominal price rigidity helps in reconciling the model’s prediction with experience. Faster growth is shown to lead to lower unemployment when inflation is relatively high, as was the case in the 1970s. In general, the sign of the effect of growth on unemployment is shown to depend on the level of steady-state inflation. There is a threshold level of inflation below (above) which faster growth leads to higher (lower) unemployment. The prediction of the model is supported by an empirical analysis based on US and European data.

JEL Classification: E24; E31

Keywords: Growth, trend inflation, unemployment.

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1 Introduction

During the 1970s, industrial countries, including the US and continental Europe, experienced a combination of slow productivity growth and high unemployment. In a seminal theoretical contribution Pissarides (1990, ch. 2) argues that the observed negative relationship is consistent with the prediction of the standard search model of unemployment. He shows that, under the assumption of an *exogenous* and constant interest rate, exogenous job destruction and disembodied technological progress, the model predicts a negative effect of growth on steady-state unemployment. This is due to a positive *capitalization effect*—by lowering the effective discount rate, higher growth raises the surplus from an employment relationship and therefore leads to higher job creation.

However, subsequent research has shown that under alternative and more plausible assumptions the standard model actually gives counterfactual predictions. For example, under the assumption of an *endogenous* interest rate and a low degree of intertemporal substitution in consumption, Aghion and Howitt (1994) and Eriksson (1997) show that faster growth leads to higher unemployment due to a negative capitalization effect: the higher real rate of interest implied by faster consumption growth raises the effective discount rate and in turn lowers the surplus from an employment relationship, implying lower job creation.¹

The present paper reexamines the impact of disembodied technological progress on unemployment in the presence of nominal rigidities and trend inflation. Whereas our analysis is motivated by the observation that the 1970s were characterized not only by a slowdown in productivity growth but also by higher inflation rates the role of trend inflation has so far not received attention within the growth-unemployment literature.²

¹This result also appears when relaxing the assumption of exogenous job destruction in Pissarides (1990). For instance, Prat (2007) shows that, by raising a worker’s outside option disembodied technological progress intensifies the rate of job separation, an effect that outweighs, for plausible parameter values, the capitalization effect so that disembodied technological progress raises unemployment. Aghion and Howitt (1994) also identify a creative destruction effect brought about by embodied technological progress: by reducing the duration of an existing job match faster growth leads to higher job destruction and therefore unemployment. Pissarides and Vallanti (2007) provide empirical evidence for a negative effect of growth on unemployment, thus supporting the view that, if unemployment is a result of search frictions, then technology must be disembodied. Nevertheless, the authors conclude that, even if one assumes technology is mainly disembodied, a significant part of the impact of growth on unemployment remains unexplained.

²Our work bridges two strands of the recent literature. The first focuses on trend inflation within the standard New Keynesian model, abstracting from unemployment (e.g., Ascari (2004), Graham and Snower (2008), Amano et. al (2009) and Snower and Tesfaselassie (2016)). The second focuses on the role of labor market frictions for inflation dynamics, abstracting from trend inflation and growth considerations (e.g., Trigari (2006), Christoffel and Kuester (2008) and Blanchard and Gali (2010)).
Our analysis is based on a balanced growth version of a two-sector New-Keynesian model with nominal price staggering, labor market frictions and exogenous disembodied technological progress. Firms in sector one produce differentiated final goods using an intermediate input but adjust prices infrequently so that price setting decisions are forward looking. Firms in sector 2 produce the intermediate input under a perfectly competitive output market and face labor hiring costs so that hiring decisions are forward looking. As in Aghion and Howitt (1994) and Eriksson (1997), we endogenize the rate of interest and assume a low degree of intertemporal substitution in consumption so that faster growth raises the effective discount rate.

In our framework, an increase in the effective discount rate leads to two opposing effects. The first is the familiar negative capitalization effect: an increase in the effective discount rate lowers the surplus from an employment relationship, thereby discouraging job creation by intermediate goods firms. The second and novel effect is what we call a markup effect: an increase in the effective discount rate lowers the price markup of final good firms, as these firms worry less about the erosion of their markups (given the staggered nature of price setting) by ongoing inflation the higher is the effective discount rate. The reduction in the price markup acts like a tax-cut on the intermediate input (i.e., raises its relative price), thereby encouraging job creation. Moreover, the markup effect is stronger the higher is the level of inflation while the effect vanishes in the limiting case of zero trend inflation.

The intuition behind the markup effect is as follows. Since prices can be adjusted only infrequently, pricing decisions of final good firms are forward-looking—future conditions matter for current price setting. In such an environment positive trend inflation erodes a firm’s markup as long as its price remains fixed, and in anticipation an optimizing firm chooses a markup higher than that implied by zero trend inflation (see e.g., King and Wolman (1996))). However, at any given level of trend inflation, the higher the effective discount rate the less firms worry about markup erosion by trend inflation and thus the lower the optimal markup.

We show that if inflation is high enough the markup effect dominates the capitalization effect so that faster growth leads to lower unemployment. More generally, there is a threshold rate of inflation below (above) which faster growth leads to higher (lower) unemployment. We use data for the US and for the four largest European economies (Ger-

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3The two-sector framework is standard in the business cycle literature (see, e.g., Trigari (2006), Christoffel and Kuester (2008) and Blanchard and Gali (2010)). The assumption that hiring costs are the source of labor market rigidity follows closely Blanchard and Gali (2010).

4We also show that, the threshold inflation level is unique under plausible parameter restrictions and
many, UK, France and Italy) to show that the model predictions are empirically plausible. Since the model predictions have implications for the long-run relation between growth and unemployment, we extract the low-frequency components of technology growth and unemployment data. The empirical results show that there is indeed clear evidence for a negative relation between growth and unemployment in a high inflation environment as predicted by the model. For a low inflation environment we find a positive relation between growth and unemployment, although the evidence here is somewhat less robust than for a high inflation environment.

While the focus of the paper is on the impact of productivity growth on unemployment, our framework can also be used to analyze the effect of trend productivity growth on the optimal steady-state inflation rate. The markup effect by which faster growth lowers price markups of final good firms suggests that the welfare cost of inflation depends negatively on the growth rate of the economy (i.e. in a fast-growing economy the welfare cost of inflation is relatively low). For this reason the optimal steady-state inflation rate is higher the higher is trend growth.

The remainder of the paper is organized as follows. In section 2 we present the model and in section 3 we discuss the model’s balanced growth path. In section 4 we undertake comparative static analysis regarding the effect of growth on unemployment and show analytical and numerical results on the existence and uniqueness of a threshold inflation rate. Section 5 presents some empirical evidence that supports the predictions of the model. In section 6 we give concluding remarks.

2 The model

Following Blanchard and Gali (2010) we use a simple two-sector framework with price staggering as well as labor market frictions. This framework is augmented to allow for productivity growth, which is labor augmenting and disembodied (e.g., as in Pissarides (1990) and Eriksson (1997)), so that productivity growth is reflected in all existing and that it depends on key labor market parameters, such as the job destruction rate and workers’ bargaining power.

In a related work Amano et al. (2009) and Snower and Tesfaselassie (2016) study the effect of trend growth on the optimal steady-state inflation rate in the presence of price and wage staggering but abstract from search frictions in the labor market.

In a separate exercise, results of which are available upon request, we find that the optimal inflation rate varies between 0.2% and 0.7%. Given our theoretical result that unemployment is negatively related to growth if trend inflation is high enough while the optimal inflation rate is quite low, one may interpret an episode of rising unemployment in response to slower productivity growth (like that of the 1970s) as a sign of a too high inflation target (to the extent that trend inflation is pinned by the inflation target).
newly employed workers. Furthermore, growth in labor productivity $A_t$ is assumed to be deterministic, where $\Gamma = A_t/A_{t-1}$ denotes gross productivity growth. As in Aghion and Howitt (1994) and Eriksson (1997), the rate of interest is endogenous and is related to consumption growth. As is standard, the economy exhibits balanced growth.\footnote{For a similar approach see, e.g., Tesfaiselassie (2013).}

2.1 Households

There is a representative household with a continuum of members over the unit interval. Similar to Aghion and Howitt (1994) and Eriksson (1997), household utility is of the form $U(C_t) = \frac{C_1^{1-\sigma}}{1-\sigma}$, where $\sigma > 1$. In a given period a fraction $N_t$ of household members are employed, who earn a nominal wage $W_t$ but forgo a nonwork nominal value $Z_t$—the opportunity of cost of employment (which may include, among others, unemployment benefits).

The household consumes a continuum of differentiated goods produced by an imperfectly competitive final goods sector (details of which are given below). $C_t$ is a Dixit-Stiglitz composite of final goods: $C_t = \left(\int_0^1 C_{k,t}^{1/\mu} dk\right)^{\mu}$ where each good is indexed by $k$, $\mu = \frac{\theta}{\theta - 1}$ and $\theta > 1$ is the elasticity of substitution between the differentiated final goods. Optimal consumption allocation across goods gives the demand equation: $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{\frac{\theta}{\theta - 1}} C_t$, where

$$P_t = \left(\int_0^1 P_{k,t}^{1-\theta} dk\right)^{\frac{1}{1-\theta}}$$

is the price index. Optimal consumption allocation across time is derived from the maximization of lifetime utility, $E_t \sum \beta^i U(C_{t+i})$, subject to the budget constraint

$$P_tC_t + B_t = W_tN_t + Z_t(1 - N_t) + R_{t-1}B_{t-1} + D_t,$$

where $\beta$ is the subjective discount factor, $R_t$ is the nominal interest rate on bond holdings $B_t$, $W_t$ is the nominal wage and $D_t$ is the aggregate nominal profit income from firm ownership. It is straightforward to derive the familiar Euler equation

$$1 = E_t \left(\frac{Q_{t,t+1}R_t}{\Pi_{t+1}}\right),$$

\footnote{\textsuperscript{8}The utility function may also include disutility from work (as for e.g., in Blanchard and Gali (2010) and Shimer (2010)). Our abstraction from work disutility follows the literature on growth and unemployment. In Appendix C we show that the presence of disutility from work leads to similar conclusions about the threshold inflation rate.}
where $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate and $Q_{t,t+1} \equiv \beta U'(C_{t+1})/U'(C_t)$ is the familiar stochastic discount factor. It can be rewritten as

$$Q_{t,t+1} = \beta \left( \Gamma c_{t+1}/c_t \right)^{-\sigma},$$

where $c_t = C_t/A_t$. The steady-state of equation (2) is $R/\Pi = \Gamma^\sigma/\beta$, which shows that higher trend growth implies a higher gross real rate $R/\Pi$ and in turn a stronger discounting of future payoffs.

### 2.2 Firms

#### 2.2.1 Intermediate goods sector

There is a continuum of firms in the intermediate goods sector. The representative firm produces output $Y^I_t$ with a linear technology using the input $N_t$ of employed workers:

$$Y^I_t = A_t N_t.$$  

Employment evolves according to the dynamic equation

$$N_t = (1 - \delta)N_{t-1} + H_t,$$  

where at the beginning of period $t$ a fraction $\delta$ of previously employed workers are separated from the firm and $H_t$ is hiring in period $t$. Thus $\delta$ represents an exogenous job separation rate. Given an exogenous job separation rate unemployment is entirely driven by hiring.

In every period, each household member can either be employed or unemployed. The size of the labor force is normalized to one so that the stock of unemployed workers in period $t$ before hiring takes place is given by $U_t = 1 - (1 - \delta)N_{t-1}$. Assuming workers start working immediately after getting hired, the unemployment rate (after hiring takes place) is $u_t = 1 - N_t$.

As in Blanchard and Gali (2010), frictions in the labor market take the form of hiring costs, $HC_t$, which are given by

$$HC_t = G_t H_t,$$  

\footnote{This section draws on Blanchard and Gali (2010). The assumption that firms can hire a worker instantaneously subject to paying hiring costs simplifies our analysis. Alternatively, one may assume vacancy posting costs as in the labor search and matching literature (see, e.g., Christoffel and Kuester (2008)). In the present paper, we do not need to track vacancies, which is necessary when one is interested, say, in the Beveridge curve (the relationship between vacancies and unemployment).}
where \( G_t \equiv \kappa A_t f_t \) is the cost per hire, \( \kappa > 0 \) and \( f_t \equiv H_t/U_t \) is the job finding rate.\(^{10}\)

Hiring costs are expressed in terms of the CES bundle of final goods. Since the model features balanced growth, the presence of \( A_t \) ensures that along the balanced growth path the cost per hire increases at the same rate as aggregate final output. For future reference the detrended version of equation (5) is

\[
h c_t = g_t H_t, \tag{6}\]

where \( g_t = \kappa f_t \). Intermediate good firms face a perfectly competitive output market and sell output at the nominal price \( P_t^I \). The presence of hiring costs makes the hiring decision intertemporal. To see this, a firm’s lifetime discounted profit is given by

\[
E_t \sum_{i=0}^{\infty} \left( p_{t+i}^I A_{t+i} N_{t+i} - w_{t+i} N_{t+i} - G_{t+i} H_{t+i} \right), \tag{7}\]

where \( p_t^I \equiv P_t^I/P_t \) is the relative price of the intermediate good and \( w_t \equiv W_t/P_t \) is the real wage. In any given period profits are equal to revenues net of the total wage bill and the total hiring cost. Maximizing the sum of discounted profits (7) subject to the employment dynamics (4) leads to the first order condition for an optimum level of hiring,

\[
p_t^I A_t = w_t + G_t - (1 - \delta) E_t \{ Q_{t,t+1} G_{t+1} \}. \tag{8}\]

The left hand side of equation (8) is the marginal revenue product of labor, while the right hand side is the cost of the marginal worker, which includes the real wage and the hiring cost net of discounted savings in future hiring costs. Dividing through by \( A_t \) and slightly manipulating the resulting equation gives

\[
p_t^I = w_t^d + g_t - (1 - \delta) E_t \{ \beta (\Gamma c_{t+1}/c_t)^{-\sigma} (\Gamma g_{t+1}) \}, \tag{9}\]

where \( w_t^d \equiv w_t/A_t \) and \( g_t \equiv G_t/A_t \) are stationary variables and the second equality follows from using equation (3) to substitute out \( Q_{t,t+1} \).

\(^{10}\)In this setup, a vacancy is filled instantaneously if the firm pays the hiring cost. As a matter of comparison, in the standard search and matching model the job-posting cost is constant for each posted vacancy. Assuming a matching function of the form \( H_t = V_t^\alpha V_t^{1-\alpha} \), where \( V_t \) is the number of posted vacancies, the hiring cost is proportional to the expected vacancy duration, which is equal to the inverse of the job-filling rate \( H_t/V_t \). It can be shown that \( V/H = f^\alpha \), where \( \alpha \equiv \alpha_0/(1 - \alpha_0) \). Our specification of the cost per hire assumes implicitly that \( \alpha_0 = 0.5 \), which is close to empirical estimates (see Blanchard and Gali (2010)).
From the right hand side of equation (9) we see that there are two offsetting effects of higher productivity growth on optimal hiring. On the one hand, it implies larger returns from current hiring since the discounted savings in future hiring costs are larger the faster they grow \((\Gamma_{g_{t+1}})\)—a positive capitalization effect, which leads to more hiring. On the other hand, it implies lower returns from current hiring since the discounted savings in future hiring costs are smaller the higher is the real interest rate due to faster consumption growth \(\left(\beta(\Gamma_{c_{t+1}/c_t})^{-\sigma}\right)\)—a negative capitalization effect, which leads to less hiring. Given the maintained assumption \(\sigma > 1\) the second effect dominates so that, given the output price \(p_I\), faster growth reduces the returns to hiring and raises unemployment.\(^{11}\)

**Wage setting.** The presence of hiring costs implies that existing employment relationships earn an economic surplus. The surplus is divided between the worker and the firm according to Nash bargaining and the wage rate is such that it maximizes the joint surplus.\(^{12}\) As in Blanchard and Gali (2010) we express the household’s surplus as the difference between the asset value of employment and the asset value of unemployment. The asset value of an employed worker is given by

\[
V^e_t = w_t + E_t \left( Q_{t+1} \left( (1 - \delta (1 - f_{t+1}))V^e_{t+1} + \delta (1 - f_{t+1})V^u_{t+1} \right) \right),
\]

where \(\delta (1 - f_{t+1})\) is the probability that an employed worker is separated from his job at the end of period \(t\) and stays unemployed in period \(t+1\) while \(1 - \delta (1 - f_{t+1})\) is the probability that an employed worker keeps his current job in period \(t+1\) or he is separated from his current job at the end of period \(t\) but finds a job in period \(t+1\).

The corresponding value of an unemployed worker is given by

\[
V^u_t = z_t + E_t \left( Q_{t+1} \left[ f_{t+1}V^e_{t+1} + (1 - f_{t+1})V^u_{t+1} \right] \right),
\]

where \(z_t = Z_t/P_t\). As is standard, \(z_t\) is assumed to be proportional to labor productivity, \(z_t = bA_t\), where \(b > 0\). The household’s surplus from an employment relationship is then given by

\[
S^h_t = w_t - bA_t + (1 - \delta)E_t \left( Q_{t+1}(1 - f_{t+1})S^h_{t+1} \right). \tag{10}
\]

Similarly, the firm’s surplus from an employment relationship is

\[
S^f_t = p_I A_t - w_t + (1 - \delta)E_t \left( Q_{t+1}S^f_{t+1} \right). \tag{11}
\]

\(^{11}\)As pointed out above our maintained assumption \(\sigma > 1\) follows, among others Eriksson (1997) and Shimer (2010). Nevertheless, it is important to bear in mind that \(\sigma > 1\) is crucial to our results.

\(^{12}\)Note here that, for the joint surplus to be positive the marginal revenue product of labor \(p_I A_t\) must be larger than the worker’s outside option \(Z_t/P_t\).
which is the sum of the current period’s profit and future expected surplus. Equations (9) and (11) imply that

$$S_t^f = G_t.$$  \hfill (12)

That is, the firm’s surplus from an additional hire is equal to the cost per hire. Under the common assumption of Nash bargaining the real wage is such that it maximizes the Nash product \((S_h^t)\eta(S_f^t)^{1-\eta}\), where \(0 < \eta < 1\) is the relative bargaining power of the household. Wage setting satisfies the optimality condition \(S_h^t = \nu S_f^t = \nu G_t\), where \(\nu \equiv \eta / (1 - \eta)\) and the second equality follows from equation (12). Then equation (10) can be rewritten as

$$w_t = bA_t + \nu (G_t - (1 - \delta)E_t \{Q_{t,t+1}(1 - f_{t+1})G_{t+1}\}).$$  \hfill (13)

Dividing equation (13) through by \(A_t\) and slightly manipulating the resulting equation gives

$$w_t^d = b + \nu \left( g_t - (1 - \delta)\beta^1 \left( \frac{c_{t+1}}{c_t} \right) - \theta \left( \frac{C_t}{Y_t} \right) \right).$$  \hfill (14)

The chosen wage is increasing in current hiring cost \((g_t)\), as this raises the firm’s surplus from an existing relationship. It is decreasing in expected future hiring costs \((g_{t+1})\) and in the probability \((1 - f_{t+1})\) of not finding a job next period in the event that the worker separates from the firm, both of which raise the continuation values to currently employed workers and hence reduce the required wage today. All else equal, the higher is productivity growth the smaller is the continuation value and hence the larger is the real wage.

2.2.2 Final goods sector

There is a continuum of firms producing differentiated final goods and face Calvo-type price staggering, where only a fraction \(1 - \omega\) of firms can reset prices in any given period. Each firm \(k\) produces a differentiated final good using the intermediate good as an input. As in Blanchard and Gali (2010) we assume a simple linear technology \(Y_{k,t} = Y_{k,t}^f\), which implies that the firm’s real marginal cost \((mc_{k,t})\) is given by \(p_{k,t}^t\). Let \(P_{k,t}\) denote firm \(k’\)s output price. Maximizing lifetime profit \(E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \left( p_{k,t}/P_{t+i} - p_{k,t}^t \right) Y_{k,t+i}\) subject to the demand for good \(k\), \(Y_{k,t+i} = \left( P_{k,t}/P_{t+i} \right)^{-\theta} Y_{t+i}\), where \(Y_{t+i} = C_{t+i} + HC_{t+i}\), leads to the optimality condition

$$p_{t+i}^{opt} = \mu^t \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i}p_{t+i}^t(Y_{t+i}/Y_t) \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i}(Y_{t+i}/Y_t) \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}.$$  \hfill (15)
where $p_{t}^{opt} \equiv P_{t}^{opt}/P_{t}$ is the relative price of optimizing firms, all of which face an identical price setting problem, and $\mu$ is the price markup in the absence of price staggering. Equation (15) can be rewritten in stationary variables

$$p_{t}^{opt} = \frac{E_{t} \sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma})^i (c_{t+i}/c_{t})^{-\sigma} p_{t+i}^{i} (P_{t+i}/P_{t})^\theta}{E_{t} \sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma})^i (c_{t+i}/c_{t})^{-\sigma} y_{t+i} (P_{t+i}/P_{t})^\theta-1}. \quad (16)$$

This is our key equation capturing the influence of steady-state inflation in the presence of price staggering. We thus discuss its relevance in more detail by looking at its steady-state version

$$p^{opt} = \mu \frac{\sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma} \Pi^{\theta})^i}{\sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1})} = \mu \frac{1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1}}{1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta}} p^{I}, \quad (17)$$

where for the sums to be convergent, we impose the restriction $\Pi < \Pi^{max} = (\beta \omega \Gamma^{1-\sigma})^{-1/\theta}. \quad 13$

When the inflation rate is zero ($\Pi = 1$), the optimal relative price is a fixed markup over real marginal cost ($p^{opt} = \mu p^{I}$, as is the case under flexible prices) and it is independent of productivity growth. When the inflation rate is positive, firms choose a markup higher than that implied by zero inflation so as to mitigate the future erosion of their markup by ongoing inflation (until they get the chance to reset their price). The underlying reason behind this markup distortion is the asymmetry in the profit function: profit declines more strongly with a markup that is below the optimum (under flexible prices) than with a markup above the optimum.14 The markup distortion is smaller the higher is the rate of productivity growth owing to a stronger discounting effect from a higher real interest rate. As will be shown below, this markup effect of productivity growth counteracts the negative capitalization effect. Finally, when the inflation rate is negative, optimizing firms choose their price so as to achieve a markup lower than that implied by zero inflation in anticipation of the fact that an ongoing deflation leaves them with a markup that is too high. As a result there is a positive markup effect of productivity growth, which reinforces the negative capitalization effect.

Under Calvo-type price staggering the price index (1) can be rewritten as

$$1 = (1 - \omega) \left(p_{t}^{opt}\right)^{1-\theta} + \omega \Pi_{t-1}^{\theta}, \quad (18)$$

which shows that in steady-state $p^{opt}$ is positively related to $\Pi$. Equations (17) and (18) imply that, given $\Pi > 1$, the higher is productivity growth the lower is the price markup

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13 For instance, assuming plausible parameter values—$\beta = 0.99, \sigma = 3, \omega = 0.75, \theta = 11$ and $\Gamma = 1.005$ (i.e., an annualized growth rate of 2%)—$\Pi^{max} = 1.028$ (i.e., an annualized inflation rate of about 11.35%).

14 See, e.g., Amano et. al (2009) for a detailed discussion.
and hence the higher is the relative intermediate good price $p^I$. Moreover, the negative markup effect of faster growth is stronger the higher is the rate of inflation. The reduction in the price markup acts like a tax-cut on the intermediate input supply and thus induces intermediate good firms to supply more output and hire more workers.

Next, aggregating both sides of the market clearing condition for the intermediate good and using the demand equation for the final good $k$ leads to a relationship between aggregate final output $y_t$ and intermediate good output $y_t^I$,

$$y_t^I = \Delta_t y_t,$$  \hspace{1cm} (19)

where $\Delta_t \equiv \int_0^1 (p_{k,t})^{-\theta} dk$ is a measure of price dispersion, which can be rewritten as

$$\Delta_t = (1 - \omega) \left(p_{opt}^t \right)^{-\theta} + \omega \Pi^\theta \Delta_{t-1}. \hspace{1cm} (20)$$

Finally, using the intermediate good production function ($y_t^I = N_t$) in equation (19) leads to a relationship between aggregate employment and aggregate final good output,

$$N_t = \Delta_t y_t. \hspace{1cm} (21)$$

Thus, higher price dispersion increases the wedge between aggregate final output and aggregate employment. Since final goods are imperfectly substitutable, a rise in the relative price (and correspondingly output) dispersion acts like a downward shift in labor productivity.

To summarize, the equilibrium of the model is determined by equations (4), (6), (9), (14), (16), (18), (20), (21), the equations determining the cost per hire ($g_t = \kappa f_t$), the job finding rate ($f_t \equiv H_t/U_t$), unemployment before hiring ($U_t = 1 - (1 - \delta)N_{t-1}$), unemployment after hiring ($u_t = 1 - N_t$), and the aggregate resource constraint ($y_t = c_t + hc_t$).

### 3 Steady-state equilibrium

In a steady-state equilibrium the flow into unemployment is equal to the flow out of unemployment. Starting with the intermediate goods sector, in steady-state the optimal hiring condition (9) becomes

$$p^I = w^d + \left(1 - \beta \Gamma^{1-\sigma} (1 - \delta) \right) g.$$  \hspace{1cm} (22)
The steady state marginal revenue from an additional worker (left-hand side) equals the steady state marginal cost of an additional worker (right-hand side), which is given by the real wage and the hiring cost net of discounted savings in future hiring costs. At any given $p^I$, faster growth decreases the discounted savings in future hiring costs, $(\beta \Gamma^{1-\sigma}(1-\delta)g)$ and thus raises the marginal cost relative to the marginal revenue. Intermediate good firms react by lowering hiring $H$, which reduces the job-finding rate $f = H/U$ and in turn the cost per hire $g = \kappa f$, thereby restoring the equality in equation (22).\footnote{Note that the effect of faster growth on optimal hiring is similar to a reduction in the subjective discount rate or an increase in the job separation rate.}

Similarly, in steady-state the wage setting equation (14) becomes

$$w^d = b + \nu \left( 1 - \beta \Gamma^{1-\sigma}(1-\delta)(1-f) \right) g,$$

(23)

which shows that faster growth increases the steady-state real wage by decreasing the continuation value to an employed worker, $(\beta \Gamma^{1-\sigma}(1-\delta)(1-f)g)$. Substituting equation (23) into equation (22) gives

$$p^I = b + \kappa fh(\Gamma, f),$$

(24)

where $h(\Gamma, f) \equiv \left( 1 - \beta \Gamma^{1-\sigma}(1-\delta) \right) + \nu \left( 1 - \beta \Gamma^{1-\sigma}(1-\delta)(1-f) \right)$ and we substitute out the cost per hire $g$ using $g = \kappa f$. The steady-state job finding rate $f$ is given by

$$f = \frac{\delta N}{1 - (1-\delta)N} \equiv f(N).$$

(25)

The term $\kappa fh(\Gamma, f)$ in equation (24) represents a labor market wedge (LMW) between the marginal revenue product $p^I$ and the opportunity cost of work $b$. The LMW is the sum of two wedges: the first is the wedge, in the presence of hiring cost, between the marginal revenue product and the real wage (see equation (22)) and the second is the wedge between the real wage and the opportunity cost of work (see equation (23)). Moreover, all else equal, the higher is productivity growth the larger is the LMW.

Next, from the final goods sector, in steady-state the aggregate price index (18) becomes

$$p^{opt} = p^{opt}(\Pi) \equiv \left( \frac{1 - \omega \Pi^{\theta-1}}{1 - \omega} \right)^{1/(1-\theta)}.$$  

(26)

Equation (26) captures the markup-eroding effect of positive trend inflation. It is easily seen that for $\Pi \geq 1$, $\partial p^{opt}/\partial \Pi > 0$—the higher is trend inflation the larger is the gap
between the optimally set nominal price and the price level. Substituting equation (26) in the steady-state optimal relative price (17) and rearranging we get

\[ p^I = \frac{p^\text{opt}(\Pi)(1 - \beta \omega \Gamma^{1-\sigma} \Pi^\theta)}{\mu(1 - \beta \omega \Gamma^{1-\sigma} \Pi^\theta - 1)} \equiv p^I(\Gamma, \Pi). \] (27)

Under the special case of zero trend rate of inflation (i.e., \( \Pi = 1 \)) equation (27) becomes \( p^I = 1/\mu \), so that the relative intermediate good price is independent of productivity growth. But when trend inflation rate is positive (\( \Pi > 1 \)), the relative intermediate good price increases monotonically with productivity growth.\(^{16}\)

By contrast, the relationship between trend inflation and the relative intermediate good price is non-monotonic. The relationship is positive (negative) when trend inflation is low (high) enough. This is a standard property of the New-Keynesian model with Calvo price staggering (see, e.g., King and Wolman (1996)). By rewriting \( p^I \equiv (P^I_t/P^\text{opt}_t)(P^\text{opt}_t/P_t) \), one can see two opposing forces at play. On the one hand, higher ongoing inflation mechanically erodes the markups of those firms whose prices are fixed in the past (from equation (26) \( P^\text{opt}_t/P_t \) rises with inflation). On the other, optimizing firms raise their prices so as to mitigate future erosion of their markups by higher inflation (from equation (17) \( P^I_t/P^\text{opt}_t \) falls with inflation). At low inflation the former effect dominates due to time discounting.

Finally, substitution of equation (27) in equation (24) leads to

\[ p^I(\Gamma, \Pi) = b + \kappa f(N)h(\Gamma, f(N)). \] (28)

The solution to the nonlinear equation (28) is an implicit function \( N^* = N(\Gamma, \Pi) \), which relates the equilibrium employment rate (and the equilibrium unemployment rate, \( u^* = 1 - N^* \)) to productivity growth \( \Gamma \) and steady-state inflation \( \Pi \). In what follows we analyze the effect of productivity growth on equilibrium unemployment and how that effect depends on the level of trend inflation. For this purpose, we work with the total derivatives \( dN^*/d\Gamma \) and \( du^*/d\Gamma = -dN^*/d\Gamma \).

### 4 Comparative statics

In this section we show our main result formally. First, we prove that for sufficiently low (high) inflation rates unemployment is positively (negatively) related to productivity.

\(^{16}\)The derivation is straightforward, as \( p^I_\Gamma \equiv \partial p^I(\Gamma, \Pi)/\partial \Gamma = (\sigma - 1)\mu^{-1} \beta \omega \Gamma^{-\sigma} (\Pi - 1)\Pi^{\theta - 1} p^\text{opt}(\Pi)/(1 - \beta \omega \Gamma^{1-\sigma} \Pi^\theta - 1)^2 > 0. \)
growth. Second, for a version of the model where either workers’ bargaining power or the equilibrium job-finding rate is not too high we prove that the threshold inflation rate is unique. Third, we discuss the underlying channels whereby the level of inflation matters for the comparative static effects of trend growth on unemployment. We then illustrate our result numerically in a calibrated version of the model. In Appendix D we discuss the sensitivity of the threshold inflation rate to alternative assumptions about labor market parameters.

We evaluate equation (28) at the implicit solution $N^*$ so that

$$F(\Gamma, \Pi, N^*) \equiv p^I(\Gamma, \Pi) - \kappa f(N^*)h(\Gamma, f(N^*)) - b \equiv 0. \quad (29)$$

By applying the implicit function theorem on the identity (29) we get an expression for the effect of trend growth on equilibrium employment,

$$\frac{dN^*}{d\Gamma} = -\frac{F_\Gamma}{F_{N^*}} = \frac{p^I_\Gamma - \kappa f^*h_\Gamma}{\Phi}, \quad (30)$$

where from equation (25) the equilibrium job-finding rate is given by $f^* = f(N^*)$, $f_N > 0$, and from equation (24) $h_\Gamma = \beta\Gamma^{-\sigma}(\sigma - 1)(1 - \delta)(1 + \nu(1 - f^*)) > 0$ and $h_f = \nu \beta \Gamma^{1-\sigma}(1 - \delta) > 0$, implying $\Phi \equiv \kappa f_N(f^*h_f + h(\Gamma, f^*)) > 0$. Moreover, $p^I_\Gamma > 0$ (cf. footnote 16).\(^{17}\) We see that the sign of $dN^*/d\Gamma$ depends on the sign of $p^I_\Gamma - \kappa f^*h_\Gamma$. The first term represents the markup effect of trend growth (it raises the intermediate good price) and the second multiplicative term represents the capitalization effect of trend growth (it raises the LMW).

We have the following propositions regarding the existence and uniqueness of a threshold rate of inflation $\tilde{\Pi}$.

**Proposition 1.** There exists a threshold level of inflation $1 < \tilde{\Pi} < \Pi^{\text{max}}$ such that in the neighborhood of $\tilde{\Pi}$ unemployment is increasing (decreasing) in trend growth for levels of inflation lower (higher) than $\tilde{\Pi}$.

**Proof.** See Appendix A.

**Proposition 2.** The threshold level of inflation $\tilde{\Pi}$ is unique if $\nu \leq 1$ or $f^* \leq 1/2$.

**Proof.** See Appendix B.

\(^{17}\)All partial derivatives are evaluated at the equilibrium steady-state employment.
For the case where $\nu > 1$ and $f^* > 1/2$, we are unable to prove uniqueness of the threshold inflation rate. In particular, the U.S. calibration falls into this category, given that the calibrated job-finding rate $f$ is 0.7 and given that the relative bargaining power of workers used in the literature ranges from as low as 0.05 ($\nu = 0.05$), as in Hagedorn and Manovskii (2008) to as high as 0.72 ($\nu = 2.6$), as in Shimer (2005). Under these alternative calibrations, we find that the inflation threshold is unique. In Section 4.2 below we demonstrate uniqueness numerically under alternative calibrations.\(^\text{18}\) But first we provide a graphical illustration of our main result.

### 4.1 Graphical illustration

Figure 1 illustrates the comparative static effects of higher growth on the equilibrium job finding rate. The left (right) hand panel of the figure shows comparative statics in an equilibrium with inflation lower (higher) than the threshold level $\tilde{\Pi}$. The equilibrium is determined by the intersection of the optimal hiring condition (22),

$$p^f(\Gamma, \Pi) = w^d + \kappa \left(1 - \beta \Gamma^{1-\sigma}(1 - \delta)\right) f,$$

and the wage setting equation (23)

$$w^d = b + \kappa \nu \left(1 - \beta \Gamma^{1-\sigma}(1 - \delta)(1 - f)\right) f,$$

where in both equations the cost per hire is substituted out using $g = \kappa f$. With knowledge of the job-finding rate, $f$, the employment rate can be inferred from equation (25).

Equations (31) and (32) are depicted in Figure 1 as a hiring curve and a wage curve, respectively. The hiring curve is downward sloping and linear in the $w^d, f$ space, while the wage curve is upward sloping and convex.\(^\text{19}\) A rise in productivity growth shifts the hiring curve rightward and rotates it clockwise. The rightward shift is due to the negative markup effect, whereby higher growth raises the relative price of the intermediate good (higher $p^f(\Gamma, \Pi)$ in equation (31)) and thus hiring. The clockwise rotation of the curve is due to the negative capitalization effect (lower $\Gamma^{1-\sigma}$ in equation (31)). In addition, a rise in productivity growth decreases the curvature of the wage curve with pivots at $f = 0$.

\(^{18}\)To our knowledge $\nu = 2.6$ is the only alternative calibration with $\nu > 1$ we could find from the labor search literature. Nevertheless, we have checked that even for values of $\nu$ as high as 14 we get a unique inflation threshold (values higher than $\nu = 14$ are not feasible since they imply that the model-based value of $b$ (value of non-work) is negative.).

\(^{19}\)This is because the rise in both the cost per hire and in the future job-finding rate reinforce each other in raising the wage rate.
and \( f = 1 \). This is due to the interaction of the negative capitalization effect with the level of the job finding rate \( (\Gamma^{1-\sigma}(1-f) \) in equation (32)).

In the left hand panel of Figure 1, with inflation lower than the threshold level, the rightward shift in the hiring curve caused by the markup effect is too small so that the clockwise rotation of the hiring curve caused by the capitalization effect dominates. In this case, the equilibrium job finding rate decreases and unemployment increases with trend growth. By contrast, in the right hand panel of the figure, with inflation higher than the threshold level, the rightward shift in the hiring curve caused by the markup effect is large enough to dominate the clockwise rotation in the hiring curve caused by the capitalization effect. In this case, the equilibrium job finding rate increases and unemployment decreases with trend growth.

### 4.2 Numerical illustration

We evaluate how the steady-state equilibrium unemployment \( u^* \) changes with productivity growth \( \Gamma \) using two alternative model calibrations at a quarterly frequency. In the first calibration the model’s equilibrium unemployment rate is relatively low and the job finding rate is relatively high (for e.g., as in the US) while in the second calibration the equilibrium unemployment rate is relatively high and the job finding rate is relatively low (for e.g., as in continental Europe). Consistent with these, the implied job separation rate is relatively

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There are two offsetting effects. On the one hand, a higher cost per hire, which is proportional to the job finding rate \( f \), strengthens the negative capitalization effect. On the other hand, a higher future job finding rate (i.e., lower \( 1-f \)) weakens the negative capitalization effect. The interaction with the negative capitalization effect vanishes as \( f \to 0 \) and as \( f \to 1 \).
low in continental Europe and relatively high in the US. Such a distinction reflects the notion that the continental European labor market is more sclerotic than that of the US labor market (Blanchard and Gali (2010)), and as we show below, the structure of the labor market affects the threshold level of inflation. In our empirical analysis the model-implied inflation thresholds are used to split the data into a low inflation and a high inflation regime.

Most of the labor market parameters are calibrated following Blanchard and Gali (2010). The exception is the value of non-market activity $b$, which we introduced following the growth-unemployment literature but which is absent in Blanchard and Gali (2010). In the US calibration the exogenous job separation rate $\delta$ is set equal to 0.12, which is consistent with an unemployment rate of 5% and a quarterly job finding rate of 0.7. In the European calibration the exogenous job separation rate is set at 0.04, which is consistent with an unemployment rate of 10% and a quarterly job finding rate of 0.25. While we choose equal bargaining power over wage setting ($\nu = 1$)—a standard assumption in the labor search literature—as a benchmark, we also allow for a higher value of $\nu = 2$ ($\eta = 0.72$ as in Shimer (2005)).

The value of the hiring cost parameter $\kappa$ is set such that in the steady-state equilibrium with a zero inflation rate the share of aggregate hiring costs in aggregate output is one percent. The implied value of $\kappa$ is 0.12 in the US calibration and one in the European calibration. Finally, the model’s implied return to non-market activity $b$ is 0.84 (0.82) in the benchmark US (European) calibration.

We set the gross productivity growth rate $\Gamma$ to 1.0075 for the US and 1.005 for Europe, implying an annualized productivity growth rate of 3% and 2%, respectively. These numbers are in line with long-term average growth rates (see, e.g., OECD (2003)). Finally, the inverse of the elasticity of intertemporal substitution $\sigma$ is set equal to 3, which is in line with Shimer (2010), who argues that reasonable values of $\sigma$ lie between 2 and 4. The rest of the model parameters take values very similar to the New-Keynesian literature: the subjective discount factor $\beta$ is set to 0.99, the elasticity of substitution between final differentiated goods $\theta$ is set to 11 (implying that firms choose a 10% price markup under flexible prices or when the inflation rate is zero) and the Calvo-parameter $\omega$ is set to 0.75 (prices are fixed on average for four quarters).

Results under US calibration. Figure 2 shows the threshold inflation rate under the bench-

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21 It is worth pointing out that the calibration of the bargaining parameter is subject to debate within the labor search literature (see, e.g., Hornstein, Krusell and Violante (2005)).

22 As Blanchard and Gali (2010) point out, one percent is a plausible upper bound given the lack of direct empirical evidence.
mark calibration (solid lines) and in the alternative case where \( \nu = 2.6 \) (dashed lines). In the left panel of the figure we plot the partial effect of productivity growth on the intermediate good price \( (p_{I}^\Gamma) \) and the partial effect of productivity on the labor market wedge \( (LMW_{\Gamma} = \kappa f^* h_{\Gamma}) \) against the (annualized) steady-state rate of inflation. As shown above, \( p_{I}^\Gamma \) captures the markup effect while \( LMW_{\Gamma} \) captures the capitalization effect. It can be seen that \( p_{I}^\Gamma \) (which is independent of the bargaining parameter \( \nu \)) increases monotonically with inflation while the \( LMW_{\Gamma} \) curves are nearly flat.\(^{23}\) There is a threshold rate of inflation of about 2.5% under the benchmark calibration, and about 3% under the alternative calibration, below which \( p_{I}^\Gamma < LMW_{\Gamma} \) (so that higher productivity growth increases equilibrium unemployment) and above which \( p_{I}^\Gamma > LMW_{\Gamma} \) (so that higher productivity growth decreases equilibrium unemployment).\(^{24}\)

![Figure 2](image)

**Figure 2:** The threshold inflation rate under the US calibration and alternative values of the bargaining power parameter (solid line \( \nu = 1 \); dashed line \( \nu = 2.6 \)). In the left panel the threshold inflation rate is the value of \( \pi \) at which the \( p_{I}^\Gamma \) and the \( LMW_{\Gamma} \) curves intersect. In the right panel the threshold inflation rate is the value of \( \pi \) at which the unemployment rate \( u^* \) is invariant to productivity growth \( \Gamma \) \( (du^*/d\Gamma = 0) \).

The right panel of Figure 2 plots the nature of the relationship between equilibrium unemployment and productivity growth \( (du^*/d\Gamma) \) as a function of the rate of inflation and illustrates our main result that the relationship between unemployment and growth depends on the level of inflation. For inflation rates below (above) 2.5% (respectively, 3%) unemployment and growth are positively (negatively) related.

**Results under European calibration.** The model with the European benchmark calibration, illustrated in Figure 3, has similar qualitative properties to that with the US calibration. The difference is quantitative: the threshold rate of inflation under the European cali-

\(^{23}\)Although not visible to the naked eye, \( LMW_{\Gamma} \) actually rises (falls) with inflation for inflation rates below (above) one percent.

\(^{24}\)See also the discussion in Appendix D.
bration, which is about 5.3%, is higher than that under the US benchmark calibration (2.5%). As can be seen from the left panel of Figure 3, the difference is mainly due to the capitalization effect (captured by the partial effect of growth on the LMW) being stronger under the European calibration.

Figure 3: The threshold inflation rate under the baseline European calibration. In the left panel the threshold inflation rate is the value of \( \pi \) at which the \( p^I_\Gamma \) and the \( LMW_\Gamma \) curves intersect. In the right panel the threshold inflation rate is the value of \( \pi \) at which the unemployment rate \( u^* \) is invariant to productivity growth \( \Gamma (du^*/d\Gamma = 0) \).

The higher inflation threshold under the European calibration reflects the fact that, relative to the US labor market, the European labor market is characterized by a more rigid labor market: a higher cost per hire \( g^* \), a lower job separation rate \( \delta \) and a lower job finding rate \( f^* \). The negative capitalization effect of growth is stronger (i.e., \( LMW_\Gamma \) is larger at any given inflation rate) the higher is the cost per hire, the lower is the job separation rate and the lower is the job finding rate (see Appendix D for details). These results show that, at any given rate of inflation, shocks, policies or institutions that contribute to a more rigid labor market also make it more likely that the negative capitalization effect dominates the markup effect. In this case it takes a higher rate of inflation for the markup effect to dominate the capitalization effect.

Table 1 below shows the steady-state unemployment rate implied by the model under the baseline calibration for a range of values of the annualized productivity growth rate and for two alternative values of the annualized trend inflation. The inflation rates (1.8% vs. 4.7% for the US and 2.4% vs. 8% for Europe) are the respective averages corresponding to low-inflation and high-inflation sub-samples (see section 5 below for details), which are constructed based on the threshold inflation rate of 2.5% under the US calibration and 5.3% under the European calibration.

We can see that in both economies, as productivity growth rises from 0% to 4%, the
Table 1: Model-implied equilibrium unemployment (%) for alternatives values of trend inflation and productivity growth rate

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<th>High-inflation regime</th>
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<td>Europe (2.4%)</td>
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<td>6.30</td>
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<td>6.50</td>
</tr>
<tr>
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<tr>
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<td>Europe (8.0%)</td>
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</table>

Notes: The inflation rates are the respective averages corresponding to low-inflation and high-inflation sub-samples, which are constructed based on the inflation thresholds—2.5% under the US calibration and 5.3% under the European calibration.

unemployment rate rises monotonically in the low-inflation regime but falls monotonically in the high-inflation regime. The unemployment effect of productivity growth is quantitatively larger in the high-inflation regime, reflecting a stronger markup effect. As the growth rate falls from 4% to 0%, the unemployment rate rises from 6.67% to 7.65% under the US calibration while it rises from 9.64% to 12.25% under the European calibration. In the low-inflation regime the response of unemployment to growth is quite small but is comparable to those found in similar studies—e.g., Aghion and Howitt (1994) and Pissarides and Vallanti (2007), who abstract from trend inflation.25

5 Empirical evidence

In this section we present some evidence regarding the dependence of the low-frequency comovement between unemployment and productivity growth on trend inflation. Using data for the US and the four largest European economies—Germany, France, Italy and the UK, we extract the low-frequency components of technology growth and unemployment data and examine their correlations.26

25In Aghion and Howitt (1994), unemployment varies between 5.7% and 6.15% as the growth rate rises from zero percent to 5%. Pissarides and Vallanti (2007) also find very small effects of growth on unemployment under the standard assumption of symmetric Nash bargaining. They find larger effects in the special case where the wage rate does not respond to labor market tightness (i.e., workers have no bargaining power over wage negotiations).

26In terms of methodology this approach follows Berentsen et al. (2011) who analyze the steady state link between unemployment and inflation by examining the correlations between their low-frequency components.
5.1 US data

For the US we use the annual purified technology series of Basu et al. (2006) to measure technology growth. The sample goes from 1949 to 1996 and is determined by the availability of the technology time series. As measures of unemployment and inflation, we use, respectively, annual data for the civilian unemployment rate from the BLS and the annual percentage change in the consumer price index. Following Ravn and Uhlig (2002) we use the HP-filter with a smoothing parameter of 6.25 to extract the low frequency trend of all three variables.\(^{27}\) We also make use of the NAIRU estimates of the Congressional Budget Office (CBO) as an alternative measure of steady state unemployment.

Based on the 2.5% trend inflation threshold computed under the baseline US calibration of the model, the 1950s and 1960s represent according to the HP-filtered inflation series a low inflation environment, while the period starting in the 1970s represents a high inflation environment.\(^{28}\) Figure 4 shows scatter plots of the two alternative measures of steady-state unemployment and HP-filtered technology growth for the low and the high inflation regime. Regression lines show results from simple OLS regressions of trend unemployment on trend growth and a constant. The slope coefficient and the respective p-values are also included in each graph. The data supports the predictions of the model.

As discussed in Appendix D the model-based inflation threshold changes depending on the labor market characteristics. For instance, under the US calibration the threshold inflation rate ranges from 2.3% to 3.2%. Therefore, we check robustness of the estimates for inflation thresholds ranging from 1.5% to 4.5%. We find that the negative relation between technology and unemployment in the high inflation regime is robust to variations of the inflation thresholds under consideration. For the low inflation regime, the positive relation between technology and unemployment holds for inflation thresholds up to 3%.\(^{29}\)

We also check robustness with respect to two alternative productivity measures, namely, labor productivity in the nonfarm business sector from the BLS and the TFP estimates by Fernald (2012). Unlike the purified technology measure these might include non-technological components like varying capital and labor utilization, non-constant returns,

\(^{27}\)This parameter used in annual data is equivalent to the standard parameter of 1600 used in quarterly data.
\(^{28}\)In this case the post-Volcker period lies in the high inflation regime. We also discuss results based on higher inflation thresholds.
\(^{29}\)Figures for all empirical results that are discussed, but not shown here, are available upon request.
imperfect competition and aggregation effects (Basu et al., 2006). Nevertheless, we find the same correlation patterns as when using the purified technology measure. Furthermore, the positive (negative) relation between growth and unemployment in the low (high) inflation regime is robust when varying the inflation threshold from 1.5% up to 4.5%.

5.2 European data

The model version calibrated to European labor markets implies an inflation threshold of 5.3%. Figure 5 shows the HP-filtered trend component of inflation based on CPI-data for Germany, France, Italy and the United Kingdom. There is quite some heterogeneity regarding trend inflation across countries. For Germany trend inflation hardly exceeds 5% even in the 1970s, while it reaches values up to 12% in France and even up to 17% in Italy and the UK. Hence, working with a 5.3% threshold for all four economies does not seem very useful. There is, however, for all four economies a period of relatively high inflation from 1970 to about 1990 and a period of relatively low inflation from 1990.
onwards. Hence, we simply split the sample in 1990.\footnote{Mean trend inflation rates in the high inflation sample are 3.9\%, 8.2\%, 11.8\%, and 10.0\% in Germany, France, Italy and the UK, respectively. Mean trend inflation rates in the low inflation regime are 2.0\%, 1.8\%, 3.1\%, and 2.6\%, respectively.}

![Trend Inflation in four European Economies](image)

**Figure 5: Trend Inflation in four European Economies**

There is no purified measure of technology growth available for a sufficiently long sample for the four European economies. Thus, we stick to measures of TFP and labor productivity. TFP is available from the Penn World Tables until 2011, which determines the end of the sample.\footnote{The data are reported relative to the US so that we combine them with the US TFP-series by Fernald (2012) to compute levels and growth rates of TFP for each of the four economies.} Data for unemployment rates are available from the OECD. As for the US case we use the HP-filter to extract the low-frequency component of the growth rate and the unemployment rate from annual data.

Figure 6 shows scatter plots of trend unemployment and trend TFP growth for the four European economies. As with the US data we can see that the negative correlation between unemployment and growth in the high inflation regime is very robust. The slope parameter of a simple OLS regression of trend unemployment on a constant and trend growth is negative and highly significant for all four economies. For the low inflation regime there is more uncertainty. We find a highly significant positive relation for France and Italy. For Germany the correlation is positive, though insignificant, while the correlation is very close to zero for the UK. Results are very similar when using labor productivity growth instead of TFP growth.

The related empirical literature is small and there is a lack of clear empirical evidence regarding the low-frequency comovement between growth and unemployment. The sign of the correlation varies depending on the time period and the country studied. For example, Caballero (1993) find a weak positive link between the low frequency components
Figure 6: Technology growth and unemployment in a low and a high inflation regime in four European countries.

of growth and unemployment for the UK, but a slightly negative link for the US. A number of studies are motivated by the simultaneous slowdown of productivity growth and rise in unemployment in industrial countries in the second half of the 1970s (see Pissarides and Vallanti, 2007, for an overview). Hence, empirical results possibly reflect this particular episode. For example, Bean and Pissarides (1993) find mild evidence for a negative link between growth and unemployment in OECD countries. They argue that the negative correlation is primarily driven by the 1975-1985 period in which in most countries productivity growth was lower and unemployment higher than in previous time. Similarly, we find a mild negative relation for the full sample for the US and the four European economies when we do not distinguish between a high and a low inflation regime.
Overall, we conclude that there is suggestive evidence that the model predictions are supported by the data. We point out, however, that one needs to be somewhat careful with the interpretation of the empirical results because of the limited number of observations, measurement problems in particular of technology growth and imperfect filtering of the data.

6 Summary and concluding remarks

Following the simultaneous slowdown in productivity growth and rising unemployment in many advanced economies during the 1970s academic research has sought to understand the effect of growth on steady-state unemployment using the standard the search model of unemployment. Past research has shown that when the intertemporal substitution in consumption is weak (a plausible assumption) search-type models of the labor market imply that disembodied technological progress leads to higher unemployment, a result at odds with the experience of the 1970s and recent empirical evidence (Pissarides and Vallanti (2007)).

Motivated by the observation that the 1970s were also characterized by high and rising inflation the present paper reexamines the effect of growth on unemployment in the presence of nominal price rigidity (implying a role for inflation). The analysis leads to a novel result: faster growth leads to lower unemployment if the rate of inflation is high enough. More generally, the paper shows that the effect of growth on unemployment may be positive or negative—there is a threshold level of inflation below (above) which faster growth leads to higher (lower) unemployment. The threshold level in turn depends on labor market characteristics—hiring efficiency, the job destruction rate, workers’ relative bargaining power and the opportunity cost of work—as is demonstrated by a model calibrated to the US and continental European economies. An empirical analysis shows that data for the US and the four largest European economies support the model predictions.

The model is kept as simple as possible (for instance, assuming an exogenous and disembodied technological progress) so as to focus on the role of nominal rigidities and present the results in a transparent way. A straightforward extension of the model is to allow for endogenous growth via learning-by-doing. For example, one could allow for a feedback from unemployment to growth (as in Aghion and Howitt (1994)) or introduce capital and assume positive externality from aggregate capital accumulation (as in Eriksson (1997)). However, the resulting model is no longer amenable to the comparative static analysis undertaken in the present paper, as then growth becomes an endogenous variable. One
can nevertheless study how growth and unemployment respond to structural parameters. While we use nominal price rigidity as a rationale for thinking about the real effects of inflation, as in much of the business cycle literature, alternative frameworks exist that rationalize the real effects of inflation. For example, Vaona (2013) studies the relationship between inflation and unemployment in a flexible price model with efficiency wages. For future work, it would be interesting to study the predictions for the growth-unemployment relation of such alternative models when augmenting them with productivity growth.

**Appendix A  Proof of Proposition 1**

From equation (30) the sign of $p^I - \kappa f^* h_f$ is the same as the sign of

$$J = \mu^{-1} \omega (\Pi - 1)^{\kappa_f^* - 1} \rho^n - \kappa f^* (1 - \delta) [1 + \nu (1 - f^*)]. \tag{A.1}$$

It is straightforward to check that $J$ is negative at $\Pi = 1$ and the first term of $J$ is positive for all $1 < \Pi < \Pi^{max}$. Moreover, the second term of $J$ approaches zero as $\Pi$ approaches $\Pi' \in (1, \Pi^{max})$. To show this we use equation (28) to rewrite the second term of $J$ as

$$\kappa f^* (1 - \delta) [1 + \nu (1 - f^*)] = \frac{(1 - \delta)(p^I - b)}{(1 + \nu)/(1 + \nu f^*) - \beta (1 - \delta) \Gamma^{1-\sigma}}. \tag{A.2}$$

where $0 < b < p^I \leq 1$, cf. (24) and (27), and the denominator is positive since $1 < (1 + \nu)/(1 + (1 - f^*)\nu) < 1 + \nu$ given an interior solution $0 < f^* < 1$. Since $\lim_{\Pi \rightarrow \Pi^{max}} p^I = 0$, cf. (27), it follows that $\lim_{\Pi \rightarrow \Pi'} p^I = b$.

Thus, $J$ becomes positive as $\Pi$ approaches $\Pi'$. Moreover, $J$ is continuous in $\Pi$ over the interval $[1, \Pi')$. It follows that there exists at least one solution $\tilde{\Pi}$ such that in the neighborhood of $\tilde{\Pi}$, $J$ is negative (positive) if $\Pi$ is lower (higher) than $\tilde{\Pi}$.

**Appendix B  Proof of Proposition 2**

From (A.1) the first term of $J$ is continuous and monotonically increasing in $\Pi$, while the second term of $J$ is increasing in $\Pi$ for $\Pi$ small enough and decreasing in $\Pi$ for $\Pi$ large enough if $\nu \leq 1$ or $f^* \leq 1/2$, since in this case the sign of

$$\frac{d\{\kappa f^* (1 - \delta) [1 + \nu (1 - f^*)]\}}{d\Pi} = \kappa (1 - \delta) [1 + \nu (1 - 2 f^*)] \frac{df^*}{d\Pi}.$$
is the same as the sign of $p_d$, as $d^*/d\Pi = p_d f_N/\Phi$, $f_N, \Phi > 0$, cf. equation (30). It follows that there exists a unique solution $\tilde{\Pi} \in (1, \Pi')$ such that $J$ is negative (positive) if $\Pi$ is lower (higher) than $\tilde{\Pi}$.

**Appendix C  Introducing disutility from work**

Following Shimer (2010) we use a functional form of the household’s utility function that is consistent with balanced growth (as first pointed out by King, Plosser and Rebelo (1988)). The household utility is derived from the optimal allocation of consumption across employed and unemployed members, where the period utility of a non-working member is $\frac{C_t^{1-\sigma} - 1}{1 - \sigma}$ and that of a working member is $\frac{C_t^{1-\sigma} (1 + (\sigma - 1) \chi) f N^{\sigma}}{1 - \sigma}$. Here, $\chi > 0$ represents the disutility of work. Under the optimal allocation of consumption the household’s utility function takes the form $U(C_t, N_t) = \frac{C_t^{1-\sigma} (1 + (\sigma - 1) \chi N_t) f N_{N_t}^{\sigma}}{1 - \sigma}$, where $C_t = N_t C_{e,t} + (1 - N_t) C_{u,t}$ is average consumption.\(^{32}\)

In the presence of disutility of work the flow value of an employed worker is the wage rate $w^d$ net of the marginal rate of substitution between consumption and work: $\text{mrs} = \frac{\sigma \chi}{1 + (\sigma - 1) \chi N_t}$. Then the analogue of the wage setting equation (23) is

$$w^d = \text{mrs} + \nu \left(1 - \beta \Gamma_{N_t}^{1-\sigma} (1 - \delta) (1 - f)\right) g,$$

(A.3)

which shows that $\text{mrs}$ represents the lower floor for the wage rate. Correspondingly, the analogue of identity (29) is

$$F(N^*, \Gamma, \Pi) \equiv p^f(\Gamma, \Pi) - \kappa f(N^*) h(\Gamma, f(N^*)) - \text{mrs}(N^*) \equiv 0,$$

(A.4)

where we have rewritten $\text{mrs}$ using the aggregate resource constraint $c = y - hc = \Delta^{-1} - \kappa \delta f(N^*) N_t$. Then $\frac{dN^*}{dt}$ takes the same form as equation (30) with $\Phi$ modified as

$$\Phi \equiv \kappa f_N (f^* h_f + h(\Gamma, f^*)) + \text{mrs}_N,$$

$$\text{mrs}_N \equiv \chi \sigma \left\{ \Delta^{-1} - \delta \kappa \left[ f^* + (1 + (\sigma - 1) \chi N^*) f_N N^* \right] \right\} / (1 + (\sigma - 1) \chi N^*).$$

A sufficient condition for $\Phi$ to be positive is $\chi N^* \leq 1.\(^{33}\) Given that $0 < N^* < 1$ this condition is not that restrictive. For instance, Shimer (2010) calibrates $\chi$ to 0.4 while in

\(^{32}\)Under the special case of $\sigma = 1$, the utility function reduces to $U(C_t, N_t) = \log C_t - \chi N_t$.

\(^{33}\)To see this rewrite $\Phi$ as

$$\frac{\chi \sigma (\Delta^{-1} - \delta \kappa f^*)}{1 + (\sigma - 1) \chi N^*} + \kappa f_N \left[ f^* h_f + h(\Gamma, f^*) - \frac{\sigma \delta \chi N^*}{1 + (\sigma - 1) \chi N^*} \right].$$

27
our case $\chi \approx 0.68$ both in the US and European calibrations. Thus, in the plausible case of $\Phi > 0$ the sign of $\frac{dN^*}{d\Gamma}$ is determined, as in the case with unemployment benefits, by the sign of the numerator of equation (30). Moreover, since the inflation threshold depends on the equilibrium level of the job-finding rate $f^*$, the disutility of work affects the level of the inflation threshold by affecting $f^*$. We find that the inflation threshold is about 2.5% in the US calibration and about 5.5% in the European calibration, values that are almost identical to those under the model with unemployment benefits.

**Appendix D  Sensitivity of the threshold level of inflation to labor market parameters**

In this appendix, we examine how the threshold level of inflation are influenced by changes in the exogenous labor market parameters—the scale parameter $\kappa$ in the cost per hire $G_t$ (see equation (5)), the job separation rate $\delta$, workers’ relative bargaining power $\nu$ and the opportunity cost of work $b$. From equation (30) these labor market parameters affect $dN^*/d\Gamma$ by influencing the magnitude of $LMW_\Gamma$, which we rewrite as follows

$$LMW_\Gamma = \beta\Gamma^{-\sigma}(\sigma - 1)(1-\delta)\kappa f^* + \beta\Gamma^{-\theta}(\sigma - 1)(1-\delta)\nu(1-f^*)\kappa f^*. \quad (A.5)$$

The first right hand side term in equation (A.5) captures the effect of growth on the wedge between the marginal revenue product and the real wage (see equation (9)). The second term captures the effect of growth on the wedge between the real wage and the opportunity cost of work (see equation (14)). From equation (9), all else equal, the expected future savings in hiring costs decline with trend growth. The decline in future savings in hiring costs (and in turn the rise in the wedge between the marginal revenue product and the real wage) is more pronounced the higher is the cost per hire, $g^* = \kappa f^*$, and the higher is the job retention rate $1 - \delta$. Likewise, from equation (14), all else equal, the continuation value to an employed worker declines with trend growth. The decline in the continuation value (and in turn the rise in the wedge between the real wage and the opportunity cost of work) is more pronounced the higher is the cost per hire, the higher is the probability of not finding a job in the case of job separation, $1 - f^*$, and the higher is the job retention rate $1 - \delta$. The total derivative of equation (A.5) with respect to parameter $z \in \{\kappa, \nu, \delta, b\}$

The first term is restricted to be nonnegative so as to rule out negative consumption. A sufficient condition for the second term to be positive is that $\chi N^* \leq 1$, since $h_f > 0$, cf. equation (30), and $h(\Gamma, f^*) > \delta$, cf. equation (24).
is given by
\[
\frac{dLMW_\Gamma}{dz} = LMW_{\Gamma,z} + LMW_{\Gamma,f} \frac{df^*}{dz}.
\]
The term \(LMW_{\Gamma,z}\) captures the direct effect of \(z\). The second multiplicative term captures the indirect effect, where \(LMW_{\Gamma,f} = \kappa \beta \Gamma^{-\sigma}(\sigma - 1)(1 - \delta)(1 + \nu(1 - 2f^*)) > 0\) if \(f^* < 0.5\) (satisfied under the European benchmark calibration) or \(\nu = 1\) (satisfied under the US and European benchmark calibrations). The overall effect depends on the signs and magnitudes of \(LMW_{\Gamma,z}\) and \(df^*/dz\). Table D below shows the sign of the effects on the unemployment rate \(u^*\), the job finding rate \(f^*\) and the threshold rate of inflation \(\tilde{\pi}\) of an increase in the value of one parameter (\(\kappa\), \(\nu\), \(\delta\) or \(b\)) while keeping the rest of model parameters at their respective baseline values, as discussed in the previous section. The comparative statics are similar under the US and European calibrations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(u^*)</th>
<th>(f^*)</th>
<th>(\tilde{\pi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\kappa)</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(\nu)</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(\delta)</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(b)</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note first the comparative static effects of the four labor market parameters on the unemployment rate and the job finding rate. For instance, the larger is the cost per hire \(\kappa\) the higher is the unemployment rate and the lower is the job finding rate. An increase in the separation rate \(\delta\), an increase in the relative bargaining power of workers \(\nu\) or an increase in the non-work value \(b\) have similar effects. These are standard properties of search models of unemployment. Below we discuss the comparative static effects on the threshold level of inflation.

**Changes in the scale parameter** \(\kappa\): The larger is the scale parameter \(\kappa\) the higher is the cost per hire and thus the stronger the negative capitalization effect of growth—the reduction in the discounted savings in future hiring costs from current hiring. It thus takes a higher inflation rate for the negative markup effect, which is independent of the labor market structure, to dominate the negative capitalization effect. To see the quantitative significance, for instance, under the US calibration, increasing \(\kappa\) from the benchmark value of 0.12 to 0.24 (so that hiring costs represent 2% of GDP compared to one percent under the benchmark calibration) increases the threshold inflation rate noticeably—from 2.5% to 3.2%.
Changes in the bargaining parameter $\nu$: All else equal, higher growth decreases the continuation value to an employed worker, and as a result, raises the wage rate at any given employment. The reduction in the continuation value is larger and the increase in the wage rate is stronger the larger is the relative bargaining power of workers (i.e., the larger is the workers’ share of the surplus from an employment relationship). Thus higher bargaining power of workers reinforces the negative capitalization effect of growth. It thus takes a higher inflation rate for the negative markup effect to dominate the negative capitalization effect. Under the US calibration, increasing $\nu$ from the standard value of one (symmetric bargaining power) to 2.6 (Shimer (2005)) increases the threshold inflation rate to 2.7%, which is quite similar to the baseline value.

Changes in the separation rate $\delta$: The higher is the separation rate the less does hiring depend on the future value of an employment relationship. As a result, the less important is discounting and thus the weaker is the negative capitalization effect of growth. It thus takes a lower inflation rate for the negative markup effect to dominate the negative capitalization effect. Under the US calibration, increasing $\delta$ from the benchmark value of 0.12 to 0.24 (Hall (2005)) decreases the threshold inflation rate to 2.3%. Thus, the threshold inflation rate is somewhat insensitive to changes in the separation rate.

Changes in the non-work value $b$: The higher is the value of non-work $b$ the less does the wage rate rise in response to higher growth, and as a result, the weaker is the negative capitalization effect of growth. It thus takes a lower inflation rate for the negative markup effect to dominate the negative capitalization effect. Again, under the US calibration, increasing $b$ from the benchmark value 0.84 to 0.86 (implying a replacement ratio of 0.95, as in Hagedorn and Manovskii (2008)) lowers the threshold inflation rate to 2.3%.

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