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DISCUSSION PAPER 08-2017

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The Lost Race Against the Machine: Automation, Education, and Inequality in an R&D-based Growth Model[∗]

Klaus Prettner† Holger Strulik‡

March 2017

Abstract. We analyze the effect of automation on economic growth and inequality in an R&D-based growth model with two types of labor: highskilled labor that is complementary to machines and low-skilled labor that is a substitute for machines. The model predicts that innovationdriven growth leads to increasing automation, an increasing skill premium, an increasing population share of graduates, increasing income and wealth inequality, a declining labor share, and (in an extension of the basic model) increasing unemployment. In contrast to Piketty's famous claim that faster economic growth reduces inequality, our theory predicts that faster economic growth promotes inequality.

Keywords: Automation, R&D-Based Growth, Inequality, Wealth Concentration.

JEL: E23, E25, O31, O33, O40.

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1. INTRODUCTION

Common wisdom in growth and labor economics suggests that technological progress is labor-augmenting. Technological progress developed by market R&D and incorporated in new machines is supposed to complement human work effort and to make workers more productive (see, for example, Jones, 2005). In this paper we look at the dark side of R&D-promoted technological change. We consider the problem that new technologies complement only some workers and render other workers redundant. Specifically, we focus on technological progress understood as automation and machines understood as robots and other devices that replace human labor. This conforms with the very definition of automation as "automatically controlled operation of an apparatus, process, or system by mechanical or electronic devices that take the place [emphasis added] of human labor" (Merriam-Webster, 2017). Examples for automation technologies that have received prominent media coverage in the last few years include industrial robots that are more and more able to replace human workers on assembly lines; driverless cars, lorries, and delivery robots that could soon transport goods and people between locations without the need for any human involvement; 3D printers that produce highly customized (and, thus, previously very labor-intensive) products such as hearing aids, prostheses, and even houses.¹ As far as industrial robots are concerned, the International Federation of Robotics refers to them as "automated, programmable and capable of movement on two or more axes. Typical applications of robots include welding, painting, assembly, pick and place for printed circuit boards, packaging and labeling, palletizing, product inspection, and testing; all accomplished with high endurance, speed, and precision" (IFR, 2015). In short, robots are built to replace human labor.

Since (at least at the current state of technology) high-skilled labor is more difficult to automate than low-skilled labor, people may avoid the perils of technological progress and enjoy its benefits by upgrading their skills. We thus integrate an education decision into an R&D-based growth theory with automation. We show that an increasing skill premium due to automation motivates an increasing share of people to obtain higher education (a college degree). However, in a heterogeneous society, not

¹ See, for example, The Economist $(2014, 2017)$, Abeliansky et al. (2015) , The Guardian (2015), and Brynjolfsson and McAfee (2016).

all people are equally able to obtain higher education. Due to time (or effort-) constraints, some individuals fail to acquire higher education and are left behind. This way, R&D-based growth leads to increasing income and wealth inequality and (in an extension of the model) to increasing unemployment.

The phenomenon of large-scale automation in manufacturing is a relatively recent phenomenon. The foundation of the first company to produce industrial robots took place in the same year as Solow (1956) founded neoclassical growth theory, which established as a key assumption that technological progress complements human labor. But only after another 40 years, industrial robots production really took off. As shown in Figure 1, this take-off happened at around the same time when the "new growth theory" of the 1990s endogenized technological progress, maintaining the basic assumption that new machines are labor-augmenting (Romer, 1990; Aghion and Howitt, 1992).

Similar to conventional R&D-based growth theory, we focus on the manufacturing sector as the driver of TFP and income growth.² We agree with the earlier literature that the notion of labor-complementing technological progress has been a reasonable assumption to describe the industrial past until the 1990s. A picture of assembly line production in Henry Ford's motor company could be a useful visual analogy. But the notion of across-theboard labor-complementing technological progress seems to be less suited to describe modern R&D-driven growth, where machines largely replaced human labor in industrial production. Here, a picture of any modern car factory could be a useful visual analogy (to see this, try "modern car factory" in google images). A future-oriented theory of R&D-based growth should thus take into account that only some workers benefit from automation and new machines, while others are left behind.³

The idea of labor-complementing technological progress is maintained in the (otherwise much related) literature on skill-biased technical change (Acemoglu, 2002). The most popular discussion of skill-biased technological progress is perhaps provided by Goldin and Katz (2009) who argue

 $^2\,$ We briefly comment on automation in the service sector in the Conclusion.

³ In this respect, the history of growth economics may appear reminiscent of Malthus' (1798) "Principle of Population", which provided a very detailed explanation of historical developments but largely failed to describe future developments because, shortly after its publication, the fertility transition set in.

Worldwide estimated operational stock of industrial robots

Source: IFR World Robotics 2015

that America has lost the "Race between Education and Technology" because high-school completion rates stagnate since the 1950s. However, as emphasized by Acemoglu and Autor (2009), this loss is of relative nature because the underlying model assumes that unskilled labor also benefits from innovations but "only" to a lower degree than skilled labor. Here, by contrast, we conceptualize tertiary educated workers as high-skilled labor (with a college degree), who are complements to machines and whose wages continued to increase throughout the 20th century, and focus on lowskilled workers as "absolute losers" of technological change in the form of new machines because they are substitutes instead of complements.

The idea of labor-substituting technological progress, understood as automation, has been popularized by Brynjolfsson and McAfee's (2011) book on another race, the "Race against the Machine". Our paper could be seen as an attempt to formalize some core ideas of the book in the language of growth economics. Specifically, Brynjolfsson and McAfee claim that recent R&D-based innovations simultaneously boost the productivity of firms and eliminate the need for many forms of human labor. Technological progress as automation thus makes people more innovative, productive and richer (as in the earlier new growth theories) but at the cost of increasing unemployment and (wealth) inequality in society. Early quantitative evidence for this view stems from Berman et al. (1998) who show that around 70 percent of the decline in production workers' share in the wage bill can be explained by R&D and computerization. More recently, Graetz and Michaels (2016) provide evidence that industrial robots lead to a reduction in the demand for low-skilled labor, Frey and Osborne (2013) show that, while a large proportion of the U.S. labor force is susceptible to automation,

the average educational attainment of an occupation and the probability of this occupation to be automated are highly negatively correlated, and Arntz et al. (2016) explain that low-skilled workers perform tasks that are typically much easier to automate than the tasks performed by high-skilled workers.⁴

Recently, a couple of theoretical papers investigated automation in the context of long-run development. Hémous and Olsen (2016) and Acemoglu and Restrepo (2016) are perhaps the most closely related ones. Like us, both studies focus on R&D-based innovations and inequality in the process of economic growth. In both studies the household side of the economy is somewhat simpler since there is no education decision and skills are taken as given for infinitely living individuals. The production side, however, is more complex in both studies and differs crucially from ours. In both studies, R&D-based innovations play a more favorable role as in our theory. Specifically, final goods are assumed to be produced by a variety of intermediate goods (Hémous and Olsen) or a variety of tasks (Acemoglu and Restrepo). Varieties are produced by labor and potentially by (low-skilled) labor replacing machines. R&D generates new varieties which start out as un-automated. The individual firms may then spend costly in-house effort to automate the production of the variety supply. As a result, (low-skilled) wages benefit from R&D-based innovations and are potentially harmed by the in-house automation process. However, more productive automation could be even good for (low-skilled) wages because it encourages more R&D.

In our theory, R&D is conceptualized as the process that creates the very machines that automate low-skilled labor in production. Maintaining the image of the car factory created above, we assume that R&D produces new machines (robots) that increase productivity and simultaneously substitute low-skilled labor in car production. Hémous and Olsen and Acemoglu and Restrepo assume that R&D creates new car parts or new tasks in the production of cars, which start out un-automated and are potentially later automated by in-house effort of the car part/task producing firm. Acknowledging that it is plausible that R&D does both of these things, our theory is complementing the existing literature. It provides a more direct and less benign view on the role of R&D, which we think is more

⁴ See David (2015) and Eden and Gaggl (2016) for further evidence.

appropriate to formalize Brynjolfsson and McAfee's (2011) ideas on the "Race against the Machine".⁵

The interaction between technology, wages, and education relates our paper to the unified growth literature, where one of the core mechanisms is the rise of education triggered by technological progress (Galor and Weil, 2000; Galor and Moav, 2002; Galor, 2005; 2011). In contrast to this literature, we focus on tertiary education, R&D-based growth, and automation through new technologies. In an earlier study (Strulik et al., 2013), we constructed an overlapping generations version of the Romer (1990)–Jones (1995) R&D-based growth model with an endogenous education and fertility decision to discuss long-run adjustment processes. However, we did not consider automation and the evolution of inequality.

Our paper also contributes to the long-standing debate on the interaction between inequality and economic growth. While the earlier theoretical literature focused mainly on the causality running from inequality to growth and empirical studies found a negative association (Persson and Tabellini, 1994; Alesina and Rodrik, 1994; Aghion et al., 1999), the literature related to skill-biased technical change (cited above) argues in favor of a causality running from growth to inequality and suggests a positive association. Recently, Piketty (2014) has popularized the view that economic growth reduces inequality in the context of the neoclassical growth model and a stratified population. Here, we argue that R&D-based growth theory in conjunction with automation provides a "non-Pikettarian" result: we show that, ceteris paribus, faster growth is predicted to lead to more inequality in labor income and wealth. This finding, however, does not imply that there is no threat from automation when the growth rate of factor productivity declines. As long as R&D-based growth is positive, automation causes inequality to rise. Along the transition we can then observe a negative association between growth and inequality because growth is declining, while inequality is on the rise. We show this outcome by simulating a calibration of the model with U.S. data.

⁵ Other, for various reasons less related studies on automation and macroeconomic performance are provided by Zeira (2010), Steigum (2011), Sachs and Kotlikoff (2012), Benzell et al. (2015), Sachs et al. (2015), Abeliansky and Prettner (2017), Gasteiger and Prettner (2017), and Prettner (2017). Most of these studies do not explain technological progress endogenously. Exceptions are Zeira (2010) and Peretto and Seater (2013), which, however, do not address inequality issues.

The paper proceeds as follows. In the next section we set up the basic model of R&D-driven automation. In Section 3 we take the education system as given and provide a series of analytical results on growth and various aspect of inequality (along the balanced growth path). The full model with growth–education interaction can only be solved numerically, which is what we do in Section 4. We discuss two alternative scenarios. First, we follow the conventional approach in growth economics and calibrate an economy with positive long-run growth and increasing growth along the (historical) adjustment path. Second, we consider an economy where total factor productivity is gradually declining. This computational experiment is important because it has been argued that automation should be observed in conjunction with rising TFP growth such that there is little threat from automation when actual TFP growth rates are declining. We show that the model refutes this view. The key insight here is that increasing automation and inequality require positive but not necessarily increasing TFP growth. In Section 5 we augment the model by a social welfare system and a labor supply decision and show how increasing automation can induce unemployment. Section 6 concludes.

2. The Model

2.1. Basic Assumptions. Consider an overlapping generations economy in which individuals live for three time periods. In the first period of their lives, individuals receive basic education and decide whether or not to acquire higher skills. In the second period, workers supply labor on the labor market and save for the third period, when they are retired. After the third period, individuals die with certainty. According to the education decision, there are two types of workers: i) c-types supply higher skills that are difficult to automate. They are complements to machines. ii) s-types supply lower skills that are easy to automate. They are substitutes for machines. A helpful (non-necessary) interpretation would be that s-types are individuals with high-school education or less. For simplicity, we ignore low-skilled, non-routine jobs that are (yet) difficult to automate (see e.g. Autor and Dorn, 2013) as well as the automation of some high-skilled jobs by artificial intelligence. Including these features would provide more realism of our stylized model but would not change the main mechanics and the main results.

There are three production factors, the two types of labor described above and physical capital in the form of machines and robots. Workers who are easy to automate can only be employed in the final goods sector for tasks that can also be performed by machines. Workers who are difficult to automate can be employed as workers in the final goods sector responsible for tasks that cannot be easily automated (managers and engineers) or as workers in the R&D sector for developing new technologies (scientists). The split of complementary labor between the final goods sector and the R&D sector is endogenous as in Romer (1990) . Time t evolves discretely with each time step capturing one generation. The working population is of size L_t and grows at rate $n \geq 0$.

2.2. Savings Decision. In the second period of their life, individuals earn a wage income from (inelastic) labor supply and make a consumptionsavings decision. In period t , the remaining lifetime utility of working-age individuals of type $j = c, s$ is given by

$$
u_t = \log(c_{j,t}) + \beta \log(\bar{R}s_{j,t}).\tag{1}
$$

in which $c_{i,t}$ is working-age consumption of the generation born at time $t-1$, R is the gross interest rate paid on savings carried over from the second period of life to the third, and $s_{j,t}$ denotes savings such that $c_{j,t+1} = \overline{R}s_{j,t}$ refers to consumption in the third period of life. For simplicity we assume that the economy is comparably small and open to international capital flows such that the interest rate is determined at the world market. The budget constraint that each individual $j = c, s$ faces in the second period of life is standard and given by

$$
w_{j,t} = c_{j,t} + s_{j,t} \tag{2}
$$

such that individuals can spend their wage income in the second period $(w_{i,t})$ on consumption in the second period or to build up assets by saving to finance consumption in the third period. Maximizing utility (1) subject to the budget constraint (2) leads to optimal consumption and optimal savings as

$$
c_{j,t} = \frac{w_{j,t}}{1+\beta}, \qquad s_{j,t} = \frac{\beta w_{j,t}}{1+\beta},
$$
 (3)

where $\beta/(1+\beta)$ is the savings rate of both types of workers.

2.3. Education Decision. In the first period of their life, individuals do not yet supply labor and receive consumption by their parents. In this phase, they decide upon their education level. While everybody receives a baseline education that allows for labor-market participation in general, the decision of whether or not to acquire higher skills so as to become a manager, an engineer or a scientist depends on the abilities of individuals and the observed wage gap between low-skilled and high-skilled labor (for a related skill-upgrading choice, see Cervellati and Sunde, 2005). Suppose that learning ability is uniformly distributed in the interval $(0, 1)$. Apparently, not all members of society are willing and capable to obtain higher education (a college degree). We model this feature conveniently be assuming that the time-cost (or effort) for higher education is a function of ability and that exerting effort (losing leisure time) causes disutility.

Specifically, suppose education effort is given by $e = \psi/a - \theta$ such that $\lim_{a\to 0} e = \infty$. Suppose young individuals have one unit of time at their disposal and that disutility from effort is given by $B \log(1 - e)$. Individuals fail to take up higher education if the disutility from exerting effort exceeds the expected utility gain that higher education provides at working age and in retirement. From (1) and (2) we obtain indirect utility $u_t = \log[w_{j,t}/(1+\beta)] + \beta \log[\bar{R}\beta w_{j,t}/(1+\beta)]$ such that the utility gain from higher education is $(1 + \beta) \log(w_{c,t}/w_{s,t})$. Thus, individuals fail to take up education if effort disutility is larger in absolute terms than the discounted consumption utility gain, i.e., if

$$
-B\log(1 - e) \ge \beta(1 + \beta)\log\left(\frac{w_{c,t}}{w_{s,t}}\right).
$$

Inserting ability-dependent effort and solving for ability, we obtain that individuals fail to take up higher education if

$$
a \le \frac{\psi}{1 + \theta - e^{\beta(1 + \beta)/B} \tilde{w}_t} \equiv f(\tilde{w}_t), \tag{4}
$$

in which $\tilde{w}_t \equiv w_{s,t}/w_{c,t}$ is the relative wage of workers with lower education and $f(\tilde{w}_t)$ is the ability threshold. The threshold is originating from $\psi/[1 + \theta - \exp[\beta(1 + \beta)/B]\tilde{w}_t]$, it is increasing and convex and exhibits a pole where $\tilde{w}_t = (1 + \theta) \exp[-\beta(1 + \beta)/B]$. To avoid unnecessary case differentiation, we assume that the weight of effort B is such that we are always to the left of the pole. Figure 1 displays the ability threshold. Recall that ability is uniformly distributed in the interval $(0, 1)$ to see that the population share of individuals with higher education is given by $1 - f(\tilde{w}_t)$ if there is higher education. This means that, generally, the workforce with higher education is given by

$$
L_{c,t} = \left[1 - f(\tilde{w}_t)\right] L_t.
$$
\n⁽⁵⁾

Individuals with ability a below the threshold $f(\tilde{w})$ remain without higher education. There are $[1-f(\tilde{w})]L_t$ individuals with higher education.

Obviously, $L_{s,t} = L_t - L_{c,t} = f(\tilde{w}_t)L_t$ individuals remain without higher education. Notice how the race between technology and education (Goldin and Katz, 2009) is captured in Figure 1. With skill-biased technological change, \tilde{w} , i.e., the inverse of the skill premium, gradually declines to zero. This means that a larger population share of individuals is motivated to take up higher education. Asymptotically, the society converges towards a situation, where a population share of $a_{min} \equiv \psi/(1 + \theta)$ remains without higher education because their learning ability is too low to obtain a college degree in finite time.

2.4. Population Growth. The evolution of the cohort size is governed by the exogenous population growth rate $n \geq 0$ and the workforce at time t evolves as

$$
L_{t+1} = (1+n)L_t.
$$

In the basic model we abstract from unemployment such that L_t refers to aggregate employment. In a growing economy, the threshold a_{min} is reached in finite time. From then onwards, the population shares $\ell_{c,t} = L_{c,t}/L_t$ and $\ell_{s,t} = L_{s,t}/L_t$ stay constant and the economy potentially grows along a balanced growth path (as discussed below).

2.5. Final Goods Production. The production side of the economy builds upon Romer (1990) and Jones (1995) whereby we distinguish between the two cases by setting the parameters accordingly. Aggregate output is produced with physical capital in the form of machines and with both types of labor according to the production function

$$
Y_t = L_{c,Y,t}^{1-\alpha} \left(L_{s,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha} \right),
$$
 (6)

where $L_{c,Y,t}$ is the part of high-skilled labor that is employed in the final goods sector, $x_{i,t}$ are machines of the specific type $i, \alpha \in (0,1)$ denotes the elasticity of output with respect to human labor that can easily be automated, and A_t is the stock of specific blueprints available for the associated machines of type i , i.e., it represents the technological frontier of the country under consideration. Technological progress is conceptualized as increasing variety of machines in production. And the growth rate of A is later associated with TFP growth.⁶

The factor rewards are

$$
w_{c,Y,t} = (1 - \alpha) L_{c,Y,t}^{-\alpha} \left(L_{s,t}^{\alpha} + \sum_{i=1}^{A_t} x_{i,t}^{\alpha} \right) \quad \Leftrightarrow \quad w_{c,t} = (1 - \alpha) \frac{Y_t}{L_{c,Y,t}},
$$
\n
$$
w_{s,t} = \alpha (L_{c,Y,t}/L_{s,t})^{1-\alpha},
$$
\n(8)

$$
\begin{array}{rcl}\n\alpha_{s,t} & = & \alpha(\Delta c, r, t / \Delta s, t) \\
\vdots & & \vdots \\
\alpha_{s,t} & = & \alpha \int_{0}^{1 - \alpha} r^{\alpha - 1} \tag{0}\n\end{array}
$$

$$
p_{i,t} = \alpha L_{c,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}, \tag{9}
$$

The key difference with respect to the related literature is that only the marginal value product of type-c labor increases with the employment of machines, while the marginal value product of type-s labor is unaffected by machines. In other words, in contrast to earlier studies, we introduce a type of labor, L_s , for which machines are a perfect substitute, which conforms to the very definition of automation. This means that technological progress (TFP growth) has a fundamentally different impact on the two types of labor. As commonly assumed, it increases the productivity of complementing labor L_c and is in this sense quasi-labor augmenting. However, it leaves productivity of substitutable labor L_s unaffected such that the relative importance of this type of labor declines with technological progress.

⁶ Alternatively, we could have used a quality-ladder model (following Aghion and Howitt (1992), which would be (in reduced-form) equivalent to the variety approach.

2.6. R&D Sector. The R&D sector produces blueprints for new machines $A_{t+1}-A_t$ by employing scientists that are recruited from high-skilled labor. The production function of the R&D sector is given by

$$
A_{t+1} - A_t = \overline{\delta}_t L_{c,A,t},
$$

where $L_{c,A,t}$ denotes scientists employed in the R&D sector and $\bar{\delta}$ is the productivity of these scientists. The productivity level of scientists itself depends on intertemporal knowledge spillovers (the standing-on-giantsshoulders externality) and on congestion effects (the stepping-on-toes externality) as described by Jones (1995). We follow the standard approach and write

$$
\bar{\delta}_t = \frac{\delta A_t^{\phi}}{L_{c,A,t}^{1-\lambda}},
$$

where $\phi \in (0, 1]$ measures the strength of intertemporal knowledge spillovers and $1-\lambda$ with $\lambda \in [0,1]$ measures the strength of the congestion externality. Notice that these parameter restrictions for ϕ and λ allow for the Romer (1990) case of $\phi = \lambda = 1$ and for the Jones (1995) case of $\phi, \lambda \in (0, 1)$.

Profits in the R&D sector are given by the revenue that R&D firms generate by selling the patents they developed net of the costs for the scientists that they employed,

$$
p_{A,t}\overline{\delta}_t L_{c,A,t} - w_{c,A,t}L_{c,A,t},\tag{10}
$$

where $p_{A,t}$ is the price of blueprints and $w_{c,A,t}$ denotes the wage rate of scientists. Due to the competitive labor market, the wage rate of scientists attains the same level as the wage rate for type-c workers in the final goods sector. R&D firms maximize profits by choosing optimal R&D employment, which provides the optimality condition $w_{c,A,t} = \bar{\delta}_t p_{A,t}$. Our overlapping generations structure allows us to introduce a finite patent length of one generation, which is reasonably close to the actual patent duration of approximately 20 years (United States Patent and Trademark Office, 2017).

2.7. Intermediate Goods Sector. The intermediate goods sector uses physical capital as variable input factor to produce machines. The production function is linear with a unitary capital input coefficient such that $x_{i,t} = k_{i,t}$, where $k_{i,t}$ is the amount of physical capital employed by each intermediate goods producer. There are 2 types of intermediate goods producers. Producers of the latest vintage of machines use a blueprint (patent) from the R&D sector as fixed input. These firms have a certain degree of market power and free entry into the intermediate goods sector implies that operating profits in period t, $\pi_{i,t}$, are equal to the entry costs consisting of the price that has to be paid up-front for the blueprint such that

$$
\pi_{i,t} = p_{A,t}.\tag{11}
$$

Producers of older vintages of machines are no longer protected by patent law and free entry ensures that a zero profit condition holds. Henceforth we index variables associated with the latest vintage of machines by i and variables associated with earlier vintages by j. Operating profits for latest vintage producers are given by

$$
\pi_{i,t} = p_{i,t}(x_{i,t})x_{i,t} - \bar{R}x_{i,t}.
$$
\n(12)

Profit maximization implies

$$
p'_{i,t}(x_{i,t})\frac{x_{i,t}}{p_{i,t}} + 1 = \frac{\bar{R}}{p_{i,t}} \qquad \Rightarrow \qquad p_{i,t} = \frac{\bar{R}}{\alpha}.
$$
 (13)

Producers of the latest vintage of machines charge a markup over marginal cost and the production of machines of type i adjusts (due to capital inflow/outflow) up to the point at which $\bar{R} = \alpha^2 L_{c,Y,t}^{1-\alpha} x_{i,t}^{\alpha-1}$. Producers of older vintages charge prices at marginal costs $p_{i,t} = \overline{R}$, for all j such that the production of machines of type j adjusts (again due to capital inflow/outflow) up to the point at which $\bar{R} = \alpha L_{c,Y,t}^{1-\alpha} x_{j,t}^{\alpha-1}$. Combining both demand functions provides the input ratio

$$
x_{j,t} = \alpha^{\frac{1}{\alpha-1}} x_{i,t}.\tag{14}
$$

Demand for older vintages is higher because prices are lower. Aggregating over all vintages and using (14) we obtain

$$
\sum_{j=1}^{A_{t-1}} x_{j,t}^{\alpha} + \sum_{i=A_{t-1}}^{A_t} x_{i,t}^{\alpha} = \tilde{A}_t x_{i,t}^{\alpha}, \qquad \tilde{A}_t \equiv \left[\alpha^{\alpha/(\alpha-1)} - 1 \right] A_{t-1} + A_t. \tag{15}
$$

Using the new notation, we can rewrite final goods production as $Y_t =$ $L_{c,t}^{1-\alpha}[L_{s,t}^{\alpha} + \tilde{A}_tx(i)^{\alpha}].$

2.8. Equilibrium. Labor market clearing for high-skilled workers that are complementary to machines implies that the total supply of type-c labor is either employed in the final goods sector or in R&D such that

$$
L_{c,t} = L_{c,Y,t} + L_{c,A,t}.\tag{16}
$$

The market-clearing wage rate is given by

$$
w_{c,A,t} = w_{c,Y,t} \quad \Leftrightarrow \quad p_{A,t} \frac{\delta A_t^{\phi}}{L_{c,A,t}^{1-\lambda}} = (1-\alpha) \frac{L_{s,t}^{\alpha} + \tilde{A}_t x_{i,t}^{\alpha}}{L_{c,Y,t}^{\alpha}}.
$$
 (17)

From equation (9) we get demand

$$
x_{i,t} = \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}} L_{c,Y,t}.
$$
 (18)

Plugging (18) and (11) into (17) provides profits of producers of the the latest vintage of machines

$$
\pi_{i,t} = \frac{\delta A_t^{\phi}}{L_{c,A,t}^{1-\lambda}} = (1-\alpha) \frac{L_{s,t}^{\alpha} + \tilde{A}_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{\alpha}{1-\alpha}} L_{c,Y,t}^{\alpha}}{L_{c,Y,t}^{\alpha}}.
$$
(19)

Using (13) we obtain profits as $\pi_{i,t} = (1 - \alpha) \bar{R} x_{i,t}/\alpha$. Inserting this expression, the price $p_{i,t} = \bar{R}/\alpha$, and Equations (18), and (16) into (19) we obtain an implicit function for the employment level of scientists $L_{c,A,t}$. If an interior solution with R&D exists, the employment level of scientists solves the equation

$$
\frac{\bar{R}}{\alpha} \left(\frac{\alpha^2}{\bar{R}}\right)^{\frac{1}{1-\alpha}} (L_{c,t} - L_{c,A,t}) \frac{\delta A_t^{\phi}}{L_{c,A,t}^{1-\lambda}} = \left(\frac{L_{s,t}}{L_{c,t} - L_{c,A,t}}\right)^{\alpha} + \tilde{A}_t \left(\frac{\alpha^2}{\bar{R}}\right)^{\frac{\alpha}{1-\alpha}}.
$$
 (20)

3. Analytical Results

The full model is recursive: young individuals need to form expectations about future wages to determine their education and future wages in turn depend on the education decision of the young. Thus, the full model is not analytically accessible and we discuss the adjustment dynamics numerically in Section 4. Here, we assume that the result of the education decision, $L_{c,t}$, is given as a positive pre-determined state variable at any time t (implying $0 < L_{s,t} < L_t$). Notice, furthermore, that A_t (and thus \tilde{A}_t) are pre-determined state variables at time t such that solving for the equilibrium boils down to solving one equation, namely (20), for one unknown, employment in R&D, $L_{c, A, t}$.

3.1. Equilibrium R&D Employment. Inspection of (20) provides the following result.

Proposition 1. At any time t the equilibrium employment level in the R&D sector exists and it is positive and unique.

For the proof notice that, for the assumed positivity constraints on parameters and state variables $R > 0$, $\delta > 0$, $\phi \in (0, 1]$, $\alpha \in (0, 1)$, $\lambda \in (0, 1)$, $A_t > 0$, $L_{s,t} > 0$, and $L_{c,t} > 0$, the left-hand side (LHS) of Equation (20) is strictly decreasing in $L_{c,A,t}$, while the right-hand side (RHS) is strictly *increasing* in $L_{c,A,t}$. Furthermore, we have that

$$
\lim_{L_{c,A,t}\to 0} LHS = \infty, \qquad \lim_{L_{c,A,t}\to 0} RHS = const. > 0,
$$
\n
$$
\lim_{L_{c,A,t}\to L_{c,t}} LHS = 0, \qquad \lim_{L_{c,A,t}\to L_{c,t}} RHS = \infty.
$$

As a consequence, there is a unique positive level of scientists in the R&D sector. Once $L_{c, A, t}$ has been found, we can solve for all other variables.

3.2. Balanced Growth Path. To establish balanced growth, we additionally assume that a_{min} has been reached. In other words, we state the common assumption that, along the balanced growth path, the population shares of workers stay constant. We can distinguish between two central cases, the Romer (1990) case with $\phi = 1$ and $n = 0$ and the Jones (1995) case with $\phi < 1$ and $n > 0$. In the Romer (1990) case, since population growth is zero, the growth rates of aggregate variables and the growth rates of their per capita counterparts coincide. Denoting the growth rate of variable x by g_x we therefore have that, along the balanced growth path, $g_C = g_A = g_Y = g_y$ with g_y being the growth rate of per capita GDP as given by $g_y = g_A = \delta L_{c,A,t}^{\lambda} - 1$. The long-run economic growth rate rises if there are more scientists employed in R&D and if these scientists have a higher productivity level (δ) , and it decreases with the extent of the duplication externality $(1 - \lambda)$.

In the Jones (1995) case with $\phi < 1$ and $n > 0$ it follows that a balanced growth path, along which the price for blueprints does not change and the sectoral allocation of type-c workers between final goods production and R&D is constant, is associated with a per capita growth rate of $g_y =$ $g_A = (1+n)^{\frac{\lambda}{1-\phi}} - 1$. As in the standard Jones (1995) case, the long-run balanced growth rate of the economy increases with the population growth

rate (n) and the extent of intertemporal knowledge spillovers (ϕ) , whereas it decreases with the extent of the duplication externality $(1 - \lambda)$.

3.3. Education and the Scale Effect. To analyze the impact of education on R&D and economic growth we re-write (20) as the implicit function

$$
F(L_{s,t}, L_{c,A,t}) = \frac{\bar{R}}{\alpha} \left(\frac{\alpha^2}{\bar{R}}\right)^{\frac{1}{1-\alpha}} (L_{c,t} - L_{c,A,t}) \frac{\delta A_t^{\phi}}{L_{c,A,t}^{1-\lambda}} - \left(\frac{L_{s,t}}{L_{c,t} - L_{c,A,t}}\right)^{\alpha}
$$

$$
-\tilde{A}_t \left(\frac{\alpha^2}{\bar{R}}\right)^{\frac{\alpha}{1-\alpha}} = 0. \tag{21}
$$

This leads to the following result.

Proposition 2. Consider two economies sharing the parameter values δ . ϕ , α , and λ that face the same interest rate $\bar{R} > 0$, and the same initial stock of blueprints for machines $A(0)$. Then, the economy with more high skilled workers allocates more workers to $R\&D$.

Proof. Implicit differentiation of Equation (21) provides $\partial F/\partial L_{c,t} > 0$ and $\partial F/\partial L_{c,A,t}$ < 0 and thus, by the implicit function theorem, $dL_{c,A,t}/dL_{c,t}$ $0.$

Proposition 2 implies that, in the Romer (1990) case of $\phi = 1$ and $n = 0$, the long-run growth rate of the economy with the larger amount of type-s labor is lower. In the Jones (1995) case of $\phi < 1$ and $n > 0$, the growth rate of the economy with the larger amount of type-s labor is lower during the transition phase but not in the long-run limit in which both economies grow at the same rate.

These results shed new light on the old debate about the scale effect.⁷ To see this, recall from (5) that the size of the highly educated workforce is

 $^7\,$ A number of remedies have been brought forward in the literature for the counterfactual prediction that larger countries grow faster than smaller ones. Jones (1995), Kortum (1997), and Segerstrom (1999) argue that, with an increasing stock of knowledge, it becomes more difficult to do the amount of R&D that is necessary to sustain a given rate of technological progress; Peretto (1998), Young (1998), and Howitt (1999) show that the product proliferation implied by horizontal innovation makes it harder to keep the number of scientists for each product line constant and therefore it becomes harder to foster vertical (quality-improving) innovation; Dalgaard and Kreiner (2001), Strulik (2005), Strulik et al. (2013), and Prettner (2014) show that it is the aggregate human capital stock that matters for long-run growth and not the sheer population size. Under reasonable assumptions, an expansion in the number of people in an economy reduces the available resources for the education of each individual to such an extent that the aggregate human capital stock declines, although the number of people increases.

a compound of population size L_t and the population share that is highly educated $1 - f(\tilde{w})$. In particular, if the skill premium is low (\tilde{w} is high) or the education system is inefficient (captured by low ψ), it can easily be that L_c is lower in a large less developed country than in a small advanced country featuring a high skill premium and an efficient education system.

3.4. Inequality. Consider the wage rates for high-skilled type-c labor and low-skilled type-s labor

$$
w_{c,t} = (1 - \alpha) \frac{Y_t}{L_{c,Y,t}}, \qquad w_{s,t} = \alpha \left(\frac{L_{c,Y,t}}{L_{s,t}}\right)^{1-\alpha}.
$$
 (22)

Notice that type-c workers enjoy wage growth in case of a growing economy (growing GDP Y_t). By contrast, wages of low-skilled type-s workers are constant on the balanced growth path, along which factor shares $\ell_{c,t}$ and $\ell_{c,Y,t}$ are constant. Notice, in this context, that the stagnation of the wages of low-skilled workers, irrespective of a growing overall economy and a growth in the wages of high-skilled workers, is a phenomenon that has been frequently bemoaned to prevail in the U.S. since the 1970s (Mishel et al., 2015; Murray, 2016). Altogether, these results prove the following proposition.

Proposition 3. In an economy populated by high-skilled type−c workers that are complementary to machines and low-skilled type-s workers who are substitutes to machines, higher growth implies higher wage inequality along the balanced growth path.

The intuition for this result is straightforward. Technological progress raises the productivity of high-skilled workers by the introduction of new machines. At the same time, however, new machines do not raise the marginal value product of low-skilled workers because these workers are substitutable by machines. Technological progress therefore increases the wage premium enjoyed by type-c workers, which proves the next proposition.

Proposition 4. Technological progress is skill-biased.

Off the balanced growth path, a growing skill premium draws some individuals – with ability levels above a^{min} but who chose to stay low-skilled before the increase in the wage gap – into higher education to upgrade their skills and to benefit from the growth in high-skilled wages. The rising relative supply of highly educated workers (rising $L_{c, Y, t}/L_{s, t}$), taken for itself, has a mitigating effect on the skill premium (as in the race between technology and education described by Goldin and Katz, 2009). Eventually, with convergence towards the steady state, this inflow into type-c labor ceases because individuals with ability below a^{min} cannot upgrade their skills. They have lost the race between technology and education. From this point onward, the wage gap increases unchecked by supply. The higher the rates of technological progress and economic growth, the faster the gap between the wages of the two types of labor increases.

Another way to illustrate the disruptive effect of technological progress on low-skilled workers is to consider the labor share in aggregate income and to decompose it between high-skilled workers and low-skilled workers.

Proposition 5. Along the balanced growth path the total labor share is declining towards $(1 - \alpha)$. The low-skilled labor share is declining to zero.

For the proof we compute the labor share as

$$
(1 - \alpha) + \frac{w_{s,t} L_{s,t}}{Y_t},\tag{23}
$$

in which $(1 - \alpha)$ is the high-skilled labor share, and note that along the balanced growth path $L_{s,t}$ and $w_{s,t}$ are constant (since population shares are constant), while Y_t is growing at a positive rate.

The declining relative income of low-skilled labor has, furthermore, a clear inequality-enhancing effect on the wealth distribution.

Proposition 6. In a growing economy, the share of wealth held by type-c workers increases and converges to one asymptotically. Ceteris paribus, faster economic growth leads to a faster increase of wealth inequality.

For the proof we insert wages (22) into savings (3) and obtain relative wealth held by high-skilled workers \tilde{s} :

$$
\tilde{s} = \frac{(1 - \alpha) \frac{Y_t}{L_{c,Y,t}}}{(1 - \alpha) \frac{Y_t}{L_{c,Y,t}} + \alpha \left(\frac{L_{c,Y,t}}{L_{s,t}}\right)^{1 - \alpha} \frac{L_{s,t}}{L_{c,t}}}. \tag{24}
$$

Along the balanced growth path, the population shares stay constant, while $Y_t/L_{c,Y,t}$ grows perpetually. This implies that the second term in the numerator becomes gradually less important from a quantitative point of view such that \tilde{s} converges to 1. Clearly, wealth inequality increases faster when

 Y_t grows at a higher rate. Notice that, off the steady state, rising higher education (declining L_s and increasing L_c) reinforces wealth inequality during the transition towards the steady state.

In our stylized framework, rising wealth inequality is simply caused by growing wages of type-c workers, stagnating wages of type-s workers, and constant saving rates. In a less stylized framework, utility functions could take into account subsistence needs or status concerns in consumption. These mechanisms, however, would further amplify wealth inequality since they imply lower saving rates for the poor. The result of Proposition 6 contrasts with the findings of Piketty (2014) who argues that, ceteris paribus, a faster economic growth rate leads to a lower capital-output ratio and that this in turn reduces inequality. With automation, by contrast, higher growth implies a higher capital-output ratio along the transition towards the steady state (where type-s labor play asymptotically no role).

4. The Race between Education and Technology

We next consider the adjustment dynamics off the steady state and the interaction between education and technology. Qualitatively, it is straightforward to see the impact of technology on education.

Proposition 7. With technological progress, the share of high-skilled labor in the population increases and converges towards $\psi/(1+\theta)$.

The proof is obvious from Proposition 4 and inspection of Equation (5). However, to fully assess the interactions in the race between education and technology (Goldin and Katz, 2009), we need to solve the model numerically. We consider two different scenarios. In the first scenario we assume, as usual in R&D-based growth theory, that the economy grows at a positive rate and converges gradually with initially low growth rates towards the steady state. In the second scenario we consider the case of a secular decline in the growth rates of TFP and per capita GDP. This case is less frequently discussed in the literature (exceptions are Jones, 2002, and Groth et al., 2010). However, it is particular relevant in the present case to address the question whether increasing automation is compatible with declining productivity growth.

4.1. Positive Steady-State Growth. We start the computation of adjustment dynamics in the year 1900 and convert the predicted growth rates per generation into annual rates. Employing the argument of finite space on earth, it can be argued that the only meaningful long-run steady state is associated with a stationary population (Strulik, 2005) such that we assume $n = 0.8$ A convenient population size is unity, since levels agree with population shares, and we set $L = 1$. Furthermore, we normalize initial A to unity. In order to have positive long-run growth, we need to impose $\phi = 1$ in this case. We set $\bar{R} = 2$ because, assuming that a generation lasts for 25 years, this value corresponds to an annual real interest rate of around 3 percent. We assume that the time preference rate equals the interest rate and set $\beta = 1/R$. Regarding the output elasticity of machines we assume that $\alpha = 0.55$ such that the long-run labor share is given by 0.45. We set the technology parameters δ and λ and the education parameters ϕ and θ such that the model predicts – for the end of the 20th century – an annual TFP growth rate of around 1.5 percent per year, an R&D share of around 2 percent and that around 30 percent of the population have acquired a college degree (which we associate with high skills). Finally, we adjust B to ensure a solution to the left of the pole in Figure 1 for the entire adjustment path. This leads to the estimates: $\lambda = 0.3$, $\delta = 1.6$, $\psi = 0.8$, $\theta = 0.55$, and $B=3$.

Solid lines in Figure 2 show the predicted adjustment dynamics. As the economy grows and skill-biased technological progress unfolds (first panel), more individuals are motivated to take up a college education (second panel). The rising supply renders high skilled labor less scarce and more high skilled labor is allocated to R&D (third panel). This in turn further amplifies technological progress such that the economy takes off with initially increasing growth rates. After a while, however, the stepping-ontoes effect becomes noticeable and the gain in growth rates levels off as the economy adjusts towards the steady state. The benefits of technological progress accrue exclusively to high-skilled labor which is, as usually assumed in standard growth theory, a complement to (new) machines. During the transition, the wage rate of low-skilled labor increases somewhat due to its declining supply such that steady state wages are around 1.4 times higher then initial wages. However, for the aggregate low-skilled wage bill, this effect is more than compensated by declining supply such that $w_s L_s$

⁸ See Strulik and Weisdorf for an R&D-based growth theory that generates this solution endogenously

declines mildly (at the steady state it is 7 percent below its initial value). Compared to these minuscule changes, wages of high skilled labor increase drastically in conjunction with TFP-growth. As a consequence, income (and wealth) inequality increases as the economy converges towards the steady state (bottom panel in Figure 2).

Parameters: $\alpha = 0.55; \ \beta = 0.5; \ \delta = 1.6, \ \phi = 1, \ \lambda = 0.3, \ \theta = 0.55, \ \psi = 0.8,$ $A(1) = 1, L = 1, B = 3.$

In order to show our "non-Piketterian" result, we next increase δ to 2 (from 1.6) and keep all other parameters and initial values from the benchmark run. Results are shown by dashed lines in Figure 2. Due to the assumed higher productivity in R&D, the alternative economy grows at a higher rate, initially and everywhere along the adjustment path (panel 1). The higher rate of skill-biased technological progress induces a faster growth of income for the high-skilled population (panel 2), which provides more labor supply for R&D (panel 3), which further spurs innovation and economic growth. Since low-skilled labor is not benefiting from these trends, inequality increases faster than in the benchmark run (panel 4). Individuals suffering from ability- (and thus time-) constraints in learning fail to achieve college graduation and are left behind. They have lost the race against technology.

4.2. Automation and Declining TFP Growth. The numerical exercise of the previous section (in line with many related studies in quantitative growth economics) predicts that TFP growth continued to grow in the second half of the 20th century. Actually, however, TFP growth declined mildly during this period. While this counter-factual prediction may be regarded as harmless in a different context, it is of particular importance for the issue of automation because it has been argued that automation should be observed in conjunction with rising TFP growth. For example, the New York Times (20 February, 2017) argued in its editorial "No, Robots Aren't Killing the American Dream":

And yet, the data indicate that today's fear of robots is outpacing the actual advance of robots. If automation were rapidly accelerating, labor productivity and capital investment would also be surging as fewer workers and more technology did the work. But labor productivity and capital investment have actually decelerated in the 2000s.

Likewise Jared Bernstein (2017) comments: "If automation were increasingly displacing workers, we'd be seeing more output produced in fewer labor hours, aka, faster productivity growth. But we see the opposite." Here we challenge the view that increasing automation is incompatible with declining TFP growth and declining investment. In particular, we calibrate the present model to fit falling TFP growth rates and falling investment rates, and then show how increasing automation causes increasing inequality (this section) and increasing unemployment (next section). In an attempt to improve on the last section's calibration, we try to fit U.S. trends for the second half of the 20th century (and beyond) for TFP growth (Fernald, 2015), the population share with college degree (U.S. Census, 2015), the Gini coefficient for before-tax monetary income (data from the U.S.

Census, 2015; computation taken from Berruyer, 2012), the R&D expenditure to GDP ratio (Ha and Howitt, 2007), and the investment rate (World Bank, 2017). The parameters used in the calibration are provided below Figure 3. The most notable change as compared to the previous exercise is the estimate of $\phi = 0.7$. A value of ϕ below unity is needed to fit a declining TFP growth trend. It implies (slow) convergence towards a steady state of zero exponential growth. We should say that we do not perform this exercise because we endorse zero long-run growth in the distant future but to focus on the implication of mildly declining TFP growth in the recent history and the near future.

Parameters: $\alpha = 0.51$; $\beta = 0.37$; $\delta = 1.35$, $\phi = 0.7$, $\lambda = 0.05$, $\theta = -0.38$, $\psi = 0.35$, $A(1) = 5, L = 1, B = 5.$

Solid lines in Figure 3 show the predicted adjustment dynamics. Dashed lines show the underlying data. The first panel shows that the calibration supports a mildly falling trend of TFP growth and gets TFP growth in the late 20th and early 21st century about right. It also shows that decreasing returns in learning from previous innovations ($\phi = 0.7$) still supports 1 percent TFP growth at the end of the 21st century.

Although TFP is declining, all the previous mechanics of the model are at work. The reason is, that they require only positive TFP growth but not increasing or constant TFP growth. In the second panel we see how the rising skill premium induces an increasing share of the population to acquire higher education. The predicted increase in the share of graduates, however, is slower than its rise in the data. This under-prediction may be due to the primitive functional forms for utility and education (log-utility does not provide much scope to manipulate the adjustment speed). It may also indicate that the rise in college education could be caused by other motives beyond the skill premium.

The middle panel in Figure 3 shows that the model gets the mildly rising trend in the R&D-share about right. The difference with respect to the previous exercise is that the rising employment in R&D does not spur further increases in the innovation rate and economic growth because it is counter-balanced by decreasing returns in learning from previous innovations.

The fourth panel shows that the model matches well the increasing trend in inequality, which is predicted to rise further and to converge to 0.57 percent when the relative income and wealth of low-skilled individuals converges to zero. The final panel in Figure 3 shows the investment rate, computed as $\beta/(1+\beta)(w_{s,t}L_{s,t}+w_{c,t}L_{c,t})/Y_t$. Since w_c grows approximately with the rate of Y and w_s is approximately constant over time, the investment rate is mildly falling over time. The model's prediction fits the actual trend reasonably well.

The computational experiment clearly refutes the view that declining productivity growth and declining investment are incompatible with increasing automation and increasing inequality. As explained above, for these trends to be simultaneously observed we only need positive TFP growth, i.e., further innovation in automation. Increasing TFP growth or increasing investment rates are not necessary for the dark side of R&Dbased innovation to materialize.

5. Automation and Rising Unemployment

The model so far explains how automation renders low-skilled labor redundant in the sense of "unnecessary" but not in the sense of "unemployed". Although their relative importance for production converges to zero, low-skilled labor stays employed. To add more realism, we next introduce unemployment caused by innovation and economic growth. In principal, there are several gateways for rising unemployment. For example, we could introduce status concerns into the utility function such that the increasing wage gap to high-skilled labor reduces the value of labor income for the poor, who would stop working for a wage considered to be inappropriate and disgraceful. Here, we follow the easiest road and introduce unemployment via the social welfare system.

In order to simplify the analysis we assume (without loss of generality) that unemployment occurs only among low-skilled individuals. For s-type workers we now distinguish between workforce L_s and employment E_s . Suppose that social benefits for the unemployed are financed by a payroll tax at constant rate τ and that the social welfare system runs a balanced budget such that

$$
\tau w_{c,t} L_{c,t} + \tau w_{s,t} E_{s,t} = b_t w_{s,t} (L_{s,t} - E_{s,t}). \tag{25}
$$

From (25) it follows that the replacement rate, denoted by b_t , increases over time because high-skilled wages are rising perpetually and low-skilled wages are approximately constant. This induces an increasing share of low-skilled workers to prefer unemployment.

To formalize this idea, we augment the utility function with individualspecific disutility from work:

$$
u_t = \log(c_{2,j,t}) + \beta \log(R_{t+1}s_{j,t}) + \xi \Omega \omega,
$$
\n(26)

in which ξ is an indicator function that assumes the value of 1 if working and 0 otherwise; Ω is a constant weight for leisure, and ω is uniformly distributed in the interval $(0,1)$. Working individuals receive the net wage $(1 - \tau)w_{c,t}$ and unemployed individuals receive social benefits $b_t w_{c,t}$, where b_t is the replacement rate. By re-solving the optimization problem, we find that individuals consume $[\xi(1-\tau) + (1-\xi)b_t]w_{c,t}/(1+\beta)$ in working age and $[\xi(1-\tau) + (1-\xi)b_t]\beta \bar{R}\tilde{b}_t w_{c,t}/(1+\beta)$ in old age. Individuals compare

utility when working and not working and opt for not working if

$$
\omega \ge -\log\left(\frac{b_t}{1-\tau}\right) \frac{1+\beta}{\Omega} \equiv g(b_t). \tag{27}
$$

Figure 4: The Unemployment Threshold

Individuals with disutility from work ω above the threshold $q(b)$ are unemployed.

The function $q(b)$ provides the unemployment threshold. It is a positive convex curve that intersects the b-axis at $1-\tau$, as shown in Figure 4. At any given time and replacement rate b_t , individuals with disutility from work $\omega > g(b_t)$ stay unemployed such that the unemployment rate is $[1 - g(b_t)]$ and there are $E_{s,t} = g(b_t)L_{s,t}$ low-skilled workers employed. When the replacement rate equals the net wage $(1 - \tau)$, all low-skilled individuals stay unemployed.

We next run the model as calibrated in the last section (Figure 3) with the unemployment extension. We set $\tau = 0.02$ and $\Omega = 1.0$ to match a replacement rate of 52 percent (OECD, 2016; for a two earners couple with two children) and an unemployment rate of around 8 percent in the early 21st century. Results are shown by solid lines in Figure 5. The most important takeaway is perhaps that the time paths in panels 1-4 are not discernibly different to the ones shown in Figure 3. Low-skilled workers are indeed redundant with regard to the evolution of the rest of the economy. The bottom panel shows the evolution of unemployment. With rising wages of high-skilled workers, the replacement rate is gradually rising (to 60 percent at the end of the 21 century) inducing an increasing share of low-skilled workers to stay unemployed.

Finally, we return to the experiment of Figure 2 and consider an otherwise identical economy that grows at a higher rate (by setting δ to 1.7). Results are depicted by dashed lines in Figure 5. They show that a falling trend of TFP growth (and investment) does not invalidate the previous result that, ceteris paribus, higher growth is associated with higher inequality. Additionally, higher growth is associated with higher unemployment. A higher rate of innovation in labor-saving technology and, thus, faster wage growth of high-skilled labor induces more people who lost the race against the machine to stay unemployed.

6. Conclusion

In this paper we proposed a new theory of endogenous technological progress and economic growth according to which R&D-based innovations in machine technology lead to more automation, a higher skill premium, and more inequality in terms of income and wealth. The model predicts

Parameters: $\alpha = 0.143$; $a = 0.013$; $A = 0.00105$; $B = 1.6 \cdot 10^{-7}$; $\gamma = 0.19$; $\omega = 1.4$; $\theta = 0.065; \mu = 0.043; w = 35,320; k(0) = \bar{k} = 0; r = 0.06; \sigma = 1.03; p = q = 1.$

that more sophisticated technology induces more education but only to a certain degree because, eventually, some individuals will be left who do not manage to obtain higher education (a college degree) due to ability constraints. This lost race against technology is reminiscent of Goldin and Katz (2009) who focused on stagnating high school education since the 1950s. Some individuals are left behind, which creates rising inequality because wages of other individuals – as commonly assumed in R&D-based growth theory – increase at the rate of technological progress. Considering the other big race mentioned in the Introduction, the model suggest that it could be hard and eventually impossible to "run with the machine" instead of against it (as suggested by Brynjolfsson and McAfee, 2011). Compared to the related literature, our theory has focused on the "dark side" of R&D-driven technological change and has for that purpose made some drastic assumptions on the substitutability of machines and labor, which are, however, consistent with the definition of automation technology and the common notion of robotics in the literature.

Similar to the related R&D-based growth literature, we focused on the manufacturing sector, which leaves, in principle, the loophole that nonroutine, low-skilled labor finds employment in an expanding service sector. However many tasks and jobs in services have a high potential for automation as well (Chui et al., 2016). Other non-routine jobs and tasks that have been thought of as non-automatable in the not so distant past, such as driving a cab in dense traffic, have already been automated successfully or may turn out to be automatable as technological progress proceeds (Brynjolfsson and McAfee, 2016). We thus expect that augmenting the model by a service sector would lead to little more insights. A more serious simplification is certainly the assumption that some (high-skilled) labor is non-automatable. In future research, the model could be generalized by assuming that more recent vintages of machines are able to substitute to a larger degree for high-skilled labor.

According to our theory, it is misleading to believe that high economic growth could be conducive to lower inequality (Piketty 2014). Yet, this is not an anti-growth study. First, we argued that the basic mechanisms are at play in an environment of declining growth as well. Second, higher growth means more aggregate value added such that it becomes easier to redistribute income from those who work at rising wages (or own robots)

to those left behind. In the unemployment section of the paper we showed an example of such a mechanism by means of an increasing replacement rate. The deeper question is perhaps whether we can envision a happy "leisure society" (Keynes, 1930a, 1930b), in which robots and some highskilled workers produce almost all the value added and increasing parts of the population are, from a production perspective, redundant.

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