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Gavrilova, Evelina; Zoutman, Floris; Hopland, Arnt Ove

Working Paper

How to Use One Instrument to Identify Two Elasticities

CESifo Working Paper, No. 6379

Provided in Cooperation with:
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Suggested Citation: Gavrilova, Evelina; Zoutman, Floris; Hopland, Arnt Ove (2017) : How to Use One Instrument to Identify Two Elasticities, CESifo Working Paper, No. 6379, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:
https://hdl.handle.net/10419/155621

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Evelina Gavrilova
Floris Zoutman
Arnt-Ove Hopland

CESIFO WORKING PAPER NO. 6379
CATEGORY 1: PUBLIC FINANCE
FEBRUARY 2017

An electronic version of the paper may be downloaded
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ISSN 2364-1428

CESifo
Center for Economic Studies & Ifo Institute

CESifo *Working Paper No. 6379*

How to Use One Instrument to Identify Two Elasticities

Abstract

We show that an insight from taxation theory allows identification of both the supply and demand elasticities with only one instrument. Ramsey (1928) and subsequent models of taxation assume that a tax levied on the demand side only affects demand through the price after taxation. Econometrically, we show that this assumption functions as an additional exclusion restriction. Under the Ramsey Exclusion Restriction (RER) a tax reform can serve to simultaneously identify elasticities of supply and demand. We develop a TSLS estimator for both elasticities, a test to assess instrument strength and a test for the RER. Our result extends to a supply-demand system with J goods, and a setting with supply-side or non-linear taxes. Further, we show that key results in the sufficient statistics literature rely on the RER. One example is Harberger's formula for the excess burden of a tax. We apply our method to the Norwegian labor market.

JEL-Codes: C360, H220, H310, H320, J220, J230.

Keywords: tax reform, instrumental variable, supply and demand elasticities, tax incidence, payroll taxation.

Evelina Gavrilova
NHH Norwegian School of Economics
& Norwegian Center for Taxation
Bergen / Norway
Evelina.Gavrilova-Zoutman@nhh.no

Floris Zoutman
NHH Norwegian School of Economics
& Norwegian Center for Taxation
Bergen / Norway
floris.zoutman@nhh.no

Arnt-Ove Hopland
NHH Norwegian School of Economics
& Norwegian Center for Taxation
Bergen / Norway
Arnt.Hopland@nhh.no

February 23, 2017
We would like to thank Etienne Lehmann, Jarle Møen, Andreas Peichl, Dominik Sachs, Kjell Salvanes, Dirk Schindler, Guttorm Schjelderup and Erik Sørensen for very helpful suggestions and comments. Further, we would like to thank seminar and congress participants of the IIPF conference 2015, the German-Norwegian conference 2015, the Annual Meeting of the Norwegian Association of Economists 2016, and NHH Norwegian School of Economics. We gratefully acknowledge a contribution by the Research Council of Norway Grant Number 239120.

"The econometric challenge in implementing any of these structural methods is simultaneity: identification of the slope of the supply and demand curves requires $2J$ instruments.".

Chetty (2009b)

1 Introduction

The quote above asserts the well-known fact that estimating the slope of both the demand and the supply curve generally requires at least two instruments (see e.g. Wright, 1928; Koopmans, 1949). Intuitively, tracing out the slope of the demand curve requires exogenous variation in the supply curve, and vice versa. However, in specific cases economic theory enables us to restrict the demand and supply equations, such that the two slopes are identified with a single instrument. In this paper we show that a relatively simple insight from taxation theory allows us to plausibly restrict the supply-demand system when a tax reform provides exogenous variation in the tax rate. Given such a reform, the tax rate can serve as the single instrument that identifies both the demand and the supply elasticity.

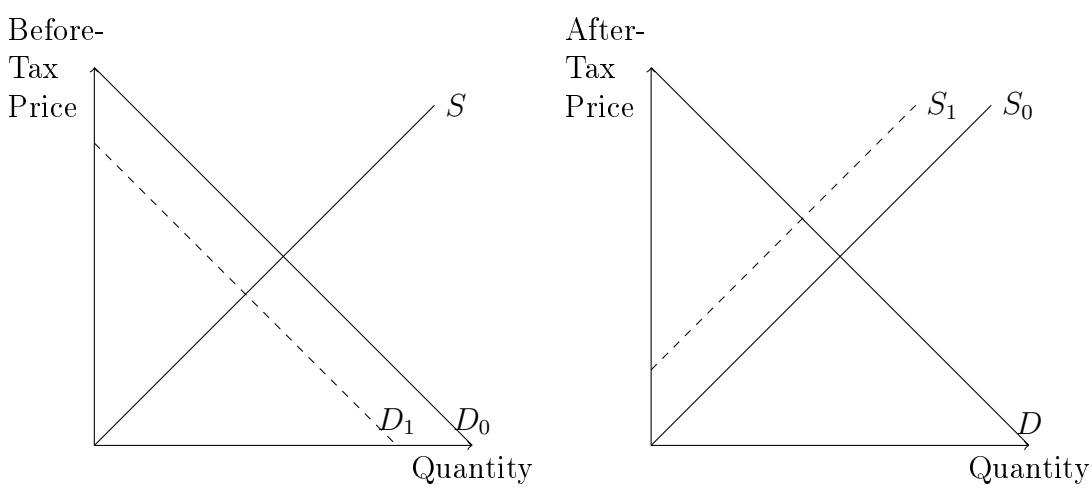


Figure 1: The Effect of a Tax Rate Increase

Our result is best understood in the context of an ad-valorem tax on a good. Figure 1 displays the effect of an increase in the tax on the price and on the traded quantity of the good. In the left panel the horizontal axis represents the traded quantity and the vertical axis represents the price excluding the tax, as the reform is usually presented in textbooks. As can be seen, in this coordinate system the tax increase shifts the demand curve inward from D_0 to D_1. For any given before-tax price, the tax increase results in a higher price for consumers, and hence reduces demand. The tax reform has no independent effect on supply, and the supply curve does not shift. Econometrically, this implies that the tax rate can serve as a valid instrument for estimating the slope of the supply curve.

The right panel shows the effect of the exact same tax reform, but the vertical axis now portrays the price including the tax, instead of the price excluding the tax. When we fix the price after taxation, the tax reform does not shift the demand curve. However, supply moves inward, because when the tax rate increases it becomes more expensive to produce the same quantity for a given after-tax price. Hence, a simple transformation of variables reveals that the exact same tax reform also allows identification of the demand elasticity.[1]

The result illustrated in Figure 1 follows from two restrictions to the supply-demand system which are regularly made in economic models of taxation. First, we make the standard exclusion restriction which states that a cost for the demand side, such as a tax, does not directly appear in the supply equation. By the standard exclusion restriction a tax reform, when represented in the left-hand panel of Figure 1, shifts the demand curve along the supply curve without simultaneously shifting the supply curve. The second exclusion restriction states that a tax on a good *only* affects demand through its impact on the after-tax price of the good. As a result of this assumption a one-percent increase in the net-of-tax rate[2] elicits the same change in demand as an (exogenous) one-percent increase in the before-tax price. From this assumption it follows that the reform, when presented in prices after taxation, shifts supply along the demand curve without simultaneously shifting the demand curve. In honor of Frank Ramsey we name the second exclusion restriction the Ramsey Exclusion Restriction (RER).

The second exclusion restriction can be justified by noting that the net-of-tax rate and the before-tax price enter the consumer's budget constraint in the same way. As a result a one-percent increase in either variable has the same effect on the consumer's budget, and hence, the RER follows from utility maximization. Because almost all models of taxation assume that consumers maximize utility, the RER implicitly plays a vital role in virtually every economic model of taxation since Ramsey (1927). [3] The RER also appears to be a particularly plausible assumption in settings where consumers only observe the price after taxation.

The standard exclusion restriction and the RER jointly ensure that the net-of-tax rate is a valid instrument for estimating both the demand and supply elasticity. To identify the elasticities we also require that the instrument has sufficient strength. Corresponding to the two exclusion restrictions, there are two conditions for instrument strength. First, strength requires that a change in the net-of-tax rate has a significant effect on the price prior to taxation. If it does not, the null hypothesis that the burden of the tax is entirely borne by the demand side cannot be rejected. Hence, the net-of-tax rate does not provide sufficient variation to allow identification of

[1] A very similar set of figures is often used in textbooks to explain the difference between a tax levied on the demand and the supply side of the market (for instance figure 19.7 in Gruber, 2010). To clarify, the textbook figure represents two different reforms in the same coordinate system, whereas in this paper both panels in Figure 1 show the same reform in different coordinate systems.

[2] In the context of an ad-valorem tax, the net-of-tax-rate is 1+the tax rate.

[3] See for instance Harberger (1964b,a); Mirrlees (1971); Diamond and Mirrlees (1971); Saez (2001). To our knowledge the only exceptions are models where the agent does not maximize utility (e.g. Chetty et al., 2009) and the case where (part of) the tax may be avoided/evaded (e.g. Chetty, 2009a).

the supply elasticity. Second, the net-of-tax rate should have a significant effect on the price after taxation. If this is not the case, the null hypothesis that the entire tax is borne by the producers cannot be rejected. Hence, there is insufficient variation in the after-tax price to estimate the demand elasticity. If the two sides of the market share the incidence of the tax, both conditions for instrument strength are satisfied asymptotically.

Our result scales up to a setting where multiple goods are traded, and each good faces an ad-valorem tax. If the net-of-tax rates on each good satisfy both the standard exclusion restriction and the RER they are valid instruments for estimating the entire set of demand and supply (cross) elasticities. In this context, instrument strength requires that the net-of-tax rates provide sufficient linearly independent variation in the vector of before- and after-tax prices.

We provide a simple estimation method that allows us to estimate the set of supply and demand (cross) elasticities, and to test the strength of the instruments. We divide the estimation of the elasticities into two different two-stage regressions. In analogue to the left panel of Figure 1, the first stage of the first regression uses the vector of prices excluding the tax as the endogenous variable, and the net-of-tax rates on each good as the instruments. The dependent variables are the equilibrium quantities of each of the goods. In the second stage the coefficients on the vector of prices measure the supply (cross) elasticities. To estimate the demand elasticities we do exactly the same, but replace the vector of prices excluding the tax with the vector of prices including the tax. The second-stage coefficients on the vector of after-tax prices denote the demand elasticities. Both regressions can be estimated using standard 2SLS or 3SLS methods, and regular strength-of-instrument tests, such as Sanderson and Windmeijer (2016), apply.

We derive two extensions to our main result. First, if the tax is levied on the supply side instead of the demand side, a simple transformation of the prices leads to the same system of equations. Hence, our result does not depend on which party bears the statutory burden of the tax. Second, we show how our result can be generalized to a setting where the tax rate is not ad-valorem, but depends non-linearly on the price and the traded quantity. If the tax is non-linear, it fundamentally cannot serve as an instrument, because its value depends on endogenous variables. However, the literature on the elasticity of taxable income has created an IV approach to deal with this issue (see e.g. Gruber and Saez, 2002; Kopczuk, 2005; Weber, 2014). We combine this approach with our method, thus extending our results to a setting with non-linear taxation.

Our methodology has important implications for both structural and sufficient statistics approaches to welfare and policy analysis. In the structural approach researchers specify economic models, and estimate or calibrate the primitives on the basis of real-world data. The calibrated models can be used to predict the impact of counterfactual policies on welfare. One important obstacle to estimating the primitives of a structural model is the simultaneity bias, as explained by Chetty (2009b) in the quote above. The RER allows researchers to overcome this obstacle and estimate the slope of the demand and supply curve using only J instruments.

The sufficient statistics approach provides an alternative method of overcoming simultaneity

bias.[4] In sufficient statistics models researchers specify a structural model for welfare analysis, but express the key formula for the welfare consequences of policy reforms in terms of reduced-form elasticities. [5] With respect to this literature, we show that our approach does not require any additional assumptions. To the best of our knowledge, all models in the sufficient statistics literature specify a priori how the policy variable enters the budget constraint of the agents. Together with utility maximization this assumption implies the RER (e.g. Harberger, 1964a,b; Saez, 2001; Hendren, 2013). For instance, we show that Harberger's canonical formula for the excess burden of a tax only applies when the RER holds, and, more generally, that the excess burden of a tax cannot be written in terms of reduced-form elasticities, unless RER holds.

One advantage of our approach is that it allows researchers to estimate the structural demand and supply elasticities underlying the reduced-form elasticity. This facilitates the comparability of estimates between different literatures. For instance in a labor-market context, the reduced-form elasticity between labor supply and the net-of-tax rate cannot be compared to the reduced-form elasticity between labor supply and a binding minimum wage. The reason is that the former reduced-form elasticity depends on both demand and supply effects, whereas the effects of binding minimum wages on labor supply solely depend on the demand elasticity. Decomposing the reduced-form elasticity from taxation into structural labor demand and supply elasticities allows researchers to compare estimates between the minimum-wage and taxation literatures.

Further, our methodology allows researchers to decompose heterogeneity in reduced-form elasticities into supply and demand-driven effects. For instance, in the labor market the reduced-form elasticity between labor supply and the net-of-income-tax rate is larger for women than for men (see e.g. Blundell and MaCurdy, 1999; Meghir and Phillips, 2010). However, to our knowledge it is not known whether this difference is driven solely by differences in the labor supply elasticity, or whether women also face a different labor demand curve (this is previously pointed out in Butler, 1982). Our approach allows the researcher to simply split the sample into men and women, and estimate the labor demand and supply elasticity for both groups, thereby decomposing the source of the heterogeneity into a supply and a demand effect. Moreover, when male and female labor supply each faces different variation in the tax rate, as is for instance the case in Gelber (2014), the multiple-good extension allows estimation of supply and demand cross elasticities as well.

We apply our method to a labor-market setting in Norway. Norway levies a payroll tax on employers, which conceptually functions as ad-valorem tax on labor. The payroll tax rate is regionally differentiated with tax rates ranging from 14.1 percent in densely populated areas in the South, to 0 percent in peripheral areas in the North. We exploit a reform in the year 2000, which

[4] The sufficient statistics approach has been pioneered by Harberger (1964a,b), and more recently popularized in Feldstein (1999); Chetty (2009b); Hendren (2013).

[5] For instance, Harberger (1964a,b) shows that the excess burden of taxation in a J-good supply-demand system can be approximated using the reduced-form elasticity between the traded quantity of each of the goods, and the J net-of-tax rates on each of the goods. The reduced-form elasticities can be obtained using only the J net-of-tax rates as instruments. Hence, like our approach, the sufficient statistics approach reduces the instrument requirement of structural welfare analysis from $2J$ to J.

raised the payroll tax rate for Norway's main manufacturing export industries to 14.1 percent, independent of where they are located. The reform is partially reversed in 2007, providing us with additional variation.

We exploit this quasi experimental variation to estimate the labor demand and supply elasticity with our methodology. We use the net-of-payroll tax rate as our instrument, the plant-average wage rate per hour (including and excluding the payroll tax) as the endogenous variable, and hours worked at the plant as the dependent variable. We include plant and sector-time fixed effects as well as control variables to control for confounding variation.

Our first-stage regression shows that the incidence of the payroll tax is shared about 50-50, as the wage prior to taxation decreases by 0.5 percent when the net-of-tax rate increases by 1 percent. In the second-stage equations we find evidence for surprisingly high labor demand and supply elasticities, equaling about -6 and 8 respectively in our main specification. All responses occur on the extensive margin, with no evidence to suggest that the hours per employee are affected by the reform.

The large labor demand elasticity could be explained by the fact that the plants in our sample are export oriented. A small increase in the wage costs may be enough to make a plant less competitive in the world market, inducing employers to cut jobs. In turn the high labor supply elasticity might be the result of unionized workers aggressively bargaining to keep workers compensation constant. However, we should also note that our estimates have been obtained with a weak instrument with F-statistics ranging between 4 and 8.

Related Literature : Our paper relates to the literature that aims to estimate the structural (labor) supply or demand elasticity using quasi-experimental variation in the tax rate as an instrument (see for estimation of the labor supply elasticity e.g. Eissa, 1995; Blundell et al., 1998, and see for estimation of the labor demand elasticity Rothstein, 2008, 2010). With respect to this literature we show that the instrument applied in these studies can be used to estimate both the demand and the supply elasticity simultaneously.

In addition, there is a large literature that estimates reduced-form elasticties between the net-of-tax rate and the traded quantity or price.[6] Most strongly related to our work are studies that estimate reduced-form elasticities, and use these to back out estimates for the structural demand, [7], supply [8], or both elasticities [9] using back-of-the-envelope calculations. These back-of-the-envelope calculations form the basis of our formal proof as well. However, our approach offers several advantages over the back-of-the-envelope calculations applied in these studies. First, our structural set-up makes it clear that the basis behind the calculations is the RER. Without the RER it is not

[6] A non-exhaustive list of empirical studies includes Hamermesh (1979); Gruber (1994); Saez et al. (2012); Lehmann et al. (2013); Blomquist (1983); Eissa (1995); Feldstein (1995); Gruber and Saez (2002); Saez (2010); Lehmann et al. (2013); Kleven and Schultz (2014); Best and Kleven (2013); Gruber and Köszegi (2001).

[7] Kramarz and Philippon (2001); Huttunen et al. (2013); Egebark and Kaunitz (2014)

[8] Rothstein (2008, 2010)

[9] Saez et al. (2012); Elias (2015)

possible to back out both elasticities from the reduced-form coefficients. Second, we propose 2SLS and 3SLS estimators for the elasticities, rather than back-of-the envelope calculations. The benefit is that our approach allows researchers to i.) estimate standard errors without the Delta method, and ii.) conveniently test the strength of the instruments. Finally, to the best of our knowledge we are the first to show that the result extends to a setting with multiple goods.

Our application contributes to the empirical literature on payroll taxation. There is a large literature that aims to estimate the incidence and employment effects of the payroll tax.[10]. We make two methodological contributions to this literature. First, using our methodology we are able to estimate the structural labor demand and supply elasticity for exporting industries in Norway. Second, we use the wage-rate per hour rather than the wage-rate per worker as our measure of the wage rate allowing us to disentangle incidence of the tax from behavioral responses on the intensive margin.

2 Methodology

2.1 The Single Good Case

We first turn to an instructive example in a market for a single good. In this setting we derive the familiar result which shows that without restrictions two instruments are required to estimate the demand and supply elasticities. We then show that the RER allows us to estimate both elasticities using only the tax rate as an instrument.

Assume that we observe panel data for good Y_{it} on the equilibrium quantity and price excluding the tax P_{it}. Here, the cross-sectional indicator i may, for instance, indicate specific regions, firms or individuals, and t denotes the time index. Assume that the good faces an ad-valorem tax rate τ_{it} and that the tax is levied on the demand side. Suppose that there is exogenous variation in the tax rate, possibly after controlling for a K-vector of control variables x_{it}.[11]

Because the tax is levied on the demand side it only affects supply through its influence on the price P_{it}. As such, the instrument appears in the demand equation, but does not directly appear in the supply equation. This setup corresponds to the left panel of Figure 1. We further assume that supply and demand are log-linear. Under these assumptions the supply-demand system can

[10] A non-exhaustive list with a focus on the Nordic countries is Hamermesh (1979); Dyrstad (1992); Gruber (1997); Johansen and Klette (1997); Johansen (2002); Carlsen and Johansen (2005); Murphy (2007); Bennmarker et al. (2009); Korkeamäki and Uusitalo (2009); Saez et al. (2012); Huttunen et al. (2013)

[11] In this section we take the exogenous variation in the tax as a given. In the real world the most likely source of variation is a natural experiment where exogenous variation in the tax can be isolated through either a difference-in-difference, or a regression discontinuity design.

be represented by the following equations:

$$y_{it} = \varepsilon^S p_{it} + \Gamma^S x_{it} + \nu_{it}^S, \quad (1)$$

$$y_{it} = \varepsilon^D p_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu_{it}^D. \quad (2)$$

where y_{it} and p_{it} denote the log of Y_{it} and P_{it}. ε^S (ε^D) denotes the supply (demand) elasticity. Standard economic theory implies that $-\varepsilon^D, \varepsilon^S > 0$. $z_{it} \equiv f(\tau_{it})$ is a monotonous transformation of the ad-valorem tax rate chosen to ensure that Y_{it} becomes log-linear in $f(\tau_{it})$. The role of the function $f(\cdot)$ will become more clear below. γ is the coefficient on the instrument z_{it}. $\Gamma^S(\Gamma^D)$ is the vector of coefficients belonging to the control variables in the supply (demand) equation. Finally, ν_{it}^S (ν_{it}^D) denotes the disturbance term in the supply (demand) equation. We assume that prices are flexible. This implies that the supply and demand equations cannot consistently be estimated with OLS due to simultaneity.

To see that we cannot identify both the demand and supply elasticity without making additional restrictions, consider the reduced-form equations:

$$\begin{bmatrix} y_{it} \\ p_{it} \end{bmatrix} = \begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} z_{it} + \Pi x_{it} + \xi_{it}. \quad (3)$$

where π_{zy} is the coefficient between the instrument and the traded quantity, π_{zp} is the coefficient between the instrument and the price, and Π is a $2 \times K$ matrix of reduced-form coefficients for each of the control variables. Since z_{it} and x_{it} are jointly exogenous, the coefficients π and Π can be recovered by estimating (3) using OLS.

The coefficients in the structural system of equations (1,2) are identified if they can be expressed in terms of reduced-form coefficients $\{\pi_{zy}, \pi_{zp}, \Pi\}$. More formally, the instrument is valid for estimating a structural coefficient when the structural coefficient can be expressed in terms of the reduced-form coefficients.

Substituting the system of equations (1,2) into the reduced-form expression we find the following link between the reduced-form and structural coefficients of interest:

$$\begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon^S \gamma}{\varepsilon^S - \varepsilon^D} \\ \frac{\gamma}{\varepsilon^S - \varepsilon^D} \end{bmatrix}.$$

The right-hand side of the expression is composed of three structural coefficients, γ, ε^S and ε^D. However, the vector on the left-hand side only has two elements. Therefore, it is impossible to identify all three structural parameters. One can only solve for the supply elasticity by noting that:

$$\varepsilon^S = \frac{\pi_{zy}}{\pi_{zp}}. \quad (4)$$

Hence, z_{it} is a valid instrument for estimating the supply elasticity. Identification requires that

the instrument is both valid and relevant. In this context, the instrument is relevant as long as $\pi_{zp} \neq 0$. Intuitively, this assumption ensures that expression (4) is defined.

The intuition behind this standard result is straightforward. The instrument is excluded from the supply equation, and can hence be used to estimate the supply elasticity. However, all exogenous variables appear in the demand equation; thus, it is impossible to estimate the structural coefficients in the demand equation.

The above reveals the standard argument for why we generally need at least two instruments to estimate both the demand and supply elasticity. However, because we use the tax rate as the source of our instrument, we can restrict the system of equations by using a standard argument from taxation theory. The relevant price in the budget constraint is the price consumers pay for the good after taxation. Hence, if consumers act rationally demand only depends on the after-tax price $P_{it}^{\tau} \equiv (1+\tau_{it})P_{it}$. As discussed in the introduction, the assumption that a tax only affects demand through the price after taxation is implicit in virtually all economic models of taxation dating back to Ramsey (1927), and we therefore label it the Ramsey Exclusion Restriction (RER).[12]

To see intuitively why the RER allows researchers to identify the demand elasticity note that the demand-supply system can be denoted either in terms of p_{it} or in terms of p_{it}^{τ}. When we specify demand in terms of p_{it} supply remains unchanged and can hence be expressed by (1). The demand equation can be written as:

$$y_{it} = \varepsilon^D p_{it} + \varepsilon^D \log(1+\tau_{it}) + \Gamma^D x_{it} + \nu_{it}^D. \tag{5}$$

As can be seen, equation (5) is a special case of (2). First, the monotonous transformation $f(\cdot)$ is chosen such that the instrument equals the log of the net-of-tax rate, $z_{it} = \log(1+\tau_{it})$. Second, the coefficient on the instrument is identically equal to the demand elasticity. Intuitively, an increase in the price before taxation P_{it} and an increase in the net-of-tax rate $1+\tau_{it}$ affect the consumer's budget in an identical manner. As a consequence, they have the same impact on demand.

In the supply-demand system (1,5) the instrument is included in the demand equation and excluded in the supply equation. Therefore, the instrument is valid for estimating the supply elasticity. However, when we rewrite the system of equations in terms of p_{it}^{τ} the supply and demand equations are given by:

$$y_{it} = \varepsilon^S p_{it}^{\tau} - \varepsilon^S \log(1+\tau_{it}) + \Gamma^S x_{it} + \nu_{it}^S. \tag{6}$$

$$y_{it} = \varepsilon^D p_{it}^{\tau} + \Gamma^D x_{it} + \nu_{it}^D. \tag{7}$$

Written this way the instrument is excluded in the demand equation, and included in the supply equation. Hence, the instrument is also valid for estimating structural demand coefficients. This provides a heuristic proof to our main result.

[12] A more formal definition of the RER follows in the next subsection.

It is also straightforward to derive a formal proof for the single-good case. Consider the system of equations given by (1,5) and relate reduced-form coefficients to structural coefficients:

$$\begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon^S \varepsilon^D}{\varepsilon^S - \varepsilon^D} \\ \frac{\varepsilon^D}{\varepsilon^S - \varepsilon^D} \end{bmatrix}. \tag{8}$$

The supply elasticity can still be expressed in terms of the reduced-form coefficients using equation (4). In addition, it is possible to express the demand elasticity in reduced-form coefficients as follows:

$$\varepsilon^D = \frac{\pi_{zy}}{1 + \pi_{zp}}.$$

Therefore, if the RER and the standard exclusion restriction hold, the instrument is valid for estimating both the demand and supply elasticity. The instrument is relevant for estimating the supply elasticity if $\pi_{zp} \neq 0$. The relevance condition for the demand elasticity is given by $\pi_{zp} \neq -1$.

The relevance conditions have a straightforward economic interpretation. If $\pi_{zp} = 0$, variation in the tax rate does not affect the price prior to taxation. In that case the entire incidence of the tax falls on the demand side. As such, the instrument does not provide variation in the price that is relevant for the supply side, and, hence, the supply elasticity is not identified. By considering equation (8), we can see that $\pi_{zp} = 0$ if $\varepsilon^D = 0$ and/or $\varepsilon^S = \infty$.

If $\pi_{zp} = -1$ the price after taxation is independent of the tax rate. Hence, the incidence of the tax falls completely on the supply side. Therefore, there is no variation in the price consumers pay for the good, and it becomes impossible to estimate the demand elasticity. By considering equation (8), we can see that $\pi_{zp} = -1$ if $\varepsilon^D = -\infty$ and/or $\varepsilon^S = 0$.

Therefore, in the market for a single good, the RER allows estimation of both the demand and the supply elasticity, as long as the incidence of the tax is shared between demand and supply.

2.2 Multiple Goods

To generalize the result to a setting with multiple goods, consider a demand-supply system of J goods. Let y_{it}^j denote the log quantity of good j. Assume that each good faces an ad-valorem tax τ_{it}^j, and assume that each of the tax rates contains exogenous and independent variation (after controlling for x_{it}). Again, we use as an instrument a known transformation of the tax rate such that $z_{it}^j \equiv f^j\left(\tau_{it}^j\right)$. We also assume that the standard exclusion restriction holds:

Standard Exclusion Restriction. *If the tax rates τ^j are levied on the demand side, they do not directly appear in the supply equations.*

Under the standard exclusion restriction the system of equations that relates the row vector of

log prices p_{it} to supply and demand is given by:

$$y_{it}^{j} = p_{it}\varepsilon^{S_j} + x_{it}\Gamma^{S_j} + \nu_{it}^{S_j} \quad \forall \quad j = 1, \ldots J, \tag{9}$$

$$y_{it}^{j} = p_{it}\varepsilon^{D_j} + z_{it}\gamma^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j} \quad \forall \quad j = 1, \ldots J, \tag{10}$$

where ε^{S_j} and ε^{D_j} are column vectors of supply and demand (cross) elasticities of good j with respect to each of the prices. z_{it} is a J row vector with instruments. x_{it} is a K row vector with linearly independent control variables. γ^{D_j} denotes a column vector of coefficients for each of the J instruments. Γ^{S_j} and Γ^{D_j} denote the column vectors of coefficients on each of the K control variables. $\nu_{it}^{S_j}$ ($\nu_{it}^{D_j}$) denotes the disturbance term in supply (demand) equation j. Equations (9,10) represent a very general supply and demand system. Demand and supply of each good can potentially depend upon the entire vector of all prices p_{it}. Moreover, the coefficients on the instruments represented by column vector γ^{D_j} are unrestricted.

The reduced-form expression can be written as follows:

$$\begin{bmatrix} y_{it} & p_{it} \end{bmatrix} = z_{it} \begin{bmatrix} \Pi_{zy} & \Pi_{zp} \end{bmatrix} + x_{it} \begin{bmatrix} \Pi_{xy} & \Pi_{xp} \end{bmatrix} + \xi_{it},$$

where y_{it} is a J row vector with all traded goods. Π_{zy} and Π_{zp} are $J \times J$ matrices of reduced-form coefficients between the instruments, and the traded quantities and prices respectively. Π_{xy} and Π_{xp} are $K \times J$ vectors of coefficients between the control variables and quantities and prices. Finally, ξ_{it} presents the $2J$ row vector of disturbance terms.

As in the single good case, the instruments z_{it} are valid for estimating the structural coefficients in each of the equations (9), when the standard exclusion restriction holds. Instrument relevance additionally requires that the matrix Π_{zp} has full rank (see e.g. Hausman, 1983). However, without imposing further restrictions it is not possible to estimate the structural coefficients in (10).

The restriction we impose is the RER. We provide two formal definitions of the RER. The strong RER restricts γ^{D_j} in each equation, and additionally sets the instrument equal to the net-of-tax rate. This version of the RER is the logical multi-good equivalent of the RER in section 2.1.

Strong Ramsey Exclusion Restriction. *An increase in the instrument z_{it}^{j} and an increase in the pre-tax price p_{it}^{j} have the same effect on demand such that γ^{D_j}=ε^{D_j} for all $j = 1, \ldots, J$. Moreover, instruments are defined as:*

$$z_{it}^{j} \equiv \log\left(1 + \tau_{it}^{j}\right) \quad \forall \quad j = 1, \ldots, J.$$

We also formulate a slightly weaker version of the RER where we restrict the γ^{D_j}s, but allow the instrument to be a general *ex ante known* transformation $f(\cdot)$ of the tax rates.

Weak Ramsey Exclusion Restriction. *The vector of coefficients on the instrument z_{it} is equal*

to the vector of coefficients on the demand elasticities, $\gamma^{D_j} = \varepsilon^{D_j}$, and the instrument is a known transformation of the tax rate, $z_{it}^j \equiv f^j(\tau_{it})$ for all $j = 1, \ldots, J$.

The difference between the strong and the weak version of the RER is that the strong version restricts the transformation $f(\cdot)$ to the net-of-tax rate. Hence, in the strong version of the RER a 1 percent change in the net-of-tax rate yields the same change in demand as a 1 percent change in the price prior to taxation. The weak RER does not restrict the monotonic transformation apart from the fact that it must be known to the econometrician. Our main result requires only that the weak RER holds.

The weak RER allows us to write the demand equations as follows:

$$y_{it}^j = p_{it}\varepsilon^{D_j} + z_{it}\varepsilon^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j} \quad \forall \quad j = 1, \ldots J.$$

To see intuitively why the weak RER allows identification of both the demand and supply elasticity define the vector of prices inclusive of the instrument as $p_{it}^z \equiv p_{it} + z_{it}$. The weak RER then allows us to rewrite the system of equations given by (9,10) as follows:

$$\begin{aligned} y_{it}^j &= p_{it}^z\varepsilon^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j} \quad \forall \quad j = 1, \ldots J, \\ y_{it}^j &= p_{it}^z\varepsilon^{S_j} - \varepsilon^{S_j} z_{it} + x_{it}\Gamma^{S_j} + \nu_{it}^{S_j} \quad \forall \quad j = 1, \ldots J. \end{aligned}$$

As can be seen, in this system of equations the instrument is excluded from the demand equation, and included in the supply equation. Hence, for appropriate conditions of instrument relevance, the structural coefficients appearing in the demand equation are identified. We have already discussed the fact that the stuctural coefficients in equation (9) are identified in the standard formulation. Therefore, both sets of elasticities are identified if the standard exclusion restriction and the weak RER hold, provided the instruments are relevant. Proposition 1 gives a formal proof of this statement, and additionally derives the conditions for instrument relevance.

Proposition 1. *The instruments z_{it} are valid for estimating all structural coefficients in the system of supply-demand equations(9,10) if the weak RER holds. They are relevant for estimating the coefficients in (9) if Π_{zp} has full rank, and relevant for estimating the coefficients in (10) if $\Pi_{zp}^z \equiv \Pi_{zp} + I_J$ has full rank.*

Proof. The proof can be found in the appendix. □

As in the single-good setting, the conditions for instrument relevance have an intuitive explanation. To see this assume that the strong RER holds such that $z_{it}^j = \log\left(1 + \tau_{it}^j\right)$, and consider the reduced-form equations that relate prices to the exogenous instruments:

$$p_{it} = \log\left(1 + \tau_{it}\right)\Pi_{zp} + x_{it}\Pi_{xp} + \xi_{it}^p.$$

If Π_{zp} does not have full rank, variation in the net-of-tax rate does not provide linearly independent variation in the prices prior to taxation, which are the relevant prices for the supply side. Hence, in that case it is impossible to independently estimate the vector of supply elasticities ε^S.

Now add the vector $\log(1+\tau_{it})$ to both sides of the equation to arrive at:

$$p_{it}^{\tau} = \log(1+\tau_{it})(\Pi_{zp} + I_J) + x_{it}\Pi_{xp} + \xi_{it}^{p}.$$

If $\Pi_{zp} + I_J$ does not have full rank, variation in the instruments does not provide linearly independent variation in the vector of prices after taxation. Therefore, it is impossible to independently estimate the vector of demand elasticities ε^D. The rank restrictions ensure that there is exogenous and independent variation in all prices before and after taxation.

2.3 Estimation

In this subsection we show how to estimate the supply and demand elasticities of the system of equations(9,10) with the RER. The estimation strategy is best described by considering both panels in Figure 1. Analogous with these two panels, we divide the estimation of the structural coefficients in (9,10) in two two-stage regressions.

In the first regression the first stage equation regresses the vector of prices prior to taxation on the instruments as follows:

$$p_{it} = z_{it}\Pi_{zp} + x_{it}\Pi_{xp} + \xi_{it}^{p}. \tag{11}$$

The regression provides a vector of instrumented prices prior to taxation which we denote by $\hat{p}_{it}$. The second-stage equation regresses traded quantities on instrumented prices prior to taxation:

$$y_{it}^{j} = \hat{p}_{it}\varepsilon^{S_j} + x_{it}\Gamma^{S_j} + \nu_{it}^{S_j} \quad \forall \quad j = 1, \ldots J, \tag{12}$$

The system of equations can be estimated with standard 2SLS or 3SLS estimators. By virtue of Proposition 1 the estimates obtained for the supply elasticities are consistent if the standard exclusion restriction holds, and Π_{zp} has full rank.

To estimate the demand elasticities it is necessary to replace prices prior to taxation with prices inclusive of the tax instrument, similar to the right-hand panel of Figure 1. Therefore, the first-stage equation is given by:

$$p_{it}^{z} = \log(1+\tau_{it})\,\Pi_{zp}^{z} + x_{it}\Pi_{xp} + \xi_{it}^{p}. \tag{13}$$

We then use instrumented prices inclusive of the instrument, $\hat{p}_{it}^{z}$ to estimate the demand elasticities with the following regression equation:

$$y_{it}^{j} = \hat{p}_{it}^{z}\varepsilon^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j} \quad \forall \quad j = 1, \ldots J. \tag{14}$$

This system of equations can again be estimated using standard 2SLS or 3SLS estimators. Note that the matrices of reduced-form coefficients fulfill mechanically the equality $\Pi^z_{zp} = \Pi_{zp} + I_J$. Hence, by Proposition 1 the demand elasticities are identified if the RER holds and Π^z_{zp} has full rank.

2.3.1 Test for Instrument Strength

For finite samples it is important to not only evaluate whether the instruments are relevant, but to also consider whether they have sufficient strength. Relevance requires that $rank(\Pi_{zp}) = J$ in order to to estimate supply elasticities, and $rank(\Pi^z_{zp}) = J$ to estimate demand elasticities. Instrument strength requires that we are able to reject the null hypothesis that $rank(\Pi_{zp}) < J$ ($rank(\Pi^z_{zp}) < J$) for the supply (demand) elasticities in a statistical sense. Fortunately, tests of instrument strength can easily be combined with our estimation method. To do so, for supply elasticities estimate the first-stage regression equation (11) and use the test of Sanderson and Windmeijer (2016) to evaluate whether the null hypothesis that Π_{zp} has insufficient rank can be rejected. Similarly, for demand elasticities estimate (13) and perform the same test on the estimated matrix Π^z_{zp}.

2.3.2 Test of the Ramsey Exclusion Restriction

The RER itself cannot be tested within the context of the reduced-form regression (3), unless a second instrument is available. In that case, the RER can be tested in the structural set of equations (1,2). To see this note that testing the RER comes down to testing the null hypothesis $\gamma = \epsilon^D$ against the alternative hypothesis $\gamma \neq \epsilon^D$. Denote the second instrument by z^2_{it} and initially assume that the instrument shifts the supply curve without affecting the demand curve. In that case (1,2) can be written as:

$$\begin{aligned} y_{it} &= \varepsilon^S p_{it} + \gamma^2 z^2_{it} + \Gamma^S x_{it} + \nu^S_{it}, \\ y_{it} &= \varepsilon^D p_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu^D_{it}, \end{aligned}$$

Without the additional instrument the null hypothesis cannot be tested because the structural coefficients in the demand equation are not identified, unless we impose that $\gamma = \varepsilon^D$. However, with an additional instrument γ and ε^D can be estimated independently. To do this, instrument the price with the instruments as follows:

$$p_{it} = \pi_{zp} z_{it} + \pi_{z^2p} z^2_{it} + \Pi x_{it} + \xi_{it}.$$

Second, use instrumented prices, $\hat{p}_{it}$, to estimate the structural parameters in the demand equation:

$$y_{it} = \varepsilon^D \hat{p}_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu^D_{it}.$$

The second stage provides consistent estimates for both ε^D and γ because the second instrument is excluded from the demand equation. In the second-stage equation the null hypothesis $\gamma = \epsilon^D$ can therefore be tested using a standard Wald-test.

Now assume that z_{it}^2 shifts demand rather than supply. In that case, the demand and supply system can be rewritten as:

$$\begin{aligned} y_{it} &= \varepsilon^S p_{it} + \Gamma^S x_{it} + \nu_{it}^S, \\ y_{it} &= \varepsilon^D p_{it} + \gamma z_{it} + \gamma^2 z_{it}^2 + \Gamma^D x_{it} + \nu_{it}^D. \end{aligned}$$

First assume that the RER holds. In that case, we can rewrite this system of equations using p_{it}^z:

$$\begin{aligned} y_{it} &= \varepsilon^S p_{it}^z - \varepsilon^S z_{it} + \Gamma^S x_{it} + \nu_{it}^S, \\ y_{it} &= \varepsilon^D p_{it}^z + \gamma^2 z_{it}^2 + \Gamma^D x_{it} + \nu_{it}^D. \end{aligned}$$

Hence, in this formulation the RER implies that the coefficient on the instrument in the supply equation equals $-\varepsilon^S$. A deviation implies that the RER does not hold. To test the null hypothesis here, first instrument p_{it}^z using the instruments:

$$p_{it}^z = \pi_{zp}^z z_{it} + \pi_{z^2p}^z z_{it}^2 + \Pi^z x_{it} + \xi_{it}^z.$$

Then use instrumented prices to estimate the supply equation:

$$y_{it} = \varepsilon^S \hat{p^z}_{it} + \gamma^S z_{it} + \Gamma^S x_{it} + \nu_{it}^S.$$

In this second-stage equation the RER can be tested through a Wald-test with the null hypothesis $\varepsilon^S = -\gamma^S$.

The fact that the RER can potentially be tested provides an important advantage of structural analysis over reduced-form analysis. There are several reasons why the RER may in practice fail to hold. For instance, Chetty et al. (2009) study the case where the tax on a good is not (fully) salient. They show theoretically and empirically that in that case a 1 percent change in the before-tax price may not elicit the same change in demand as a 1 percent change in the net-of-tax rate. In addition, tax evasion or tax avoidance may also result in a failure of the RER (see e.g. Chetty (2009a); Doerrenberg et al. (forthcoming)). In this case, a change in the price and a change in the net-of-tax rate do not elicit the same change in demand because part of the tax can be evaded or avoided. It is therefore useful to test whether the RER holds, and such a test can only be performed in a structural setting.

2.4 Extensions

In this subsection we discuss several straightforward extensions to Proposition 1. In our main analysis we focus on ad-valorem taxes that are levied on the demand side. In the extensions we consider taxes that are levied on the supply side and non-linear taxes.

2.4.1 Supply Side Taxes

Consider the case in which an ad-valorem tax on the goods is levied on the supply rather than the demand side. This requires an adaptation of the exclusion restrictions. When taxes are levied on the supply side, the standard exclusion restriction states that the instruments do not directly appear in the demand side equation. The (weak) RER states that a 1 percent change in the before-tax price, yields the same change in supply as a 1 percent change in the instrument $z_{it}^j \equiv f^j(\tau_{it}^j)$. For the purpose of this extension we denote prices (before taxation) by $\tilde{P}_{it}$. Under these exclusion restrictions, the system of supply and demand equations can be written as:

$$\begin{aligned} y_{it}^j &= \tilde{p}_{it}\varepsilon^{S_j} + z_{it}\varepsilon^{S_j} + x_{it}\Gamma^{S_j} + \nu_{it}^{S_j}, \\ y_{it}^j &= \tilde{p}_{it}\varepsilon^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j}, \end{aligned}$$

where z_{it} is again a known transformation of the tax rate. To see that Proposition 1 applies in this section denote the price including the instrument by $p_{it} = \tilde{p}_{it} + z_{it}$, and rewrite the system of equations in terms of p_{it}:

$$\begin{aligned} y_{it}^j &= p_{it}\varepsilon^{S_j} + x_{it}\Gamma^{S_j} + \nu_{it}^{S_j}, \\ y_{it}^j &= p_{it}\varepsilon^{D_j} - z_{it}\varepsilon^{D_j} + x_{it}\Gamma^{D_j} + \nu_{it}^{D_j}. \end{aligned}$$

As can be seen, this system of equations is equivalent to (9,11). Therefore, Proposition 1 applies. Hence, we obtain a similar result for taxes that are levied on the supply side as we do for taxes levied on the demand side.

2.4.2 Non-linear taxes

Consider the case where a tax is levied on the demand side and the tax rate varies non-linearly with prices and quantities. Specifically, let the tax rate on good j be given by the (known) tax function $\tau_{it}^j = T_t^j(p_{it}^j, y_{it}^j)$. [13]

With non-linear taxes the standard approach outlined in section 2.2 fails. To see this, note that the instrument $z_{it}^j = f^j(\tau_{it}^j) = f^j(T^j(y_{it}^j, p_{it}^j))$ depends on endogenous variables and hence,

[13] Note that here we allow for a very general non-linear tax system, because the tax rate may independently depend both on y_{it}^j and on p_{it}^j. This allows us to capture the standard format $\tau_{it}^j = T_t^j(p_{it}^j y_{it}^j)$, as a special case (see e.g. Mirrlees, 1971). In addition, the formulation allows us to capture specific taxes by letting $T_t^j(p_{it}^j y_{it}^j) = \frac{\theta_t}{p_{it}^j}$, where θ_t denotes the tax per unit of the good at time t.

even reduced-form equations cannot be estimated consistently using OLS.

Fortunately, there is a literature that attempts to estimate reduced-form elasticity of taxable income with respect to the non-linear income tax (Gruber and Saez, 2002; Kopczuk, 2005; Weber, 2014). The idea in this literature is to create synthetic instruments by taking the new tax rules and applying them to lagged quantities and prices. That is, a synthetic instrument is created by letting $s_{it}^j \equiv T_t^j(y_{i,t-L}^j, p_{i,t-L}^j)$ where L denotes the number of lags. The synthetic instruments are plausibly exogenous provided L is large enough (see Weber, 2014 for a discussion).

To combine this approach with our method the first step is to create instrumented tax rates by regressing actual tax rates on the synthetic instrument:

$$\tau_{it}^j = s_{it}^j \beta + x_{it} \Lambda^j + \phi_{it}^j,$$

which is the standard first-stage regression in the literature on the elasticity of taxable income. The next step is to estimate structural demand and supply elasticities replace actual tax rates with instrumented tax rates in the estimation method outlined in section 2.3.

3 The Ramsey Exclusion Restriction and the Sufficient Statistics Literature

Our approach allows researchers to estimate J structural demand and supply elasticities using only J instruments. Effectively, we reduce the instrument requirement of structural methods of welfare analysis by 50%. An alternative approach to reducing the instrument requirement is formed by the sufficient statistics literature. This literature specifies structural models of welfare analysis, but expresses the key formulae for welfare analysis and policy evaluation in terms of reduced-form elasticities.

In two famous articles Harberger (1964b,a) pioneers the sufficient statistics approach. He shows that the excess burden of a tax can be calculated using only reduced-form elasticities. In the most basic setup Harberger considers the market for a single good with a representative agent. For simplicity we also focus on this single-good case in this section. In his analysis Harberger assumes that the budget constraint of the agent only depends on the price after taxation. This assumption and utility maximization jointly imply that the strong RER holds in his setup. The easiest method of deriving Harberger's formula is by first imposing the strong RER on the demand curve, and then calculating the deadweight loss associated with a tax (e.g. Gruber, 2010 provides a textbook version of this proof). The formula for the marginal deadweight loss is derived by taking the derivative between the size of the deadweight loss triangle and the tax rate.

In this paper, we also derive Harberger's formula, but reverse the order by only imposing the RER in the very last step of the proof. The purpose of reversing the order is to show that a

reduced-form formulation of the excess burden of the tax does not exist unless we impose the RER. Hence, the starting point to derive the deadweight loss is the unrestricted supply-demand system given in equations (1,2). For this supply-demand system the excess burden when $z_{it} = z$, is given by the deadweight loss triangle in Figure 2.

To calculate the size of the triangle note that, with respect to *laissez-faire*, demand has shifted down by $\frac{\gamma}{\varepsilon^D} z$. The traded quantity has decreased by $\frac{\gamma \epsilon^S}{\epsilon^S - \epsilon^D} z$. Hence, the area of the deadweight loss triangle is given by:

$$EB = \frac{1}{2} \frac{\varepsilon^S \gamma^2 z^2}{(\varepsilon^S - \varepsilon^D)\, \varepsilon^D}.$$

The marginal excess burden associated with a change in the instrument can be found by differentiating the above expression with respect to z:

$$\frac{dEB}{dz} = \frac{\varepsilon^S \gamma^2 z}{(\varepsilon^S - \varepsilon^D)\, \varepsilon^D}.$$

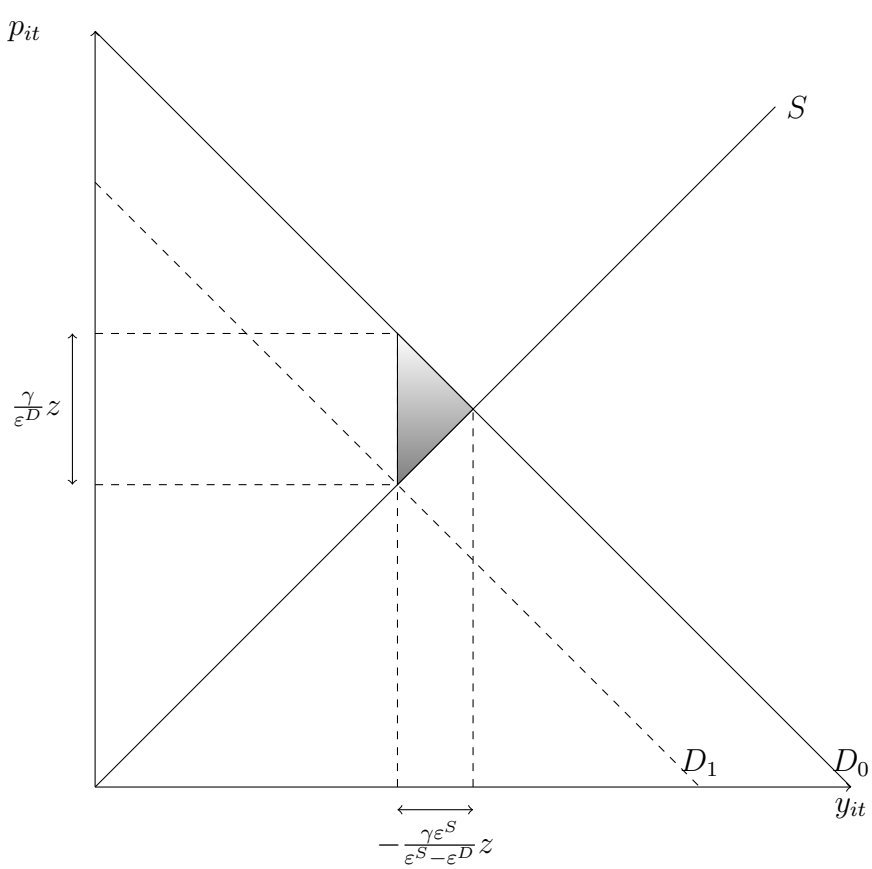


Figure 2: The Deadweight Loss of a tax in an unrestricted supply and demand system

As can be seen, the expression depends on all three structural parameters and therefore cannot be expressed in terms of the reduced-form elasticities. Hence, unless we impose additional restrictions, the reduced-form elasticity is not a sufficient statistic for calculating the excess burden of

the tax. To calculate the excess burden in an unrestricted supply-demand system, we require at least one additional instrument, that allows for identification of the demand coefficients γ, ε^D.

However, if we impose the strong RER such that $\gamma = \epsilon^D$, and $z = \log(1+\tau)$, we arrive at Harberger's formula:[14]

$$\frac{dEB}{d\tau_{it}} = \frac{\varepsilon^S \varepsilon^D}{\varepsilon^S - \varepsilon^D}\tau = \pi_{zp}\tau.$$

Hence, the reduced-form elasticity is *only* a sufficient statistic for welfare analysis when the RER applies.

Harberger (1964b,a) is not the only study in the sufficient statistic literature that relies on the RER in the derivation of its key formula. Saez (2001) considers taxation of income from labor. Labor supply in his model depends only on the wage rate net-of-taxes. Hence, labor supply satisfies the RER for supply-side taxes discussed in section 2.4. Hendren (2013) considers both demand and supply-side taxes and similarly, his assumptions imply that the RER holds.[15]

To our knowledge Chetty et al. (2009) and Chetty (2009a) provide the only exceptions to this rule. Chetty et al. (2009) do not assume rationality. They show that in this case the excess of the burden depends on both the reduced-form elasticity π_{zp} and the structural elasticity ε^D. Hence, welfare analysis in that case requires two instruments. Similarly, the RER fails to hold in Chetty (2009a), because he assumes that part of the tax can be evaded. Also, in that case the reduced-form elasticity π_{zp} can no longer serve as a sufficient statistic for welfare analysis. Hence, our approach and the sufficient statistics literature are similar in the sense that key formulae for welfare analysis can only be expressed in terms of reduced-form elasticities if the RER (or a variant thereof) holds.

4 The Ramsey Exclusion Restriction in Action

In this section we explore two advantages our approach offers over the reduced-form literature. First, we show that our approach allows researchers to compare estimates from the labor income and payroll tax literature to outcomes from the minimum-wage literature. Second, we look at the advantage of our approach in studying heterogeneity in the response to taxation.

4.1 Extrapolation: Taxation and Minimum Wages

As discussed in the introduction, there is a large reduced-form literature that aims to estimate the reduced-form elasticity between either a payroll or a labor income tax and equilibrium labor supply. There is a similarly large reduced-form literature that considers the effect of minimum

[14] In our derivation we use the fact that $dz = d\log(1+\tau) = d\tau$ on the left-hand side of the equation, and the approximation $\log(1+\tau) \approx \tau$ on the right-hand side.

[15] It goes beyond the scope of this paper to show that the formulas derived in these studies fail to hold when the RER does not hold.

wages on equilibrium labor supply (see e.g. Card and Krueger, 2015 for an overview). However, to our knowledge there has not been any recent attempt to compare estimates between the two literatures.

The problem of comparing estimates between the two literatures is that a binding minimum wage leads to labor market rationing. Because demand does not equal supply, the effect of a change in the minimum wage on equilibrium labor supply only depends on the labor demand elasticity. On the other hand, as can be seen by considering Figure 1, or equation (8), the reduced-form elasticity between the equilibrium quantity and the net-of-tax rate depends on both the demand and the supply elasticity. Therefore, the reduced-form elasticities obtained in each of the two literatures are fundamentally incomparable.

However, our approach allows researchers to decompose the reduced-form elasticity obtained from variation in the tax rate into a demand and a supply elasticity. Hence, through our approach researchers can compare labor market elasticities obtained through variation in a labor income tax or payroll tax to estimates obtained using variation in a minimum wage. Thus, our approach allows researchers to compare elasticities between two of the largest empirical literatures in economics.

4.2 Analyzing Heterogeneity in the Response to Taxation

From the reduced-form literature it is well known that some groups are more responsive to taxation than others. In a labor-market context we know that the reduced-form elasticity between labor supply and the net-of-tax rate is higher for women than for men (see e.g. Blundell et al., 1998; Meghir and Phillips, 2010). We also know there are differences between income groups (see e.g. Gruber and Saez, 2002). What we know less about, is whether the differences in the reduced-form elasticities are driven by differences in the labor supply elasticity, the labor demand elasticity or both.

Our approach offers a convenient method for answering this question. Splitting the sample by gender allows researchers to estimate demand and supply elasticities for both groups. Even more interesting is the case where males and females each face independent variation in the tax rate. For instance, Gelber (2014) studies couples in Sweden where males and females each face different variation in their tax rate. Conceptually, this problem can be studied in the multi-good model we develop in section 2.2. Here, female labor supply serves as the first good, while male labor supply serves as the second good. The price variables are the wage rates of each of the spouses. Proposition 1 applies because the reform Gelber (2014) studies provides independent variation in the tax rate on both goods. This implies that variation in the tax rate can be used to estimate own-demand and supply elasticities for each spouse in the couple, as well as cross-demand and cross-supply elasticities. The former two elasticities can be used for studying the mechanisms underlying the gender wage gap, while the latter elasticities are of great value for researchers interested in, for instance, optimal taxation of couples.

5 Application: Payroll Taxation in Norway

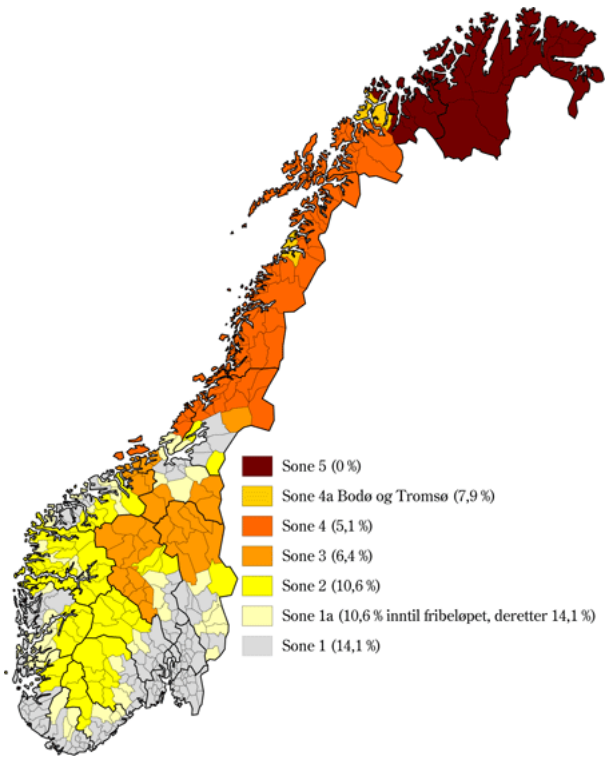
We now turn to an application in the Norwegian labor market. Norway levies a payroll tax, *arbeidsgiveravgift*, at the employer level. Tax revenue from the payroll tax is used to finance social security.[16] A payroll tax levied at the employer level is conceptually an ad-valorem tax levied on (labor) demand. As such, the conceptual framework we developed in section 2 can be applied to this setting.

Over the years the payroll tax system in Norway has been reformed multiple times. Below, we discuss the institutional setting, as well as the payroll tax reform that provides the source of exogenous variation in the payroll tax.

5.1 Background

Payroll tax rates in Norway are differentiated across regions. The policy aim is to stimulate employment in the Norwegian periphery. Employers in the urban areas in the South of Norway face a tax rate of 14.1 percent. Employers in the rural North of the country are exempt from the tax, and in between are several zones with rates ranging between about 5 and 11 percent. Figure 3 provides an overview of the seven current tax zones.

Figure 3: Payroll Tax Zones in Norway



Source: Statistics Norway

[16]Social benefits are unrelated to the tax rate an employer faces. As such, the payroll tax functions as as a pure tax, rather than an insurance premium.

In our application we isolate variation from one specific reform to the payroll tax system. Before the year 2000 all employers within a specific zone were charged the same tax rate. After the reform firms in Norway's most important exporting industries faced a 14.1 percent tax rate regardless of their location. The affected industries were Steel Production, Mining and Shipyards. The reform was introduced in order to comply with EU legislation, and is hence plausibly exogenous to local labor market conditions.

In 2000 Norway had five tax zones. Therefore, the reform allows us to divide plants in the affected industries into one control and four different treatment groups. The control group consists of firms in the affected industries that already faced a 14.1 percent tax rate, because they were located in urban areas in the South of Norway. Exporting firms in the other four tax zones each face a different treatment, depending on the tax rate they faced prior to the reform. Firms in the rural North face the largest shock, because their payroll tax rate increases from 0 to 14.1 percent. The shock in the rural areas in the South is smallest at about 3.5 percentage points.

Part of the 2000 reform was undone in 2007, as some of the industries affected by the first reform were again allowed to pay the tax rate corresponding to their location. We use the 2007 reform as an additional source of variation. By that time the number of tax zones had also increased from five to seven. As a result, the 2007 reform divides the country into one control, and six treatment groups. Figure 4 gives an overview of the average tax rate in each of the tax zones.

5.1.1 Labor Markets in Norway

Labor markets in Norway are strongly unionized. The share of employees in the private sector who are members of a union is 43% and about 50 % of the private labor force is covered by collective agreements (see Wallerstein et al., 1997; Stokke et al., 2003).[17]

The collective wage setting process takes place on both the national (or industry) and the firm level. The central level handles issues such as collective agreements, wage regulations, working hours, working conditions, medical benefits and pensions, while local negotiations handle local adjustments. The local negotiations usually take place under a peace clause, preventing strikes and lockouts throughout the duration (usually two years) of the collective agreements (see Holden, 1998; Hunnes et al., 2009).

5.2 Data description

We use plant-level data from the Annual Manufacturing Census of Statistics Norway, covering the period from 1996 to 2012. The dataset contains administrative data on the universe of production plants in Norway. Plants are defined as production facilities for a given firm in a given municipality. For the purposes of our analysis we limit our sample to plants with multiple employees in industries

[17] The difference between bargaining coverage and union density arises because firms that are covered by a collective agreement implement this for all employees, not only those who are unionized.

that have been affected by the payroll tax reform. These industries are Steel Production, Mining and Shipyards. Table 1 presents summary statistics for treatment and control groups of plants, where we aggregate the information of the four treatment groups into one treatment category.

5.3 Methodology

Our methodology requires that we observe a panel of traded quantities Y_{it}, logged prices P_{it}, tax rates τ_{it}, a transformation $f(\cdot)$ that transforms the tax rate into the instrument z_{it}, and control variables x_{it}. In our application we study a yearly panel of production plants. Hence, the cross-sectional indicator i denotes a production plant and t denotes years. In our labor market setting, Y_{it} denotes the number of effective units of labor at plant i in year t. To distinguish between extensive and intensive margin responses, we use both the number of employees and the average number of hours per employee as a dependent variable.

The price variable p_{it} is the average wage rate per hour at the plant. τ_{it} is the payroll tax rate. We assume that the strong RER holds such that $z_{it} = f(\tau_{it}) = \log(1 + \tau_{it})$. Finally, for control variables, we use plant-fixed effects, sector$\times$ time fixed effects, and municipality level controls for unemployment and the average wage rate.

To estimate the labor demand and supply elasticity, we follow the strategy outlined in section 2.3. That is, we first estimate the first-stage equations (11) and (13), and use the instrumented values of p_{it} and p^z_{it} to estimate equations (12) and (14).

5.3.1 Identification

In our empirical framework identification consists of two parts. First, the instrument should be exogenous, implying that the coefficients in the system of reduced-form equations (3) can be estimated using OLS. Conceptually, by controlling for plant-fixed effects and sector$\times$ time fixed effects we use a difference-in-difference approach. This implies that identification of the reduced-form coefficients relies on the following common-trend assumption. In each tax zone, both the wage rate per hour and hours worked should follow a similar trend in the absence of treatment. We verify whether this assumption holds through a placebo test. In the placebo test we simulate treatment for all manufacturing plants that are not in the main export industry and hence, not treated in reality. For these plants reduced-form coefficients on the instrument should equal zero. This verifies that wages and employment follow a common trend among treatment and control zones in the absence of treatment.

Second, to identify the labor supply and the labor demand elasticity we additionally require that both the standard exclusion restriction and the RER hold. In our context, these assumptions appear rather plausible. Because the payroll tax is only charged on the plant side, it is unlikely that it has a direct effect on labor supply. This implies the standard exclusion restriction. In addition, the wage a firm pays to a worker equals the price inclusive of the payroll tax. Hence, if

Table 1: Summary Statistics

	Treated Plants	Control Plants	Total
Wage Rate	195.5	213.5	208.3
	(66.35)	(72.33)	(71.13)
Payroll Tax	0.114	0.141	0.133
	(0.0404)	(1.31e-09)	(0.0248)
Employees	42.30	78.64	68.20
	(71.11)	(194.1)	(169.0)
Hours per Employee	1625.9	1679.2	1663.9
	(274.2)	(266.4)	(269.7)
Gross Wage per Employee	316.0	357.1	345.3
	(115.1)	(129.7)	(127.0)
Municipal Average Sales	10.70	11.92	11.57
	(1.159)	(1.207)	(1.316)
Municipal Unemployment Rate	2.246	2.201	2.214
	(1.079)	(0.774)	(0.873)
Observation	2,210	5,483	7,693

Notes: All wage variables are denominated in 1000 Norwegian Krones. Statistics reported are means with standard deviations in parenthesis. The control group is defined as plants that reside in zone 1. The treatment group are plants in zones 2-5.

firms act rationally the strong RER should hold. Unfortunately, we cannot test the RER in this setting because we do not have an additional instrument.

5.4 First Stage: Incidence

Column 1 of Table 2 presents estimates for the first-stage equation. In panel A we present results for the supply side equation (11), and Panel B presents evidence for the demand-side equations (13). We find that a one percent increase in the net-of-tax rate results in a .49 reduction in the wage rate excluding the payroll tax. This implies the payroll tax is shared roughly 50-50 between employers and employees. This result is very useful for our methodology, as it implies that changes in the net-of-tax rate affect both the relevant wage rate for the demand side, and the relevant wage rate for the supply side. However, standard errors are also quite large. A 95-percent confidence interval suggests that the burden of the payroll tax borne by the demand side ranges between 12 % and 89 %.

5.5 Elasticities

In columns 2 and 3 of Table 2 we consider the reduced form effect of the payroll tax on both hours worked per employee and the number of employees. An increase in the net-of-tax rate leads to a decrease in the number of employees at the plant level. Surprisingly, an increase in the net-of-tax rate also results in an increase in the number of hours worked per employee.

We proceed to recover labor supply and demand elasticities. Column 4 presents intensive-

Figure 4: Statutory Payroll Tax Rates per Zone

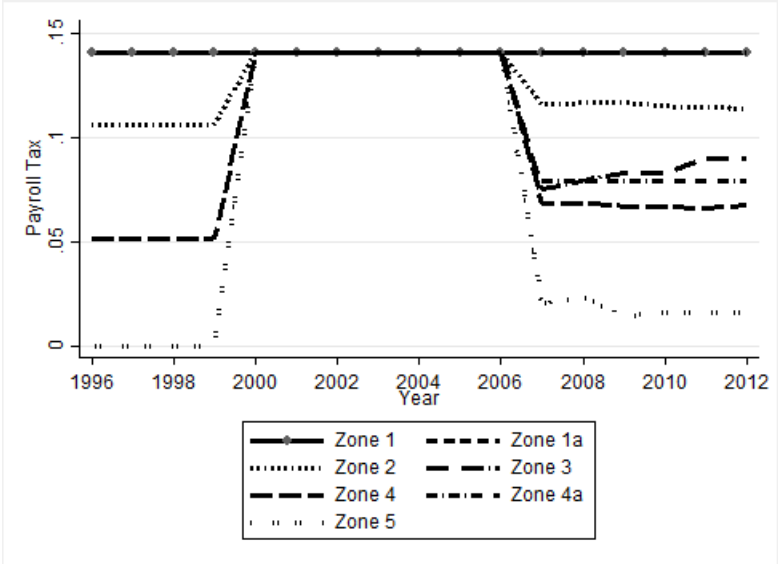


Table 2: The Effect of the Payroll Tax

Panel A. Supply	(1) Wage Rate excl. Payroll Tax per Hour	(2) Hours per Employee	(3) Employees	(4) Hours per Employee	(5) Employees	(6) Wage Rate excl. Payroll Tax per Employee	(7) Employees
Net of Payroll Tax	-0.494**	0.342*	-2.728***			-0.152	
	(0.229)	(0.194)	(1.015)			(0.248)	
ε^S Intensive				-0.693*			
				(0.419)			
ε^S Extensive					5.523**		
					(2.625)		
ε^S Extensive, Literature							17.984
							(29.499)
N	7693	7693	7693	7693	7693	7693	7693
F-Statistic				4.639	4.639		0.375
Panel B. Demand	(1) Wage Rate incl. Payroll Tax per Hour	(2) Hours per Employee	(3) Employees	(4) Hours per Employee	(5) Employees	(6) Wage Rate incl. Payroll Tax per Employee	(7) Employees
Net of Payroll Tax	0.506**	0.342*	-2.728***			0.848***	
	(0.229)	(0.194)	(1.015)			(0.248)	
ε^D Intensive				0.677			
				(0.563)			
ε^D Extensive					-5.392		
					(3.693)		
ε^D Extensive, Literature							-3.216**
							(1.591)
N	7693	7693	7693	7693	7693	7693	7693
F-Statistic				4.866	4.866		11.718

Notes: The dependent variable in each regression is shown at the top of the column. Statutory Payroll Tax is transformed as $\log(1 + Payroll \quad Tax \quad Rate)$. Regressions include plant × firm, sector and year fixed effects. The estimates are weighted by the number of employees at the plant. All regressions include as control variables the average sales and the unemployment rate at the municipality level. All variables are in logs. Robust standard errors. Asterisks denote: $***p < 0.01, **p < 0.05, *p < 0.1$.

margin elasticities, whereas column 5 presents results on the extensive margin. As can be seen, the labor supply elasticity on the intensive margin is negative. This is consistent with the idea that workers in the main export industries in Norway find themselves on the backward-bending part of the labor supply curve. Intensive labor demand responses are not significantly different from zero. This is unsurprising because it is difficult for employers to adjust contracted hours.

In column 5 we measure elasticities on the extensive margin. Panel A shows that an increase in the wage rate for workers results in a strong increase in labor supply. Similarly, panel B shows that an increase in the wage rate inclusive of the payroll tax results in a strong decrease in labor demand. Both elasticities have a similar magnitude of around 5.4. The estimated elasticities are rather large in size relative to the previous literature (Blundell and MaCurdy, 1999; Keane, 2011; Elias, 2015). This could be due to the sample being composed of export industries. To remain competitive on the world market, plants need to downsize their operations significantly when wage costs increase. The high labor supply elasticity may in turn be the result of aggressive wage bargaining by unions. However, F-statistics for instrument strength are somewhat below conventional levels for both regressions. Weak instruments can potentially introduce bias in our estimates.

Columns 6 and 7 estimate the same first- and second-stage equations, but, in line with previous literature, use the number of employees, rather than hours worked as a measure for effective units of labor. This implies that P_{it} denotes the wage rate per worker, and Y_{it} denotes the number of workers. As can be seen, both incidence and labor demand elasticities are radically different in this approach. The reason is that workers in our sample respond to changes in the payroll tax on the intensive margin. This biases both the estimated incidence parameter, and the labor supply and demand elasticities when we use the number of employees as our measure for effective units of labor.

Robustness Checks. In Table 3 we do a placebo test by simulating the same tax reform for plants outside of the main export industries, and, hence, not affected by the tax reform. We estimate the reduced-form expression (3). As can be seen, in the sectors that are not affected by the tax, neither the wage rate nor hours worked are affected by the simulated changes in the payroll tax. Therefore, it is plausible to assume that both variables follow a common trend in treatment and control regions.

6 Conclusion

In this paper we outline a novel methodology which allows econometricians to estimate both supply and demand elasticities using only (exogenous variation in) the tax rate as an instrument. We define the restriction that allows identification of the two elasticities and coin it the Ramsey Exclusion Restriction (RER). The RER restricts demand such that a 1 percent increase in the net-of-tax rate elicits the same change in demand as a one-percent increase in the before-tax price.

In models of taxation the RER follows from rational behavior by consumers. We show that our method extends to settings where J goods are traded, where taxes are levied on the supply side, and when taxes are non-linear. We link our results to the sufficient-statistics literature and show various possible applications of our method. Finally, we demonstrate our approach through an application for the Norwegian payroll tax.

Our results are of interest for structural and reduced-form approaches to welfare analysis. Our approach allows researchers to decompose reduced-form responses to taxation into demand and supply elasticities. The elasticities that are obtained through our methodology can be used in calibrating structural models, in comparing estimates between literatures, and in studying heterogeneity.

The RER has several limitations. First, our approach requires researchers to observe both the traded quantity and the price of the good. This can be problematic in some labor market settings where researchers only observe labor income.

Second, identification of the two elasticities requires that both demand and supply share the incidence of the tax. If incidence falls on only one side, our methodology allows estimation of the elasticity for the side that pays the tax. Alternatively, as is the case in our application, if the correlation between the net-of-tax rate and before- and after-tax prices is weak, neither elasticity is identified. The weak instrument problem we encounter in this study should not pose a big problem to researchers that have access to larger datasets.

Finally, our methodology requires that the RER holds. As we show, the RER is in principal testable, but this requires a second valid instrument, which in reality might not be available. The RER may not hold in settings where consumers are irrational (e.g. Chetty et al., 2009), and in settings where the tax can be (partially) avoided (e.g. Chetty, 2009a). In those cases two instruments are required to estimate the demand and supply elasticity, and to estimate the excess burden of a tax.

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A Proof to Proposition 1

Proof. To link the system of equations (9,10) to the literature on identification in simultaneous equation models it is useful to stack the equations. Let N denote the total number of observations. y^j is the $N\times 1$ vector of observations of the log quantity of good j, and p^j is the corresponding vector of prices. $Y = \left[y^1, \ldots, y^J, p^1, \ldots, p^J\right]$ denotes the $N\times 2J$ matrix of endogenous variables. Similarly, let z denote the $N\times J$ matrix of instruments and x the $N\times K$ matrix of control variables. $Z = [z, x]$ denotes the $N\times(J+K)$ matrix of exogenous variables. Finally let $\nu = \left[\nu^{D_1}, \cdots, \nu^{D_J}, \nu^{S_1}, \ldots, \nu^{S_J}\right]$ denote the $N \times 2J$ matrix of disturbance terms. The ordering implies that the demand equations are stacked on the left side, while supply equations appear on the right side. We can now present the system of equations, (9,10), as follows:

$$YB + Z\Gamma = \nu, \tag{15}$$

where B is a $2J \times 2J$ matrix of coefficients for the endogenous variables in Y, and Γ is the $(J+K) \times 2J$ matrix of coefficients for the exogenous variables.

We prove that all coefficients in the system of equations (9,10) are identified by showing that the structural coefficients in each of the $j = 1, \ldots, 2J$ individual equations are identified. Denote by $\{B_j, \Gamma_j\}$ the j-th column of B and Γ. The two column vectors contain the full set of structural coefficients in equation j. Denote the restrictions on the coefficients in matrix form as follows:

$$\left[\Phi^{B_j}, \Phi^{\Gamma_j}\right] \begin{bmatrix} B_j \\ \Gamma_j \end{bmatrix} = \phi_j,$$

where Φ^{B_j} (Φ^{Γ_j}) is a $g \times 2J$ ($g \times (J+K)$) matrix with restrictions on the coefficients in B_j (Γ_J), ϕ_j is a g-vector and g is the total number of restrictions on B_j and Γ_j. In this case, the structural coefficients in equation j are identified if the system of equations:

$$\begin{bmatrix} \Pi & I_{J+K} \\ \Phi^{B_j} & \Phi^{\Gamma_j} \end{bmatrix} \begin{bmatrix} B_j \\ \Gamma_j \end{bmatrix} = \begin{bmatrix} 0 \\ \phi_j \end{bmatrix},$$

has a single solution, or is overidentified (see Hausman (1983)). The necessary and sufficient condition for this to be true is known as the rank condition and can be written as:

$$rank \begin{bmatrix} \Pi & I_{J+K} \\ \Phi^{B_j} & \Phi^{\Gamma_j} \end{bmatrix} = 3J + K.$$

We first show that the rank condition is satisfied. To see this, note that only one good appears in each equation, and the coefficient on the good is restricted to equal -1. Moreover, instruments do not appear in supply equations. The restriction matrix is thus given by:

$$\begin{bmatrix} \Phi^{B_j} & \Phi^{\Gamma_j} \end{bmatrix} = \begin{bmatrix} I_J & 0_{J\times J} & 0_{J\times J} & 0_{J\times K} \\ 0_{J\times J} & 0_{J\times J} & I_J & 0_{J\times K} \end{bmatrix},$$

for all supply equations. The rank condition for a given supply equation j is therefore:

$$rank \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J\times K} \\ \Pi_{xy} & \Pi_{xp} & 0_{J\times J} & I_K \\ I_J & 0_{J\times J} & 0_{J\times J} & 0_{J\times K} \\ 0_{J\times J} & 0_{J\times J} & I_J & 0_{J\times K} \end{bmatrix} = 3J + K.$$

The matrix on the left-hand side is $(3J+K) \times (3J+K)$. Hence, we have to prove that it has full rank. To do this, consider whether linear row-operations can be used to fully cancel out rows. If this is not possible, the matrix has full row rank, and since, it is square, full rank.

Consider the partitions from top to bottom. Rows from the second partition cannot be used

to cancel out rows in any of the other partitions, as it is the only partition with non-zero elements in the right-most partition. The second partition has full row rank by virtue of the fact that we have assumed that the control variables are linearly independent. It can therefore be removed from consideration. The rank restriction thus simplifies to:

$$rank \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J\times K} \\ I_J & 0_{J\times J} & 0_{J\times J} & 0_{J\times K} \\ 0_{J\times J} & 0_{J\times J} & I_J & 0_{J\times K} \end{bmatrix} = 3J. \tag{16}$$

The right-most partition no longer contributes to the rank as it consists of zeros, and can therefore be removed from consideration as well. Furthermore, multiply the second partition from the top in equation (16) by $-\Pi_{zy}$ and add it to the first partition to arrive at:

$$rank \begin{bmatrix} 0_{J\times J} & \Pi_{zp} & I_J \\ I_J & 0_{J\times J} & 0_{J\times J} \\ 0_{J\times J} & 0_{J\times J} & I_J \end{bmatrix} = 3J.$$

The second partition from the top has full row rank, and cannot be formed through linear combinations of the other partitions. The rank condition thus simplifies to:

$$rank \begin{bmatrix} \Pi_{zp} & I_J \\ 0_{J\times J} & I_J \end{bmatrix} = 2J.$$

Now multiply the bottom partition by -1 and subtract from the top partition to arrive at:

$$rank \begin{bmatrix} \Pi_{zp} & 0_{J\times J} \\ 0_{J\times J} & I_J \end{bmatrix} = 2J.$$

Both the bottom and the top partition have full row rank, provided Π_{zp} has full rank. Furthermore, we clearly cannot use operations from the first partition to cancel out the second partition or vice versa. Therefore, the rank condition is satisfied.

For demand equations the additional restrictions come from the RER. The matrix of restrictions on demand equations can be written as:

$$\begin{bmatrix} \Phi^{B_j} & \Phi^{\Gamma_j} \end{bmatrix} = \begin{bmatrix} I_J & 0_{J\times J} & 0_{J\times J} & 0_{J\times K} \\ 0_{J\times J} & I_J & -I_J & 0_{J\times K} \end{bmatrix},$$

The rank condition is hence given by:

$$rank \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J\times K} \\ \Pi_{xy} & \Pi_{xp} & 0_{J\times J} & I_K \\ I_J & 0_{J\times J} & 0_{J\times J} & 0_{J\times K} \\ 0_{J\times J} & I_J & -I_J & 0_{J\times K} \end{bmatrix} = 3J + K.$$

Applying the same operations as above, we can simplify this to:

$$rank \begin{bmatrix} \Pi_{zp} & I_J \\ I_J & -I_J \end{bmatrix} = 2J,$$

Finally, add the bottom partition to the top partition to arrive at:

$$rank \begin{bmatrix} \Pi_{zp} + I_J & 0_{J\times J} \\ I_J & -I_J \end{bmatrix} = 2J.$$

This rank condition is satisfied under the assumption that $\Pi_{zp} + I_J$ has full rank. □

B Additional Tables

Table 3: The Effect of the Payroll Tax, Placebo

	(1) Wage Rate excl. Payroll Tax per Hour	(2) Hours per Employee	(3) Employees
Net of Payroll Tax	-0.040	-0.027	-0.312
	(0.064)	(0.058)	(0.260)
N	165553	165553	165553
F-Statistic			

Notes: The dependent variable in each regression is shown at the top of the column. Statutory Payroll Tax is transformed as $log(1 + Payroll \quad Tax \quad Rate)$. Regressions include plant × firm, sector and year fixed effects. The estimates are weighted by the number of employees at the plant. All regressions include as control variables the average sales and the unemployment rate at the municipality level. All variables are in logs. Robust standard errors. Asterisks denote: $***p < 0.01, **p < 0.05, *p < 0.1$.