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**OPTIMUM SAVING AND GROWTH:  
HARROD ON DYNAMIC WELFARE ECONOMICS**

BY

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**CENTER FOR THE  
HISTORY OF POLITICAL ECONOMY**  
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# Optimum saving and growth: Harrod on dynamic welfare economics

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**Abstract.** In the 1960s and 1970s Harrod shifted the emphasis of his research in economic dynamics from the study of business cycles (instability principle) to the investigation of the growth process. As part of that, he restated his concept of the natural growth rate as an optimum welfare rate. The present paper examines Harrod's dynamic welfare economics, built around his concept of optimum saving developed as a reaction to Ramsey's approach to capital accumulation. It is shown that, according to Harrod, the saving rate does not affect the long-run growth rate of per capita income, which is determined by technical progress. Moreover, the economy will grow at the natural (full employment) rate only if economic policy is able to bring saving to its "optimum" level in macroeconomic equilibrium. Harrod's interest in optimal growth was motivated by his double concern with growth policy in mature economies and economic development in poor countries.

**JEL classification.** B22, O10

**Key words.** Harrod, optimum saving, growth, dynamic welfare economics

I would affirm that maximum economic growth in accordance with the potential supply capacity of the country is the primary objective and that all other objectives are subordinate to this ... I do not think that this is merely a personal view. I hold that it expresses what economics has always been thought to be about. (Harrod 1969, p. 334)

## 1. Introduction

Roy Harrod's (1939, 1948) seminal contributions to economic dynamics have been discussed mostly from the perspective of his 'instability principle', that is, the proposition that departures of the economic growth rate  $G$  from its equilibrium ("warranted") growth path  $G_w$  are not self-correcting but cumulative in effect (Kregel 1980; Besomi 2001; Harcourt 2006; Bruno and Dal Pont Legrand 2014; Blume and Sargent 2015; Halsmayer and Hoover 2016). Although the momentary instability of the actual growth rate remained a topic of permanent interest for the Oxford economist, he would in the 1960s shift the focus of his research agenda to the investigation of what he called the natural growth rate  $G_n$  governed by population growth plus technical progress (that is, the rate of increase of the labor force in efficiency units). In fact, signs of change were already visible in Harrod (1948, pp. 22-28, 87-89). The concept of the natural rate as determined by "fundamental conditions" received more attention in that book than in Harrod (1939, p. 30), when it was first formally introduced into economics. Reacting to Joan Robinson's critical interpretation, Harrod (1970, p. 741) pointed out that 'this instability principle, much stressed by Mrs. Robinson, is only one part, and perhaps by now a minor part, of my total growth theory'.

The present paper investigates Harrod's welfare economic dynamics, built around his concept of optimum saving developed as a reaction to Ramsey's (1928) approach to accumulation. As it is well-known, Harrod (1939, 1948) distinguished between the *actual* growth rate per unit of time  $G = s/C$  (an ex-post tautology), the *warranted* rate  $G_w = s/C_r$  that enables investment demand to absorb the savings agents want to make and brings the goods market to balance, and the *natural* rate  $G_n$  that equilibrates the labor market – where  $s$  is the propensity to save,  $C$  is the

accretion of capital divided by the increment of goods produced in the same unit of time, and  $C_r$  is the requirement for new capital divided by the increment of output to sustain which new capital is required, that is, the required (accelerator) capital coefficient for a fixed interest rate. In order for  $G_w = G_n$  on the full-employment path, the required (or optimum) saving ratio  $s_r$  must be equal to  $C_r G_n$ . The variables  $s$  and  $s_r$  differ in the essential aspect that the latter is a dependent variable, decided by the capital coefficient and the natural growth rate (Harrod 1960). The optimal saving-income ratio  $s_r$  is such that capital accumulates at the same rate as the increase of labor supply in efficiency units in balanced equilibrium.

$G_n$  is a ceiling growth rate with capital requirements (at a given rate of interest) decided by population growth and Harrod-neutral technical progress. In Harrod's model, equality between  $G_n$  and  $G_w$  cannot be achieved by automatic working of the market mechanism, but through economic policy only. Differences between those two rates bring about inflation ( $G_n > G_w$ ) or secular stagnation ( $G_w > G_n$ ), which are not self-corrected. In the former situation  $G$  would exceed  $G_w$  for most of the time, and in the latter  $G$  must in average lie below  $G_w$ , with ensuing destabilizing reactions by entrepreneurs. Harrod's  $G_w$  attracted much attention, but his  $G_n$  concept broke new ground as well. Unlike other approaches, such as classical or Schumpeterian growth economics, Harrod (1948, pp. 20-22) assumed that both population growth and technical progress are exogenous variables. By treating technical progress as a 'built in propensity' in the economy, instead of occasional shocks that shift the equilibrium position of the system, he set growth economics off on a new trail (Robinson 1962, p. 98; Leon-Ledesma and Thirlwall 2002, sec. 2).

Harrod never abandoned the analysis of divergences between the actual and warranted growth rates (or thought that problem had been solved). Nevertheless, he would state that 'for some time now I have come to consider my concept of a "natural" rate of growth ( $G_n$ ) as more important [than the concept of] a "warranted" rate of growth ( $G_w$ )' (1964, p. 905). This reflected the emergence in the 1950s and 1960s of growth and development economics as new fields that attracted increasing theoretical and political interest, unlike Harrod's (1936, 1939) earlier focus on fluctuations (Boianovsky and Hoover 2014, sec. 1). Growth became part of Harrod's overall concern with economic policy and welfare, which came to dominate his agenda in the postwar years (Phelps Brown 1980, pp. 25-26). Harrod (1963a, p. 421)

declared in his Royal Economic Society address that ‘many economists are finding, as I do, their attention attracted to the developing countries. That partly accounts for my continuing interest in growth economics’. This is illustrated by his organization of a conference on trade and development and a contribution to the American Committee for Economic Development (Harrod 1963d; [1958] 1961).

By the 1960s, when the neoclassical Solow-Swan model came to dominate growth economics, Harrod’s subtle distinction between three growth rates and its dynamic implications were largely gone. This helps to explain why his approach to optimum saving attracted only scattered critical attention from contemporary commentators (Graaff 1960, Sen 1961, Robertson 1961, Chakravarty 1962, Asimakopulos & Weldon 1963, Wan 1971, Guillaumont 1971). Asimakopulos and Weldon discussed Harrod’s (1960) new equations in some detail, but missed Harrod’s attempt to break away from Ramsey. Harrod tried to engage Robert Solow in correspondence about Ramsey and optimal saving, with only limited success. Except for brief mentions by Eltis (1987, p. 600), Young (1989, pp. 182-83 and 192-93) and Collard (2011, p. 268), Harrod’s discussion of optimal growth has gone unnoticed by historians of thought. A close examination of Harrod’s dynamic welfare economics, developed as a unique mix of Keynesian growth theory and aspects of Ramseyan approach to saving, may illuminate not just hitherto unexplored aspects of his approach to growth, but also contribute to a better understanding of the history of growth economics.

Harrod’s post-war contributions to growth economics are examined here from the perspective of the analytical content and intellectual context of his notion of optimal growth. Maximization of economic welfare should be considered ‘through time’, something economists had asserted long before him.<sup>1</sup> What was new, claimed Harrod (1969, p. 334), was the recognition of the need to elaborate new tools appropriate for the analysis of how economic welfare can be maximized through time. This involved going back to Ramsey’s (1928) classic article on optimal saving. Harrod (1960) was one of the first attempts to combine Ramsey’s intertemporal utility maximization model with growth economics. Harrod, however, moved away

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<sup>1</sup> Malthus’s (1820, p. 9) argument, that there is an ‘intermediate’ saving rate such that the ‘encouragement to the increase of wealth is the greatest’, may be seen as the first exploration into optimum saving akin to Harrod’s. Malthus regretted, though, that ‘the resources of political economy may not be able to ascertain it’, which was precisely Harrod’s goal.

from Ramsey's framework by rejecting the relevance of the notion of 'Bliss' (maximum permanent consumption per capita) for optimal growth. Instead, just as in his treatment of  $G$  and  $G_w$ , Harrod (1963a) dealt with the determinants of the optimum rate of saving and  $G_n$  as 'operating *at a given point of time*', without any reference to the future course of events.

Unlike Ramsey, the role of the intertemporal utility function in Harrod (1960, 1963a) was essentially to determine what (natural) interest rate  $r_n$  – as an expression of intertemporal preference – is consistent with the natural welfare optimum rate of growth, not to decide the growth rate itself along the accumulation path. As discussed below, Harrod put forward the formula  $r_n = \frac{pcG_n}{e}$ , where  $pcG_n$  is the natural rate of growth per capita and  $e$  is the (inverse of the) elasticity of the diminishing marginal utility of income schedule, called 'second fundamental equation'. That formula illustrated the 'supply' side of economic growth – in the sense of deciding the saving supply ratio  $s_r$  compatible with utility maximization and full employment – whereas his better-known 1939 fundamental equation for the warranted rate expressed the 'demand' dimension. Divergences between  $G_w$  and  $G_n$  were associated to differences between the actual and optimum (or required) rates of savings when the interest rate is not at its natural level.

Harrod's position was unique insofar as he was a pioneer in both the positive theory of growth and a major contributor to the post-war development of dynamic welfare economics. He wrote extensively on the concept of the 'welfare optimum' growth rate and how to achieve it (Harrod 1953, 1955, 1957, [1958] 1961, 1960, 1963a, 1963b, 1963c, 1964, 1966, 1968, 1969 chapter 8, 1973 chapters 5 and 7, 1974, 1976). As documented below, his elaboration of the concept of  $G_n$  as welfare optimum led him to discuss topics such as the work/leisure choice, the possibility of growth satiety in rich economies, the role of expected  $G_n$  in indicative planning, and the priority of skilled labor (over physical capital) in the development of poor countries. Harrod argued that economic growth is not determined by saving, though the latter is instrumental for its implementation. The 'natural' rate of interest corresponding to  $s_r$  should be the target of monetary policy. Moreover, fiscal policy and especially indicative planning could affect  $s_r$  directly by fixing demand expectations on the underlying natural rate of growth, Harrod (1964) claimed.

Discussions about optimal growth started to come out in the 1950s (Tinbergen 1956; Horvat 1958), based on the so-called Harrod-Domar model but without any use of Harrod's  $G_n$  – a concept missing from Evsey Domar's framework and development economics in general (Boianovsky 2015, 2017). It was only in the 1960s that optimal growth became part and parcel of growth economics. The Solow-Swan model led to the result that the steady state rate of growth of income per capita is governed by technical progress. Because of diminishing returns to capital accumulation, increases in saving rates would have no permanent effect on the rate growth of income, only on its level. Optimal growth was essentially about choosing the saving ratio that maximizes permanent consumption per capita, as expressed by the so-called 'golden rule'.<sup>2</sup> Harrod (1960, 1963a) shared the view that the natural growth rate is exogenous in respect with the rate of saving, although it did play a role in implementing  $G_n$  in macroeconomic equilibrium through the formulation of economic policy.

## 2. Economic welfare and the natural rate of growth

Harrod's (1953, pp. 553-55) first identification of the optimum growth rate with  $G_n$  appears in his reaction to Pilvin's (1953) note about the Harrod-Domar model. Harrod (op. cit.) suggested that there are two distinct analogues between static economic theory and economic dynamics. In contrast with statics, the equilibrium concept of dynamics – that is, the 'steady rate of growth' – is not stable but 'surrounded by a field of centrifugal forces'. The analogue that Harrod considered pertinent in the context of Pilvin's criticism was another one.

Economists had discussed in what circumstances the pattern of stable equilibrium constitutes an 'optimal arrangement'. Whereas the warranted rate corresponded to the equilibrium pattern of statics, Harrod (1953, p. 554) explained that 'in dynamics I have used the expression "natural rate of growth" for something that may be regarded as corresponding to an optimal static pattern'. Harrod (1939, p. 30) had defined the natural rate as the 'maximum rate of growth allowed by the

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<sup>2</sup> Koopmans (1965) and others would soon give neoclassical optimal growth its final formulation by combining intertemporal utility maximization *à la* Ramsey with the golden rule (Spear and Young 2014).



increase in population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing there is always full employment in some sense'. The reference to capital accumulation as a determinant of natural growth was incorrect and later removed (Harrod 1948, p. 87). Adding the rate of increase of capital equipment as a determinant of  $G_n$  would be putting 'the car before the horse' (Harrod 1973, p. 21). He reacted against growth accounting exercises based on the aggregate production function, which measured the growth of potential output by the increase of workers, the increase of output per worker and the increase of the amount of capital. This entailed splitting technical progress into two parts, which he found 'not operational or statistically identifiable' (Harrod 1967, p. 7). The association between  $G_n$  and (Harrod-neutral) labour-augmenting technical progress – that keeps the capital-output ratio constant at a given interest rate – was introduced in the 1948 restatement of his dynamic economics.

Apart from securing full-employment, growth at the optimal natural rate is conducive to the adaptation of methods of production to the 'latest findings in technology, so as to secure utilization of the cheapest current methods' (Harrod 1957, pp. 2-3; 1960, p. 279). Although fully employed, workers may not be supplied with the optimum amount of up-to-date capital goods. 'Employers may continue to employ ... old-fashioned equipment because their estimate of the prospect of rising demand is not sufficient to justify them in going to the cost of introducing more highly productive methods'. (Harrod 1969, p. 187)<sup>3</sup> Textual evidence suggests that Harrod was similarly unwilling to acceptance the existence of tradeoffs, in calling for a target rate of 0% unemployment (Harrod 1967, p. 17).

In mature economies the greatest feasible increase in output per head is governed by technical progress. In developing countries, on the other hand, there are in general extensive fields for the application of knowledge created in developed economies. What limits the natural rate of growth in those countries is 'the maximum possible rate at which cadres of qualified personnel can be increased' (Harrod 1966, p. 170; 1963b, p. 113). As Harrod pointed out in correspondence with Solow of 26 November 1963, although the fundamental determinants of the natural rate of growth

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<sup>3</sup> Harrod (1948, pp. 73-74) had argued in favor of a 'steady rate of progress conformable with fundamental conditions' as the target of economic policy instead of Keynes's goal of full employment.

differ in developed and developing countries, ‘*both* presuppose the presence of sufficient *demand*’ (Harrod 1960-1963).

Maximizing economic growth is not the same as maximizing the rate of growth of GDP, as the latter does not provide a precise index of economic welfare (Harrod 1969, pp. 334-35; 1973, pp. 168-69). Leisure is an economic good, bought in exchange for money income foregone. An increase in (voluntary) leisure involves an increase in happiness, although it may cause a reduction of GDP growth. Economic welfare is broader than GDP, but more limited than the notion of happiness. As seen by Harrod (1963a, p. 417), economics had been based on the ‘greatest happiness principle’ since the classical school and up to Jevons and Edgeworth. He described himself as ‘an unrepentant believer in inter-personal comparisons and cardinal measurements’. Whereas Harrod agreed with the Cambridge approach (Boianovsky 2014), he parted company with Pigou’s assumption that economic welfare moves in the same direction as total welfare. According to Harrod (1963a, p. 420), ‘happiness does not depend only on *economic* welfare’. That applied to the view, criticized by Harrod (1963a, pp. 420-21), that social and cultural institutions should be changed to bring about higher growth in developing countries.

In contrast with poverty in developing nations, rich countries faced the ‘possibility of economic satiety’ (Harrod ([1958] 1961). Harrod rejected the view that needs or desires are unlimited, for the maximum income level that can give satisfaction to the rich minority of population (i.e. its satiation point) is necessarily above the satiation point of the majority. His argument was based on the distinction between *oligarchic* and *democratic wealth*. The former comprises a large fraction of the consumption of the rich, formed by the purchase of personal services and service-intensive consumption goods. An individual whose income is well above average can enjoy those services, but an overall rise in the incomes of the majority of population will not enable them to reach such consumption standard. It is ‘impossible for one man”, however high the average income per capita, ‘to engage in any one year the services of more than one man-year of the labour of others’, which puts a ‘severe limitation upon the range of possible satisfactions achievable’ through a general increase of income (Harrod, p. 8).<sup>4</sup> *Democratic wealth* comprises consumption

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<sup>4</sup> As put by Hirsch (1976, p. 24) in his discussion of Harrod’s concept, ‘one man’s servant is another man’s service’. Behind the argument is the implicit assumption that the productivity of an hour’s service remains constant. The average man, no

available to all, and increases according with average productivity. It ‘can be raised only by more material things, capable of mass production, being made available’ – not in respect of direct personal services or of the enjoyment of intrinsically scarce goods.

The threshold of economic satiety must always be lower for society as a whole than for the rich minority, which may end the ‘economic struggle’ of scarcity and bring growth process to a halt. But that is not the end of history, since leisure takes the place of effort time devoted to production. As observed by Hirsch (op. cit.), ‘paradoxically, and rather nobly ... Harrod ... led the field in pinpointing what growth could not achieve ... He identified the economic wants and not just the cultural or spiritual values that would go unmet by economic growth’.<sup>5</sup> Regardless of the prospect of economic satiety, Harrod continued to press economic growth at the natural optimal rate as the grand objective of economic policy.

We need growth to mitigate hardships, which abound even in richer countries, and to give citizens fuller opportunities; and, if and when countries become rich to the point of satiety, there is still the great challenge of the poverty-stricken famished countries of the world that an over-plus in the rich countries could help to meet. (Harrod 1965, pp. 77-78)

Optimal natural growth rate is not a ‘distant ideal’, but rather objective and feasible in close connection with the achieved ‘degree of civilization’ (Harrod 1955, p. 358). It is not constant in time, since it is governed by determinants operating at a *given point of time* (Harrod 1963a, p. 406). In principle, according to ‘sophisticated theoretical analysis’, it may vary daily due to changes in the rate of growth of population and technical progress. In practice, one may consider the natural rate as an average over the period of analysis (1955, *ibid*).

Amartya Sen (1961, p. 496) regarded Harrod’s approach to take the natural growth rate, given by a set of purely objective factors, as the normative welfare

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matter how high his income, can never command the service of more than one other average man. Abramovitz (1979, p. 13) clarified, in his interpretation of Harrod’s paradox, that ‘there is an absolute fixed amount of “services” which average men can command, no matter how rich they become’.

<sup>5</sup> There are some striking similarities between Harrod’s American essay and Keynes’s ([1930] 1963) well-known pamphlet. Harrod, however, did not refer to Keynes but to Galbraith (1958).

optimum a neat and ingenious method of ‘solving this [optimal] intertemporal problem using value judgments that are sufficiently spread ... to be taken for granted’. Harrod (1963a, pp. 416-20), however, rejected the view that economic recommendations in general implied some form of value judgments. The ‘happiness principle’ should be enough to warrant the natural growth rate as welfare optimum. What Sen found unacceptable was Harrod’s assumption that technical progress and population growth are independent of capital accumulation. Sen contrasted that with Tinbergen’s (1956) discussion of optimal saving – based on a traditional Harrod-Domar equation with a given capital-output ratio – and its result that a rise in the saving rate leads to a rise in the growth rate in the same proportion. Tinbergen (1956) fitted well in the postwar development economics, which tended to disregard Harrod’s  $G_n$ .

Harrod, of course, was not the only economist supporting growth as the goal of economic policy. Domar (1957, p. 14) deemed growth the ‘healthier objective’, as full employment may coexist with inefficiency. According to Domar ([1952] 1957, p. 30), the ‘optimum allocation of resources does not mean the maximum output under given conditions ... but a maximum achievable rate of growth over time’. Growth as the policy goal was the topic of Tobin’s (1964) presidential address. Unlike Domar and Harrod, Tobin relied on the neoclassical Solovian adjustment mechanism to argue that, asymptotically, policy makers have no choice over the rate of growth, but can choose only among parallel paths of different levels according to the golden rule. The natural rate is also exogenous in Harrod’s framework, but the convergence to  $G_n$  is a matter of economic policy, through its effect on the rate of saving. Hence, there is only one saving ratio compatible with  $G_n$ , unlike Tobin’s neoclassical perspective of choosing, among an infinite number of saving ratios consistent with  $G_n$ , the one that maximizes permanent consumption (Ramsey’s bliss).

When introducing the notion of  $G_n$  rate as optimum, Harrod (1953, p. 554) recognized that there was some ‘ambiguity’ in his concept in relation to the optimal distribution of effort in the provision of present and future goods. At first, he thought the distribution determined by savings individuals prefer to perform when fully employed could be regarded as optimal. Harrod (1957a, p. 3) still regarded the optimum natural growth rate idea ambiguous, as ‘nothing is stipulated about the

behavior of the rate of interest'. It was only in his 1960 "Second essay" that Harrod came up with an answer, as discussed next.

### 3. Saving, the rate of interest and Ramsey

Harrod stressed the causality issues involved in the relation between  $s$  and  $G_w$  on one hand, and  $s_r$  and  $G_n$  on the other. The fraction of income saved  $s_r$  becomes a '*desideratum*' along the optimal full-employment growth path. While the natural rate of growth is (mostly) exogenous in relation to the variables of the equations and 'is therefore taken to *require* a specified amount of savings ( $s_r$ ), the warranted rate of growth is taken to be determined by the actual rate of saving ( $s$ )' (Harrod 1960, p. 286). In the warranted equation it is assumed that economic agents are free to save the amount they prefer, which decides the equilibrium growth rate.<sup>6</sup> The 'essential doctrine' of a natural growth rate, Harrod (1969, p. 197) claimed, was that 'there should not be a failure to achieve it because individuals do not choose to save as much as would be required'. This did not prevent discrepancies between  $G_n$  and  $G_w$ .

This assumption of the independence of  $s$  is appropriate in its context. Indeed, the fact that the 'warranted' rate of growth may not be equal to the 'natural' rate depends precisely on the assumption that private motives may not yield the amount of saving that complies nicely with the needs of society; they may generate too much or too little saving. The Keynesian origin of this idea is obvious. (Harrod 1960, p. 279)

The relation between capital accumulation and the natural growth rate, and its change between the 1930s/40s and the 1960/70s definitions, is not straightforward. In

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<sup>6</sup> A high  $s$  means that the rate  $G_w$  is high, because only with a high rate of growth there will be enough investment to absorb (through the accelerator mechanism) available saving. Whether a high  $s$  actually brings about a high growth rate  $G$  is another matter, pertaining to the stability properties of Harrod's model (Hahn and Matthews 1964; Boianovsky 2015). See Samuelson (1964, pp. 743-46) for a rare treatment of the distinction between  $s$  and  $s_r$ .

correspondence with Harrod, Dennis Robertson grasped the notion that, although  $G_n$  is independent of  $s_r$ , the latter is the saving ratio required to *implement* or finance growth at its natural rate: ‘Harrodian man, unlike Ramseyan man, won’t get his technical progress unless he saves for it’. This comes from the way Harrod deduced  $s_r$  as the result of the equality between  $G_w$  and  $G_n$ . As Robertson (1961, pp. 14-15) remarked, growth theorists treated technical progress as something that ‘just happens’ and increases output regardless of whether capital is growing or not. For Harrod, technical progress will only bring about economic growth if ‘additional capital to “support” it is forthcoming on the requisite scale’ (ibid).<sup>7</sup> In Harrod’s (1939, 1948) initial formulation the natural growth rate was the highest rate of output continuously permitted by the rate of growth of population and the rate of technical progress. ‘For it to be maintained capital must grow at such a rate as to keep the rate of interest constant; but the question of how such a rate of growth of capital was achieved, if it was achieved at all’, did not affect the definition of  $G_n$  (Robertson, 1961, p. 15). This changed in Harrod’s (1960) restatement of  $G_n$  as welfare optimum. In Robertson’s (p. 15) enlightening interpretation, Harrod redefined  $G_n$  ‘as the maximum rate of growth of output which would be compatible’ not only with the fundamental conditions of population growth and technical progress, but ‘with a particular behaviour on the part of the community as regards saving’.

Robertson’s (1961) remarks reflected his correspondence with Harrod. In a letter of 13 August 1960, Robertson observed that, whereas most authors treated the rate of technical progress as a residual remaining after the influence of capital-growth on output has been sweated out, in Harrod’s treatment technical advance is ‘completely non-operative on output unless it is “molded” or “supported” by appropriate capital-growth’. Harrod agreed in his reply dated 17 August 1960: ‘one wants as much saving as will finance the new equipment that is suggested by the progress of technology and to support such labor as is released by this new equipment with other equipment’ (all passages quoted from Besomi 2004, pp. 39-40). The optimal growth rate determines what fraction of income agents ‘ought to save in order to provide the capital required for that rate of growth’ (Harrod 1974, p. 242).

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<sup>7</sup> This is distinct from the notion of embodied technical progress, an assumption Harrod never made.

The topic of the Harrod-Robertson exchange was the former's attempt to explain saving under the double assumption of the law of diminishing marginal utility and absence of net time preference, both borrowed from Ramsey (1928). Robertson was puzzled by the fact that Harrod reached a different result from Ramsey about the optimal proportion of income saved. Harrod replied that Ramsey thought of a Bliss obtainable from the mere accumulation of capital, without technical progress. Ramsey's problem was 'what is the optimal number of years over which to spread the saving? When one approaches the problem in this way, such questions as one's attitude to one's heirs and the value one puts on future pleasures comes into the forefront of the picture' (Harrod to Robertson, 17 August 1960).

As Harrod pointed out, if one supposes that Bliss is obtainable by saving only and 'proceeds to eliminate all [net] time preference, then one may find that utility is maximized at a very high rate of savings', as in Ramsey. But if the path to bliss is decided by technical progress, optimal saving is determined by the expected rate of increase of potential output per Harrod's formula. Given that one cannot expect an increase of per capita income of more than 5%, 'the amount of saving that can be fruitfully employed is strictly limited'. That distinguished Harrod's "point of time" approach from Ramsey's accumulation path:

It is because one is thinking of a maximum rate of progress available through advancing technology, that the whole question of our attitude to our heirs or remote descendants becomes irrelevant. To my mind the Ramsey analysis of bliss is in terms of the longest of Marshall's long periods. Whereas I would claim ... that we get a new approach to the problem of thinking in "dynamic" terms, that is, in terms of rates of progress currently achieved or achievable (Harrod to Robertson, 17 August 1960).

Harrod had long been attracted to Ramsey's (1928) essay. The two men corresponded in 1929 about that model (Ramsey's letter of 27 March 1929 to Harrod, reproduced as letter # 158 in Besomi 2003). Harrod ([1936] 1965, pp. 108-09; 1938, p. 404, n.1) endorsed Ramsey's result that if an individual is not biased in favor of the present – that is, has zero net time preference – he should save at such a pace that his net income rises at such a rate that the marginal utility of consumption falls at a rate equal to the current rate of interest (that result had been advanced by Wicksell; Boianovsky 1998). Harrod's (1948) chapter on the supply of saving was Ramseyan,

including the notion that pure time preference is a ‘polite expression for rapacity and the conquest of reason by passion’ (p. 40).

Harrod’s supply equation was derived by rewriting Ramsey’s condition as  $r = \frac{G(con)}{e}$ , where  $G(con)$  is the rate of growth of consumption (Asimakopulos and Weldon 1963; Wan 1971, p. 22, n. 6). Harrod then replaced  $G(con)$  for  $G$  on the grounds that consumption and income increase at the same rate in steady growth. Unlike Harrod, Ramsey had assumed constant population. In order to apply the expression to a community (instead of a representative individual as in Ramsey), Harrod assumed a given income distribution, interpreted  $e$  as a weighted average of individual elasticities, and expressed consumption in per capita terms. The concept and measurement of  $e$  implies, as in Ramsey, cardinal measurement of utility (Harrod 1963a, p. 417; 1973, p. 79). Hence,  $r = \frac{pcG}{e}$ , which was not yet Harrod’s supply equation. He next switched the role of the interest and growth terms, and replaced  $G$  for  $G_n$ , exogeneously determined by population growth and technical progress. Corresponding to the natural or welfare growth rate was Harrod’s natural rate of interest, which led to his supply equation  $r_n = \frac{pcG_n}{e}$ . As observed by Harrod (1960, p. 282),  $G_n$  is only ‘mainly’ exogenous, since the capital coefficient  $C_r$  is a negative function of the rate of interest. This means that  $r_n$  is determined by a system of three simultaneous equations:  $G_w = s/C_r$ ,  $r_n = \frac{pcG_n}{e}$ , and  $C_r = f(r_n)$ .

In growing economies it is possible to formulate the optimum rate of saving ‘without reference to the whole journey to Bliss’ (Harrod 1963b, p. 121). Moreover, Harrod (1960, p. 280; 1963a, p. 410; 1966, p. 169) deemed Ramsey’s Bliss unsatisfactory for welfare economics, as it involved the ‘impracticable’ comparison of the total utility achieved at “Bliss” with total and marginal utility currently achieved.<sup>8</sup>

I define the optimum rate of saving (equals investment) as that required to implement the maximum growth of production rendered possible by the growth of factors other than capital disposal ... This is in conflict with views

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<sup>8</sup> Writing  $B$  for Bliss,  $x$  for the current daily rate of saving,  $c$  for the current daily rate of consumption,  $U$  for its total utility and  $V$  for its marginal utility, the so-called Keynes-Ramsey rule establishes that the optimal rate of saving is given by  $x = \frac{B-U}{V}$  (see Robertson [1957-1958-1959] 1963, p. 252 for a simple derivation). ‘This does not look very promising from the practical point of view’, Robertson noted.



that have been much canvassed recently in the theoretical literature. In some of these writings it is implied that the growth of income depends primarily on the extent to which a community is prepared to sacrifice present consumption in favor of future consumption ... Once we recognize that the increase of future income obtainable by a sacrifice of consumption now is a function of the rate at which non-capital factors are growing, it appears that Ramsey's solution lacks a dimension. (Harrod 1963b, pp. 113-14)

Indeed, Harrod (1957b, p. 193) criticized Kurihara (1956, p. 211) for suggesting that the major problem facing underdeveloped economies was 'increasing capital accumulation'. 'Is this not a wrong emphasis, not confined to Professor Kurihara?' asked Harrod. Surely, he claimed, more important limitations came from factors, such as lack of know-how, which are behind the natural rate of growth of per capita output. If an attempt were made to get growth in output per head 'merely by increasing the capital/labor ratio, when no other factors of increase were operating, it is likely that one would run into sharply diminishing returns, and very soon a nil return, from any further rise in the capital/labour ratio' (1966, p. 169).

It is interesting – and perhaps surprising to many, in view of prevailing accounts of the history of growth economics – to find Harrod in the unlikely position of criticizing Solow, in correspondence of 26 November 1963, for 'following Ramsey [and taking] the growth potential to be mainly determined by the rate of investment' instead of technical progress. In a previous letter (August 30 1960) to Solow, Harrod claimed that in his 1960 essay he was 'breaking away' from Ramsey's line of thought. 'I now entirely repudiate' the notion that in assessing the optimum rate of saving one must take into account utilities in the state of Bliss in the remote future.<sup>9</sup>

Solow was not convinced that Harrod (1960) had successfully broken away from Ramsey. In his reply of 23 September 1960, he argued that Harrod's second fundamental equation assumed that the future values of the rate of interest are '*independent* of the present and future decisions about saving and capital formation. And this in turn follows only if the marginal productivity of capital is constant'. If the

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<sup>9</sup> Harrod (1960), differently from Ramsey, dealt with a finite economic horizon, but overlooked the question of valuing terminal capital equipment (Chakravarty 1962, p. 339, n. 4).

return on capital depends on the stock of accumulated capital, then today's investment decision influence the terms in which present and future consumption will be exchangeable in next periods, 'and the welfare decision reverts to a more complicated Ramseyan form'. Solow concluded that Harrod's attempt – to apply the new equation for the natural rate to an economy with a variable capital coefficient – failed in the sense that the equation could only be deduced from Ramsey's saving function if the capital coefficient was fixed. As seen by Solow, Harrod's second fundamental equation was built on a two-period Ramsey equation, which could be applied to a long-run succession of periods only if the marginal productivity of capital (or  $C_r$ ) was given.

The matter was related to J. de V. Graaff's (1960) objection to Harrod's (1960) identification of the growth rate of per capita income with the growth rate of per capita consumption. That was only valid under the 'restrictive' assumption of a constant capital-output ratio, as Harrod (1963a, p. 405) acknowledged. This led him to redraft his second fundamental equation in terms of the natural rate of growth of consumption per capita as a component of the natural growth of output (ibid):  $r_n = \frac{pcG(con)n}{e}$ . This was solved simultaneously with equations expressing  $C_r$  as a function of  $r_n$  and  $pcG(con)n$  as a function of  $C_r$ . Harrod (1963a, p. 406) illustrated the simultaneous determination of  $r_n$  and  $C_r$  by means of supply – as a function of  $pcG(con)n$  – and demand curves of capital disposal. Harrod's reformulation seemed to meet some of Solow's criticism, even though Harrod (ibid) insisted that, unlike Ramsey, his equations and diagram showed how forces operating 'at a point of time' determine optimal saving.

As observed by Wan (1971, p. 22, n. 6), Harrod (1960) had grafted the constant saving/income ratio assumption of Keynes onto the optimal saving model of Ramsey. 'But under what conditions will the optimal saving ratio be constant over time?' The answer is that, in an optimal saving model with constant capital/output ratio and constant elasticity of marginal utility  $e$ , the optimal output path and the optimal consumption path will grow at the same rate only if  $e = 1$ , as in the utility function  $u = \log c$ , a quite restrictive case (Wan 1971). Such criticism did not apply to Harrod's (1963a, pp. 405-06; 1964, p. 910) reformulation, which assumed that the capital-output ratio changes at a steady rate at any given point of time.

#### 4. Economic dynamics, growth policy and planning

Harrod (1964, p. 905) eventually regarded the concept of the natural growth rate as ‘more important’ than the warranted rate, as it was ‘*realistic* in relation to what is *quantitatively* important in determining the system’, and anticipated that  $G_n$  would be of increasing relevance in economic dynamics. He claimed that  $r_n$  could be empirically estimated, once measurements of  $e$  and  $pcG_n$  were made. Econometric investigations of  $e$  mentioned by Harrod (1963a, p. 408) indicated a value around 0.5. Harrod (1976, p. 74) acknowledged that in practice there must be ‘some haziness’ about the rate at which the income of the representative person will grow in the future, but suggested 5% as a good guess (1973, p. 77). Hence, the estimated value of the natural rate of interest was around 10% for the UK, which should guide banking policy (Harrod 1976, p. 74).<sup>10</sup>

The increasing relevance of  $G_n$  in Harrod’s framework should be seen in the context of his methodology. According to Harrod (1960, p. 277; 1968), a model is a formulation that has ‘adjustable parameters’, that is, can be statistically verified and tested. He doubted his growth equations matched that requirement. Harrod’s basic equation for the actual growth rate  $G = s/C$  is a tautology based on the truism that ex-post saving is equal to ex-post investment. Harrod tended to interpret his first fundamental equation as a *definition* of  $G_w$  and therefore non testable in principle. On the other hand, he believed the equation  $s_r = C_r G_n$  could become ‘model-like’, if empirical investigation were able to figure out the optimal rate of interest corresponding to optimum saving  $s_r$ , as mentioned in his discussion of economic policy.

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<sup>10</sup> The origins of Harrod’s  $r_n$  concept may be traced back to a letter from J.M. Keynes of 17 August 1938 about a draft of Harrod’s 1939 essay. According to Keynes, the ‘ideal is, of course, to establish a rate of interest, a state of confidence and a propensity to save so related to one another that the normal warranted rate of growth is equal to the natural growth corresponding to full employment’ (Moggridge 1973, p. 326). It was only in the 1960s that Harrod attempted an analytical formulation of those ideas.

Since his first ‘Essay’, Harrod (1939, pp. 31-33) had distinguished between two problems of policy: (i) the short-term one of preventing deviations from the warranted growth rate and (ii) the long-term one of bringing the warranted rate into equality with the natural rate. In the 1960s and 1970s he focused on the second problem. However, both fiscal and monetary policies have their limits in bringing the economy to its natural growth rate. Assuming the propensity to save is weak in relation to the growth potential of the economy, to obtain  $G_n$  the rate of interest should in principle be high enough to encourage such labor intensive methods ‘as to leave enough savings over to finance a “widening” of production in accordance with the growth potential of the economy’ (Harrod 1963c, p. 75). He doubted the rate of interest could rise so high.

It was in that context that Harrod (1963c, pp. 77-78; 1964, pp. 914-15; 1973, pp. 119-21) advocated the use of indicative planning in the UK, along the lines of the successful French experience (see Estrin and Holmes 1983). The creation of the British National Economic Development Council in 1962 provided a political context for Harrod’s (1964, p. 915) plea. Indicative planning should be used to give entrepreneurs the necessary assurance that their markets will in fact increase over a period of time, based on the forecast of the potential growth rate estimated by economic planners. From that perspective,  $G_n$  could ground demand expectations and make producers ‘sufficiently adventuresome’, even if there is no compulsion whatever in the plan (Harrod 1963c, p. 77; 1964, p. 914).

By setting demand expectations on the estimated natural rate of growth, Harrod believed indicative planning would be able to ‘jack up’ the actual and warranted growth rates to this optimal figure (see also Estrin and Holmes, pp. 19-21). Harrod (1964, p. 914) was aware that the expectational effect of planned targets depended on the existence of some ‘guarantee’ by policy makers that monetary and fiscal policy would be deployed to ensure that the economy reaches its growth potential.<sup>11</sup> As he realized, this created a theoretical dilemma. If authorities have the policy instruments to give this guarantee to ensure natural optimum growth, why is indicative planning necessary? On the other hand, if they lack such instruments, how can they give the required guarantee? However, even if monetary and fiscal policies

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<sup>11</sup> Domar ([1946] 1957, pp. 79-81) had already suggested ‘guaranteed growth of income’ as the best way to achieve full employment growth (Boianovsky 2017).

are able to ensure growth of aggregate demand in accordance with the supply potential, they will likely do so at the price of inflation and balance of payments problems. That is why Harrod (1973, pp. 119-21) kept supporting indicative planning as a ‘weapon of superior sophistication’. The fashion for indicative planning had been superseded in the 1970s by ‘quite unacceptable doctrines’ put out by the ‘monetarist school’, which, in his view, simplified the complex problems of the theory of potential growth.

Planning in developing countries had a different goal, as discussed by Harrod (1963a, 1963b). It was not so much a matter of reaching the natural growth rate, but changing the natural path itself. Developing countries lacked enterprise, in the sense of ‘pure will power or initiative’, which restricted taking up potential production projects. The main determinant of  $G_n$  per capita in developing countries is the maximum rate at which skilled personnel can be increased. The supplement to previous entrepreneurship through planning may raise that rate and thereby entail an upward inflection in the natural growth curve, accompanied by a higher  $s_r$ . Harrod (1963a, p. 410; 1963b, pp. 121-22) asked whether consumption per capita must be reduced. He rejected the view of the ‘stern moralist’ that a developing country should reduce consumption in order to build up its capital (1963b, p. 121). This would have a perverse effect on ‘incentives’ provided for higher forms of work associated with skilled personnel.

The problem for the developing countries is how, out of an increasing national product, to provide *both* saving at a higher level (the  $s_r$  as increased by the plan) *and* sufficient incentives, involving increasing consumption for the increasing cadres of qualified personnel (1963a, pp. 410-11).

In order to take into account the restriction of the amount of feasible saving caused by the increased incentives, Harrod (p. 411) suggested the concept of ‘optimum transitional saving’  $s_{rtr}$ , which is less than  $s_r$  during the economic development breakthrough. However, it might happen that the increase of consumption, essential to give the necessary incentives, was such as to prevent an increase in  $s_{rtr}$ . This is the case for foreign aid, since an unaided take-off into sustained growth would be impossible (Harrod 1963b).

It is the shortage of skilled personnel that accounts for low growth in developing economies, rather than a low  $s$  as compared to  $s_r$ . If, for those reasons, a high  $G_n$  is not feasible,  $s_r$  will be low, and  $s$ , though probably low in developing countries, may not fall short of  $s_r$ . Nevertheless, Harrod (1960, pp. 289-91) did discuss the case where  $s$  is below  $s_r$ , that is,  $G_w < G_n$ . If actual growth is below natural growth only because of the shortage of saving, this implies that there is available qualified personnel to implement improved methods of production. In such a case, to obtain ‘social optimum’ it is necessary to raise  $s$  towards  $s_r$  by a budget surplus or compulsory levy. Such increase of saving would bring the economy to a ‘transitional period’ in which actual growth is above the natural rate, owing to the exploiting of previously underemployed skilled personnel. Once this limited fund of workers is taken up, transitional growth is merged into regular natural growth. There are clear limits to the effect of higher saving on the per capita rate of growth, since its ‘very essence ... is the education by practice and the gradual drawing out of the latent potentialities of personnel’ (p. 291). Harrod’s concept of natural optimum growth differed sharply from simple recipes that stressed saving as the key to economic growth.

It is important to avoid over-simplified ideas about the consequences of the mere provision of additional saving. This view might be regarded as pessimistic in that it sets a limit to what can be done by one simple recipe (more saving); but it is optimistic in that it enables us to curb the impulses of those who believe Utopia can be gained solely by the imposition of harsh austerity. (Harrod 1960, p. 291)

‘Utopia’ in this context means achieving the natural welfare optimum rate of growth and possibly bringing it to a higher level.

## 5. Conclusions

‘Dynamic welfare economics’ became an important area of research after the development of optimal growth models in the 1960s. Apparently, Arrow (1951, p. 926) was the first to use the term. Hicks (1965, pp. 6-7, 201-02) deployed it in the

sense of optimal growth theory. As shown above, Harrod saw the extension of his concept of natural rate of growth to welfare economics as a task pertaining to the core of economic theory and policy. From that perspective, Harrod overlapped with aspects of neoclassical optimal growth models. This has led Asimakopulos and Weldon (1965, p. 71) to call Harrod (1960) ‘Harrod II’, as distinguished from ‘Harrod I’ of 1939 and 1948. In their view, ‘Harrod II’ was the same model of natural growth as Solow-Swan. Although Harrod did shift the focus of his research, a sharp contrast between ‘Harrod I’ and ‘Harrod II’ may be more distracting than revealing, as there are elements of continuity on his way to the formulation of welfare economic dynamics. Harrod (1973, chapter 7) continued to frame his economic dynamics in terms of differences between his three growth rates. Indeed, Harrod (1973, pp. 167-68) would criticize ‘current writings’ for not recognizing that there are ‘two normative growth rates’ ( $G_n$  and  $G_w$ ) and that they ‘may be *entirely different*’.

Harrod’s optimal growth was largely overlooked by Keynesian and neoclassical economists alike, because it was seen as a departure from his 1939 instability analysis and from the golden rule. Moreover, Harrod’s perspective did not square with the notion of unlimited supply of labor that dominated development economics (Boianovsky 2015). Hence, Harrod’s effort to establish a conversation with growth and development economists failed. There have been attempts to reformulate Harrod’s  $G_n$  as a partly endogenous variable (Leon-Ledesma and Thirlwall 2002). Harrod’s discussion of the role of incentives represented a step into that direction. Collard (2011, p. 268) has described Harrod (1960) as a ‘valiant (and rather neglected) attempt to bring “optimality” back to center stage’, despite the fact that the attempt to marry up Ramsey and the natural rate theories was ‘not fully worked out’. The neglect of Harrod’s optimal growth by neoclassical economists cannot be ascribed to any lack of effort from his part to integrate – if critically – Ramsey’s model with his own dynamic framework. The Harrod-Solow correspondence suggests that part of the difficulty came from Harrod’s notion that there is no market mechanism bringing  $r_n$  to reconcile  $G_w$  and  $G_n$  even in the long run. It is only through economic policy that  $r_n$  can be such as to bring about an optimum saving ratio  $s_r$  and, by that, equality between  $G_w$  and  $G_n$ .

As much as in his treatment of  $G$  and  $G_w$ , Harrod’s  $G_n$  applied to a point of time, not to a growth path. That is behind his effort to break away from Ramsey, and

partly explains as well the critical reactions from Graaff, Solow and Wan to Harrod (1960). Harrod (1963a) reformulated his second fundamental equation in an attempt to make it consistent with a variable capital-output ratio, but that did not contribute to bridge the gap between his and the neoclassical research program. In fact, Harrod would never refer to the neoclassical optimal growth literature – or to the Solow-Swan result that the growth rate is not permanently affected by the saving ratio. The absence of references to neoclassical optimal growth theory probably reflected Harrod's conclusion that the two research programs were essentially dissimilar, as illustrated by his 1973 (pp. 167-68) quotation about distinct growth rates cited above.

The reading of Harrod's notion of optimum saving provided here goes against the widespread belief that he (and Domar) founded the idea, later dubbed 'capital fundamentalism', that capital accumulation is the engine of growth (King and Levine 1994; Ray 1998). One of the main implications of his analysis is the proposition that the maximum rate of increase of qualified workers represents a more important limitation than the supply of saving in determining the maximum growth rate in developing countries. Harrod (1962, p. 10) expressed his belief that 'if all other factors were known to be present in adequate supply (and if the markets for products were known to be available) there would be seldom any difficulty in obtaining the capital-disposal required'. The training and qualification of workers was related to education, the 'most important problem in the whole range of development economics'.

All that pertained to 'dynamic theory', in the sense of propositions about rates of increase of certain variables in a growing economy. It was part of 'pure economics', whereas 'growth theory', which encompassed political, cultural and social elements such as political security and moral codes, was better described as the 'political economy of growth' (Harrod 1960, pp. 277-78). Harrod's optimal growth represented the culmination of his long lasting investigation of economic dynamics. Revisiting Harrod's search for optimal growth brings to light his unique effort to formulate welfare economic dynamics in a Keynesian framework, even if departing to some extent from his previous dominant concern with economic instability. It provides parallels and contrasts with the neoclassical optimal growth literature, which illuminate not just Harrod's intellectual trajectory but also the history of post-war growth economics in general.



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