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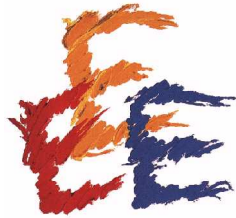
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Fondazione Eni Enrico Mattei

## **Multiple Politico-Economic Regimes, Inequality and Growth**

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# Multiple Politico-Economic Regimes, Inequality and Growth

Alain Desdoigts\* and Fabien Moizeau\*\*

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## Abstract

In this paper, we assume that redistribution modifies the community structure of an economy and generates both intra and intergenerational mobility. In a world in which neighborhood attributes and family backgrounds are important to determine the investment effort and the productivity of education, we abandon the stylized median voter in favor of more nuanced pressure-group reasoning and study (i) the conflicts of material interest, (ii) both the income distribution and growth dynamics. We find multiple politico-economic regimes where the pressure for redistribution is highly non-linear depending on social affinity. In some range of inequality, the ends may oppose high rates of redistribution against the middle and more unequal and segmented economies may redistribute less, not more. Our framework displays multiple steady states describing either a situation of equality and integration or segregation with persistent inequality depending on historical economic discrimination. Redistribution increases the number of candidates for the integrated equilibrium, but an economy's institutions will have first to overcome ranges of inequality in which voting cycles are likely to emerge.

**Keywords:** Clubs formation, growth, human capital, redistribution, and social mobility.

**JEL Classification:** D31, H2, I22, J62, P16.

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## 1. Introduction

The pattern of income distribution in a society reflects a history of class bargains and struggles which is specific to each economy. On the other hand, recent years have witnessed that economic clustering remains an essential feature of most economies and that inequality is on average and in most cases harmful for growth. As a consequence, it is often argued that there is a need for state action to arise to limit the degree of group inequality which in turn is expected to enhance growth. In this paper, we shed light on how redistribution or government transfers may (i) modify the community structure of an economy, (ii) generate both intra and intergenerational mobility, (iii) influence accumulation of human capital and the growth process of an economy along its transitional path and the associated steady state. Our modelling is based on two types of arguments which have recently been corroborated with robust empirical evidence.

The first argument is the following. In some countries like most European countries, the school-finance system is largely centralized and financed by the government. The aim is to provide the same quality and amount of education to everybody everywhere. In contrast, in others like the United States, education is more privately provided and when it is publicly provided, the level of school funding is mostly determined at the local level. However, in both systems, the parental background and neighborhood attributes remain important factors that determine the individual investment effort into education and its productivity.

Even in Europe, human capital investment in a child still highly depends on the income of his parents. (Evidence for the U.S. is provided in now well-known studies by Solon [35], and Zimmerman [38], among others.) In Italy, poor and non-educated families are less likely to invest in the education of their children despite an offer of equal opportunities to attend college and university (see, Checchi *et al.* [10]). In France, a manager's child spends on average 7.6 years at university compared to 3.5 for a worker's child (see *le Monde*, 02/27/2001). More generally, the propensity to attend university which is almost free of charge is higher among richer-income individuals and still strongly depends on the family income (see, Galland and Rouault [21]). Higher-income individuals in Europe therefore benefit in a larger extent of public spending in education than poorer-income individuals do, especially at the highest level of education.

It may be the case even within a centralized and public school-finance system, because poorer-income individuals

may not have a good perception of the relationship between income and schooling so that they deliberately choose not to attend college or university (See for instance, Streufert [36] who discusses under which conditions underclass social isolation indeed depresses schooling). It may also be true that the returns to education differ across classes of income so that lower-income individuals hesitate to invest into higher-education (see, Checchi *et al.*, respectively Galland and Rouault, for such an empirical evidence in the Italian, respectively the French case). In our model, we therefore argue, in contrast to Glomm and Ravikumar [23] , and Saint Paul and Verdier [34] , that even in the case of centralized public education, not all individuals obtain the same level of education. Instead, following Fernandez and Rogerson [17] , we rather consider education expenditure as a non rival but partially excludable good.

Moreover, in all countries, there always exists a set of private alternatives and even in the case of public-provision of education richer individuals may be more willing and able to spend extra effort to ensure that their children are progressing well in school, e.g., they may be more willing to pay high housing prices to benefit from the local externalities (local networks or “social capital”) associated with a given neighborhood. In France, 44% of company heads (30% of senior managers) children attend private schools. Among all those families whose children attend private schools, 77% choose this alternative because “the right sort of people go there” (see *le Monde*, 02/06/2001). These neighborhood effects have been found to play a major role beside parental status in generating positive local feedbacks on wealth accumulation and social mobility (see for instance, Borjas [9] , and Cooper *et al.* [11] ). As a consequence, the acquisition of human capital can not be considered as a discrete choice where you are able or not to obtain education. In contrast to Perotti [28] and Fernandez & Rogerson [17] , the models closest to ours, we assume that the costs and benefits of education are endogenous and do not rely on the existence of an exogenous threshold; that is, investment opportunities are divisible. We also assume along the lines emphasized by for instance, Bénabou [4] , Durlauf [13] , and Fernandez & Rogerson [18] that beside parental background, peer-group effects are a key factor which lead the all population of an economy to sort itself into relatively homogeneous communities.

If there are capital market imperfections, it is often argued that a way to fight against underclass social isolation from schooling and at the same time to extend to the largest possible portion of the population the benefits of the

local externalities which operate through peer influences, may consist of strengthening the forces that make for greater equality among the initially different socioeconomic classes. There is some scope for raising income of poorer-income individuals through reallocation of existing resources. However, in a democracy, the level of redistribution must be chosen by majority voting, and therefore most countries face the traditional conflicts of economic interest between the different income classes whose degree of altruism both within and across generations is only limited.

The second argument which partially drives our model is based on three robust empirical findings. First, more equal societies have on average higher rates of investment in education which are reflected in higher rates of growth. Second, the effect of transfers on growth across nations has been found to be significantly positive. (See for instance Perotti [29] and [30], Figini [19], and the critical survey by Bénabou [5].) We therefore require our model to be consistent with this class of models where credit constraints prevent poorer-income individuals to invest into education which in turn is expected to foster economic growth in the presence of diminishing returns. (See Glomm and Ravikumar [23], Galor and Zeira [22], Bénabou [4], and Durlauf [13], among many others.) The third empirical evidence remains more like a puzzle: more unequal countries tend to redistribute less, not more! As Bénabou [6] argues: how is it possible that there is more pressure for redistributive policies and more upward social mobility in Scandinavian countries like for instance Sweden compared to the United States although the former are at the same time more equal (see Björklund and Jäntti [7])? See also Saint Paul [33] who studies the impact of income inequality on fiscal conservatism. Or, following Bolton and Roland [8], how can we explain that the United Kingdom favors lower taxes and less redistribution, while others like the Netherlands favor higher taxes to protect their welfare state?

In this paper, we first study the redistributive conflicts of interest which may arise in an economy composed of heterogeneous individuals who vote over a level of pure redistribution. Our model reflects a world in which neighborhood differences and family backgrounds are important in determining the individual investment effort and the returns on education. This allows us to provide an explanation to the above puzzle; that is, why inequality can be associated with less, rather than more, redistributive taxation? Second, we characterize both the growth process and the income distribution dynamics along the transitional path to the steady state for any given initial pattern of income distribution.

More specifically, we focus on an economy initially composed of three different income classes. Investment in education is not modelled as an exogenous discrete choice. Instead, the investment effort into education of an individual depends on her posttax income. A key feature of our model is that once a lower-income class reaches a threshold level of development such that it becomes homogeneous enough to the next higher-income class, they can form a community in which the lower type experiences a peer-group effect. This threshold level depends on the characteristics of all the members of the community.

The local connection which causes the formation of relatively homogeneous communities has been analyzed in now seminal papers by Bénabou [3] , and Fernandez and Rogerson [18] , among others. In both these studies, authors focus on games where agents first choose the community in which they will live and then decide on how much to spend on education which is publicly provided at the community level through local taxation. Our model also consists of a two-stage game but where agents first vote over a level of pure redistribution which determines how the entire economy stratifies into communities. All agents of a club then benefit from a similar level of quasi-public resources where the richer the community is on average the higher is this level. Moreover, there is an explicit local trickle down mechanism which reflects for instance the social networks built by higher-income individuals and which become available to those poorer-income agents who live in the same community.

The possibility either to form clubs or to block them within a pure redistribution framework, and endogenizing the opportunity cost of education yield conclusions which are more intricate compared to the existing and associated literature. Suppose that the middle-income class is well-aware that redistribution would allow them to benefit from higher local externalities like social networks as they might be available to the high-income class or to be more able and willing to attend college or university whatever the latter is privately or publicly provided. Then, what matters for the former is its prospect of upward social mobility; that is, to eventually be able to be affiliated to the community of the higher-income individuals, and not only whether it is richer or poorer relative to the mean as argued in standard political economy models like for instance Alesina and Rodrik [1] , and Persson and Tabellini [31] . Even though the low income-class may be excluded from this new community, as long as it is better off with that level of redistribution

compared to any lower tax rate, the political outcome which prevails is the one most preferred by the middle-income class. However, there also might exist *ex ante* levels of inequality such that the low-income class opposes the high levels of redistribution preferred by the middle-income class. This may occur in some range of inequality where the pressure for redistribution most preferred by the middle-income class leads the low-income class to lag further behind the two other income classes while they could catch up with a lower level of redistribution. Notice that within our majority voting framework, these conflicts of interest can also give rise to Condorcet cycles.

The analysis of the conflicts of interest across our three initially differently endowed income classes allows us to infer that: (i) the pressure for redistributive policies is not necessarily smaller in a rich economy where the income of the middle class is high relative to the average compared to the pressure that prevails in a poorer economy where the income of the middle class may be smaller than the average, (ii) there exist in some range of inequality, political equilibria where both the low and the high-income classes agree on a similar level of redistribution which is smaller than the one most preferred by the middle-income class, (iii) in a completely segregated economy, a rise in the wealth bias against the poor or an increase in the Gini coefficient may lead to lower the equilibrium level of redistribution, (iv) finally, more unequal and segmented economies may redistribute less, not more.

The analysis would not be complete without considering the dynamics of inequality and the growth process underlying our modelling. Our model displays multiple history-dependent steady states, describing either a situation of equality and integration or segregation with persistent inequality. Along the transitional path, a large range of initial conditions may lead economies to cross Condorcet cycles regions. Finally, introducing redistribution increases the number of candidates for the integrated equilibrium and fosters growth.

The paper is organized as follows. Section 2 presents the structure of the model. In Section 3, we define what is an equilibrium partition and provide information about the key conflicts of interest which may occur depending on the initial pattern of income distribution. We also examine under which conditions multiple politico-economic regimes may occur. Section 4 discusses the dynamic properties of the model. Finally, Section 5 concludes and most of the technical apparatus is left to the appendix.



## 2. The Model

Our objectives are similar to those of Saint Paul [33] and Bénabou [6] although our analytical approach differs greatly. It proposes an alternative explanation as to why different initial patterns of income distribution associated with imperfect capital market can lead to multiple politico-economic regimes.

Our modelling strategy consists in a two-stage game which is repeated every period and the ingredients of the model which allow us to explore this puzzling feature across nations are: (i) a level of pure redistribution determined in a first stage by majority vote and which involves deadweight losses as for instance in Perotti [28], and Bolton and Roland [8] (ii) which modifies the community structure of the whole economy and gives rise to a decentralized formation of clubs of agents who differ only with respect to their posttax income, (iii) and who decide in the second stage of the game on how much to invest into skill-acquisition, (iv) a level of quasi-public resources which depends on the characteristics of the community's members and which is distributed uniformly to people in the affiliational club. Non members are outliers and may partially be excluded from its benefits. (v) Finally, both the parental background and those quasi-public resources drive accumulation of human capital (see Bénabou [3] and [4]). Note that for a given level of redistribution, the equilibrium partition which emerges in the absence of Condorcet cycles belongs to the core; that is, there is no other partition of the economy which is Pareto-improving.

### 2.1 Preferences, Technologies, and the Tax System

We consider an infinity of non-overlapping generations, each living one period. Let there be a large number of individuals  $i \in \Omega$  within each generation, and total population  $\Omega$  lives in a world composed of three groups of agents of equal size  $n^1$ , and characterized by different pre-tax human capital endowment,  $h_t^i$ , with  $i = h, m, \text{ or } l$ , which is the only source of heterogeneity. At  $t = 0$ , pre-tax human capital endowments are characterized by the following inequalities:

$$0 < h_0^l < h_0^m < h_0^h.$$

Agents may group themselves into clubs that are defined as follows:

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<sup>1</sup> In fact, all what we require to solve the model is that no single class is able to impose its most preferred outcome on the others and that the size of two income classes amounts at least to fifty percent of the entire population.

**Definition 1:** A nonempty subset  $S_{jt}$  of  $\Omega$  is called a club or a community and  $P_t = (S_{1t}; \dots; S_{Jt})$  for  $j = 1, \dots, J$ ,

is called a partition of  $\Omega$  if:

- (1)  $\bigcup_{j=1}^J S_{jt} = \Omega$ ;
- (2)  $S_{jt} \cap S_{j't} = \emptyset$  for  $j \neq j'$ .

Each agent is affiliated to one and only one community.

Following Saint Paul and Verdier [34], each individual maximizes a strictly concave utility function of the form

$$U(c_{j,t}^i, h_{j,t+1}^i) = \ln c_{j,t}^i + \ln h_{j,t+1}^i \quad (1)$$

where  $c_{j,t}^i$  is the consumption level of individual  $i$  at time  $t$  and  $h_{j,t+1}^i$  is the human capital stock left to her offspring when she belongs to the community  $S_{jt}$ .

Individuals have no access to credit. They accumulate human capital as follows:

$$h_{j,t+1}^i = \kappa \cdot G_{jt}^\beta \cdot \bar{h}_t^\alpha \cdot (h_t^i)^{1-\alpha-\beta} \quad (2)$$

Following Bénabou [4], the accumulation of human capital reflects the influence of family through the pre-tax human capital inherited from parents ( $h_t^i$ ), the influence of economy-wide knowledge spillovers through  $\bar{h}_t$  which is the average stock of human capital in the entire society at time  $t$ . Finally, agents are also concerned about the level of quasi-public resources supplied in the club ( $G_{jt}$ ) and which are provided uniformly to people of the same community. We assume that  $\alpha$ ,  $\beta$ , and  $1 - \alpha - \beta \in (0, 1)$  so that all factors exhibit diminishing returns.

Let  $G_{jt}$  be the quantity of public good available in the club  $S_{jt}$ . Following Bahram *et al.* [2], we assume that  $G_{jt}$  is the sum of the individual investment efforts in education of the members of the community, denoted by  $g_{jt}^i$ , given some linear congestion costs denoted by  $a$ .

$$G_{jt} = \sum_{i \in S_{jt}} g_{jt}^i - n_{jt}a \quad (3)$$

where  $n_{jt}$  is the number of agents in the community  $S_{jt}$ .

Notice that although  $G_{jt}$  will be determined within a non cooperative game, this simple formulation allows us to

shed light on how the community structure which emerges from the first stage of the game interacts with the quality of education across communities and the individual investment effort into education as well as to incorporate all the above discussed ingredients in an easily tractable fashion.

At the beginning of the period agents vote over the level of income taxes that are proportional to pre-tax income. Total tax revenues are redistributed as a lump sum transfer that is constant across individuals. The task of redistribution is therefore to modify economic disparities across heterogeneous groups of persons. The government budget is always balanced. We also assume that taxation involves deadweight losses. More specifically, there are convex costs in collecting taxes: if  $\tau$  is the tax rate,  $\tau\bar{h}_t$  is collected but only  $(\tau - \tau^2)\bar{h}_t$  can be redistributed to each individual.

## 2.2 Optimal Individual Investment Effort into Education

The model is solved by working backwards from the second stage of the game. We first determine the optimal individual investment effort into education given her posttax income and the equilibrium community structure of the economy which emerges from the first stage of the game. The political equilibrium and the process of club formation is discussed in the next section.

The choice of individual  $i$ 's effort of investment into education  $g_{jt}^i$  when she belongs to a community  $S_{jt}$  is found by maximizing

$$U(c_{j,t}^i, h_{j,t+1}^i) = \ln c_{j,t}^i + \ln h_{j,t+1}^i$$

subject to :

$$c_{j,t}^i + g_{jt}^i \leq (1 - \tau)h_t^i + (\tau - \tau^2)\bar{h}_t$$

$$h_{j,t+1}^i = \kappa \cdot G_{jt}^\beta \cdot \bar{h}_t^\alpha \cdot (h_t^i)^{1-\alpha-\beta}$$

$$G_{jt} = \sum_{i \in S_{jt}} g_{jt}^i - n_{jt}a$$

$$g_{jt}^i \geq 0, \text{ given } h_t^i, \bar{h}_t, \text{ and } \tau.$$

Each agent faces the same trade off. Her posttax income is devoted either to private consumption or to investment into education. Further, the chosen optimal individual investment effort is such that a threshold level of after-tax income

is necessary for an agent to be able to benefit from the level of public good provided in the community  $S_{jt}$ . It is such that

$$g_{jt}^i = \max \left\{ 0, (1 - \tau)h_t^i + (\tau - \tau^2)\bar{h}_t - \frac{1}{\beta}G_{jt} \right\} \quad (4)$$

Literally, the higher the posttax income of an agent in a given community, the higher the level of her investment into education in that community. The posttax income does not determine whether an individual does acquire education or not, it rather determines how much education she will receive which is given by

$$G_{jt} = \frac{\beta}{\beta + n_{jt}} \sum_{i \in S_{jt}} ((1 - \tau)h_t^i + (\tau - \tau^2)\bar{h}_t - a) \quad (5)$$

Note that not all agents of a same club need to contribute to the same extent to the provision of the public good available in that club. Instead, two types of agents may belong to the same community where both types experience a peer group effect. On the one hand, the equilibrium level of  $G_{jt}$  reflects the traditional effect where a low-income class (or the weak students) may derive more benefit from educational spending when higher-income individuals (or strong students) are present in the community (classroom). On the other hand, richer communities benefit from higher levels of educational spending<sup>2</sup>.

Replacing the expression of  $G_{jt}$  into the above individual  $i$ 's utility function, we get the following indirect utility function

$$V^i(S_{jt}; \tau) = (1 + \beta) \ln G_{jt} + \ln(\beta^{-1} \cdot \kappa \cdot \bar{h}_t^\alpha \cdot (h_t^i)^{1-\alpha-\beta}) \quad (6)$$

At this stage, it is worth noticing that the second term in the sum is determined at the beginning of the period and therefore at the beginning of the game for each individual. Hence, all that matters to compute the utility gain or loss of the different agents after redistribution is the level of public good available in the club to which they belong; that is,  $\ln G_{jt}$  which reflects the capacity or ability of the individual to afford the costs involved in education through her posttax income but also the incentive to do so through her affiliation to a community.

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<sup>2</sup> See also Epple and Romano [15] and Lloyd-Ellis [25] who emphasizes an empirical finding by Mingat and Tan [26] : in developing countries, an average of 71 percent of the population receives primary schooling, but only receive 22 percent of the resources devoted to education, while the 6 percent of the population who attain higher education receive 39 percent of the resources. On the other hand, Filmer and Pritchett [20] find within a set of 35 countries that educational attainment is strongly positively correlated with the household wealth.

### 3. Equilibrium Partition, Preferred Tax Rate, and Political Outcome

We now move back up the game tree. First, we define what is an equilibrium partition. Second, we provide information about the trade-off faced by the different income classes and the conflicts of interest which may arise for any given *ex ante* pattern of income distribution. Third, we solve the first stage of the game and determine both the political equilibrium if it exists<sup>3</sup>, and the associated equilibrium partition.

#### 3.1 Equilibrium Partition

The first stage of the game consists of determining the community structure which emerges from the political equilibrium given the after-tax income distribution. We require that any equilibrium partition, denoted  $P_t^E$ , of agents into clubs be in the core. That is, there exists no other group that could block  $P_t^E$  (see for instance, Farrell and Scotchmer [16]).

It then follows that because of the existence of crowding costs linked to the size of a club, an agent of type  $i$  belongs to a community  $S_{jt}^E$  of the equilibrium partition if her posttax income has reached a threshold level such that she is able to invest into education a strictly positive amount of her posttax income in that community. From equations (4) and (5), for an agent of type  $i$  in  $S_{jt}^E$ , this condition yields the following inequality:

$$(1 - \tau)h_t^i + (\tau - \tau^2)\bar{h}_t > \frac{1}{\beta + n_{jt}} \sum_{i \in S_{jt}^E} ((1 - \tau)h_t^i + (\tau - \tau^2)\bar{h}_t - a) \quad (7)$$

In other words, no non contributing individual can be accepted in a community  $S_{jt}^E$  of the equilibrium partition. Consider a club in a partition in which at least one of its members does not contribute. This agent increases the congestion cost. Obviously, all the contributors of this club are better off by excluding this agent. Hence, this partition is blocked by the set of contributors and is not an equilibrium partition. Notice that preferences across agents are homogeneous and that the marginal congestion cost is constant. When an agent of a given type is able to positively contribute to the provision of the public good in a given community all members in that community are better off when all agents of the same type join that club. An equilibrium partition is such that all agents of a same type belong to the

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<sup>3</sup> The final decision about the tax rate is traditionally taken by majority rule; that is, the only admissible outcomes are those which defeat every other in a binary majority contest (see for instance, Moulin [27]).

same club. Hence, we focus on the heterogeneity within communities rather than on their sizes. Finally, the clubs in the equilibrium partition are consecutive. Suppose they are not and let us consider the partition  $P_t = (\{h, l\}; \{m\})$ ,  $P_t$  is blocked by  $\{h, m\}$  because both the high and the middle-income classes are strictly better off in this new club.

Therefore, we define the partitions which are candidates for the core of the economy as,

**Definition 2:** At each date, if it exists, the equilibrium partition is one of these four cases:

$$(i) P_t^I = (\{h, m, l\}), (ii) P_t^{II} = (\{h, m\}; \{l\}), (iii) P_t^{III} = (\{h\}; \{m\}; \{l\}), (iv) P_t^{IV} = (\{h\}; \{m, l\})$$

### 3.2 Conflicts of Interest over Tax Rates

A first set of relevant tax rates which must be considered to determine the political equilibrium at each time  $t$ , is the set of tax rates that maximize an agent  $i$ 's indirect utility function or equivalently the level of quasi-public resources provided in the community she may have access to after redistribution. It is defined as<sup>4</sup>

$$\lim_{a \rightarrow 0} \tau_{S_{jt}}^* = \max \left\{ 0, \arg \max \left( \frac{\beta}{n_{jt} + \beta} \sum_{i \in S_{jt}} (1 - \tau) h_t^i + (\tau - \tau^2) \bar{h}_t \right) \right\}$$

and each corresponding community's optimal tax rate is

$$\lim_{a \rightarrow 0} \tau_{S_{jt}}^* = \max \left\{ 0, \frac{1}{2} \left( 1 - \frac{\sum_{i \in S_{jt}} h_t^i}{n_{jt} \bar{h}_t} \right) \right\} \quad (8)$$

Notice that the definition of  $\tau_{S_{jt}}^*$  is such that whatever the *ex ante* income distribution is, we always have the following ranking of tax rates:

$$0 = \tau_{\{h\}}^* = \tau_{\{h, m\}}^* = \tau_{\{h, m, l\}}^* \leq \tau_{\{m\}}^* < \tau_{\{m, l\}}^* < \tau_{\{l\}}^* < \frac{1}{2}$$

Agents in poorer communities tend to favor higher tax rates, while the members affiliated to richer neighborhoods have their well-being maximized at lower tax rates. This reflects the traditional conflict of interest between rich and poor.

Second, as in Perotti [28] and Fernandez and Rogerson [17], various discontinuities arise in the indirect utility functions of both the middle and the low-income groups which lead their preferences in some range of inequality not to be single-peaked. Indeed, for a given pattern of *ex ante* income distribution, there might exist a redistributive pol-

<sup>4</sup> From now on, we assume without loss of generality that congestion costs are infinitesimal.

icity which modifies the community structure of the economy and leads a lower-income class to join a higher initially endowed income group. In that case and in the absence of serious distortions, the benefits for the former are straightforward as the level of quasi-public resources or the social capital available in this new community is enhanced by the presence of the other group.

For instance, the level of redistribution required for the low-income group to join the middle-income class so that they form a Pareto-improving community  $S_{jt} = \{m, l\}$  compared to the situation where they both remain on their own for the same level of redistribution, is defined as follows:

$$V^m(\{m, l\}; \tilde{\tau}_{\{m, l\}}) = V^m(\{m\}; \tilde{\tau}_{\{m, l\}})$$

Using equation (7),  $\tilde{\tau}_{\{m, l\}}$  tends to

$$\lim_{a \rightarrow 0} \tilde{\tau}_{\{m, l\}} = \max \left\{ 0, \frac{nh_t^m - (n + \beta)h_t^l}{\beta \bar{h}_t} \right\} \quad (9)$$

$\tilde{\tau}_{\{m, l\}}$  is therefore defined as the marginal tax rate such that the middle-income class is indifferent between the two clubs  $\{m\}$  and  $\{m, l\}$ .

Similarly, we must also consider the following relevant candidates:

$$\lim_{a \rightarrow 0} \tilde{\tau}_{\{h, m\}} = \max \left\{ 0, \frac{nh_t^h - (n + \beta)h_t^m}{\beta \bar{h}_t} \right\} \quad (10)$$

$$\lim_{a \rightarrow 0} \tilde{\tau}_{\{h, m, l\}} = \max \left\{ 0, \frac{n(h_t^h + h_t^m) - (2n + \beta)h_t^l}{\beta \bar{h}_t} \right\} \quad (11)$$

where  $\tilde{\tau}_{\{h, m\}}$ , respectively  $\tilde{\tau}_{\{h, m, l\}}$ , are the tax rates such that  $V^h(\{h, m\}; \tilde{\tau}_{\{h, m\}}) = V^h(\{h\}; \tilde{\tau}_{\{h, m\}})$ , respectively  $V^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}}) = V^i(\{h, m\}; \tilde{\tau}_{\{h, m, l\}})$ , for  $i = h, m$ <sup>5</sup>.

Literally, from equation (11), when the *ex ante* pattern of income distribution is such that  $n(h_t^h + h_t^m) - (2n + \beta)h_t^l < 0$ , no redistribution is needed for the partition  $P_t^I = (\{h, m, l\})$  to emerge. The whole economy is homogeneous enough so that the three income classes are able to attain equal educational opportunities. The same result applies for  $P_t^{II} = (\{h, m\}, \{l\})$  ( $P_t^{IV} = (\{h\}; \{m, l\})$ ) when the initial income distribution is such that  $nh_t^h - (n + \beta)h_t^m < 0$  ( $nh_t^m - (n + \beta)h_t^l < 0$ ). In that case, no redistribution is needed for the middle- (low-) income group to join the high-

<sup>5</sup> Notice that the tax rate  $\tau$  such that  $V^h(\{h\}; \tau) = V^h(\{h, m, l\}; \tau)$  is an irrelevant candidate. Indeed, whatever the level of inequality, we have  $\tilde{\tau}_{\{h, m\}} < \tau$ . Hence, the community  $\{h, m\}$  already exists with levels of redistribution smaller than  $\tau$  and the above trade off is irrelevant.

(middle-) income group. Notice that as long as the *ex ante* distribution is such that  $\tilde{\tau}_{\{h,m\}} \leq \tilde{\tau}_{\{m,l\}}$ , then  $\tilde{\tau}_{\{h,m\}}$  is preferred by a majority in pairwise comparison to  $\tilde{\tau}_{\{m,l\}}$ . This reflects our assumption that a positive intra community externality is exerted by the relatively higher-income individuals on the poorer-income ones.

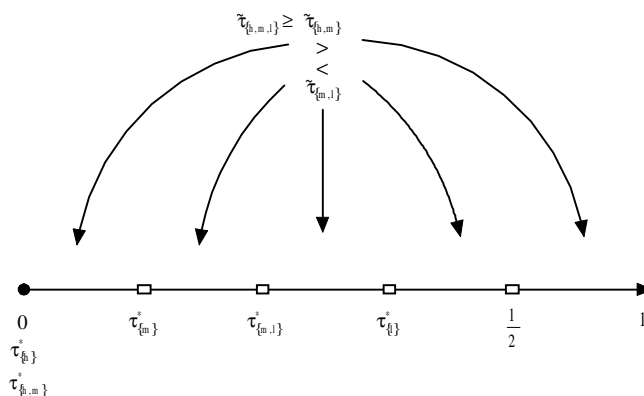


Figure 1: The relative position of the different candidates provide information about the initial pattern of inequality.

In our discussion,  $\tau_{S_{jt}}^*$  corresponds to the tax rate which maximizes the level of public good in the club  $S_{jt}$ . However, for a given initial distribution, this tax rate may not be large enough for the corresponding community to endogenously form. As a consequence, the political outcome depends on the relative levels of all the tax rates which are candidates for the political equilibrium; that is,  $\tilde{\tau}_{S_{jt}}$  and  $\tau_{S_{jt}}^*$  (see Figure 1).

Consider the trade off for an agent who initially belongs to the middle-income group such as displayed in Figure 2; the tax rate that maximizes her utility function if she forms a club only with agents of the same kind is  $\tau_{\{m\}}^*$ . Note that a small increase in the tax rate causes a reduction in her utility. However, further increase in the tax rate until  $\tilde{\tau}_{\{h,m\}}$  allows her to have access to a richer community. As shown in Figure 2, the middle-income class is now better off by forming a club with agents of type  $h$ . It most prefers  $\tilde{\tau}_{\{h,m\}}$  to  $\tau_{\{m\}}^*$  and the former is permissible as long as the level of distortions associated with this fiscal policy remains reasonable; that is,  $\tilde{\tau}_{\{h,m\}} \leq 1/2$ . More generally, a jump in the indirect utility function of both the middle- and the low-income groups arises whenever the tax rate reaches a level such that one of these groups' posttax income becomes large enough to form a relatively homogeneous community



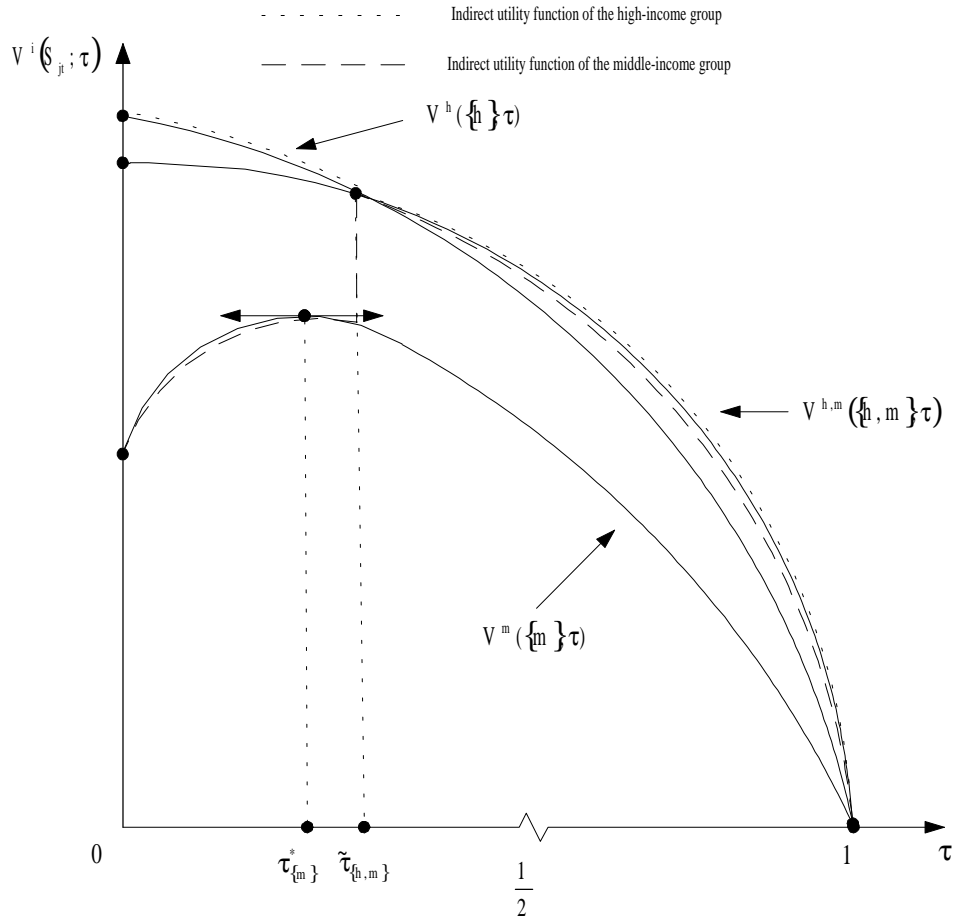


Figure 2: Indirect utility functions for the  $\{h, m\}$ , and  $\{h\}$ , and  $\{m\}$  communities. A discontinuity arises in the utility of an agent  $m$  when the tax rate reaches the marginal level  $\tilde{\tau}_{\{h,m\}}$ . The display of the indirect utility function of the different income groups only considers that part of  $V^i(S_{jt}; \tau)$  which depends on  $G_{jt}$ .

with the next higher-income class. This leads their preferences not to be single-peaked in some range of inequality.

Because of non single peakedness, the usual conditions for the existence of a stable majority cannot be applied directly and the most preferred tax rate in the economy is not always associated with the agent of type  $m$ 's most preferred tax rate. The low-income class may in some cases, depending on the initial pattern of income distribution, prefer a lower tax rate compared to the preferred tax rate by the middle-income group. An example as displayed in Figure 3 where we consider the indirect utility functions of agents  $h$ ,  $m$ , and  $l$ , illustrates the possibility for Condorcet paradoxes to arise given a particular initial pattern of income distribution. Consider the following profile of relevant preferences for the three income classes associated with the initial pattern of income distribution as depicted in Figure 3:

$$\begin{aligned} V^h(\{h\}; \tau_{\{m\}}^*) &> V^h(\{h\}; \tau_{\{m,l\}}^*) > V^h(\{h,m\}; \tilde{\tau}_{\{h,m\}}) > V^h(\{h,m\}; \tau_{\{l\}}^*) \\ V^m(\{h,m\}; \tilde{\tau}_{\{h,m\}}) &> V^m(\{h,m\}; \tau_{\{l\}}^*) > V^m(\{m\}; \tau_{\{m\}}^*) > V^m(\{m,l\}; \tau_{\{m,l\}}^*) \\ V^l(\{m,l\}; \tau_{\{m,l\}}^*) &> V^l(\{l\}; \tau_{\{l\}}^*) > V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) > V^l(\{l\}; \tau_{\{m\}}^*) \end{aligned}$$

It is easily checked that there is no Condorcet winner. Therefore, in contrast, for instance, to Perotti [28], initial patterns of income distribution may occur so that there is no value of  $\tau$  which is stable against the rule of majority.

### 3.3 Political Equilibrium

Let us start the discussion by recalling that levels of inequality where  $\frac{h_t^h}{h_t^m} \leq 1 + \left(\frac{n}{\beta}\right)^{-1}$  or  $\frac{h_t^m}{h_t^l} \leq 1 + \left(\frac{n}{\beta}\right)^{-1}$ ; that is  $\tilde{\tau}_{\{h,m\}} = 0$ , or  $\tilde{\tau}_{\{m,l\}} = 0$ , are such that no redistribution is needed for either the middle- or the low-income class to form a club with respectively, either the high- or the middle-income group. Then, we can easily conclude that without redistributive policies, as soon as  $\frac{h_t^h}{h_t^m} > 1 + \left(\frac{n}{\beta}\right)^{-1}$ , and  $\frac{h_t^m}{h_t^l} > 1 + \left(\frac{n}{\beta}\right)^{-1}$  the equilibrium partition is  $P_t^E = (\{h\}; \{m\}; \{l\})$ . Note also that without redistribution, the conditions on the pattern of relative inequality between the three income groups required for the all economy to benefit from the same level of public good is even more restrictive. Indeed, it is such that  $\frac{h_t^h}{h_t^m} \leq \left(2 + \left(\frac{n}{\beta}\right)^{-1}\right) \left(\frac{h_t^m}{h_t^l}\right)^{-1} - 1$ .

Therefore, two questions arise: what are the consequences of introducing a pure redistributive scheme on the formation of economic communities; that is, on the stratification of the income distribution? And, what is the impact of

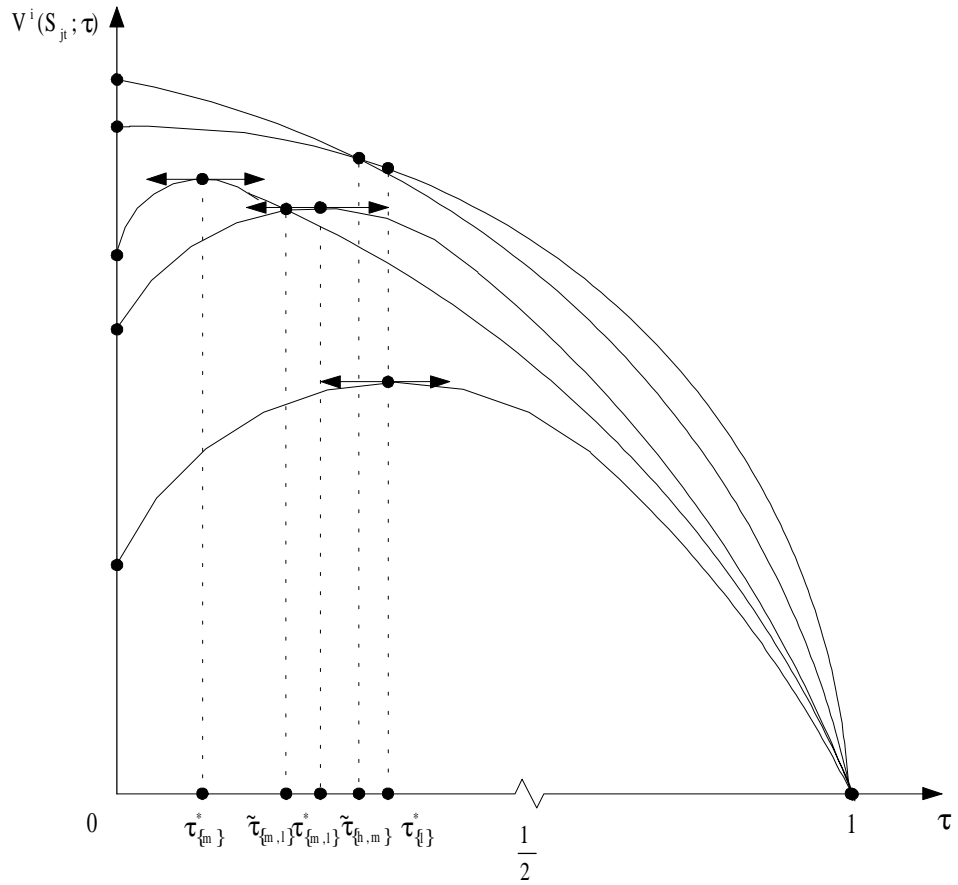


Figure 3: Indirect utility functions for the  $\{h, m\}$ ,  $\{m, l\}$ ,  $\{h\}$ ,  $\{m\}$ , and  $\{l\}$  communities. An initial pattern of income distribution that yields a Condorcet paradox. The display of the indirect utility function of the different income groups only considers that part of  $V^i(S_{jt}; \tau)$  which depends on  $G_{jt}$ .

inequality on redistribution both within and across the different equilibrium partitions? In a world where an individual's capacity to invest in education and to benefit from high local spillovers or from higher educational services depends on her posttax income, it turns out that the link between inequality and redistribution is more complex than advocated in conventional models of political economy. We now highlight the role of income distribution in setting the uneven levels of redistribution across nations.

Propositions 1 to 5 characterize for any *ex ante* pattern of income distribution the tax rate which is chosen by a majority and the equilibrium economic partition which emerges from the elected redistributive policy as well as the range of inequality where Condorcet cycles are likely to emerge. The propositions discussed below in the text are better illustrated as depicted in Figure 4 which displays the ranges of *ex ante* inequality where the different equilibrium partitions emerge (some information about the construction of this figure is provided in Appendix B<sup>6</sup>). Whenever there is a Condorcet winner, the associated equilibrium partition is unique and it belongs to the core.

The following proposition describes Region *I* displayed in Figure 4.

**Proposition 1.** *First, the equilibrium partition  $P_t^E = (\{h, m, l\})$  occurs when the posttax income inequality between  $h_t^h, h_t^m, h_t^l$  is such that  $n(h_t^h + h_t^m) - (2n + \beta)h_t^l < 0$ . In that case, the whole population agrees to vote in favor of no redistribution. Second,  $P_t^E = (\{h, m, l\})$  is also the equilibrium partition when both the low- and the middle-income classes are similarly endowed ( $h_t^m = h_t^l$ ) and as long as they are both better off in a community  $\{h, m, l\}$  rather than a community  $\{m, l\}$ . In that case, the most socially preferred tax rate is  $\tilde{\tau}_{\{h, m, l\}} \geq 0$ .*

**Proof.** See Appendix A ■

Rather naturally, redistribution increases the range of *ex ante* relative inequality where the grand coalition belongs to the core. The three other possible equilibrium partitions at least require that  $n(h_t^h + h_t^m) - (2n + \beta)h_t^l \geq 0$ . The

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<sup>6</sup>  $AA \equiv V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}})$ .  $BB \equiv V_t^l(\{l\}; \tau_{\{m\}}^*) = V_t^l(\{l\}; \tilde{\tau}_{\{h, m\}})$ .  $CC \equiv V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ .  $DD \equiv V_t^m(\{m, l\}; \tilde{\tau}_{\{h, m\}}) = V_t^m(\{m\}, \tau_{\{m\}}^*)$ .  $EE \equiv V_t^i(\{m, l\}; \tilde{\tau}_{\{h, m\}}) = V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$  for  $i = m, l$ .  $FF \equiv V_t^i(\{m, l\}; \tau_{\{m, l\}}^*) = V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$  for  $i = m, l$ .

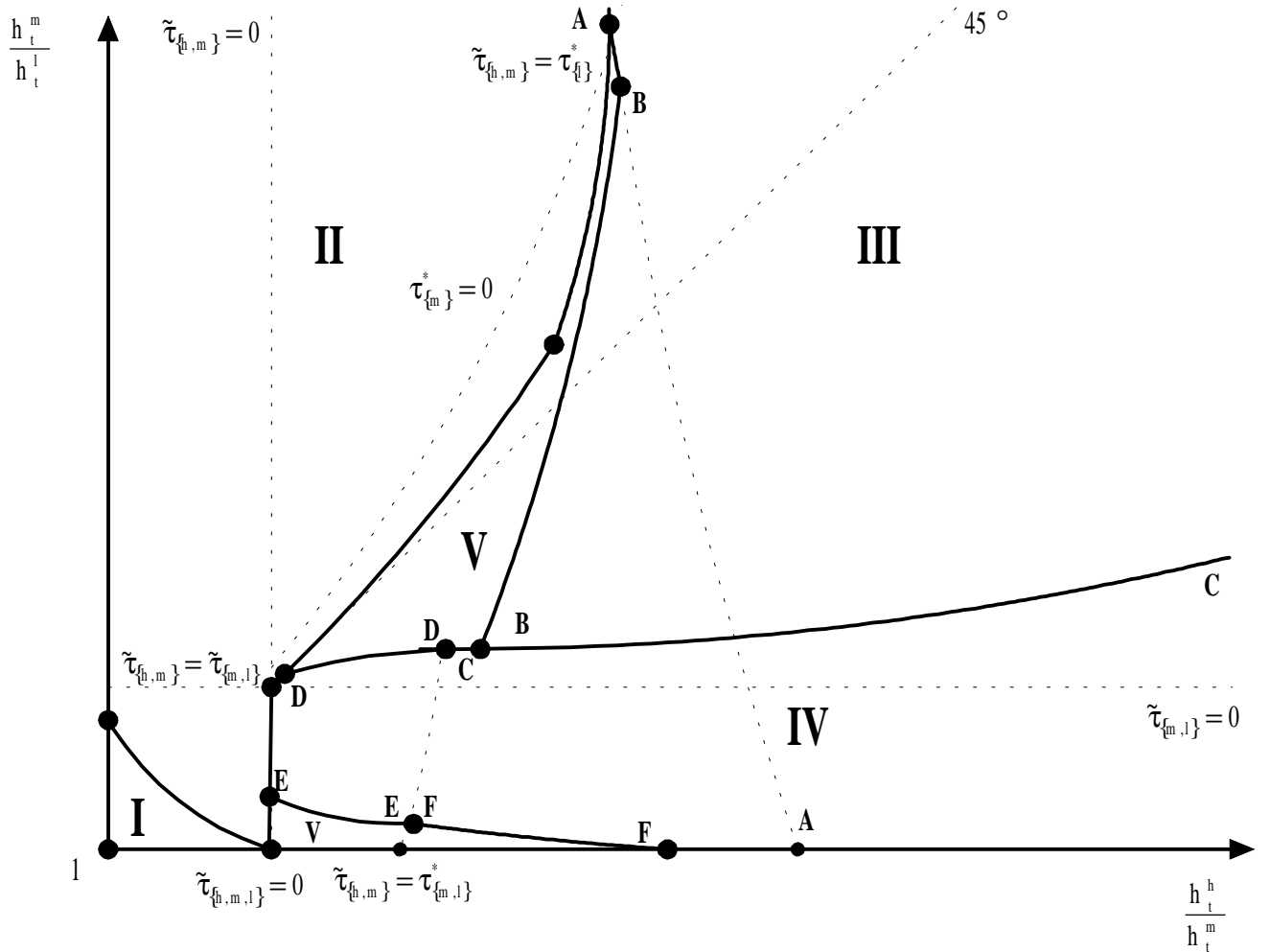


Figure 4: Equilibrium partition regions in the plane  $\left(\frac{h_t^h}{h_t^m}, \frac{h_t^m}{h_t^l}\right)$ . Region *I* corresponds to the equilibrium partition  $P_t^E = (\{h, m, l\})$  which is associated with the equilibrium tax rate  $\tilde{\tau}_{(\{h, m, l\})}$ . Region *II* corresponds to  $P_t^E = (\{h, m\}; \{l\})$  and either  $\tau_{\{h, m\}}^*$  or  $\tilde{\tau}_{\{h, m\}}$ . Region *III* corresponds to  $P_t^E = (\{h\}; \{m\}; \{l\})$  and  $\tau_{\{m\}}^* \geq 0$ . Region *IV* corresponds to  $P_t^E = (\{h\}; \{m, l\})$  and either  $\tau = (nh^h - (n + \beta)h^m) / \beta \bar{h}$  or  $\tau_{\{m, l\}}^*$ . Finally, Region *V* depicts ranges of inequality where there are Condorcet cycles. The loci which partition our plane are defined in Appendix B.

first one, depicted in Region *II* of Figure 4, is described in the following proposition,

**Proposition 2.** *When the ex ante pattern of relative income inequality between  $h_t^h$ ,  $h_t^m$ ,  $h_t^l$  is such that the three following conditions are satisfied simultaneously: (i) the marginal tax rate required for the middle-income class to form a club with the high-income class is smaller or equal to the tax rate required for the low-income group to join the middle-income one ( $0 \leq \tilde{\tau}_{\{h,m\}} \leq \tilde{\tau}_{\{m,l\}}$ ), (ii)  $\tilde{\tau}_{\{h,m\}}$  is smaller or equal to the most preferred tax rate by the low-income class when it remains on its own ( $\tau_{\{l\}}^*$ ), (iii) the middle-income class is better off by forming a club with the high-income class rather than remaining on its own; that is,  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}}) \geq V_t^m(\{m\}; \tau_{\{m\}}^*)$ , the equilibrium partition is  $P_t^E = (\{h, m\}; \{l\})$  and a majority emerges in favor of  $\tilde{\tau}_{\{h,m\}} \geq 0$ , whatever  $h_t^m \geq \bar{h}_t$ .*

**Proof.** See Appendix A ■

Region *II* in the plane  $\left(\frac{h_t^h}{h_t^m}, \frac{h_t^m}{h_t^l}\right)$  is always located above the 45°-line when  $\tilde{\tau}_{\{h,m\}} > 0$ ; that is, for the equilibrium partition  $(\{h, m\}; \{l\})$  to emerge, the economy must initially exhibit a clear wealth bias against the poor. Notice that by definition, when  $\tilde{\tau}_{\{h,m\}} > 0$ , it increases with both  $\frac{h_t^h}{h_t^m}$  and  $\frac{h_t^m}{h_t^l}$ . First, the higher the distance  $\frac{h_t^h}{h_t^m}$ , the lower the income of the middle class relative to the mean, and therefore the more redistribution is required toward the middle-income class to form the club  $\{h, m\}$ . Second, a rise in  $\frac{h_t^m}{h_t^l}$  yields an increase of the income of the middle class relative to the average and also increases the pressure in favor of higher redistribution. This result may appear somehow surprising, especially in light of the early political economy theory and conventional wisdom (see Alesina and Rodrik [1], and Persson and Tabellini [31], who predict that redistribution will be higher the poorer the median voter is relative to the mean). However, in our setting the underlying argument is straightforward. Let there be two economies 1 and 2, and suppose first that  $h_{1t}^h = h_{2t}^h$  and  $h_{1t}^m = h_{2t}^m$ , but  $h_{1t}^l = 2h_{2t}^l$ ; that is, in the second economy, there is a higher wealth bias against the poor while at the same time the income of the middle class relative to the average is higher. This also means that the second economy is on average poorer than the first one which is itself more homogeneous than the former. As a consequence, in the second economy, more redistribution is required for the middle-income class to join the high-income class so that they can benefit from the same level of public good. Second, suppose now that  $h_{1t}^l = h_{2t}^l$

and that  $h_{1t}^h = \frac{1}{2}h_{2t}^h$  and  $h_{1t}^m = \frac{1}{2}h_{2t}^m$ , the second economy is now on average richer than the first one. Moreover, its middle-income class is richer relative to the average. What is important in this case to understand why  $\tilde{\tau}_{\{h,m\}}$  is higher in the second compared to the first economy is the fact that the absolute income gap between both the middle- and the high-income class is now higher in the second compared to the first economy. In both cases, the middle-income class must extract more resources from the high-income class to join it into the club  $\{h, m\}$ .

**Corollary 1.** *The pressure for redistributive policies is not necessarily smaller in a rich (poor) economy where the income of the middle class is high relative to the average compared to the pressure that prevails in a poorer (richer) economy where the income of the middle class may be smaller than the average.*

We now turn to Region III depicted in Figure 4 and which corresponds to ranges of inequality where the political equilibrium is characterized as follows:

**Proposition 3.** *When the income distribution is such that the middle-income class prefers to remain on its own rather than forming a club  $\{m, l\}$ ,  $\tau_{\{m\}}^*$  is the Condorcet winner which yields the equilibrium partition  $P_t^E = (\{h\}; \{m\}; \{l\})$  as soon as the ex ante pattern of income distribution is such that  $V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$  or  $V_t^l(\{l\}; \tau_{\{m\}}^*) > V_t^l(\{l\}; \tilde{\tau}_{\{h,m\}})$ .*

**Proof.** See Appendix A ■

In Region III where  $\tilde{\tau}_{\{h,m\}} > \tau_{\{m\}}^*$ , it is also true that the equilibrium partition  $(\{h\}; \{m\}; \{l\})$  is not necessarily the most preferred outcome for the middle-income class. Indeed, in that region there exist levels of inequality such that even though the middle-income class may most prefer a partition  $(\{h, m\}; \{l\})$  associated with a level of redistribution  $\tilde{\tau}_{\{h,m\}}$ , both agents of type  $h$  and  $l$  prefer a lower level of redistribution  $\tau_{\{m\}}^*$  which can be defeated in pairwise comparison by no other level of redistribution (see the proof of Proposition 3 in Appendix A).

**Corollary 2.** *Because of non-single peakedness, there exist in some range of inequality political equilibria where both the low- and the high-income classes agree on a similar level of redistribution which can not be defeated in binary contest. In other words, the pivotal voter does not always belong to the middle-income class.*

Moreover, this level of redistribution is smaller than the one most preferred by the middle-income class.

Intuitively, this result occurs for the following reason. The low-income class agrees with the high-income class to oppose high levels of redistribution because they want to prevent the middle-income class to form a club  $\{h, m\}$  from which they are implicitly excluded. This scenario occurs in our framework when the low-income class can be better off by remaining on its own with a lower level of redistribution than required for the middle to join the high-income class. In other words, when high levels of redistribution which are associated with a high level of distortions, lead the low-income class to lag further behind the other two income classes while they could catch up with a lower level of redistribution, they may oppose high rates of redistribution together with the high-income class. (See also Fernandez and Rogerson [17] who argue in favor of this kind of political outcomes, and Epple and Romano [15] and the so-called “ends against the middle” problem but in the latter case in a context of dual provision of education where an agent can consume either public or private education but not both, and where all agents who choose public school services obtain the same level of education.)

Recall that the equilibrium partition  $(\{h\}; \{m\}; \{l\})$  in Region III is associated with  $\tau_{\{m\}}^* = \max \left\{ 0, \frac{1}{2} \left( 1 - \frac{h_t^m}{h_t} \right) \right\}$ . By definition, a decrease in  $\frac{h_t^h}{h_t^m}$ , respectively an increase in  $\frac{h_t^m}{h_t^l}$ , leads to a decline in  $\tau_{\{m\}}^*$ . At first sight, this corroborates the view that the richer the middle-income class relative to the average, the lower the pressure for redistribution. However, notice that this increase in  $\frac{h_t^m}{h_t^l}$  may have two sources. First, it may be the result of a middle-income class catching-up with the high-income class. Second, it may be issued by the low-income class lagging further behind both the middle- and the high-income groups. Furthermore, both  $\frac{h_t^h}{h_t^m}$  and  $\frac{h_t^m}{h_t^l}$  may increase so that the Gini coefficient also increases<sup>7</sup>, this will not necessarily lead to more redistribution if the rise in inequality deteriorates relatively more the situation of the low-income class relative to the middle-income class compared to the deterioration of the situation of the middle-income relative to the high-income class. This strengthens the argument of fiscal conservatism proposed by

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<sup>7</sup> Using the implicit function theorem, the iso-redistribution curve  $\tau_{\{m\}}^* = \bar{\tau}$ , is strictly increasing, convex in our plane and of slope:

$$\left. \frac{d(h^m/h^l)}{d(h^h/h^m)} \right|_{\tau_{\{m\}}^* = \bar{\tau}} = \left( \frac{h^m}{h^l} \right)^2.$$



Saint Paul [33] who argues that the joint rise of inequality and fiscal conservatism that one could observe in the 1980s and 1990s in industrialized countries such as the United States and the United Kingdom and in less developed countries such as Mexico and Argentina may be explained by the increase in inequality which disproportionately affected the bottom portion of income distribution.

**Corollary 3.** *In a segmented economy, a rise in the wealth bias against the poor or even an increase in the Gini coefficient may yield a lower or similar equilibrium level of redistribution.*

We shall now be interested in Region *IV* displayed in Figure 4 where the following political equilibrium occurs

**Proposition 4.** *When the ex ante pattern of income distribution is such that: (i) the level of redistribution required to form a club  $\{h, m\}$  is higher than the level of redistribution required to form a club  $\{m, l\}$ ; that is,  $\tilde{\tau}_{\{h, m\}} > \tilde{\tau}_{\{m, l\}}$ , and (ii) the middle-income class is better off in a club  $\{m, l\}$  associated with either a level of redistribution  $\tau_{\{m, l\}}^*$  or  $(nh^h - (n + \beta)h^m)/\beta\bar{h}$  compared to both clubs  $\{m\}$  and  $\{h, m, l\}$  associated respectively with tax rates  $\tau_{\{m\}}^*$  and  $\tilde{\tau}_{\{h, m, l\}}$ , the equilibrium partition is  $P_t^E = (\{h\}; \{m, l\})$ . The Condorcet winner is either  $\tau_{\{m, l\}}^*$  or  $(nh^h - (n + \beta)h^m)/\beta\bar{h}$  depending on whether  $\tau_{\{m, l\}}^* \gtrless (nh^h - (n + \beta)h^m)/\beta\bar{h}$ , even though the middle-income class may be better off in that range of inequality in a club  $\{h, m\}$  associated with a higher level of redistribution  $\tilde{\tau}_{\{h, m\}}$ .*

**Proof.** See Appendix A ■

Region *IV* is located below the 45° -line; that is, the low- and the middle-income classes are now more homogeneous and there is a strong wealth bias in favor of the high-income class compared to Region *II*. Whenever there is a rise in  $\frac{h_t^h}{h_t^m}$  and/or  $\frac{h_t^m}{h_t^l}$ , redistribution increases although one can now easily understand that when the income share of the low-income class decreases relative to the others, the pressure for redistribution increases for the club  $\{m, l\}$  to become feasible. (Corollary 1 therefore also applies in Region *IV*.)

“Ends against the middle” situations as described in Corollary 2 also arise in Region *IV*. Indeed, it is not necessarily the case that the partition  $(\{h\}; \{m, l\})$  corresponds to the middle-income agent’s most preferred outcome of the

political process. Even though the middle-income class might be best off within a club  $\{h, m\}$  associated with the tax rate  $\tilde{\tau}_{\{h, m\}}$ , the club  $\{m, l\}$  associated with a level of redistribution  $\tau_{\{m, l\}}^*$  or  $(nh^h - (n + \beta)h^m)/\beta\bar{h}$  is preferred by both the low- and the high-income classes and therefore may block the partition  $(\{h, m\}; \{l\})$ . It is also worth noticing that when  $(nh^h - (n + \beta)h^m)/\beta\bar{h} < \tau_{\{m, l\}}^*$ , then the club  $\{m, l\}$  emerges with a level of redistribution which does not provide the maximum level of quasi-public resources which could be available to agents of type  $m$ , and  $l$ . Indeed, in that range of inequality, the partition  $(\{h\}; \{m, l\})$  associated with  $\tau_{\{m, l\}}^*$  is blocked by the coalition  $\{h, m\}$ . Further, the emergence of a political coalition between the low- and the high-income classes does not necessarily require that the most preferred tax rate of the middle-income class involves high deadweight costs.

Finally, we can provide the following:

**Proposition 5.** *All alternative ex ante income distributions which do not enter the above 4 propositions yield Condorcet cycles.*

The range of inequality where there is no political equilibrium under pure majority rule is displayed in Region  $V$  of Figure 4. It is well-known that in this case there are strong incentives to strategic manipulation, either of the agenda itself or of the preferences revealed in the voting process. We can think of two ways to solve this problem. First, imposing additional or more institutional structure on the political process can give rise to a well-defined equilibrium (see for instance, Chapter 2 of Persson and Tabellini's reference book [32]). Second, another voting rule could be chosen, namely the Borda rule (see for instance Moulin [27]). However, at this stage, we leave unexplored this issue and just emphasize how likely Condorcet cycles may arise in some key range of inequality.

We now analyze levels of redistribution across the different regions and provide an explanation to why government transfers may differ so greatly between countries. Let us consider three economies,  $E_{II}$ ,  $E_{III}$ , and  $E_{IV}$  characterized by levels of relative inequality such as displayed in Figure 5. Keep in mind that we have no information about whether  $E_{III}$  characterizes a richer or a poorer economy compared to  $E_{II}$ . In this figure, we first plot iso-redistribution

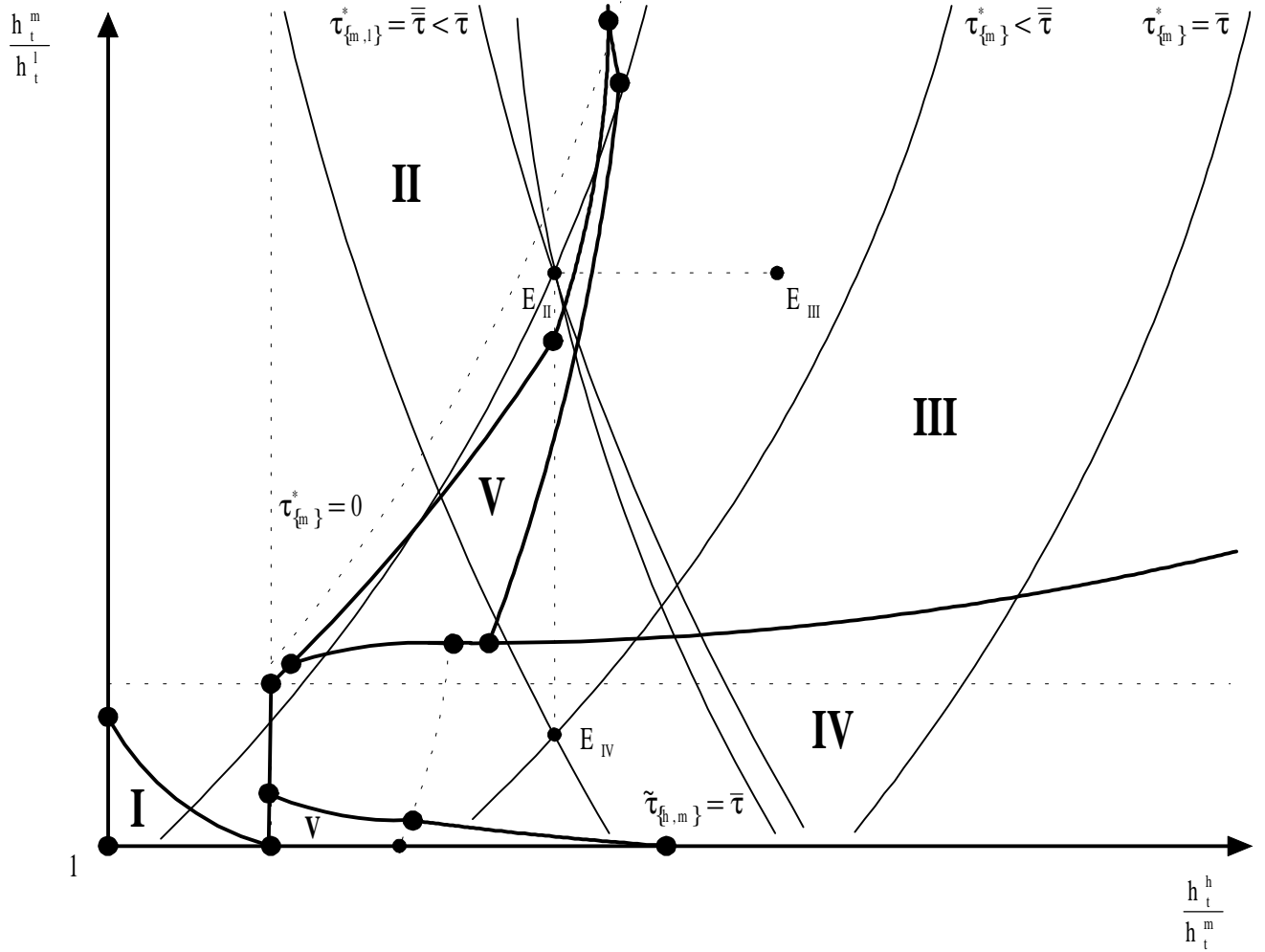


Figure 5: Equilibrium partition regions in the plane  $\left(\frac{h_t^h}{h_t^m}, \frac{h_t^m}{h_t^l}\right)$ , and iso-redistribution curves.

curves such that for instance  $\tilde{\tau}_{\{h,m\}} = \bar{\tau}$  and  $\tau_{\{m\}}^* = \bar{\tau}$ , and such that  $\tau_{\{m,l\}}^* = \bar{\tau} < \bar{\tau}^8$ . Notice first that  $E_{II}$  ( $E_{III}$ ) is located in region  $II$  ( $III$ ) where the corresponding equilibrium partition is  $(\{h, m\}; \{l\})$  ( $(\{h\}; \{m\}; \{l\})$ ). Second, the position of  $E_{II}$  and  $E_{III}$  in the plane, is such that the middle-income class in  $E_{III}$  is poorer relative to the average compared to the middle-income class in  $E_{II}$ . However, notice that in both economies  $E_{II}$  and  $E_{III}$ ,  $\tilde{\tau}_{\{h,m\}} > \tau_{\{m\}}^*$ , it is then clear that the equilibrium tax rate  $\tau_{\{m\}}^* \Big|_{E_{III}}$  which defeats in binary contest all alternative redistribution patterns in region  $III$  is lower than  $\tilde{\tau}_{\{h,m\}} \Big|_{E_{II}}$  which is the Condorcet winner in region  $II$ . Finally,  $E_{IV}$  located in region  $IV$  is also an economy characterized by a middle-income class which is poorer relative to the mean than it is in the first economy ( $E_{II}$ ). Provided that  $E_{II}$  is located in the plane where the pattern of income distribution is such that  $\tilde{\tau}_{\{h,m\}} > \tau_{\{m,l\}}^*$ , it is also straightforward given the slopes of the iso-redistribution curves, that  $\tau_{\{m,l\}}^* \Big|_{E_{IV}} < \tilde{\tau}_{\{h,m\}} \Big|_{E_{II}}$ . Finally, whatever the level of inequality,  $\tau_{\{m,l\}}^* > \tau_{\{m\}}^*$ , therefore, the level of redistribution is also lower in  $E_{III}$  compared to  $E_{IV}$ . The link between the pressure for corrective policy action in favor of more equality and income distribution is nothing but automatic. High levels of redistribution are not always associated with large inequalities yielding multiple politico-economic regimes. We can therefore provide the following:

**Corollary 4.** *More unequal and segmented economies may redistribute less, not more.*

Bénabou [4] provides an insightful review of the empirical findings about the relationship between inequality and the share of transfers. In fact, no empirical regularity can be inferred from the traditional regressions across a worldwide set of countries. In our model, when the wealth bias against the poor or when the income of the middle class increases relative to the mean, redistribution increases in Regions  $II$  and  $IV$  and decreases in Region  $III$ . Perotti [29] and [30], among others, tests the relationship between inequality and the share of transfers within a cross-section of countries where the share of the 3<sup>rd</sup> quintile is used as a proxy for the income of the median voter relative to the average. The relationship is positive and not significant even when controlling for democracy. In our model, this relationship is indeed expected to be highly non linear because of the existence of multiple politico-economic regimes as discussed

<sup>8</sup> Let us here consider the iso-redistribution curves  $\tilde{\tau}_{\{h,m\}} = \bar{\tau}$  and  $\tau_{\{m,l\}}^* = \bar{\tau}$ . It is easily shown that both are strictly decreasing and convex in our plane. Moreover, we also have  $-\frac{d(h^m/h^l)}{d(h^h/h^m)} \Big|_{\tau_{\{m,l\}}^* = \bar{\tau}} < -\frac{d(h^m/h^l)}{d(h^h/h^m)} \Big|_{\tilde{\tau}_{\{h,m\}} = \bar{\tau}}$ .

above. More care should therefore be devoted to the choice of the variables used as proxies for inequality and of the controls. Patterns of redistribution are richer than that proposed by the standard political economy models, and the empirics of inequality and redistribution should definitely take into consideration that patterns of redistribution can be non linear (see for instance Lindert [24] who provides empirical evidence in favor of the “social affinity” hypothesis such as emphasized in this article among OECD countries). A test of the existence of multiple redistributive regimes as proposed in our model and as opposed to conventional political economy models could then be implemented by endogenously splitting the sample according to the relative levels of inequality as depicted and suggested by Figure 5<sup>9</sup>, or by adopting a non or semiparametric approach.

## 4. Dynamics

We now concentrate on how inequality and redistribution influence the dynamic evolution of income distribution through time, human capital accumulation, and growth. We also provide a scenario which illustrates the forces setting the limits of government transfers. The evolution of the economy is characterized by the dynamics of  $\frac{h_t^h}{h_t^m}$  and  $\frac{h_t^m}{h_t^l}$ . Recall that at time  $t$ , the income distribution not only determines the after-tax equilibrium, but also determines next period *ex ante* distribution of human capital stocks. As the pattern of income distribution changes over time, the political equilibrium shifts endogenously along the transitional path<sup>10</sup>. Similarly to Durlauf [13], intergenerational group formation implies that income trajectories may be very different depending on the initial pattern of income distribution leading to the existence of multiple steady states. In our simple deterministic framework, we characterize intergenerational mobility along the transitional path. For instance, the evolving pattern of income distribution from Region *IV* to Region *II* leads the middle-income class to experience upward mobility. It becomes homogeneous enough to the high-income class so that, in this range of inequality, there exists a political outcome which allows it to benefit from better local interaction in the community  $\{h, m\}$ .

More specifically, the dynamics of income inequality across agents of type  $i$  and  $k$  where  $h_t^i > h_t^k$ , who belong

<sup>9</sup> This could be done by letting the data speak by themselves as, for instance, in Durlauf and Johnson [14].

<sup>10</sup> Remind that, due to a limited bequest motive, our framework is not a dynamic game in which agents would care about future political outcomes. See for instance Saint Paul [33] for a dynamical voting game.

respectively to a club  $S_{jt}$  and  $S_{j't}$  at equilibrium, depend on the combination of two effects. Let us consider the following expression:

$$\frac{h_{j,t+1}^i}{h_{j',t+1}^k} = \left( \frac{h_t^i}{h_t^k} \right)^{(1-\alpha-\beta)} \left( \left( \frac{\sum_{z \in S_{jt}} (h_t^z + \tau \bar{h}_t)}{n_{jt}} \cdot \frac{n_{j't}}{\sum_{z' \in S_{j't}} (h_t^{z'} + \tau \bar{h}_t)} \right) \left( \frac{n_{j't} + \beta}{n_{j't}} \cdot \frac{n_{jt}}{n_{jt} + \beta} \right) \right)^\beta \quad (12)$$

The first effect is the traditional income convergence result due to diminishing returns in  $G_{jt}$  and  $h_t^i$ , as emphasized by Tamura [37], and Glomm and Ravikumar [23], among others. Second, the neighborhood effects associated with the equilibrium partition which emerges also influence income dynamics in three specific ways captured in the second term of Equation (12): (i) the per capita posttax income ratio between  $S_{jt}$  and  $S_{j't}$  forces divergence, (ii) but this ratio is negatively related to  $\tau$ ; that is, redistribution drives convergence. This reflects the effect of redistribution which brings about more equality of opportunity to save and invest a greater amount of income into education or to benefit from higher levels of educational services and local externalities. (iii) An ambiguous size effect which yields convergence, respectively divergence, when the higher-income community is smaller, respectively larger, compared to the lower-income community.

The underlying global dynamics of the economy depend on the interplay between these two effects. Given Propositions 1 to 5 and information about the evolution of income dynamics provided in Appendix C, we can construct the phase diagram as displayed in Figure 6.

As can be seen from Figure 6, the long run behavior of the income distribution is such that there exist two steady states. They describe either a situation of equality and integration or segregation with persistent inequality. Denoting  $\Pi$  by

$$\Pi = \ln \left( \frac{2n + \beta}{2n} \right) / \ln \left( \frac{2(n + \beta)}{2n + \beta} \right) > 1 \quad (13)$$

we can provide the following:

**Proposition 6.** *If and only if  $\beta/\alpha \geq \Pi$ ; that is, the isokine  $y_{t+1} = y_t|_{\bar{\tau}_{\{h,m\}}=0}$  is always located above the locus  $\tilde{\tau}_{\{h,m,l\}} = 0$ , there exist two steady states:*

1/ *One called the integrated equilibrium (IE) and characterized by a completely homogeneous population which*

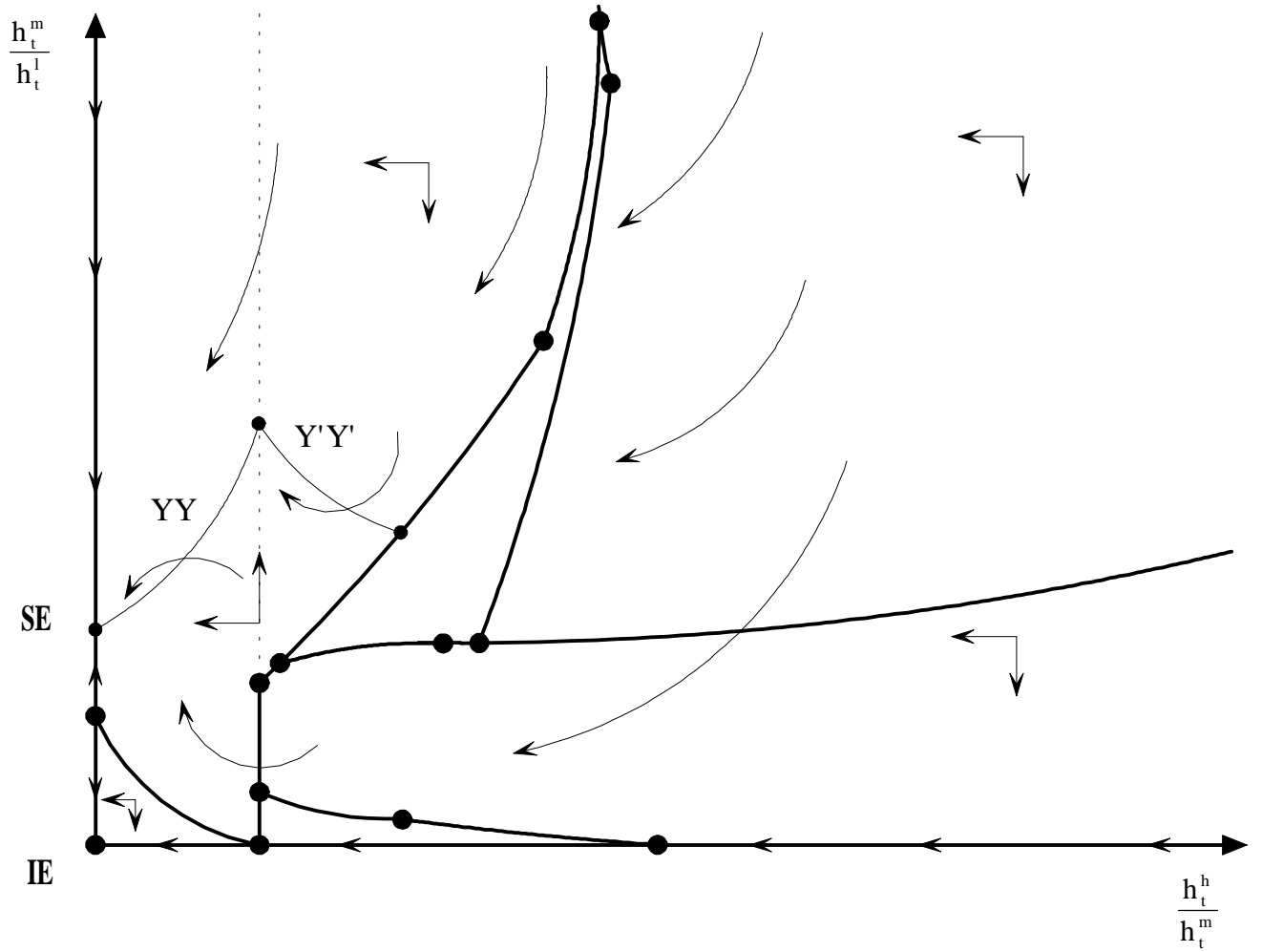


Figure 6: Inequality dynamics. SE, respectively IE, denotes the segregated, respectively the integrated, equilibrium. YY (Y'Y') corresponds to the isokine  $y_{t+1} = y_t |_{\bar{\tau}_{\{h,m\}}=0}$  ( $y_{t+1} = y_t |_{\bar{\tau}_{\{h,m\}}>0}$ ).

*belongs to the same community. It is a globally stable steady-state within Region I.*

*2/ The second, called the segregated equilibrium (SE), is such that the initially high- and middle-income classes consist of an homogeneous community while the low-income class remains isolated. In this case, there is persistent inequality. It is a locally stable steady state within Region II.*

*Otherwise, there is a unique steady state which is the integrated equilibrium.*

**Proof.** See Appendix C ■

Income inequalities are not necessarily completely eliminated in the long run. The multiple steady states situation occurs when the local versus global externalities ratio ( $\beta/\alpha$ ) reaches a threshold level as defined in Equation (13). Otherwise, if  $\beta/\alpha < \Pi$ , then the economy-wide spillovers are strong enough and inequality vanishes in the long run. (See also Durlauf [12] who stresses how much the relative size of the local spillovers is important to obtain multiple steady states.)

The integrated equilibrium is characterized by a completely homogeneous population where individual incomes are all equalized. When the three income classes interact in the grand coalition, this equilibrium partition remains forever; that is, as soon as the economy is in that range of inequality defined by Region I, the positive feedbacks spread uniformly over the whole population with no need for redistributive policies. On the contrary, at the segregated equilibrium, inequality is persistent. While the high- and the middle-income classes benefit from the same level of local externalities and are similarly endowed, the poorer individuals remain stuck into a wealth trap where they are not able to benefit from the local linkage available in the richer community. The long-run inequality is characterized by a permanent income gap between the rich community and the low-income class equal to  $\left(\frac{2(n+\beta)}{2n+\beta}\right)^{\beta/\alpha}$ . Notice that this steady state corresponds to a situation where no redistribution is now preferred by a majority of the population. As a consequence, there is still room for a central planner to influence this locally stable steady state through one-shot redistribution policies until the pattern of income distribution reaches Region I.

Finally, in a segregated equilibrium polarization leads the above size effect to lower growth compared to the integrated equilibrium. Moreover, as the specified local public good technology is characterized by perfect substitutability



between the individual investment effort and that the economy-wide knowledge spillovers are simply an arithmetic average, steady-state heterogeneity reduces growth<sup>11</sup>.

**Proposition 7.** *The integrated equilibrium has higher growth than the segregated one.*

**Proof.** See Appendix C ■

Clearly, this proposition also stresses the long-run inefficiency of social polarization as all dynasties are hurt by a slower aggregate human capital accumulation.

We now turn to the description of the global dynamics of our framework. Then, we address the issue of the impact of the introduction of redistribution on income trajectories and growth.

Let us consider an economy starting in Region *III*. From Proposition 3, inequality is relatively high such that each income class forms its own community and the associated equilibrium level of redistribution is  $\tau_{\{m\}}^*$ . As displayed in Figure 6, local and global spillovers lead both ratios  $\frac{h_t^h}{h_t^m}$  and  $\frac{h_t^m}{h_t^l}$  to decrease. Over a finite time, the community structure of the economy evolves towards a new equilibrium partition. Depending on the initial income distribution, the economy may enter in Region *II*, *IV*, or *V*. Suppose that the new regime is Region *IV*; that is, the low-income dynasty experiences an upward mobility movement as she has reached a posttax income high enough to interact with agents of type  $m$  in the community  $\{m, l\}$ . The global effect on the dynamics of income is that the three income classes become more and more homogeneous. Three possibilities can then arise. First, the low- and middle-income classes become similar and the grand community is now preferred by a majority. Second the economy enters in the Condorcet cycle region. Third, the economy reaches Region *II* as the middle-income class catching up the rich agents is now accepted in their club while the poorer individuals remain in a community with only people of the same kind. In this case, the middle-income dynasty is upward moving while the poor dynasty experiences a downward social mobility. Suppose dynamics are such that the economy enters in Region *II*. The ratio  $\frac{h_t^m}{h_t^l}$  increases because the local interaction in community  $\{h, m\}$  combines two divergence forces: a richer club and a bigger size than in the club  $\{l\}$ . After a finite

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<sup>11</sup> See Bénabou [4] who studies the effect of stratification when the degree of complementarity between individuals' human capital stock varies depending on both local and global interactions.

time, two transitions are possible. Either social polarization is enhanced and the economy converges to the segregated equilibrium, or, despite increasing  $\frac{h_t^m}{h_t^l}$ , the low-income class is rich enough to be accepted in a club  $\{h, m, l\}$  and the economy enters in Region *I*. In this case, the economy evolves toward the integrated equilibrium.

Global dynamics of inequality are characterized by history-dependent steady states. The initial political equilibrium which emerges, determines the human capital accumulation of all subsequent generations. Thus, when the initial pattern of income distribution exhibits a strong wealth bias against the poor (extreme North West of Figure 6), the economy ends up polarized. On the opposite, when the initial pattern of income distribution is such that the low- and middle-income classes are homogeneous but very far from the high-income-class (extreme South East of Figure 6), the economy ends up integrated.

It is also worth noticing the crucial feature of the Condorcet regions: many economies starting from Region *III* and *IV* can be confronted with these political conflicts along their transitional path. Nevertheless the chance to achieve the integrated equilibrium decreases with the relative importance of local externalities. Indeed, there exists a critical value of  $\beta/\alpha$  denoted  $\Gamma$  with

$$\Gamma = \ln \left( \frac{\frac{2n+\beta}{n}}{1 + \left(\frac{n+\beta}{n}\right)^{(1-\alpha-\beta)}} \right)^{\frac{1}{\alpha}} / \ln \left( 1 + \left(\frac{n}{\beta}\right)^{-1} \right) > \Pi \quad (14)$$

such that,

**Proposition 8.** *When  $\frac{\beta}{\alpha} \geq \Gamma$ , whatever the political outcome that would emerge in Region *V*, only economies with an initial pattern of income distribution located in Region *I* converge towards the integrated equilibrium.*

**Proof.** See Appendix C ■

Thus, when the local determinants of education are relatively important, whatever the intergenerational mobility which occurs along the transitional path, the only possibility to obtain long run equality is that everybody initially benefits from the same neighborhood spillover effects. At the extreme when  $\alpha = 0$ , any initial pattern of income distribution leading to separation of agents results in an income gap between a rich club with size  $2n$  and the poor class which nevertheless keeps growing. Further, despite the fact that the area in which  $(\{h, m\}; \{l\})$  emerges as the

equilibrium partition spreads out, the introduction of redistribution enhances the probability to reach the integrated equilibrium at the steady state. We can indeed provide the following

**Proposition 9.** *Introducing redistribution increases the number of candidates for the integrated equilibrium.*

*Moreover, redistribution favors growth.*

**Proof.** See the above discussion ■

Compared to the case without any fiscal policy, redistribution enhances aggregate human capital accumulation despite deadweight losses. This result relies on the fact that *ex ante* income heterogeneity is reduced leading human capital technology to be more efficient.

## 5. Conclusion

In a world where the level of educational services available to an individual strongly depends on her capacity or ability to invest into education but also on the incentive to do so through the affiliational community she may belong to, the conflicts of interest which arise are more complex than advocated in the conventional political economy literature. More specifically, we abandon the stylized median voter in favor of more nuanced pressure-group reasoning. Depending on the social stratification, we find that the pressure for redistributive taxation is not necessarily higher in more unequal and segmented economies, therefore limiting government spending and redistributive programs.

The present community structure strongly depends on historical backgrounds and shocks which can explain why across a worldwide set of nations, we observe such a highly non-linear and uneven redistributive pattern. Moreover, most democracies can be expected to face ranges of inequality where voting cycles may occur. Additional structure should be imposed in our model by restricting the institutions which govern how policy decisions are made. At this stage, this issue is left unexplored and postponed to a future research agenda.

## Appendix A. Proofs of Propositions 1 through 5

The voting rule which solves the collective decision problem is issued by the Condorcet method where the political

outcome of the vote is the candidate which defeats every other candidate in pairwise comparison. Each candidate is associated with an equilibrium partition which may differ according to the *ex ante* pattern of income distribution. Recall first that the high-income class has no incentive to redistribute so that we always have for agents of type  $h$ :  $0 \succ \tau_{\{m\}}^* \succ \tau_{\{m,l\}}^* \succ \tau_{\{l\}}^* \succ \frac{1}{2}$ . Second, our assumption about the distortions associated with the redistributive policy leads a level of redistribution  $\tau \geq 1/2$  to be always preferred in binary contest by all the other candidates which are:

$$\tilde{\tau}_{\{h,m,l\}}, \tilde{\tau}_{\{h,m\}}, \tilde{\tau}_{\{m,l\}}, 0, \tau_{\{m\}}^*, \tau_{\{m,l\}}^*, \tau_{\{l\}}^*$$

**Proof of Proposition 1** First, when the *ex ante* pattern of inequality is such that  $n(h_t^h + h_t^m) - (2n + \beta)h_t^l < 0$ , all income classes are able to positively contribute to the provision of the public good in the club  $\{h, m, l\}$  for any level of redistribution. Hence, for any tax rate  $\tau \geq 0$ , the club  $\{h, m, l\}$  provides the highest level of public good to all income classes. Notice that in that range of inequality,  $\tau_{\{h,m,l\}}^* = \tilde{\tau}_{\{h,m,l\}} = 0$ , then the level of public good available in  $\{h, m, l\}$  is maximum when no distortionary redistribution occurs. As a consequence, the equilibrium partition is  $P_t^I = (\{h, m, l\})$  and all income classes vote in favor of no redistribution. Second, notice that when  $h_t^m = h_t^l$  the low- and the middle-income classes are similarly endowed *ex ante*. It implies that  $\tilde{\tau}_{\{h,m,l\}} = \tilde{\tau}_{\{h,m\}} \geq 0$  and  $\tau_{\{m,l\}}^* = \tau_{\{m\}}^* = \tau_{\{l\}}^*$ . As long as  $V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h,m,l\}}) \geq V_t^i(\{m, l\}; \tau_{\{m,l\}}^*)$  for  $i = m, l$  the political outcome of the vote is  $\tilde{\tau}_{\{h,m,l\}}$  associated with the equilibrium partition  $P_t^I = (\{h, m, l\})$ . ■

In order to prove the four other propositions, we need some information provided in Lemma 1.

**Lemma 1** *When the initial pattern of income distribution is such that  $n(h_t^h + h_t^m) - (2n + \beta)h_t^l \geq 0$  and  $h^m > h^l$ ,  $\tilde{\tau}_{\{h,m\}}$  is most preferred by both the middle- and the high-income classes compared to  $\tilde{\tau}_{\{h,m,l\}}$ . Hence, in that range of inequality,  $\tilde{\tau}_{\{h,m,l\}}$  cannot be a Condorcet winner. It can just be a candidate which yields Condorcet cycles. Moreover, when the *ex ante* pattern of inequality is such that  $nh_t^h - (n + \beta)h_t^m \geq 0$  and  $nh_t^m - (n + \beta)h_t^l \geq 0$ ,  $\tilde{\tau}_{\{h,m,l\}}$  is not a relevant candidate any more because it implies that  $\tilde{\tau}_{\{h,m,l\}} > 1$ .*

**Proof.** In that range of inequality, it is always true that  $\tilde{\tau}_{\{h,m\}} < \tilde{\tau}_{\{h,m,l\}}$ . In that case, the club  $\{h, m\}$  blocks the partition  $P_t^I = (\{h, m, l\})$  because  $\ln G_{jt}$  available to agents of type  $h$ , and  $m$  in the partition  $P_t^{II} = (\{h, m\}; \{l\})$  is

higher compared to the level of public good they could benefit in the partition  $P_t^I = (\{h, m, l\})$ ; that is,

$$\frac{n(1 - \tilde{\tau}_{\{h, m, l\}})(h_t^h + h_t^m) + 2n(\tilde{\tau}_{\{h, m, l\}} - \tilde{\tau}_{\{h, m, l\}}^2)\bar{h}_t}{2n + \beta} < \frac{n(1 - \tilde{\tau}_{\{h, m\}})(h_t^h + h_t^m) + 2n(\tilde{\tau}_{\{h, m\}} - \tilde{\tau}_{\{h, m\}}^2)\bar{h}_t}{2n + \beta}$$

Finally, notice that by definition,  $\tilde{\tau}_{\{h, m, l\}} = \frac{n(h_t^h + h_t^m) - (2n + \beta)h_t^l}{\beta h_t}$ . Hence, in that range of inequality where  $h_t^h \geq \frac{(n + \beta)}{n}h_t^m \geq \left(\frac{n + \beta}{n}\right)^2 h_t^l$ , it is easily checked that  $\tilde{\tau}_{\{h, m, l\}} > 1$ . ■

**Proof of Proposition 2** Suppose that the *ex ante* pattern of income distribution is such that: (i)  $0 \leq \tilde{\tau}_{\{h, m\}} \leq \tilde{\tau}_{\{m, l\}}$ , (ii)  $\tilde{\tau}_{\{h, m\}} \leq \tau_{\{l\}}^*$ , and (iii)  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*)$ , are satisfied simultaneously. First, (i) and (ii) imply that  $\tilde{\tau}_{\{h, m\}}$  defeats in binary contest the candidates  $\tau_{\{l\}}^*$  and  $\tilde{\tau}_{\{m, l\}}$ . Second, consider the case where  $\tilde{\tau}_{\{h, m\}} > \tau_{\{m, l\}}^*$ , then (i) implies that the club  $\{m, l\}$  is unfeasible with  $\tau_{\{m, l\}}^*$ . Provided that (ii) and (iii) are also satisfied, the preferences for both the low- and the middle-income classes are,  $V_t^l(\{l\}; \tilde{\tau}_{\{h, m\}}) > V_t^l(\{l\}; \tau_{\{m, l\}}^*)$ , respectively  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{m\}; \tau_{\{m, l\}}^*)$ . On the other hand,  $\tilde{\tau}_{\{h, m\}} \leq \tau_{\{m, l\}}^*$  yields the following preference ordering for the middle-income class:  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) \geq V_t^m(\{h, m\}; \tau_{\{m, l\}}^*)$ . Therefore, (i), (ii), and (iii) also lead  $\tilde{\tau}_{\{h, m\}}$  to defeat  $\tau_{\{m, l\}}^*$  in pairwise comparison whatever  $\tilde{\tau}_{\{h, m\}} \gtrless \tau_{\{m, l\}}^*$ . Finally, using Lemma 1 and considering the above result and (iii) allows us to infer that the middle-income class is best off in the club  $\{h, m\}$  associated with  $\tilde{\tau}_{\{h, m\}}$ . There is no other issue than  $\tilde{\tau}_{\{h, m\}}$  which is preferred by both the high- and the low-income groups. Therefore, in that range of *ex ante* inequality, the equilibrium partition is  $P_t^{II} = (\{h, m\}; \{l\})$  associated with  $\tilde{\tau}_{\{h, m\}} \geq 0$ . ■

The following lemma allows us to prove Propositions 3 and 4.

**Lemma 2** *The middle-income class may be indifferent between clubs  $\{m, l\}$  and  $\{m\}$  associated with tax rate  $\tau_{\{m, l\}}^*$ , respectively  $\tau_{\{m\}}^*$ ; that is,  $V_t^m(\{m, l\}; \tau_{\{m, l\}}^*) = V_t^m(\{m\}; \tau_{\{m\}}^*)$ , if and only if the *ex ante* pattern of inequality is such that:  $\tau_{\{m\}}^* < \tilde{\tau}_{\{m, l\}} < \tau_{\{m, l\}}^*$ .*

**Proof.** By definition, notice that  $V_t^m(\{m, l\}; \tilde{\tau}_{\{m, l\}}) - V_t^m(\{m\}; \tilde{\tau}_{\{m, l\}}) = 0$ . Then, for any tax rate  $\tau > \tilde{\tau}_{\{m, l\}}$ , the middle-income class is better off in the club  $\{m, l\}$  compared to the club  $\{m\}$ . On the one hand, if  $\tilde{\tau}_{\{m, l\}} \leq \tau_{\{m\}}^*$ , it is obvious that we have  $V_t^m(\{m\}; \tau_{\{m\}}^*) < V_t^m(\{m, l\}; \tau_{\{m\}}^*) < V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ . On the other hand, when  $\tilde{\tau}_{\{m, l\}} \geq \tau_{\{m, l\}}^*$ , we also always have  $V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{m\}; \tau_{\{m, l\}}^*) > V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ . ■

**Proof of Proposition 3** Consider an *ex ante* pattern of income distribution such that  $V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ . Using Lemma 2, it is straightforward that  $\tilde{\tau}_{\{m, l\}} > \tau_{\{m\}}^*$  and therefore  $\tau_{\{m\}}^*$  defeats in binary contest  $\tilde{\tau}_{\{m, l\}}$ ,  $\tau_{\{m, l\}}^*$ , and *a fortiori*  $\tau_{\{l\}}^*$ . Therefore, the only trade off faced by the middle-income class is between the candidates  $\tau_{\{m\}}^*$  and  $\tilde{\tau}_{\{h, m\}}$ . Two possibilities may arise. First, the pattern of inequality is such that  $V_t^m(\{m\}; \tau_{\{m\}}^*) \geq V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}})$  which implies that  $\tilde{\tau}_{\{h, m\}} > \tau_{\{m\}}^*$ , and whatever  $V_t^l(\{l\}; \tau_{\{m\}}^*) \geq V_t^l(\{l\}; \tilde{\tau}_{\{h, m\}})$ , the Condorcet winner is  $\tau_{\{m\}}^*$ . Second, the *ex ante* income distribution is such that  $V_t^m(\{m\}; \tau_{\{m\}}^*) < V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}})$  but  $V_t^l(\{l\}; \tau_{\{m\}}^*) > V_t^l(\{l\}; \tilde{\tau}_{\{h, m\}})$ . This pattern of preferences may occur either when  $\tilde{\tau}_{\{h, m\}} < \tau_{\{m\}}^*$ , or when  $\tilde{\tau}_{\{h, m\}} > \tau_{\{m\}}^*$ . Let us consider the case where  $\tilde{\tau}_{\{h, m\}} < \tau_{\{m\}}^*$ , and notice that the starting condition  $V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$  implies that  $\tilde{\tau}_{\{m, l\}} > \tau_{\{m\}}^*$  which in turn implies that  $\tilde{\tau}_{\{h, m\}} < \tilde{\tau}_{\{m, l\}}$ . Then, we are back to the situation described in Proposition 2. On the other hand, when  $\tilde{\tau}_{\{h, m\}} > \tau_{\{m\}}^*$  and  $V_t^l(\{l\}; \tau_{\{m\}}^*) > V_t^l(\{l\}; \tilde{\tau}_{\{h, m\}})$ , both the low- and the high-income classes prefer  $\tau_{\{m\}}^*$  compared to  $\tilde{\tau}_{\{h, m\}}$ . Finally, notice that in these cases,  $\tilde{\tau}_{\{h, m\}}$  and  $\tilde{\tau}_{\{m, l\}}$  are both strictly positive. Thus, using Lemma 1, we can ignore the candidate  $\tilde{\tau}_{\{h, m, l\}}$ . Hence, in that range of inequality, the Condorcet winner is  $\tau_{\{m\}}^* \geq 0$  which yields the equilibrium partition  $P_t^{III} = (\{h\}; \{m\}; \{l\})$ . ■

The proof of Proposition 4 requires another lemma.

**Lemma 3** *The middle-income class may be indifferent between clubs  $\{m, l\}$  and  $\{m\}$  associated with tax rate  $\tilde{\tau}_{\{h, m\}}$ , respectively  $\tau_{\{m\}}^*$ ; that is,  $V_t^m(\{m, l\}; \tilde{\tau}_{\{h, m\}}) = V_t^m(\{m\}; \tau_{\{m\}}^*)$ , when the *ex ante* pattern of inequality is such that  $\tau_{\{m\}}^* \leq \tilde{\tau}_{\{m, l\}} \leq \tilde{\tau}_{\{h, m\}} \leq \tau_{\{m, l\}}^*$ .*

**Proof.** Recall that by definition, we always have  $V_t^m(\{m, l\}; \tilde{\tau}_{\{m, l\}}) - V_t^m(\{m\}; \tilde{\tau}_{\{m, l\}}) = 0$ . Hence, when  $\tau_{\{m\}}^* \leq \tilde{\tau}_{\{m, l\}}$  we also have  $V_t^m(\{m\}; \tau_{\{m\}}^*) \geq V_t^m(\{m, l\}; \tilde{\tau}_{\{m, l\}})$ . Using Lemma 2, it is easily checked that in this range of inequality, there may exist  $\tilde{\tau}_{\{m, l\}}$  such that  $V_t^m(\{m, l\}; \tau_{\{m, l\}}^*) \geq V_t^m(\{m\}; \tau_{\{m\}}^*)$ . In that case, because  $\tilde{\tau}_{\{h, m\}} \leq \tau_{\{m, l\}}^*$ , there also exists  $\tilde{\tau}_{\{h, m\}}$  such that  $V_t^m(\{m, l\}; \tilde{\tau}_{\{h, m\}}) = V_t^m(\{m\}; \tau_{\{m\}}^*)$ . ■

**Proof of Proposition 4** When the *ex ante* pattern of income distribution is such that: (i)  $\tilde{\tau}_{\{h, m\}} \geq \tau_{\{m, l\}}^*$ , (ii)  $V_t^m(\{m\}; \tau_{\{m\}}^*) \leq V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$  and (iii)  $V_t^i(\{m, l\}; \tau_{\{m, l\}}^*) > V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$  for  $i = m, l$  are satisfied simultaneously, notice first that  $\tau_{\{m, l\}}^*$  is strictly preferred by a majority to 0 and  $\tau_{\{m\}}^*$ . From (ii) and using

Lemma 2, we know that  $0 \leq \tilde{\tau}_{\{m,l\}} < \tau_{\{m,l\}}^*$  which leads the club  $\{m, l\}$  to be feasible with tax rate  $\tau_{\{m,l\}}^*$  which, by definition, maximizes the level of public good in that club. Hence,  $\tau_{\{m,l\}}^*$  defeats in binary contest  $\tilde{\tau}_{\{m,l\}}$  and  $\tau_{\{l\}}^*$ . Second, provided that (i) is satisfied,  $\tau_{\{m,l\}}^*$  is always strictly preferred to  $\tilde{\tau}_{\{h,m\}}$  by the low- and the high-income classes although the middle-income class may prefer to form a club  $\{h, m\}$  associated with  $\tilde{\tau}_{\{h,m\}}$ . Finally, given (i) and (iii),  $\tau_{\{m,l\}}^*$  is unanimously preferred to  $\tilde{\tau}_{\{h,m,l\}}$ . Thus, in that range of *ex ante* income distribution, the equilibrium partition is  $P_t^{IV} = (\{h\}; \{m, l\})$  associated with  $\tau_{\{m,l\}}^*$ .

This equilibrium partition also occurs when (i)  $\tau_{\{m,l\}}^* \geq \tilde{\tau}_{\{h,m\}} > \tilde{\tau}_{\{m,l\}} \geq 0$ , (ii)  $V_t^m(\{m, l\}; \tilde{\tau}_{\{h,m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*)$ , and (iii)  $V_t^i(\{m, l\}; \tilde{\tau}_{\{h,m\}}) > V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h,m,l\}})$  for  $i = m, l$  are satisfied simultaneously. Let us first consider the case  $0 < \tilde{\tau}_{\{h,m\}} \leq \tau_{\{m\}}^*$ , then it is clear that  $\tilde{\tau}_{\{h,m\}}$  defeats in binary contest  $\tau_{\{m\}}^*$ ,  $\tau_{\{m,l\}}^*$ ,  $\tau_{\{l\}}^*$ , and 0. The level of redistribution therefore lies between  $\tilde{\tau}_{\{m,l\}} \geq 0$ , and  $\tilde{\tau}_{\{h,m\}}$ . Notice that the level of public good available in club  $\{m, l\}$  increases in that range with the level of redistribution.  $\tau = \frac{nh^h - (n+\beta)h^m}{\beta h} > \tilde{\tau}_{\{m,l\}}$  is the tax rate which provides the highest level of public good in club  $\{m, l\}$  and which prevents the middle-income class to join the high-income class in a club  $\{h, m\}$ . Hence,  $\tau = \frac{nh^h - (n+\beta)h^m}{\beta h}$  is strictly preferred by both the low- and the high-income classes to  $\tilde{\tau}_{\{h,m\}}$ . Second, when  $\tilde{\tau}_{\{h,m\}} > \tau_{\{m\}}^*$ , provided that (ii) is satisfied and using Lemma 3, the same reasoning as above leads  $\frac{nh^h - (n+\beta)h^m}{\beta h}$  to be strictly preferred to  $\tilde{\tau}_{\{h,m\}}$ . Finally, as long as (iii) is satisfied  $\frac{nh^h - (n+\beta)h^m}{\beta h}$  is a Condorcet winner in that range of inequality and the equilibrium partition is  $P_t^{IV} = (\{h\}; \{m, l\})$ . ■

Finally, the following lemma provides information used to prove Proposition 5.

**Lemma 4** *Let the middle-income class be indifferent between  $\tilde{\tau}_{\{h,m\}}$  and  $\tau_{\{m\}}^*$  associated with club  $\{h, m\}$ , respectively  $\{m\}$ ; that is,  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}}) = V_t^m(\{m\}; \tau_{\{m\}}^*)$  and suppose that this indifference requires that  $\tilde{\tau}_{\{h,m\}} > \tau_{\{l\}}^*$  for  $\tau_{\{m\}}^* > 0$ . Then, as long as  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*)$ , we also have  $V_t^m(\{h, m\}; \tau_{\{l\}}^*) > V_t^m(\{m\}; \tau_{\{m\}}^*)$ .*

**Proof.** See Appendix B ■

**Proof of Proposition 5** All remaining *ex ante* patterns of income distribution yield Condorcet cycles. Keep in mind that whenever,  $\tilde{\tau}_{\{h,m\}}$  and  $\tilde{\tau}_{\{m,l\}}$  are both positive we can ignore  $\tilde{\tau}_{\{h,m,l\}}$  because all income classes strictly prefer any other candidate.

- In Proposition 4, we just showed that  $(\{h\}; \{m, l\})$  emerges as an equilibrium partition when (i)  $\tau_{\{m, l\}}^* \geq \tilde{\tau}_{\{h, m\}} > \tilde{\tau}_{\{m, l\}} \geq 0$ , (ii)  $V_t^m(\{m, l\}; \tilde{\tau}_{\{h, m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*)$ , and (iii)  $V_t^i(\{m, l\}; \tilde{\tau}_{\{h, m\}}) > V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$  for  $i = m, l$  are satisfied simultaneously. As soon as (iii) is not satisfied  $\tilde{\tau}_{\{h, m, l\}}$  becomes a key competitor compared to  $\frac{nh^h - (n+\beta)h^m}{\beta h}$ . Indeed, in that case, we have

$$V^h(\{h\}; \tau) > V^h(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V^h(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$$

$$V^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V^m(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}}) > V^m(\{m, l\}; \tau)$$

$$V^l(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}}) > V^l(\{m, l\}; \tau) > V^l(\{l\}; \tilde{\tau}_{\{h, m\}})$$

$$\text{with } \tau = \frac{nh^h - (n+\beta)h^m}{\beta h}.$$

Notice that the same is true when (i)  $\tilde{\tau}_{\{h, m\}} \geq \tau_{\{m, l\}}^*$ , (ii)  $V_t^m(\{m\}; \tau_{\{m\}}^*) \leq V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ , but (iii)  $V_t^i(\{m, l\}; \tau_{\{m, l\}}^*) \leq V_t^i(\{h, m, l\}; \tilde{\tau}_{\{h, m, l\}})$  for  $i = m, l$ . In that case,  $\tilde{\tau}_{\{h, m, l\}}$  becomes a key competitor compared to  $\tau_{\{m, l\}}^*$  which is not anymore a Condorcet winner.

- A second type of cycle occurs when (i)  $\tau_{\{m\}}^* \leq \tilde{\tau}_{\{m, l\}} < \tilde{\tau}_{\{h, m\}} < \tau_{\{m, l\}}^*$ , and (ii)  $V_t^m(\{m, l\}; \tilde{\tau}_{\{h, m\}}) \leq V_t^m(\{m\}; \tau_{\{m\}}^*)$  are satisfied simultaneously. Notice, that this range of inequality is very similar to the above case described in Proposition 4 except that (ii) implies now that  $\tau_{\{m\}}^*$  becomes a serious competitor in pairwise comparison to  $\frac{nh^h - (n+\beta)h^m}{\beta h}$ . Indeed, in that case we have the following ordering of relevant preferences which yields a Condorcet cycle:

$$V^h(\{h\}; \tau_{\{m\}}^*) > V^h(\{h\}; \tau) > V^h(\{h, m\}; \tilde{\tau}_{\{h, m\}})$$

$$V^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V^m(\{m\}; \tau_{\{m\}}^*) > V^m(\{m, l\}; \tau)$$

$$V^l(\{m, l\}; \tau) > V^l(\{l\}; \tilde{\tau}_{\{h, m\}}) > V^l(\{l\}; \tau_{\{m\}}^*)$$

$$\text{with } \tau = \frac{nh^h - (n+\beta)h^m}{\beta h}.$$

- A third type of cycle occurs when (i)  $\tau_{\{m, l\}}^* \leq \tilde{\tau}_{\{h, m\}} \leq \tau_{\{l\}}^*$ , (ii)  $\tilde{\tau}_{\{h, m\}} > \tilde{\tau}_{\{m, l\}}$ , (iii)  $V_t^m(\{m\}; \tau_{\{m\}}^*) > V_t^m(\{m, l\}; \tau_{\{m, l\}}^*)$ , and (iv)  $V_t^m(\{h, m\}; \tilde{\tau}_{\{h, m\}}) > V_t^m(\{m\}; \tau_{\{m\}}^*)$  are satisfied simultaneously. In that range of inequality, the preferences of the different income groups are either as displayed in Figure 3 in the text where the most preferred tax rate of the low-income class is  $\tau_{\{m, l\}}^*$ , i.e.,  $\tilde{\tau}_{\{m, l\}} \leq \tau_{\{m, l\}}^*$ , or such that the most preferred tax



rate of the low-income class is  $\tilde{\tau}_{\{m,l\}}$ , i.e.,  $\tilde{\tau}_{\{m,l\}} > \tau_{\{m,l\}}^*$ , everything else being equal. In that case, it can easily be checked that there is no Condorcet winner. For instance, using Lemma 4, when  $\tilde{\tau}_{\{m,l\}} \leq \tau_{\{m,l\}}^*$ , the preference ordering of the different relevant candidates for each income class is as follows,

$$\begin{aligned} V^h(\{h\}; \tau_{\{m\}}^*) &> V^h(\{h\}; \tau_{\{m,l\}}^*) > V^h(\{h, m\}; \tilde{\tau}_{\{h,m\}}) > V^h(\{h, m\}; \tau_{\{l\}}^*) \\ V^m(\{h, m\}; \tilde{\tau}_{\{h,m\}}) &> V^m(\{h, m\}; \tau_{\{l\}}^*) > V^m(\{m\}; \tau_{\{m\}}^*) > V^m(\{m, l\}; \tau_{\{m,l\}}^*) \\ V^l(\{m, l\}; \tau_{\{m,l\}}^*) &> V^l(\{l\}; \tau_{\{l\}}^*) > V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) > V^l(\{l\}; \tau_{\{m\}}^*) \end{aligned}$$

On the other hand, when  $\tilde{\tau}_{\{m,l\}} > \tau_{\{m,l\}}^*$  everything else being equal, the preference ordering is the same as above but the club  $\{m, l\}$  becomes now unfeasible, and the same type of Condorcet paradox occurs where  $V^i(S_{jt}; \tau_{\{m,l\}}^*)$  must be replaced by  $V^i(S_{jt}; \tilde{\tau}_{\{m,l\}})$ .

Let us now consider an *ex ante* pattern of income distribution defined as above where (ii), (iii), and (iv) are satisfied, but where  $\tilde{\tau}_{\{h,m\}} > \tau_{\{l\}}^*$ . Then, the candidate  $\tau_{\{l\}}^*$  which now yields the partition  $(\{h\}; \{m, l\})$  is defeated for each income class by  $\tau_{\{m,l\}}^*$ , when  $\tilde{\tau}_{\{m,l\}} \leq \tau_{\{m,l\}}^*$  and by  $\tilde{\tau}_{\{m,l\}}$  when  $\tau_{\{l\}}^* > \tilde{\tau}_{\{m,l\}} > \tau_{\{m,l\}}^*$ . In both these cases, there are Condorcet paradoxes between  $\tilde{\tau}_{\{h,m\}}$ ,  $\tau_{\{m\}}^*$ , and either  $\tau_{\{m,l\}}^*$  or  $\tilde{\tau}_{\{m,l\}}$  as long as (v)  $V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) > V^l(\{l\}; \tau_{\{m\}}^*)$  is satisfied. Similarly, a Condorcet cycle arises when  $\tilde{\tau}_{\{m,l\}} \geq \tau_{\{l\}}^*$  as long as  $V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) > V^l(\{l\}; \tau_{\{m\}}^*)$ . More specifically, we have

$$\begin{aligned} V^h(\{h\}; \tau_{\{m\}}^*) &> V^h(\{h\}; \tau) > V^h(\{h, m\}; \tilde{\tau}_{\{h,m\}}) \\ V^m(\{h, m\}; \tilde{\tau}_{\{h,m\}}) &> V^m(\{m\}; \tau_{\{m\}}^*) > V^m(\{m, l\}; \tau) \\ V^l(\{m, l\}; \tau) &> V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) > V^l(\{l\}; \tau_{\{m\}}^*) \end{aligned}$$

with  $\tau = \tau_{\{m,l\}}^*$ , when  $\tilde{\tau}_{\{m,l\}} \leq \tau_{\{m,l\}}^*$ ,  $\tau = \tilde{\tau}_{\{m,l\}}$  when  $\tau_{\{l\}}^* > \tilde{\tau}_{\{m,l\}} > \tau_{\{m,l\}}^*$ , and  $\tau = \tilde{\tau}_{\{m,l\}}$ , or  $\tau = \tau_{\{l\}}^*$  when  $\tilde{\tau}_{\{m,l\}} > \tau_{\{l\}}^*$  depending on whether  $V^l(\{m, l\}; \tilde{\tau}_{\{m,l\}}) \geq V^l(\{l\}; \tau_{\{l\}}^*)$ .

Finally, in that range of inequality where (iii), (iv), and (v) are still satisfied simultaneously, but where  $\tilde{\tau}_{\{h,m\}} \leq \tilde{\tau}_{\{m,l\}}$  and  $\tilde{\tau}_{\{h,m\}} > \tau_{\{l\}}^*$ , a Condorcet cycle also occurs between the candidates  $\tau_{\{m\}}^*$ ,  $\tilde{\tau}_{\{h,m\}}$ ,  $\tau_{\{l\}}^*$ . Notice that, everything else being equal, whenever  $V^l(\{l\}; \tilde{\tau}_{\{h,m\}}) \leq V^l(\{l\}; \tau_{\{m\}}^*)$ , we are back to Proposition 3. ■

The discussion is now complete as we considered all the possible rankings of the potential candidates as well as their underlying preference orderings for the three different income classes whatever the *ex ante* initial pattern of income distribution is.

## Appendix B. Construction of Figure 4

This appendix provides some information about the construction of Figure 4. From now on, we denote  $x_t = h_t^h/h_t^m$ ,  $y_t = h_t^m/h_t^l$ ,  $\eta = \frac{n}{\beta}$ , and consider that  $\eta > 6$ .

1. First, notice that the zero-tax rate locus  $\tau_{\{m\}}^* = 0$  is such that  $y_t = \frac{1}{2-x_t}$ . This locus is defined for  $x_t \in [1, 2[$ , upward sloping and convex. The display of the zero-tax rates loci  $\tilde{\tau}_{\{h,m,l\}} = 0$ ,  $\tilde{\tau}_{\{h,m\}} = 0$ ,  $\tilde{\tau}_{\{m,l\}} = 0$  is also straightforward.

Second, by definition, the locus  $\tilde{\tau}_{\{h,m\}} = \tilde{\tau}_{\{m,l\}} > 0$  is given by all pairs  $(x_t, y_t)$  that satisfy the following equality:

$$y_t = \frac{\eta + 1}{2\eta + 1 - \eta x_t}$$

It is defined in  $[1 + \eta^{-1}, 2 + \eta^{-1}[$ , upward sloping and convex.

Finally, given the definition of the different tax rates, a similar reasoning allows us to depict the loci  $\tilde{\tau}_{\{h,m\}} = \tau_{\{l\}}^*$ , and  $\tilde{\tau}_{\{h,m\}} = \tau_{\{m,l\}}^*$  as displayed in Figure 4 and to show that the locus  $\tau_{\{m\}}^* = 0$  is always located above the locus  $\tilde{\tau}_{\{h,m\}} = \tilde{\tau}_{\{m,l\}}$ .

2. First, we are now interested in the loci comparing indifferent welfare levels between different community's structures for our three income classes. We provide information about the Locus *AA* in Figure 4 which characterizes the initial pattern of income inequality such that  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$ .

Notice that this locus belongs to the region of the plane where  $\tilde{\tau}_{\{h,m\}} > 0$ , i.e.  $x_t > 1 + \eta^{-1}$  and  $y_t \geq 1$ .

Given the definitions of  $\tau_{\{m\}}^*$ , and  $\tilde{\tau}_{\{h,m\}}$ , this locus is defined as

$$\frac{n(1 - \tau_{\{m\}}^*)h_t^m + n(\tau_{\{m\}}^* - (\tau_{\{m\}}^*)^2)\bar{h}_t}{n + \beta} = \frac{n(1 - \tilde{\tau}_{\{h,m\}})h_t^h + n(\tilde{\tau}_{\{h,m\}} - (\tilde{\tau}_{\{h,m\}})^2)\bar{h}_t}{n + \beta}$$

When  $\tau_{\{m\}}^* > 0$ , replacing  $\tau_{\{m\}}^*$  and  $\tilde{\tau}_{\{h,m\}}$  by their expressions yields an implicit function  $f(x_t, y_t) = 0$  which

is downward sloping in the plane  $(x_t, y_t)$ . Indeed, notice that  $f(x_t, y_t) = 0$  has two real roots  $x_t'$  and  $x_t''$

$$x_t' = A + By_t^{-1} \text{ or } x_t'' = C + Dy_t^{-1}$$

As the solution  $A + By_t^{-1}$  is defined in our plane where  $\tilde{\tau}_{\{h,m\}} \leq 0$ , we only take into consideration the locus

$$x_t = C + Dy_t^{-1}$$

where  $C = -(11 - 36\eta^2 - 24\eta)^{-1} (36\eta^2 + 54\eta + 30\sqrt{(1+\eta)} + 14)$ ,

and  $D = -(11 - 36\eta^2 - 24\eta)^{-1} (6\eta + 6\sqrt{(1+\eta)} + 5)$ .

We can easily conclude that it is monotonically decreasing and convex. On the other hand, considering initial patterns of income distribution such that  $\tau_m^* = 0$ , i.e.  $h_t^m \geq \bar{h}_t$ , the locus  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$  becomes

$$\frac{nh_t^m}{n + \beta} = \frac{n(1 - \tilde{\tau}_{\{h,m\}})h_t^h + n(\tilde{\tau}_{\{h,m\}} - (\tilde{\tau}_{\{h,m\}})^2)\bar{h}_t}{n + \beta}$$

Replacing  $\tilde{\tau}_{\{h,m\}}$  by its definition, similar straightforward algebra allows us to conclude that this locus is monotonically decreasing and convex in our plane.

Second, recall that we assumed in Lemma 4 that the locus  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$  is always located to the right of the locus  $\tilde{\tau}_{\{h,m\}} = \tau_{\{l\}}^*$  when  $\tau_{\{m\}}^* > 0$ . We now prove this result. On the one hand, both the loci  $\tau_{\{m\}}^* = (1/2)(1 - h_t^m/\bar{h}_t) = 0$  and  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$  intersect where

$$x_t = \left(6\eta^2 + 11\eta + 7\sqrt{(1+\eta)} + 4\right) \left(6\eta^2 + 5\eta + \sqrt{(1+\eta)} - 1\right)^{-1}$$

On the other hand, the locus  $\tau_{\{m\}}^* = (1/2)(1 - h_t^m/\bar{h}_t) = 0$  and the locus  $\tilde{\tau}_{\{h,m\}} = \tau_{\{l\}}^*$  intersect where

$$x_t = \left(\eta + \frac{1}{2}\right) \left(\eta - \frac{1}{2}\right)^{-1}$$

which is smaller than the prior intersection for any  $\eta \geq 1$ . Because  $\tilde{\tau}_{\{h,m\}} = \tau_{\{l\}}^*$ , respectively  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$ , is increasing, respectively decreasing, the locus  $V_t^m(\{m\}; \tau_{\{m\}}^*) = V_t^m(\{h, m\}; \tilde{\tau}_{\{h,m\}})$  is always located to the right of the locus  $\tilde{\tau}_{\{h,m\}} = \tau_{\{l\}}^*$  in our plane as long as the initial pattern of income inequality is such that  $\tau_{\{m\}}^* > 0$ .

**3.** Finally, some tedious but straightforward algebra allows us to depict in Figure 4 the other loci as defined in

Footnote 5 in the text. Further information about the construction of Figure 4 is available upon request.

## Appendix C. Dynamics of Inequality

### Evolving income distribution

(i) When  $(x_t, y_t) \in \text{Region } I$ , we have

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \\ y_{t+1} = (y_t)^{1-\alpha-\beta} \end{cases}$$

(ii) When  $(x_t, y_t) \in \text{Region } II$ , we have the following equations

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \\ y_{t+1} = (y_t)^{1-\alpha-\beta} \left( \frac{x_t + \bar{\tau}_{\{h, m\}} (x_t + y_t^{-1} + 1)/3}{y_t^{-1} + \bar{\tau}_{\{h, m\}} (x_t + y_t^{-1} + 1)/3} \right)^\beta \end{cases}$$

(iii) When  $(x_t, y_t) \in \text{Region } III$ , the dynamics of inequality are described by the following dynamical system

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \left( \frac{x_t + \tau_{\{m\}}^* (x_t + y_t^{-1} + 1)/3}{1 + \tau_{\{m\}}^* (x_t + y_t^{-1} + 1)/3} \right)^\beta \\ y_{t+1} = (y_t)^{1-\alpha-\beta} \left( \frac{1 + \tau_{\{m\}}^* (x_t + y_t^{-1} + 1)/3}{(1/y_t) + \tau_{\{m\}}^* (x_t + y_t^{-1} + 1)/3} \right)^\beta \end{cases}$$

(iv) Finally, when  $(x_t, y_t) \in \text{Region } IV$ , the dynamical system is

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \left( \frac{x_t + \tau (x_t + y_t^{-1} + 1)/3}{\eta + 1} \right)^\beta \left( \frac{2\eta + 1}{1 + 1/y_t + 2\tau (x_t + y_t^{-1} + 1)/3} \right)^\beta \\ y_{t+1} = (y_t)^{1-\alpha-\beta} \end{cases}$$

with  $\tau = \tau_{\{m, l\}}^*$  if  $(nh_t^h - (n + \beta)h_t^m)/\beta\bar{h}_t < \tau_{\{m, l\}}^*$  and  $\tau = (nh_t^h - (n + \beta)h_t^m)/\beta\bar{h}_t$  otherwise.

As a consequence, the global dynamics that describe the evolution of the income distribution are such that,

**Lemma 5** *The sequence  $\{x_t\}_{t=0}^\infty$  is monotonically decreasing in the plane  $(x_t, y_t)$ .*

**Proof.** Recall that  $\alpha + \beta < 1$ , and having in mind that  $\frac{x_t + A}{1 + A} \leq x_t$  for  $A \geq 0$ , it is then straightforward that  $x_{t+1} < x_t$  everywhere in the plane  $(x_t, y_t)$ . ■

The sequence  $\{y_t\}_{t=0}^\infty$  is slightly more intricate.

**Lemma 6** *The global dynamics of  $y_t$  are given by*

(i) if  $(x_t, y_t) \in \text{Region } I, III, \text{ and } IV$ , then  $\frac{y_{t+1}}{y_t} < 1$ .

(ii) if  $(x_t, y_t) \in \text{Region } II$ , then

$$\frac{y_{t+1}}{y_t} \begin{cases} < 1 & \text{if } (x_t, y_t) > (x_t, y(x_t)) \\ = 1 & \text{if } (x_t, y_t) = (x_t, y(x_t)) \\ > 1 & \text{if } (x_t, y_t) < (x_t, y(x_t)) \end{cases}$$

where  $y(x_t)$  is a single-valued function such that  $(x_t, y(x_t)) \in \left\{ (x_t, y_t) \mid \frac{y_{t+1}}{y_t} = 1 \right\}$ .

**Proof.** (i) is straightforward. (ii) Let us focus on the locus  $y(x_t)$  such that  $\frac{y_{t+1}}{y_t} = 1$  in Region *II*.

When  $\tilde{\tau}_{\{h, m\}} > 0$ , it is such that

$$y_t^{\frac{\alpha}{\beta} + 1} = \frac{(\eta + 1)(x - 1)}{\eta x_t + y_t^{-1} - (\eta + 1)}$$

Using the implicit function theorem, it is easily shown that this locus is monotonically decreasing and convex for  $x_t \in [1 + \eta^{-1}, \infty[$ . It corresponds to the locus  $Y'Y'$  in Figure 6 in the text.

When  $\tilde{\tau}_{\{h, m\}} = 0$ , we have the following expression

$$y_t^{\frac{\alpha}{\beta}} = \frac{(\eta + 1)(x_t + 1)}{2\eta + 1}$$

and the corresponding locus denoted  $YY$  in Figure 6 is monotonically increasing and convex for  $\beta > \alpha$ . ■

**Proof of Proposition 6** First, using the definition of  $\tilde{\tau}_{\{h, m\}}$ , the dynamics in Region *II* are the following

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \\ y_{t+1} = (y_t)^{1-\alpha} \left( \frac{\eta+1}{2\eta+1} \right)^\beta (x_t + 1)^\beta \end{cases}$$

Log-linearizing in the neighborhood of the steady state,  $x_\infty = 1$  et  $y_\infty = \left( \frac{2(n+\beta)}{2n+\beta} \right)^\frac{\beta}{\alpha}$ , we obtain two real eigenvalues  $\lambda_1 = (1 - \alpha - \beta)$  and  $\lambda_2 = (1 - \alpha)$ , both smaller than 1. It is then obvious that these dynamics are locally stable within Region *II*. Notice that if and only if  $\frac{\beta}{\alpha} \geq \Pi$ , then the pair  $\left( 1, \left( \frac{2(n+\beta)}{2n+\beta} \right)^\frac{\beta}{\alpha} \right)$  belongs to Region *II* and the global dynamics exhibit multiple steady states.

Second, it follows directly from the dynamics in Region *I* defined above that the integrated steady state is stable. ■

**Proof of Proposition 7** Let us denote  $\gamma^*$  the growth rate at the integrated equilibrium which is equal to

$$\gamma^* = \kappa \left( \frac{3n\beta}{3n + \beta} \right)^\beta$$

While  $\lambda^*$ , the growth rate at the segregated equilibrium can be expressed by

$$\lambda^* = \kappa \left[ \left( \frac{2n\beta}{2n + \beta} \right)^\beta f + \left( \frac{n\beta}{n + \beta} \right)^\beta g \right]$$

with  $f = \frac{2}{3} \left( \frac{3\bar{h}_{S_{jt}}}{2\bar{h}_{S_{jt}} + h_t^i} \right)^{1-\alpha}$ ,  $g = \frac{1}{3} \left( \frac{3h_t^i}{2\bar{h}_{S_{jt}} + h_t^i} \right)^{1-\alpha}$ , and where  $S_{jt} = \{h, m\}$ .

Due to Jensen's inequality, we have  $f + g = \frac{E(h_t^{1-\alpha})}{[E(h_t)]^{1-\alpha}} < 1$ . We can easily conclude that  $\gamma^* > \lambda^*$ . ■

**Proof of Proposition 8** Let us define  $d_t$  the distance between a pair  $(x_t, y_t)$  evolving in Region *II* where  $x_t \leq 1 + \eta^{-1}$  and the frontier between Regions *I* and *II* ( $\tilde{\tau}_{\{h, m, l\}} = 0$ ). It is given by

$$d_t = y_t \left( \frac{2 + \eta^{-1}}{1 + x_t} \right)^{-1}$$

We can rewrite the dynamical system in Region *II* where  $x_t \leq 1 + \eta^{-1}$  in the space  $(x_t, d_t)$

$$\begin{cases} x_{t+1} = (x_t)^{1-\alpha-\beta} \\ d_{t+1} = (d_t)^{1-\alpha} \left( \frac{\eta+1}{2\eta+1} \right)^\beta \left( \frac{1+(x_t)^{1-\alpha-\beta}}{(1+x_t)^{1-\alpha-\beta}} \right) (2 + \eta^{-1})^{-\alpha} \end{cases}$$

Notice that  $d_{t+1}$  is increasing with  $d_t$  and decreasing with  $x_t$ . Thus,  $d_{t+1}$  is minimum for  $d_t = 1$  and  $x_t = 1 + \eta^{-1}$ .

Then,

$$\min \{d_{t+1}\} = (1 + \eta^{-1})^\beta \left( \frac{1 + (1 + \eta^{-1})^{1-\alpha-\beta}}{2 + \eta^{-1}} \right)$$

Notice that if the following condition is satisfied which ensures that the global dynamics exhibit multiple steady states,

$$\frac{\beta}{\alpha} \geq \frac{\ln \left( \frac{2+\eta^{-1}}{1+(1+\eta^{-1})^{(1-\alpha-\beta)}} \right)^{\frac{1}{\alpha}}}{\ln(1 + \eta^{-1})} \equiv \Gamma > \Pi$$

then  $d_{t+1} \geq 1$ . Given the evolving pattern of  $x_t$  in that range of inequality in Region *II* where  $x_t \leq 1 + \eta^{-1}$ , we also have  $d_{t+n} \geq 1$ , whatever  $n \geq 1$ . ■

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