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**Basinwide Water Management:  
A Spatial Model**

Ujjayant Chakravorty and Chieko Umetsu

**NOTA DI LAVORO 45.2000**

# **BASINWIDE WATER MANAGEMENT: A SPATIAL MODEL**

by

Ujjayant Chakravorty and Chieko Umetsu \*

**Proposed Running Head:** Basinwide Water Management

## **ABSTRACT**

This paper develops a spatial model for a water basin that allows for surface water allocation and re-use of the water that is lost. The analytical solution suggests specialization of production over space - upstream farmers use canal water and downstream farmers pump groundwater that is lost upstream. Groundwater emerges as an endogenous “backstop.” The empirical results suggest that when traditional conservation technologies are used, optimization over the entire basin leads to significant increases in aggregate output, project area and water use. Somewhat counter-intuitively, rents from water decrease if farmers switch from traditional to modern irrigation technology.

**Key words:** *Water Allocation, Spatial Models, Conjunctive Use, Irrigation Technology*

**JEL Classification:** *Q25, Q15, R30*

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## **BASINWIDE WATER MANAGEMENT: A SPATIAL MODEL**

### **1. Introduction**

Most water projects are characterized by significant distribution losses. It is estimated that gravity irrigation systems typically lose about 40 percent of their water, while drip and sprinkler systems waste 30 and 10 percent respectively (Seckler, 1996). Most often, however, the water “lost” is captured by users at the bottom of the river basin. It has often been argued that increasing these on-farm efficiencies may only result in “paper” water savings, since their impact on reducing return flows elsewhere in the basin may be significant (Huffaker and Whittlesey, 1997).

Several papers have dealt with the problem of the effect of upstream water diversions by firms on downstream users (e.g., see Griffin and Hsu, 1993; Hartman and Seastone, 1970; Meyers and Posner, 1971). In particular, Johnson, Gisser and Werner (1981) have extended this literature to examine third-party impacts when streamflows are (in)sufficient relative to diversions. They suggest that if water rights are defined as consumptive use (i.e., water diversions net of return flows) market transactions will lead to optimality if streamflows are sufficient relative to diversions at each location. Kanazawa (1991) uses a similar framework to examine declining water quality from consumptive use and notes that appropriative rights may need to be redefined to address the third party effects arising from non-point source pollution.

Another strand of the literature (e.g., Chakravorty, Hochman and Zilberman (1995), henceforth called CHZ) has focused on endogenizing distribution losses by choosing optimal investments in water conveyance and its impact on quasi-rents accruing to water users uniformly located over

space. This literature however, implicitly assumes that the water lost in distribution and on-farm is irretrievable and is lost to the system. In this paper, we address the issue of return flows by extending the above framework to include third-party impacts. Essentially, we include return flows in a CHZ framework. In a brief paper recently, Umetsu and Chakravorty (1998) outlined a CHZ model with return flows. However, their primary focus was to demonstrate the empirical solution and they do not go beyond outlining the necessary conditions of the basic spatial model with return flows. The present paper adopts the Umetsu and Chakravorty framework and substantially extends the analyses through the rigorous development and characterization of the analytical solution. In terms of the empirical analysis, the present paper focuses on the role of irrigation efficiency in the form of traditional and modern technologies on return flows and shows that under traditional technology, basinwide optimization leads to significant increases in aggregate output and project area.

To keep the model analytically tractable, we use a stylized one-dimensional model where all water users are assumed to lie along a straight line and a fixed proportion of return flows are available to water users for pumping. We show that if return flows replenish the groundwater table, then the optimal solution suggests specialization: upstream firms draw surface water while those located downstream use groundwater. Because of conveyance costs, water prices increase with distance and are bounded above by the price of groundwater. Thus groundwater emerges as an endogenous “backstop” technology.

The theory is illustrated with data from Western U.S. agriculture. We compare two models: one

where both surface and groundwater use are jointly maximized over the entire basin; and another where surface water allocation and return flows are optimized separately. The comparisons suggest some interesting conclusions. For instance, if modern technology is used on-farm, there are no significant benefits to optimizing water allocation over the entire river basin. However, under traditional technology, basinwide optimization leads to significant increases in aggregate output and project area. If property rights to water diversions are held on the basis of consumptive use and these rights are fully transferable, we show that surface water rights holders enjoy significantly higher quasi-rents relative to groundwater users. On the other hand, use of efficient irrigation technology on the farm decreases rents accruing to surface and groundwater users, although the effect of modern technology adoption is a substantial increase in the area serviced by the system and an increase in aggregate economic benefits.

These results have several policy implications. They suggest that in regions where farmers already use modern technology, the benefits from basinwide water allocation may be small. In this case, it would be a reasonable approximation to optimize over surface water use and let the groundwater be recovered downstream. However, when on-farm technological efficiencies are low, optimization over the entire basin may lead to an expansion of project area upstream, since that implies more water will be available to downstream locations. Explicit consideration of the positive externalities of return flows allows more water to be generated at the source, and aggregate output is increased substantially. If surface water users are charged prices net of the value of the return flow, water prices decrease in the project leading to increased quasi-rents from surface water. Thus, upstream farmers “lose” from increased water charges in the expanded system while other users benefit from

being “connected” to the grid.

The proposed basinwide model provides a useful methodology for the analysis of water allocation, technological change on-farm as well as in conveyance, and the distribution of water rents in the entire hydrological system. The spatial shadow prices could be interpreted as permit prices if water markets were introduced. Issues relating to basin dynamics could be explored, although not attempted here. For instance, the model could be run for multiple periods with uncertainty in initial water supplies and the resulting spatial-dynamic pattern of water use could be examined. Although we only consider the use of return flows in agriculture, urban and nonagricultural uses such as recreation, fish and wildlife habitat and hydropower generation can be included if their demand parameters are known. For instance, if the return flows were to accumulate in a wetland system with spatially variable effects from water quality, the resulting analysis could be performed with suitable modification of the framework proposed in this paper. The tail end of river basins is often characterized by a high urban demand for water return flows – over 20 percent of the world’s population lives in coastal areas (WRI, 1994) – as well as for agricultural and environmental uses, as in the San Francisco Bay and Delta. The proposed model could be used to determine optimal allocation of fresh water and return flows among competing sectors.

Section 2 develops the spatial model with conjunctive use and characterizes the solution. A comparison is made to a model where the benefits from return flows are not explicitly incorporated. Section 3 provides an illustration with secondary data. Section 4 concludes the paper.

## 2. The Model

The model used here is similar in spirit to that in CHZ except that we have surface and groundwater extraction so that technically, we have an additional state and control variable. To ensure tractability, we assume that the level of on-farm technology used is given, which may be reasonable to expect within the same river basin. This issue is an important one and is discussed further below. The model is kept simple by abstracting from uncertainty and dynamics. Water is supplied by a central planner (e.g., a water utility or manager) to farms located on both sides along a straight line canal, as shown in the layout in Figure 1. Our results translate readily to a situation where distribution canals are located on a grid, provided we retain the assumption that access to groundwater flows is independent of a user's location. However in that case, the spatial distribution of prices and water use may become somewhat less transparent.

Farms are assumed to lie in a continuum and are homogenous in all respects (e.g., use of inputs other than water, land quality) except their location from the water source, denoted by  $x$ . Only one crop is assumed to be grown in this project, although an extension to an index of crops is straightforward. Part of the water flowing through the canal is lost through seepage and percolation and is collected as recharge in the groundwater aquifer. The amount lost can be reduced by investing in conveyance (i.e., lining) while the recharge can be pumped up by individual farmers for irrigation. For ease of exposition, we assume that the lost water collects in the groundwater aquifer lying below the project and can be pumped up anywhere in the system. However, in reality, the water could seep into another aquifer remote from the system, and as the solution below makes clear, the analysis in this paper will still hold without any change. In fact this latter representation



may be more typical in real-world water projects where third-party impacts often occur remote from the project area. We further abstract from water quality differentials between freshwater and return flows. As is common knowledge, return flows are likely to be lower quality because of salt and other chemical contamination. In essence, we are implicitly assuming that this quality difference can be ignored in farming. If return flows had recreational or amenity values or was used for drinking purposes, this would not be the case. Similarly, possible time delays between freshwater irrigation and pumping the percolated water are not considered in this static model.

Let the subscripts  $s$  and  $g$  denote surface and ground water respectively. Then the aggregate stock of surface water is given by  $z_s(0)$ . Let  $g(z_s(0))$  be the total cost of generating water for the project at the source. This may include annualized capital costs and the cost of operation and maintenance of the headworks. The cost of water generation is assumed to be an increasing, twice differentiable and convex function with  $g'(z_s(0)) > 0$  and  $g''(z_s(0)) > 0$ . Let the water flowing in the canal at any location  $x$  be denoted by  $z_s(x)$ . It is decreasing because of conveyance losses and withdrawals at each location. Let the fraction of water lost at any location be represented by  $a(x)$  with  $a(x) \geq 0$ . The volume of surface water used per unit land area at any location is  $q_s(x)$ . Then the change in the residual stock of water is given by the following differential equation:

$$z_s'(x) = -q_s(x)\alpha - a(x)z_s(x) \quad (1)$$

where  $\alpha$  is the width of the irrigated area and is assumed to be constant. The first and second terms on the left-hand side of (1) represents the water applied and lost at any location 'x'.

As in CHZ, the fraction of water lost,  $a(x)$ , depends on  $k(x)$  - conveyance expenditure per unit surface area of the canal, which may vary with location  $x$ . This relationship is given by

$$a(x) = a_0 - m(k(x)), \quad a_0 \in [0,1], \quad m \in [0,a_0), \quad a(x) \in (0,1] \quad (2)$$

where  $a_0$  is the constant base loss rate that would result from no investment in canal lining, i.e.,  $k=0$ . With non-zero investment, the base loss  $a_0$  is reduced by the factor  $m(k(x))$ , where  $m(k(x))$  is assumed to be an increasing, twice differentiable function and exhibits decreasing returns to scale with respect to  $k$ , i.e.,  $m(k) \in [0,a_0)$ ;  $m'(k) > 0$ ;  $m''(k) < 0$ ;  $\lim_{k \rightarrow \infty} m(k) = a_0$ ;  $\lim_{k \rightarrow 0} m'(k) = \infty$ . Thus, investment in conveyance will decrease the water loss rate, hence  $a(x) \leq a_0$ . If no investment in conveyance expenditures is made, i.e.,  $k(x)=0$ , then the fraction of water lost is the same as the base loss rate,  $a_0$ . Since  $m(k)$  approaches the base loss rate only at the limit, the fraction of water lost is zero in the limit. The limit on the derivative of  $m(k)$  ensures that conveyance expenditures are always non-zero.

Let  $q_g(x)$  be the volume of groundwater applied per unit area at any location  $x$ . Then the total volume of water applied on unit land area at any location is  $q_s(x) + q_g(x)$ . To keep the model simple, we assume that the marginal pumping cost denoted here by  $w$ , does not depend on the quantity of water pumped at any location. However, pumping costs could be sensitive to the depth of water in the aquifer, i.e., the vertical distance between the land surface and the water table. Less water in the aquifer may increase the pumping depth and require more energy per unit of pumped volume. In

this situation the pumping cost may be written as  $w(z_g(0))$  with  $w' < 0$ . However such a formulation does not change the solution below in any way except that it produces an endogenous marginal pumping cost  $w$  that is faced uniformly by all locations. Moreover, we assume instantaneous spatial leveling of the water table, which is standard in the literature.

Losses in carrying water from the farm-gate to the root zone of the plant are given by an increasing, concave function  $h(I)$  where  $I$  is investment in on-farm technology (e.g., sprinkler or drip irrigation). We assume that if  $I=0$ , farmers choose furrow irrigation, for which  $h(0)=0.6$ , i.e., 60 percent of the water withdrawn ( $q_s+q_g$ ) is taken by the plant. The remaining 40 percent is assumed to be lost. However, if investment is made in improved on-farm technology such as sprinkler or drip technology,  $h(I)$  may approach unity. On-farm technology is assumed to be fixed over space, i.e.,  $h(I)$  is a constant. Endogenizing firm-level technology choice may provide for differential levels of technology adoption over space. However, this is a reasonable approximation, as demonstrated empirically by Umetsu and Chakravorty (1998). Conveyance investments reduce water losses, thereby reducing spatial differences in water prices and on-farm technology choice. Near-constant spatial prices imply similar levels of investment in on-farm technology over space. Even though technology is held constant over space, the *level* of technology and hence on-farm efficiency is important, as we will see in section 3. There we will compare resource allocation under alternative assumptions of a low (e.g., furrow) and a high (sprinkler or drip) level of on-farm technology.

The actual amount of water  $e$  used by the crop, often called "effective water" (see Caswell and

Zilberman (1986)) is obtained as  $e(x)=(q_s(x)+q_g(x))h$ . The crop production function is defined as  $y = f(e(x))$  where  $y$  is output per unit land area. The production function is assumed to be increasing, twice differentiable and exhibits decreasing returns to scale with respect to effective water, which together imply that  $f(e) > 0$ ;  $f'(e) > 0$ ;  $f''(e) < 0$ .

We assume that a proportion,  $\beta$  of the water lost from the system (canal and individual fields) accumulates as return flow in the groundwater aquifer and is available for pumping. We abstract from considering natural recharge and it is assumed that the water flowing into the groundwater table is available for pumping instantaneously. Thus there is no change in the level of the aquifer over time. The model considered here is static and can be thought of as the steady-state of a dynamic system. In a dynamic framework not considered here for tractability reasons, the level of the water table may be important. For instance, if the water table is high, then percolated water may be recovered immediately and at low cost, while if the level was low, then there may be a long time lag (up to several years) before water could be recovered possibly at a higher cost. If  $\beta=1$ , all the water lost is available as return flow. If  $\beta=0$ , there is no return flow in the system. We avoid these polar cases by assuming that  $0 < \beta < 1$ .

Denote  $z_g(x)$  as the amount of water available for pumping at any location  $x$  with  $z_g(0)=0$ . Then  $z_g'(x)$ , the change in the groundwater stock is the sum of the recharge from the canal  $\beta a(x)z_g(x)$  and from the field,  $\beta(1-h)(q_s(x)+q_g(x))\alpha$  net of the extraction of groundwater,  $q_g(x)\alpha$ , and can be written as follows:

$$z_g'(x) = \beta a(x)z_s(x) - q_g(x)\alpha + \beta(1-h)(q_s(x)+q_g(x))\alpha. \quad (3)$$

Notice that the above equation allows the inflow and outflow from the groundwater stock to be measured. It does not mean that the stock of groundwater is actually changing over distance since we assume that all additions and withdrawals happen simultaneously.

### *The Optimization Problem*

Let  $p$  be the constant output price of the crop and  $F$  the fixed cost of farming per unit area. This may include the non-irrigation costs of production, equipment, and set-up of an irrigation system, and may vary depending on the technology in use, as well as land. For simplicity, fixed costs are taken as constant. The optimization is performed in two stages: first we assume that given a fixed value of on-farm technology  $I$ , the social planner chooses  $q_s(x)$ ,  $q_g(x)$ ,  $k(x)$ , and  $X$ , the boundary of the project area, to maximize net benefits from the water project given a fixed stock of water,  $z_s(0)$ .

Next we optimize over the value of  $z_s(0)$ . The first stage is written as follows:

$$\text{maximize } NB(z_s(0)) = \int_0^X \{ [pf](q_s+q_g)h - I - F - wq_g \} \alpha - k \} dx \quad (4)$$

$q_s(x), q_g(x), k(x), X$

subject to state constraints (1) and (3). Define  $X^*$  as the optimal length of the project area. Let  $\lambda_s(x)$  and  $\lambda_g(x)$  be co-state variables attached to equations (1) and (3) respectively. With  $z_s(x)$  and  $z_g(x)$  as state variables, and  $q_s(x)$ ,  $q_g(x)$ , and  $k(x)$  as control variables, the Hamiltonian can be written as:

$$H(q_s, q_g, k, \lambda_s, \lambda_g) = [pf[(q_s+q_g)h] - I - F - wq_g]\alpha - k - \lambda_s[q_s\alpha + az_s] + \lambda_g[\beta az_s - q_g\alpha + \beta(1-h)(q_s+q_g)\alpha]. \quad (5)$$

Let \* denote the optimal values for this problem. Assume that the Hamiltonian is concave in  $q_s, q_g, k, z_s, z_g$  and that the sufficiency conditions are met. Then the necessary conditions for optimality are given as follows:

$$pf' h \leq \lambda_s - \lambda_g \beta(1-h) \quad (= \text{if } q_s > 0) \quad (6)$$

$$pf' h \leq w + \lambda_g[1-\beta(1-h)] \quad (= \text{if } q_g > 0) \quad (7)$$

$$(\lambda_s - \beta\lambda_g)z_s m'(k) \leq 1 \quad (= \text{if } k > 0) \quad (8)$$

$$\lambda_s'(x) = a(\lambda_s - \beta\lambda_g) \quad (9)$$

$$\lambda_g'(x) = 0 \quad (10)$$

where  $\lambda_s'(x)$  and  $\lambda_g'(x)$  indicate total derivatives. Since this is a free terminal point problem with the only restriction that the residual stocks of surface and groundwater be non-negative, the transversality condition states that the Hamiltonian at the terminal point  $X^*$  is zero, and the shadow prices  $\lambda_s(X^*)$  and  $\lambda_g(X^*)$  must satisfy the following constraints:

$$[H(q_s, q_g, k, \lambda_s, \lambda_g)]_{x=X^*} = 0.$$

$$\lambda_s(x) \geq 0, \lambda_g(x) \geq 0, \lambda_s(X^*)(z_s(X^*)) = \lambda_g(X^*)(z_g(X^*)) = 0. \quad (11)$$

Interpretation of the necessary conditions is straightforward. In (6), the left hand side represents the

marginal value product of surface water which is less than or equal to its net marginal cost on the right. The net marginal cost consists of the shadow price of surface water net of the term  $\lambda_g \beta(1-h)$  which represents the positive externality effects of leakage -- the shadow price of groundwater times the fraction that seeps down into the aquifer and is available for pumping. Thus the marginal cost of surface water is less than or equal to the sum of the marginal benefit from crop production and from return flows.

Equation (7) can be rewritten as:

$$pf \cdot h \leq w + \lambda_g - \lambda_g \beta(1-h) \quad (= \text{if } q_g > 0) \quad (12)$$

which suggests on comparison with (6) that the marginal value product of groundwater is less than or equal to the sum of the shadow price of groundwater and the unit pumping cost net of the externality effect, since a fraction of the loss from groundwater pumping ends up in the aquifer and is available as return flow. Thus under both surface water and groundwater use, the price of water that may be charged by the utility which is the true social cost (given by the right hand side of (6) and (7)) is less than the private cost of supplying water at that location, given by  $\lambda_s$  and  $\lambda_g + w$  respectively.

Equation (8) equates the marginal value of water saved (net of the groundwater recharge) to its marginal cost, the unit cost of conveyance. Equation (9) gives the change of shadow price of water with location. It is interesting to note that with return flows, the shadow price increases over space

at a rate slower than the pure cost of water conveyance, given by  $a\lambda_s$ . Thus, if return flows were not explicitly considered in determining water prices at each location, they would be higher than optimal. Not only will shadow prices increase at a faster rate and hence would be higher at each location, the externality terms in (6) and (7) will be ignored. This in turn would imply lower than optimal water use by farmers and lower return flows. Finally, (10) indicates that the shadow price of groundwater is constant over space. This is a result of our simplifying assumption that the water in the aquifer can be pumped up at constant unit cost from any location in the basin.

In the second stage of the optimization process, let us define the optimal net benefit function given the initial stock of surface water  $z_s(0)$  as  $NB^*(z_s(0))$ . The optimal initial stock of water,  $z_s^*(0)$ , is obtained as follows:

$$\underset{z_s(0)}{\text{maximize}} NB^*(z_s(0)) - g(z_s(0)) \quad (13)$$

from which we obtain the necessary condition,  $NB^*(z_s^*(0)) = g'(z_s^*(0))$ . From the first stage, partial differentiation of  $NB^*$  with respect to the given optimal initial stock of surface water  $z_s^*(0)$  gives:

$$\partial NB^* / \partial z_s^*(0) = \lambda_s^*(0). \quad (14)$$

Equation (14) also yields:

$$\lambda_s^*(0) = g'(z_s^*(0)). \quad (15)$$



That is, the shadow price of surface water at the source must equal the marginal cost of generating water at the optimal capacity.

Using the necessary conditions, we can proceed to obtain the following results that determine resource allocation in the project:

*Lemma 1: There exists no non-zero interval  $I = [X_1, X_2] \subset [0, X^*]$  such that  $q_s(x) > 0, q_g(x) > 0 \forall x \in I$ .*

*Proof:* Suppose there exists such a non-zero interval  $I$ . Then both (6) and (7) must hold with equality, so that  $\lambda_s - \lambda_g \beta(1-h) = w + \lambda_g[1-\beta(1-h)] \forall x \in [X_1, X_2]$ . On simplification, we obtain  $\lambda_s(x) = w + \lambda_g$ , which implies that  $\lambda_s > \lambda_g$  since  $w > 0$ . Since this equality holds at each  $x \in [X_1, X_2]$ , we can differentiate it to obtain  $\lambda_s'(x) = 0$  since  $\lambda_g$  is constant. Then from (9),  $\lambda_s'(x) = a(\lambda_s - \beta\lambda_g) = 0$  which gives  $\lambda_s = \beta\lambda_g$  because  $a > 0$ . Since  $0 < \beta < 1$ , the last equality gives  $\lambda_s < \lambda_g$ . This contradicts the previous inequality  $\lambda_s > \lambda_g$ , so our initial hypothesis that both  $q_s$  and  $q_g$  are strictly positive on  $I$  is false. ■

Define  $p_s = \lambda_s - \lambda_g \beta(1-h)$  and  $p_g = w + \lambda_g[1-\beta(1-h)]$ . Intuitively, these are the location-specific “net” prices of surface water and groundwater respectively. We make the further reasonable assumption that  $0 < w < pf'h \big|_{q_s+q_g=0}$  that is, pumping costs are positive and less than the maximum (“choke”) price farmers are willing to pay for water. If pumping costs were higher than this choke price, there would be no groundwater pumping and we would revert back to the strictly surface water allocation model of CHZ. Then

*Lemma 2:*  $z_g(X^*) = z_s(X^*) = 0$ .

*Proof:* If  $z_g(X^*) \neq 0$ , then by (11),  $\lambda_g(X^*) = 0$ . For arbitrarily small  $\epsilon > 0$ , at  $x = X^* + \epsilon$ ,  $q_g(x) = 0$ ,  $q_s(x) = 0$  and  $\lambda_g(x) = 0$  by (10). Then by (7),  $pfh|_{q_s+q_g=0} \leq w + \lambda_g[1 - \beta(1-h)] = w < pfh|_{q_s+q_g=0}$  by assumption,  $\therefore z_g(X^*) = 0$  and by corollary,  $\lambda_g(X^*) > 0$  which from (10) suggests that  $\lambda_g(x)$  is a positive constant.

If  $z_s(X^*) \neq 0$ ,  $\lambda_s(X^*) = 0$ . Since  $\lambda_s(0) = C'(z_0) > 0$ , and  $\lambda_s'(x) = a(\lambda_s - \beta\lambda_g)$ , with  $\beta\lambda_g$  a constant by (10),  $\lambda_s'(x) \geq (<) 0$  iff  $\lambda_s(0) \geq (<) \beta\lambda_g \forall x \in [0, X^*]$ . But  $\lambda_s(0) > 0$ ,  $\lambda_s(X^*) = 0$  implies that  $\lambda_s'(x) < 0$ . Since  $\lambda_s(x)$  is continuous by (9),  $\exists \delta > 0$ ,  $\epsilon > 0$ , such that for  $\forall x$  for which  $|X^* - x| < \delta$ ,  $0 < \lambda_s(x) < \epsilon < \lambda_g\beta(1-h)$ , the last term being a constant greater than zero. By (6) and the last inequality,  $pfh \leq \lambda_s - \lambda_g\beta(1-h) < 0$  which is a contradiction since the left most term must be non-negative.  $\therefore z_s(X^*) = 0$  and  $\lambda_s(X^*) > 0$ .

■

The above results imply that both surface and groundwater stocks must ultimately be exhausted.

*Lemma 3:*  $\lambda_s(x)$  is monotone increasing and convex, i.e.,  $\lambda_s'(x) > 0$ ,  $\lambda_s''(x) > 0$ .

*Proof:* Differentiation of (9) yields  $\lambda_s''(x) = a\lambda_s' = a^2(\lambda_s - \beta\lambda_g)$ . Thus  $sgn(\lambda_s') = sgn(\lambda_s'') = sgn(\lambda_s - \beta\lambda_g)$  which implies that  $\lambda_s(x)$  is monotone increasing, constant or decreasing depending on the difference  $(\lambda_s - \beta\lambda_g)$ . Suppose that  $p_s'(x) \leq 0$ . Then by definition, since  $\lambda_g(x)$  is a constant, this implies that  $\lambda_s'(x) \leq 0 \forall x \in [0, X^*]$ . By (9), this gives  $\lambda_s(x) \leq \beta\lambda_g < \lambda_g < w + \lambda_g$  so that by subtracting terms,  $\lambda_s - \lambda_g\beta(1-h) < w + \lambda_g - \beta\lambda_g(1-h)$  which gives  $p_s < p_g \forall x \in [0, X^*]$ . From (6) and (7),  $pfh \leq p_s < p_g \forall x \in [0, X^*]$  hence  $q_g(x) = 0$ . Thus groundwater will never be used, which violates Lemma 2. So  $p_s'(x) > 0$  and hence  $\lambda_s'(x) > 0 \forall x \in [0, X^*]$ . ■

A corollary to the above lemma is that  $\lambda_s(x) > \beta\lambda_g \forall x \in [0, X^*]$  and  $p_s(x)$  is monotone increasing. However, the relative magnitude of  $\beta\lambda_g$  and  $p_g$  is indeterminate and will depend on parameter values. Finally we have:

*Lemma 4:*  $\exists X_c^* \in [0, X^*]$  such that  $q_s(x) > 0 \forall x \in [0, X_c^*]$  and  $q_g(x) > 0 \forall x \in [X_c^*, X^*]$ .

*Proof:* Define  $X_c^* = \{x \mid \lambda_s(x) = w + \lambda_g\}$ . If  $\lambda_s(0) \geq w + \lambda_g$  then by lemma 3,  $\lambda_s(x) \geq w + \lambda_g \forall x \in [0, X^*]$  so that  $\lambda_s - \lambda_g \beta(1-h) \geq w + \lambda_g - \beta\lambda_g(1-h)$  which implies that  $p_s \geq p_g$  and surface water will never be used except at a location with measure zero ( $x=0$ ), violating Lemma 2. Hence  $\lambda_s(0) < w + \lambda_g$ . By Lemma 3,  $\lambda_s(x)$  is increasing so that by definition,  $\exists X_c^*$  such that  $\lambda_s(x) < w + \lambda_g$  and  $p_s(x) < p_g \forall x \in [0, X_c^*]$ . This implies by (6) and (7) that  $pf^h \leq p_s(x) < p_g$  so that  $q_g = 0$  and only surface water is used. A similar argument shows that  $\forall x \in [X_c^*, X^*]$ ,  $\lambda_s(x) > w + \lambda_g$ ,  $p_s(x) > p_g(x)$ ,  $q_s(x) = 0$  and only groundwater is used. ■

We have thus proved the following proposition which suggests that there is specialization in production, i.e., upstream farmers use surface water and downstream farmers use groundwater for production:

*Proposition 1:* There exists  $X_c \in [0, X^*]$  such that all firms located on  $[0, X_c]$  use surface water, while those located on  $[X_c, X^*]$  use groundwater.

Figure 2 shows the price paths for surface water,  $p_s$  and groundwater,  $p_g$  respectively. The shadow price of surface water  $\lambda_s$  at source is equal to the marginal cost of water generation,  $C'(z_0)$  and

increases exponentially with distance as shown. The “real” price of surface water  $p_s$  at any location is  $\lambda_s - \lambda_g \beta(1-h)$  which lies at a constant distance below  $\lambda_s$ . Similarly, the real price of groundwater is less than the shadow price of groundwater  $\lambda_g$  by a constant. Between  $0$  and  $X_c^*$ ,  $p_s < p_g$  hence only surface water is used for production. However, with increasing distance, transporting surface water becomes more expensive and groundwater becomes cheaper beyond  $X_c^*$ , and is used exclusively until the project boundary.  $X_c^*$  is the optimal length of the canal. Intuitively, since production with surface water entails conveyance losses, it is best done close to the water source in the upstream reaches of the canal. Similarly, as production with groundwater is independent of location, it is optimally located downstream of the canal. Thus groundwater emerges as an endogenous "backstop" technology in this simplified framework.

Finally, resource allocation within the two distinct regions is obtained as follows:

*Proposition 2:* (i)  $k'(x) < 0$ ,  $q_s'(x) < 0 \quad \forall x \in [0, X^*]$ ; (ii)  $q_g(x)$  is constant  $\forall x \in [X_c^*, X^*]$ .

Proof: Note that  $k(x) > 0 \quad \forall x \in [0, X_c^*]$  since LHS of (8) is strictly positive and  $\lim_{k \rightarrow 0} m'(k) = \infty$ . Differentiating (8) which now holds with equality gives  $\lambda_s z_s m' + (\lambda_s - \beta \lambda_g) z_s' m'(k) + (\lambda_s - \beta \lambda_g) z_s m''(k) k' = 0$ . Using (1) and (9), we get  $a(\lambda_s - \beta \lambda_g) z_s' m' + (\lambda_s - \beta \lambda_g)(-q_s \alpha - a z_s) m'(k) + (\lambda_s - \beta \lambda_g) z_s m''(k) k' = 0$  which on cancelling terms gives  $z_s m''(k) k' = (\lambda_s - \beta \lambda_g) q_s \alpha m'(k)$  and we obtain  $k' < 0$  since  $m'(k) > 0$ ,  $m''(k) < 0$ . Next, differentiating (6) which holds with equality in  $[0, X_c^*]$ , gives

$pf''h^2 q_s'(x) = \lambda_s'(x)$  which by Lemma 3 implies  $q_s'(x) < 0$ .

(ii)  $\forall x \in [X_c^*, X^*]$ ,  $pf'h = w + \lambda_g[1 - \beta(1-h)]$  so that by differentiation using (10),  $pf''h^2 q_g'(x) = 0$  implies  $q_g'(x) = 0$  and  $q_g$  constant. ■

Denote quasi-rents to land as  $R_L$ , which accrues to firms at location  $x$  as follows:

$$R_L(x) = \{pf[(q_s(x)+q_g(x))h] - [\lambda_s(x)-\lambda_g \beta(1-h)]q_s(x) - (w+\lambda_g(x)[1-\beta(1-h)])q_g(x) - I - F\} \alpha \quad (16)$$

Land rents decrease with distance until  $X_c^*$ , since the price of water increases and output, which depends on the amount of water withdrawn  $q_s(x)$ , also decreases. Both rents and output are constant in the region using groundwater beyond  $X_c^*$ .

It is helpful to compare water allocation in the conjunctive use model above with the more restrictive surface water model of CHZ in which return flows cannot be reused, although the solution is somewhat ambiguous and depends on parameter values. Since the surface water model can be derived as a special case of the conjunctive use model by restricting  $q_g=0$ , by Le Chatelier's Principle the shadow price of water at the source in the latter model must be greater than in the former, as shown in Fig. 3(a), where superscripts '\*' and 's' denote the conjunctive use (solid lines) and surface water (dashed lines) models respectively. This also implies that aggregate water use under conjunctive use will be higher, since water is used more efficiently (see equation (15)). Thus in the upstream reaches of the canal, shadow prices of water are higher under conjunctive case. However, it is not clear if the “net” shadow prices faced by the upstream firms is higher or lower than for surface water, i.e., the relative values of  $p_s^*=\lambda_s-\lambda_g\beta(1-h)$  and  $\lambda_s^s$ . There is a trade-off between the increased scarcity value of water under conjunctive use and the value of the externality from water reuse that drives down the real price of water in the upstream region. In Fig.3 we have

drawn the case in which  $\lambda_s^s(0)$  is greater than  $p_s(0)$ . This seems to be the more pervasive case, as suggested by our simulation with data from California agriculture (see section 3).

Furthermore, under no return flows, there is likely to be greater investment in conveyance in the surface water model resulting in lower distribution losses (see equation (8)). One policy implication of this analysis is that if the economic value of the lost irrigation water is not taken into account, overinvestment in distribution facilities may occur. It is however unclear if the surface water shadow price will grow slower than in the conjunctive use model, since that depends on the relative magnitudes of the conveyance loss rate and the shadow price of groundwater (see equation (9)). For example, if the seepage rate  $\beta$  is low and close to zero, the shadow price in the conjunctive use model will rise faster than in the surface water case which in an extreme situation may cause the canal length in the conjunctive use model  $X_c^*$  to be smaller than that in the surface water model,  $X_c^s$ . On the other hand, if  $\beta$  is relatively high, the shadow price in the conjunctive use model will rise slower than that of the surface water model. It may then be possible for the canal length under conjunctive use to be greater than that of the latter, as shown in the figure. Intuitively, if the recharge rate  $\beta$  is high, it makes sense to use higher amounts of surface water since the benefits from reuse will be correspondingly larger. Since more water will be available for groundwater pumping, more land will be under groundwater irrigation. Because of a larger aggregate stock of water, the total length of the project under conjunctive use is likely to be larger than under pure surface water allocation.

Relative water use at each location is indeterminate and depends on the relative values of  $p_s^*$  and

$\lambda_s^s$ . One plausible solution is shown in Fig. 3(b). The conjunctive model uses a higher aggregate water stock and distributes it through a longer canal. Although locational shadow prices are higher under conjunctive use, water prices faced by firms is lower because of the positive externality of groundwater use so that actual water withdrawals are higher at each location leading to a higher output, as shown in Fig. 3(c).

### 3. Simulation

In this section we apply the above model to California cotton production by developing a simulation model that approximates the above continuous framework over discrete space. The parameters of the model are similar to that in CHZ except for those relating to seepage losses and groundwater extraction. The production function for cotton is defined as a quadratic function of effective water. It yields a maximum of 1,500 *lbs.* with 3.0 acre-feet of effective water and 1,200 *lbs.* with 2.0 acre-feet of effective water. The revenue function in US\$ is given by the following function:

$$Pf(e) = - 0.2224 + 1.0944 \cdot e - 0.5984 \cdot e^2 \quad (17)$$

where output price of cotton is taken as US\$0.75 per lb. and the effective water,  $e$ , is in  $m^3$ . Partial differentiation of equation (17) with respect to  $e$  provides the value of marginal product function:

$$Pf'(e) = 1.0944 - 1.1968 \cdot e. \quad (18)$$

On-farm water conservation at each location is a function of on-farm investment. The conservation function is estimated from available data on expenditures in irrigation technologies in California (University of California, 1988). It is constructed such that we assume that in the case of traditional furrow irrigation, no investment is made, i.e.,  $I=0$  and the proportion of water delivered to the field for use in production,  $h(0)$  is 0.6. Increasing the level of on-farm investment increases the efficiency closer to unity. The function is defined as follows:

$$h(I) = 0.6 + 21.67 \cdot I - 333.3 \cdot I^2 \quad (19)$$

where  $\partial h/\partial I > 0$  and  $\partial^2 h/\partial I^2 < 0$  and  $I$  is in US\$/m<sup>2</sup>. In this study,  $I$  was fixed at zero for traditional technology and US\$0.02/m<sup>2</sup> for modern technology, which give water efficiencies of 0.6 and 0.9, respectively. The modern technology averages the efficiency for sprinkler irrigation ( $h=0.85$ ) and drip irrigation ( $h=0.95$ ) as in Caswell and Zilberman (1986). Irrigated farming also requires fixed costs of US\$0.107/m<sup>2</sup> (US\$433/acre), assumed to be constant regardless of the level of on-farm technology (University of California, 1988). In reality, fixed costs may vary according to the type of technology adopted. Zilberman, MacDougall, and Shah (1994) provide varying fixed costs of irrigation technology for California cotton production. According to their estimates, fixed costs increase from US\$0.124/m<sup>2</sup> (US\$500/acre) for furrow irrigation to US\$0.156/m<sup>2</sup> (US\$633/acre) for drip irrigation.

The water loss function is a quadratic function of conveyance expenditure that was constructed from data on average lining and piping costs in 17 states in the western United States. For example,



a piped canal with a conveyance expenditure of US\$200/m length of canal is assumed to result in zero conveyance loss. An investment of US\$100/m in concrete lining gives a total loss rate of  $10^{-5}/\text{m}$  with a loss rate of 0.1 for a 10-km length of the canal, or a conveyance efficiency of 0.9 per 10 km. If there is no conveyance expenditure, no reduction in loss is achieved. Then the loss rate is equal to the base loss,  $4 \times 10^{-5}/\text{m}$ , which is equivalent to a loss rate of 0.4 per 10 km or a conveyance efficiency of 0.6 per 10 km. The water loss function at each location  $x$  is defined as:

$$a(x) = 4 \cdot 10^{-5} - (4 \cdot 10^{-7}k - 10^{-9}k^2) \quad (20)$$

where base loss  $a_0 = 4 \times 10^{-5}$ , and the loss reduction function is defined as:

$$m(k) = 4 \cdot 10^{-7}k - 10^{-9}k^2, \quad 0 \leq k \leq 200 \quad (21)$$

which is increasing at a decreasing rate with respect to conveyance expenditure  $k$ , i.e.,  $\partial m / \partial k \geq 0$ ;  $\partial^2 m / \partial k^2 \leq 0$ .

The long-run marginal cost function for water supply was estimated from the average cost of water supply in 18 irrigation projects in the western United States and is given as:

$$g'(z_s(0)) = 0.003785 + (3.785 \times 10^{-11} z_s(0)) \quad (22)$$

where marginal cost is in US\$ and the initial stock of water  $z_s(0)$  is in  $\text{m}^3$ .

For convenience, the width of the project area  $\alpha$  is assumed to be  $10^5$ m, although our results are independent of project width. For the seepage rate  $\beta$ , Gisser and Sanchez (1980) use a return flow coefficient of 0.27 for the Pecos River Basin in New Mexico. Kim, Moore, and Hanchar (1989) use 0.20 for irrigated production in the Texas High Plains. In this model, seepage rate was assumed to be 0.7 of the water lost from the canal and the field. This results in a seepage rate of 0.28 of total water applied with traditional furrow irrigation where 40% of water applied is lost, and 0.07 for modern technology where 10% of water applied is lost.

In many groundwater studies, the pumping cost of water is defined as a linear function of pumping lift (Gisser, 1983; Kim, Moore, and Hanchar, 1989). These studies assume that the marginal cost of pumping does not depend on the amount of water pumped. Kanazawa (1992) suggested in his study for Central California that the marginal pumping cost of groundwater may increase as the amount of pumping increases. For simplicity, the assumption of constant marginal cost of pumping is adopted here. The marginal pumping cost, US\$0.0128/m<sup>3</sup> (US\$15.80/acre-foot) as estimated by Negri and Brooks (1990), excludes the cost of pressurization and is considered appropriate for this model which separates pumping cost from on-farm investment.

### *The algorithm*

A computer algorithm was written to solve the above model. First, we guess the initial stock of surface water,  $z_s(0)$ , and  $\lambda_s(0)$  is computed from the salvage value condition (15). The initial stock of groundwater  $z_g$  at  $x=0$  is zero. Next we guess  $\lambda_g$ , which is constant, and condition (8) gives  $m'(k)$  at  $x=0$ . The marginal reduction function derived from (21) defines the conveyance expenditure  $k(0)$  and by substituting it into (20), water loss from the canal  $a(0)$  is computed. Given  $\lambda_s(0)$  and  $\lambda_g$ , condition (6) yields  $q_s(0)$ , which is used to compute  $e(0)$ ,  $y(0)$  and  $R_L(0)$ .

In the next cycle when  $x=1$ ,  $a(0)$ ,  $\lambda_s(0)$ , and  $\lambda_g$  are substituted into (9) to give  $\lambda_s(1)$ . The residual stock of surface water  $z_s(1)$  is obtained from the state constraint (1) by subtracting the surface water use and water loss in the canal in the previous period  $x=0$ . Similarly, the residual groundwater stock  $z_g(1)$  is estimated from (3). Given  $\lambda_s(1)$  and  $z_s(1)$ , the previous cycle is repeated to yield  $q_s(1)$ ,  $e(1)$ ,  $y(1)$  and  $R_L(1)$ . This procedure for surface water production is repeated at  $x=2,3,\dots$  and is terminated when one of the following conditions is satisfied: (1) the shadow price of surface water is greater than or equal to the shadow price of groundwater, (2) the land rent is non-positive, or (3) the residual surface water stock  $z_s$  is exhausted.

On completion of the above sequence, the extraction of groundwater begins. The groundwater cycle repeats the same process with the values of  $z_g(X_c)$  and  $\lambda_g$  from above. It terminates when the residual groundwater stock is exhausted. Finally, aggregate net benefit, output and land rents from surface and groundwater production are computed by summing over the entire project.

The above procedure is repeated with new guesses of  $z_s(0)$  and  $\lambda_g$ . The optimal  $z_s(0)$  and  $\lambda_g$  are those that maximize the total net benefit for the project, i.e., the aggregate net benefit less the cost of generating surface water at the source given by (13).

### *Simulation Results*

Simulations were run for the conjunctive use model with traditional and modern on-farm technology (see Table 1). For comparison, we modify the CHZ surface water model by assuming that the water that is lost is actually recovered by a third party for farming. This is because if we were to assume that the water in the surface water model was completely lost to the system the conjunctive model would obviously dominate because it was reusing the lost water and the surface water model was not. Therefore, what we show in Table 1 is joint optimization of surface and groundwater (conjunctive use) compared with optimization of the surface water distribution with use of the return flows as a third party effect. Solution to these cases under two different levels of on-farm technology are presented.

Note that when modern technology (drip or sprinkler) is adopted, the difference between the two models is quite small. This is because when firms use water-conserving technology, only a small fraction of water is available as return flow. However when traditional technology is used, there is a significant difference between joint optimization of the basin relative to the case in which return flows are considered separately. The former provides larger aggregate benefits, project area, initial stock of water, water use, aggregate output and land rents. Shadow prices of surface water are higher in the conjunctive use model as is to be expected. The main results are highlighted below:

1. When traditional technology is used, 20 percent more aggregate water is used under conjunctive use. Almost 70 percent of the total area is serviced by surface water and the remainder by groundwater. Note that aggregate output is almost 70 percent higher under conjunctive use than when surface and groundwater systems are modeled separately, although water use is only 20 percent higher. That is, basinwide optimization leads to significant increases in the efficiency of water use. However, under modern technology, the difference between the two models is much smaller. Canal length is approximately the same in both cases under modern technology. Recall our earlier conjecture that the ordering of canal lengths is indeterminate.

2. In conjunctive use, the "gross" shadow price is higher but the net price of surface water is in both cases lower relative to the shadow price of the surface water model. The net price is the price farmers actually pay for the water withdrawn from the canal. Part of the water in the conjunctive use model is recycled, lowering its true economic cost and leading to increased use. Water loss rates are also several times higher under conjunctive use because of lower investment in distribution, given the positive externality of groundwater recharge. From a policy perspective this suggests that optimal conveyance investment should be computed through optimization at the river basin level and not at the level of the individual project. Conveyance investments that ignore basinwide impacts, as in CHZ are likely to be significantly higher than optimal.

3. The results also indicate that recycling of water creates a wedge between the marginal cost of water at the head and the prices firms actually pay for water. Under traditional technology, the

marginal cost of water generation at the head of the canal is US\$0.1363/m<sup>3</sup> (US\$168.1/acre-feet) for the utility, while the net water price faced by farmers is only US\$0.0999/m<sup>3</sup> (US\$123.2/acre-feet). Note that the real price groundwater farmers pay is also lower than the supply price because return flows from groundwater use replenish the aquifer. For example, the net price at the head under traditional technology is US\$123.2/acre-feet while at the tail it is US\$131.3/acre-feet (which equals the supply price of groundwater). The figures for the corresponding surface water model are US\$140.2 and US\$140.4/acre-feet respectively. Notice that under basinwide management, the price gradient is steeper simply because it becomes optimal to “lose” water that can be used at another location. Thus, incorporating basinwide considerations in water pricing will imply that uniform pricing within a water district may be suboptimal. At least from the parameter values we have used, the gradient does not seem to depend significantly on whether traditional or modern technology is used.

Compared to the surface water model where losses are low, hence shadow prices are approximately equal over space, the basinwide model creates a somewhat skewed distribution of rents in favor of head firms due to larger spatial differences in shadow prices from head to tail of the project area. However, the above numbers suggest that groundwater prices are only a modest 6 percent higher than what head farmers pay. Therefore, even though theory predicts a rising net price of water bounded above by a backstop, in reality spatial differences in water prices are small. This may reduce the administrative difficulties of implementing first-best prices.

4. Aggregate land rents are approximately three to four times higher in the conjunctive use model

because of a lower net price of water, higher water use and increased output. More pertinently, land rents in the upstream reaches are higher under conjunctive use because firms benefit from reuse by paying less than the spot shadow price of water. Implicitly, we have assumed that the property rights to canal water lie with the firms located along the canal and they are compensated for the value of the water re-used downstream. In a water market context, these farmers would equivalently, sell their unused water rights to users located downstream. Similarly, groundwater farmers too could have tradable rights over return flows from their land. The net prices of surface and ground water could be interpreted as permit prices for water.

5. When on-farm losses are low as with modern technology, the difference between the conjunctive use model and the surface water model is small as is to be expected. Total net benefit and area irrigated is almost the same. The small amount of water lost allows for a small fraction of area to be under groundwater irrigation.

6. Table 2 provides the summary statistics for the surface water model that shows the breakdown of net benefits, quasi-rents and area served under traditional and modern on-farm technology. For example, adoption of modern technology reduces return flows by more than 70 percent. Output and service area is reduced by a half. Although the magnitude of aggregate water use (sum of surface water and return flows) is not markedly different across the two levels of technology, how the water is distributed between surface and groundwater regions is highly sensitive to technology choice.

7. As discussed earlier, we do not incorporate endogenous technology choice on the farm. Note that technology choice is strictly a function of water prices: higher “net” prices will encourage the adoption of conservation technology. In locations where the “net” price gradient is relatively flat, there will not be significant spatial differences in technology adoption within an irrigation district. Table 1 suggests that the conjunctive use framework will tend to promote increased conservation among downstream water users, although in a discrete technology choice (e.g., drip, sprinkler and furrow) framework, there may be no spatial differences in the type of on-farm technology adopted.

#### **4. Concluding Remarks**

This paper develops a spatial model for conjunctive use of water and obtains an analytical solution that determines optimal water diversions, investment in conveyance, and leads to specialization between surface and groundwater use. An empirical illustration suggests that incorporating return flows in the allocation of water resources and in determination of pricing and investment rules over a basin is important especially when irrigation efficiencies are low. Although it is well known that return flows have significant economic value, we show that including them in the modeling framework increases output, acreage and water use efficiency, and alters the spatial distribution of resource use, output and quasi-rents.

The basinwide optimization framework presented here can be used to compute the effects of economic policies in the entire basin. For example, the basinwide impacts of alternative pricing or technology (e.g., subsidy) policies in the upstream can be determined through this framework once the hydrological characteristics of the return flow are known. The economic consequences of



proposed legislation currently being passed in several Western U.S. states that encourages farmers to invest in on-farm technology, and apports the saved water between upstream farmers and in-stream (Huffaker and Whittlesey, 1997) can be computed using the integrative framework proposed in this paper. The results obtained above create opportunities for analysis of institutions and strategic behavior. For example, independent water users' associations could be formed for (i) canal management and (ii) groundwater use. Since the amount of water available as groundwater is dependent on canal maintenance and surface water allocation, this generates the possibility of Nash bargaining between the two groups over the amount of distribution losses and on-farm investment in the upstream region. These extensions into the effect of alternative property rights regimes on water allocation and project size and benefits could be handled in future work.

Our results highlight the relationship between investment in water distribution and project size in light of recent efforts to promote the formation of water-user associations to maintain and rehabilitate project facilities promoted by agencies such as the World Bank (World Bank, 1994, p.117). For example, in regions where the transactions costs of forming beneficiary groups to upgrade and improve distribution systems is high, it may be cost-effective to allow for high distribution losses (opt for a bare-bones distribution and maintenance system) together with groundwater pumping. This strategy may be preferred in regions where there is political pressure to extend the project to a larger geographical area.

Insights from the model can be used by water managers to promote canal maintenance and rehabilitation as well as groundwater pumping by farmers, especially in downstream regions

which very often do not have access to secure supplies of canal water (Wade, 1984). Water utilities can adopt policies such as providing subsidies for purchase of pumps or energy and removing legal impediments to the digging of wells, which facilitate reuse of canal water. Of course, for precise implementation in a project, the above methodology may need to be tailored to fit the specific topographical and production characteristics.

For reasons of analytical convenience, the model proposed here does not incorporate several important factors. First, negative environmental problems from water losses such as waterlogging, salinization, chemical loss, and groundwater contamination were not considered. Incorporating these negative impacts would increase conveyance expenditures in our model and decrease groundwater pumping. On the other hand, if waterlogging is a serious problem, this may cause a decrease in conveyance investments and an increase in pumping. Including the cost of drainage would also result in an increase in groundwater pumping. If contaminated groundwater needs to be purified, the supply price of groundwater will increase. Another important issue is the possibility of using the groundwater aquifer as a storage device. This problem could be examined in future research using a dynamic model with supply uncertainty (as in Tsur and Graham-Tomasi, 1991).

Although the framework presented above suggests a specialization in water use over space, most growers may be using a mix of surface and groundwater supplies. There may be several factors not included in our model that may contribute to this gap between theory and practice. A critical issue may be the uncertainty of assured surface water supplies. If surface water availability were

constrained because of climatic or other reasons, farmers may choose to invest in groundwater pumping facilities for use in lean periods. For example, if rights to uncertain water supplies are determined by location, with upstream (downstream) farmers given senior (junior) rights, then it is likely that one would observe monotonicity in the sense that while all farmers may resort to pumping some of the time, the proportion of surface water withdrawals relative to groundwater declines along the canal.

The model could also be extended through more realistic specification of the agronomic relationship between applied water and percolation flows. In particular, the amount of water percolated could be a nonlinear function of the water applied in which case the “net” prices for surface water and groundwater would be related in a more complicated fashion. In one sense, therefore, the “net price” is an administratively convenient second-best policy instrument.

In terms of further research, the above framework paves the way for an integration of the spatial model with the dynamic groundwater management literature. The latter deals with the strategic dynamic interaction between multiple users of a common groundwater aquifer (Negri, 1989). Aggregating across users in the spatial model - upstream vs. downstream or users located in separate water districts but a common water basin – will allow for modeling of dynamic interactions over space and time using solution concepts such as Markov equilibria (Dockner and Long, 1993).

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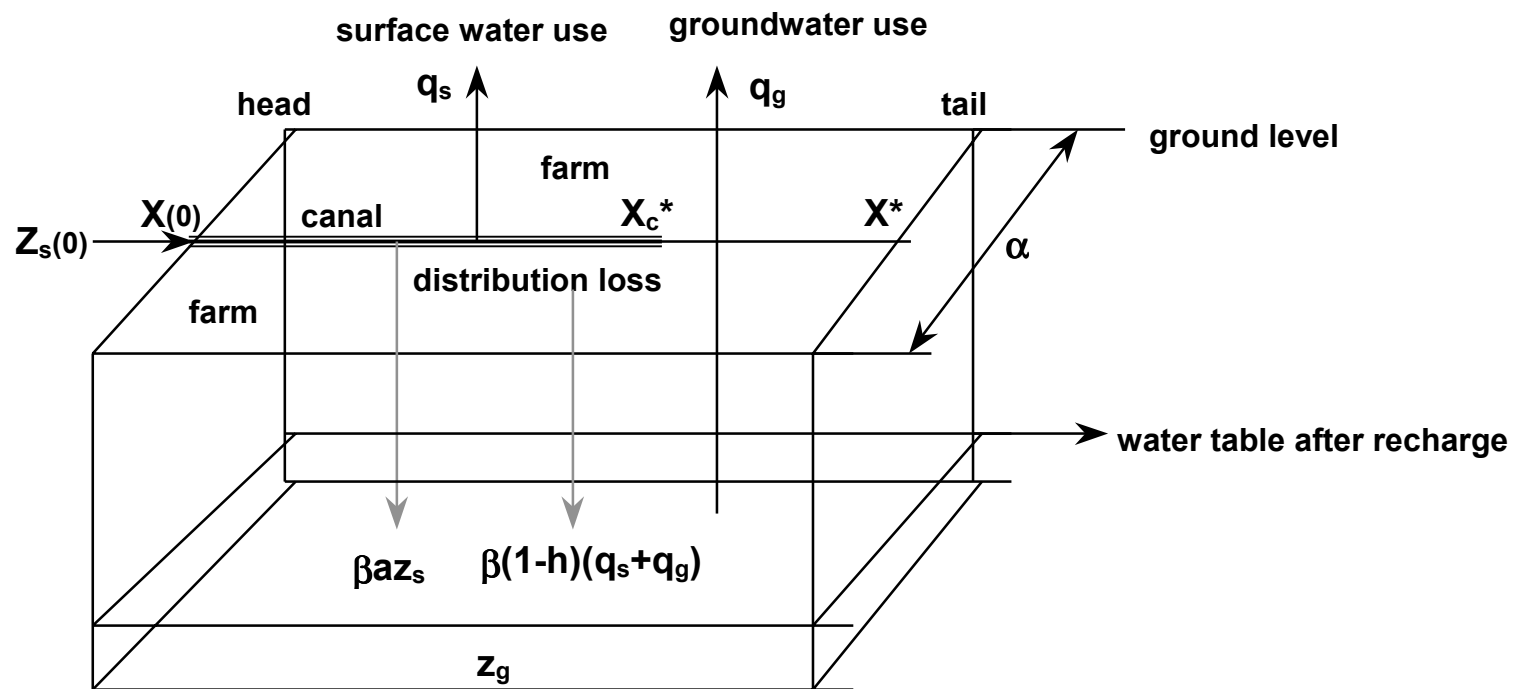
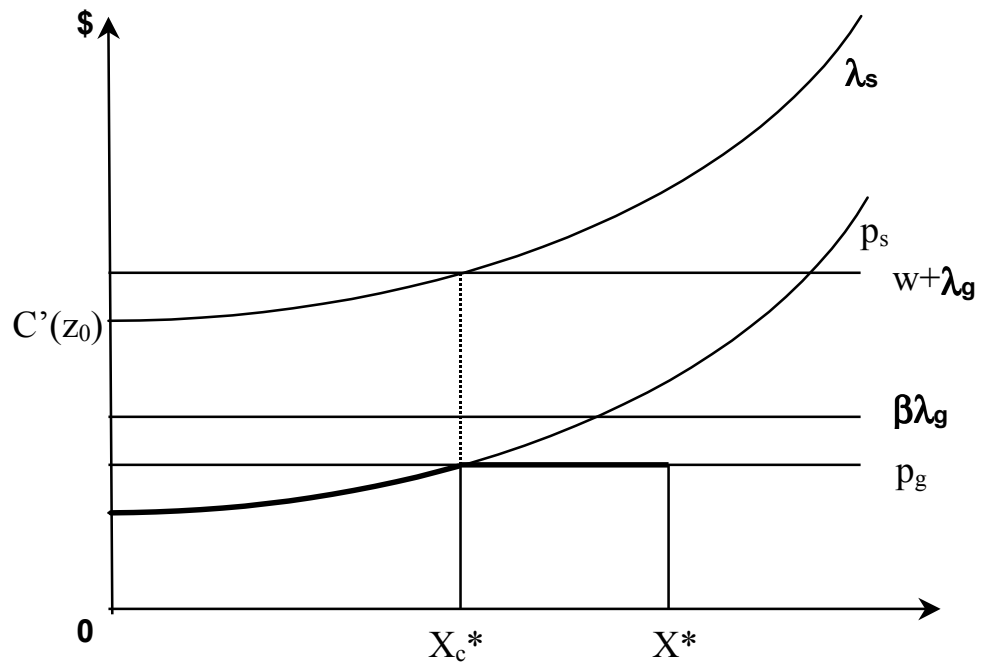
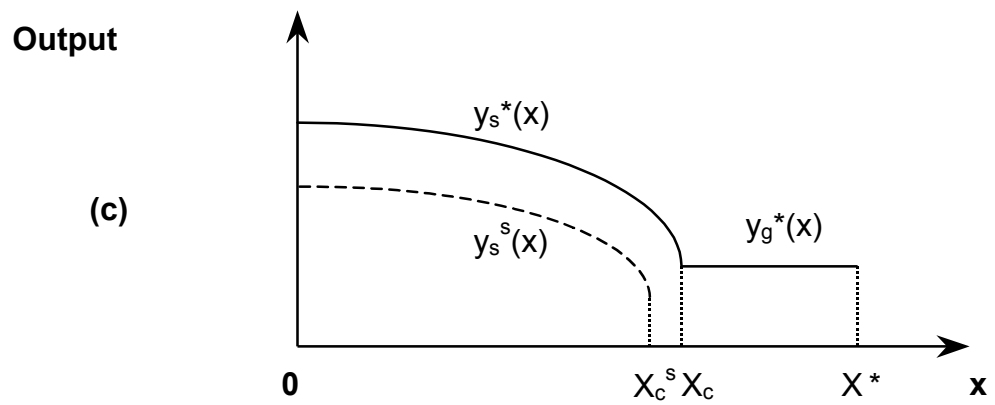
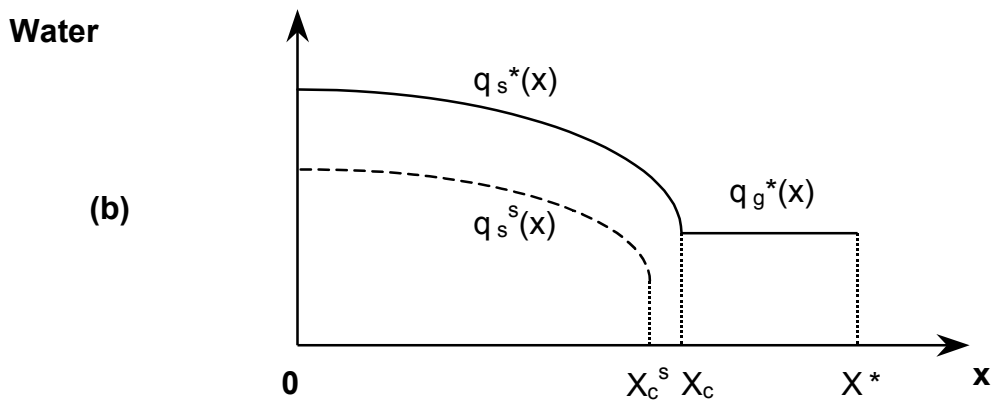
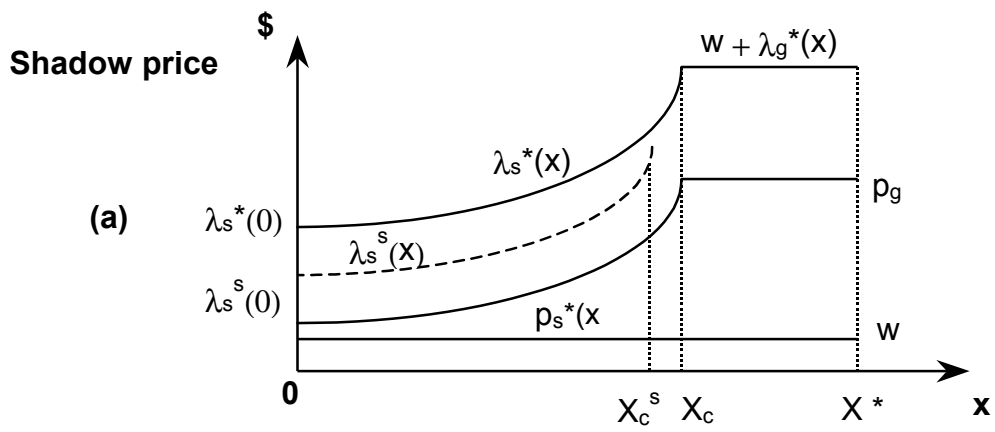


Figure 1. Box diagram of water project area



**Figure 2. Surface and Groundwater price paths**



—— conjunctive use      - - - - surface water

**Figure 3. Spatial distribution of shadow price, water use, and output**



**Table 1. Simulation Results with Traditional and Modern Technology**

Variable	Unit	Traditional Technology (h=0.6)		Modern Technology (h=0.9)	
		Conjunctive Use Model	Surface Water Model	Conjunctive Use Model	Surface Water Model
Total net benefit	(10 <sup>8</sup> US\$)	3.66	3.21	3.88	3.85
Area irrigated	(10 <sup>3</sup> ha)	390	310	520	520
Length of canal (X <sub>c</sub> *)	(km)	27	23	48	48
Length of project area (X*)	(km)	39	31	52	52
Initial water stock z <sub>s</sub> (0)	(10 <sup>8</sup> m <sup>3</sup> )	35	29	41.4	41.0
Groundwater stock z <sub>g</sub>	(10 <sup>8</sup> m <sup>3</sup> )	14	11.47	3.2	3.15
Aggr. output	(10 <sup>8</sup> US\$)	10.64	6.31	14.12	12.98
Aggr. land rent (surface water)	(10 <sup>8</sup> US\$)	0.84	0.31	0.51	0.12
Aggr. land rent (groundwater)	(10 <sup>8</sup> US\$)	0.27	1.22	0.02	0.52
Shadow price of surface water (head)	(US\$/m <sup>3</sup> ) [US\$/AF]	0.1363 [168.1]	0.1136 [140.2]	0.1605 [198.1]	0.1590 [196.2]
Shadow price of surface water (tail)	(US\$/m <sup>3</sup> ) [US\$/AF]		0.1138 [140.4]		0.1593 [196.6]
Net price of surface water (head)	(US\$/m <sup>3</sup> ) [US\$/AF]	0.0999 [123.2]	n.a.	0.1497 [184.7]	n.a.
Supply price of groundwater	(US\$/m <sup>3</sup> ) [US\$/AF]	0.1428 [176.2]	0.0128	0.1669 [206.0]	0.0128
Shadow price of groundwater	(US\$/m <sup>3</sup> ) [US\$/AF]	0.1300 [160.4]	n.a.	0.1541 [190.2]	n.a.
Net price of groundwater	(US\$/m <sup>3</sup> ) [US\$/AF]	0.1064 [131.3]	n.a.	0.1561 [192.6]	n.a.
Surface water use (head) q <sub>s</sub> (0)	(m <sup>3</sup> /m <sup>2</sup> )	1.2922	1.2603	0.8616	0.852
Surface water use (tail) q <sub>s</sub> (X <sub>c</sub> *)	(m <sup>3</sup> /m <sup>2</sup> )		1.2599		0.852
Groundwater use (tail) q <sub>g</sub>	(m <sup>3</sup> /m <sup>2</sup> )	1.2771	1.4943	0.8550	1.0028
Investment in canal (head) k(0)	(US\$/m)	196.85	198.48	197.71	199.23
Investment in canal (tail) k(X <sub>c</sub> *)	(US\$/m)	120.19	165.26	91.82	167.00
Water loss rate (head) a(0)	(10 <sup>3</sup> /km)	0.0099	0.0023	0.0053	0.0006
Water loss rate (tail) a(X <sub>c</sub> *)	(10 <sup>3</sup> /km)	6.369	1.207	11.704	1.089
Land rent at head	(US\$/m <sup>2</sup> )	0.0303	0.0129	0.0104	0.0025
Land rent at tail (X <sub>c</sub> *)	(US\$/m <sup>2</sup> )	0.0220	0.0126	0.0049	0.0022
Land rent (groundwater)	(US\$/m <sup>2</sup> )	0.0220	0.1517	0.0049	0.1301

Notes: n.a. = not applicable; supply price of groundwater is the sum of the shadow price of groundwater and pumping cost; AF = acre-foot; the numbers at X<sub>c</sub>\* and the tail are to the nearest kilometer.

**Table 2. Summary Statistics for Surface Water Allocation with Residual Groundwater Use**

		Traditional Technology <b>h=0.6</b>			Modern Technology <b>h=0.9</b>		
	Unit	Surface water	Ground water	Total	Surface water	Ground water	Total
Net Benefit	(10 <sup>8</sup> US\$)	1.99	1.22	3.21	3.33	0.52	3.85
Land Rent	(10 <sup>8</sup> US\$)	0.31	1.22	1.53	0.12	0.52	0.64
Output	(10 <sup>8</sup> US\$)	6.31	2.22	8.53	12.98	1.11	14.09
Area Irrigated	(10 <sup>3</sup> ha)	23	8	31	48	4	52
Aggr. Water	(10 <sup>8</sup> m <sup>3</sup> )	29	11.47	40.47	41	3.15	44.15