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Fondazione Eni Enrico Mattei

## **Capital Subsidies Versus Labour Subsidies: A Trade-Off between Capital and Employment?**

Alberto Petrucci and Edmund Phelps

NOTA DI LAVORO 32.2000

# **CAPITAL SUBSIDIES VERSUS LABOR SUBSIDIES: A TRADE-OFF BETWEEN CAPITAL AND EMPLOYMENT?**

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This version: February 2000

## **Abstract**

This paper examines the consequences of capital and labor subsidies for employment, capital formation and other macroeconomic variables within an OLG small open economy model of wealth accumulation. Two cases, the neoclassical-equilibrium one and the modern-equilibrium one, have been analyzed. We discover that the employment effects of the subsidies studied differ significantly, while whether the subsidy hike is financed by an increase of payroll taxation or a decrease of employment subsidization is immaterial for the qualitative effects on the macroeconomic system. In the neoclassical-equilibrium theory, a capital subsidy causes a temporary increase in hours worked which vanishes in the new long-run, while an increase in labor subsidy has no aggregative effects on the macroeconomic equilibrium. The key finding of the modern-equilibrium case is the existence of a negative relationship between capital formation and employment. Capital subsidies boost investment and aggravate unemployment, while labor subsidies stimulate employment and may depress capital accumulation.

*JEL classification:* E24; J32; J38

*Keywords:* Capital subsidy; Employment subsidy; Structural unemployment; Wealth accumulation

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## 1. Introduction

The persistence of high structural unemployment rates in many industrial economies has been the source of increasing attention and concern among public opinion, scholars and policy-makers during the last years. Hence the identification and adoption of proper therapies to cut unemployment are widely felt to be a necessity. At the same time, there is also a widespread concern about the poor investment performance of many western countries.

Policy measures to reduce unemployment that focus on the labor market are sometime supposed to have positive effects on capital accumulation; policies of this type are the reduction of taxes on labor or the introduction of labor subsidies. See Tullio (1987), Begg-Portes (1993), Drèze-Malinvaut (1994), and Daveri-Tabellini (1997).

Other scholars suggest that rather than adopt measures that impact only/mainly on the labor market it would be better to implement policies to enhance capital formation, and thus raise labor productivity, which they presume would increase employment. See, for example, Drèze-Bean (1990), Rowthorn (1995), and Snower (1997). A typical action of this sort is the introduction of subsidies on capital or investment as means to stimulate capital accumulation. Auerbach-Kotlikoff (1987, chapter 9), for example, discover in a numerical intertemporal equilibrium model with finite-lived Samuelson-Diamond agents that investment incentives in a closed economy increase labor temporarily though leaving it unchanged in the long-run. Sen-Turnovsky (1990) show that in a small open economy model with a representative infinite-lived household the introduction of a permanent investment tax credit though initially reducing employment, leads to higher employment in the steady-state equilibrium. The common feature of studies on capital subsidies is that they consider a neoclassical labor market, where positive equilibrium unemployment is absent. The consequences of investment-promoting policies on equilibrium unemployment have received very little specific attention within a proper theoretical framework.

The purpose of this paper is to compare the consequences for employment and capital formation of balanced-budget capital and labor subsidies in an OLG model of wealth accumulation and exogenous growth in a small open economy. Two cases are considered for the labor market: the neoclassical-equilibrium theory, where changes in labor determined by the shock depend on variation in hours worked and not in the unemployment rate which is absent, and the modern-equilibrium theory, characterized by the existence of a positive

structural rate of unemployment due to efficiency wage considerations of the shirking type. Particular attention is devoted to problems of financing the policy shifts that can neutralize the expansionary effect of inputs stimuli when there is simultaneously a compensatory adjustment of distortionary taxes/subsidies that hinge on the labor market.

We discover that in the neoclassical analysis capital subsidies spur capital formation, but exerts only temporary positive effects on manhours supplied; labor subsidy is neutral for the macroeconomic equilibrium, both in the short and long-run.

The key finding under a generalized real-wage rigidity is the existence of a negative relationship between capital formation and employment. It appears that factor incentives increase the subsidized factor, but may diminish the non-subsidized factor. Therefore capital subsidies boost investment and can aggravate unemployment, while labor subsidies stimulate employment and may depress capital accumulation. The financing regime of the policy shocks plays no qualitative role for the macroeconomic equilibrium.

The paper is organized as follows. Section 2 sets out the neoclassical model and analyzes the long-run and comparative dynamics effects of capital and labor subsidies hikes. Section 3 presents the modern-equilibrium model and investigates the macroeconomic implications of balanced-budget factors incentives. Section 4 concludes.

## **2. The neoclassical-equilibrium case**

### *2.1 The model*

Consider a nonmonetary small open economy that produces a single tradable good, which is perfectly substitutable with the foreign-produced good. Domestic production is obtained by using capital and labor. There are only two assets in the economy: real capital and net foreign assets. The economy is populated by three sectors: finite-lived consumers, competitive firms and the government. Time is continuous and agents are endowed with perfect foresight.

The behavior of consumers is derived by adopting the OLG approach of Blanchard (1985) with no intergenerational bequest motive and an endogenous labor-leisure choice, as in Kanaginis-Phelps (1994) and Phelps (1994, chapter 16). All consumers are identical and face uncertainty on the duration of their lives, since a constant probability of death  $\theta$  is assumed. In every instant of time a large new cohort, whose size is normalized to one, is

born. The population, composed of cohorts of different ages, is constant, as the birth rate is assumed to equal the death rate.

Assuming that individual utility is logarithmic in consumption of the good,  $c$ , and leisure,  $\bar{l} - l$  (where  $\bar{l}$  is the time endowment and  $l$  is the labor supply), at each instant  $t$  a consumer born at time  $s \leq t$  solves the following problem

$$\max \int_t^{\infty} [\ln c(s, j) + \ln(\bar{l} - l(s, j))] \exp[-(\theta + \rho)(j - t)] dj \quad (1)$$

subject to the instantaneous budget constraint

$$\frac{d}{dt} w(s, t) = [r^h(t) + \theta]w(s, t) + v^h(t)l(s, t) - c(s, t) \quad (2)$$

and the solvency condition precluding Ponzi schemes

$$\lim_{j \rightarrow \infty} w(j, t) \exp\left\{-\int_t^j [r^h(\mu) + \theta] d\mu\right\} = 0 \quad (3)$$

where  $w$ =nonhuman wealth;  $r^h$ =real interest rate faced by households;  $v^h$ =hourly real wage; and  $\rho$ =rate of time preference (exogenous).

The optimal conditions for the individual problem (1)-(3) are

$$c(s, t) = (\theta + \rho)[w(s, t) + h(s, t)]$$

$$\bar{l} - l(s, t) = \frac{c(s, t)}{v^h(t)}$$

$$\frac{d}{dt} c(s, t) = [r^h(t) - \rho]c(s, t)$$

where  $h(s, t)$  represents consumer's human wealth, given by

$$h(s, t) = \int_t^{\infty} [l(s, j)v^h(j)] \exp\left\{-\int_t^j [r^h(\mu) + \theta] d\mu\right\} dj$$

Aggregating over all the cohorts and omitting the time index, the demand side of the model can be expressed as

$$C = (\theta + \rho)(W + H) \quad (4a)$$

$$\bar{L} - L = \frac{C}{v^h} \quad (4b)$$

$$\dot{H} = (r^h + \theta)H - v^h L \quad (4c)$$

$$\dot{W} = r^h W + v^h L - C \quad (4d)$$

where capital letters denote aggregate variables of the corresponding individual variables.

From system (4), the Blanchard-Yaari dynamic equation for aggregate consumption can be easily derived

$$\dot{C} = (r^h - \rho)C - \theta(\theta + \rho)W \quad (4a')$$

Individual nonhuman wealth consists of real capital and the stock of net foreign assets. In aggregate terms, we have

$$W \equiv K + F$$

where  $K$  is physical capital stock and  $F$  is the stock of net foreign assets.

The production side of the economy is populated by many identical firms operating in a competitive environment in the output and factor markets. Domestic output,  $Z$ , is produced by using capital and labor as inputs. The production function has positive, but diminishing, marginal physical products of inputs, and is linearly homogeneous in its arguments

$$Z = G(K, L) = LG(k, 1) = Lg(k), \quad g' > 0, \quad g'' < 0 \quad (5)$$

where  $g(k)$  is the output per manhour and  $k \equiv \frac{K}{L}$  is the capital-manhour ratio. For simplicity, we assume that capital stock does not depreciate and capital accumulation does not involve adjustment costs.

The first order conditions for maximum profit entail

$$g'(k) = r^f \quad (6a)$$

$$g(k) - kg'(k) = v^f \quad (6b)$$

where  $r^f$  represents the cost of capital and  $v^f$  is the hourly labor cost, both after taxes and subsidies. Factor prices faced by firms are defined as

$$r^f = (1 + \tau^K)r^h - s^K \quad (7a)$$

$$v^f = (1 + \tau^L)v^h - s^L \quad (7b)$$

$\tau^K$  and  $\tau^L$  are ad valorem tax rates on capital and labor respectively and  $s^K$  and  $s^L$  are the corresponding quantity subsidy rates.

The government keeps the budget balanced. Therefore its instantaneous budget constraint is

$$\tau^L v^h + \tau^K r^h k = s^L + s^K k + \omega \quad (8)$$

where  $\omega$  represents the fixed per hours government purchases. Our analysis will consider the macroeconomic effects of exogenous changes in either  $s^K$  or  $s^L$  financed through a

compensatory accomodation of  $\tau^L$  and shifts in the subsidy composition, i.e. an increase in  $s^K$  accompanied by a reduction in  $s^L$ .<sup>1</sup>

The current account is given by the the trade balance, equal to the excess of production over absorption, plus the interest income earned on foreign bond holdings

$$\dot{F} = G(K, L) - C - \dot{K} - \omega L + r^h F \quad (9)$$

As foreign assets are perfectly tradable at world level, the interest rate available to households,  $r^h$ , is equal to the given world interest rate,  $r^*$ ,

$$r^h = r^* \quad (10)$$

The full model of the economy is obtained combining the optimality conditions for consumers, firms and the equations of accumulation. The basic structure of the economy can be expressed as:

$$g'(k) = (1 + \tau^K)r^* - s^K \quad (11a)$$

$$v^h = g(k) - kr^* - \omega \quad (11b)$$

$$\tau^L v^h + \tau^K r^* k - (s^L + s^K k) = \omega \quad (11c)$$

$$\bar{L} - L = \frac{C}{v^h} \quad (11d)$$

$$\dot{C} = (r^* - \rho)C - \frac{\theta(\theta + \rho)}{(r^* + \theta)} y^W \quad (11e)$$

$$\dot{H} = (r^* + \theta)H - v^h L \quad (11f)$$

$$\dot{y}^W = r^* y^W + (r^* + \theta)(v^h L - C) \quad (11g)$$

$$k \equiv \frac{K}{L} \quad (11h)$$

$$y^W \equiv (r^* + \theta)(K + F) \quad (11i)$$

where  $y^W$  is nonwage income of households, given by income from nonhuman wealth,  $r^*(K + F)$ , plus the actuarial dividend on wealth,  $\theta(K + F)$ . The endogenous variables are:

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<sup>1</sup> There is no point in exploring the effects of an increase in the capital subsidy coupled with an increase in the tax rate on domestic capital,  $\tau^K$ , since the one would undo the other.



$k, v^h, L, C, H, y^w, K, F$  and, depending on the mode of financing the exogenous  $s^K$  or  $s^L$  shock,  $\tau^L$  or  $s^L$ .<sup>2</sup>

In order to analyze the dynamic properties of model (11), linearize equations (11a), (11b) and (11d) around the long-run equilibrium, and solve for  $C$ ; we obtain the following implicit semi-reduced form<sup>3</sup>

$$C = C(L, s^K), \quad C_L = -\tilde{v}^h < 0; \quad C_{s^K} = -\frac{(\bar{L} - \tilde{L})(\tau^K r^* - s^K)}{g''} \quad (12)$$

Therefore substituting equation (12) into the equation of motion of nonhuman wealth for  $C$  and using (11d) yields

$$\dot{y}^w = r^* y^w + (r^* + \theta) \left[ \frac{C(L, s^K)}{(\bar{L} - L)} L - C(L, s^K) \right]$$

By linearizing the above equation around the steady-state equilibrium and substituting out  $C$  from the Blanchard-Yaari Euler equation through (12), the core dynamics of the model is described by the autonomous system

$$\begin{bmatrix} \dot{L} \\ \dot{y}^w \end{bmatrix} = \begin{bmatrix} (r^* - \rho) & \frac{\theta(\theta + \rho)}{\tilde{v}^h(r^* + \theta)} \\ 2\tilde{v}^h(r^* + \theta) & r^* \end{bmatrix} \begin{bmatrix} (L - \tilde{L}) \\ (y^w - \tilde{y}^w) \end{bmatrix} \quad (13)$$

where tildes denote long-run values.

The basic features of the dynamics for  $L$  and  $y^w$  can be easily grasped by using the phase diagram associated with system (13), represented in Fig. 1. The locus labelled  $\dot{L} = 0$  describes the combinations of labor and income from nonhuman wealth at which hours worked (as well as consumption) remain constant or, put in another way, the combinations at which the capital market is in equilibrium. It is downward sloping in the  $(y^w, L)$  diagram with slope  $\left. \frac{dL}{dy^w} \right|_{\dot{L}=0} = -\frac{\theta(\theta + \rho)}{(r^* - \rho)\tilde{v}^h(r^* + \theta)}$ . If  $L$  is above (below) the  $\dot{L} = 0$  locus, manhours are increasing (decreasing) as shown by the vertical arrows of motion in Fig.1. The schedule labelled  $\dot{y}^w = 0$  shows points at which the cash-flow from nonhuman wealth is constant (hence saving vanishes). It is negatively-sloped, being its slope

<sup>2</sup> Notice that equation (11b) has been obtained using relationships (6b) and (7b) and bringing in public budget balance and the world interest rate. The balance of payments (9) is redundant by the Walras's law.

<sup>3</sup> Other fiscal policy parameters, irrelevant for our purposes, have been omitted from equation (12).

$\frac{dL}{dy^W} \Big|_{\dot{y}^W=0} = -\frac{r^*}{2\tilde{v}^h(r^* + \theta)}$ . For  $y^W$  above the  $\dot{y}^W = 0$  schedule, cash-flow from wealth increases as indicated by the horizontal arrows in Fig. 1. The reverse is true for  $y^W$  below  $\dot{y}^W = 0$  locus.

Since the determinant of the coefficients' matrix in (13) is unambiguously negative,<sup>4</sup> the long run equilibrium is saddle-point stable, as  $L$  (hence  $C$ ) is a forward-looking variable, – i.e.  $L(0)$  is free– and  $y^W$  a predetermined one – i.e.  $y^W(0) = y_0^W$  as  $W(0) = W_0$ . Therefore the  $\dot{L} = 0$  schedule is steeper than the  $\dot{y}^W = 0$  locus.

The equation of the saddle-path  $SS$  is

$$L - \tilde{L} = -\frac{(r^* - \eta_1)}{2\tilde{v}^h(r^* + \theta)}(y^W - \tilde{y}^W) \quad (14)$$

where  $\eta_1 < 0$  denotes the stable eigenvalue of the matrix of the coefficients in (13). Equation (14) represents the unique path that converges to the long-run equilibrium. The saddle-path  $SS$  has a slope that lies in between the slopes of the  $\dot{L} = 0$  and  $\dot{y}^W = 0$  loci.

INSERT FIG. 1

## 2.2 Comparative statics and dynamics

The analysis studies the following policy shifts: an increase in  $s^K$  financed through higher  $\tau^L$ , an increase in  $s^L$  under  $\tau^L$  financing, and finally a change in the subsidies mix in favor of  $s^K$ .

### i) increase in $s^K$ under $\tau^L$ financing

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<sup>4</sup> The determinant of the state matrix in (13) is

$$\Delta_N = r^*(r^* - \rho) - 2\theta(\theta + \rho) = -\frac{(r^* - \rho)(r^* + \theta)}{\tilde{y}^W}[\tilde{v}^h\tilde{L} + \tilde{C}] < 0.$$

Consider the long-run, when all the dynamic variables remain constant through time. The capital subsidies hike results in an increase in the capital-labor ratio –namely  $\frac{d\tilde{k}}{ds^K} = -\frac{1}{g''} > 0$ – since it reduces the after-tax and subsidy cost of capital for firms,  $r^f$ , and the marginal product of capital is diminishing in capital intensity.<sup>5</sup>

The implications of the shock on manhours can be analyzed employing the labor supply and capital market equilibrium schedules, as in Hoon-Phelps (1996). The stock of human wealth  $\tilde{H} = \frac{\tilde{v}^h \tilde{L}}{r^* + \theta}$  –implied by equation (11f)– together with relationships (11d) and (4a) yields the following labor-supply function

$$\frac{\tilde{L}}{\bar{L}} = \frac{1 - \left( \frac{\theta + \rho}{r^* + \theta} \right) \left( \frac{\tilde{v}^h \bar{L}}{\tilde{y}^w} \right)^{-1}}{1 + \left( \frac{\theta + \rho}{r^* + \theta} \right)} \quad (15)$$

Equation (15) gives the steady-state labor supply of finite-lived consumers in terms of the ratio of wage to cash-flow from wealth. According to equation (15) a reduction in the wage-non-wage-income ratio brings about a diminution of manhours worked because it increases leisure. This equation is depicted in Figure 2 and represented by the  $L^S$  schedule. It is positively sloped and not affected by subsidies and taxes.

Furthermore, using the long-run Blanchard-Yaari “modified golden rule” together with the consumption function (4a) and the expression for  $\tilde{H}$ , we obtain the relation

$$\frac{\tilde{v}^h \bar{L}}{\tilde{y}^w} = \frac{(\theta + \rho - r^*)}{(r^* - \rho)} \left( \frac{\tilde{L}}{\bar{L}} \right)^{-1} \quad (16)$$

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<sup>5</sup> There are no crowding out effects on capital intensity stemming from the compensatory change in the payroll tax as equation (11a) represents a reduced form for  $\tilde{k}$ .

This equation gives for any level of manhours the corresponding  $\frac{\tilde{v}^h \bar{L}}{\tilde{y}^w}$  compatible with the capital market equilibrium.<sup>6</sup> This relationship, downward-sloping in the  $(\frac{\tilde{L}}{\bar{L}}, \frac{\tilde{v}^h \bar{L}}{\tilde{y}^w})$  diagram since  $\theta > r^* - \rho > 0$ , is represented by the *BY* schedule in Figure 2. The *BY* curve is not affected by fiscal variables. The intersection between the two schedules,  $L^s$  and *BY*, determines the long-run equilibrium values of labor and wage-to-income-from-wealth ratio.

INSERT FIG. 2

Since both equations (15) and (16) are independent of the capital subsidy and the payroll tax rate, any change in the capital incentive under the current financing regime leaves hours worked and the wage-nonwage-income ratio unchanged. However, the stimulus to capital boosts gross domestic product, because domestic capital is increased with the increase in capital intensity; in the special case  $g' = r^*$ , which implies that the initial  $s^K$  and  $\tau^L$  were exactly offsetting, there is no effect on gross national product and national income; in the case  $g' > r^*$ , there is a positive effect on those variables as a result of the improved allocation of national wealth. As  $\frac{\tilde{v}^h \bar{L}}{\tilde{y}^w}$  remains unchanged after the shock, hourly wage and income from wealth therefore change in the same proportion when  $s^K$  changes. And since  $\frac{\tilde{C}}{\tilde{y}^w}$  is also invariant (from equation (11e)), consumption also changes in equal proportion to  $\tilde{v}^h$ .

It is crucial for exploring what the effects of  $s^K$  on the most of the macroeconomic variables are to detect the forces exerted on  $v^h$ . The net effect, given by  $\frac{d\tilde{v}^h}{ds^K} = -\frac{(\tau^K r^* - s^K)}{g''}$ , is ambiguous, as it depends on whether the capital's before-tax

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<sup>6</sup> Equation (16) is a long-run arbitrage condition; it can be written as  $r^* = \rho + (\theta + \rho - r^*) \left( \frac{\tilde{L}}{\bar{L}} \frac{\tilde{v}^h \bar{L}}{\tilde{y}^w} \right)^{-1}$ . This equation asserts that the rate of return on wealth must equal the rate of return on consumption, given by the discount rate plus a premium proportional to the ratio of income from wealth to income from labor.

marginal product ( $g'$ ) is above or below the world interest rate ( $r^*$ ) or equivalently whether  $\tau^K r^* > < s^K$ . Households' wage increases (diminishes) if taxes on capital are relatively higher (lower) than subsidies, i.e.  $\tau^K r^* > (<) s^K$ . If, for example,  $\tau^K = 0$ ,  $\frac{d\tilde{v}^h}{ds^K} \leq 0$  depending on whether  $s^K$  is initially equal to or greater than zero. With a positive capital tax rate, the capital subsidy raises the wage rate when, for example, the capital subsidy is introduced *ex novo*. Both cases are plausible. We shall assume, in order to draw some definitive conclusions on the effect of the capital incentive, that  $\tau^K r^* > s^K$ , namely  $\frac{d\tilde{v}^h}{ds^K} > 0$ .<sup>7</sup> In this circumstance, the pay rate  $\tilde{v}^h$ , income from wealth, consumption, and nonhuman wealth all increase unambiguously. (Hence  $\tilde{v}^f$  is also raised by the capital subsidy.) In addition, since the current account must be balanced and the trade balance may either deteriorate or improve as  $\tilde{v}^h$  rises, the stock of net foreign assets may be either reduced or increased:

$$\frac{d\tilde{F}}{ds^K} = -\frac{1}{\tilde{v}^h g''} \left[ \frac{\tilde{y}^w (\tau^K r^* - s^K)}{(r^* + \theta)} - \tilde{v}^h \tilde{L} \right].$$

Consider now the comparative dynamics of the model. The analysis of the short-run adjustment studies only a permanent unanticipated fiscal disturbance. As a rise in  $s^K$  results in a long-run increase of the hourly wage, consumption and income from wealth, the unexpected permanent shock shifts the saddle-path upward to  $S'S'$ ,<sup>8</sup> leading to a short-run increase of labor (despite the long-run invariance), which overshoots its long-run equilibrium value (see Figure 1). The equilibrium moves suddenly from point  $E_0$  to point  $E'_0$  on the new saddle-path. Since leisure diminishes less than the household wage is increased, an initial jump of consumption occurs as well. The capital stock also jumps up in response to the jump in capital intensity and to the increase in hours worked, provided we suppose for simplicity that physical capital is instantaneously and costlessly mobile across borders.

After the shock has taken place, the system, placed on the new stable arm  $S'S'$ , converges monotonically to the long-run equilibrium with a reduction in labor (necessary to restore the initial equilibrium value) and an incipient accumulation of wealth, which increases the cash-flow from wealth.

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<sup>7</sup> This assumption is also retained below.

<sup>8</sup> It is assumed for simplicity that the slope of the saddle-path is unaffected by the exogenous shock.

When the capital stimulus brings about a long-run drop in household wage, consumption and income from wealth, i.e.  $\tau^K r^* < s^K$ , the economic system follows an opposite adjustment process, driven by an impact reduction of manhours and consumption. In this case, it is worth observing that labor increases along the convergence toward the new long-run equilibrium.

ii) increase in  $s^L$  financed through higher  $\tau^L$

A rise in the labor subsidy exerts no long-run effects on the economy as capital intensity is pinned down by the given cost of capital,  $r^f$ , which is not changed by this type of shock, and the labor costs remain constant as the higher  $s^L$  is completely offset by the increase in  $\tau^L$ , leaving  $v^f$  unaffected. All other variables stay unchanged. Therefore as the transitional dynamics is expectationally driven, no short-run effects of the disturbance occur, since agents correctly foresee that there will be no future changes in the economy.

iii) increase in  $s^K$  under  $s^L$  financing

This shock generates the same qualitative results of case i) both for the short-run adjustment and the steady-state equilibrium.<sup>9</sup>

It is worth noticing that the consequences of capital subsidies are independent of the way of financing, because the compensatory changes of either the labor subsidy or the payroll tax rate do not feed back into other equations of the model, as they only affect the government budget constraint in isolation (which gives residually the solution for  $s^L$  or  $\tau^L$  respectively).

### 3. The modern-equilibrium case

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<sup>9</sup> The endogenous adjustment of  $s^L$  required to support a given increase of  $s^K$  is

$$\frac{d\tilde{s}^L}{ds^K} = -\tilde{k} - \frac{(1 + \tau^L)(\tau^K r^* - s^K)}{g''}. \text{ If taxation of physical capital is relatively high compared to capital}$$

subsidy, as assumed, the employment subsidy could increase (instead of diminish) as a result of the higher capital subsidy, because the revenue effect of capital taxation could be quite robust.

### 3.1 The model

The neoclassical model gives no insight regarding the consequences of fiscal incentives for capital formation or labor use on unemployed workers, since it does not provide an explanation of equilibrium unemployment. Changes in the labor input are only due to variations of hours worked and wages adjust to equate labor supply and demand. In order to investigate the implications of subsidies on the natural rate of unemployment and its time path, we use the incentive or efficiency wage theory, based on the assumption of the shirking behavior of workers begun by Calvo (1979) and Solow (1979). The employees' effort is imperfectly monitored by firms and requires some costs in order to be observed; it is then optimal for firms to set wages above the competitive level to raise the cost of being fired for workers, this stimulating their effort and reducing firms' costs of monitoring.

The model below is a one-sector version of the (two-sector) model in Phelps (1994, chapter 9), in which the role of wealth and asset prices are at center-stage.

The production function is

$$Z^i = G(K^i, \varepsilon^i N^i) = \varepsilon^i N^i g(\mathbf{k}^i) \quad g' > 0, g'' < 0 \quad (17)$$

where  $G$  satisfies the usual neoclassical properties of regularity and is constant returns to scale,  $\varepsilon^i$  represents the effort or efficiency of a single worker in the  $i$ -th firm,  $N^i$  is the number of workers employed by the  $i$ -th firm and  $\mathbf{k}^i \equiv \frac{K^i}{\varepsilon^i N^i}$  represents the efficiency-adjusted capital-labor ratio.

We postulate that the effort of employees at individual firm level depends on the worker wage earned within the firm, on expected wage and nonwage income, according to the following relationship

$$\varepsilon^i = \varepsilon^i\left(\frac{z}{v_i^h}, \frac{y^w}{v_i^h}\right), \quad \varepsilon_1^i < 0, \varepsilon_2^i < 0, \varepsilon_{11}^i < 0, \varepsilon_{22}^i < 0, \varepsilon_{12}^i < 0 \quad (18)$$

where  $z$  is the expected income obtainable elsewhere if the worker is fired,  $v_i^h$  gives the wage per employee paid in the  $i$ -th firm and  $y^w$  is the average nonwage income of workers, taken as ratio to the worker population (whose size is unity).

In this context, firms are wage setters. Assuming that the optimal behavior of all firms is identical, so that we can omit the superscript  $i$ , the first order conditions for the maximum profit are

$$g'(\mathbf{k}) = (1 + \tau^K) r^* - s^K \quad (19a)$$

$$\varepsilon[g(\mathbf{k}) - \mathbf{k}g'(\mathbf{k})] = (1 + \tau^L)v^h - s^L \quad (19b)$$

$$- [g(\mathbf{k}) - \mathbf{k}g'(\mathbf{k})] \left( \varepsilon_1 \frac{z}{v^h} + \varepsilon_2 \frac{y^w}{v^h} \right) = (1 + \tau^L)v^h \quad (19c)$$

By following Calvo (1979) and Salop (1979), we can set  $z = Nv^h$  –where  $N$  also represents the employment rate, since the population has been normalized to one.

Combining equations (19b) and (19c) and using the Calvo-Salop indicator for  $z$ , we obtain the modified Solow condition

$$-\left( \frac{\varepsilon_1}{\varepsilon} N + \frac{\varepsilon_2}{\varepsilon} \frac{y^w}{v^h} \right) = \frac{(1 + \tau^L)v^h}{(1 + \tau^L)v^h - s^L} \quad (19d)$$

According to condition (19d) the sum of the partial elasticities of the effort function, taken in absolute value, must be greater than one at the optimum. The hypothesis  $s^L > 0$ , assuring that the sum of the partial elasticities is greater than one, is crucial in order to have the effects of subsidies and taxes on the incentive-wage equation and therefore on the whole macroeconomic equilibrium.

The government budget constraint is given by

$$\tau^L v^h + \tau^K r^* \varepsilon \mathbf{k} = s^L + s^K \varepsilon \mathbf{k} + \hat{w}$$

where  $\hat{w}$  represents the fixed public spending-employment ratio.

The rest of the model is the same as in the neoclassical-equilibrium case. The substantial difference from the previous model is that now  $L$  is replaced with  $N$ .

The investigation of the short-run adjustment requires the discussion of the dynamic properties of the model. As shown in the Appendix, the modern-equilibrium model exhibits saddle-point stability under any financing regime considered.

### 3.2 Steady-state and transitional dynamics

#### i) increase in $s^K$ financed through an increase in $\tau^L$

As for the analysis developed under the neoclassical-equilibrium case, the Blanchard-Yaari capital market equilibrium condition along with the incentive-wage equation (which is for the modern-equilibrium model the equivalent of the labor-supply schedule) can be used



to study the comparative statics effects of  $s^K$  on employment and wage-to-cash-flow-from-wealth ratio.

From the arbitrage condition between returns on wealth and consumption, the following relation is obtained

$$\frac{\tilde{v}^h}{\tilde{y}^w} = \frac{(\theta + \rho - r^*)}{(r^* - \rho)} \tilde{N}^{-1} \quad (20)$$

This function is depicted in fig. 3 e labelled  $BY$ .

The wage-setting condition must be expressed in a convenient way. By substituting equation (19b) for  $(1 + \tau^L)v^h$  into equation (19c), the optimal condition for wage can be expressed as

$$-\left( \varepsilon_1 \tilde{N} + \varepsilon_2 \frac{\tilde{y}^w}{\tilde{v}^h} \right) = \varepsilon + \frac{s^L}{g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})}$$

From the previous equation, by using the relationship  $\tilde{\mathbf{k}} = \mathbf{k}(s^K)$  (which is implied by the reduced form (19a), with  $\mathbf{k}' = -\frac{1}{g''} > 0$ ), the following functional relation is obtained

$$\tilde{N} = \Psi\left(\frac{\tilde{y}^w}{\tilde{v}^h}, s^K, s^L\right) \quad \Psi_1 < 0, \Psi_2 < 0, \Psi_3 > 0, \quad (21a)$$

Expressions for the partial derivatives of this function are reported in the Appendix.

The incentive-wage equation (21a) states that  $\frac{\tilde{v}^h}{\tilde{y}^w}$  positively depend upon  $\tilde{N}$  and  $s^K$ , and negatively on  $s^L$ . Notice that if  $s^L = 0$ ,  $\Psi_2 = 0$ ; in this case the incentive-wage equation becomes independent of fiscal variables, since the Solow elasticity condition implies that the sum of the partial elasticities must be equal to one. Equation (21a) can be purposefully employed for the steady-state analysis as  $s^L$  is fixed.

Consider the steady-state effects of the capital subsidy using the  $(\frac{\tilde{v}^h}{\tilde{y}^w}, \tilde{N})$  diagram of fig. 3. The  $IW$  schedule represents equation (21a). If  $s^L$  is positive, the capital-promoting policy shifts the incentive-wage relation upwards and, for an unchanged  $BY$  schedule, leads to lower employment and higher ratio of wage to income from wealth. This result is to be attributed to the fact that the greater capital intensity resulting from the higher capital subsidy induces firms to pay higher wages, compared to nonwage income, in order to elicit

the optimal effort of employees. In turn, the increase of unemployment and  $\frac{\tilde{v}^h}{\tilde{y}^w}$  raise the effort of each worker ( $\varepsilon$ ). It follows from our results and the elasticity condition in (19d) that labor input expressed in efficiency units ( $\varepsilon \tilde{N}$ ) rises on balanced as  $s^L > 0$ . The increase of  $\varepsilon$  pulls up both the steady-state wage rate (hence also steady-state wealth) and, on balance, pulls up output (hence capital stock) in spite of the induced decrease of employment. It can be shown that the wage-bill and consumption are likewise increased. It is not clear whether the stock of net foreign assets increases or not, since the trade balance can either improve or worsen.<sup>10</sup> In a world with no employment subsidy, i.e.  $s^L = 0$ , an increase in  $s^K$  would leave employment, nonhuman wealth-wage ratio and the effort of each worker unaffected.

### INSERT FIG. 3

Consider now the dynamic adjustment for  $N$  and  $y^w$  following an unexpected permanent increase in the capital subsidy when  $s^L > 0$ . Fig. 4 describes the short-run behavior of the economy. Here the  $N_E$  schedule represents the equilibrium rate of employment as a function of income from wealth.<sup>11</sup> This is a static equation that is negatively sloped and shifted downward on the left by higher capital subsidies. The  $\dot{y}^w = 0$  schedule exhibits the points at which the cash-flow from wealth is constant; it can be either positively or negatively sloped and is shifted downward by a rise in  $s^K$ .<sup>12</sup> We shall consider the case of the  $\dot{y}^w = 0$  schedule positively sloped.<sup>13</sup>

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<sup>10</sup> The principal long-run multipliers are given in the Appendix.

<sup>11</sup> See the Appendix for mathematical details. This schedule can be derived by using equation (19b) along with the government budget and the incentive-wage equation (21a).

<sup>12</sup> The  $\dot{y}^w = 0$  schedule is constructed by using the private budget constraint together with the equilibrium wage (as a function of income from wealth) and the saddle-path equation that describes the behavior of  $C$  in terms of  $y^w$ . See equation (24b) in the Appendix. When this schedule is negatively sloped, its slope, taken in absolute value, is lower than the slope of the  $N_E$  line by the saddle-path stability condition. In such a case the downward shift of the  $\dot{y}^w = 0$  line induced by the higher  $s^K$  is bigger than the vertical shift of the  $N_E$  schedule.

<sup>13</sup> The same type of behavior is observed when the  $\dot{y}^w = 0$  schedule is downward sloping.

After the unexpected permanent change in  $s^K$ , a sudden downward adjustment of employment from  $E_0$  to  $E_0'$  occurs. Now in contrast to the neoclassical-equilibrium model, the employment drop undershoots the long-run contraction. The decrease in employment by dampening the rate of shirking in turn drives up the take-home wage received by employees; this can either overshoot or undershoot its final equilibrium value. If  $s^L > 0$ , employment expressed in efficiency units is actually increased and capital stock increases proportionally more, since capital intensity defined in efficiency terms is increased on impact by the subsidy. Consumption jumps upwards.

INSERT FIG. 4

Soon after the impact adjustment of the economy, an accumulation of wealth begins, accompanied by further reduction of employment and increase of consumption, which monotonically converge toward the long-run equilibrium.

ii) increase in  $s^L$  under  $\tau^L$  financing

The long-run effects of an increase in  $s^L$  can be analyzed using equations (20) and (21a). Since  $s^K$  is now constant, the higher labor subsidy shifts the  $IW$  schedule downward on the right at  $IW''$  in Fig. 3. Therefore, we have higher employment and a lower wage-to-income-from-wealth ratio. There are two cases. If  $s^L$  is initially equal to zero, then the rise of employment comes at the expense of an inversely equiproportionate decrease of the wage, as a result of the increased shirking. Employment in efficiency units is unchanged on balance (as  $\varepsilon$  has a global elasticity with respect to  $\tilde{N}$  in (20) just equal to one) and since the efficiency-adjusted capital intensity remain constant (as the cost of capital is unchanged) the capital stock is unchanged. If initial  $s^L$  is positive, then employment in efficiency units is actually decreased on balance (as the global elasticity of  $\varepsilon$  with respect to  $\tilde{N}$  is now greater than one), and the capital stock is decreased as well. Wage, nonwage income, consumption, output and the stock of net foreign assets are all reduced unambiguously.<sup>14</sup>

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<sup>14</sup> Since the increase in  $\tau^L$  to finance the increase in  $s^L$  must reduce take-home wage by the amount of the increase in the subsidy, any change in the take-home wage is matched by an equal change in the cost of labor per employee.

The system shows a saddle-path behavior, characterized by the instantaneous jump of employment, wage and consumption and lagged adjustment of the predetermined variables. It is not difficult to show that no perverse-shootings occur.

iii) increase in  $s^K$  financed through a reduction in  $s^L$

In this case it is convenient for the aim of simplifying the analysis to find an expression for the incentive-wage that is independent of  $s^L$ . By substituting the government budget constraint into (19b) and using (19a), the wage-setting condition becomes

$$-\left(\varepsilon_1 \tilde{N} + \varepsilon_2 \frac{\tilde{y}^w}{\tilde{v}^h}\right) = \frac{\{ \varepsilon [g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}r^*] - \hat{\omega} \} (1 + \tau^L)}{g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})}$$

This equation may be solved for  $\tilde{N}$ , using the reduced-form  $\tilde{\mathbf{k}} = \mathbf{k}(s^K)$ , to yield the following relationship

$$\tilde{N} = \Phi\left(\frac{\tilde{y}^w}{\tilde{v}^h}, s^K, \tau^L\right) \quad \Phi_1 < 0, \quad \Phi_2 < 0, \quad \Phi_3 > 0, \quad (21b)$$

where the partial derivatives are given in the Appendix. Equation (21b) represents another characterization of the incentive-wage relationship.<sup>15</sup> By inverting equation (21b), we see that  $\frac{\tilde{v}^h}{\tilde{y}^w}$  depends positively on employment and capital subsidy, and negatively upon the ad

valorem payroll tax rate.<sup>16</sup> Since  $\tau^L$  is fixed, equation (21b) together with the capital market equilibrium condition describes the steady-state consequences of the capital subsidy in the quasi-Marshallian diagram of Fig. 3.

The long-run effects and the short-run dynamics of the unanticipated increase in the capital subsidy under the current financing regime are qualitatively the same as those seen under  $\tau^L$  financing. Therefore, what was said before remains valid when describing them. However it is not difficult to show that now the steady-state multipliers taken in absolute

<sup>15</sup> Equations (21a) and (21b) characterize the incentive-wage with different emphasis on fiscal subsidy/tax rates. The main difference is that equation (21a) is based on the optimal conditions for employment and wage, while equation (21b) includes the government budget constraint and the optimal condition for capital and wage.

<sup>16</sup> Note that in principle the effect of  $s^K$  on the wage-to-income-from-wealth ratio is unclear, namely the sign of  $\Phi_2$  is ambiguous, because the rise in capital subsidy could result in an increase in public revenues due to capital taxation that overwhelms the corresponding increase in public expenditures for investment subsidy. Therefore an increase in employment subsidy rather than a decrease could happen, if the revenue effect from capital taxation is strong enough. If we impose the mild restriction that prevents this perverse effect from occurring, i.e.  $(1 + \tau^L)(\tau^K r^* - s^K) + \tilde{\mathbf{k}}g'' < 0$ , then  $\Phi_2$  will be unambiguously negative.

value are bigger than those in case i), as the channels of transmission of the shock are reinforced by the reduction of the employment subsidy.

#### 4. Concluding remarks

The paper has examined the consequences of capital and labor subsidies for employment, capital formation and other macroeconomic variables within a nonmonetary small open economy model of wealth accumulation with new generations. Two cases, the neoclassical-equilibrium one and the modern-equilibrium one, have been analyzed. We discovered that the employment effects of the subsidies studied differ significantly from the case analyzed and the type of subsidy, while the financing procedure is immaterial for the qualitative effects on the macroeconomic system.

In the neoclassical-equilibrium theory, a capital subsidy at first causes an increase in hours worked which is eroded and vanishes along the transition to the new long-run equilibrium, provided that taxation of capital was and remains higher than capital subsidies. In this case national income, consumption, output and capital stock are all increased by the shock. An increase in labor subsidy has no effects on the macroeconomic equilibrium.

Within the modern-equilibrium theory, a capital subsidy, financed by either an increase of  $\tau^L$  or a decrease of  $s^L$  is contractionary for employment if  $s^L > 0$ , despite the positive effect on capital intensity and hence the capital stock, and is neutral for employment if  $s^L = 0$ . The reason for the negative consequences of the capital stimulus on employment comes entirely from the incentive-wage offered by firms, which is increased by the shock. In the short-run the sudden drop in employment is followed by a further reduction along the transient path toward the equilibrium. The capital subsidy exerts positive effects on output, capital stock and consumption. An increase in the employment subsidy raises employment, and in so doing reduces the wage. If  $s^L$  is initially zero, a small subsidy has no first-order effect on national income, consumption, output and capital stock. If  $s^L$  is initially positive, the subsidy reduces national income, consumption and capital.

It is not new to find that, when labor finances its own subsidy, the gain of employment achieved by the subsidy lowers the wage; proposals for low-wage subsidies envision financing by all wage earners, so low wages may be pulled up alongside employment. What is new here is that employment subsidies may reduce output and capital stock, while capital subsidies may reduce employment.

## Appendix

### I. *Partial derivatives of the incentive-wage functions*

#### I.A Equation (21a)

$$\Psi_1 = \frac{[g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})](2\varepsilon_2 + N\varepsilon_{12} + \frac{\tilde{y}^W}{\tilde{v}^h} \varepsilon_{22})}{\Sigma} < 0; \Psi_2 = -\frac{\varepsilon \tilde{\mathbf{k}} s^L}{\Sigma[(1 + \tilde{\tau}^L)\tilde{v}^h - s^L]} < 0;$$

$$\Psi_3 = \frac{1}{\Sigma} > 0; \text{ and } \Sigma = -[g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})](2\varepsilon_1 + \tilde{N}\varepsilon_{11} + \frac{\tilde{y}^W}{\tilde{v}^h} \varepsilon_{21}) > 0.$$

#### I.B Equation (21b)

$$\Phi_1 = \frac{\frac{\tilde{s}^L \varepsilon_2}{\varepsilon} + [g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})](2\varepsilon_2 + \tilde{N}\varepsilon_{12} + \frac{\tilde{y}^W}{\tilde{v}^h} \varepsilon_{22})}{\Xi} < 0;$$

$$\Phi_2 = \frac{(1 + \tau^L)\varepsilon(\tau^K r^* - s^K) - \tilde{\mathbf{k}}g''(\tilde{N}\varepsilon_1 + \frac{\tilde{y}^W}{\tilde{v}^h} \varepsilon_2)}{\Xi} < 0; \Phi_3 = \frac{\tilde{v}^h}{\Xi} > 0; \text{ and}$$

$$\Xi = -\left\{ \frac{\tilde{s}^L \varepsilon_1}{\varepsilon} + [g(\tilde{\mathbf{k}}) - \tilde{\mathbf{k}}g'(\tilde{\mathbf{k}})](2\varepsilon_1 + \tilde{N}\varepsilon_{11} + \frac{\tilde{y}^W}{\tilde{v}^h} \varepsilon_{12}) \right\} > 0.$$

### II. *Analysis of stability*

#### II.A Case of $\tau^L$ financing

By expressing the wage per employee as  $v^h = \varepsilon\{g[\mathbf{k}(s^K)] - \mathbf{k}(s^K)r^* - \hat{\omega}\}$  and using the effort function (18) with the Salop-Calvo indicator, we obtain the real wage demand relation

$$v^h = \Lambda(N, \frac{y^W}{v^h}, s^K)$$

$$\text{where } \Lambda_1 = [g(\mathbf{k}) - \mathbf{k}r^* - \hat{\omega}]\varepsilon_1 < 0; \Lambda_2 = [g(\mathbf{k}) - \mathbf{k}r^* - \hat{\omega}]\varepsilon_2 < 0; \Lambda_3 = -\frac{\varepsilon(\tau^K r^* - s^K)}{g''} > 0.$$

Equation (21a) and the real wage demand can be solved together for  $N$  and  $v^h$  in terms of the state variable  $y^W$ , and the fiscal parameters as

$$N = N(y^W, s^K, s^L) \quad N_1 < 0, N_2 < 0, N_3 < 0 \quad (22a)$$

$$v^h = v^h(y^W, s^K, s^L) \quad v_1^h ?, v_2^h > 0, v_3^h < 0 \quad (22b)$$

$$\text{where } N_1 = \frac{\Psi_1 (1 - \frac{\tilde{y}^W \Lambda_2}{\tilde{v}^{h^2} \Omega})}{\Gamma} < 0; \quad N_2 = \frac{(\Psi_2 - \frac{\Psi_1 \tilde{y}^W \Lambda_3}{\tilde{v}^{h^2} \Omega})}{\Gamma} < 0; \quad N_3 = \frac{\Psi_3}{\Gamma} < 0;$$

$$v_1^h = \frac{(\Psi_1 \Lambda_1 + \Lambda_2)}{\Gamma \Omega} ?; \quad v_2^h = \frac{(\Psi_2 \Lambda_1 + \Lambda_3)}{\Gamma \Omega} > 0; \quad v_3^h = \frac{\Psi_3 \Lambda_1}{\Gamma \Omega} < 0; \quad \Gamma = 1 + \frac{\Psi_1 \tilde{y}^W \Lambda_1}{\tilde{v}^{h^2} \Omega} > 0;$$

$$\Omega = 1 + \frac{\tilde{y}^W \Lambda_2}{\tilde{v}^{h^2}} > 0.$$

Equations (22a) and (22b) give the equilibrium unemployment and wage as function of income from wealth and fiscal parameters, respectively.

By substituting equations (22) into the equation of motion of income from wealth, i.e. the consumers budget constraint, and linearizing around the long-run equilibrium, we get:

$$\dot{y}^W = (r^* + \tilde{N}v_1^h + \tilde{v}^h N_1)(y^W - \tilde{y}^W) - (C - \tilde{C})$$

This equation, together with the law of motion of aggregate consumption, describes the basic dynamics of the economy. Saddle-path stability requires that the determinant of the core dynamic system satisfies the following inequality

$$\Delta_{ME} = (r^* - \rho) \left( r^* + \tilde{N}v_1^h + \tilde{v}^h N_1 - \frac{\tilde{C}}{\tilde{W}} \right) = -(r^* - \rho) \left( \frac{\tilde{v}^h \tilde{N}}{\tilde{y}^W} - \tilde{N}v_1^h - \tilde{v}^h N_1 \right) < 0.$$

The short-run behavior of the model can be inferred from the saddle-path equation in the  $y^W - C$  plane, given by

$$C = \tilde{C} + \frac{\vartheta(\vartheta + \rho)}{[(r^* - \rho) - \mu_1](r^* + \theta)} (y^W - \tilde{y}^W) \quad (23)$$

where  $\mu_1 < 0$  is the unique stable eigenvalue of the 2x2 autonomous system of differential equations in  $C$  and  $y^W$ .

The linearized version of equation (22b) along with the private budget constraint and equation (23) describes the dynamic behavior of employment and nonwage income

$$N - \tilde{N} = N_1(y^W - \tilde{y}^W) \quad (24a)$$

$$\dot{y}^W = \left\{ r^* + (r^* + \theta)\tilde{N}v_1^h - \frac{\vartheta(\vartheta + \rho)}{[(r^* - \rho) - \mu_1]} \right\} (y^W - \tilde{y}^W) + (r^* + \theta)\tilde{v}^h(N - \tilde{N}) \quad (24b)$$

Equations (24a) and (24b) give the  $N_E$  and  $\dot{y}^W = 0$  schedules of Fig. 4 respectively. The slope of  $\dot{y}^W = 0$  schedule can be either positive or negative.

The impact effect of capital subsidy on employment is

$$\frac{dN(0)}{ds^K} = N_2 = \frac{d\tilde{N}}{ds^K} - N_1 \frac{d\tilde{y}^W}{ds^K} < 0.$$

The short-run effect of  $s^L$  on employment is

$$\frac{dN(0)}{ds^L} = N_3 = \frac{d\tilde{N}}{ds^L} - N_1 \frac{d\tilde{y}^W}{ds^L} > 0.$$

## II.B Case of $s^L$ financing

The dynamic properties of the model in this case can be analyzed similarly to case II.A. Now the employment and wage semi-reduced forms can be derived using the real wage demand relation along with equation (21b); the partial derivatives of these equations correspond to those of (22a) and (22b) once  $\Psi_i$  is replaced with  $\Phi_i$ , for  $i=1,2,3$ .



### III. Long-run multipliers

#### III.A Increase in $s^K$ under $\tau^L$ financing

The basic multipliers are

$$\begin{aligned}\frac{d\tilde{N}}{ds^K} &= \frac{\Psi_2}{(1 - \frac{\tilde{y}^W}{\tilde{v}^h \tilde{N}} \Psi_1)} < 0; \quad \frac{d(\varepsilon \tilde{N})}{ds^K} = -\frac{\varepsilon \tilde{s}^L}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L]} \frac{d\tilde{N}}{ds^K} > 0; \\ \frac{d\tilde{K}}{ds^K} &= -\frac{\varepsilon \tilde{N}}{g''} - \frac{\tilde{s}^L \varepsilon \tilde{\mathbf{k}}}{[v^h(1 + \tau^L) - \tilde{s}^L]} \frac{d\tilde{N}}{ds^K} > 0; \\ \frac{d\tilde{v}^h}{ds^K} &= -\frac{\varepsilon(\tau^K r^* - s^K)}{g''} + [g(\mathbf{k}) - \mathbf{k}r^* - \hat{\omega}](\varepsilon_1 + \frac{\tilde{y}^W}{\tilde{v}^h \tilde{N}} \varepsilon_2) \frac{d\tilde{N}}{ds^K} > 0; \\ \frac{d\tilde{y}^W}{ds^K} &= -\frac{\tilde{y}^W \varepsilon(\tau^K r^* - s^K)}{\tilde{v}^h g''} - \frac{\tilde{s}^L \tilde{y}^W}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L] \tilde{N}} \frac{d\tilde{N}}{ds^K} > 0; \quad \frac{d\tilde{C}}{ds^K} = \frac{\theta(\theta + \rho)}{(r^* + \theta)(r^* - \rho)} \frac{d\tilde{y}^W}{ds^K} > 0; \\ \frac{d\tilde{F}}{ds^K} &= -\frac{\varepsilon \tilde{W}[(\tau^K r^* - s^K) - 1]}{(\tilde{v}^h \tilde{N}) g''} - \frac{\tilde{s}^L \tilde{F}}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L] \tilde{N}} \frac{d\tilde{N}}{ds^K} ?\end{aligned}$$

where it has been assumed  $\tau^K r^* > s^K$ .

#### III.B Increase in $s^L$ under $\tau^L$ financing

The basic multipliers are

$$\begin{aligned}\frac{d\tilde{N}}{ds^L} &= \frac{\Psi_3}{(1 - \frac{\tilde{y}^W}{\tilde{v}^h \tilde{N}} \Psi_1)} > 0; \quad \frac{d(\varepsilon \tilde{N})}{ds^L} = -\frac{\varepsilon \tilde{s}^L}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L]} \frac{d\tilde{N}}{ds^L} < 0; \\ \frac{d\tilde{K}}{ds^L} &= -\frac{\tilde{s}^L \varepsilon \tilde{\mathbf{k}}}{[v^h(1 + \tau^L) - \tilde{s}^L]} \frac{d\tilde{N}}{ds^L} < 0; \quad \frac{d\tilde{v}^h}{ds^L} = [g(\mathbf{k}) - \mathbf{k}r^* - \hat{\omega}](\varepsilon_1 + \frac{\tilde{y}^W}{\tilde{v}^h \tilde{N}} \varepsilon_2) \frac{d\tilde{N}}{ds^L} < 0; \\ \frac{d\tilde{y}^W}{ds^L} &= -\frac{\tilde{s}^L \tilde{y}^W}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L] \tilde{N}} \frac{d\tilde{N}}{ds^L} < 0; \quad \frac{d\tilde{C}}{ds^L} = \frac{\theta(\theta + \rho)}{(r^* + \theta)(r^* - \rho)} \frac{d\tilde{y}^W}{ds^L} < 0; \\ \frac{d\tilde{F}}{ds^L} &= -\frac{\tilde{s}^L \tilde{F}}{[\tilde{v}^h(1 + \tau^L) - \tilde{s}^L] \tilde{N}} \frac{d\tilde{N}}{ds^L} < 0.\end{aligned}$$

### III.C Increase in $s^K$ under $s^L$ financing

The basic multipliers are given by those of section III.A when  $\Psi_2 < 0$  is replaced with  $\Phi_2 < 0$ .

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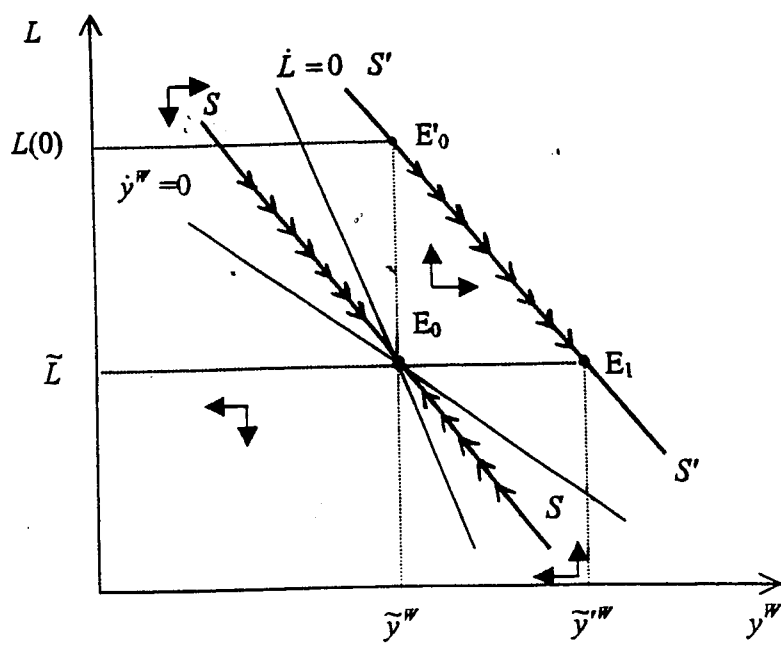


Fig. 1

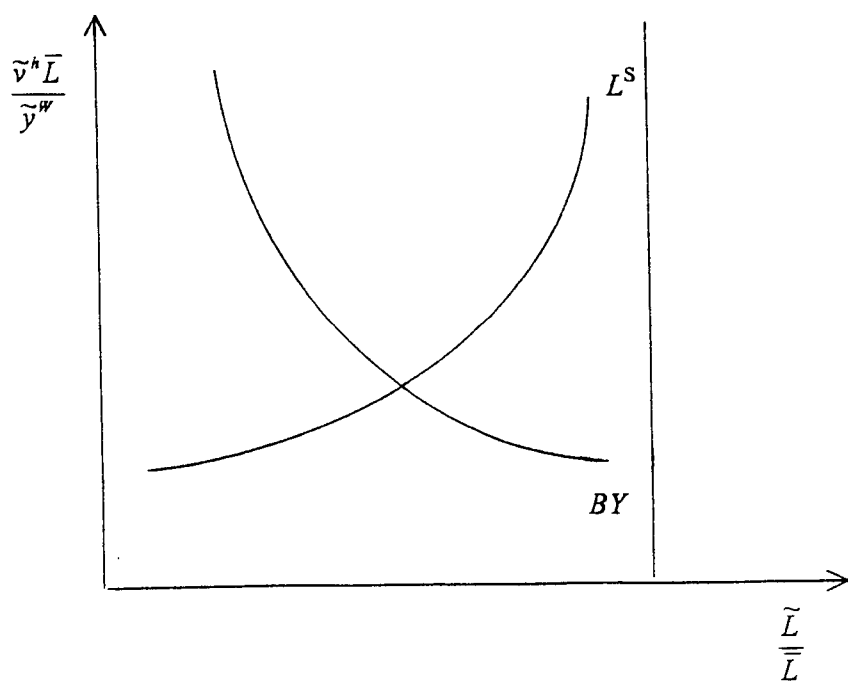


Fig. 2

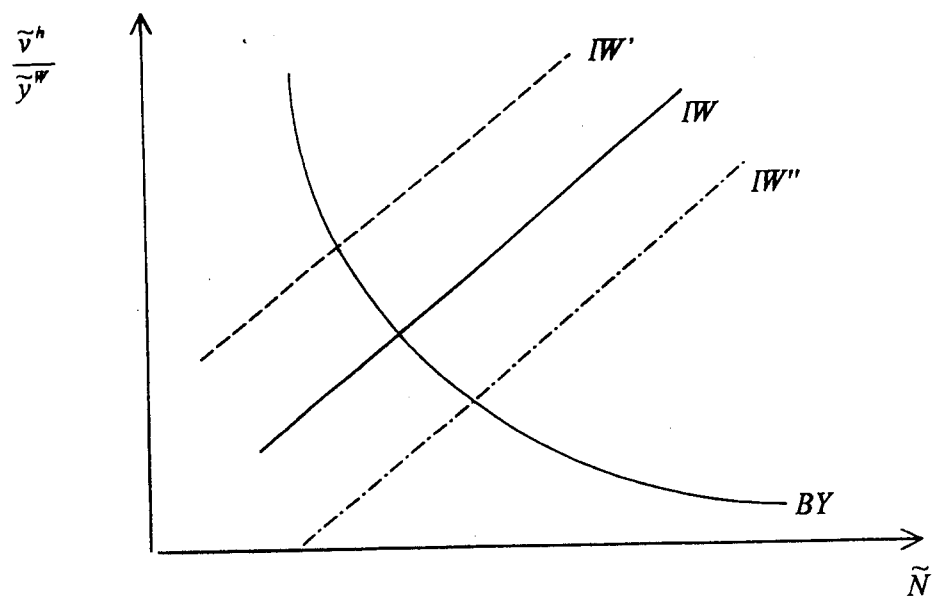


Fig. 3

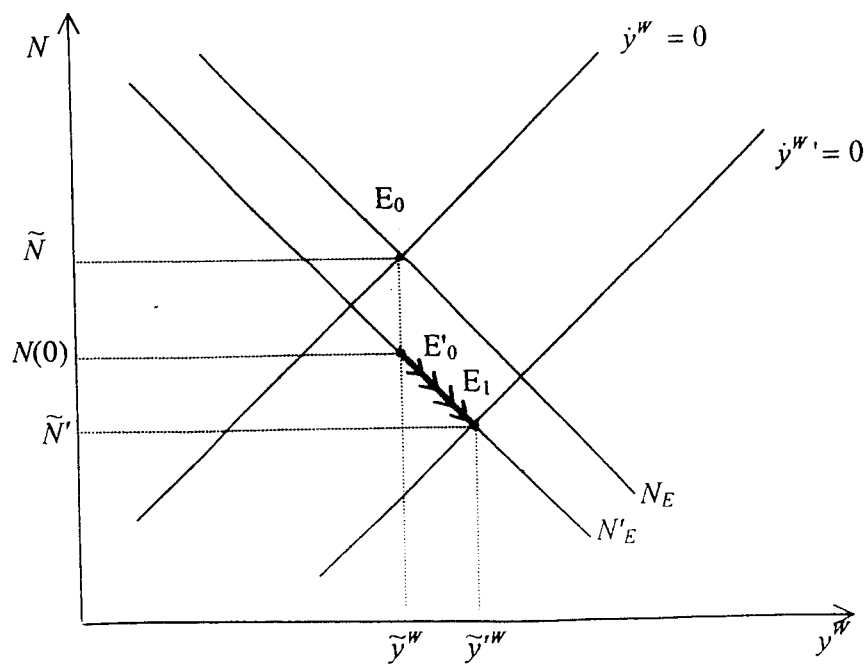


Fig. 4