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**Working Paper**

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Nota di Lavoro, No. 25. 2000

**Provided in Cooperation with:**

Fondazione Eni Enrico Mattei (FEEM)

*Suggested Citation:* Rubio, Santiago J.; Aznar, Juana (2000) : Sustainable growth and environmental policies, Nota di Lavoro, No. 25. 2000, Fondazione Eni Enrico Mattei (FEEM), Milano

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# Sustainable growth and environmental policies<sup>x</sup>

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27/10/1999

## Abstract

A model of ecologically sustainable endogenous growth is presented, in which environmental quality has a positive influence on individual welfare and on the productivity of capital. The effect of different environmental policies on the long-run growth of the economy is studied in the framework of this model. The results establish that an optimal policy which taxes production and subsidizes pollution abatement has a favorable effect on environmental quality, and could increase the growth rate if the positive external effects of the environment on the productivity are important. Furthermore, it is shown that this kind of environmental policy is neutral in budgetary terms, i.e. tax receipts are equal to subsidies. Finally, it is demonstrated that a policy based on emission control will only have a positive effect on the growth rate if the initial level of environmental quality is sufficiently low.

**Key words:** sustainable growth, external effects, Pigouvian taxes, emissions control.

**JEL Classification:** H23, O41, Q28.

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<sup>x</sup>This paper was presented at the 3rd Seminar on Environmental and Resource Economics held in Girona, Spain, May 24-25, 1999, and at the XXIV Simposio de Análisis Económico, Barcelona, December 15-17, 1999. It has also been accepted for presentation at the Tenth Annual Conference of the European Association of Environmental and Resource Economists, Rethymnon, Greece, June 30-July 2, 2000. Financial support from the Instituto Valenciano de Investigaciones Económicas and from the Generalitat Valenciana under grant GV98-08-104 is gratefully acknowledged.

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# 1 Introduction

A great many papers have been written on the relationship between economic growth and environmental preservation since R.C. d'Arge published his Essay on economic growth and environmental quality in 1971. The principal questions occupying economists have been: Is long-run economic growth compatible with environmental preservation? That is to say, is sustainable growth viable? Under what conditions? What would be the effect of greater concern for the environment over economic growth? How do environmental externalities influence growth rate, and thus, what is the effect of environmental policy on economic growth?

These questions have been analyzed in many of these papers in the framework of stationary models and exogenous growth models.<sup>1</sup> In this type of models, either there is no long-run growth or if there is long-run growth it is exogenously determined, so that environmental quality may have a negative effect on capital accumulation or no effect on the growth rate. In addition, in this literature the emphasis has been put on analysis of the efficient growth path without paying much attention to growth based on market equilibrium.

However, since the appearance of the new theory of growth at the end of the eighties and the start of the nineties, a series of papers has been published in which these questions are addressed in the framework of endogenous growth models.<sup>2</sup> Among these, we would like to draw particular attention to those of Gradus and Smulders (1993), Ligthart and Ploeg (1994), Huang and Cai (1994), Hung, Chang and Blackburn (1994), Ewijk and Wijnbergen (1995), Bovenberg and Smulders (1995) and (1996), Michel and Rotillon (1995), Smulders (1995b), Smulders and Gradus (1996), Mohtadi (1996), Rosendahl (1996) and Stokey (1998). These contributions can be classified into two groups. The first, comprised of the work of Ligthart and Ploeg (1994), Huang and Cai (1994), Hung, Chang and Blackburn (1994), Michel and Rotillon (1995) and Mohtadi (1996), would include models which predict ecologically unsustainable growth. In these models growth is achieved at the cost of continuous deterioration in environmental quality. In the second group, which contains the remaining papers, appears models which predict ecologically sustainable growth. According to these models, under determined conditions, the economy could follow a path of balanced growth with stable emissions, and therefore a constant level of environmental quality, and in some cases growth could even be compatible with decreasing emissions.<sup>3</sup>

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<sup>1</sup>See Rubio and Fisher (1994) for a review of this literature.

<sup>2</sup>A thorough survey of these papers can be found in Smulders (1995a).

<sup>3</sup>In Stokey's (1998) paper, stabilization of the emissions has a negative effect on the rate of return of capital and makes long-run sustainable growth inviable. Reference can

Within this latter group we can discern three sub-groups according to the effect on growth rate of a greater concern of the individuals for the environment. Using an extension of Rebelo's (1991) basic model, Gradus and Smulders (1993) show that a higher environmental quality reduces the rate of growth when environmental quality only affects consumers' welfare. The same authors and Ewijk and Wijnbergen (1995) conclude that the effect is positive using different versions of Lucas' (1988) model, even when environmental quality affects utility, as demonstrated by Gradus and Smulders (1993).<sup>4</sup> This result is based on the fact that environmental quality has a positive effect on the accumulation of human capital. Finally, Bovenberg and Smulders (1995) and (1996), and Smulders and Gradus (1996) show that the effect is ambiguous and that it depends on the relative importance of environmental quality as amenity value or as productive value. For the same reason, when the effect of a tighter environmental policy based on emission control is addressed, they also get an ambiguous effect (Bovenberg and Smulders (1996)), so that only if the productive value of environmental quality is higher than the amenity value will a tighter environmental policy have positive effects, not only on environmental quality but also on economic growth. Furthermore, Smulders and Gradus (1996) show that while the market equilibrium growth rate can be higher or lower than the efficient rate, in the latter case an environmental policy based on Pigouvian taxes and subsidies could result in an improvement in both environmental quality and growth rate.

Our contribution continues along the line of the papers published by Mohtadi (1996) and Smulders and Gradus (1996). We are particularly interested in the design of optimal environmental policy, as set out by Mohtadi (1996), but in the framework of a model of ecologically sustainable growth.<sup>5</sup> For this purpose we present a model of endogenous growth à la Rebelo, in which the productivity of capital depends positively on environmental quality but is constant for a given level of environmental quality, and we assume that firms can devote resources to pollution abatement but that the productivity

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also be made to the papers of John and Pecchenino (1994), Fisher and van Marrewijk (1998) and Elbasha and Rose (1996). In John and Pecchenino (1994) and Fisher and van Marrewijk (1998), the relationship between environmental quality and growth is analyzed in the framework of an overlapping generations model, and this subject is addressed with the inclusion of technological change and international trade in Elbasha and Rose (1996).

<sup>4</sup>In Rosendahl's (1996) paper the effect is null because environmental quality only affects the production of consumer goods and not the accumulation of human capital.

<sup>5</sup>See Bovenberg and de Mooij (1997) and Hettich (1998) for a complementary approach. In these papers is studied how an environmental tax reform affects pollution and economic growth in endogenous growth models with pre-existing tax distortions.

of these resources is decreasing.<sup>6</sup> Our main purpose is to study the effect of environmental policy on the growth rate, focussing on two types of policy. First, we are interested in an environmental policy based on Pigouvian taxes and subsidies, the objective of which is to restore the allocative efficiency of the market, which we can define as a pricing policy. In the second place, we focus on a sub-optimal policy of pollution control, that is, a policy based on standards, and we analyze the effect on the growth rate of a tighter environmental policy, consisting of requiring a lower emission level from firms. In line with Mohtadi (1996) this paper is solely concerned with long-term effects as long as the AK models of endogenous growth has no transitional dynamics.

Our results indicate that the optimal environmental policy, which consists of establishing a tax on production and a subsidy on abatement pollution, has a positive effect on growth only if the external effects on productivity are sufficiently large in comparison with the external effects on consumers' welfare. This conclusion can be also found in Smulders and Gradus's paper. Moreover, we show that the optimal policy is self-financing, in the sense that tax receipts are sufficient to pay for subsidies needed to restore the allocative efficiency. This result is of interest since one of the limitations of Pigouvian taxes (subsidies) is that lump sum subsidies (taxes) are required to distribute the gains resulting from the restoration of the efficiency. Finally, we conclude that a tighter environmental policy based on emissions control can have a positive effect on growth if the initial level of environmental quality is low, although it is appropriate to mention that very low levels of environmental quality can bring the very process of economic growth into question. This result shows that in those countries in which emissions control is very low or non-existent a tighter environmental policy promote economic growth.

The paper is structured as follows: in Section 2 we present the model of endogenous growth and environmental quality; and in Section 3 we develop a first approach to the issues which interest us, on the assumption that environmental quality only affects utility. This approach allows us to evaluate in Section 4, by comparison, the importance of the productive value of environmental quality. This Section includes four sub-sections. In the first the efficient balanced growth path is studied, in the second the market equilibrium path, in the third the Pigouvian taxes are calculated, and finally

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<sup>6</sup>This assumption differentiates our paper from Mohtadi's (1996), which does not take into consideration that firms can allocate resources to pollution abatement. Smulders and Gradus (1996) have developed a model where growth could be compatible with decreasing emissions. In our paper we do not consider this possibility which requires that the pollution function can be homogeneous of a degree lower than zero in the pollution abatement and capital stock.

in the fourth policy based on emissions control is analyzed. Last come the conclusions.

## 2 The model

Let us consider an endogenous growth model for a closed economy with a rational representative consumer and two goods: a private good,  $C(t)$ , and a public good,  $E(t)$ , which represents the level of environmental quality. The preferences are described by the following utility function

$$U(C(t); E(t)) = \frac{1}{1 - \frac{1}{\sigma}} (C(t)E(t)^\theta)^{\frac{1}{\sigma}} \quad (1)$$

where  $\theta$  and  $\frac{1}{\sigma}$  are positive constants. The social welfare is given by

$$W = \int_0^Z \frac{1}{1 - \frac{1}{\sigma}} (C(t)E(t)^\theta)^{\frac{1}{\sigma}} e^{-\rho t} dt; \quad (2)$$

where  $\rho$  is the discount rate. For this specification the elasticities of marginal utility, both that with respect to consumption and that with respect to environmental quality, are constant and the intratemporal elasticity of substitution between  $C(t)$  and  $E(t)$  in utility is unity. In this function the parameter  $\theta$  lets us characterize the individual preferences with respect to the two goods, such that the greater is the value of  $\theta$  the greater the importance of environmental quality is in the preferences. Keep in mind that for a given combination of  $(C(t); E(t))$ ,  $\theta$  determines the willingness to pay for environmental quality which is defined by the Marginal Rate of Substitution,  $MRS_{CE} = \theta C(t)^\theta E(t)^{\theta-1}$ ; so that an increment in  $\theta$  increases the willingness to pay for environmental quality.

Next we establish the relationship between the environment and economic activity. We assume that the environmental quality is related negatively to the capital stock,  $K(t)$ ; and positively to the pollution abatement,  $A(t)$ <sup>7</sup>. Given the linear dependency we are going to establish between production and capital stock, the negative effect of accumulation of capital on environmental quality represents the environmental damage caused by the productive activity. In this paper we adopt the pollution function proposed by Gradus and Smulders (1993, page 31), but we interpret it in the terms appearing in Mohtadi (1996).

$$E(t) = \frac{\bar{A} A(t)}{K(t)}; \quad 0 < \gamma < 1; \quad (3)$$

<sup>7</sup>In this paper we exclude the pollution associated with consumption activity.

The principal characteristic of this function is that it shows decreasing returns for pollution abatement,  $E_{AA} < 0$ :

The technology is linear with respect to  $K(t)$ ; but concave with respect to  $E(t)$ ,

$$Y = f(K(t); E(t)) = BK(t)E(t)^{\bar{\alpha}}; \quad 0 < \bar{\alpha} < 1; \quad (4)$$

So that environmental quality is considered, although indirectly, to be a production factor, in the sense that a higher level of environmental quality positively affects the productivity of the direct production factors, in the Rebelo's model, the capital stock dynamics is given by the following differential equation

$$\dot{K} = BK(t)E(t)^{\bar{\alpha}} - C(t) - A(t); \quad (5)$$

We do not take into account the depreciation of capital stock, although incorporating it into the analysis through a constant depreciation rate would not modify the qualitative results of the paper.

### 3 The sustainable balanced growth rate and the optimal environmental policy.

In this section we are interested in determining the conditions which make economic growth compatible with environmental preservation, and in the Pigouvian taxes which lead the economy to the efficient growth path when the environment only affects consumers' welfare. The results of this section will enable us to clarify what effects environmental quality has on the growth path of the economy through productivity.

#### 3.1 The efficient path.

First we calculate the efficient growth path which we use later both to evaluate the market equilibrium path and to calculate the Pigouvian taxes which correct the allocative distortions caused by the external effects associated with the environment.

When the environment does not affect productivity the parameter  $\bar{\alpha}$  is equal to zero, and the efficient path is found by internalizing the external effects associated with consumption, which is done by maximizing for  $C(t)$  and  $E(t)$  the social welfare function (2) subject to the restrictions (3) and (5) for a given initial value of capital stock. Eliminating  $E(t)$  using pollution

or environmental quality function (3), the control variables of the optimal control problem we have just defined are  $C(t)$  and  $A(t)$  and the current Hamiltonian associated with the problem is<sup>8</sup>

$$H(K; \lambda; C; A; t) = \frac{1}{1 - \beta} C^{1-\beta} \frac{\mu A^{\beta(1-\beta)}}{K} \lambda^{1-\beta} [BK - C - A]; \quad (6)$$

where  $\lambda$  is the co-state variable.

The first-order conditions establish that

$$C^{-\beta} \frac{\mu A^{\beta(1-\beta)}}{K} = \lambda; \quad (7)$$

$$\beta \frac{C^{1-\beta} \mu A^{\beta(1-\beta)-1}}{K} = \lambda; \quad (8)$$

On the margin, the marginal utility of consumption must be equal to the marginal valuation of pollution abatement which is given by the product of the marginal utility of environmental quality and the marginal effect of the pollution abatement on environmental quality. The second-order condition is satisfied if the utility function,  $U(C; E(A; K)) = U(C; E; K)$ ; is strictly concave with respect to control variables which implies that the marginal utility of consumption and pollution abatement are decreasing and, moreover, that  $\beta + \beta(1-\beta)$  is positive. This inequality defines a lower bound, less than unity, on the elasticity of marginal utility of consumption,  $\beta = (1 + \beta) < \beta$ :

Also, for the co-state variable, we get

$$\dot{\lambda} = \lambda (\beta - B) + \beta \frac{C^{1-\beta} \mu A^{\beta(1-\beta)}}{K}; \quad (9)$$

Thus, equations (5) and (7)-(9), together with the transversality condition  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda K = 0$  implicitly describe the efficient path for any given initial capital stock value. The easiest way to characterize the optimal paths is to look for the sustainable balanced growth paths which we define as a solution  $f(K; \lambda; C; A)$  to the optimization problem for an initial condition  $K(0) = K_0$ , such that the growth rates of  $K; \lambda; C; A$  and the ratios  $Y=K; C=K$  and  $A=K$  are constant. We call this sustainable growth because the growth is compatible with preservation of the environment; that is, with a constant level of environmental quality. Remember that environmental quality is a function of the ratio  $A=K$ .

<sup>8</sup>Without loss of generality, the time reference of the variables will be omitted, provided it is not required for correct interpretation of the expressions.

Let  $g_C$  be the growth rate of consumption,  $\dot{C}=C \cdot g_C$ . Then from (7), by differentiation, we get  $\dot{u}_C = -\frac{1}{2}g_C$ , where  $\frac{1}{2}$  is the elasticity of marginal utility of consumption. In the process of differentiating (7), we treat the ratio  $A=K$  as a constant, just as we have established above in the definition of sustainable balanced growth. From (9) we get

$$B = \frac{1}{2} + \frac{1}{2}g_C + \frac{\frac{1}{2}C^{1-\frac{1}{2}} \mu A^{\frac{1}{2}}}{K} : \quad (10)$$

Thus, along the path of balanced growth, the marginal product of capital is equal to its opportunity cost, which in our model includes an additional term in comparison with the standard growth model which represents the negative effect that capital stock has on utility through its influence on environmental quality

$$p_{EK} = \frac{\frac{1}{2}C^{1-\frac{1}{2}} \mu A^{\frac{1}{2}}}{K} : \quad (11)$$

To have an homogeneous expression this term is divided in (10) by the price of consumption good defined by the marginal utility of consumption,  $U_C = u_C$ :

Using (8) to eliminate  $u_C$  from the right-hand side of (10) and reordering terms, we obtain

$$g_C = \frac{1}{2} B - \frac{1}{2} \frac{A}{K} \quad (12)$$

which, interpreting (10), we name the asset market equilibrium condition.

From (7) and (8) we have  $C = A^{\frac{1}{2}}$ ; and substituting in the dynamic restriction (5) yields

$$K = BK - \frac{A}{2} : \quad (13)$$

adding terms and dividing by  $K$  we get

$$g_K = B - \frac{1}{2} \frac{A}{K} : \quad (14)$$

which we name the goods market equilibrium condition. Imposing  $g_C = g_K$ ; the system of equations (12) and (14) allows us to calculate the growth rate and the level of environmental quality which correspond with the efficient path of sustainable balanced growth of the economy. If we also keep in mind that  $g_A = g_K$ ; given that the level of environmental quality must remain constant, we conclude that all the variables in our model must grow at the same rate  $g$ .

Solving the system (12) and (13) we obtain

$$\frac{\mu_A}{K_P} = \frac{\tau^{\otimes} [\frac{1}{2} + B(\frac{3}{4} i - 1)]}{\frac{3}{4} + \tau^{\otimes} (\frac{3}{4} i - 1)}; \quad (15)$$

$$g_P = \frac{B i^{-\frac{1}{2}} (1 + \tau^{\otimes})}{\frac{3}{4} + \tau^{\otimes} (\frac{3}{4} i - 1)}; \quad (16)$$

Where  $\frac{3}{4} + \tau^{\otimes} (\frac{3}{4} i - 1)$  is positive by the strict concavity of the utility function. Then  $(A=K)_P$  and  $g_P$  have strictly positive values if the following conditions on the parameters are fulfilled<sup>9</sup>

$$\frac{B i^{-\frac{1}{2}}}{B} < \frac{3}{4}; \quad \tau^{\otimes} < \frac{B i^{-\frac{1}{2}}}{\frac{3}{4}}; \quad (17)$$

where the terms of the inequalities are positive, provided that  $B i^{-\frac{1}{2}} > 0$ : The first condition guarantees that the environmental quality is positive and the second condition that the rate of growth is also positive. These two conditions impose certain restrictions on the preferences to get a sustainable balanced growth for the economy. In particular, the elasticity of marginal utility of consumption cannot be very low and the willingness to pay for environmental quality cannot be very high. Nevertheless, our results show that a sustainable balanced growth is compatible with a elasticity of marginal utility of consumption lower than unity since  $(B i^{-\frac{1}{2}}) = B < 1$ : This also means that is not necessary to establish any particular assumption on the sign of the cross-effects on marginal utilities to reach a path of sustainable growth. Finally, we may add that it is easy to prove that if condition  $(B i^{-\frac{1}{2}}) = B < \frac{3}{4}$  is met, the transversality condition is also fulfilled.

Next, we evaluate the impacts of variations in the parameters on the quality of the environment and on the growth rate of the economy. Let us consider first the effects of variations in  $\tau^{\otimes}$

$$\frac{\partial \frac{\mu_A}{K_P}}{\partial \tau^{\otimes}} = \frac{\tau^{\frac{3}{4}} [\frac{1}{2} + B(\frac{3}{4} i - 1)]}{(\frac{3}{4} + \tau^{\otimes} (\frac{3}{4} i - 1))^2} > 0; \quad (18)$$

$$\frac{\partial g_P}{\partial \tau^{\otimes}} = i \frac{\tau^{\frac{1}{2}} [\frac{1}{2} + B(\frac{3}{4} i - 1)]}{(\frac{3}{4} + \tau^{\otimes} (\frac{3}{4} i - 1))^2} < 0; \quad (19)$$

The signs of the derivatives establish that an increase in (marginal) willingness to pay for environmental quality (a increase in  $\tau^{\otimes}$ ) results in an increase

<sup>9</sup>P denotes the Pareto efficient intertemporal allocation.

in environmental quality and a decrease in the sustainable balanced growth rate.

This result leads us to the conclusion that preferences which imply a higher valuation of the environment are associated with a lower growth rate. In other words, a higher environmental conservation will only be achieved at the cost of a reduction in growth. This result is implicit in conditions (12) and (14), which show an inverse relationship between growth rate and environmental quality, and define a clear trade-off between these two variables. Observe that  $\beta$  determines the ratio  $C=A$  and the allocation of output between the investment and the expenditure in consumption and pollution abatement. This is very clear if we rewrite condition (13) as

$$Y = BK = K + \frac{1 + \beta}{\beta} A \quad (20)$$

Then, for given values of  $K$  and  $A$ , an increase in  $\beta$  reduces the expenditure and increases the investment resulting in an increase of the growth rate. With  $g_C = g_K$ ; this change causes a disequilibrium in the asset market since the opportunity cost of capital increases. In order to reach again the equilibrium in the asset market, without affecting the equilibrium in the goods market, the rate of growth must decrease and the environmental quality increase. Notice that the adjustment in the asset market cannot be reached by a reduction both the growth rate and the environmental quality because this would cause a disequilibrium in the goods market.

Changes in  $\beta$  affect the productivity of pollution abatement, and have the same effects on the growth rate and the environmental quality that the ones caused by a variation of  $\beta$  as long as both parameters determines the allocation of the output between the investment and the expenditure in consumption and pollution abatement (see (20) again). Finally, we emphasize the positive effect of an increase in the discount rate on the environmental quality.

### 3.2 The equilibrium path

In this first part we assume that the environmental quality, which depends on decisions of the firms, does not affect productivity, on account of which the market allocation is seen to be distorted by the presence of unidirectional external effects, from the production activity to the consumption activity. In this framework individuals determine the demand for consumption goods, taking the level of environmental quality as given, and the firms have no incentive to spend in pollution abatement, for which reason we expect that market equilibrium leads to an ecologically unsustainable growth, as occurs,

for example, in the paper of Ligthart and Ploeg (1994) and Mohtadi (1996). Hence ecologically sustainable growth will only be possible through public intervention.

Next we show that the efficient intertemporal allocation can be replicated through a proportional tax on production. Moreover, tax receipts are exactly sufficient to finance the public expenditure required to stabilize the emissions and achieve the level of environmental quality corresponding to the efficient outcome, so that the resulting public intervention is neutral from the budgetary perspective.

Let us assume that the representative consumer maximizes the utility function over an infinite horizon, choosing consumption subject to the usual intertemporal budgetary restriction and taking the level of environmental quality as given. From this maximization problem the well-known Keynes-Ramsey rule is obtained:  $r = \frac{1}{2} + \frac{3}{4}g_C$ ; where  $r$  is the market interest rate. Let us also assume that a great number of firms exist which produce a final homogeneous good under conditions of perfect competition. The firms maximize profits, and the first-order condition for a maximum establishes that  $r = (1 - \tau)B$ ; where  $\tau$  stands for the rate of taxation on production. These two conditions allows us to establish the asset market equilibrium condition for a decentralized economy

$$g_C = \frac{1}{\frac{3}{4}} [(1 - \tau)B - \frac{1}{2}]; \quad (21)$$

If we impose the balanced budget condition,  $G_A = \tau Y = \tau BK$ , where  $G_A$  is the public expenditure allocated to pollution abatement, the goods market equilibrium condition is written

$$g_K = (1 - \tau)B - \frac{C}{K}; \quad (22)$$

and the economy grows at the rate  $g_M = g_C = g_K$  given by (21)<sup>10</sup>. Observe that the growth rate and the consumption-capital relationship depend on the tax rate. To determine the optimum policy we establish that  $G_A = A_P$ ; and using the balanced budget condition we obtain the optimal tax rate:  $\tau^* = \frac{1}{B} (\frac{A}{K})_P$ : Substituting this optimal value in the equilibrium condition for the assets market we get  $g_M(\tau^*) = g_P$ :

Next we show that the optimal tax rate does not only ensure that the environmental quality and the growth rate corresponding to the equilibrium path are socially optimal but that the decentralized solution exactly repro-

<sup>10</sup> $g_M$  stands for the sustainable balanced growth rate of market equilibrium.

duces the efficient allocation. If we calculate the initial values, we obtain

$$G_{A0} = \frac{\tau^{\text{e}} [\frac{1}{2} + B(\frac{3}{4} i - 1)]}{\frac{3}{4} + \tau^{\text{e}}(\frac{3}{4} i - 1)} K_0; \quad (23)$$

which is obtained from the balanced budget condition, substituting the optimal value for  $\zeta$ , and we get

$$C_{M0} = \frac{\frac{1}{2} + B(\frac{3}{4} i - 1)}{\frac{3}{4} + \tau^{\text{e}}(\frac{3}{4} i - 1)} K_0; \quad (24)$$

which is obtained from the goods market equilibrium condition, substituting the optimal value for  $\zeta$  and  $g_P$  for  $g_K$ : It can easily be proved that these initial values correspond with those for the efficient solution, so that when the tax is set to obtain  $g_P = g_M$  and  $(A=K)_P = (A=K)_M$ ; we have that  $A_{P0} = G_{A0}$  and  $C_{P0} = C_{M0}$ ; and the equilibrium path coincides with the efficient path.

### 3.3 Regulation of emissions

In this last subsection we address the study of environmental policy based on emission control. If the government fixes the level of environmental quality, using this variable as an instrument of environmental policy, we have that the ratio  $A=K$  is exogenously determined, and the firms support the cost of pollution abatement. Then the environmental quality function (3) defines a linear dependence between  $A$  and  $K$ ;  $A = \hat{E}^{1-\tau} K$ ; where  $\hat{E}$  is the level of environmental quality fixed by the regulatory authority. In this case, the firm's profits are given by

$$\pi = BK i - rK i - A = BK i - rK i - \hat{E}^{1-\tau} K; \quad (25)$$

and the first order condition for the maximization of profits can be written as  $r = B i - \hat{E}^{1-\tau}$ , whereas the asset market equilibrium condition is

$$g_C = \frac{1-\tau}{3/4} B i - \frac{1}{2} i - \hat{E}^{1-\tau}; \quad (26)$$

and the goods market equilibrium condition is

$$g_K = B i - \hat{E}^{1-\tau} i - \frac{C}{K}; \quad (27)$$

If we set the balanced growth condition, which requires a constant capital stock growth rate, we get that the market equilibrium growth rate is given

by (26) and the  $C=K$  relationship by (27). As was to be expected, we find a negative relationship between the growth rate and the environmental policy instrument. Thus, any tightening of environmental policy has a negative effect on the growth rate of the economy, independently of the initial level of environmental quality.

## 4 The environment affects productivity

In this section we show that if the environment affects productivity, there exists a positive relationship between growth rate and environmental quality, provided that the level of environmental quality is not very high. This happens because the increment of production due to an improvement in environmental quality is greater than the increment in resources needed to reduce the pollution and stabilize the environmental quality. For this reason a more restrictive environmental policy could have positive effects on both environmental quality and growth rate.

### 4.1 The efficient path

To calculate the efficient solution we eliminate  $E(t)$  by substitution, using the pollution function (3), which yields the Hamiltonian associated with this problem

$$H(K; \lambda; C; A; t) = \frac{1}{1 - \alpha} C^{1-\alpha} \frac{\mu_A}{K} \pi^{(1-\alpha)\alpha} + \lambda \left[ BK \frac{\mu_A}{K} \pi^{1-\alpha} - C - A \right] \quad (28)$$

and from the first-order conditions we obtain

$$C^{1-\alpha} \frac{\mu_A}{K} \pi^{(1-\alpha)\alpha} = \lambda \quad (29)$$

$$\alpha \frac{C^{1-\alpha} \mu_A}{K} \pi^{(1-\alpha)\alpha - 1} + \lambda^{-1} B \frac{\mu_A}{K} \pi^{1-\alpha - 1} = 0 \quad (30)$$

Again, the valuation of resources on the margin must be the same. However, when the environment affects productivity, the marginal valuation of pollution abatement presents an additional term which measures the effect that pollution abatement has on production through its effect on environmental quality:  $\lambda^{-1} \frac{\partial H}{\partial A}$ . Thus, condition (30), in terms of the functions of the model,

may be written as  $U_E E_A + \lambda f_E E_A = \lambda$  or  $(U_E E_A = U_C) + f_E E_A = 1$ ; where 1 is the opportunity cost of pollution abatement in terms of consumption.<sup>11</sup> This condition requires that  $f_E E_A < 1$ , which implies that

$$B \frac{\mu_A \pi^{1-\alpha}}{K} < 1; \quad (31)$$

The dynamic of the state variable is given by the following differential equation

$$\dot{\lambda} = \frac{1}{2} \lambda + \frac{\alpha C^{1-\alpha} \mu_A \pi^{\alpha(1-\alpha)}}{K} - \lambda B (1 - \alpha) \frac{\mu_A \pi^{1-\alpha}}{K}; \quad (32)$$

As (29) is equal to (7), from (32) we obtain

$$B (1 - \alpha) \frac{\mu_A \pi^{1-\alpha}}{K} = \frac{1}{2} + \alpha g_C + \frac{\alpha C^{1-\alpha} \mu_A \pi^{\alpha(1-\alpha)}}{K}; \quad (33)$$

which requires that the net marginal product of capital must be equal to its opportunity cost along the path of sustainable balanced growth. If we compare this condition with the one which we obtained in the previous section (see (10)), we find that they differ only on the left-hand side. When the environment affects productivity, a variation in capital has two effects, one direct and positive for a given level of environmental quality, and the other indirect and negative as a consequence of the dependence of environmental quality on the capital stock. So that the net marginal productivity of capital is the sum of these two effects:  $f_K + f_E E_K$ ; which for the functions in our model gives us the expression that appears on the left-hand side of (33).<sup>12</sup>

Using (29) to eliminate  $\lambda$  from the right-hand side of (33), we get

$$B (1 - \alpha) \frac{\mu_A \pi^{1-\alpha}}{K} = \frac{1}{2} + \alpha g_C + \frac{\alpha C}{K}; \quad (34)$$

Furthermore, eliminating  $\lambda$  substituting (29) in (30) yields

$$C = \frac{A}{\alpha} (1 - \alpha) B \frac{\mu_A \pi^{1-\alpha}}{K}; \quad (35)$$

which we can use to substitute for C in (34) and obtain the asset market equilibrium condition

<sup>11</sup> Given the strict concavity of the environmental quality function and of the production function, the strict concavity of the utility function,  $U(C; E(A; K)) = U(C; A; K)$ , with respect to control variables guarantees the fulfillment of the second-order condition.

<sup>12</sup> Observe that for the assumptions of the model the net marginal productivity is positive, since we have established that  $\alpha < 1$  and  $\pi < 1$ , see (3) and (4).

$$g_C = \frac{1}{\frac{3}{4}} B \frac{\mu_A \pi^{1-\alpha}}{K} i^{\frac{1}{2}} i \frac{A}{K} \quad (36)$$

Finally, using the relationship between C and A defined by (35) and the capital stock dynamic restriction, we calculate the goods market equilibrium condition

$$g_K = \frac{\frac{\alpha}{\alpha-1} + \frac{\alpha}{\alpha-1}}{\frac{\alpha}{\alpha-1}} B \frac{\mu_A \pi^{1-\alpha}}{K} i \frac{1 + \frac{\alpha}{\alpha-1} A}{K} \quad (37)$$

For  $g_C = g_K$ , the system of equations (36) and (37) is defined for the growth rate and the level of environmental quality corresponding to the efficient path and the solution for the level of environmental quality is given by the following equation

$$i \frac{\frac{\alpha}{\alpha-1} (\frac{3}{4} i + 1) + \frac{\alpha}{\alpha-1}}{\frac{\alpha}{\alpha-1}} B \frac{\mu_A \pi^{1-\alpha}}{K} i^{\frac{1}{2}} + \frac{\frac{3}{4} + \frac{\alpha}{\alpha-1} (\frac{3}{4} i + 1)}{\frac{\alpha}{\alpha-1}} \frac{\mu_A \pi^{1-\alpha}}{K} i^{\frac{1}{2}} = 0; \quad (38)$$

which has an unique solution if the second-order condition for the maximization of the Hamiltonian is satisfied.

However, the fact that  $(A=K)_P$  is positive does not guarantee that the growth rate is positive, since (38) only implies that the growth rates are equal. If we study functions (36) and (37) to determine whether the intersection point is associated to a positive growth rate, we find two concave functions which present a unique maximum for the following values

$$\frac{\mu_A \pi^{1-\alpha}}{K}_C = \left[ \frac{\alpha}{\alpha-1} B \right]^{\frac{1}{1-\alpha}}; \quad \frac{\mu_A \pi^{1-\alpha}}{K}_K = 4 \frac{\alpha}{\alpha-1} B \frac{\frac{\alpha}{\alpha-1} + \frac{\alpha}{\alpha-1}}{\frac{\alpha}{\alpha-1} + \frac{1}{\alpha}} \frac{3}{1-\alpha}; \quad (39)$$

which, keeping in mind that  $\frac{\alpha}{\alpha-1} < 1 = \frac{1}{\alpha}$ ; yields  $(A=K)_K^{\frac{\alpha}{\alpha-1}} < (A=K)_C^{\frac{\alpha}{\alpha-1}}$ . Then, if the productivity parameter, B, is enough large, the maximum for both functions gives a positive value and the intersection point defines a positive value for the growth rate. Notice that for a given value of  $A=K$ , functions (36) and (37) are increasing with respect to B:

Furthermore, we can establish that  $g_C^0(A=K)$  is negative at the intersection point: The derivative of this function is

$$g_C^0 \frac{\mu_A \pi^{1-\alpha}}{K} = \frac{1}{\frac{3}{4}} \frac{\alpha}{\alpha-1} B \frac{\mu_A \pi^{1-\alpha}}{K} i^{\frac{1}{2}} i^{\frac{1}{2}}; \quad (40)$$

which is negative if condition (31) is fulfilled. We can also conclude that  $g_C^0(A=K) > g_K^0(A=K)$  for  $(A=K)_P$ . Let us assume that  $g_C^0(A=K) \cdot g_K^0(A=K)$  then it is obtained

$$\frac{\frac{3}{4} + \tau^{\otimes}(\frac{3}{4} i - 1)}{\tau} \cdot (\tau^{\otimes}(\frac{3}{4} i - 1) + \frac{3}{4} \tau^{-})^{-1} B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} : \quad (41)$$

On the other hand, we can rewrite (38) as

$$\frac{1}{\tau} \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} \cdot (\tau^{\otimes}(\frac{3}{4} i - 1) + \frac{3}{4} \tau^{-}) B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} + \frac{\frac{3}{4} + \tau^{\otimes}(\frac{3}{4} i - 1)}{\tau} = \frac{1}{2} > 0;$$

which implies that

$$\frac{\frac{3}{4} + \tau^{\otimes}(\frac{3}{4} i - 1)}{\tau} > (\tau^{\otimes}(\frac{3}{4} i - 1) + \frac{3}{4} \tau^{-}) B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} :$$

Then given that  $\tau^{-} < 1$ , we obtain that

$$\frac{\frac{3}{4} + \tau^{\otimes}(\frac{3}{4} i - 1)}{\tau} > (\tau^{\otimes}(\frac{3}{4} i - 1) + \frac{3}{4} \tau^{-})^{-1} B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} : \quad (42)$$

But this inequality defines a contradiction with respect to (41), so that we have to conclude that  $g_C^0(A=K) > g_K^0(A=K)$  for  $(A=K)_P$ .

Thus we obtain the following graphical representation of the solution to the problem

[ FIGURE 1 ]

Next we evaluate the effects of a variation of the willingness to pay for environmental quality on environmental quality and the growth rate of the economy. We will use expression (38) to study the effect on environmental quality. Differentiating the left-hand side of equation (38), we obtain

$$\begin{aligned} & \frac{\frac{3}{4} + \tau^{\otimes}(\frac{3}{4} i - 1)}{\tau} \cdot (\tau^{\otimes}(\frac{3}{4} i - 1) + \frac{3}{4} \tau^{-})^{-1} B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} \cdot \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} \\ & = \frac{1}{2} i (\frac{3}{4} i - 1) \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} + (\frac{3}{4} i - 1) B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} \cdot \tau^{\otimes} : \end{aligned} \quad (43)$$

The sign of the parenthesis of the left-hand side is positive given (42). To know the sign of the parenthesis of the right-hand side, we use again (38) which we rewrite as

$$\frac{1}{2} i (\frac{3}{4} i - 1) \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} + (\frac{3}{4} i - 1) B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} = \frac{3}{4} \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} \cdot \tau^{-1} \cdot B \frac{\mu_A \Pi^{\tau^{-1} i - 1}}{K_P} ; \quad (44)$$

where the sign of the parenthesis of the right-hand side is positive as long as condition (31) must be satisfied for the optimal solution. The result is

that  $\partial(A=K)_P^{\partial\theta} > 0$ : The effect on growth rate is given by  $\partial g_P^{\partial\theta} = \partial g_P^{\partial(A=K)_P} \partial(A=K)_P^{\partial\theta}$ ; which is negative because  $\partial g_P^{\partial(A=K)_P}$  is negative at the intersection point of the functions  $g_C$  and  $g_K$  (see Figure 1). Hence, we have the same qualitative results as in Section 3, which confirms the long-term conflict existing between the preservation of nature and economic growth, given that we again find that a change in preferences in favor of the environment (an increase in  $\theta$ ) has a negative effect on the growth rate, even though the positive influence of the environment on productivity is taken into account in the model.

## 4.2 The equilibrium path

When the environment also affects productivity, we find that, because of its nature as a public good, external effects on both consumption and production are present, as a result of which firms' decisions affect both consumer welfare and other firms' profits. In order to take these external effects on production into account we distinguish between an internal effect and an external effect. We represent the external effect, which the firms consider to be exogenously determined by  $E_e$ , and the internal effect, which depends on the firm's decisions by  $E$ .<sup>13</sup> In this case the firm's profits are given by

$$\pi = B \frac{\mu_A \pi^{\alpha} - \mu_A \pi^{\alpha} (1 - \alpha)}{K} K - A - rK;$$

where  $E = (A=K)^{\alpha}$ ;  $E_e = (A=K)_e^{\alpha}$  and  $\alpha < 1$ : The first-order conditions for the maximization of profits are

$$B(1 - \alpha) \frac{\mu_A \pi^{\alpha} - \mu_A \pi^{\alpha} (1 - \alpha)}{K} = r; \quad (45)$$

$$\alpha B \frac{\mu_A \pi^{\alpha} - \mu_A \pi^{\alpha} (1 - \alpha)}{K} = 1; \quad (46)$$

The first condition establishes that the firm uses capital until its net marginal productivity equals the interest rate, and the second that the firm spends in pollution abatement until its marginal productivity is equal to its opportunity cost.

The market is in equilibrium when the value of  $E$  which maximizes profits coincides with the value that the firm consider to be exogenously determined,

<sup>13</sup>This approach was used by Lucas (1998) to analyze the external effects of human capital on the technology of the economy. Here, we adapt this approach to represent the external effects of environmental quality.

$E_e$ , such that the expected and the present behaviour of this variable are the same. Setting this condition,  $E = E_e$ ; the previous conditions become

$$B(1 - i^e) \frac{\mu_A \pi^{1-\alpha}}{K} = r; \quad (47)$$

$$i^e B \frac{\mu_A \pi^{1-\alpha}}{K} = 1; \quad (48)$$

Taking the Keynes-Ramsey rule into account, we now obtain from (47) the equilibrium condition in the asset market

$$B(1 - i^e) \frac{\mu_A \pi^{1-\alpha}}{K} = \frac{1}{2} + \frac{3}{4}g_C; \quad (49)$$

which gives us the following expression for the growth rate of consumption

$$g_C = \frac{1}{\frac{3}{4}} \left[ B(1 - i^e) \frac{\mu_A \pi^{1-\alpha}}{K} - \frac{1}{2} \right]; \quad (50)$$

Furthermore, the goods market equilibrium condition can be expressed as

$$g_K = B \frac{\mu_A \pi^{1-\alpha}}{K} - i \frac{C}{K} - \frac{A}{K}; \quad (51)$$

and if we set  $g_M = g_K = g_C$ ; the market equilibrium path is determined by conditions (48), (50) and (51). Specifically, (48) defines the level of environmental quality, (50) the balanced growth rate and (51) the consumption-capital ratio. Thus for the decentralized solution, the level of environmental quality can be obtained in explicit form

$$\frac{\mu_A \pi^{1-\alpha}}{K}_M = \frac{1}{B} \frac{1}{1 - i^e}; \quad (52)$$

and hence the values of growth rate and the consumption-capital ratio.

If we continue by comparing the two growth paths, we obtain that the comparison between the levels of environmental quality is immediate. Observe that the efficient level is higher than the level  $(A=K)_C^*$  defined by expression (39), see Figure 1, and that this expression is higher than the environmental quality level associated with the equilibrium path (52), so that we get

$$\frac{\mu_A \pi^{1-\alpha}}{K}_M < \frac{\mu_A \pi^{1-\alpha}}{K}_C < \frac{\mu_A \pi^{1-\alpha}}{K}_P; \quad (53)$$

This difference is explained by the external effects of environmental quality. To evaluate the allocative distortion which these external effects generate, we

rewrite the optimality condition (30), which characterizes the efficient path, as follows

$$\frac{r + \frac{C}{K}}{K} \mu_A \pi^{(1-\alpha)i-1} + (1-\alpha)B \frac{\mu_A \pi^{i-1}}{K} + \alpha B \frac{\mu_A \pi^{i-1}}{K} = 1;$$

or in terms of the functions of the model

$$\frac{U_E}{U_C} E_A + f_E^e E_A^e + f_E^i E_A^i = 1; \quad (54)$$

where  $f_E^e E_A^e$  is the increase in production (external effect) caused by a marginal increment of pollution abatement and  $f_E^i E_A^i$  is the increase of own production (internal effect), and we compare this expression with (48). The result is that, when the firms decide on pollution abatement, they do not take into account the positive external effects of their decisions on consumer welfare and on the productivity of other firms. The positive external effect on consumer welfare is represented by the first term on the left-hand side,  $U_E E_A = U_C$ ; and is given by the increment in utility due to the increment in environmental quality resulting from the marginal increase in pollution abatement. The positive external effect on productivity is represented by the second term on the left-hand side,  $f_E^e E_A^e$ ; and measures the increment in the productivity of capital owing to the increment in environmental quality caused by the marginal increase in pollution abatement. Thus, the positive effect of A is undervalued in the decentralized solution, as a result of which less resources are allocated to pollution abatement in comparison with the Pareto efficient allocation.

If we now rewrite condition (33), we can compare it with (49) and find out what the bias existing in the accumulation of capital is

$$B(1-\alpha) \frac{\mu_A \pi^{i-1}}{K} + (1-\alpha)B \frac{\mu_A \pi^{i-1}}{K} = r + \frac{r + \frac{C}{K}}{K} \mu_A \pi^{(1-\alpha)i-1}; \quad (55)$$

or in terms of the functions of the model

$$f_K + f_E^i E_K^i + f_E^e E_K^e = r + \frac{U_E}{U_C} E_K; \quad (56)$$

where  $f_K + f_E^i E_K^i$  is the increase of own production (internal effect) caused by a marginal increment of capital stock whereas  $f_E^e E_K^e$  represents the external effect on productivity. Moreover,  $U_E E_K = U_C$  is the negative external effect on consumer welfare due to the decrease in environmental quality resulting from the marginal increase in capital stock. For this reason the negative effect of K is undervalued in the decentralized solution and the firms keep

their capital stock above the efficient level in each moment of time. This explains why the level of environmental quality is lower for the equilibrium path.

To compare the growth rates of both solutions, we proceed to determine the relative position of the functions  $g_C(A=K)$ ; which are given by expressions (36) and (50). To clarify our notation, we represent the function corresponding to the efficient solution by  $g_{PC}$  and that corresponding to the market solution by  $g_{MC}$ : If we subtract one function from the other, we obtain

$$g_{PC} - g_{MC} = \frac{1}{\beta} \left( \frac{A}{K} \right)^{\alpha} - \frac{\mu}{\beta} \left( \frac{A}{K} \right)^{\alpha-1} \frac{1}{\beta} \quad (57)$$

This difference is zero for  $A=K = 0$  and  $(A=K)_M$ ; since for the environmental quality level corresponding to the equilibrium path, condition (48) is satisfied, so that the difference is positive for values of  $A=K$  lower than  $(A=K)_M$  and negative for values greater than  $(A=K)_M$ : Given these relationships we obtain the following graphical representation

[ FIGURE 2 ]

In the graph, the equilibrium growth rate is higher than the efficient growth rate. However, we cannot rule out a different relationship between these two growth rates, because the results only establish that the intersection point between the two functions must be on the increasing section of the function  $g_{PC}$ ; which does not exclude that the equilibrium growth rate may be lower than the efficient rate. This will depend on the magnitude of the external effects on production. In the model the magnitude of the external effects on production is represented by  $\beta$ : If the value of this parameter is close to  $\beta$  the magnitude of the external effects on production is low and the environmental quality level of the equilibrium path is not far from  $(A=K)_C$ ; so that we can expect that the market growth rate is above the efficient level. However, if the external effects are important,  $\beta$  is close to zero, and both the environmental quality level and the growth rate are lower than the efficient levels.

### 4.3 Pigouvian taxes-subsidies

The previous analysis of the equilibrium and efficient paths allows us now to address the design of the environmental policy which lets the economy reach the efficient path as a market equilibrium. Thus, this subsection deals with illustrating how Pigouvian taxes can be used to promote efficiency, which moreover in this case means promoting environmental conservation.

Let us consider the following taxation scenario on firms: a proportional tax on production ( $\zeta_Y$ ) combined with a proportional subsidy on pollution abatement ( $\zeta_A$ ): In this case the firm's profits are given by

$$\pi = (1 - \zeta_Y)B \frac{\mu_A \pi^{\alpha}}{K} - \frac{\mu_A \pi^{\alpha(1-\beta)}}{K} - rK - \tau; \quad (58)$$

where  $\tau$  is a lump-sum tax or subsidy, which is exogenously determined by the government, equal to the budgetary balance resulting from the taxation scenario which we have just defined:  $\tau = \zeta_Y Y - \zeta_A A$ : In this case, the first order conditions for the maximization of profits (47) and (48) can be written as

$$(1 - \zeta_Y)B(1 - \beta) \frac{\mu_A \pi^{\alpha-1}}{K} = r; \quad (59)$$

$$(1 - \zeta_Y)^{\beta} B \frac{\mu_A \pi^{\alpha-1}}{K} = 1 - \zeta_A; \quad (60)$$

and the asset market equilibrium condition is

$$g_M(\zeta_Y) = \frac{1}{\beta} (1 - \zeta_Y)B(1 - \beta) \frac{\mu_A \pi^{\alpha-1}}{K} - \frac{\beta}{2}; \quad (61)$$

Using the efficient values for  $g$  and  $A=K$ , the conditions (60) and (61) allow us to calculate the optimal values for  $\zeta_Y$  and  $\zeta_A$ : We can rewrite condition (61), making  $g_M(\zeta_Y^*) = g_P$ ;  $(A=K) = (A=K)_P$  and changing the order of the terms,

$$g_P = \frac{1}{\beta} B \frac{\mu_A \pi^{\alpha-1}}{K}_P - \frac{\beta}{2} = \frac{1}{\beta} \zeta_Y^* B \frac{\mu_A \pi^{\alpha-1}}{K}_P + (1 - \zeta_Y^*)^{\beta} B \frac{\mu_A \pi^{\alpha-1}}{K}_P;$$

Substituting (36) for  $g_P$  and dividing by  $(A=K)_P$  yields

$$1 = \zeta_Y^* B \frac{\mu_A \pi^{\alpha-1}}{K}_P + (1 - \zeta_Y^*)^{\beta} B \frac{\mu_A \pi^{\alpha-1}}{K}_P;$$

As the second term on the right-hand side is equal to  $1 - \zeta_A^*$  according to (60), we obtain the following relationship between the optimal values

$$\zeta_A^* = \zeta_Y^* B \frac{\mu_A \pi^{\alpha-1}}{K}_P; \quad (62)$$

This relationship allows us to conclude that the proposed taxation scenario is self-financing ( $\tau = 0$ ): Multiplying the previous equality by  $A_P$ ; we get

$$\zeta_A^* A_P = \zeta_Y^* B \frac{\mu_A \pi^{\alpha-1}}{K}_P K_P = \zeta_Y^* Y_P;$$

Finally, we eliminate  $\lambda_A^a$  from (60) using (62), and we solve to obtain the optimal value for the tax rate

$$\lambda_Y^a = \frac{1}{1 - \lambda_Y^a} \left[ \frac{1}{B} \frac{\mu_A}{K} \frac{\eta_{i_1}}{P} \right] \lambda_A^a \quad (63)$$

This value is positive because  $\lambda_Y^a < 1$  and the difference within the bracket is positive. Remember that  $\frac{1}{B} \frac{\mu_A}{K} \frac{\eta_{i_1}}{P} < 1$ ; see (31) in Subsection 4.1, which implies that  $\lambda_Y^a < \frac{1}{B} \frac{\mu_A}{K} \frac{\eta_{i_1}}{P}$  and as  $\lambda_Y^a < 1$ ; the result is that the difference within the bracket in (63) is positive. Now, substituting (63) in (62), we get the optimal value for the subsidy rate

$$\lambda_A^a = \frac{1}{1 - \lambda_Y^a} \left[ \frac{1}{B} \frac{\mu_A}{K} \frac{\eta_{i_1}}{P} \right] \lambda_Y^a \quad (64)$$

Observe that the system of equations to calculate the rates is completed with equation (38), which defines implicitly the environmental quality level corresponding to the efficient path.

Conditions (59) and (60) can be also written as

$$(1 - \lambda_Y^a)(f_K + f_E^i E_K^i) = r; \quad (65)$$

$$(1 - \lambda_Y^a)f_E^i E_A^i = 1 - \lambda_A^a; \quad (66)$$

so that if we compare them with the optimality conditions of efficient solution (54) and (56) we obtain the following expressions for the optimal values of the tax and subsidy rates

$$\lambda_Y^a (f_K + f_E^i E_K^i) = \lambda_Y^a f_E^e E_K^e + \frac{U_E E_K}{U_C} \eta; \quad (67)$$

$$\lambda_A^a = f_E^e E_A^e + \lambda_Y^a f_E^i E_A^i + \frac{U_E E_A}{U_C}; \quad (68)$$

Thus, we can check that the proportional tax on output reduces the net productivity of capital for an amount equal to the negative external effects of capital on production and consumers' welfare, whereas the proportional subsidy on pollution abatement is equal to the positive external effects of pollution abatement plus an additional term that compensates to the harm of the reduction of pollution abatement productivity caused by the tax on output.

Finally, it is easy to show from (63) and (64) that an increase of the marginal willingness to pay for environmental quality requires a higher tax and subsidy because of the external effects on consumers' welfare increase

with the marginal willingness to pay. However, an increase of the external effects on production, a decrease of  $\bar{e}$ ; has an ambiguous effect on taxation. The reason of this ambiguity is that the internal net productivity of the capital, in the left-hand side of (67), also increases with  $\bar{e}$ ; see the first term of the left-hand side of (55), since the negative effect of capital on its own productivity, through its negative effect on environmental quality, decreases. Notice that we assume that the parameter  $\bar{e}$ ; which determines the total productivity of environmental quality, see (4), does not change.

#### 4.4 Regulation of emissions

Finally, we study what the effects of an environmental policy based on direct control of pollution are. As occurred in Subsection 3.3, this kind of policy establishes a linear relationship between  $A$  and  $K$ ,  $A = \bar{e}^{1-\alpha} K$ ; so that the firm's profits are given by

$$\pi = BK\bar{e}^{1-\alpha} - rK - A = BK\bar{e}^{1-\alpha} - rK - \bar{e}^{1-\alpha} K; \quad (69)$$

and the first order condition for the maximization of profits can be written as  $r = B\bar{e}^{1-\alpha} - \bar{e}^{1-\alpha}$ : In this case, the equilibrium condition in the asset market determines the balanced growth rate of the economy

$$g_M(\bar{e}) = \frac{1}{3} B\bar{e}^{1-\alpha} - \frac{1}{2} \bar{e}^{1-\alpha}; \quad (70)$$

As this is a concave function with respect to the instrument of environmental policy, for a enough high value of  $B$  the function has a unique maximum associated with a positive growth rate, so that between the critical values for which  $g_M(\bar{e}) = 0$ , the relationship between  $\bar{e}$  and  $g_M$  is positive if  $\bar{e}$  is lower than the maximum and negative when  $\bar{e}$  is higher. Based on this relationship we find that a tighter environmental policy, starting from low levels of pollution control, has beneficial effects not only on environmental quality but also on growth rate. Thus, for low levels of pollution control, there is no conflict between growth and environment, so that a higher environmental preservation is compatible with a higher economic growth. Furthermore, if the initial level of environmental quality is very low and  $\bar{e}$  is increased and fixed at the efficient level, there can be a positive effect on growth rate, although the efficient combination  $[(A=K)_P; g_P]$  is located on the decreasing section of the function  $g_M(\bar{e})$ : However, if the initial level of environmental quality is high but lower than the efficient one, an increase in pollution control could have negative effects on the growth rate. This result also establishes that very low levels of environmental quality can bring the very process of economic growth into question.

## 5 Conclusions

In this paper we have developed a model of endogenous growth a la Rebelo in which environmental quality, which depends positively on pollution abatement and negatively on capital stock, has positive effects on the utility of consumers and on the productivity of capital. We have analyzed in the framework of this model the effect of different environmental policies on the growth of the economy.

Our results establish that greater concern of individuals for the environment reduces the long-term growth rate. This is so because, for the optimal values, the marginal effect of environmental quality on the productivity of capital is lower than the marginal effect of environmental quality on the opportunity cost of capital, so that an increase of environmental quality has a negative effect on the growth rate since the marginal cost of capital increases more than its marginal productivity. For this reason the growth rate must decrease in order to recover the equilibrium in the asset market when the willingness to pay for the environmental quality increases..

Furthermore, we show that the level of environmental quality associated with the market equilibrium path is below the efficient level, while the growth rate may be higher or lower depending on the extent of the external environmental effects on productivity. In this case, a policy which taxes production and subsidizes pollution abatement would have a favorable effect on environmental quality, and could increase the growth rate of the economy if the external effects are important. In addition, we prove that this policy is neutral since it does not affect the budgetary balance of the government because tax receipts are equal to expenditures, for this reason it could be implemented without having to resort to lump-sum taxes/subsidies. Finally, we show that a policy based on emissions control has a positive effect on growth rate only if the initial level of environmental quality is sufficiently low, since this is the only case where the positive effect on productivity compensates for the negative effect on the accumulation of capital which results from the use of resources for pollution abatement.

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