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# **Negotiation and Optimality in an Economic Model of Global Climate Change**

**Hans W. Gottinger**

Abstract.

We suggest a two-country , two-sector model as a basis for the control of global climate change in which the dynamic time path of the world economy is analyzed under the provision that the outcomes of a negotiation game generate the global optimal solution.

JEL Classification D0,D6,H4,Q3

**Negotiation and Optimality in an Economic Model of Global Climate Change**

## **1. Introduction**

In a related work (Gottinger, 1998, chapt.7) we have explored conditions under which the time path of the world economy and climate may be a cyclical or even chaotic, in a two-sector competitive equilibrium model. There are two major sets of questions that were left unanswered:

(1) What are the characteristics of the optimal time path of the world economy and climate? In particular, will the optimal time path converge to a steady state? (2) What will happen if the governments decide to correct the problem of global warming? Specifically, will they be able to achieve global (Pareto) optimality in competitive economies?

The purpose of this paper is to answer these questions . Using a two-country, two-sector general equilibrium model, it is shown that the global optimal time path of outputs and temperature will converge to a unique steady state provided that consumers care enough about the future. To answer the second set of questions, we study the equilibrium outcome in a bargaining game where two countries negotiate an agreement on future consumption and production plans for the purpose of correcting the problem of global warming. It is demonstrated that the agreement that arises from such a negotiation process achieves global optimality. It is also shown that the agreement can be implemented in decentralized economies by a system of taxes and transfers.

While most of the discussion in the literature about global environmental problems by economists points out the importance of international cooperation in coping with these problems (see, for example, Barrett 1990, Nordhaus 1990), there is few formal economic modeling of these issues in the literature. Examples include Barrett (1992), Heal (1993), Radner (1998) and Uzawa (1999). [Various game tools have been employed to approach a solution, here.] By employing recent advances in non-cooperative bargaining theory, the agreement between two and more countries is derived endogenously through a well specified bargaining procedure.

The paper is organized as follows. Section 2 describes the economic and natural environments of the model. Section 3 studies the characteristics of the global optimal path of temperature and outputs by solving a world planner's problem. Sections 4 and 5 demonstrate how the global optimality can be achieved through an agreement negotiated by the two governments and the characteristics of such an agreement. It is shown in Section 6 that such an agreement can be implemented in decentralized economies by means of a system of taxes and international transfers. Section 7 summarizes the conclusions.

## **2. The Background: Economic and Natural Environment**

In this section, the production, consumption, and climatic aspects of the model are specified. As in a previous paper (Gottinger, 1998) we shall consider a world that consists of two countries. The two countries have the same preferences, the same production technology, the same climate, but (maybe) different population sizes. Two goods can be produced, one of which is an agricultural good and the other a manufactured good. The productivity of the agricultural sector is affected by the global temperature. The manufacturing activities, on the other hand, affect temperature level.

The formal specification of the model follows.

Time, denoted  $t$ , is discrete and the horizon is infinite:  $t \in \{0, 1, \dots\}$ .

The world consists of two countries: Country  $H$  and Country  $F$ . Population in each country is constant over time.

Let the size of the world population be normalized to one country. Country  $H$  has a population of "size"  $\alpha$  while Country  $F$  has  $1 - \alpha$ . Population is immobile between countries. The two countries are assumed to have identical production technology, identical preferences and identical climate. In what follows, the production and consumption size of the model is specified for Country  $H$ . The variables of Country  $F$  which will be denoted by attaching a superscript " $*$ " can be specified in the same way.

On the production side of the world, two non-storable goods are produced: an agricultural good and a manufactured good, with quantities being denoted by  $S_1$  and  $S_2$  for  $H$ . Goods can be transported at zero cost.

There is a fixed continuum of firms in each industry in each country. Hence both industries are perfectly competitive. Labor is the only input of production. At each date a representative firm in industry  $i$  ( $i = 1, 2$ ) in Country  $H$  chooses the level of employment in the industry,  $\phi_{it}$ .

The production technology of both goods exhibits constant return to scale. The productivity of labor in the manufacturing sector does not depend on climate and is denoted by  $b$ . Country  $H$ 's output of the manufacturing sector at date  $t$  can then be written as  $S_{2t} = b\phi_{2t}$ . In the agricultural sector, however, the productivity of labor depends on one aspect of the climate, namely the global temperature. Let  $a(\tau_t)$  denote the productivity coefficient of the agricultural sector, i.e.,

$S_{1t} = a(\tau_t)\phi_{1t}$ , where  $\tau_t$  is the world average temperature level in period  $t$ . It is assumed that  $a(\tau_n) > 0$ ;  $a'(\tau_t) > 0$  if  $\tau_t < \bar{\tau}$  and  $a'(\tau_t) < 0$  if  $\tau_t > \bar{\tau}$ ;  $a(\tau_u) = 0$ ; and  $a''(\tau_t) < 0$ .  $\tau_n$  denotes the "natural" temperature level, i.e., the level at which the global temperature would stay in the absence of any manufacturing activities.  $\bar{\tau}$  is some critical level of temperature for the agriculture sector,  $\tau_n < \bar{\tau}$ .  $\tau_u$  ( $> \bar{\tau}$ ) is the temperature level at which the agricultural productivity equals zero. Therefore, by assumption the agricultural productivity is positive when there have been no manufacturing activities. A higher level of temperature improves the agricultural productivity as long as the temperature is below the critical value  $\bar{\tau}$ . As temperature level exceeds  $\bar{\tau}$ , however, higher temperature will reduce the productivity of the agricultural sector. The agricultural productivity eventually approaches zero as temperature level reaches  $\tau_u$ .

The consumer side of the economy comprises a fixed continuum of identical consumers. A representative consumer is endowed with one unit of labor endowment which is supplied inelastically. He has no initial wealth. His preference over consumption of the agricultural good and the manufactured good to date  $t$ , denoted by  $C_{1t}$  and  $C_{2t}$ , respectively, is represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}), \text{ where } U(\cdot) \text{ satisfies:}$$

(A1)  $U(C_1, C_2) \in C^2$  satisfies  $\lim_{C_1 \rightarrow 0} U_1(C_1, C_2) = +\infty$ ,  $\lim_{C_2 \rightarrow 0} U_2(C_1, C_2) = +\infty$  and that  $U(0,0) = 0$ ,

(A2)  $U(C_1, C_2)$  is concave and homogenous of degree  $\gamma$ .

(A3)  $U_{12}(C_1, C_2) \geq 0$ .

The law of motion of the global average temperature is characterized by a variation of the zero-dimensional climate system model presented in Dickinson (1986):

$$\tau_{t+1} = (1 - c)(\tau_t - \tau_n) + \tau_n + g(S_{2t} + S_{2t}^*) \quad (2.1)$$

where  $c \in (0,1)$ ,  $g(\cdot) \in C^2$ ,  $g(0) = 0$ ,  $g'(\cdot) > 0$  and  $g''(\cdot) \leq 0$ .

(2.1) states that the manufacturing activities raise temperature. When temperature is above its natural level,  $\tau_n$ , nature has the ability of absorbing a percentage of the excess greenhouse gases and cooling down the climate towards its natural level at a rate  $c$ . Since it is assumed that  $\tau_n < \bar{\tau}$ , starting from the point of time where no manufacturing activities had taken place in the past "some" manufacturing activities would be good for the production of the agriculture good by assumption. Without loss of generality, we choose the unit of temperature level such that  $\tau_n = 0$ .

### 3. The Global Optimal Solution

In this section, the problem faced by a hypothetical "world government" is presented and solved. It is demonstrated that the sequence of optimal outputs converges to a unique steady state as long as consumers care enough about the future. The world welfare possibility frontier is derived.

The objective of the world government is to maximize the weighted sum of per capita utilities of the two countries over an infinite time horizon. Let  $\psi$  ( $<1$ ) denote the weight attached to H's per capita welfare and  $1 - \psi$ , the weight attached to F's per capita welfare. Let  $L_{1t}$ ,  $L_{2t}$  be the labor input allocated to sector  $i$  ( $i = 1, 2$ ). The world government problem can then be written as:

$$\text{Max}_{\{C_{1t}, C_{2t}, C_{1t}^*, C_{2t}^*, L_{1t}, L_{2t}, \tau_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \{ \psi U(C_{1t}, C_{2t}) + (1 - \psi) U(C_{1t}^*, C_{2t}^*) \} \quad (3.1)$$

$$\alpha C_{1t} + (1 - \alpha) C_{1t}^* = a(\tau_t) L_{1t} = S_{1t} + S_{1t}^* \quad (3.2)$$

$$\alpha C_{2t} + (1 - \alpha) C_{2t}^* = b L_{2t} = S_{2t} + S_{2t}^* \quad (3.3)$$

$$L_{1t} + L_{2t} = 1 \quad (3.4)$$

$$\tau_{t+1} = (1 - c)\tau_t + g(S_{2t} + S_{2t}^*) \quad (3.5)$$

and  $\tau_0 \in [0, \tau_u)$  given.  $\beta$  is the discount factor in  $t$ .

In the above problem, given the sequence of the world outputs  $\{S_{1t} + S_{1t}^*, S_{2t} + S_{2t}^*\}_{t=0}^{\infty}$ , the allocation of the consumption goods among the two countries is a static problem and is governed by the following standard first-order conditions:

$$\psi U_1(C_{1t}, C_{2t}) = (1 - \psi) \frac{\alpha}{1 - \alpha} U_1(C_{1t}^*, C_{2t}^*) \quad (3.6)$$

$$\psi U_2(C_{1t}, C_{2t}) = (1 - \psi) \frac{\alpha}{1 - \alpha} U_2(C_{1t}^*, C_{2t}^*) \quad (3.7)$$

Define  $\omega = \frac{1-\psi}{\psi} \frac{\alpha}{1-\alpha}$ . Notice that  $\omega = 1$  if  $\psi = \alpha$ . (3.6)-(3.7) together with the assumption of homogeneity on  $U(\cdot)$ , of degree  $\gamma$  ( $\gamma < 1$ ), implies

$$C_{1t} = \frac{S_{1t} + S_{1t}^*}{\alpha + (1 - \alpha)\omega^{\frac{1}{1-\gamma}}} \quad (3.8)$$

$$C_{1t}^* = \frac{\omega^{\frac{1}{1-\gamma}}(S_{1t} + S_{1t}^*)}{\alpha + (1 - \alpha)\omega^{\frac{1}{1-\gamma}}} \quad (3.9)$$

$$C_{2t} = \frac{S_{2t} + S_{2t}^*}{\alpha + (1 - \alpha)\omega^{\frac{1}{1-\gamma}}} \quad (3.10)$$

$$C_{2t}^* = \frac{\omega^{\frac{1}{1-\gamma}}(S_{2t} + S_{2t}^*)}{\alpha + (1 - \alpha)\omega^{\frac{1}{1-\gamma}}} \quad (3.11)$$

Since  $g(\cdot)$  is monotonically increasing in its argument, one can define its inverse function  $G(\cdot) = g^{-1}(\cdot)$ . (3.5) can be written as

$$S_{2t} + S_{2t}^* = G(\tau_{t+1} - (1 - c)\tau_t) \quad (3.12)$$

The assumption on  $g(\cdot)$  implies that  $G(0) = 0$ ,  $G'(\cdot) > 0$  and  $G''(\cdot) \geq 0$ .

Using (3.2)-(3.4), (3.8)-(3.12) and the homogeneity property of  $U(\cdot)$ , one can rewrite (3.1) as

$$W(\tau_0; \psi) = \max_{\{\tau_{t+1}\}_{t=0}^{\infty}} \left[ \psi \left( \frac{1}{a + (1-a)w^{\frac{1}{1-g}}} \right)^{\gamma} + (1-\psi) \left( \frac{\omega^{\frac{1}{1-\gamma}}}{\alpha + (1-\alpha)\omega^{\frac{1}{1-\gamma}}} \right)^{\gamma} \right] \cdot \sum_{t=0}^{\infty} b^t U[a(\tau_t)(1 - \frac{1}{b}G(\tau_{t+1} - (1-c)\tau_t)), G(\tau_{t+1} - (1-c)\tau_t)] \quad (3.13)$$

Notice that if  $\psi = \alpha$ ,  $W(\tau_0; \psi)$  is independent of  $\psi$  and  $\alpha$ . Therefore, we can define  $W(\tau) \equiv W(\tau; \alpha)$ .

**Proposition 3.1.** Assume (A.1)-(A.2). The global optimal sequence of outputs  $\{S_{1t} + S_{1t}^*, S_{2t} + S_{2t}^*\}_{t=0}^{\infty}$  is independent of  $\psi$  and  $1 - \psi$ .

Proof: The solution to (3.13) is the same as the solution to

$$W(\tau_0) = \max_{\{\tau_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U[a(\tau_t)(1 - \frac{1}{b}G(\tau_{t+1} - (1-c)\tau_t)), G(\tau_{t+1} - (1-c)\tau_t)] \quad (3.14)$$

which is independent of  $\psi$ .

The assumption of homogeneity of  $U(\cdot)$  is crucial to obtaining Proposition 3.1. The homogeneity of  $U(\cdot)$  implies that in any period the two countries will consume the two goods at the same proportion. When the weight parameter  $\psi$  changes, what the world planner has to do is reducing the consumption of both goods in one country and increasing the consumption of both goods in the other country by the same proportion, leaving the optimal world output mix unchanged.

Proposition 3.1 suggests that the sequence of optimal temperature can be solved from (3.14).

Define

$$V(\tau_t, \tau_{t+1}) = U[a(\tau_t)(1 - (1/b)G(\tau_{t+1} - (1-c)\tau_t)), G(\tau_{t+1} - (1-c)\tau_t)] \quad (3.15)$$

In order to apply the technique of dynamic programming, we define two sets. Let  $T$  denote the set of feasible values for the state variable  $\tau_t$ .  $T = [0, \tau_u]$  by definition. Let  $\Gamma(\tau)$  be the set of feasible values for the state variable next period given the current state  $\tau \in T$ . (3.3)-(3.5) implies that  $\Gamma(\tau) = [(1-c)\tau, \min\{(1-c)\tau + g(b), \tau_u\}]$ . Let  $A$  be the graph of  $\Gamma$ .

$$A = \{(\tau, \tau'), \tau \in T \times T : \tau' \in \Gamma(\tau)\} \quad (3.16)$$

The following assumptions are needed in establishing the concavity of  $V(\cdot)$ :

$$(A.4) \quad -\frac{g''(S)S}{g'(S)} < 1-\gamma.$$

$$(A.5) \quad -\frac{a(t)a''(t)}{(a'(t))^2} > \frac{1}{1-g}.$$

**Lemma 3.2.**  $T$  is a convex subset of  $\mathbb{R}$  (real line), and the correspondence  $\Gamma: T \rightarrow T$  is nonempty, compact-valued and continuous. Furthermore,  $\Gamma$  is convex in the sense that, for any  $\theta \in (0,1)$  and  $\tau_a, \tau_b \in T$ ,  $\tau'_a \in \Gamma(\tau_a)$  and  $\tau'_b \in \Gamma(\tau_b)$  implies

$$\theta \tau'_a + (1-\theta)\tau'_b \in \Gamma[\theta \tau_a + (1-\theta)\tau_b].$$

Proof: Obvious from the definition of  $T$  and  $\Gamma$ .

**Lemma 3.3.** Assume (A1)-(A5). The function  $V: A \rightarrow \mathbb{R}$  is continuous, bounded and strictly concave. Furthermore,  $V$  is continuously differentiable in the interior of  $A$ .

Proof: The continuity and differentiability of  $V$  is obvious given (A1) and the specifications of  $g(\cdot)$  and  $a(\cdot)$ .  $V(\cdot)$  is bounded below by 0 and bounded above by  $U(a(\bar{\tau}), b)$ .

$$\begin{aligned} V_{22} = & U_{11}(\cdot) \left(\frac{a}{b}\right)^2 [G'(\cdot)^2 - 2U_{12}(\cdot) \frac{a}{b} G'(\cdot)]^2 - \frac{1}{1-\gamma} U_{12}(\cdot) C_1 G''(\cdot) \\ & - U_{11}(\cdot) \frac{a}{b} G''(\cdot) - U_{22}(\cdot) C_2 G''(\cdot) \left[ -\frac{g'(S)}{S g''(S)} - \frac{1}{1-\gamma} \right] < 0 \end{aligned} \quad (3.17)$$

It can be shown that  $V_{11}V_{22} - V_{12}^2 > 0$  given (A1)-(A5). Therefore,  $V(\tau_t, \tau_{t+1})$  is concave in  $(\tau_t, \tau_{t+1})$ .

Consider the functional equation

$$W(\tau) = \max_{\tau' \in \Gamma(\tau)} [V(\tau, \tau') + \beta W(\tau')] \quad (3.18)$$

**Proposition 3.4.** Assume (A1)-(A5). There exists a continuous, single-valued function  $h_\beta(\tau)$  that solves the functional equation (3.18). Furthermore,  $W$  is bounded and strictly concave.

Proof: Lemmas 3.2 and 3.3 establish conditions needed to apply Theorems 4.6 and 4.8 in Stokey and Lucas (1989).

**Proposition 3.5.** Assume (A1)-(A5).  $\tau_{t+1} = h_\beta(\tau_t)$  solves (3.14).



Proof: Since  $W(\tau)$  is bounded,  $\lim_{n \rightarrow \infty} \beta^n W(\tau) = 0$ . The result follows from Theorem 4.3 of Stokey and Lucas (1989).

The first order condition to (3.18) can be written as

$$-V_2(\tau_t, \tau_{t+1}) = \beta W'(\tau_{t+1}) = \beta V_1(\tau_{t+1}, \tau_{t+2}) \quad (3.19)$$

Propositions 3.4 and 3.5 state that the solution to (3.19),  $\tau_{t+1} = h_\beta(\tau_t)$ , is continuously differentiable and that  $W(\tau)$  is concave.

**Lemma 3.6.** Assume (A1)-(A5). There exists  $\tau_s \in (0, \tau_u)$  such that  $\tau_s = h_\beta(\tau_s)$ .

Proof. Substitute  $\tau_s$  for  $\tau_{t+i}$  ( $i = 0, 1, 2$ ) in (3.19)

$$-V_2(\tau_s, \tau_s) = \beta V_1(\tau_s, \tau_s) \quad (3.20)$$

Using (3.15) one can rewrite (3.20) as:

$$\begin{aligned} & [1 - \beta(1-c)]G'(c\tau_s) \left[ \frac{a(\tau_s)}{b} - \frac{U_2(a(\tau_s)(1 - b^{-1}G(c\tau_s)), G(c\tau_s))}{U_1(a(\tau_s)(1 - b^{-1}G(c\tau_s)), G(c\tau_s))} \right] \\ & = \beta a'(\tau_s) \left( 1 - \frac{1}{b}G(c\tau_s) \right) \end{aligned} \quad (3.21)$$

Denote the left-hand side of (3.21) by LHS and the right-hand side by RHS. Since  $a(0) > 0$  and  $G(0) = 0$ , then  $S_1 + S_1^* > 0$  and  $S_2 + S_2^* = 0$  if  $\tau_s = 0$ , which implies that  $\lim_{\tau \rightarrow 0} \frac{U_2(\cdot)}{U_1(\cdot)} = +\infty$ . Notice both  $G'(\cdot)$  and  $a'(\cdot)$  are finite. Therefore,  $LHS < 0 < RHS$  as  $\tau_s$  approaches 0.

$$a(\tau_u) = 0 \text{ implies that } S_1^* + S_1 = 0 \text{ and that } S_2 + S_2^* > 0 \text{ if } \tau_s = \tau_u. \text{ Thus } \lim_{\tau \rightarrow \tau_u} \frac{U_2(\cdot)}{U_1(\cdot)} = 0.$$

Hence,  $LHS > 0 > RHS$  as  $\tau_s$  approaches  $\tau_u$ .

By continuity there exists  $\tau_s \in (0, \tau_u)$  such that (3.21) holds.

**Lemma 3.7.** Assume (A1)-(A5). There exists  $\bar{\beta} < 1$  so that if  $\beta > \bar{\beta}$ ,  $\tau_s$  is unique.

Proof: The strict concavity of  $V$  implies that there exists a  $\bar{\beta} < 1$  so that  $\beta V_{11} + V_{22} + (1 + \beta)V_{12} < 0$  for all  $\beta > \bar{\beta}$ . Therefore, for all  $\beta \in (\bar{\beta}, 1)$ ,  $V_2(\tau, \tau) + \beta V_1(\tau, \tau)$  is a decreasing function in  $\tau$ , which implies that  $\tau_s$  is unique.

**Lemma 3.8.** Assume (A1)-(A5). There exists  $\hat{\beta} < 1$  so that if  $\beta > \hat{\beta}$ ,  $h_\beta(\tau)$  has no periodic point of

period  $n \geq 2$ .

Proof: This result is essentially the same as Lemma 3 in Deneckere and Pelikan (1986). Since  $V(\tau, \tau')$  is strictly concave and  $h_\beta(\tau)$  is continuous, Deneckere and Pelikan's (1986) proof can be directly applied with proper changes in notations.

**Proposition 3.9.** Assume (A1)-(A5). There exists  $\bar{\beta} < 1$  so that if  $\beta > \bar{\beta}$ ,  $\{h_\beta^j(\tau)\}_{j=1}^\infty$  converges to a unique steady state,  $\tau_s$ , for all  $\tau_0 \in [0, \tau_u]$ .

Proof:  $h_\beta(\tau)$  is a continuous function that maps  $[0, \tau_u]$  into itself. Define  $\bar{\beta} = \max(\bar{\beta}, \hat{\beta})$ . The result follows Lemmas 3.7 and 3.8 and the main theorem in Coppel (1955).

Proposition 3.9 states that the global optimal temperature will converge to a unique steady state in the long-run provided that consumers care enough about the future.

Given the sequence of optimal temperatures, the sequence of optimal outputs can be determined accordingly using (3.2)-(3.4) and (3.12). Next we shall study how the optimal outputs are allocated among the two countries. Specifically, we shall find out how the per capita welfare of each country

varies with  $\psi$ . Define  $\pi \equiv \sum_{t=0}^\infty \beta^t U(C_{1t}, C_{2t})$  and  $\pi^* \equiv \sum_{t=0}^\infty \beta^t U(C_{1t}^*, C_{2t}^*)$ .  $\pi$  and  $\pi^*$  are the

discounted sum of the per capita utility for H and F, respectively. Using (3.8)-(3.11) and homogeneity of  $U(\cdot)$ , one obtains

$$\pi = \left( \frac{1}{\alpha + (1-\alpha)\omega^{\frac{1}{1-\gamma}}} \right)^\gamma W(\tau_0) \quad (3.22)$$

$$\pi^* = \left( \frac{\omega^{\frac{1}{1-\gamma}}}{\alpha + (1-\alpha)\omega^{\frac{1}{1-\gamma}}} \right)^\gamma W(\tau_0) \quad (3.23)$$

Given  $\tau_0$ ,  $W(\tau_0)$  is a constant. The value of  $\pi$  and  $\pi^*$  depends on  $\psi$

**Proposition 3.10 .** Assume (A1)-(A5).  $\pi = \pi^* = W(\tau_0)$  if  $\psi = \alpha$ .

$$\pi > \pi^* \text{ if } \psi > \alpha.$$

$$\pi < \pi^* \text{ if } \psi < \alpha.$$

Proof: Obvious from (3.22) and (3.23)

If the world government assigns the welfare weight in proportion to the country's population size, every citizen in the world will enjoy the same level of welfare. If a country is assumed a weight that is larger than her proportion in the world population, however, a citizen in this country will have a higher welfare

level than in the other country.

It is easy to verify that as  $\psi$  increases,  $\pi$  increases but  $\pi^*$  decreases. Therefore, we can define "the world welfare possibility frontier" by  $\pi^* = v^*(\pi)$  where  $v^*(\pi)$  is solved from (3.22) and (3.23):

$$\pi^* = v^*(\pi) = (1 - \alpha)^{-\gamma} (W^{\frac{1}{\gamma}}(\tau_0) - \alpha \pi^{\frac{1}{\gamma}})^{\gamma} \quad (3.24)$$

It is clear that  $\pi^*$  is decreasing in  $\pi$ . Notice that if  $\gamma = 1$ , (3.24) is a linear function that can be written as

$$\alpha\pi + (1 - \alpha)\pi^* = W(\tau_0) \quad (3.25)$$

#### 4. Negotiation

In the next two sections, the outcome of a negotiation process between the two countries are studied. In these two sections, the assumption of competitive economies is abandoned temporarily and is replaced by the assumption that there is a central planner in each country. The two central planners can negotiate a binding agreement on future production and consumption plans. The equilibrium outcome of this negotiation process is derived in Section 4 while the properties of the equilibrium outcome is analyzed in Section 5.

Consider a world where competitive equilibrium has been prevailing in the past. To simplify analysis, assume that the world has reached a steady state under competitive equilibrium (as derived in Gottinger (1999)). Let  $\tau_c$  denote the steady state temperature under competitive equilibrium.  $\tau_0 = \tau_c$  by assumption. Assume that in each country, there is a central planner who has the authority to choose consumption and production plans for the citizens in his country. The objective function of the central planner is to maximize the per capita utility of his citizens over an infinite time horizon. The central planners are identified by the country they represent (ie. H or F)

Suppose that in period 0, the two planners suddenly realize the importance and urgency of the global warming problem and decide to negotiate an agreement on future production and consumption plans over an infinite time horizon for both countries. F immediately replies "Yes" or "No". If F says "Yes", an agreement is reached and the two planners will implement the agreement from period 0 on. If F replies "No", the planners will continue to allow the competitive equilibrium to prevail in period 0 and wait until period 1 when the second round of negotiation begins. At the beginning of period 1, the same process is repeated except that now F makes a proposal to which H immediately replies, and so on. We shall seek the subgame perfect equilibrium of this game, as outlined by Fudenberg and Tirole (1989).

Define  $\pi(s) \equiv \sum_{t=s}^{\infty} \beta^t U(C_{1t}, C_{2t})$  and  $\pi^*(s) \equiv \sum_{t=s}^{\infty} \beta^t U(C_{1t}^*, C_{2t}^*)$ .  $\pi(s)$  and  $\pi^*(s)$  are the payoffs received by the two countries in a subgame starting from period  $s$  ( $s = 0, 1, 2, \dots$ ). In any subgame

starting from period  $s$  ( $s = 0, 2, 4, \dots$ ), the problem faced by H is:

$$\text{Max } \pi(s) \quad (4.1)$$

subject to (3.2) - (3.5) and

$$\pi^*(s) \geq \bar{\pi}^* \quad (4.2)$$

where  $\bar{\pi}^*$  is the minimum payoff that H believes F must obtain in a subgame perfect equilibrium. The problem faced by F in any subgame starting from period  $s$  ( $s = 1, 3, \dots$ ) is symmetric to the problem (4.1) – (4.2) faced by H.

**Proposition 4.1.** Assume (A.1) – (A.5). Any subgame perfect equilibrium in this game is globally optimal.

Proof: It can be verified that the problem (4.1) – (4.2) generates the same set of first –order conditions as the world planner’s problem (3.1) with appropriately chosen  $\psi$ . Therefore, the proposal made by H or F in any subgame is globally optimal

Therefore, any subgame perfect equilibrium must be a point on the world welfare possibility frontier as given by (3.24). In other words, any subgame perfect equilibrium will generate the sequence of optimal world outputs, denoted by  $\{\hat{S}_{1t} + \hat{S}_{1t}^*, \hat{S}_{2t} + \hat{S}_{2t}^*\}_{t=0}^{\infty}$ .

Let  $\pi$  and  $\pi^*$  be the subgame perfect equilibrium payoff to H and F, respectively. Suppose the sequence  $\{\hat{C}_{1t}, \hat{C}_{2t}, \hat{C}_{1t}^*, \hat{C}_{2t}^*\}_{t=0}^{\infty}$  yields a payoff  $\hat{\pi}$  to H and  $\hat{\pi}^*$  to F. (3.8) – (3.11) and Proposition 4.1 imply that  $\hat{C}_{it}, \hat{C}_{it}^*$  ( $i=1,2$ ) are proportional to the world output of good  $i$ . Define  $\lambda$  as Country H’s share of consumption of good  $i$  in the world total output of good  $i$ , i.e.

$$\alpha \hat{C}_{it} = \lambda (\hat{S}_{it} + \hat{S}_{it}^*) \quad (4.3)$$

Then,

$$(1-\alpha) \hat{C}_{it}^* = (1-\lambda) (\hat{S}_{it} + \hat{S}_{it}^*) \quad (4.4)$$

Using (4.3) – (4.4) and the homogeneity assumption of utility function, one obtains:

$$\frac{\hat{\pi}}{\hat{\pi}^*} \equiv \frac{\sum \beta^t U(\hat{C}_{1t}, \hat{C}_{2t})}{\sum \beta^t U(\hat{C}_{1t}^*, \hat{C}_{2t}^*)} = \frac{(1-\alpha)^\gamma \lambda^\gamma}{\alpha^\gamma (1-\lambda)^\gamma} \quad (4.5)$$

which implies that

$$\lambda = \frac{\alpha \hat{\pi}^{\frac{1}{\gamma}}}{\alpha \hat{\pi}^{\frac{1}{\gamma}} + (1 - \alpha)(\hat{\pi}^*)^{\frac{1}{\gamma}}} \quad (4.6)$$

Therefore, if we are given a pair of equilibrium payoffs  $(\hat{\pi}, \hat{\pi}^*)$ , we can uncover the sequence of consumptions that are implied by these payoffs. Hence we can characterize the outcome of the game by the equilibrium payoffs.

The subgame perfect equilibrium in this game can be derived by using similar arguments as in Sutton (1986). Let  $\bar{u} (>0)$  be the per period utility a consumer obtains under the competitive steady state.  $\bar{u}$  is the same for both countries since the two countries have the same per capita consumption level under competitive equilibrium. If the world stayed in the competitive equilibrium forever, the payoff received by each country would be  $\sum_{t=0}^{\infty} \beta^t \bar{u} = \frac{1}{1-\beta} \bar{u}$ .

$W(\tau_c) > \frac{1}{1-\beta} \bar{u}$  by the definition of  $W(\tau_c)$ .

For the simplicity of notation, we define  $W \equiv W(\tau_c)$ .

Let  $\pi_m$  be the maximum payoff H will obtain in any subgame perfect equilibrium. Therefore,  $\pi_m$  is also the maximum payoff H will obtain in the subgame perfect equilibrium that starts from period 2. It is obvious that  $\pi_m$  must be greater than the payoff received from staying in competitive equilibrium. In other words,  $\pi_m > \frac{1}{1-\beta} \bar{u}$

Now consider period 1. If H rejects a proposal made by F, he obtains  $\bar{u}$  in period 1, plus  $\beta \pi_m$  at maximum in the future. Hence, any proposal that gives H a payoff higher than  $\bar{u} + \beta \pi_m$  will certainly be accepted, which implies that the minimum payoff F will obtain in period 1 is  $v^*(\bar{u} + \beta \pi_m)$ .

In period 0, F will reject any proposal that offers him a payoff smaller than  $\bar{u} + \beta v^*(\bar{u} + \beta \pi_m)$ , which means that the maximum payoff H will obtain is  $v^{*-1}[\bar{u} + \beta v^*(\bar{u} + \beta \pi_m)]$ . Therefore, by the definition of  $\pi_m$ ,

$$\pi_m = v^{*-1}[(\bar{u} + \beta v^*(\bar{u} + \beta \pi_m))] \quad (4.7)$$

Alternatively,

$$v^*(\pi_m) = \bar{u} + \beta v^*(\bar{u} + \beta \pi_m) \quad (4.8)$$

It can be shown that (4.8) holds when  $\pi_m$  is defined instead as the minimum payoff H receives in a subgame perfect equilibrium. Therefore (4.8) characterizes the subgame perfect equilibrium in this game.

**Proposition 4.2.** Assume (A.1) – (A.5). The unique subgame perfect equilibrium in this game is characterized by

$$(W^{\frac{1}{\gamma}} - \alpha \hat{\pi}^{\frac{1}{\gamma}})^{\gamma} - \beta [W^{\frac{1}{\gamma}} - \alpha(\bar{u} + \beta \hat{\pi})^{\frac{1}{\gamma}}]^{\gamma} = (1 - \alpha)^{\gamma} \bar{u} \quad (4.9)$$

and  $\hat{\pi} \geq \frac{1}{1-\beta} \bar{u}.$

Proof: (4.9) is obtained by rewriting (4.8) using (3.24). Define the left-hand-side of (4.9) as  $F(\pi)$ . For  $\pi \geq \frac{1}{1-\beta} \bar{u}$ ,

$$F'(\pi) = \frac{\alpha \pi^{\frac{1-\gamma}{\gamma}}}{(W^{\frac{1}{\gamma}} - \alpha \pi^{\frac{1}{\gamma}})^{1-\gamma}} + \frac{\alpha \beta^2 (\bar{u} + \beta \pi)^{\frac{1-\gamma}{\gamma}}}{[W^{\frac{1}{\gamma}} - \alpha(\bar{u} + \beta \pi)^{\frac{1}{\gamma}}]^{1-\gamma}} < 0 \quad (4.10)$$

Since,

$$F\left(\frac{1}{1-\beta} \bar{u}\right) = (1 - \beta) \left(W^{\frac{1}{\gamma}} - \alpha \left(\frac{1}{1-\beta} \bar{u}\right)^{\frac{1}{\gamma}}\right)^{\gamma} > (1 - \alpha)^{\gamma} \bar{u} \quad (4.11)$$

and

$$F\left(\frac{W}{\alpha^{\gamma}}\right) = \beta W \left[1 - \frac{\alpha^{\gamma} \bar{u}}{W} + \beta\right]^{\frac{1}{\gamma}} < 0 < (1 - \alpha)^{\gamma} \bar{u}, \quad (4.12)$$

there exists a unique  $\pi$  that satisfies (4.9) for  $\hat{\pi} \geq \frac{1}{1-\beta} \bar{u}$ .

## 5. Negotiation : Further Analysis

The game presented in Section 4 differs from the standard Rubinstein bargaining game in two aspects. First, the two players are two countries with possibly different population sizes rather than two individuals. Second, the status quo yields positive payoffs to both players. This section is devoted to the investigation of the question that how these differences affect the bargaining solution. We shall start with the analysis on the special case where the utility function is homogenous of degree one ( $\gamma=1$ ), followed by an intuitive discussion of the results. We then derive the analogous results for the more general case  $\gamma \in (0,1]$ .

If  $\gamma = 1$ , the subgame perfect equilibrium can be solved explicitly from (4.9), together with (3.24)

$$\hat{p} = \frac{W}{a(1+b)} + \frac{a(1+b)-1}{a(1-b^2)} \bar{u} \quad (5.1)$$

$$\hat{\pi}^* = \frac{\beta W}{(1-\alpha)(1+\beta)} + \frac{\alpha(1+\beta)-1}{(1-\alpha)(1-\beta^2)} \bar{u} \quad (5.2)$$

In the standard Rubinstein bargaining game with identical time discount factors and linear bargaining frontier, the player that moves first obtains a larger share of the "pie" than the other player. In this model, however, it is not always the case. From (5.1) and (5.2) one can verify that when  $\gamma = 1$ ,  $\hat{p} > W > \hat{\pi}^*$  if  $\alpha < \frac{1}{1+b}$ ;  $\hat{p} < W < \hat{\pi}^*$  if  $\alpha > \frac{1}{1+\beta}$  and  $\hat{p} = \hat{\pi}^* = W$  if  $\alpha = \frac{1}{1+\beta}$ . The equilibrium per capita welfare of a country depends on her relative population size. In fact, when  $\gamma = 1$ ,

$$\frac{d\hat{p}}{da} = \frac{1}{a^2(1+b)} \left( W - \frac{\bar{u}}{1-b} \right) < 0. \quad (5.3)$$

$$\frac{d\hat{\pi}^*}{da} = \frac{b}{(1-a)^2(1+b)} \left( W - \frac{\bar{u}}{1-b} \right) > 0. \quad (5.4)$$

In other words, the per capita welfare of a country is decreasing in her relative population size. One might wonder what will happen to the aggregate welfare of a country as the relative population size changes.

$$\frac{d(a\hat{p})}{da} = \frac{\bar{u}}{1-b} > 0. \quad (5.5)$$

$$\frac{d((1-\alpha)\hat{\pi}^*)}{d\alpha} = \frac{\bar{u}}{1-\beta} < 0. \quad (5.6)$$

Therefore, the aggregate welfare of a country is increasing in her relative population size.

In the standard Rubinstein game, the disagreement point is (0,0). In this model, however, a planner obtains a positive  $\bar{u}$  for the period of no agreement is reached. While  $\bar{u}$  is calculated endogenously from the competitive equilibrium of the model, it is exogenous to the bargaining game in question. Therefore, we can perform comparative statics on (5.1) and (5.2) to find out how  $\bar{u}$  affects equilibrium outcomes.

$$\frac{d\hat{\pi}}{d\bar{u}} = \frac{\alpha(1+\beta) - 1}{\alpha(1-\beta^2)} \begin{cases} > 0, \alpha > \frac{1}{1+\beta}; \\ = 0, \alpha = \frac{1}{1+\beta}, \\ < 0, \alpha < \frac{1}{1+\beta}. \end{cases} \quad (5.7)$$

That is, Country H's payoff is increasing in  $\bar{u}$  if  $\alpha > \frac{1}{1+\beta}$  and is decreasing in  $\bar{u}$  if  $\alpha < \frac{1}{1+\beta}$ . The reverse is true for Country F.

The above results can be portrayed in Figures 5.1 – 5.2. In Figure 5.1,  $(1-\beta)\pi$  and  $(1-\beta)\pi^*$  are the average per period payoffs measured in terms of per capita welfare of H and F, respectively. FF' is the welfare possibility frontier. D is the disagreement point. R is the bargaining solution. The slope of DR is  $\frac{\alpha\beta}{1-\alpha}$ . In Figure 5.1 DR is below the 45° line, which corresponds to the case  $\alpha < \frac{1}{1+\beta}$ . If  $\alpha > \frac{1}{1+\beta}$ , DR is above the 45° line, as illustrated in Figure 5.2.

## 6. Decentralization

In Section 4, the negotiation outcome is derived under the assumption that the two planners have the authority to choose consumption and production plans for their countries. How can governments in the two countries implement such an agreement in decentralized economies?

The agreement negotiated by the two planners specifies a sequence of consumption and production plans

$$\left\{ \hat{C}_{1t}, \hat{C}_{2t}, \hat{C}_{1t}^*, \hat{C}_{2t}^*, \hat{L}_{1t}, \hat{L}_{2t}, \hat{S}_{1t}^* + \hat{S}_{1t}, \hat{S}_{2t}^* + \hat{S}_{2t} \right\}_{t=0}^{\infty}$$

Since the two countries have identical production technology and identical climate, the location of production can be chosen arbitrarily. The implementation of the agreement in competitive economies is to devise a system of tax and international transfers so that the competitive equilibrium generates the same sequence of outputs and consumption for the two countries as required by the agreement.

Consider a tax/transfer system where there is a worldwide pollution tax on the unit cost of the manufacturing production and a country specific lump-sum tax (transfer) on (to) consumers. The total revenue from the pollution tax is distributed equally among the world residents in a lump-sum fashion. The revenue from the lump-sum tax on the consumers of one country is used to finance the lump-sum transfer to the consumers in the other country. Let  $\eta_t$  be the period  $t$  world pollution tax rate. Set

$$\eta_t = \frac{b\beta(1-c)G'(t+1)}{a(\tau_t)G'(t)} \frac{U_1(\hat{S}_{1t+1} + \hat{S}_{1t+1}^*, \hat{S}_{2t+1} + \hat{S}_{2t+1}^*)}{U_1(\hat{S}_{1t} + \hat{S}_{1t}^*, \hat{S}_{2t} + \hat{S}_{2t}^*)}.$$



$$\begin{aligned}
& \left[ \frac{U_2(\hat{S}_{1t+1} + \hat{S}_{1t+1}^*, \hat{S}_{2t+1} + \hat{S}_{2t+1}^*)}{U_1(\hat{S}_{1t+1} + \hat{S}_{1t+1}^*, \hat{S}_{2t+1} + \hat{S}_{2t+1}^*)} - \frac{a(\tau_{t+1})}{b} \right] \\
& - \frac{\beta U_1(\hat{S}_{1t+1} + \hat{S}_{1t+1}^*, \hat{S}_{2t+1} + \hat{S}_{2t+1}^*)}{U_1(\hat{S}_{1t} + \hat{S}_{1t}^*, \hat{S}_{2t} + \hat{S}_{2t}^*)} a'(\tau_{t+1})(1 - \hat{L}_{2t+1}) \frac{b}{a(\tau_t)G'(t)}. \tag{6.1}
\end{aligned}$$

Define  $\rho_t$  and  $\rho_t^*$  as the lump-sum tax (transfer) in Country H and Country F, respectively

$$\rho_t = (\hat{S}_{1t} + \hat{S}_{1t}^* - \hat{C}_{1t}) + \frac{U_2(\hat{C}_{1t}, \hat{C}_{2t})}{U_1(\hat{C}_{1t}, \hat{C}_{2t})} (\hat{S}_{2t} + \hat{S}_{2t}^* - \hat{C}_{2t}) \tag{6.2}$$

$$\rho_t^* = (\hat{S}_{1t} + \hat{S}_{1t}^* - \hat{C}_{1t}^*) + \frac{U_2(\hat{C}_{1t}^*, \hat{C}_{2t}^*)}{U_1(\hat{C}_{1t}^*, \hat{C}_{2t}^*)} (\hat{S}_{2t} + \hat{S}_{2t}^* - \hat{C}_{2t}^*) \tag{6.3}$$

The sequences of  $\eta_t$ ,  $\rho_t$  and  $\rho_t^*$  can be calculated using the consumption and production sequences specified by the agreement.  $\eta_t$  is the same in both countries so that the pollution tax will not cause price disparities across countries. It is easy to verify that

$$\alpha \rho_t + (1 - \alpha) \rho_t^* = 0 \tag{6.4}$$

**Proposition 6.1.** Assume (A.1) – (A.5). The agreement by the two central planners can be implemented in competitive economies by the tax/transfer system specified above.

*Proof:* The idea behind this proof is to show that given the tax/transfer system, the competitive equilibrium yields the same optimization conditions as the world planner's problem. We shall prove the result in terms of the competitive equilibrium in Country H. The competitive equilibrium in Country F is completely symmetric. Let  $\omega_t$  denote the period  $t$  wage rate in Country H and  $P_t$  the period  $t$  relative price of the manufactured good in terms of the consumption good. Since there is no capital good in this model, the firms' objective is to maximize their profits in each period. The optimization problem of a representative firm in the agricultural sector in Country H is:

$$\text{Max}_{f_{1t}} \quad a(t_t) f_{1t} - w_t f_{1t} \tag{6.5}$$

Similarly, the optimization problem of a representative firm in the manufacturing sector is

$$\text{Max}_{f_{2t}} P_t b f_{2t} - (1 + h_t) w_t f_{2t} \quad (6.6)$$

Since both goods are produced in the world in equilibrium, we have

$$P_t b = (1 + \eta_t) a(\tau_t) \quad (6.7)$$

A representative consumer in Country H receives the rebate of the pollution tax revenue on the one hand, and faces a lump-sum tax (transfer)  $\rho_t$  on the other. Let  $\mu_t$  be the pollution tax rebate he receives in period  $t$ . Let  $Z_t$  denote the lending he made in period  $t$ .  $Z_t$  is measured in the unit of the period  $t$  agricultural good. The interest rate prevailing between period  $t - 1$  and period  $t$  is denoted by  $r_t$ . His optimization problem can be written as:

$$\text{Max}_{\{C_{1t}, C_{2t}, Z_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}) \quad (6.8)$$

subject to

$$C_{1t} + P_t C_{2t} + Z_t = \omega_t + \mu_t - \rho_t + (1 + r_t) Z_{t-1} \quad (6.9)$$

(6.8) – (6.9) gives the standard optimization condition

$$P_t = \frac{U_2(C_{1t}, C_{2t})}{U_1(C_{1t}, C_{2t})} \quad (6.10)$$

$$U_1(C_{1t}, C_{2t}) = \beta(1 + r_t) U_1(C_{1t+1}, C_{2t+1}) \quad (6.11)$$

One can derive analogous first-order conditions for Country F. Since the two countries face the same prices  $P_t$ ,

$$\frac{U_2(C_{1t}, C_{2t})}{U_1(C_{1t}, C_{2t})} = \frac{U_2(S_{1t}, +S_{1t}^*, S_{2t}, +S_{2t}^*)}{U_1(S_{1t}, +S_{1t}^*, S_{2t}, +S_{2t}^*)} \quad (6.12)$$

and

$$\frac{U_1(C_{1t+1}, C_{2t+1})}{U_1(C_{1t}, C_{2t})} = \frac{U_1(S_{1t+1}, +S_{1t+1}^*, S_{2t+1}, +S_{2t+1}^*)}{U_1(S_{1t}, +S_{1t}^*, S_{2t}, +S_{2t}^*)} \quad (6.13)$$

Substitute (6.1), (6.10), and (6.12) – (6.13) into (6.7) and re-arrange:

$$G'(t) \left[ \frac{a(\tau_t)}{b} - \frac{U_2(C_{1t}, C_{2t})}{U_1(C_{1t}, C_{2t})} \right] = \beta \frac{U_1(C_{1t+1}, C_{2t+1})}{U_1(C_{1t}, C_{2t})} a'(\tau_{t+1})(1 - L_{2t+1}) \\ + \beta(1-c)G'(t+1) \frac{U_1(C_{1t+1}, C_{2t+1})}{U_1(C_{1t}, C_{2t})} \left[ \frac{a(\tau_{t+1})}{b} - \frac{U_2(C_{1t+1}, C_{2t+1})}{U_1(C_{1t+1}, C_{2t+1})} \right] \quad (6.14)$$

(6.14) is equivalent to the world planner's optimization condition(3.19). Therefore, the competitive equilibrium with the tax/transfer system will generate the optimal production and temperature sequences.

Next we show that the tax system generates the consumption sequence for Country H as specified by the agreement. In equilibrium,  $\mu_t = \eta_t \omega_t L_{2t}$ . Suppose that there is no borrowing and lending in equilibrium, ie.  $Z_t = 0$ . Substitute the equilibrium value of  $\mu_t$  and  $\rho_t$  into (6.9), we have:

$$C_{1t} + P_t C_{2t} = \hat{C}_{1t} + P_t \hat{C}_{2t} \quad (6.15)$$

It is easy to verify that  $\{\hat{C}_{1t}, \hat{C}_{2t}, \hat{C}_{1t}^*, \hat{C}_{2t}^*\}_{t=0}^{\infty}$  satisfies the first-order conditions (6.10)-(6.11) for both countries given appropriately chosen prices and interest rates  $\{P_t, r_t\}_{t=0}^{\infty}$ . Therefore  $\{\hat{C}_{1t}, \hat{C}_{2t}, \hat{C}_{1t}^*, \hat{C}_{2t}^*\}_{t=0}^{\infty}$  can be supported as an equilibrium consumption sequence if borrowing and lending among consumers is prohibited.

Finally, we prove that under the tax/transfer system specified above,  $Z_t=0$ . In other words, there is no borrowing and lending among consumers in a competitive equilibrium. In equilibrium there is no borrowing and lending within a country since all consumers in the country are identical and have the same income level. We need only to prove that there is no international borrowing and lending in equilibrium.

In equilibrium, the two countries face the same interest rates,  $r_t$ . (6.11) and the homogeneity of utility function imply

$$\frac{C_{1t}}{C_{1t+i}} = \frac{C_{1t}^*}{C_{1t+i}^*}, i = 1, 2, \dots, \quad (6.16)$$

Notice that  $\omega_t, \mu_t$  are determined by the production side of the economy and  $\rho_t$  is specified by the agreement between the two countries. They are not influenced by the amount of borrowing and lending among consumers.

Consider a sequence of consumption that involves non-zero borrowing and lending among countries,  $\{\hat{C}_{1t}, \hat{C}_{2t}, \hat{C}_{1t}^*, \hat{C}_{2t}^*\}_{t=0}^{\infty}$ . Let period  $s$  be the earliest period in which borrowing and lending occurs.

Without any loss of generality, assume that  $\hat{Z}_s > 0 > \hat{Z}_s^*$ , ie, H lends to F. From (6.9) we know that the borrowing in period  $s$  will be used to finance F's consumption in period  $s$ . Then  $\bar{C}_{js} < \hat{C}_{js}$  and

$\bar{C}_{js} > \hat{C}_{js}^*$  ( $j = 1, 2$ ). Since F has to pay back the debt in some future period  $s + i$  ( $i \geq 1$ ),  $\bar{C}_{js+i} > \hat{C}_{js+i}$  and  $\bar{C}_{js+i} < \hat{C}_{js+i}^*$  ( $j = 1, 2$ ). Recall from (4.3) – (4.4) that

$$\frac{\hat{C}_{1t}}{\hat{C}_{1t+i}} = \frac{\hat{C}_{1t}^*}{\hat{C}_{1t+i}^*} \quad i = 1, 2, \dots \quad (6.17)$$

by construction, Then

$$\frac{\bar{C}_{1t}}{\bar{C}_{1t+i}} < \frac{\bar{C}_{1t}^*}{\bar{C}_{1t+i}^*} \quad (6.18)$$

which violates (6.16). Therefore, there is no borrowing and lending in equilibrium.

Proposition 6.1 implies that, instead of negotiating the consumption and production plans for the two countries, the two governments can negotiate a tax/transfer system that generates exactly the same outcome.

**Proposition 6.2.**  $\rho_t > 0 > \rho_t^*$  if  $\hat{\pi} < W < \hat{\pi}^* < W < \hat{\pi}^*$ , and  $\rho_t < 0 < \rho_t^*$  if  $\hat{\pi} > W > \hat{\pi}^*$ .

Proof: By Proposition 4.1, equations (3.8) – (3.11) and the definitions of  $\pi$ ,  $\pi^*$  and  $W$ ,

$$\hat{C}_{jt} < \hat{S}_{jt} + \hat{S}_{jt}^* < \hat{C}_{jt}^*, \quad (j = 1, 2) \quad (6.19)$$

if  $\hat{\pi} < W < \hat{\pi}^*$ ; and the reverse is true if  $\hat{\pi} > W > \hat{\pi}^*$ . The result follows from (6.2) and (6.3).

In the tax / transfer regime discussed above, the pollution tax revenue from the two countries is pooled and is distributed equally among the world residents. In this process, some transferring of the tax revenue from one country to the other may occur. To see this consider the Country H's government budget of the pollution tax. The pollution tax revenue is equal to  $\eta_t \omega_t \phi_{2t}$  while the payment to her citizens is  $\alpha \mu_t = \eta_t \omega_t \alpha L_{2t}$ . Therefore, if Country H's share of the output of the manufactured good is larger than her share in the world population, ie.,  $\frac{\phi_{2t}}{L_{2t}} > \alpha$ , part of the pollution tax revenue will be transferred to Country F to finance the payment to her consumers.

Therefore, in equilibrium, in general we will observe two kinds of international transfers. First, the transfers that are made due to the disproportional distribution of the manufacturing productions. We will observe that the country that produces more than her share of manufacturing output and hence emits more than her share of carbon dioxide transfers part of her pollution tax revenue to finance the compensation in the other country. Second, the transfers that are made due to the asymmetry in bargaining power. The gain from international cooperation is not shared equally among the two countries as a result of asymmetry in the bargaining procedure and in relative population sizes.

## Conclusions

In this paper the global optimal time path of consumption, production and temperature is characterized. Furthermore, it is demonstrated that the global optimality can be achieved under competitive equilibrium through a binding agreement between two countries that specifies a system of taxes and transfers.

One important issue discussed in most of the existing literature on global environmental problems is the side payments between countries. In the existing analyses, side payments are made either for compensating the victim of pollution for damages or for compensating the polluter for not polluting (See, for example, Barrett, 1990, Mäler, 1990). In this paper, it is shown that in general international transfers from one country to the other will be observed in equilibrium. Not all of these transfers, however, are made by the heavy-polluting country for the purpose of compensating the damages in the other country. Part of these transfers are made purely due to the asymmetry in bargaining power. Therefore, the observed international transfers are not necessarily made for the "right" reason, i.e., for compensating the victim country.

Economists have pointed out many difficulties associated with controlling climate change (Barrett 1990, Nordhaus 1990). Among them, the two most important ones are: (1) the cooperation among countries ; and (2) the uncertainty and the lack of information about future climate changes. Our analysis shows that if governments have perfect information, the cooperation problem may be solved by negotiating a binding agreement on an international tax/transfer system. In the real world, of course, the governments do not have perfect information. Given the current state of scientific and economic research on global warming, the governments do not have enough information on the possible losses (or gains) that will arise from future climate changes. Therefore, it is the lack of information that is preventing governments from cooperating on the global warming issue. The key to solving the global warming problem is information rather than cooperation.

It suggests that we may not expect equilibria (with Pareto optimal properties) in realistic situations that are in some sense superior to a "business-as-usual" situation (Radner, 1998).

The Montreal Protocol for the protection of the ozone layer offers an example that supports our arguments that international cooperation can be achieved if sufficient information is available. In contrast to the uncertainties with regard to the existence and the effects of global warming, the evidence on the depletion of ozone layer by CFCs is clear and convincing.

In this paper, the non-cooperative bargaining theory is employed in deriving the agreement between the two countries. The Rubinstein bargaining solution is derived from a well-specified economic environment. In the process, some interesting new results for the non-cooperative bargaining theory are generated.

In the standard Rubinstein game, the players' time preference is the only factor that affects their bargaining power. In this model, because of its richer structure, in addition to the countries' time discount factors, the other factors, such as the relative size of population and the payoffs at status quo, also affect a country's bargaining power. While the first mover's advantage still exists, the country that moves first will not necessarily obtain a higher payoff in per capita terms than the other country because

of the effects of other factors.

It is shown that the aggregate bargaining power of a country improves (in the sense that the aggregate payoff increases) as the relative population size of the country increases. Such an increase, however, is less than proportional to the augmentation in relative population size. As a consequence, the per capita payoff of a country decreases as the country's relative population size increases.

Another feature of this bargaining game is that the status quo point in this model generates non-zero payoffs to the players. Intuition suggests that an increase in the payoffs at the status quo for both countries should diminish the first mover's advantage because the second mover has less to lose from delaying an agreement. It is shown that this conjecture is not always true. The opposite is true if the relative population size of the country that makes the first move is larger than certain critical value.

## References

Barrett, S. (1990) "The Problem of Global Environmental Protection," *Oxford Review of Economic Policy*, 6:68-79

Barrett, S. (1992) "Self-Enforcing International Environmental Agreements", CSERGE WP 6EC 92-34, London.

Coppel, W.A. (1955), "The Solution of Equations by Iteration," *Proceedings Cambridge Philosophical Society*, 51: 41-43.

Deneckere, R. and Pelikan S., (1986), "Competitive Chaos", *Journal of Economic Theory*, 40: 13-25.

Dickinson, R.E. (1986), "How Will Climate Change? The Climate System and Modelling of Future Climate", in Bolin, et al. Eds., *The Greenhouse Effect, Climate Change and Ecosystems*, John Wiley & Sons, New York.

Fudenberg, D. and J. Tirole, (1989) "Noncooperative Game Theory for Industrial Organization: An Introduction and Overview", Chap. 5 in R. Schmalensee and R.D. Willig (eds), *Handbook of Industrial Organization*, Vol. I, North Holland: Amsterdam 1989, 259-327.

Gottinger, H.W. (1998) , *Global Environmental Economics*. Kluwer Academic Publishers: Boston.

Heal, G., (1993) "Formation of International Environmental Agreements", in S. Carraro (ed.), *Trade*,

Innovation, Environment, Kluwer Academic Publishers: Dordrecht.

Hoel, M. (1991) "Global Environmental Problems: The Effects of Unilateral Actions Taken by One Country", *Journal of Environmental Economics and Management*, 20, 1991, 55-70.

Mäler, Karl-Göran (1990), "International Environmental Problems," *Oxford Review of Economic Policy*, 6: 80-108.

Nordhaus, W.D., (1990), "Global Warming: Slowing and Greenhouse Express," in Henry J. Aaron, ed., *Setting National Priorities: Policy for the Nineties*, The Brookings Institution, Washington D.C.

Radner, R.(1998) "A. Game-Theoretic Approach to Global Warming" Stern School of Business, New York Univ. (Mimeo)

Rubinstein, A. (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50:97-109.

Sutton, J. (1986), "Non-Cooperative Bargaining Theory: An Introduction," *Review of Economic Studies* 53: 709-724

Stokey, N. and R. Lucas, Jr. (1989), *Recursive Method in Economic Dynamics*, Harvard University Press, 1989.

Uzawa, H. (1999), "Global Warming as a Cooperative Game", UNU-IAS Working Paper No. 47, Tokyo