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Working Paper

Joint costs in network services: The two-way problem in the case of unbalanced transport markets

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Abstract.

Markets for transport are often characterised by unequal demand in both directions: every morning during peak hours the trains are crowded while moving towards the direction of large cities, whereas they may be almost empty in the other direction. In this paper we discuss the implications of these imbalances for price setting of transport firms. From the viewpoint of economic theory, two regimes can be distinguished: one where –owing to price discrimination- the flows are equal, and one where unequal flows are the result. Special attention is paid to the case where the transport firm does not apply price discrimination, as is the case in most railway firms in Europe. We find that in the case of substantial joint costs, the introduction of price discrimination not only leads to an increase of profits, but also to positive effects on consumer surplus. This result differs from the standard result in the literature on industrial economics. The standard result purports that with linear demand functions price discrimination has a negative impact on the welfare of the average consumer and that this negative impact dominates the positive effect on profits of the producer.
1. Introduction.

Multiproduct firms often benefit from the fact that the production of several different goods in one firm is cheaper than outputs produced by separate monoproduct firms. A special example of multiproduct firms are transport companies: a transport company serving a certain market between points A and B will also produce services between B and A when the vehicles make the return journey. The structure of transport cost is such that once the capacity costs of the AB trip have been made, the additional costs of transporting passengers in the opposite direction are relatively small. An interesting feature of transport markets is that demand in both directions may be rather unbalanced. This holds true for both freight and passenger transport. This imbalance is one of the reasons why load factors in transport are often so low. In this paper we address the question to what extent differentiated prices can help to overcome this problem and explore the (possible) welfare implications.

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<td>CD</td>
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</tr>
<tr>
<td>KL</td>
<td>train</td>
<td>756</td>
<td>461</td>
</tr>
</tbody>
</table>

Source: NSR (train data), ZWN (bus data)

Table 1.1 Number of daily public transport passengers in the Netherlands during the peak (weekdays) in both directions for a small sample of links.

To give an illustration of the magnitude of the problem we present data on a small sample of public transport links in the Netherlands during the morning peak (see Table 1.1)\(^1\). Substantial

---

\(^1\) Although we use abstract symbols like A and B to represent links between nodes, we want to emphasise that they represent real data about real cities; the reason we do not give the actual names is that the data concerned are confidential. Also the demand and cost function used in section 4 are taken from applied models currently
differences in the number of passengers in both directions can be observed. These differences are especially large for bus transport (up to a factor of 20). An explanation of the difference between bus and train is the large share of school children travelling by bus from villages to cities where most of the schools for secondary education are located. Public transport operators can apply several strategies to accommodate this imbalance such as waiting until the afternoon peak for the return trip of some of the vehicles or using some of the vehicles on other links between the morning and the afternoon peak. In this paper we discuss the strategy of differentiated price setting to cope with the lack of balance.

In the economic literature various terms have been used for the phenomenon that production of one type of output implies that also other types of outputs are produced; for example; *joint products, joint costs or cost interdependence* (Gravelle and Rees, 1992, Tirole, 1988). In the transport literature it is known as the *back haul problem* (Felton, 1981). The problem has received relatively little attention in the transport economics literature. In addition to contributions by Ferguson (1972) and Felton (1981), it is briefly mentioned in publications of Mohring (1976), Korver et al. (1992), Button (1993) and Blauwens et al. (1995). The joint cost phenomenon is obviously related to the notion of *economies of scope*. A joint cost function is said to exhibit economies of scope when the costs of producing given amounts of two different outputs $q_1$ and $q_2$ are lower when it is done in an integrated firm compared with when it is done by two specialist firms, one producing only $q_1$ and the other one only $q_2$. In the context of the back-haul problem this condition is obviously satisfied.²

We will give a micro-economic analysis of profit maximisation by a monopolist serving unbalanced markets (section 2). Based on this we will investigate two regimes: an equal quantity regime (where despite non-identical demand functions, one arrives at equal quantities transported) and an unequal quantity regime. In section 3 we will investigate the consequences of a price setting strategy where the supplier adopts the constraint that *prices are equal* in both directions. Such a price setting strategy is typical in public transport firms in many countries. In section 4 some numerical examples will be given for the case of the Netherlands

² Note that there is still another form of economies of scope that is not related to joint costs: A firm producing complementary goods may have higher profits compared with supply of the products by separate firms, even when the costs would be independent.
railways. Special attention will be given to the welfare effects of the price equality constraint: it certainly reduces profits of the monopolist, but what is its effect on consumer surplus of travellers in both directions and how does it affect total welfare?

2. Price setting in the presence of joint costs

Consider the case of a monopolist serving two markets (AB and BA) and having part of the production costs dependent on the maximum quantity sold in one of the two markets. Her profit function could in this case be expressed as:

\[ \Pi = p_{AB} q_{AB} (p_{AB}) + p_{BA} q_{BA} (p_{BA}) + C(Q) + c(q_{AB}) + c(q_{BA}) \]  

(2.1)

where \( p \) are prices, \( q(.) \) are demand functions, \( C(.) \) and \( c(.) \) are cost functions with standard properties. Furthermore:

\[
Q \geq q_{AB} \\
Q \geq q_{BA}
\]  

(2.2)

If the monopolist can price-discriminate between the two markets, optimal prices would be determined on the basis of the following first-order conditions:

\[
p_{AB} = \left(1 - \frac{1}{\varepsilon_{AB}}\right)^{-1} (c'(q_{AB}) + \lambda_{AB}) \]

(2.3a)

\[
p_{BA} = \left(1 - \frac{1}{\varepsilon_{BA}}\right)^{-1} (c'(q_{BA}) + \lambda_{BA}) \]

(2.3b)

\[
C'(Q) = \lambda_{AB} + \lambda_{BA}
\]

(2.3c)

\[
\lambda_{AB}(Q - q_{AB}) = 0 \lambda_{AB} \geq 0 Q - q_{AB} \geq 0 \\
\lambda_{BA}(Q - q_{BA}) = 0 \lambda_{BA} \geq 0 Q - q_{BA} \geq 0
\]

(2.3c)

3 We assume here that the cost function is separable and that the sub-functions related to the two markets are equal. Both assumptions, however, are not necessary to obtain the results illustrated in this section.
Conditions (2.3a) are standard rules setting the price on the basis of a mark-up over marginal costs, where the mark-up is determined by the price elasticity of the demand in each market (\( \varepsilon \)). Here, however, marginal costs are given by two components: “direct” marginal costs \( c'(.) \) and a Lagrange multiplier. From (2.3b) it can be seen that at least one of the two multipliers must be positive, which means that at least in one market the quantity \( q \) is equal to \( Q \).

When only one multiplier is positive, marginal common costs \( C'(Q) \) are completely assigned to the larger market, and the supplier acts as if she were in two separate markets in which marginal costs are higher in the larger market.

A more interesting case emerges when quantities \( q \) are equal (and equal to \( Q \)). Common costs are then shared, the multipliers are both positive and they are set in order to satisfy (2.3a). Clearly, nothing ensures that the multipliers expressing the share of common marginal costs allocated to the two markets are equal. This may be interpreted as a sort of cross-subsidisation between the two markets.

To better understand how a regime of equal quantities (EQ) may emerge from an unconstrained profit maximisation with joint costs, it may be useful to think about the problem of price setting as being composed of two stages: first, for each level of total quantity produced, the production is efficiently allocated in the two markets and, second, the optimal activity level is determined.

The first stage can be easily studied by means of isocost-isorevenue diagrams. An isocost function includes all the pairs \((q_{AB}, q_{BA})\) associated with the same level of total production costs. Analogously, an isorevenue curve identifies all the pairs associated with the same revenue level. Examples of isocost and isorevenue functions are depicted in figures 2.1 and 2.2.
Isocost curves appear to be kinked here because of the presence of common costs, and in this case the kink is found where the quantities are equal. Higher common costs, relative to direct marginal costs, make isocost functions “more kinked”.

Isorevenue functions are instead concentric circles with inner circles associated with higher revenue levels. The centre of the concentric curves is situated on the quantity pair that would be chosen in the absence of production costs.

It is straightforward to see that a necessary condition for profit maximisation is given by the tangency between an isorevenue and an isocost curve\(^4\). The set of all tangency points therefore defines an “optimal expansion path”, a one-dimensional space along which total profits vary\(^5\). Since the profit maximising quantity pair is a point on the expansion path, the two possible outcomes of profit maximisation, equal (EQ) or unequal (UQ) quantities, appear as shown in figures 2.3 and 2.4.

A similar result with two possible regimes (EQ versus UQ) has been obtained by Felton (1981) for the case of a price-taking transport company.

\(^4\) To see this, choose an isocost curve and find the highest revenue obtainable at the given cost level or, vice versa, minimise costs for each level of revenue.

\(^5\) Isocost-isorevenue diagrams can also be usefully employed to illustrate cases of constrained profit maximisation. For example, in this paper the case of uniform price setting is considered. For each single price level, the quantities sold in the two markets would then be exogenously determined. This defines an alternative “expansion path”, along which profits can be maximised.
By looking at the two diagrams, it is immediately verifiable that the following conditions make the emergence of an EQ solution more likely:

a) relatively higher common costs, shifting the kink point above and rightward. A special example is given by the absence of direct marginal costs \( c(.) \), where isocost curves would be “squared”;

b) market sizes relatively equal, where market size is defined as the maximum quantity associated with positive marginal revenue. Similar markets have the centre of isorevenue functions close to the line \( d_{AB} = q_{BA} \);

c) relatively high price elasticity in one or both markets. High elasticities flatten the part of the isorevenue circle where the tangency point can be found. In the limit case of exogenously fixed prices, isorevenue functions would become downward sloping parallel lines.

The latter point suggests that it is straightforward to extend the analysis to non-monopolistic market structures. The demand function adopted in (2.1) could be re-interpreted as an individual demand function encountered by each competitor, on the basis of her conjectures about the competitors’ behaviour. For example, in an oligopolistic market “à la Cournot”, production volumes chosen by concurrent firms are taken as given, so that the individual demand curve is the residual of the market demand when the quantity produced by the competing firms is subtracted.

It is interesting to note that, in a monopolistic competition model, the price elasticity of the individual demand functions would be affected by the entry of new competitors. Since entry increases demand elasticity in a market for non-perfect substitute goods or services, market
deregulation and increased competition may bring about the choice of an EQ price strategy. In the analysis given above we did not pay explicit attention to the existence of congestion externalities in transport; yet, congestion may be quite relevant in an analysis of a ‘busy’ versus a ‘non-busy’ direction. Two types of congestion can be distinguished in this respect: external versus internal. In the first case congestion occurs in a competing mode (say road transport) implying a partial shift towards the public transport mode considered here. This would already be absorbed in the parameters of the demand function \( q_{AB}(p_{AB}) \) for public transport: the high level of demand for public transport in the busy direction may be partly due to travellers who want to avoid road congestion.

In the case of ‘internal’ congestion the speed and/or comfort in the busy direction may be adversely affected by the number of travellers. When this leads to higher costs of producing the transport services this will be reflected by the pertaining cost function \( c(q_{AB}) \) which has the property of decreasing returns to scale. The analysis of section 2 allows for such cost functions.

Internal congestion may also have an impact on the travellers, however. Slow or crowded public transport will discourage potential users during busy periods. To take this into account the demand function \( q_{AB}(p_{AB}) \) would have to be generalised so that it incorporates indicators of travel time \( t_{AB} \) and comfort \( v_{AB} \) (for example, the probability of getting a seat), which are themselves functions of travel demand \( q_{AB} \). Note that the present formulation of the demand function \( q_{AB}=q_{AB}(p_{AB}) \) can be considered as a rewritten form of \( q_{AB} = f[p_{AB}, t_{AB}(q_{AB}), v_{AB}(q_{AB})] \) so that congestion has been incorporated in an implicit way. Obviously, when the number of travellers has an impact on travel time and comfort this would have a dampening effect on demand in the busy direction. Given the externality involved, profit maximising public transport operators would apply an upward shift of the price in the busy direction to let the travellers pay for the external costs imposed on other travellers, similar to the case of congestion pricing on roads.

We conclude that the various forms of congestion considered here all tend to lead to a higher price set in the busy direction:

- congestion on the road makes public transport demand less price elastic (because the competing mode is less attractive) so that there are more opportunities for a mark-up (see 2.3a),
-higher marginal costs $c'(q_{AB})$ imply higher prices (see 2.3a),
-crowding and travel time losses among travellers induce congestion pricing strategies such that travellers in the busy direction pay a congestion charge to cover the disutility imposed on other travellers.

3. Discriminatory vs. uniform pricing: a welfare analysis.

The analysis conducted thus far has made clear that a network operator, if she is free to set prices in order to maximise profits, would not - in general - choose equal prices for two markets linked by cost interdependence. Yet, several examples can be found, especially in transportation markets, where prices are made dependent on distance but not on the direction, so that the same price applies to different markets. In this section, we shall consider the implications in terms of variations in profits and welfare of discriminatory (differentiated) and uniform pricing. To this end, we shall continue to assume that firms set prices according to profit maximisation, even in the presence of a constraint of price equalisation.

First of all, it is apparent that profits cannot increase if prices must be equalised. If a firm is free to set different prices, it can “at worst” replicate the outcome obtained under uniform pricing. Mathematically, a constrained maximisation cannot lead to better results than an unconstrained one.

On the other hand, the welfare of consumers in the two markets is affected by the direction of price changes. In this respect, it can be demonstrated under fairly general assumptions about demand and cost functions$^6$ that the single price, which would be chosen for both markets, must lie between the two prices chosen under discrimination.

---

$^6$ The assumptions needed regard the quasi-concavity of profit functions in the two markets. For example, this is ensured if demand and cost functions are both linear. We shall also assume here that the two demand curves do not intersect, so that for each price the market associated with the largest quantity remains the same.
Fig. 3.1 - Profit functions in the two markets

To see this, consider the example of figure 3.1, in which profit functions for the two markets are depicted in terms of price. The two functions are drawn under the assumption that common costs are considered as the production costs of the largest market.

If prices are allowed to differ and it is not optimal to sell equal quantities, prices are chosen to achieve, independently, the maximum profit in the two markets. If, instead, a uniform price has to be set, the firm maximises the sum of the two profit functions. By recalling that total profits are increasing (decreasing) where both profit functions are increasing (decreasing)\(^7\), it is clear that the optimal uniform price can neither be found in regions X and X’ nor in regions Y and Y’. The price must therefore lie between the two differentiated prices. In turn, welfare of consumers in the large market increases with a single price, whereas consumers in the small market are made worse off.

Since the price in the small market tends to increase, it is also possible that the price would be too high to have a positive demand there. In this case, the small market would “disappear”. In the large market the price and the consumer surplus would remain unchanged. In other words, price discrimination would lead to a Pareto improvement with gains for both the firm and the remaining consumers.

When prices are set in the two markets so as to equalise the quantity sold, from (2.3) it can be noted that the marginal profit in the small market is positive in the optimum, whereas it is

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\(^7\) Or, equivalently, one is increasing and the other is constant.
negative for the large market. Prices are then set lower in the small market and higher in the large market, in comparison with the UQ case. Nonetheless, the argument applied above can be extended here as well, so that the optimal uniform price can neither be found in region X nor in region Y.

A difference with the UQ regime emerges when the small market is not served under uniform pricing, because there would be no more need to equalise quantities, and this would make the price in the large market somewhat lower. Again we have here losses for consumers in the small market and gains for those in the large market, but with a different causal mechanism.

To sum up the results obtained in the different cases, we can conclude that, in addition to the reduction in profits, the imposition of a uniform price always reduces the consumers’ welfare in the small market, whereas the large market always receives benefits, except in one case where its position is unaffected.

In order to better understand which are the factors affecting the magnitude of the welfare gains, it may be useful to consider a special case where demand functions are linear, there are no direct marginal costs, and marginal common costs \( C \) are constant:

\[
\begin{align*}
q_{AB} &= A - Bp_{AB} \\
q_{BA} &= a - bp_{BA} \\
C(q_{AB}) &= Cq_{AB} \\
A &> aA / B > a / b
\end{align*}
\]  

It can be easily verified that optimal discriminatory and uniform prices are here determined as the following:

\[
\begin{align*}
p_{AB} &= \frac{A + BC}{2B} \\
p_{BA} &= \frac{a}{2b} \\
p &= \frac{(A + a) + BC}{2(B + b)}
\end{align*}
\]  

---

*Recall that here the profit functions are computed by allocating all common costs to the large market.*

*This restriction is irrelevant if direct marginal costs are constant, because the parameters of the demand function can be normalised. To obtain results for the more general case, substitute parameters \( A \) and \( a \) in (3.2), (3.3), (3.4) with \( A' + Bc \) and \( a' + bc \), if \( c \) is the direct marginal cost (possibly different in the two markets).*

*We are assuming here that parameter values are consistent with the emergence of an UQ solution in the*
Furthermore, because demand functions are linear and there are constant returns to scale, it can be checked that the total quantity $q_{AB} + q_{BA}$ is the same under the two price regimes (Robinson, 1933). This has important implications in terms of welfare. If the total quantity to be allocated is a given, the maximum total consumers’ utility is achieved when the marginal utilities are equal. But in turn each consumer equates her marginal utility to the ratio of the price and the marginal utility of income; if the latter is not too different among the consumers, the imposition of a single price means that welfare gains in the large market always exceed welfare losses in the small market, if the small market continues to be served.

Since profits are higher under discriminatory pricing, the variation in profits caused by the imposition of a uniform price is unambiguously negative:

$$\Delta \Pi = -\frac{(Ab - aB + bBC)^2}{4bB(b + B)}$$

(3.3)

Looking at (3.3), it is interesting to notice that:

- profit losses are higher the larger the difference in the size of the markets, expressed in terms of maximum willingness to pay, or in terms of maximum quantity. This is intuitively clear, as there should be more scope for setting different prices when the basic conditions are different;
- profit losses are higher the larger the common cost component $C$. In the case of a UQ discrimination, common costs are considered as production costs of the large market: their inclusion amplifies the asymmetry of the two markets. When an EQ solution emerges, there is an incentive to differentiate the prices even more to achieve the equalisation of quantities.

By charging discriminatory prices, the quantity sold in the large market decreases, making it possible to save on production costs. This “cost saving effect” is important because it overlaps with the fundamental motivation of price discrimination: the extraction of consumer surplus by the monopolist.

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11 We are not aware of systematic evidence on differing marginal utilities of income between passengers in busy versus non-busy directions. Note that these marginal utilities may also vary within the two groups; for example commuters and students, that are probably over-represented in the busy direction compared with travellers in the non-busy direction may have quite different marginal utilities of income.
Consequently, there are two separate reasons why price discrimination is profitable for a monopolist. The standard case is that with different price elasticities, price discrimination serves to exploit a larger part of the consumer surplus. In addition, with joint costs, price discrimination helps to avoid high costs for the production on the large AB market.

What are the implications for total welfare? The standard result has been formulated by Tirole (1988): with linear demand functions price discrimination is bad for total welfare as long as both markets are served. The introduction of joint costs makes the expression of variations in total welfare, defined as the sum of profit and consumer surplus, quite complex and difficult to analyse in general terms. However, following Tirole (1988), it is possible to compute lower and upper bounds for this variation. These are:

\[
-C\Delta q_{AB} \leq \Delta W \leq (p_{AB} - C)\Delta q_{AB} + p_{BA}\Delta q_{BA} - \frac{C((Ab - aB) + bBC)}{2(b + B)} \leq \Delta W \leq \frac{(Ab - aB)^2 - (bBC)^2}{4bB(b + B)}
\] (3.4)

Observe that if \( C = 0 \), total welfare would increase if a uniform price is imposed. This means that, if common costs are not very large, the gains for the consumers in the large market can compensate for the losses in profits for the firm and in surplus for the consumers of the small market. However, if \( C \) is sufficiently high, the “cost saving effect” may dominate, bringing about losses in profits so large that the variation in total welfare becomes negative. This can be seen in (3.4) because increases in \( C \) reduce both the lower and upper bounds.


Railways are a good example of suppliers facing joint costs. In this section we will give a short illustration of the consequences of joint costs on price setting. We take as an example the link between two medium sized cities (A and B)\(^{12}\), where in the current situation of direction independent prices in the morning peak, the daily number of travellers in one direction is about 35% higher than in the other direction.

The linearised demand equations for both directions are:

\(^{12}\) The parameters are based on confidential data provided by the Netherlands Railways (NSR).
\[ q_{AB} = 2460 - 60p_{AB}, \]
\[ q_{BA} = 1820 - 44p_{BA} \]

where monetary units are in dfl. The difference in scale in both markets is reflected by the difference in the constants in this demand function. The highest willingness to pay equals about dfl 41 in both markets (2460/60 and 1820/44). It is a coincidence that this value is equal for both markets. An implication is that for a given price, the price elasticity of demand is equal in both markets.

The cost function for the service between A and B and vice versa is assumed to be linear; the parameters have been estimated to be:

\[ C = (0.86)(q_{AB} + q_{BA}) + (9.69)[\max(q_{AB}, q_{BA})] + \text{fixed costs}. \]

The cost factor 0.86 is the marginal cost of a passenger when we assume that there is sufficient capacity; this cost relates to passenger dependent services (such as ticket windows, train guards, etc). The cost factor of 9.69 relates to the costs of moving seats (regardless of whether they are occupied or empty), plus the cost for the national railways of owning seats; it is assumed that additional seats are only used during the peak and that they do not generate receipts during the rest of the day.

Profit maximisation without imposing a price equality constraint leads to an optimum as follows:

\[ p_{AB} = 25.78, q_{AB} = 913.2 \]
\[ p_{BA} = 21.11, q_{BA} = 891.2. \]

Note that the marginal cost in market BA is very low, so that the price is near the price level where marginal returns are zero (p=20.5). The price paid by travellers in the low demand direction is only based on the marginal cost of 0.86 per passenger; the capacity costs are completely taken into account in the price charged to the passengers travelling in the opposite direction. The price differentiation clearly leads to a substantial convergence of volumes of travellers in both directions compared with the current situation. Actually, the optimum found in this case is quite close to the equal quantity regime discussed in sections 2 and 3.
Profit maximisation under the *price equality constraint* would result in:

\[ p = 23.80, q_{AB} = 1032, q_{BA} = 772.6. \]

Under this constraint the difference in the number of travellers in both directions is substantial\(^{13}\).

What are the welfare consequences of the introduction of the equal price? For AB travellers welfare (measured by means of consumer surplus) increases owing to the price decrease (+1945). For BA travellers the opposite occurs: (-2246). Thus, the average consumer loses when the monopolist uses the self-imposed constraint that prices are equal in both directions. In addition, the profits of the railway company would decrease by an amount of 548. Thus, the net aggregate change in welfare when the price equality constraint is imposed is -849, the distribution being such that the firm and BA travellers are negatively affected and AB travellers are positively affected. This case underlines the importance of joint costs in the welfare analysis. Without joint costs the introduction of a uniform price would have a positive effect on total welfare (see section 3\(^{14}\)). But here the opposite is found: strong cost interdependence calls for differentiated prices from a social welfare perspective, even in the context of monopolistic price setting practices.

Of course this result depends on the specific parameters of demand and cost functions. Below we show the results of two other cases using different sets of parameters. In *case 1* a rather large difference exists between the AB and the BA market, both in terms of size of market and willingness to pay. The cost function remains unaltered:

\[
q_{AB} = 2000 - 50p_{AB},
q_{BA} = 600 - 20p_{BA}
\]

\[
C = (0.86)(q_{AB} + q_{BA}) + (9.69)[\max(q_{AB}, q_{BA})] + \text{fixed costs}.
\]

The results can be found in Table 4.1. A large difference in the number of passengers in both directions is observed. The quantity difference is substantially larger when the price equality constraint is imposed; this would lead to considerable positive welfare effects in market AB.

---

\(^{13}\) Note that as already shown in section 4, under the given specifications of the demand and cost functions, the imposition of the price equality constraint does not affect the sum of travelers in both directions.  

\(^{14}\) As indicated in section 3 this result holds with linear demand functions. It is contingent on the assumption that both markets are served. Both conditions are satisfied in this example.
The welfare loss of passengers in the other direction is smaller, therefore the average consumer benefits from the self-imposed price equality constraint. However, the sum of changes in consumer surpluses and profits is negative, since it appears that the advantage for the average consumer does not outweigh the decrease in profits for the supplier.
### Table 4.1 Effects of profit maximisation under the price equality constraint, with and without cost interdependence.

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<td></td>
</tr>
<tr>
<td>Consumer surplus$_{AB}$</td>
<td>2179</td>
<td>1452</td>
</tr>
<tr>
<td>Consumer surplus$_{BA}$</td>
<td>-1462</td>
<td>-915</td>
</tr>
<tr>
<td>Profit</td>
<td>-1157</td>
<td>-357</td>
</tr>
<tr>
<td>Sum</td>
<td>-440</td>
<td>180</td>
</tr>
</tbody>
</table>

In case 2 we use the same demand functions but drop the cost interdependence by using the cost function:

$$C = (0.86)(q_{AB} + q_{BA}) + \text{fixed costs.}$$

In this case the imposition of the price equality constraint means that the benefits for the passengers in the large market exceed the disadvantages for the small market passengers and the transport company itself. This is a well-known result from the standard literature (see section 3). It implies that in the absence of cost interdependencies and with the given linear...
specifications, a self-imposed constraint for a monopolist (which by definition leads to lower profits), leads to benefits for consumers that outweigh the profit decrease.

We conclude that the outcome of the welfare analysis strongly depends on the size of the cost interdependence. When the interdependence is low, the imposition of the price equality constraint leads to a smaller decrease in costs for the monopolist, compared with the case where the price interdependence is high. The reason is that price equality induces larger differences in flows in both directions which has a consequent strong cost impact via the cost interdependence term $c \cdot \max(q_{AB}, q_{BA})$ when parameter $c$ is large. Since in the case of transport firms the cost interdependence is usually substantial, we conclude that the self-imposed price equality constraint may not be expected to have positive effects on total welfare.

One may wonder to what extent it is appropriate to model the Dutch national railway company as a monopoly. A first point is that there is competition between public transport and the car. A more complete formulation of the demand functions would indeed indicate that the price of the competing mode (car) would be incorporated. However, one may consider the price of car use as exogenous in our model -there is no risk that some agent would adjust the price of car use as a consequence of price policies of the railway company- so that competition between car and railways can be safely ignored. A second question is to what extent competition on the Dutch railway tracks has to be considered. The Dutch railway system has been in a period of transition from a national state controlled company towards a system where some form of competition may be allowed. During a couple of years there has been a second supplier of services on the Dutch railways (Lovers Rail) versus the incumbent Netherlands Railways (NSR), but services provided by Lovers Rail were very insignificant in size and consumer response has been disappointing (a market share of less than 0.1%), so that Lovers Rail has decided to terminate its services in 1999. In the meantime the national government is changing its policy from stimulating competition towards franchising of regional networks based on competitive bidding. On the intercity network (the link AB analysed in the empirical case study being part of it) competition is not allowed according to the current rules. In a legal sense NSR is free to set its prices in order to maximise profits but it has been reluctant to do so because the Dutch public still considers NSR as a public
company. For example, changes in the system of annual travel cards have led to a strong negative response in public discussions. The empirical study given here serves as an investigation of the consequences a profit-oriented differentiation of prices would have.

5. Concluding remarks.

Joint costs in the situation of unbalanced markets pose a challenge to transport firms. In this paper we have investigated the implications for price strategies of transport firms. We find that two regimes may emerge when cost interdependence is present: an equal quantity regime (EQ), where despite the difference in demand in both markets, differentiated prices lead to equal quantities, and the UQ regime, where price differences will not be able to yield equal quantities. Thus, equal quantities are not necessarily a sign of balanced markets. The following conditions make the emergence of an EQ result more likely:
- a small difference in the dimension of both markets
- high price elasticity in the large market
- a high level of cost interdependence.

In many countries suppliers of transport services apply an equal price in both markets. This obviously has welfare implications. In the unconstrained case optimal quantities would be different, and the imposition of equal prices will lead to lower profits, a higher consumer surplus in the large market, and a lower surplus in the small market. An important question is of course what a combination of the three effects would look like.

A numerical illustration using Dutch railway data shows that the aggregate effect of the price equality constraint on total welfare may easily be negative: for plausible values of the parameters we find a decrease in profits that is not off-set by an increase in the surplus of the average consumer. In the particular case considered, we even find that the loss of consumer welfare in the small market is larger than the increase in consumer welfare in the large market. Based on the numerical exercises in section 4, we conclude that the price equality constraint is most likely to have a positive effect on total welfare when the difference in size of the two markets is large and the cost interdependence is low.

On the basis of the empirical illustrations we conclude that the introduction of direction dependent prices during peak hours may easily have positive effects on total welfare.
Depending on the parameters of the demand function even the average consumer may benefit. Given the positive effect of price discrimination on profits it is no surprise that several European railway companies are considering non-uniform pricing schemes where the prices are no longer simply based on the number of kilometres travelled, but where prices are time and direction dependent. The present paper demonstrates that given the issue of joint costs this price discrimination may also be beneficial for the average consumer.

It is clear that the above results are based on a rather stylised model of transport markets. To bring the models formulated here closer to reality, we could introduce richer network structures (for example, people using the AB link in reality will travel from A to C via B). Travellers may express a dislike for crowding and accordingly have a different willingness-to-pay. The introduction of direction dependent prices implies the introduction of peak load pricing in public, and hence leads to the issue of choice of period of travel (peak versus off-peak). Issues of frequency choice by public transport suppliers have not been included. These are promising directions for further research in this context.
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