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## Money, Endogenous Fertility and Economic Growth

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## MONEY, ENDOGENOUS FERTILITY AND ECONOMIC GROWTH

### 1. Introduction

In his seminal (1967) contribution Sidrauski shows through an intertemporal optimizing monetary growth model that, in the long-run, money is superneutral -i.e. all real variables (with the exception of real money balances) are independent of the money supply growth rate- in contrast to the analysis of Tobin (1965), who finds, using a framework based on ad hoc behavioral relationships, that a higher money growth permanently increases capital intensity.

According to the voluminous literature that has examined the effects of anticipated inflation on capital accumulation, the result established by Sidrauski is due to the restrictive features of the model adopted rather than to a general consequence of monetary growth on capital-labor ratio and other real variables. In fact, the necessary assumptions incorporated in Sidrauski's model that support the steady state money superneutrality are: i) an infinitely-lived representative consumer; ii) a time-separable utility function with a constant subjective discount rate; iii) an identical rate of time preference for all individuals; iv) a rate of return on money that varies with the quantity held (this property follows from either inserting money in the utility function or considering a consumption "shopping costs" approach); v) a rate of return on capital that is independent of real money balances; vi) an exogenous labour supply; and vii) the absence of uncertainty<sup>1</sup>.

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<sup>1</sup> Many articles written in the last three decades explore the role of these assumptions in determining the superneutrality of money. See, for example, for the removal of

Sidrauski's result strongly hinges on the validity of the modified golden rule -i.e. the equality between the marginal product of capital and the sum of the rate of time preference and the rate of population growth- which alone determines the capital stock when all the conditions previously listed are satisfied and, in addition (this is an assumption not mentioned above), the population growth rate is -as always assumed in neoclassical growth models of inflation- constant.

Surprisingly, to the best of my knowledge no attempts have been made to incorporate and analyze the hypothesis of an endogenous population growth rate within theoretical monetary models of capital accumulation, although this issue has received some attention from an empirical perspective. Barro (1996), for example, shows panel regressions where fertility, i.e. a proxy of population growth, and inflation jointly explain, amongst other explanatory factors, per capita output growth rate. Both inflation and fertility account for a negative effect on per capita output growth and rate of capital accumulation. But the channels of interaction between inflation, population expansion rate and output growth are not clear and need to be investigated theoretically.

The purpose of the present paper is to explore this neglected aspect of the theoretical literature through a simple

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each specific condition of Sidrauski's neoclassical monetary growth model, the following references: i) Drazen (1981), Carmichael (1982), Van der Ploeg-Marini (1988) and Weil (1991); ii) Epstein-Hynes (1983), and Hayakawa (1992); iii) Carmichael (1982); iv) Stockman (1981), Abel (1985), Feenstra (1986), Wang-Yip (1991) and (1992); v) Dornbusch-Frenkel (1973) and Fischer (1983); vi) Brock (1974) and Wang-Yip (1992); vii) Danthine-Donaldson-Smith (1987).

monetary growth model *à la* Sidrauski, where a variable population growth rate is considered, and investigate how the effects of anticipated inflation on capital accumulation and growth are affected.

The paper considers a model with an endogenous fertility choice, where the number of children enters directly into the utility function of the infinitely-lived representative consumer, along with private consumption and real money balances, and represents a control variable. The fertility rate coincides with population growth given a zero death rate and the absence of immigration. In this regard, we employ the standard practice of the literature on optimizing real growth models with endogenous fertility; see, for example, Razin-Ben-Zion (1975), Barro-Becker (1989), Wang-Yip-Scotese (1994), Barro-Sala-i-Martin (1995), Palivos (1995), and Nerlove-Raut (1997)<sup>2</sup>.

The framework developed here can be regarded as an exogenous growth model when per capita variables are considered, since there are decreasing returns to scale with respect to productive factors that can be accumulated, but at the same time it can be viewed as an endogenous growth model in terms of aggregate variables (i.e. variables expressed in levels), as their rate of expansion depends on the fertility rate, which is endogenously determined.

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<sup>2</sup> Some of these papers employ a different demographic setup from ours, but they have in common with the present paper that the fertility rate enters the utility function of consumers.

It is shown in the paper that the assumption of a variable population represents an additional source of money non-superneutrality that was not considered before, and adds another case to the variegated taxonomy of theoretical results concerning the effects of inflation on capital accumulation<sup>3</sup>. In our framework, however, despite the fact that the origin of money non-superneutrality comes from the consumption-side (since fertility is a choice variable for the consumers, even though there are also some external effects on the production side), there is a reverse Tobin effect, i.e. steady state capital stock per capita is reduced by an increase in the growth of money supply<sup>4</sup>. However, when population growth is augmented by the monetary shock, we have a positive effect on the growth rate of aggregate capital and output.

The paper is organized as follows. Section 2 builds the theoretical model and presents its general features. Section 3 analyzes the property of stability. Section 4 examines the relationship between the steady state rate of money growth and macroeconomic equilibrium, paying special attention to the effects on fertility, capital intensity, and consumption. Section 5 determines the optimal monetary policy rule and section 6 draws conclusions.

## 2. The model

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<sup>3</sup> See Orphanides-Solow (1990) and Danthine (1992) for comprehensive surveys.

<sup>4</sup> In Sidrauski's model, when violation of superneutrality is introduced on the consumption-side, either by considering a finite horizon for households (see, among others, Drazen, 1981, and Van der Ploeg-Marini, 1988) or by making the rate of time preference endogenous (*à la* Uzawa; see, for example, Hayakawa, 1992), the effect of anticipated inflation on capital stock is positive. However, in the case of an endogenous labour-leisure choice (see, e.g., Wang-Yip, 1992), the direction of non-superneutrality introduced, stemming simultaneously from both the consumption and production sides, becomes in principle ambiguous.

Consider a monetary economy populated by identical agents who are infinitely-lived, behave competitively and have perfect foresight. This economy is closed. Production and consumption activities are consolidated, so that the economic agent simultaneously plays the double role of consumer and entrepreneur. The paradigm of the representative agent is used. The total wealth of economic agents is divided between two assets: money and real capital.

The population size grows at an endogenously determined rate of expansion. The introduction of a variable population growth by the insertion of the fertility rate into the utility function of the representative agent allows fertility to be endogenously chosen<sup>5</sup>. The fertility rate corresponds to population growth because the mortality rate is zero and the economy is closed (i.e. there is no immigration from the rest of the world).

Money is introduced into the economy by introducing real balances into the utility function of economic agents, since we want to keep the analysis as close as possible to Sidrauski's (1967)<sup>6</sup>.

We assume that the representative agent makes consumption, fertility, and savings decisions in order to maximize the

$$\int_0^{\infty} [u(c,n) + v(m)] e^{-\delta t} dt$$

following intertemporal utility function

$$l + h(n) = 1$$

subject to the time allocation constraint

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<sup>5</sup> See, for the same demographic structure, Wang-Yip-Scotese (1994), Barro-Sala-i-Martin (1995), Palivos (1995), and Yip-Zhang (1997).

<sup>6</sup> The introduction of money balances into the system through consumption "shopping costs" (see Feenstra, 1986) would not change our qualitative results.

$$c + \dot{m} + \dot{k} = f(k, l) + s - (n + p)m - nk$$

the flow budget constraint

and the initial condition  $k(0) = k_0$ ,

where  $c$ =per capita consumption;  $n$ =fertility rate;  $m$ =per capita real money balances;  $l$ =labor;  $h(\cdot)$ =time spent for child-rearing;  $k$ =per capita capital stock;  $f(\cdot, \cdot)$ =per capita output;  $s$ =per capita public lump-sum transfers;  $p$ =actual inflation rate; and  $d$ =rate of time preference (exogenous).

The instantaneous utility function, which depends on consumption, fertility, and real money balances, is additively separable on consumption and fertility, on the one hand, and money holdings, on the other<sup>7</sup>. We postulate that the sub-utility function  $u$  is twice-continuously differentiable, increasing and strictly concave in their arguments.  $c$  and  $n$  are assumed to be normal goods. No special assumption on the sign of  $u_{cn}$  is made. Regarding the sub-utility function  $v$ , we assume that it is twice-continuously differentiable, strictly-concave and the marginal

$$\text{sgn}[v'(m)] = \text{sgn}(m^s - m)$$

utility of money satisfies<sup>8</sup>

where  $m^s$  gives the satiation level of real money balances, as in Friedman (1969). Therefore, the marginal utility of money is positive (negative), if  $m < (>) m^s$ .

According to the time allocation constraint (2), the fixed time endowment (normalized to one) can be divided between labor

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<sup>7</sup> The analysis is kept as simple as possible in order to focus on the specific source of non-superneutrality introduced through the hypothesis of endogenous fertility. Therefore, besides the strong separability between consumption-fertility and real money balances in the instantaneous utility function (1), labor supply is assumed to be inelastic.

<sup>8</sup> See Turnovsky-Brock (1980) for an identical hypothesis.

and child-rearing<sup>9</sup>. The function  $h(n)$  represents the time cost of child-rearing, with  $h(0)=0$  and  $h>0$  for  $n>0$ ,  $h'>0$ , and  $h''>0$ <sup>10</sup>.

Per capita output is obtained by using capital and labor as inputs. The production function,  $f(\cdot)$ , is assumed to have the usual neoclassical properties of positive, but diminishing, marginal products, exhibits constant returns to scale and satisfy the Inada conditions:  $f_k>0$ ,  $f_l>0$ ,  $f_{kk}<0$ ,  $f_{ll}<0$ ,  $f_{kk}f_{ll}=f_{kl}^2$ . Therefore, capital and labor are Edgeworth complements:  $f_{kl}>0$ . Capital is assumed not to depreciate.

Savings can take the form of both money and capital accumulation. All capital accumulation (or decumulation) occurs at a continuous rate and does not incur adjustment costs. Total

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<sup>9</sup> We follow Wang-Yip-Scotese (1994) and Palivos (1995) on using a time allocation constraint where the fertility rate indirectly enters. Alternatively, Barro-Sala-i-Martin (1995) consider explicitly a child-rearing cost function (depending directly on  $n$  and  $k$ ) in the budget constraint of the representative consumer. None of our results would change under the Barro-Sala-i-Martin approach.

<sup>10</sup> When  $h''<0$ , there are economies of scale in child-rearing; see Palivos (1995) for a discussion.

income available for consumption and savings takes into account the inflation tax on money holdings and the population growth rate, which reduces per capita wealth, and must include government lump-sum transfers. Since perfect foresight has been assumed, the expected inflation rate in equation (3) has been replaced with the actual one.

After inserting the time constraint (2) into the production function for  $l$ , the present value Hamiltonian for the dynamic

$$H = u(c, n) + v(m) + \lambda \{ f[k, l - h(n)] + s - (n + p)m - nk - c \}$$

optimization program of the representative agent is given by where  $\lambda$  is the co-state variable representing the shadow value of wealth in the form of real money balances and physical capital.

The first-order conditions for the consumer's optimization problem are

$$u_c(c, n) = \lambda$$

$$u_n(c, n) = \lambda \{ f_l[k, l - h(n)] h'(n) + k + m \}$$

$$\dot{\lambda} + \lambda \{ -k - f_k[k, l - h(n)] h'(n) - (n + p) \} = 0$$

$$\lim_{t \rightarrow \infty} \lambda m e^{-\rho t} = \lim_{t \rightarrow \infty} \lambda k e^{-\rho t} = 0$$

In addition, the conventional transversality conditions must be respected:

The first two equations (4a)-(4b) are the static efficiency conditions. According to equation (4a), the marginal utility of consumption must equal the marginal utility of wealth. Equation (4b) asserts that the marginal rate of substitution of consumption for fertility must equal the opportunity cost of one unit of fertility in output terms, given by the marginal product

of labour times the marginal time-cost of child-rearing plus the per capita stock of total wealth.

Equations (4c) and (4d) derive from intertemporal arbitrage relationships, which implicitly state that in equilibrium the rate of return on consumption, given by  $d_{c,t}/c_t$ , has to be equal to the real return on each asset, which are given by  $v'(m)/l - (p+n)$ , for real money balances, and  $f_k[k, 1-h(n)] - n$ , for capital<sup>11</sup>.

Conditions (4) are necessary, but not sufficient for the optimum, since the production function  $f(k, n)$  in the budget constraint (3) may no longer be concave because of the external effect of  $n$ . The sufficient condition for a unique maximum

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<sup>11</sup> Joint consideration of equations (4c) and (4d) leads to the equalization of the marginal rate of substitution between consumption and real balances to the marginal product of capital plus the inflation rate:

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Equation (4c') represents the implicit money demand equation.

requires that the Hessian of  $H$  is negative definite. This condition is satisfied if the instantaneous utility function and the production function are strictly concave in their arguments (as postulated) and if the opportunity cost of children in terms of output is strictly increasing in  $n$ , i.e.  $f_{11}(h')^2 - f_1 h'' < 0$ . We assume that the latter condition holds throughout.

The goods market equilibrium condition assures that full employment output always equals consumption,  $c$ , plus total

$$f[k, 1-h(n)] = c + \dot{k} + nk$$

investment,  $\dot{k} + nk$ :

where the time allocation constraint (2) has been used.

$$\dot{m} = m(q - \rho - n)$$

$$s = qm$$

We assume that the monetary authority allows the nominal money supply to grow at a constant rate, given by  $q$ . Therefore, the evolution of real money balances per capita is

$$\dot{m}'(a, n) = \{ f[k, 1-h(n)] - \rho(n) + k + m \}$$

Finally, the seigniorage from money creation is transferred to the private sector in a lump-sum fashion:

### 3. Stability analysis

The general macroeconomic equilibrium -obtained by combining the optimality conditions (4) together with equations (5), (6) and (7)- is given by the following set of equations

$$\dot{l} = l \{d + n - f_k[k, l - h(n)]\}$$

$$\dot{m} = m(q - p - n)$$

$$f[k, l - h(n)] = c + \dot{k} + nk$$

$$s = qm$$

together with the transversality conditions (4f).

Equations (8a)-(8c) may be solved for  $c$ ,  $n$  and  $p$  in terms

$$c = c(l, m, k), \quad c_l = \frac{\bar{E}}{\Sigma} < 0; \quad c_m = -\frac{\bar{l} u_{cn}}{\Sigma}; \quad c_k = -\frac{\bar{l} \Omega u_{cn}}{\Sigma};$$

$$n = n(l, m, k), \quad n_l = -\frac{\Psi}{\Sigma} < 0; \quad n_m = \frac{\bar{l} u_{cc}}{\Sigma} < 0; \quad n_k = \frac{\bar{l} \Omega u_{cc}}{\Sigma} < 0;$$

$$p = p(l, m, k), \quad p_l = -\frac{\bar{l} f_{kl} h' \Psi + (d + q) \Sigma}{\Sigma} < 0; \quad p_m = \frac{v'' \Sigma + \bar{l}^2 f_{kl} h' u_{cc}}{\Sigma} < 0;$$

$$p_k = -\bar{l} \frac{f_{kk} \Sigma - \bar{l} f_{kl} h' \Omega u_{cc}}{\Sigma};$$

of the endogenous dynamic variables  $(l, m, k)$  in the form:

where overbar variables denote the long-run equilibrium values and the various combinations of parameters employed in the partial derivatives of these functions are defined in the appendix.

Substituting the short-run solutions for  $c$ ,  $n$  and  $p$ , given by equations (9), in the arbitrage condition (8d), the evolution of real money balances (8e) and the product market equilibrium

$$\dot{m} = m \{q + p(l, m, k) - f_k[k, l - h(n(l, m, k))]\}$$

condition (8f), we obtain the following autonomous dynamic system<sup>12</sup>:

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<sup>12</sup> Lump-sum transfers are omitted from the general equilibrium, since they can be solved residually.

$$\dot{k} = f\{k, l - h[n(l, m, k)]\} - c(l, m, k) - n(l, m, k)k$$

Linearizing the sub-system (10) around the long-run

$$\begin{bmatrix} \dot{l} \\ \dot{m} \\ \dot{k} \end{bmatrix} = \frac{1}{\Sigma} \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} l - \bar{l} \\ m - \bar{m} \\ k - \bar{k} \end{bmatrix}$$

equilibrium, the dynamic model may be written in matrix form as

$$h_{11} = -\bar{l}\Psi\Omega < 0; \quad h_{12} = \bar{l}^{-2}\Omega u_{cc} < 0; \quad h_{13} = \bar{l}(\bar{l}\Omega^2 u_{cc} - \Sigma f_{kk})?;$$

$$h_{21} = \frac{\bar{m}[\bar{l}\Psi\Omega + (f_k + \bar{p})\Sigma]}{\bar{l}} > 0; \quad h_{22} = -\frac{\bar{m}(v''\Sigma + \bar{l}^{-2}\Omega u_{cc})}{\bar{l}} > 0;$$

$$h_{23} = -\bar{m}(\bar{l}\Omega^2 u_{cc} - \Sigma f_{kk})?;$$

$$h_{31} = \Gamma\Psi - \Xi > 0; \quad h_{32} = \bar{l}(u_{cn} - \Gamma u_{cc})?; \quad h_{33} = d\Sigma + \bar{l}\Omega(u_{cn} - \Gamma u_{cc})?.$$

where<sup>13</sup>

Since the economy has two jump variables,  $l$  and  $m$  -i.e.  $l(0)$  and  $m(0)$  are free determined-, and one predetermined variable,  $k$  -i.e.  $k(0)=k_0-$ , saddle-point stability requires that the matrix of the coefficients of the short-run dynamic model (10) must have a negative determinant, since it must admit two positive and one negative eigenvalues. Thus for this requisite to be satisfied, it is required that

$$\Delta = -v''[\Psi(d\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l}\Omega^2] + \bar{l}(f_k + \bar{p})[f_{kk}(u_{cn} - \Gamma u_{cc}) + d\Omega u_{cc}] > 0$$

The negative determinant condition (12) does not guarantee that we have two positive characteristic roots and a negative one, as it could be also satisfied with three negative roots leading to a globally stable economy instead of a saddle-point

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<sup>13</sup> See the appendix for the combinations of parameters used.

equilibrium. Since the trace of the state matrix in (10) is unambiguously positive<sup>14</sup>, we are ensured that, once condition (12) holds, the dynamic system is saddle-point stable.

#### 4. Long-run effects of the money supply growth rate

In the steady state equilibrium, where  $\dot{c} = \dot{m} = \dot{k} = \dot{n} = 0$ , the

$$u_c(\bar{c}, \bar{n}) = \bar{1}$$

$$\frac{u_n(\bar{c}, \bar{n})}{u_c(\bar{c}, \bar{n})} = f_1[\bar{k}, 1 - h(\bar{n})]h'(\bar{n}) + \bar{k} + \bar{m}$$

$$\frac{v'(\bar{m})}{u_c(\bar{c}, \bar{n})} = d + \bar{n} + \bar{p}$$

$$f_k[\bar{k}, 1 - h(\bar{n})] = d + \bar{n}$$

$$\bar{p} = q - \bar{n}$$

$$f[\bar{k}, 1 - h(\bar{n})] = \bar{c} + \bar{n}\bar{k}$$

$$\bar{s} = q\bar{m}$$

model is described by the following system

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<sup>14</sup> In fact the trace of the matrix of the coefficients in (10) is

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There are some special features of this long-run equilibrium model (13) that must be emphasized. First, according to equation (13e), the endogeneity of the population growth makes the inflation rate endogenous, breaking its one-to-one correspondence with the money supply growth rate (which is a typical feature of models with constant population growth and exogenous economic growth). Second, equation (13d), i.e. the modified golden rule, no longer determines capital intensity by itself, but establishes an inverse relationship between capital intensity and population growth rate. Third, if no exogenous shocks occur, per capita variables -like output, capital and consumption- remain constant in the long-run, while aggregate variables (namely variables in levels) grow at rate  $n$ , which is endogenously determined. Finally, the demands for fertility, derived from equation (13b), and for real balances, given by equation (13c), are strictly interconnected with the rest of the economy. These are the elements, as we shall see in a moment, that violate money superneutrality.

The steady state effects of the money supply growth rate are

$$\frac{d\bar{k}}{dq} = -\frac{\bar{\Gamma}\Omega u_n}{\Delta} < 0;$$

$$\frac{d\bar{n}}{dq} = -\frac{\bar{\Gamma} f_{kk} u_n}{\Delta} > 0;$$

described by the following multipliers:

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This is the exact opposite of Malthus's prediction (see, e.g., Barro-Sala-i-Martin, 1995, and Yip-Zhang, 1997).

The demands for fertility and real money balances can implicitly be stated as:

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Money demand depends on the population growth rate as well as the usual variables (i.e. consumption and the nominal interest rate). If  $n$  were exogenous, we would have  $m, \tilde{n}=0$ .

$$\frac{d\bar{c}}{dq} = -\frac{\bar{l}(\bar{d}\Omega - \Gamma f_{kk})u_n}{\Delta} < 0;$$

$$\frac{d\bar{m}}{dq} = -\frac{[\Psi(\bar{d}\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l}\Omega^2]}{\Delta} < 0;$$

$$\frac{d\bar{l}}{dq} = \bar{l}u_n \frac{(u_{cc}\Gamma - u_{cn})f_{kk} - \bar{d}u_{cc}\Omega}{\Delta} > 0;$$

$$\frac{d(\bar{k}/\bar{l})}{dq} = -\frac{\bar{l}u_n}{\Delta} < 0;$$

$$\frac{d\bar{s}}{dq} = \bar{m} + q \frac{d\bar{m}}{dq} = \bar{m}(1 - |e_{\bar{m},q}|) > 0.$$

$$\frac{d\bar{p}}{dq} = \bar{l} \frac{-\frac{v''}{l}[\Psi(\bar{d}\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l}\Omega^2] + (\bar{d} + q)u_{cc}[f_{kk}(\frac{u_{cn}}{u_{cc}} - \Gamma) + \bar{d}\Omega] + u_n f_{kk}}{\Delta} > 0;$$

An increase in the rate of monetary growth raises the implicit cost of holding money -i.e.  $f_k + p = \bar{d} + q$ - and hence reduces real money balances, which in turn stimulates, for a given level of consumption, fertility, because the opportunity cost of one unit of fertility in output terms, namely  $f_l h' + k + m$ , diminishes (for a given capital stock). The latter effect generates, through the modified golden rule, a decline in the demand for capital that brings about a reduction of per capita output and consumption. The monetary shock also reduces the labor effort (even if inelastically supplied), since the rise in fertility implies a larger consumption of time for child-rearing which lowers the time that can be devoted to work. As the capital-labor ratio declines because labor diminishes in percentage terms less

than capital, the marginal product of capital, which in equilibrium is equal to  $d+n$ , increases.

Inflation is increased by the shock, but less than the rise in  $q$  because of the higher fertility rate. Since the real interest rate is augmented, the Fisher relation is invalidated. The nominal interest rate follows one-to-one the increase of the money supply growth, even if it has risen more than the inflation rate.

The effects on seigniorage are unclear, since they depend on whether the elasticity of real money balances with respect to  $q$ , taken in absolute value, is less or greater than one.

Moreover, it is easy to show that the money supply growth rate unambiguously lowers steady state welfare, that is, the negative consequences on consumption and real money balances overcompensate the positive effect exerted on fertility.

Finally, it is worth noticing that while a higher money growth rate lowers capital intensity -i.e. a reverse Tobin effect in per capita terms occurs- and leaves its long-run rate of expansion unaffected, it increases the growth rate of capital and output levels, that is, we have a sort of Tobin effect in growth terms.

#### **4. Optimal monetary growth rule**

Determination of steady state optimal monetary growth is based on the assumption that the government seeks to maximize the welfare function of the representative agent subject to the model of the economy, given by system (13) as a constraint.

We can make some simplifications. If we express the population growth rate (employing equation 13d) and consumption

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Note that in principle the inflation rate could also be reduced if the increase in  $n$  were higher than the increase in  $q$ . We consider this case a rather implausible event according to the parameters of the linearization involved in the inflation multiplier.

(using simultaneously the modified golden rule and the resources constraint) as implicit functions of capital stock -namely  $n, \bar{n} = n, \hat{n}(k, \bar{m})$  and  $c, \bar{c} = c, \hat{c}(k, \bar{m})$  respectively- and capital stock as a function of real money balances (using equations 13b, 13d, and 13f) -i.e.  $k, \bar{k} = k, \hat{k}(m, \bar{m})$ - , the determination of the optimal  $q$  can

$$\max_{\bar{k}, \bar{m}, q} \{u[\hat{c}(\bar{k}), \hat{n}(\bar{k})] + v(\bar{m})\}$$

be found by solving the following problem

$$\bar{k} = \hat{k}(\bar{m})$$

subject to

$$v'(\bar{m}) = (d + q)u_c[\hat{c}(\bar{k}), \hat{n}(\bar{k})]$$

$$\hat{c}_k = \frac{d\Omega - \Gamma f_{kk}}{(\Xi - \Gamma\Psi) f_{kk} + \Omega(d\Psi - \bar{\Gamma}\Omega)} > 0; \quad \hat{n}_k = \frac{f_{kk}}{(\Xi - \Gamma\Psi) f_{kk} + \Omega(d\Psi - \bar{\Gamma}\Omega)} < 0; \quad \hat{k}_m = \frac{\bar{\Gamma}\Omega}{(\Xi - \Gamma\Psi) f_{kk} + \Omega(d\Psi - \bar{\Gamma}\Omega)} > 0.$$

and

where

The first order conditions for the optimum are where  $\bar{m}_1$  is the Lagrange multiplier associated with constraint (14b) and asterisks denote the socially optimal steady state values. The endogenous variables of system (15) are  $k, \bar{k}^*, m, \bar{m}^*, \bar{m}_1$  and  $q^*$ .

Substituting relationships (15a), (15b) and (15c) into equation (15d), the optimal monetary growth rate can be expressed as

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The maximization problem (14) admits another necessary condition, namely  $\bar{m}_2 u_c = 0$ , where  $\bar{m}_2$  is the Lagrange multiplier associated with constraint (14c). This condition implies  $\bar{m}_2 = 0$ , and has been included in system (15). The alternative case contained in such a condition, i.e.  $u_c = 0$ , is not possible, since it would imply from model (13) that  $u_c = u_n = v' = 0$ , which yields optimal values for  $c, \bar{c}^*, n, \bar{n}^*, m, \bar{m}^*$ , and (through the modified golden rule)  $k, \bar{k}^*$  inconsistent with the resources constraint (13f).

$$q^* = \bar{l}^* \frac{d \frac{\partial(f_k - \bar{n})}{\partial n} - \frac{\partial f_k}{\partial k} \left[ \frac{u_n}{u_c} + \frac{\partial(f - \bar{n}k)}{\partial n} \right]}{[\Psi(d\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l}^* \Omega^2]} - d = \frac{\bar{l}^* (d\Omega + \bar{m}^* f_{kk})}{[\Psi(d\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l}^* \Omega^2]} - d > < 0$$

From equation (15d'), we see that in our context the Friedman full liquidity rule -i.e.  $q^{*F} = -d$  or  $f_k + p = 0$ - is not optimal, since the money supply growth rate affects the fertility choice, which distorts the long-run capital intensity. The optimal monetary policy rule requires a balancing for the partial effects exerted by the population growth on both the net and the gross return on capital. These effects are negative, but enter equation (15d') with both positive and negative signs. The rule specified in (15d') can prescribe either a contraction or an expansion rate of money supply. Accordingly, the nominal interest rate corresponding to  $q^*$  can be negative or positive. If  $d + q^*$  is negative (positive), the optimum quantity of money balances must be pushed beyond (kept below) the satiation level -since  $v' < (>) 0$ , i.e.  $m > (<) m^s$ - raising the utility of agents correspondingly.

## 5. Concluding remarks

This paper has examined the steady state consequences of the money supply growth rate in an optimizing monetary model of capital accumulation with endogenous population. The economy is characterized by infinitely-lived agents, with a fixed discount rate, an inelastic labor supply and money balances -together with

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When there is an endogenous labor-leisure choice with proportional income taxation and exogenous population growth, as in Turnovsky-Brock (1980), the optimal monetary policy, which is a "distorted" Friedman rule, requires that the direct effect of money in the utility function must be balanced with the indirect effect resulting from its interaction with consumption and leisure. Thus, it is possible to have either an optimal inflation tax or an optimal inflation subsidy.

the fertility rate and consumption- inserted into the utility function. This paper is an attempt to understand a source of non-superneutrality not studied before.

The analysis carried out in the paper proves that, when an endogenous population growth rate is incorporated into Sidrauski's monetary growth model, the modified golden rule is incapable of fully determining the capital-labor ratio (a basic feature of the Sidrauski's analysis), since capital intensity becomes interdependent with other variables of the model because of the endogenous fertility choice. By reducing the real balances holdings, the higher monetary growth rate diminishes the opportunity cost of one unit of fertility, which in turn stimulates people to increase fertility and hence results in a reduction of the capital intensity. This inverse relation between per capita capital stock and population growth is the crucial element at the root of the non-superneutrality of money discovered in the present context. The reverse Tobin effect on per capita output and capital is matched by an increase in aggregate output and capital growth rates.

In this framework, the optimal monetary growth rule, which must take into account the effect of fertility on the net and the gross return of capital and corresponds to a modified Friedman full liquidity rule, can call for either a contraction or an expansion of money supply. If the optimal rule is implemented, the nominal interest rate can be either positive or negative, therefore implying that the optimum quantity of money attained can be lower or greater than the satiation level.

## **Appendix**

## A. Combinations of parameters

$$\Xi = u_{nn} - \frac{u_n}{u_c} u_{cn} - u_c [f_l h'' - f_{ll} (h')^2] < 0; \quad \Psi = u_{cn} - \frac{u_n}{u_c} u_{cc} > 0;$$

$$\Omega = 1 + h' f_{kl} > 0; \quad \Sigma = \Xi u_{cc} - \Psi u_{cn} > 0; \quad \Gamma = f_l h' + \bar{k} > 0;$$

$$\Delta = -v'' [\Psi (d\Omega - \Gamma f_{kk}) + \Xi f_{kk} - \bar{l} \Omega^2] + \bar{l} (f_k + \bar{p}) [f_{kk} (u_{cn} - \Gamma u_{cc}) + d\Omega u_{cc}] > 0.$$

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The assumption that  $f_l h'$  is decreasing in  $n$ , i.e.  $f_{ll} (h')^2 - f_l h'' < 0$ , required for satisfying the second order conditions for the consumers optimization problem (1)-(3), has been used to sign some combinations of the parameters.

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