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Environmental advertisement:  
An alternative policy to control consumption pollution

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Abstract
This paper examines the efficiency enhancing potential of supplementing existing policies of controlling consumption pollution with environmental advertisement. Our definition of environmental advertisement includes both information dissemination and persuasion. While incentive-based regulations that are based on coercion are effective immediately, environmental advertisement that is based on inducing voluntary action, requires time. We formalize this argument by assuming that the shift of consumers’ preference towards the desirable environmentally friendly goods or behaviour depends on the stock rather than the flow of advertisement. We assume homogeneous consumers, having the choice of consuming two goods one of which generates pollution. We treat environmental advertisement as a separate good within consumers’ utility. Within a dynamic partial equilibrium framework, we analyse regulators’ optimal control problem. We find that at the optimal steady state there is a positive stock of environmental advertisement. Thus, if we start with some small stock of advertisement, the stock increases monotonically while the rate of taxation decreases along the optimal path.

JEL Classifications: D62, Q28  
Keywords: environmental advertisement, pollution taxation, dynamic optimization, second best optimum.

Non-technical summary
Consumption generated pollution includes household pollution (solid waste generation) and product pollution (use of cars, aerosol sprays, energy-related household activities, and water use). The increasing importance of consumption pollution has prompted regulatory action in the form of incentive based policies. Examples of incentive-based policies include municipal charges per unit of waste, deposit-refund systems, and carbon taxes. Incentive-based approaches have resulted in high administrative costs due to the complexity of substances to be controlled and they also involve indirect costs such as those resulting from illicit dumping. Given these costs, should regulators rely exclusively on incentive-based policies to control consumption pollution or they should explore potential supplements to these policies? This paper addresses this question by examining the efficiency enhancing properties of using environmental advertisement to

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complement unit charges.
Our definition of environmental advertisement is broad in scope and includes both information enhancing and persuasive advertisement. We believe that there is significant educational and information disseminating scope for environmental advertisement. This is important in situations where consumers are little or not concerned about the pollution aspect of their consumption behaviour, and/or the existing pollution control programs are not well-entrenched, such as newly introduced waste management and car-pooling programs. We also believe that at least equally important is the persuasive role of advertisement. In situations of well-informed consumers some of which might even have positive attitudes towards environmentally-friendly behaviour but are reluctant to behave accordingly, persuasion is by far the most important aspect. Finally, we believe that the effect of environmental advertisement on pollution reduction is truly dynamic in nature. Consumers need to be informed at a general as well as a program-specific level so that they are aware of the consequences of their action as well as the availability of pollution control programs, and they also have to be convinced of the effectiveness of their action and the worthiness of their participation. While incentive-based regulations that are based on coercion are effective immediately, environmental advertisement that is based on inducing voluntary action, requires time. The effectiveness of environmental advertisement increases over time not only because of its cumulative effect that strengthens the message to each consumer, but also because of the increased social influence to the consumers as the number of participants to the program increases over time.

We build an analytical framework using the following assumptions. We assume homogeneous consumers that have the choice of two consumption goods one of which generates waste/pollution that adversely affect environmental quality. In order to internalize the environmental externality, regulators, in each time period, use a combination of two policy instruments; a per unit charge on the dirty good and some level of environmental advertisement. The per unit charge involves an indirect cost to the society. Environmental advertisement is considered a good in households’ utility and is assumed to be a complement to the clean good, i.e. it promotes the relatively cleaner alternative of consumption. Based on our belief that environmental advertisement has a cumulative effect on consumers’ behaviour, we assume that it is the stock rather than the flow of environmental advertisement that affects consumers’ behaviour. Within this framework we derive the optimal environmental policy mix in the form of environmental taxes and environmental advertisement expenses. Starting with a sufficiently low stock of environmental advertisement, we find that the stock of advertisement increases continuously towards the steady state while the optimal tax rate monotonically decreases. Given the costs of implementing the unit charge, there is always a positive level of environmental advertisement at the steady state. The optimal combination of environmental advertisement and unit charges at the steady state depends on the costs of implementing the unit charge. Our results suggest that the use of environmental advertisement to complement unit charges in inducing environmentally friendly behaviour on the part of consumers enhances efficiency.
Introduction

Pollution can be a byproduct of either the production or the consumption process. Consumption generated pollution includes household pollution (solid waste generation) and product pollution (use of cars, aerosol sprays, energy-related household activities, and water use). In this paper we are concerned with policies to control consumption pollution. Traditionally pollution control relied on emission standards and other forms of command and control policies, while more recently there is considerable support for the use of incentive-based instruments. Examples of incentive-based policies for the control of consumption pollution include municipal charges per unit of waste, deposit-refund systems, and carbon taxes. Although incentive-based approaches, when implemented, have increased cost efficiency of pollution control, they have not been able to fully explore all potential gains in the area of pollution control. This is because the number and the complexity of the substances to be controlled results in high administrative costs, while in the same time direct charges involve indirect costs such as those resulting from the interaction with the existing set of taxes and the inducement of illicit dumping.

Continuing the search for improving the design of pollution control policies economists have proposed approaches that could either broaden the scope or complement incentive-based instruments. Such proposals include the deposit-refund systems that apply to all consumption goods (see for example Fullerton and Kinnaman (1994)), revenue-recycling programs (see for example Goulder (1995)), information strategies (see Tietenberg (1997)), and voluntary compliance programs (see for example Glachant (1994) and Boyd et al. (1998)). This paper contributes to this discussion by examining the use of environmental advertisement as an alternative policy instrument to complement existing regulation in the case of consumption pollution. We define environmental advertisement to include both information dissemination and persuasion, the two aspects of advertisement that have been traditionally treated separately in the economic literature.

Some economists have emphasized the information disseminating role of advertisement (both direct provision of information or indirect in the form of signalling quality), assuming that consumers are not fully informed and they receive through advertisement complete, costless and instantly validated information (see for example Nelson (1974), Kotowitz and Mathewson (1979a), Kihlstrom and Riordan (1984) and Stigler (1961)). Others have advocated that advertisement alters consumers’
tastes resulting in higher demand or lower demand elasticity for the advertised product (see for example Galbraith (1958) and Dixit and Norman (1978)). Kotowitz and Mathewson (1979b) examine the “persuasive” aspect of advertisement without including advertisement into consumers’ utility. They consider goods that are differentiated by quality which is unknown to consumers. Consumers learn through experience about the unknown quality so that their quality expectations follow an adaptive rule. Persuasive advertisement affects consumers’ perception of experience quality; it maintains expectations when they exceed actual experience quality and it raises expectations when they fall short of actual experience quality. Kotowitz and Mathewson derive the profit maximizing time path of advertisement and proceed with the welfare analysis. More recently Becker and Murphy (1993) proposed that advertisement should be treated as a good rather than as a “taste shifter” within the utility function. Advertisement is considered as a complement to the good advertised and includes all activities that give “favourable notice” to the advertised good. Advertisement could have a positive or a negative price depending on the direct effect it has on consumers’ utility. Most of the advertisement delivered through mass media bears a negative indirect price, in the sense that consumers get compensated for the exposure to advertisements in the form of higher quality programming.1 Consumers can control the quantity of advertisement that they receive and their decisions depend on their marginal utility of advertisement and the indirect compensation they receive.

We believe that there is significant educational and information disseminating scope for environmental advertisement. This is important in situations where consumers are little or not concerned about the pollution aspect of their consumption behaviour, and/or the existing pollution control programs are not well-entrenched, such as newly introduced waste management and car-pooling programs. In such situations, the informative/educational aspect of advertisement increases consumers’ environmental awareness and understanding of both what can be done to help the environment and how it can be done. It can also influence consumers’ perception of how easy it is to perform the suggested behaviour. Informative/educational advertisement can be at the general and/or the program-specific level. For example, governments (at any level) can inform the public about the contribution of consumption pollution to aggregate pollution problems, the mounting problems of

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1 Since advertisement is not sold separately, it is difficult to talk about the price of a specific advertisement.
garbage disposal, the effects of car pollution, etc. It could also provide information about available recycling programs, various waste composting techniques and products that can be used in the composting process, the availability of public transportation and car-pooling programs.

We also believe that at least equally important is the persuasive role of advertisement. In situations of well-informed consumers some of which might even have positive attitudes towards environmentally-friendly behaviour but are reluctant to behave accordingly, persuasion is by far the most important aspect.\(^2\) The persuasive aspect of advertisement attempts to (1) further improve attitude toward environmental behaviour; (2) stress the personal as well as the societal benefits of the suggested behaviour;\(^3\) (3) improve the perceived consumer effectiveness of performing the suggested behaviour;\(^4\) and (4) emphasize the accessibility of resources that minimize effort, time and cost. For example, environmental advertisement could stress the significance of individual participation as well as the potential aggregate effect of such programs as waste composting, energy conservation and car pooling, so that consumers believe that their actions can be effective. Environmental advertisement could also try to increase the appeal of these programs, and shift consumers away from atomistic behaviour by making them part of the community life.\(^5\)

Finally, we believe that the effect of environmental advertisement (in both its informative and persuasive aspect) on pollution reduction is truly dynamic in nature. Consumers need to be informed at a general as well as a program-specific level so that they are aware and concerned, and they also have to be convinced of the effectiveness of their action and the worthiness of their participation.

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\(^2\) What we describe as persuasive aspect of environmental advertisement share some characteristics with what in the discipline of Marketing is called “social marketing”. For a recent review and definition of social marketing see Andreasen (1994). It should be clear though that we do not emphasize the distinction between changes in attitude and behaviour, but we rather emphasize the difference between information and persuasion.

\(^3\) Taylor and Todd (1995) find that consumers react more to societal rather than personal benefits.

\(^4\) There is extensive literature on the role of perceived consumer effectiveness in motivating environmentally friendly behaviour. For example, Berger and Corbin (1992) find that self-efficacy is a key parameter in translating positive environmental attitudes into behaviour.

\(^5\) Currently there is a number of advertisements aired on television with strong emphasis on persuasive arguments. For example, in the province of British Columbia, Canada the public electricity producer’s tv advertisements emphasise the aggregate effect of energy conservation. The regional environmental authority’s tv advertisements emphasize the accessibility of recycling programs, the effectiveness of composting and the community aspect of both these programs. These programs have already had a well documented positive effect.
Habits have to be broken and consumers have to be alarmed not to return back to them. While incentive-based regulations that are based on coercion are effective immediately, environmental advertisement that is based on inducing voluntary action, requires time. The effectiveness of environmental advertisement increases over time not only because of its cumulative effect that strengthens the message to each consumer, but also because of the increased social influence to the consumers as the number of participants to the program increases over time.

This paper examines the efficiency enhancing potential of supplementing existing pollution control policies with environmental advertisement. We have argued above that environmental advertisement requires time to induce the desirable behaviour on the part of consumers. We formalize this argument by assuming that the shift of consumers’ preference towards the desirable environmentally friendly goods or behaviour depends on the stock rather than the flow of advertisement. We assume homogeneous consumers, having the choice of consuming two goods one of which generates pollution. We assume that regulators can implement a unit charge and/or environmental advertisement in order to control pollution. We assume that unit charges have negative indirect effects on social welfare. Following Becker and Murphy (1993) we treat environmental advertisement as a separate good within consumers’ utility. We assume that environmental advertisement has a negative direct effect on consumers’ utility, and thus consumers have to be compensated to accept advertisement. Thus, the cost of environmental advertisement includes the cost of production as well as that of distribution.

Within a dynamic partial equilibrium framework, we analyse regulators’ optimal control problem. We find that at the optimal steady state (OSS) there is a positive stock of environmental advertisement. For any initial stock of advertisement, we always converge to the OSS, since it exhibits the saddle point property. Thus, if we start with some small stock of advertisement, the stock increases monotonically along the optimal path towards the OSS. As the stock of advertisement increases over time, the rate of taxation decreases along the optimal path. The OSS rate of tax depends on the indirect costs associated with environmental taxation. Thus, our results show that it is optimal to complement unit charges with environmental taxation.

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6 It is worth noting that we treat the many aspects of environmental advertisement as a homogeneous good.
To the best of our knowledge, there are no studies examining advertisement as an environmental policy instrument. However, our work relates to the recent proposal by Tietenberg (1997) of expanding environmental policy into the provision of information. Tietenberg reviews the different settings (household, consumption, employment and community) under which provision of information is important. He further assesses the role of government in detecting environmental risks, assuring reliable information, and providing new channels --when the existing are not enough-- through which information can be used. This study provides substantial support to our argument in favour of informative advertisement. An earlier study by Kennedy, Laplante and Maxwell (1994) examines the provision of information in the presence of environmental externalities. They show that when conventional environmental policies are not available, publicly provided information may improve welfare. However, when conventional policies are available, the situations that call for public provision of information are very limited.

**II. The model**

We assume there are only two consumption goods, $D$ and $C$. We normalize the generation of waste/pollution such that consumption of good $C$ generates zero waste/pollution, while consumption of good $D$ generates waste/pollution that adversely affect environmental quality, $d = d(D)$, with $d > 0$, $d'' > 0$. In the absence of any regulation, households consume more than the socially optimum level of good $D$, either because they do not have information on the environmental consequences of their actions, or because they do not believe that their individual action can be effective. Thus, there is room for policy intervention to shift consumption towards the relatively cleaner good $C$. Although, for simplicity, we will refer to polluting and environmentally friendly goods throughout the paper, our analysis applies to consumption behaviour as well. Examples of environmentally-based differentiation between consumption behaviours include, consumption without waste reduction versus consumption with composting and recycling; transportation in one-passerenger vehicle versus car-pooling or using public transportation; residential use of electricity or fuel versus use of alternative technologies such as natural gas.

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7 Since most consumption activities generate some waste/pollution, this should be considered just as a simplifying assumption.
In order to internalize the environmental externality, regulators, in each time period, use a combination of two policy instruments; a per unit charge, $\tau$ on $D$ and some level of environmental advertisement $a$. The per unit charge on $D$ involves an indirect cost to the society. In order to simplify the analysis while concentrating on environmental advertisement, we use a partial equilibrium framework. Thus, we assume that the indirect cost of the unit charge, $\phi(\tau)$ is determined exogenously, and we further assume that $\phi'(\tau) > 0$ and $\phi''(\tau) > 0$.\(^8\)

Following Becker and Murphy (1993) we treat advertisement as a good in households’ utility. Based on our belief that environmental advertisement has a cumulative effect on consumers’ behaviour, we assume that it is the stock rather than the flow of environmental advertisement that affects consumers’ behaviour. The stock of environmental advertisement changes over time as described by a first-order differential equation, $\dot{A} = a - \delta A$, where $A$ is the stock of advertisement and $\delta$ is the rate at which the stock depreciates. Consumers may forget the environmentally friendly behaviour and return to their old habits that involve lower costs. Since we assume that in the absence of any policy, households do not decrease the consumption of $D$, households’ utility is

\[ U(D, C, A) . \] (1)

We assume that advertisement directly lower households’ utility, $U_A < 0$, and thus regulators have to compensate households in order to deliver advertisement. We further assume that $U_{AA} < 0$. Thus the utility function is jointly concave in $(D, C, A)$. Consumers are compensated indirectly through enhanced quality of programs and coverage of news on radio, television, magazines and newspapers. Since advertisements can be easily ignored (particularly in the case of printed media), utility-decreasing advertisements are not sold in discrete units. Although there is no explicit price per unit of advertisement, the marginal disutility of advertisement $U_A$ determines in principle the required compensation and thus the cost of communicating the advertisement. The quantity of advertisement

\[ \text{Alternatively, a general equilibrium model, explicitly modelling the labour market, could be employed to capture the negative effect of unit charges on factor prices. The reader is directed to the literature on revenue recycling policies for an extensive discussion of the interaction effects (see Goulder 1994 for a recent review of this literature). Or we would have to explicitly model households’ consumption “technologies” to internalize the cost of illicit dumping (see for example Fullerton and Kinnaman 1995).} \]
is controlled by regulators and consumers get enough compensation to accept the quantity delivered.\(^9\) The cost of environmental advertisement \(w(a)\), with \(w'(a) > 0\) and \(w''(a) > 0\), includes both the cost of producing as well as the cost of communicating the message to the consumers.

Environmental advertisement is considered a complement to good \(C\), i.e. it promotes the relatively cleaner alternative of consumption, and thus, \(U_{CA} > 0\) and since \(C\) and \(D\) are substitutes, \(U_{DA} < 0\). Symmetry in utility implies that as consumers change over to environmentally-friendly good or behaviour, the marginal utility of environmental advertisement increases, i.e. \(U_{AC} > 0\).

Households’ single period utility maximization problem is,

\[
\max \quad U(D, C, A) \\
\text{subject to} \quad I = p_c C + (1 + \tau) p_D D,
\]

where \(I\) denotes households’ income. Assuming that the second order conditions hold, we derive the demand for \(C\) and \(D\) as functions of the policy parameters. Substituting the demand functions into households’ utility, we express the indirect utility function as a function of the policy parameters, omitting prices in order to simplify notation, as \(V = V(A, \tau) = V(C(A, \tau), D(A, \tau), A)\), where

\[
V(A, \tau) = \max_{C,D} \left\{ U(C, D, A) \quad \text{s. t.} \quad p_c C + (1 + \tau) p_D D - I = 0 \right\}.
\]

The curvature properties of the indirect utility function and the monotonicity characteristics of the demand functions in terms of the policy parameters are important in the development of the optimal policy mix. It can be shown that since the utility function \(U(C,D,A)\) is concave in \(A\) and the constraint does not contain \(A\), then the indirect utility function \(V(A, \tau)\) is concave in \(A\).\(^{10}\) Furthermore, the demand function for the clean good \(C\) is monotonically increasing in \(A\), while the demand function for the dirty good \(D\) is monotonically decreasing in \(A\). This can be shown by taking the comparative static matrix of the household’s utility maximization problem (2), which is written as:

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\(^9\) For a detailed discussion on the pricing of advertisements see Becker and Murphy (1993).

\(^{10}\) See Beavis and Dobbs (1990).
The comparative static derivatives are obtained as

\[
H = \begin{bmatrix}
\frac{\partial C}{\partial A} \\
\frac{\partial D}{\partial A} \\
\frac{\partial \lambda}{\partial A}
\end{bmatrix} = \begin{bmatrix}
-U_{CA} \\
-U_{DA} \\
0
\end{bmatrix}, \quad \text{where} \quad H = \begin{bmatrix}
U_{CC} & U_{CD} & -p_C \\
U_{DC} & U_{DD} & -(1+\tau)p_D \\
-p_C & -(1+\tau)p_D & 0
\end{bmatrix}, \quad \text{with} \quad |H| > 0.
\]

An increase in the tax rate, \(\tau\), can be regarded as equivalent to a price increase for the dirty good \(D\), so its demand will decrease while the demand of the substitute clean good will increase.

### III. Optimal policy mix

To derive the optimal policy mix we solve the social welfare maximization problem. We examine regulators’ problem starting from a time period at which there is a positive unit charge \(\tau_0 > 0\) and a positive stock of environmental advertisement \(A_0 > 0\). A reduced form social welfare function is defined as

\[
W(A, \tau, a) = V(A, \tau) - h(C(A, \tau)) - g(D(A, \tau)) - \phi(\tau) - w(a) - d(D(A, \tau))
\]

where \(h(C)\) and \(g(D)\) are the cost of production for goods \(C\) and \(D\) respectively and \(d(D)\) is the external social environmental cost from the consumption of the dirty good, which is assumed to be monotonically increasing and convex. In order to have a well defined optimization problem we assume that the social welfare function is concave in \((A, \tau, a)\).

We also assume that from the social point of view advertisement as a stock and environmental taxes are substitutes. The substitutability assumption imposes restrictions on the welfare function. In particular, substitutability implies that \(W(A, \tau, a)\) has decreasing differences in \((A, \tau)\), or that \(W(A, \tau') - W(A, \tau)\) is decreasing in \(A\) for all \(\tau' > \tau\). Decreasing differences means that increasing the tax rate reduces the marginal welfare gains from advertisement and requires that \(W_{A_\tau} \leq 0\).
In this framework the regulator uses as controls the advertisement rate and the tax rate to maximize discounted welfare. Thus the regulators’ optimal control problem is

$$\max_{(a(t),\tau(t))} \int_0^\infty e^{-\rho t} W(A,\tau,a)\,dt$$

$$\text{s.t. } \dot{A} = a - \delta A, \quad A(0) = A_0 > 0.$$ 

The current value Hamiltonian for the problem is:

$$H(A,\tau,a,q) = W(A,\tau,a) + q(a - \delta A).$$

By the maximum principle we have:

$$\frac{\partial H}{\partial \tau} \leq 0 \quad \text{or} \quad W_\tau \leq 0, \quad \tau \geq 0,$$  \hspace{1cm} (3)  

$$\frac{\partial H}{\partial a} \leq 0 \quad \text{or} \quad -w'(a) \leq q, \quad a \geq 0,$$  \hspace{1cm} (4)  

and

$$\dot{q} = pq - \frac{\partial H}{\partial A} = (\rho + \delta)q - W_A(A,\tau,a).$$  \hspace{1cm} (5)  

along with the transversality condition \(\lim_{t \to \infty} e^{-\rho t} q(t)A(t) = 0\).

For interior solutions, we can use condition (3) to derive the optimal tax rate at each point in time as a function of the advertisement stock, \(\dot{\tau} = \ddot{\tau}(A)\). Using the properties of the welfare function we have:

$$\frac{d\ddot{\tau}}{dA} = -\frac{W_{A\tau}}{W_\tau} < 0$$  \hspace{1cm} (6)  

From condition (4) we obtain the optimal short run advertisement flow \(\dot{a} = \ddot{a}(q)\), with \(\dot{a} > 0\). Substituting \(\ddot{a}, \ddot{\tau}\) into condition (5) and the differential equation for \(A\) we obtain the modified Hamiltonian dynamic system (MHDS)

$$\dot{A} = \ddot{a}(q) - \delta A,$$  \hspace{1cm} (7)  

$$\dot{q} = (\rho + \delta)q - W_A(A,\ddot{\tau}(A)).$$  \hspace{1cm} (8)  

The steady state for the MHDS is the point \((\ddot{A}, \ddot{q})\): \(\dot{A} = \dot{q} = 0\). The optimal steady state (OSS) for the control problem is defined as the solution of the implicit programming problem.
The Lagrangean for the OSS problem is

$$\max_{(A,a,\tau)\geq 0} W(A,\tau,a)$$

subject to

$$0=a-\delta A - \rho(A-\bar{A}).$$

(9)

By the implicit programming theorem $\lambda=\bar{q}$. Assuming

$$\bar{q} = \frac{dJ}{d\bar{A}} \geq 0, \quad J = \max \int_0^\infty e^{-\rho t} W(A,\tau,a) \, dt,$$

accumulated advertisement is not harmful at the steady state, the following proposition can be stated.

**Proposition 1:** If for any $(\tau,a)\in 1 \times \# \subseteq \mathbb{R}^2$, $W_A(0) > \bar{q}(\rho+\delta)$, $\lim_{A \to 0} W_A = 0$. Then there exist a unique positive advertisement stock at the OSS, or $\bar{A} > 0$.

**Proof.** The FOC for the implicit programming problem (9) imply

$$W_A \leq \lambda (\rho+\delta), \quad \bar{A} \geq 0, \quad \lambda = \bar{q},$$

$$W_\tau \leq 0, \quad \bar{\tau} \geq 0,$$

$$W_a - \lambda \leq 0, \quad \bar{a} \geq 0.$$ 

From the assumptions about $W_A$, we have that $W_A$ is monotonically decreasing and lies above the $\bar{q}(\rho+\delta)$ line in the neighborhood of zero, and below the line for sufficiently large $A$. Thus there is a unique positive OSS for $A$, as shown in figure 1 **Q.E.D.**
To examine the stability properties of the steady state, we assume that the stock of advertisement exhibits global diminishing returns (non-increasing returns) with respect to the social welfare or \( W_{AA} + W_{tA} \tau_A \leq 0 \). This implies that diminishing returns are associated both with the direct effect on welfare, \( W_{AA} \), and the indirect effect \( W_{tA} \tau_A \). Under this assumption the following can be stated.

**Proposition 2:** The steady state \((\bar{A}, \bar{q})\) has the local saddle point property.

**Proof.** Taking the Jacobian determinant of the MHDS in the neighbourhood of the steady state we have:

\[
\begin{vmatrix}
(\rho + \delta) & -(W_{AA} + W_{tA} \tau_A) \\
\hat{\alpha}' & -\delta
\end{vmatrix} < 0.
\]

Then the steady state has the local saddle point property **Q.E.D.**
**Proposition 3:** The steady state $(\bar{A}, \bar{q})$ is globally asymptotically stable (GAS), for all bounded solutions $A(t), q(t)$.

**Proof.** The curvature matrix in the neighbourhood of the steady state is defined as

\[
Q = \begin{bmatrix}
\dot{\bar{a}}' & \frac{\rho}{2} \\
\frac{\rho}{2} & -\left(V_{\dot{A}A} + V_{\tau A}A'\right)
\end{bmatrix}.
\]

The matrix is positive definite for sufficiently small $\rho$ and the steady state is GAS\textsuperscript{11}. Q.E.D.

This implies that there exist a one dimensional manifold such that for any initial stock of advertisement, $A(0)$, there is an initial shadow value $q(0)$ such that the solutions $A(t), q(t)$, converge to the steady state. Since $q(t)$ relates to the flow of advertisement per unit of time $a(t)$ through $\dot{a} = \hat{a}(q)$, the above proposition implies that for any initial stock of advertisement there is an initial advertisement expenditure, $a(0)$, such that the solutions $A(t), a(t)$, converge to the steady state $(\bar{A}, \bar{a})$.

**Concluding Remarks**

The above analysis suggests that if the stock of environmental advertisement is a substitute to environmental taxation, then an optimal environmental policy mix in the form of environmental taxes and environmental advertisement expenses can be derived. Under the scheme, and provided that the economy starts with a sufficiently low stock of environmental advertisement, the stock of advertisement increases monotonically along the optimal path towards the steady state and the optimal tax rate monotonically decreases along this optimal path.

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\textsuperscript{11} See Brock and Malliaris 1989.
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