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Working Paper
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ECB Working Paper, No. 1973

Provided in Cooperation with:
European Central Bank (ECB)


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Working Paper Series

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The optimal conduct of central bank asset purchases

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Abstract

We analyse the effects of central bank government bond purchases in an estimated DSGE model for the euro area. In the model, central bank asset purchases are relevant in so far as agency costs distort banks asset allocation between loans and bonds, and households face transaction costs when trading government bonds. Such frictions in the banking sector induce inefficient time-variation in the term premia and open up for a credit channel of central bank government bond purchases. Considering first ad hoc asset purchase programmes like the one implemented by the ECB, we show that their macroeconomic multipliers are stronger as the lower bound on the policy rate becomes binding and when the purchasing path is fully communicated and anticipated by economic agents. From a more normative standpoint, interest rate policy and asset purchases feature strong strategic complementarities during both normal and crisis times. In a lower bound environment, optimal policy conduct features long lower bound periods and activist asset purchase policy. Our results also point to a clear sequencing of the exit strategy, stopping first the asset purchases and later on, lifting off the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategies bring sizeable benefits and have the potential to largely offset the costs of the lower bound on the policy rate.

Keywords: Portfolio optimisation, Banking, Quantitative Easing, DSGE

JEL Classification: C61, E52, G11
Non-technical summary

Through the global financial crisis, central banks have embarked on various forms of unconventional monetary policies, one of which being asset purchase programmes. In most cases, central bank asset purchases were deployed once the room for more accommodative monetary policy stance through interest rate cuts was exhausted. Moreover, evidence has built up on the effectiveness of such unconventional policies in affecting financial prices, credit conditions and expenditure decisions through a variety of channels. In the euro area, which would constitute the empirical case for this paper, the pass-through of asset purchases on sovereign yields and on broader financing costs, notably bank lending rates, appeared significant and might put the emphasis on bank-based transmission channels. From a normative standpoint, whereas the early literature largely dismissed the usefulness of quantitative easing policies at the lower bound of interest rates, the potential benefits of targeted asset purchases have been revisited and more recently, some contributions would even explore the scope for active asset purchase strategies also in normal times.

Against this background, the aim of this paper is to discuss the optimal conduct of unconventional monetary policies within an estimated DSGE model for the euro area. In our model, central bank asset purchases are relevant in so far as agency costs distort banks asset allocation between loans and bonds, and households face transaction costs when trading government bonds. The banking frictions indeed limit arbitrage in the sovereign bond market and lead to endogenous time-variation in the term premium which might complicate macroeconomic stabilisation through conventional monetary policy. In this case, central bank asset purchases can be used as an instrument of monetary policy to affect long-term rates.

In the first part of the paper we evaluate the macroeconomic effects of ad hoc central bank asset purchase programmes when the policy rate reached its effective lower bound. Such an occasionally binding constraint brings some non-linearity into the model and makes the macroeconomic multipliers quite sensitive to the underlying crisis scenario. It turns out that central bank asset purchases are more powerful in a lower bound environment, and the longer the duration of the lower bound period. Besides, at the lower bound, the programme is more effective when fully communicated and anticipated and when complemented by forward guidance extending the lift-off date for the policy rate.
In the second part, we take a normative perspective and derive an optimal rule-based portfolio management strategy by the central bank which would be conditional on the state of the economy. The optimal policy conduct exploits the strategic complementarities between the two policy instruments. Within the confines of the model validity, the optimal allocation in the presence of the effective lower bound on interest rate displays long period of binding lower bound constraint, a strong use of forward guidance and activist asset purchase strategies. The model also points to a sequencing of the exit strategy, stopping first asset purchases and later on lifting off the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategy brings sizeable benefits and has the potential to largely offset the costs associated with the lower bound constraint on the policy rate.
1 Introduction

Through the global financial crisis, central banks have embarked on various forms of unconventional monetary policies, one of which being asset purchase programmes. In most cases, central bank asset purchases were deployed once the room for more accommodative monetary policy stance through interest rate cuts was exhausted. They were adopted in conjunction with some form of forward guidance on the future path of the key policy interest rates, much beyond the practice in normal times. Some monetary authorities also gave some clear indications on the sequencing of the exit strategy. Although the general effects of asset purchases have been quite intensively discussed, less is known about the consequences of asset purchasing programmes’ form.

The aim of this paper is to discuss the optimal conduct of unconventional monetary policy within an estimated DSGE model for the euro area. The objective of the paper is twofold: on the one hand we evaluate various quantity-based government bonds purchase programmes regarding their time profile and the information content. This is done for a realistic lower bound scenario for the policy rate, where realistic means that it bases upon observed (and expected) shocks. On the other hand we discuss the optimality of asset purchases following a welfare-based approach. In this regard, we show in a lower bound case for the policy rate how optimal asset purchases would look like and how they stabilise the economy. A focus is directed to the interplay between forward guidance on the policy rate and asset purchases.

Regarding the welfare evaluation we first ignore the lower bound constraint on the policy rate before we explicitly focus on it.

In the first part of the paper we consider ad hoc asset purchase programmes of the central bank, instead of a rule-based portfolio management policy which would be conditional on the state of the economy. Although programmes recently introduced by central banks like the asset purchasing programme of the ECB have been re-calibrated along with material changes in the inflation outlook, the first implementation can be regarded as a regime shift in the policy conduct due to their unprecedented nature. Therefore, our prime interest goes towards evaluating the unexpected announcement of a one-off purchase programme. We follow in this regard the literature on government output multipliers (Christiano et al., 2011a). Concretely, we analyse
the macroeconomic transmission of ad hoc programmes in an unconstrained environment as well as in the presence of the lower bound on the policy rate. Such an occasionally binding constraint brings some non-linearity into the model and makes the macroeconomic multipliers of central bank asset purchases quite sensitive to the underlying crisis scenario. Furthermore, we discuss the specific modalities of the ad hoc programme. Altogether, our results show that central bank asset purchases are more powerful i) in an environment in which the policy rate reached its effective lower bound, ii) the longer the duration of the lower bound period, iii) when, at the lower bound, the programme is fully communicated and anticipated, and iv) when it is complemented by forward guidance extending the lift-off date for the policy rate beyond agents expectations formulated on the basis of normal times policy conduct.

In the second part, we take a normative perspective and apply similar optimal policy concepts as proposed by Eggertsson and Woodford (2003) to derive a path for government bond purchases. Optimal paths for government bonds and the short rate are derived under commitment (similar to Adam and Billi (2006)). The optimal policy including asset purchases exploits the strategic complementarities between the two policy instruments. They feature distinctive propagation channels, different macroeconomic stabilisation properties and should not be considered perfect substitute. Within the confines of the model validity, the optimal policy conduct in the presence of the effective lower bound on interest rate displays: i) longer period of binding lower bound constraint and a strong use of forward guidance, ii) activist asset purchase policy and iii) a sequencing of the exit strategy, stopping first asset purchases and later on, lifting off the policy rate. In terms of macroeconomic stabilisation, optimal asset purchase strategy brings sizeable benefits and has the potential to largely offset the costs associated with the lower bound constraint on the policy rate.

Evidence has built up on the effectiveness of such unconventional policies in affecting financial prices, credit conditions and expenditure decisions through a variety of channels: direct effects on the price of assets in the targeted market segment (see for example Hancock and Passmore (2011) or Altavilla et al. (2014)), changes in expectations due to the signalling effect of the programmes (see inter alia Krishnamurthy and Vissing-Jorgensen (2011), Gilchrist and Zakrjasev (2013) or Joyce et al. (2011) and more indirect effects via the portfolio decisions of banks and other financial institutions. In the euro area, which would constitute the empirical
basis for this paper, the pass-through of asset purchases on sovereign yields and on broader financing costs, notably bank lending rates, appeared significant and might put the emphasis on bank-based transmission channels (see notably ECB (2015)). For this reason, we focus on government bond purchases and on their impact on the bank credit channel through portfolio rebalancing.

In our model banking frictions affect the pricing of long-term government bonds and create a term premium. Conventional monetary policy has an impact on the economy by affecting consumption and savings decisions as in traditional models without a banking sector. It also transmits to banks’ funding costs and bank asset valuation. Through these channels conventional monetary policy influences the provision of loans to non-financial agents. In equilibrium, banks’ capital structure and asset composition are jointly determined with the excess returns on loans and government bonds. The banking frictions indeed limit arbitrage in the sovereign bond market and lead to endogenous time-variation in the term premium which might complicate macroeconomic stabilisation through conventional monetary policy. This is particularly true if the policy rate reaches its lower bound. Monetary policy can nevertheless have an impact on long-term rates via the expectation hypothesis of the term structure by communicating the future path for the policy rate (forward guidance). But, term premia is not directly affected by forward guidance and central bank asset purchases can be used as an instrument of monetary policy to affect long-term rates if the lower bound hold for short-term rates.

More precisely, our modelling strategy consists in introducing the minimal set of frictions into established DSGE models with satisfactory empirical properties in order to account for bank portfolio decisions between sovereign holdings and loan contracts. The specification of the DSGE model is first inherited from Smets and Wouters (2007) for the non-financial blocks and the estimation strategy. We introduce a segmented banking sector à la Gerali et al. (2010) and Barraqu París et al. (2011) and allow for risky corporate debt contract à la Bernanke et al. (1999) with pre-determined lending rates. Finally, for the bank portfolio allocation frictions, we follow the approach of Gertler and Karadi (2013). In our model, central bank asset purchases are relevant in so far as agency costs constrain the asset allocation of banks between loans and bonds, and households face transaction costs when trading government
The estimation of the DSGE model enriches the analysis of this paper along two dimensions. First, it enables to design crisis scenarios which are more realistic than the ones contemplated in the closely related literature. We would argue that the macroeconomic multipliers evaluated for the *ad hoc* central bank asset purchase programme have in this respect satisfactory empirical plausibility. Second, the estimation provides a realistic set of structural business cycle shocks for the euro area. Such shock distributions are instrumental for quantifying the stabilisation gains from the optimal policy conduct.

Our paper is linked to the normative debate on monetary policy frameworks which has been intensified through the crisis. At the beginning of this discussion, [Eggertsson and Woodford (2003)](#) dismissed the usefulness of "pure" quantitative easing policies (i.e. policies aiming at replacing short-term assets with excess reserves) at the lower bound of interest rates provided that some appropriate form of forward guidance was implemented. Later on, [Cúrdia and Woodford (2011)](#) revisited the potential benefits of targeted asset purchases, to the extent that the financial system was significantly disrupted and the unconventional policies could deliver adequate credit easing. Our model also consists of various frictions which create wedges between risk-free interest rates and ultimate borrowing rates. These wedges are determined endogenously in the general equilibrium in our model. Our paper is also close to [Ellison and Tischbirek (2014)](#) or [Jones and Kulish (2013)](#) who provide arguments for using an active asset purchase strategy as an additional instrument in normal times and when the policy rate hits the effective lower bound. We contribute to this discussion by explicitly deriving the optimal path of asset purchases jointly with the optimal path for the policy rate. To the best of our knowledge, we are the first who discuss the optimal interplay between the policy measures based upon an estimated model with an elaborated banking sector which resembles some real world features.

The rest of the paper is organised as follows. Section 2 describes the main features of the DSGE model, highlighting the key frictions which are essential to the transmission of central bank asset purchases as well as to the empirical performance of the model. Section 3 presents the estimation of the DSGE model and discusses the relative propagation mechanism of standard and non-standard monetary policy shocks. Section 4 then explores the macroeconomic
multipliers of central bank asset purchases when the monetary policy rate is constrained at its effective lower bound and the central bank implements an ad hoc asset purchase programme. Some sensitivity analysis regarding the implementation design of the programme is also performed. Section 4 derives some optimal policy concepts and elaborates on the desirability of combining both instruments through the cycle and in crisis time.

2 The model economy

The model consists of households, goods producers, capital producers, non-financial firms (called entrepreneurs) investing into capital projects, and banks who funds the projects of non-financial firms. Since households cannot provide their savings directly to the real sector, banks need to intermediate these funds. Both entrepreneurs and banks are exposed to endogenous borrowing constraints. Additionally, the loan market operates under imperfect competition. Hence, financial frictions and market power in the loan market create inefficiencies in borrowing conditions. The real sector is rather standard and features staggered prices and wages.

The decision problems illustrating the transmission of central bank asset purchases are reported below while details on the rest of the economic environment is presented in the appendix. The model bases upon Smets and Wouters (2007) regarding the real sector and combines elements in the banking sector from Gertler and Karadi (2011, 2013), Kerali et al. (2010), and Darracq Paries et al. (2011) with elements from Christiano et al. (2014) as similarly done by Rannenberg (2010 and 2014). The model economy evolves along a balanced-growth path driven by a positive trend, $\gamma$, in the technological progress of the intermediate goods production and a positive steady state inflation rate, $\pi^*$. In the description of the model, stock and flow variables are expressed in real and effective terms (except if mentioned otherwise): they are deflated by the price level and the technology-related balanced growth path trend.

2.1 Households

The economy is populated by a continuum of heterogenous infinitely-lived households. Each household is characterized by the quality of its labour services, $h \in [0,1]$. At time $t$, the
intertemporal utility function of a generic household $h$ is

$$W_t(h) = E_t \sum_{j=0}^{\infty} (\beta \gamma^{1-\sigma_c})^j c^h_{t+j} H \left( C_{t+j}(h) - \eta C_{t+j-1}(h) / \gamma, N^S_t(h) \right)$$

with $\beta$ as the time preference rate. Household $h$ obtains utility from consumption of an aggregate index $C_t(h)$, relative to an internal habit depending on its past consumption $\eta$, while receiving disutility from the supply of their homogenous labour $N^S_t(h)$. Utility also incorporates a consumption preference shock $\epsilon b_t$. $\tilde{L}$ is a positive scale parameter. $\sigma_c$ is the intertemporal elastasticity of substitution.

Each household $h$ maximizes its intertemporal utility under the following budget constraint:

$$D_t(h) + Q_{B,t} \left[ B_{H,t}(h) + \frac{1}{2} \chi_H (B_{H,t}(h) - \bar{B}_H)^2 \right] + C_t(h) = R_{D,t} - \frac{1}{\pi_i} (1 - \tau_{w,t}) W_t^h N^S_t(h) + A_t(h) + T_t(h) P_t + \Pi_t(h)$$

where $P_t$ is an aggregate price index, $R_{D,t}$ is the one period ahead nominal gross deposit rate, $D_t(h)$ is a deposit, $Q_{B,t}$ is the price of the government bond and $B_{H,t}(h)$ is the quantity of government bonds with $\bar{B}_H$ as the corresponding steady state value. $W_t^h$ is the nominal wage, $T_t(h)$ are government transfers (both expressed in effective terms) and $\tau_{w,t}$ is a time-varying labor tax. $\Pi_t(h)$ corresponds to the profits net of transfers from the various productive and financial segments owned by the households. $\chi_H$ is the households’ portfolio adjustment cost. A positive value of $\chi_H$ prevents full (frictionless) arbitrage of the returns on securities by the household sector. Finally $A_t(h)$ is a nominal stream of income (both in effective terms) coming from state contingent securities and equating marginal utility of consumption across households $h \in [0, 1]$.

In equilibrium, households’ choices in terms of consumption, hours and deposit holdings are identical. The first order condition of the household problem with respect to government bond holdings is

$$E_t \left[ \Xi_{t+1} \left( \frac{R_{G,t+1} - R_{D,t}}{\pi_{t+1}} \right) \right] = \chi_H (B_{H,t} - \bar{B}_H)$$

$$(1)$$
where $\Xi_{t,t+1}$ is the period $t$ stochastic discount factor of the households for nominal income streams at period $t + 1$.

2.2 Banks

The banking sector is owned by the households and is segmented in various parts: Bankers, retail branches and loan officers. First, bankers collect household deposits and provide funds to the retail lending branches. As in Gertler and Karadi (2011, 2013), bankers can divert funds and depositors enforce on them an incentive constraint which forces the bankers to hoard a sufficient level of net worth. This creates a financing cost wedge related to bank capital frictions. Second, retail lending branches receive funding from the bankers and allocate it to the loan officers. In the retail segment, a second wedge results from banks operating under monopolistic competition and facing nominal rigidity in their interest rate setting. In the third segment of the banking sector, loan officers extend loan contracts to entrepreneurs as explained previously which implies a third financing cost wedge related to credit risk compensation (see in the Appendix for details on the entrepreneurs decision problem).

2.2.1 Bankers

Every period, a fraction $(1 - f)$ of household’s members are workers while a fraction $fe$ are entrepreneurs and the remaining mass $(1 - e)$ are bankers. Bankers face a probability $\zeta_b$ of staying banker over next period and a probability $(1 - \zeta_b)$ of becoming a worker again. When a banker exits, accumulated earnings are transferred to the respective household while newly entering bankers receive initial funds from their household. Overall, households transfer a real amount $\Psi_{B,t}$ to new bankers for each period $t$. As shown later in this section, bankers’ decisions are identical so we will expose the decision problem for a representative banker.

Bankers operate in competitive markets providing loans to retail lending branches, $L_{BE,t}$, and purchasing government securities, $B_{B,t}$, at price $Q_{B,t}$. To finance their lending activity, Bankers receive deposits, $D_t$, from households, with a gross interest rate, $R_{D,t}$, and accumulate net worth, $NW_{B,t}$. Their balance identity, in real terms, reads

$$L_{BE,t} + Q_{B,t}B_{B,t} = D_t + NW_{B,t}. \tag{2}$$
The accumulation of the bankers' net worth from period \( t \) to period \( t + 1 \) results from the gross interest received from the loans to the retail lending bank, the gross return on government bond holdings, \( R_{G,t+1} \), the lump-sum share of profits (and losses) coming from retail lending and loan officers activity, \( W_{BL,t+1}^B \), per unit of each banker's net worth, minus the gross interest paid on deposits:

\[
NW_{B,t+1} = \frac{RB_{N,t+1}^B}{\pi_{t+1}} - \gamma.
\]

with

\[
\begin{align*}
R_{BL,t+1}^B &\equiv (R_{BLE,t}^B - R_{DL,t}) \kappa_{BL,t}^B + (R_{G,t+1}^B - R_{DL,t}) \kappa_{BL,t}^B + R_{DL,t}^B + W_{BL,t+1}^B, \\
\kappa_{BL,t}^B &\equiv \frac{LB_{E,t}^B}{NW_{B,t}} \quad \text{and} \quad \kappa_{BL,t}^B \equiv \frac{QB_{B,t}^B}{NW_{B,t}},
\end{align*}
\]

Iterating this equation backward implies

\[
NW_{B,t+1} = \tilde{R}_{N,t+1-x,t+1}^B NW_{B,t+1-x}/\gamma^x
\]

where \( \tilde{R}_{N,t+1-x,t+1}^B = \prod_{i=0}^{x-1} \left[ \frac{RB_{N,t+1}^B}{\pi_{t+1-i}} \right] \) and \( \tilde{R}_{N,t+1-x,t+1-x}^B = 1 \). The bankers' objective is to maximise their terminal net worth when exiting the industry, which occurs with probability \( (1 - \zeta_b) \) each period. The value function for each banker is therefore given by

\[
V_{B,t} = (1 - \zeta_b) \sum_{k=0}^{\infty} (\zeta_b)^k \Xi_{t+k,t+1} NW_{B,t+k+1}
\]

Using (5), the value function can be written recursively as follows

\[
V_{B,t} = (1 - \zeta_b) NW_{B,t} (X_{B,t} - 1)
\]

with

\[
X_{B,t} = 1 + \zeta_b E_t \left[ \frac{\Xi_{t,t+1} R_{N,t+1}^B}{\pi_{t+1}} X_{B,t+1} \right].
\]

As in Gertler and Karadi (2013), bankers can divert a fraction of their assets and transfer them without costs to the households. In this case, the depositors force the default on the intermediary and will only recover the remaining fraction of the asset. The corresponding
The incentive compatibility constraint is

\[ V_{B,t} \geq \lambda_b (L_{BE,t} + \delta_b Q_{B,t} B_{R,t}) \geq \lambda_b \left( \kappa_l B_{t} + \delta_b \kappa_g B_{t} \right) NW_{B,t}. \] (6)

The diversion rate for private loans is \( \lambda_b \) and \( \lambda_b \delta_b \) for government securities. We allow \( \delta_b \) to be time-varying, driven by an exogenous AR1 process. Under the parameter values considered thereafter, the constraints are assumed to always bind in the vicinity of the steady state.

Given their initial net worth, the end-of-period contracting problem for bankers consists in maximising \( V_{B,t} \) for the exposures to private sector loans \( \kappa_l B_{t} \) and government securities \( \kappa_g B_{t} \) subject to the incentive constraint (6):

\[ V_{B,t} = \max \{ \kappa_l B_{t}, \kappa_g B_{t} \} \] (7)

where we denoted \( \tilde{X}_{B,t} \equiv (X_{B,t} - 1) \left( \frac{1 - \zeta_b}{\zeta_b} \right) \) and \( \tilde{X}_{B,t} \) follows

\[ \tilde{X}_{B,t} = \mathbb{E}_t \left[ \sum_{j=1}^{\infty} \frac{R_{N,t+1}^B}{\tau_{t+1}} \left( \zeta_b \tilde{X}_{B,t+1} + (1 - \zeta_b) \right) \right]. \] (8)

Note that the stream of transfers \( \Pi_{B,t+1}^R \) is considered exogenous by bankers in their decision problem which implies that \( \frac{\partial \Pi_{B,t+1}^R}{\partial \kappa_l B_{t}} = 0 \).

The first order conditions for this problem can then be formulated as

\[ \mathbb{E}_t \left[ \sum_{j=1}^{\infty} \frac{\partial R_{N,t+1}^B}{\partial \kappa_l B_{t}} \left( \zeta_b \tilde{X}_{B,t+1} + (1 - \zeta_b) \right) / \tau_{t+1} \right] = \mu_t \lambda_b \] (9)

\[ \mathbb{E}_t \left[ \sum_{j=1}^{\infty} \frac{\partial R_{N,t+1}^B}{\partial \kappa_g B_{t}} \left( \zeta_b \tilde{X}_{B,t+1} + (1 - \zeta_b) \right) / \tau_{t+1} \right] = \mu_t \lambda_b \delta_b \] (10)

where \( \mu_t \) is the lagrange multiplier related to the incentive constraint.

Aggregating across bankers, a fraction \( \zeta_b \) continues operating into the next period while the rest exits from the industry. The new bankers are endowed with starting net worth, proportional to the assets of the old bankers. Accordingly, the aggregate dynamics of bankers’
net worth is given by

\[ NW_{B,t} = \zeta B,NW_{B,t-1}^{1/\gamma} + \Psi_{B,t}. \]  

(11)

2.2.2 Retail lending branches and loan officers

A continuum of retail lending branches indexed by \( j \), provide differentiated loans to loan officers. The total financing needs of loan officers follow a CES aggregation of differentiated loans

\[ L_{E,t} = \left[ \int_0^1 L_{E,t}(j) \frac{\mu_{RE}}{\mu_{RE} - 1} \, dj \right] ^{1-\mu_{RE}}. \]

Differentiated loans are imperfect substitutes with elasticity of substitution \( \mu_{RE} > 1 \). The corresponding average return on loans is

\[ R_{LE} = \left[ \int_0^1 R_{LE}(j) \frac{1}{\mu_{RE}} \, dj \right] ^{1-\mu_{RE}}. \]

Retail lending branches are monopolistic competitors which levy funds from the bankers and set gross nominal interest rates on a staggered basis a la Calvo (1983), facing each period a constant probability \( 1 - \xi_{RE} \) of being able to re-optimize. If a retail lending branch cannot re-optimize its interest rate, the interest rate is left at its previous period level:

\[ R_{LE,t}(j) = R_{LE,t-1}(j) \]

The retail lending branch \( j \) chooses \( \hat{R}_{LE,t}(j) \) to maximize its intertemporal profit

\[ E_t \left[ \sum_{k=0}^{\infty} (\beta^\gamma - \sigma_{RE})^k \frac{\Lambda_t + k}{\mu_{RE} - 1} \right] \left( R_{LE,t}(j)L_{E,t+k}(j) - R_{BLE,t+k}(j)L_{E,t+k}(j) \right) \]

where the demand from the loan officers is by

\[ L_{E,t+k}(j) = \left( \frac{R_{LE,t}(j)}{R_{LE,t}} \right) ^{-\sigma_{RE}} \left( \frac{R_{LE,t}}{R_{BLE,t+k}} \right) ^{-\frac{\mu_{RE}}{\mu_{RE} - 1}} L_{E,t+k} \]

and \( R_{BLE,t} \) is the gross funding rate on the loans from the bankers.

The staggered lending rate setting acts in the model as maturity transformation in banking activity and leads to imperfect pass-through of market interest rates on bank lending rates.

Finally, loan officers operate in perfect competition. They receive one-period loans from the retail lending branches, which cost an aggregate gross nominal interest rate \( R_{LE,t} \), set at the beginning of period \( t \). They extend loan contracts to entrepreneurs which pay a state-contingent
return \( \tilde{R}_{LE,t+1} \) (see the Appendix for details on the decision problem of entrepreneurs). Loan officers have no other source of funds so that the volume of the loans they provide to the entrepreneurs equals the volume of funding they receive. Loan officers seek to maximise its discounted intertemporal flow of income so that the first order condition of its decision problem gives

\[
E_t \left[ \frac{\tilde{R}_{LE,t+1} - R_{LE,t}}{u_{t+1}} \right] = 0 \quad (12)
\]

Profits and losses made by retail branches and loan officers are transferred back to the bankers.

2.3 Entrepreneurs

As explained before, every period, a fraction \( fe \) of the representative household’s members are entrepreneurs. Like bankers, each entrepreneur faces a probability \( \zeta_e \) of staying entrepreneurs over next period and a probability \( (1 - \zeta_e) \) of becoming a worker again. To keep of share of entrepreneurs constant, we assume that similar number of workers randomly becomes entrepreneur. When an entrepreneur exits, their accumulated earnings are transferred to the respective household. At the same time, newly entering entrepreneurs receive initial funds from their household. Overall, households transfer a real amount \( \Psi_{E,t} \) to the entrepreneurs for each period \( t \). Finally, as it will become clear later, entrepreneurs decisions for leverage and lending rate are independent from their net worth and therefore identical. Accordingly, we will expose the decision problem for a representative entrepreneur.

At the end of the period \( t \) entrepreneurs buy the capital stock \( K_t \) from the capital producers at real price \( Q_t \) (expressed in terms of consumption goods). They transform the capital stock into an effective capital stock \( u_{t+1}K_t \) by choosing the utilisation rate \( u_{t+1} \).

The adjustment of the capacity utilization rate entails some adjustment costs per unit of capital stock \( \Gamma_u(u_{t+1}) \). The cost (or benefit) \( \Gamma_u \) is an increasing function of capacity utilization and is zero at steady state, \( \Gamma_u(u^*) = 0 \). The functional forms used for the adjustment costs on capacity utilization is given by \( \Gamma_u(X) = \frac{\phi}{\phi'}(\exp[\phi'(X - 1)] - 1) \).

The effective capital stock can then be rented out to intermediate goods producers at a
nominal rental rate of $r_{K,t+1}$.

Finally, by the end of period $t + 1$, entrepreneurs sell back the depreciated capital stock $(1 - \delta)K_t$ to capital producer at price $Q_{t+1}$.

The gross nominal rate of return on capital across from period $t$ to $t + 1$ is therefore given by

$$R_{KK,t+1} \equiv \pi_{t+1} \frac{r_{K,t+1}u_{t+1} - \Gamma_u (u_{t+1}) + (1 - \delta)Q_{t+1}}{Q_t},$$

(13)

where $\pi_{t+1}$ is the inflation rate.

Each entrepreneur’s return on capital is subject to a multiplicative idiosyncratic shock $\omega_{e,t}$. These shocks are independent and identically distributed across time and across entrepreneurs. $\omega_{e,t}$ follows a lognormal CDF $F_e(\omega_{e,t})$, with mean 1 and variance $\sigma_{e,t}$ which is assumed to be time-varying. By the law of large number, the average across entrepreneurs (denoted with the operator $\mathbb{E}$) for expected return on capital is given by

$$\mathbb{E}[\mathbb{E}_{t}(\omega_{e,t+1}R_{KK,t+1})] = \mathbb{E}_{t}(\int_{0}^{\infty} \omega_{e,t+1}dF_e(\omega) R_{KK,t+1}) = \mathbb{E}(R_{KK,t+1}).$$

Entrepreneur’s choice over capacity utilization is independent from the idiosyncratic shock and implies that

$$r_{K,t} = \Gamma_u^T(u_t).$$

(14)

Entrepreneurs finance their purchase of capital stock with his net worth $NW_{E,t}$ and a one-period loan $L_{E,t}$ (expressed in real terms, deflated by the consumer price index) from the commercial lending branches:

$$Q_tK_t = NW_{E,t} + L_{E,t}.$$

(15)

In the tradition of costly-state-verification frameworks, lenders cannot observe the realisation of the idiosyncratic shock unless they pay a monitoring cost $\mu_e$ per unit of assets that can be transferred to the bank in case of default. We constrain the set of lending contracts available to entrepreneurs. They can only use debt contracts in which the lending rate $R_{LLE,t}$ is pre-determined at the previous time period.

Default will occur when the entrepreneurial income that can be seized by the lender falls short of the agreed repayment of the loan. At period $t + 1$, once aggregate shocks are realised,
this will happen for draws of the idiosyncratic shock below a certain threshold $\Xi_{e,t}$, given by

$$\Xi_{e,t+1} = (R_{LLE,t} + 1) (\kappa_{e,t} - 1)$$  \hspace{1cm} (16)

where $R_{LLE,t}$ is the nominal lending rate determined at period $t$ and $\kappa_{e,t}$ is the corporate leverage defined as

$$\kappa_{e,t} = \frac{Q_{t}T_{t}}{N_{E,E,t}}$$  \hspace{1cm} (17)

$\chi_{e}$ represents the share of the entrepreneur’s assets (gross of capital return) that banks can recover in case of default. When banks take over the entrepreneur’s assets, they have to pay the monitoring costs.

The ex post return to the lender on the loan contract, denoted $\tilde{R}_{LLE,t}$, can then be expressed as

$$\tilde{R}_{LLE,t} = G(\omega_{e,t}) \chi_{e} R_{KK,t+1} \kappa_{e,t}^{-1}$$  \hspace{1cm} (18)

where

$$G_e(\omega) = (1 - F_{e} (\omega) ) \omega + (1 - \mu_e) \int_0^{\omega} \omega dF_{e} (\omega) .$$

We assume that entrepreneurs are myopic and the end-of-period $t$ contracting problem for entrepreneurs consists in maximising the next period return on net worth for the lending rate and leverage:

$$\max_{\{R_{LLE,t}, \kappa_{e,t}\}} \mathbb{E}_t \left[ (1 - \chi_{e} \Gamma_{e}(\Xi_{e,t+1})) R_{KK,t+1} \kappa_{e,t} \right]$$

subject to the participation constraint of the lender (12), the equation (16) for the default threshold $\Xi_{e,t+1}$, and where

$$\Gamma_{e}(\Xi) = (1 - F_{e} (\Xi) ) \Xi + \int_0^{\Xi} \omega dF_{e} (\omega) .$$

After some manipulations, the first order conditions for the lending rate and the leverage lead to

$$\mathbb{E}_t \left[ (1 - \chi_{e} \Gamma_{e}(\Xi_{e,t+1})) R_{KK,t+1} \kappa_{e,t} \right] = \mathbb{E}_t \left[ \chi_{e} \Gamma_{e}(\Xi_{e,t+1}) R_{KK,t+1} \kappa_{e,t} \right]$$

$$\mathbb{E}_t \left[ (1 - \chi_{e} \Gamma_{e}(\Xi_{e,t+1})) R_{KK,t+1} \kappa_{e,t} \right] = \frac{\mathbb{E}_t \left[ \chi_{e} \Gamma_{e}(\Xi_{e,t+1}) \right]}{\mathbb{E}_t \left[ \chi_{e} \Gamma_{e}(\Xi_{e,t+1}) \right]} \mathbb{E}_t \left[ \Xi_{e,t+1} \right] R_{LLE,t}$$  \hspace{1cm} (19)
where
\[ \Gamma(e) = (1 - F(e)) \] and \[ G(e) = (1 - F(e)) - \mu F(e) \].

As anticipated at the beginning of the section, the solution to the problem shows that all entrepreneurs choose the same leverage and lending rate. Moreover, the features of the contracting problem imply that the ex post return to the lender \( R_{LE,t} \) will differ from the ex ante return \( R_{KE,t-1} \). Log-linearising equation (19) and the participation constraint (12), one can show that innovations in the ex post return are notably driven by innovations in \( R_{KE,t} \).

Finally, aggregating across entrepreneurs, a fraction \( \zeta_e \) continues operating into the next period while the rest exits from the industry. The new entrepreneurs are endowed with starting net worth, proportional to the assets of the old entrepreneurs. Accordingly, the aggregate dynamics of entrepreneurs’ net worth is given by

\[ \text{NW}_{E,t} = \zeta_e (1 - \chi_e \Gamma(e)) \frac{R_{KE,t}}{\gamma} - \Gamma_{e,t-1} \text{NW}_{E,t-1} \gamma + \Psi_{E,t}. \tag{20} \]

In the estimation, we also introduce a shock on the net worth of entrepreneurs which can be rationalised either as time-varying transfers to new entrepreneurs \( \Psi_{E,t} \), or as a multiplicative shock on the survival probability of entrepreneurs, \( \zeta_{E,t}^2 \).

### 2.4 Government sector and monetary policy instruments

Public expenditures \( G^* \), expressed in effective terms, are subject to random shocks \( e_t^g \). The government finances public spending with labour tax, product tax and lump-sum transfers so that the government debt \( Q_{BG,t} \), expressed in real effective terms, accumulates according to

\[ Q_{BG,t} = \frac{R_{G,t}}{\tau_L} Q_{BG,t-1} \frac{1}{\gamma} + G^* e_t^g - \tau_L w_t L_t - \tau_p Y_t - T_t. \tag{21} \]

In the empirical analysis, we neglect the dynamics of public debt and assume that lump-sum taxes \( T_t \) are adjusted to ensure that

\[ \forall t > 0, \quad B_{G,t} = \frac{R_{G,t}}{\gamma}. \]
In order to introduce long-term sovereign debt, we assume that government securities are perpetuities which pay geometrically-decaying coupons \((c_g, \text{ the first period, } (1 - \tau_g)c_g, \text{ the second one, } (1 - \tau_g)^2c_g, \text{ the third one, etc...})\). The nominal return on sovereign bond holding from period \(t\) to period \(t + 1\) is therefore

\[
R_{G,t+1} = \epsilon_{R_G} c_g + (1 - \tau_g)Q_{B,t+1} Q_{B,t}. \tag{22}
\]

For the purpose of the empirical analysis, we introduced an ad hoc government bond valuation shock, \(\epsilon_{R_G}\). This "reduced-form" shock is meant to capture time-variation in the excess bond return not captured by our bank-centric formulation of the term premium. In particular, the rise in sovereign risk pricing during the euro area financial crisis is not be accounted for within the micro-foundation of the model. Note that the estimation period stops before the start of the ECB’s asset purchase programme so that the introduction of the government bond valuation shock does not partially substitute for an unconventional monetary policy shock in the estimation.

Within the government sector, the monetary authority controls the deposit interest rate \(R_{D,t}\). Similar to Smets and Wouters (2007), the monetary authority follows an interest rate feedback rule which incorporates terms on lagged inflation, lagged output gap and its first difference. The output gap is defined as the log-difference between actual and flexible-price output. The reaction function also incorporates a non-systematic component \(\epsilon_{r,t}\).

Written in deviation from the steady state, the interest rule used in the estimation has the form:

\[
\hat{R}_{D,t} = \rho\hat{R}_{D,t-1} + (1 - \rho) \left[ r_s \hat{y}_{t-1} + r_y \hat{y}_{t-1} + r_{\Delta y} \Delta \hat{y}_{t} + \log(\epsilon_{r,t}) \right] \tag{23}
\]

where a hat over a variable denotes log-deviation of that variable from its deterministic steady-state level.

Finally, we assume as in Gertler and Karadi (2013) that the monetary authority can manage a bond portfolio \(B_{CB,t}\).
2.5 Clearing conditions on debt markets

On the private credit market, due to nominal rigidity in the setting of interest rate by retail banking branches, the following conditions holds

\[ L_{BE,t} = \Delta_{EL}^{B} L_{E,t} \]  

(24)

where \( \Delta_{R}^{E,t} = \int_{0}^{1} \left( \frac{R_{E,t}(j)}{\Delta_{E,t}} \right) \frac{R_{E,t}}{\Delta_{E,t}} dj \) is the dispersion index among retail bank interest rates. Moreover, in equilibrium the lump-sum transfer to bankers per unit of net worth from retail lending and loan officer profits and losses is given by

\[ R_{B,t+1}^{R} = \left( \bar{R}_{LE,t+1} - R_{BLE,t} \right) \kappa_{B,t}. \]  

(25)

We can now rewrite the recursive formulation of the bankers value function \( V_{B,t} \) from equation (8) using bankers incentive constraint (6) and first order conditions (9)-(10). This gives a relationship between bank leverage and intermediation spreads:

\[ \lambda_{B} \frac{\kappa_{B,t}}{\zeta_{B}} = E \left[ \frac{R_{BLE,t+1} - R_{D,t}}{\pi_{t+1}} \bar{R}_{LE,t+1} + \frac{R_{BLE,t+1} - R_{BLE,t} \kappa_{B,t} + R_{L,t}}{\pi_{t+1}} (\lambda_{B} \kappa_{B,t+1} + (1 - \zeta_{B})) \right] \]  

(26)

where we denoted \( \kappa_{B,t} = \kappa_{B,t}^{L} + \delta_{B,t} \kappa_{B,t}^{g} \).

Finally, on the government bond market, the fixed supply is distributed across holdings by households, bankers and the central bank:

\[ B_{H,t} + B_{B,t} + B_{CB,t} = B_{G}. \]

3 Transmission of standard and non-standard monetary policy shocks in the estimated DSGE model

In this section, we present the estimation of the DSGE model as in Smets and Wouters (2007). The model is estimated on euro area data using Bayesian likelihood methods. We consider 10 key macroeconomic quarterly time series from 1995q1 to 2014q2: output, consumption,
fixed investment, hours worked, real wages, the GDP deflator inflation rate, the three-month short-term interest rate, bank loans, bank lending spreads and the (GDP-weighted) 10-year euro area sovereign spread. The data are not filtered before estimation with the exception of loans which are linearly detrended. We limit the number of shocks to be equal to the number of observed variables. As in Smets and Wouters (2007), we introduce a correlation between the government spending shock and the productivity shock, $\rho_{a,g}$.

The exogenous shocks can be divided in three categories:

1. Efficient shocks: AR(1) shocks on technology $\epsilon^{r}_t$, investment $\epsilon^{I}_t$, public expenditures $\epsilon^{g}_t$ and consumption preferences $\epsilon^{b}_t$.

2. Inefficient shocks: ARMA(1,1) shocks on price markups $\epsilon^{p}_t$, and AR(1) on wage markups $\epsilon^{w}_t$.

3. Financial shocks: AR(1) shock on entrepreneurs idiosyncratic risk $\epsilon^{\sigma}_t$, on entrepreneurs net worth accumulation $\epsilon^{\zeta}_t$, as well as on government bond valuation $\epsilon^{R}_t$ in equation 22.

4. Policy shocks: AR(1) shock on the Taylor-rule residual $\epsilon^{r}_t$.

### 3.1 Posterior distributions for the key portfolio rebalancing parameters

Most parameters of the model are left free in the estimation. As most of the data used in the estimation are not filtered, some of the deep parameters, notably on the financial side, capture both steady state and cyclical properties of the model. In the Appendix, we document at length the calibration strategy and the choice of prior distributions for the financial block. Regarding the other structural parameters, the prior distributions are similar to Smets and Wouters (2007).

The posterior distribution of estimated parameters, characterised by the mean and the 80% density intervals, are reported in Tables 3 and 4. We focus here on the key parameters which drive the transmission of central bank asset purchases: the bankers relative diversion rate on government bonds, $\delta_b$, households portfolio frictions, $\chi_H$, and the rigidity parameter $\chi_{R}$. All the AR(1) processes are written as $\log(\epsilon^r_t) = \rho_{r} \log(\epsilon^r_{t-1}) + \epsilon^r_t$ where $\epsilon^r_t \sim N(0, \sigma_{\epsilon^r}).$ ARMA(1,1) are of the form $\log(\epsilon^r_t) = \rho_{r} \log(\epsilon^r_{t-1}) - \eta_{r} \epsilon^r_{t-1} + \epsilon^r_t$. All shock processes $\epsilon^r_t$ are equal to one in the steady state.
on retail lending rates, $\xi_E^R$. All these parameters are relatively well-identified in the estimations posterior distributions are sizeably narrower and shifted compared with the prior distributions.

As explained in Gertler and Karadi (2013), $\delta_b$ and $\chi_H$ are crucial parameters for the transmission of central bank asset purchases. When $\delta_b$ goes to 0, bank portfolio constraints on holdings of sovereign bonds weakens and the macroeconomic impact of asset purchases vanishes. The authors considered the value of $\delta_b = 0.5$ to match the fact that the level of sovereign spreads in the data is half the intermediation spreads measured with mortgage and corporate bonds. In our model, $\delta_b$ ties a link between sovereign spreads and bankers loan rate which corresponds to the quarterly return on the bank loan book, net of expected losses and net of the monopolistic margin. Therefore, higher values of $\delta_b$ than in Gertler and Karadi (2013) can still be consistent with the sample mean of the sovereign spread and lending rate spread introduced in the estimation, as the latter includes credit risk compensation and a retail margin. Moreover the diversion rate of sovereign holdings introduced in the bankers incentive compatibility constraint is $\lambda_b \delta_b$ and with calibrated $\lambda_b$ at around 0.3, $\delta_b$ could in principle take values significantly higher than 1. Those considerations explain the choice of a relative loose prior distribution for $\delta_b$ which does not constrain strongly the inference towards low levels.

Turning to $\chi_H$, Gertler and Karadi (2013) set it to 1 in order to broadly match empirical evidence on the impact of QE2 on output and sovereign spreads. But $\chi_H$ also affects the distribution of sales of sovereign securities between households and banks in the context of central bank asset purchases. And for values higher than 0.1, the macroeconomic propagation of central bank asset purchases becomes relatively insensitive to $\chi_H$: in particular, households would almost not sell any bonds to the central bank which is at odds with empirical evidence from the US or the UK. Consequently, the support of the prior distribution covers low values for this parameter.

The posterior distribution of the adjustment cost on household portfolio decisions $\chi_H$ is low (with mean values below the one of the prior distribution, at less than 0.01). Finally, the bankers diversion rate for sovereign bond holdings $\delta_b$ features a mean posterior distribution

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see for example Carpenter et al. (2013) in the case of the Federal Reserve and Joyce et al. (2013) regarding the Bank of England.
around 2. The calibration values of Gertler and Karadi (2013) for $\chi_H$ and $\delta_b$ are far from our posterior mean estimates, and are not even covered by the 80% shortest density interval.

Finally, we introduced in the DSGE staggered lending rate setting in the retail banking segment, not present in Gertler and Karadi (2013), which significantly affects the pass-through of bankers’ required return on loans to the marginal lending rate for entrepreneurs, and therefore the effectiveness of the portfolio rebalancing channel. To control the size of asset purchase output multipliers, lower values of $\delta_b$ which would increase the multiplier, everything else being equal, can be compensated by a higher level of $\xi^{RE}$. The posterior mode for $\xi^{RE}$ is around 0.3 which is consistent a relatively fast lending rate past-through of corporate loans. Obviously there are differences across retail bank products in terms of the speed and degree with which banks pass-through changes in policy rates due to the maturity of the interest rate fixation in the loan contract, the degree of market power of the bank or other indexation scheme on the interest paid through the course of the loan. Darracq-Pariès et al. (2014) for example summarise existing time-series evidence showing a more sluggish pass-though of monetary policy rate to mortgages than to corporate lending rates.

3.2 Interest rate cuts versus asset purchases

Standard monetary policy accommodation on the one hand and central bank asset purchases on the other lead to different credit channels and bank balance sheet conditions. Two sets of IRFs are contrasted in this section regarding their transmission to the broad macroeconomic landscape, together with bank profitability and capital position. In the first one, the central bank unexpectedly cuts its key interest rates while in the second one, the central bank announces an asset purchase programme. We implement it in the DSGE model like Gertler and Karadi (2013):

$$B_{CB,t}^{QE} = \epsilon^{QE}$$

(27)

where the asset purchase shock $\epsilon^{QE}$, expressed as a percentage of the fixed government bond supply, follows an AR(2) process,

$$\log(\epsilon^{QE} t) = \rho_1^{QE} \log(\epsilon^{QE} t-1) + \rho_2^{QE} \log(\epsilon^{QE} t-2) + \nu^{QE}$$

(28)
QE corresponds to unexpected innovations to the purchase strategy of the central bank. For a programme which is announced and completely communicated to the agents $\nu^{QE}$ is non-zero in one period and has the interpretation that the programme is activated. This can be seen as rough an approximation of the first features of ECB’s asset purchase programme (APP). The next section will investigate in greater details the design of one-off asset purchase programmes and would propose more accurate ways of implementing it within the DSGE model. Nonetheless, for the sake of comparability with the relevant literature, we also present the simulations associated with the AR(2) process.

In normal times, policy rate cuts are favourable to bank profitability both through higher net interest income as well as general equilibrium effects. Figure 16 presents the IRFs for a one-standard deviation negative shock on the Taylor rule residual (see blue dotted lines). Temporarily lower short-term interest rates shift and steepen the term structure and directly support the profitability of maturity transformation activities of the banking system. In the model, lending rates respond sluggishly to money market rates due to nominal rigidities in lending rate setting. Besides, the decline in short-term interest rates leads to higher price of sovereign bonds which provides some mild holding gains for the banks. Finally, improving economic conditions and increasing asset prices are beneficial to firms creditworthiness, with receding delinquency rates. Such favourable developments in credit quality allow banks to scale down the credit risk compensation when setting lending rates. Turning to the macroeconomic multipliers of the monetary policy impulse, output increases by 0.3% at the peak while the rise in the quarterly inflation rate reaches 0.05%. Standard monetary policy interventions entail powerful transmission channels beyond the banking system, on the real side through the intertemporal substitution of spending decisions, and on the financial side, through the discount factor of asset pricing decisions. Therefore the credit multiplier is relatively low with real loans increasing by 0.25% while corporate lending rates moderate by more than the policy rate as the pass-through is almost full over two-years in the model and credit risk compensation is lower.

By contrast, the APP entails a strong portfolio rebalancing channel, incentivising banks to ease credit conditions, foregoing profit margins on loans and originating more credit exposures. Figure 16 presents the IRFs of a central bank asset purchase programme mimicking the January
2015 ECB’s APP (see black lines and grey shaded areas). The modelled frictions in bank capital structure decisions embed a constrained portfolio allocation between securities and loans. In this context, the central bank asset purchases do have an impact on government bond yields and compress the excess return on this asset class. The term spread is compressed by around 60 bps (in annual terms). Lower government bond yields urge banks to shed sovereign bonds and increase loan exposures. Over the course of the programme, bank sales of government bonds account for roughly one third of the central bank asset purchases. Banks therefore benefit from sizeable holding gains on their securities portfolio. This rebalancing mechanism leads in equilibrium to narrower “required” excess return on loan books by intermediaries. The pass-through of sovereign spreads to the required return on loans by retail lender is around 0.8. Credit expansion through lower borrowing cost is a key propagation mechanism of the central asset purchases in the model, compared with standard monetary policy easing. Net interest income therefore declines over the first two years of the simulation. As with the standard monetary policy shock, credit quality improves alongside with economic activity and asset prices, which contributes to bank profitability. Overall, the easing in financial conditions spurs investment and output, generating inflationary pressures and countercyclical monetary policy adjustment. The output multiplier peaks at 0.35%, and the quarterly inflation rate increases up to 0.04 pp. Compared with the standard monetary policy shock, the APP transmission features relatively less inflation and more output: indeed, the APP mainly propagates by compressing the overall external finance premium in the economy and thereby entails stronger cost channel than the standard monetary policy shock. In the simulation, we allowed the monetary policy rate to respond in line with the estimated Taylor rule. The increase in the policy rate partially mitigates the expansionary effects of the central bank asset purchases.

By comparing the transmission of interest rate cuts and asset purchases, we have illustrated the potential strategic complementarities between the two policy instruments. They feature distinctive propagation channels, different macroeconomic stabilisation properties and should not be considered perfect substitute. Against this background, the rest of the paper aims at exploring the optimal combination of standard and non-standard monetary policy measures. We first start with the specific configuration where the monetary policy rate is constrained at its effective lower bound and the central bank implements a specific asset purchase programme.
This exercise enables to evaluate the macroeconomic multipliers of central bank asset purchases and to perform some sensitivity analysis regarding the implementation design of the programme. In a second stage, we adopt a broader and more normative perspective, looking at the desirability of combining both instruments through the cycle (see section 4). All in all, we first evaluate the efficiency of programmes similar to the ones introduced by central banks during the crisis before we investigate the optimal policy setting.

4 Modalities of ad hoc central bank asset purchase programmes at the effective lower bound on interest rate

In this section we try to shed some light on the general effects of central bank government bond purchase programmes in the presence of the lower bond constraint on the policy rate. In this respect we lean on the parallel literature focusing on fiscal multipliers at the lower bound as we consider central bank asset purchases as an exogenous process (Christiano et al., 2011b; Woodford, 2011b). The effective lower bound on interest rate is implemented in the following way

\[
\hat{R}_{D,t} = \text{max} \left( R, \hat{R}^*_{D,t} \right)
\]

\[
\hat{R}^*_{D,t} = \rho \hat{R}^*_{D,t-1} + (1 - \rho) \left[ \rho \hat{y}_{t-1} + r_y \hat{y}_{t-1} \right] + r_{\Delta y} \Delta \hat{y}_t
\]

where \( \hat{R} \) is the effective lower bound (see Christiano et al., 2015 for a similar approach). This specification enables to endogenously determine the length of time for which the lower bound constraint is binding.

When central banks first introduced asset purchase programmes as an additional policy tool, they primarily focussed on giving guidance on the purchases path instead of providing a specific-contingency as usually done for setting the policy rate. Although asset purchasing programmes have also been contingent on specific targets, our approach resembles those programmes which have been introduced by some central banks. The Federal Reserve, for instance, announced a purchase path with equally distributed purchases across the months. From this communicated path, the expected central bank balance sheet could be derived. The
Eurosystem chose a similar approach when announcing the expanded asset purchasing programme in January 2015. For the underlying path for the stock of government bonds held by the central bank, we assume that there is a build up period until the point is reached when purchases stop which coincides with the maximum stock of bonds held. Following this period in time the stock is unwound by a specific speed which may be determined by the maturity profile of the portfolio.

The section presents first the construction of realistic crisis scenarios with endogenous periods of binding lower bound constraint on the policy rate. This occasionally binding constraint brings some non-linearity to the model and makes the macroeconomic multipliers of central asset purchases sensitive to the underlying crisis scenario. Thereafter, we focus on specific modalities of the ad hoc programme. Altogether, our results show that central bank asset purchases are more powerful i) in an environment in which the policy rate reached its effective lower bound, ii) the longer the duration of the lower bound period, iii) when, at the lower bound, the programme is fully communicated and anticipated, and iv) when it is complemented by forward guidance extending the lift-off date for the policy rate beyond agents expectations.

4.1 Constructing realistic lower bound scenario(s)

Asset purchase programmes were usually introduced as an additional policy tool when the short-term interest rate reached its effective lower bound and thus the room for further easing of the monetary stance through standard measures has been exhausted. To analyse such a policy configuration, we simulate an endogenous lower bound scenario. This requires the selection of shocks which can severely depress economic conditions so that the policy rate reaches its lower bound. Specifying the central bank interest rate policy as in equation 29 implies that the length of the lower bound period becomes endogenous: as shocks vanish over time, the economy recovers and the policy rate returns to its steady state value. In most of the literature, the lower bound scenario is generated by a single shock, a discount factor shock as done by [Eggertsson and Woodford (2003)] for example. Since our model has satisfactory data consistency, it allows for well-founded shocks located in the financial sphere which are combined on the basis of real observations.
A more realistic design of the lower bound scenarios is achieved through the lens of the estimated model for the euro area, first constructing shocks which reproduce the salient features of the euro area crisis, and then injecting those shocks in the model with an occasionally binding constraint on the short-term interest rate. This approach is similar to Christiano et al. (2015), for instance. However, we do not rely on narrative shocks outside the model. Instead, we prolong historical data with official forecasts as available by end-2014 and we use the estimated first order approximation of the model to back engineer the structural shocks consistent with our set of observable variables, in the absence of lower bound constraint on the short-term interest rate. We consider for the scenarios all shocks except the monetary policy shock (i.e. the Taylor rule residual). A filtering technique which allows for an occasionally binding constraint is beyond the scope of this paper. More importantly, we are interested in showing the sensitivity of macroeconomic multipliers to the underlying lower bound scenarios before the decision of the ECB public securities purchase programme.
and the degree of plausibility of the shock selection through our procedure is largely sufficient for this purpose. Regarding the modelling of the ELB we make use of anticipated shocks and basically follow [Laseen and Svensson 2011]. In this respect, we make endogenous the period in which the effective zero lower is binding similar to [Holden 2016].

Along those lines, we define three different scenarios which varying in terms of sample for the shocks and dates for reaching the effective lower bound. For entering the lower bound we treat two periods (Panel (a) in Figure 1) and follow a similar approach as done by [Christiano et al. 2015] who define a threshold interpreted as the zero lower bound. The first one is when the rate for the deposit facility in the euro area reached a level of zero. Although it has been reduced even further, we take this as the first period. The second period starts with the statement of ECB’s president Draghi “...the key ECB interest rates have reached their lower bound” 4. Given the official projections, the first period is five years long, which we refer to as the long lower bound period. The second period is shorter and comprises roughly three years. In both cases the underlying shocks are the same. We simulate the economy by starting at the end of 2007. To provide further robustness for the results, we also make use of all shocks starting at the beginning of our sample (Panel (b) in Figure 1). Here, we only look at the long lower bound period. By targeting specific dates for hitting the lower bound, we can implicitly control the duration for which the constraint on the policy rate is binding. Both the entry to and the exit from the lower bound are endogenous in this exercise and driven by our estimated shocks. The three scenarios are then used for evaluating asset purchases which brings to our approach large elements of realism and pragmatism at the same time.

4.2 Asset purchases multipliers at the lower bound on interest rates

We intend now to examine in greater details the distinct features of central asset purchases at the lower bound of interest rate, using our estimated DSGE model for the euro area as well as the lower scenarios of the previous section. The benchmark programme in the forthcoming analysis is meant to reproduce ECB’s APP announced in January 2015. Government bonds are purchased gradually over one year. The bond portfolio then dissipates over time as securities are held to maturity.

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Conversely to the AR(2) process specification of Gertler and Karadi (2013) examined in section 3.2, the law of motion for the purchases can be expressed as an AR(1) process with news as to more precisely match the time profile of the programme: announcement about future purchases can indeed be interpreted as news about future innovations. The formal description is

$$\log(\epsilon_{QE,t}) = \rho_{QE1} \log(\epsilon_{QE,t-1}) + \sum_{i=0}^{T} \nu_{QE,i} t_{i}. \quad (30)$$

where the last term on the right-hand side reflect the announcement about the future increase in the stock of government bonds held by the central bank. For \( i = 0 \) the purchases come as a surprise and for \( i > 0 \) the agents now at time \( t \) the future increase in the stock. Since the entire law of motion is known to the agents, agents can also build expectations about the future path of the stock and the parameter \( \rho_{QE1} \) controls the maturity profile of central bank’s portfolio.

The simulation of the central bank asset purchase programme is presented as the difference between the lower bound scenario including asset purchases on the one hand and the lower bound scenario without asset purchases on the other hand. In Figure 2 we show the responses of the selected macroeconomic variables under the three different lower bound scenarios: in each case the profile of asset purchases generated by equation (29) is exactly the same. The black lines correspond to the case from Panel (b) in Figure 1 i.e. a long lower bound period generated by the full series of structural shocks. The dashed blue lines (dashed red lines ) refer to the case where we start the counterfactual simulation after the financial crisis and reflects a long (short) lower bound period. Beyond this, the dotted purple lines represent the case without the lower bound constraints and are broadly similar to the IRFs of section 3.2.

Qualitatively, the responses are broadly similar across the various lower bound scenarios but display some notable quantitative differences. As is known from the literature on government expenditures, the lower bound environment leads to higher output multipliers (Christiano et al., 2011; Woodford, 2011). This property extends to central bank asset purchases, which has also been documented by Gertler and Karadi (2011, 2013) or Chen et al. (2012).

The intuition behind these results is that government bond purchases positively affect output.
and eventually boost inflation. Since the policy rate is constrained at its lower bound, the countercyclical effects from policy rate increases are missing. Thus, the deterioration of banks’ funding costs, through an increase in the short rate, is absent which stimulates bank equity and the origination of loans as a consequence. Indeed, the lending rate spread compression is almost 3 times smaller when the policy rate is unconstrained, declining by 20 bps (in annual terms), compared to 50 bps in the other scenarios. The increase in loans is also muted and is twice weaker than in the lower bound simulations. In addition, countercyclical monetary policy reins on the rise of inflation expectations and limit the decline in real interest rates, curtailing the overall output multipliers threefold in comparison with the other scenarios.

Figure 2: Comparison of the benchmark programme at the ZLB for different scenarios

Across the various lower bound configurations, the quantitative differences in the macroeconomic multipliers seem partly related to the duration of the lower bound period. For the scenario in which the shocks are induced not before 2007 and the policy rate remains at its lower bound for longer, government bond purchases have the strongest macroeconomic effects,
stimulating output by more than 1% at the peak and annual inflation by around 0.5 pp. At the same time, the responses also differ between the two other scenarios despite similar duration of binding lower bound constraint. Hence, we have illustrated some relevant dimensions of non-linearity: the macroeconomic multipliers of government bond purchases depend on the length of the lower bound period and more generally on the underlying crisis shock typology.

4.3 Communication strategy and anticipation effects

We now examine the sensitivity of our previous results with respect to asymmetric information between the central bank and private agents.

For this purpose, we consider the same actual path of purchases but assume different announcement strategies. The various simulations are conducted in a lower bound environment as it is the relevant policy configuration for evaluating asset purchases. The underlying lower bound scenario corresponds to the long binding period of Panel (a) in Figure 1.

Figure 3 presents all results. The benchmark case (bold black lines) assumes that agents are fully aware of all the features of the programme and is the same as the dashed blue lines of Figure 2: the announcement comes as a surprise but the purchase path and the unwinding path are known completely. This comes the closest to the ECB’s APP of January 2015 as market participants anticipated that purchased asset would be held to maturity. In the second case, the unwinding path is known to the agents but the purchases come as a surprise every period (blue dashed lines). In the third case, the entire programme comes as a surprise: both the purchases and the unwinding path are unknown and come as a surprise period by period (red dashed lines). The fourth case is similar to the second one with the difference that agents expect the unwinding to take place at a slower pace while the actual unwinding is faster (cyan dotted lines). This means that they are faced with less positive surprises during the front loading and with negative surprises during the unwinding period. In the last case, we combine positive surprises during both the purchase path and the unwinding path (line with stars in magenta)6. We restrict our analysis to the case in which there is a steady surprise (positive or negative) which means that agents cannot learn the actual features of the programme. A more explicit learning formulation would be worth investigating but this is beyond the scope

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6 A formal description how we implement these programmes technically is provided in the appendix.
Our results show that the correct anticipation of the programme results in the largest macroeconomic responses in the short run (bold black lines). This is specially true for the bond market: only in this case are sovereign yields sharply declining on the announcement of the programme and much before the actual purchases are actually implemented. By contrast, the smallest effects occur when all the features of the programme come as a surprise (red dashed lines). This scenario is an extreme case because agents see the purchases but they do not know that the purchases continue in the next period. Nevertheless, this scenario allows to demonstrate the importance of the knowledge about the offloading path on the one hand. On the other hand, the responses can be interpreted as the flow effects of the programme since agents do not take future stock holdings into account. The closest to the fully announced programme are the ones where agents are “positively” surprised only (see dashed blue lines and lines with stars in magenta). Here, output rises with a short delay. The difference can
be traced back to the anticipation effect of the programme. As the programmes include some elements of surprise the bond market adjusts along with purchases so that the portfolio rebalancing takes place with some delay. Smaller surprises during the front loading dampens the output expansion (see cyan dotted lines). This is also true during the unexpectedly faster unwinding of the portfolio.

Overall the transmission of government bond purchases to the real economy does change by assuming different information sets, although, based on a visual inspection of the responses in Figure 2 some configurations yield results which are very close to each other.

In order to measure and compare results across specifications, we develop more quantitative indicators of output multipliers. However, we focus on numerical multipliers and do not provide the analytical expression for the multipliers as done by Woodford (2011b), for example. Given the rich structure of our model, an analytical solution would be difficult to obtain and interpret. The output multiplier is therefore defined as the cumulated gains in output relative to the cumulated stock, i.e. the balance sheet profile of the central bank. This present-value multiplier (PVMP) weights the future with the time preference rate. This concept leans on Mountford and Uhlig (2009) for fiscal multiplier and reflects the idea that output gains today are more valuable for the agents than output gains tomorrow. We compute two multipliers over an horizon of one year and of ten years.

\[
PVMP = \frac{\sum_{t=1}^{T} \beta^{t-1} \Delta Y_{t+i}}{\sum_{t=1}^{T} \beta^{t-1} CBStock_{t+i}}
\]

The multipliers for the various information sets are presented in Panel (a) of Table 1 based on different underlying lower bound scenarios (i.e. with either short or long duration of binding lower bound constraint). The results confirm our previous findings: the full anticipation of the programme yields the strongest short- and medium-run improvements in output. This qualitatively holds for both lower bound scenarios, albeit the effects being stronger for the longer lower bound duration. Conversely, the fully unexpected programme provides the smallest output multiplier over both horizons. Agents cannot build expectations about future purchases and the evolution of the stock. Accordingly, it seems that "stock effects" are key for the effectiveness of the asset purchase programme. Turning the other information sets,
the short-run effects of the unexpected front-loading are obviously identical because the programmes mainly differ regarding the off-loading part. As expected, the output gains from the programme with positive surprises are larger than in the "unexpected frontloading" case (without surprises on the offloading strategy) but they are quantitatively very close to each other.

Table 1: Output multipliers from government bond purchases for different programmes calculated over ten years - endogenous lower bound period

<table>
<thead>
<tr>
<th>Panel A: Purchases over one year with different information sets</th>
<th>Short ZLB period</th>
<th>Long ZLB period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>over 1 year</td>
<td>over 10 years</td>
</tr>
<tr>
<td>Full anticipation (benchmark model)</td>
<td>0.0604</td>
<td>0.0798</td>
</tr>
<tr>
<td>Unexpected front loading</td>
<td>0.0303</td>
<td>0.0401</td>
</tr>
<tr>
<td>Completely unexpected</td>
<td>0.0029</td>
<td>0.0037</td>
</tr>
<tr>
<td>Unexpected front loading and negative surprises while offload-in</td>
<td>0.0207</td>
<td>0.0282</td>
</tr>
<tr>
<td>Unexpected front loading and positive surprises while offload-in</td>
<td>0.0304</td>
<td>0.0439</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Purchases over time</th>
<th>Short ZLB period</th>
<th>Long ZLB period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>over 1 year</td>
<td>over 10 years</td>
</tr>
<tr>
<td>Purchases over one year</td>
<td>0.0604</td>
<td>0.0798</td>
</tr>
<tr>
<td>Purchases over two years</td>
<td>0.0986</td>
<td>0.132</td>
</tr>
<tr>
<td>Purchases over 1 year with 1 year reinvestment</td>
<td>0.065</td>
<td>0.085</td>
</tr>
<tr>
<td>One-off</td>
<td>0.0319</td>
<td>0.0785</td>
</tr>
</tbody>
</table>

Note: The present value multiplier weights the periods with the discount rate.

4.4 Time profile of asset purchases and reinvestment strategy

While the preceding sensitivity analysis has always assumed the same time profile for the asset purchase programme, we put now more emphasis on the path of central bank’s balance sheet, discussing first the anticipated frontloading of purchases and turning afterwards to the offloading strategy.

In the benchmark programme, the purchases are announced and distributed across the first year (recall that the purchases in the first quarter come as a surprise). In Figure 4, we contrast the benchmark programme (black solid lines) with three other programmes. The first one implements quarterly asset purchases equally distributed over two years (blue dashed lines) with the same maximum stock effect as in the benchmark programme (the purchases are therefore smaller in every quarter). Second, we allow for a reinvestment policy which is
combined with the initial path of the benchmark model (red dashed lines). Reinvestment means that the maximum stock of government bonds is kept constant over one year before the unwinding starts. The reinvestment programme shares the purchase path with the benchmark model, and has the same unwinding path as the two-year programme. Third, we show one-off purchases with similar stock effect (cyan dotted lines), noting that the current analysis abstracts from any implementation constraints for the purchase programme that might become binding in this case (like issue or issuer limits for example).

![Graphs showing different effects of ad hoc APP over time and the zero lower bound - long ZLB period](image)

Figure 4: Different distribution of ad hoc APP over time and the zero lower bound - long ZLB period

Among all the cases, the reinvestment programme shows the largest peak effects on output and inflation, followed by the two-year programme. The one-off purchase programme however has a lower average impact on the main variables. The ranking of the different programmes according to their macroeconomic effects might be simply related to larger or smaller average increase in central bank balance sheet.

In order to control for differences across programmes in the average increase in central bank
balance sheet, we look again at the present value output multipliers. They are presented in Panel (b) of Table 1 for the different front-loading paths. Different anticipated purchase paths do not alter much the (relative) effectiveness of government bond purchases on output in the medium run. The output multipliers over an horizon of ten years are widely the same for the benchmark (one-year) programme, the two-year programme, and the one-year with reinvestment programme. However, the quantitative effects differ more evidently in the short run. Among these three programmes, the two-year programme generates the largest short-term response on output per average unit of purchase. Obviously, anticipation effects about the time profile of the programme affects the output multipliers particularly in the short run. This argument can also be seen by comparing the announced programmes with the one-off programme. In this case, the short-run multipliers fall well below those from the announced programmes although the long-run effects are broadly similar. This lets us conclude that the average size of central banks’ balance sheet in fully anticipated asset purchase programmes is important for generating output (and inflation) gains. In this respect, for the same peak effect on central bank balance sheet, a re-investment policy can provide meaningful macroeconomic amplification. At the same time, output multipliers per unit of purchase are not very sensitive to the time profile of the programmes, even at the lower bound.

Figure 5: Output multipliers of one-off asset purchases
We turn now to the discussion of the offloading strategy. Figure 5 shows the present-value multipliers on output (y-axis) as a function of the offloading pace of one-off purchases (i.e. the half-life of the portfolio on the unwinding path which is a function of $\rho_{QE}^1$ from equation 2). The figure clearly points to non-monotonicity in the relationship. Starting from very short periods of holdings the output multiplier increases quickly. After reaching a maximum at around 4-year half-life, the output multiplier falls. For very long periods of holdings there seems to be a stabilisation in terms of output multipliers. The results are mostly insensitive to the duration of the long-term government bonds in our economy, at least from 5 to 10 years. This result indicates that there is an optimal offloading path in terms of effectiveness per unit of purchases which does not correspond to the duration of the underlying bond. In other words, hold-to-maturity central bank portfolios may not be a dominant unwinding strategy.

Altogether, considering that the "unit" efficiency of a programme is related to the cumulative holdings over its lifetime, this section suggests that holding-to-maturity the assets might be more efficient for the central bank than earlier offloading strategies which would precisely reduce the cumulative holding. Faster unwinding of the bond portfolio seems to weaken the impact of programme on asset prices and its pass-through to the broader economy. By contrast, for the same cumulative holding, a very persistent programme would imply much smaller asset purchases in the initial phases of the programme, when the funding constraints are tighter and the lower bound on interest rate is binding, thereby reducing the macroeconomic multipliers.

4.5 Forward guidance on the interest rate

We have studied government bond purchases in a lower bound environment which has been created endogenously. Part of the results obtained is related to the length of the period in which the lower bound is binding, i.e. is expected to hold. The expectations about the duration of the lower bound are related to the structure of the underlying shocks, their impact on the economy and the policy conduct at the lower bound formulated in equation 29.

Policymakers can nonetheless influence the economy at the lower bound by using forward guidance on the policy rate: they can commit to keep the policy rate unchanged beyond the

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The half-life of the underlying AR process in years is computed as $\text{half-life} = \frac{\ln(2)}{\ln(\rho)}$. 

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lift-off date expected by agents on the basis of the state of the economy and the historical monetary policy conduct. To investigate the effects of forward guidance on the outcomes of government bond purchases we depart from the endogenous zero lower bound scenario and assume that the policy rate is being fixed for a given period of time. To disentangle the effects of forward guidance for the interest rate, we consider the benchmark programme from the previous sections and vary the period for which the interest rate is held fixed. Regarding the modelling of forward guidance, we again lean on Laseen and Svensson (2011).

The responses of the economy for 2 quarters, 4 quarters, and 6 quarters of forward guidance are presented in Figure 6 in deviation from the transmission of the asset purchase programme under the estimated Taylor rule, i.e. we compare the effects under forward guidance with those without forward guidance. The relative perspective allows us to only highlight the effects coming from forward guidance. The corresponding output multipliers are given in Table 2. Government bond purchases become significantly more effective - in terms of resources used - by extending the period of constant interest rate: the longer the period of forward guidance, the more effective the asset purchase programme. Forward guidance turns out to have outrageously strong macroeconomic amplification effects on asset purchases. This result is also related to the "forward guidance puzzle" in DSGE model as explained in Carlstrom et al. (2015) for example. The effectiveness of government bond purchases is much stronger.

By comparing Table 2 with Table 1, it turns out that combining government bond purchases with forward guidance on the policy rate beyond few quarters, increases their effectiveness

Figure 6: Effects of forward guidance on the interest rate
by more than the endogenous lower bound environment. It is worth dwelling on the reasons explaining this result. The underlying crisis shocks used to create the endogenous lower bound environment, imply first that the standard monetary policy rule is constrained: all agents perceive that the policy rate cannot go down further and would eventually rise again in line with the Taylor rule as soon as the economic conditions improve tangibly. The binding lower bound constraint can actually be interpreted as "tightening" forward guidance, keeping rates higher than they should. In this context, the asset purchase programme interact with the underlying shock dynamics at the lower bound and are amplified because they do remove some of the "tightening" forward guidance. Only this mitigation of the "tightening" forward guidance should be compared to the previous simulations. Instead, the "easing" forward guidance implemented in this section solely impacts on the asset purchase programme by keeping the interest rate fixed and supporting its effects.

Table 2: Present value output multipliers from government purchases for different period of forward guidance on interest rates

<table>
<thead>
<tr>
<th>Period</th>
<th>1 Quarter</th>
<th>2 Quarters</th>
<th>3 Quarters</th>
<th>4 Quarters</th>
<th>6 Quarters</th>
<th>8 Quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over 1 year</td>
<td>0.0489</td>
<td>0.0608</td>
<td>0.0774</td>
<td>0.1008</td>
<td>0.1851</td>
<td>0.4261</td>
</tr>
<tr>
<td>Over 10 years</td>
<td>0.0339</td>
<td>0.0403</td>
<td>0.0496</td>
<td>0.0628</td>
<td>0.1121</td>
<td>0.2569</td>
</tr>
</tbody>
</table>

Note: The present value multiplier weights the periods with the discount rate.

From this point of view, one reading of the results is that the central bank can increase the effectiveness of its asset purchase programme in an endogenous lower bound environment by combining it with forward guidance on the policy rate. Although coming from a different perspective, this argument resembles the exposition of Woodford (2012) on the strategic complementarities between "quantitative easing" and forward guidance.

5 Asset purchases under an optimal policy design

After discussing the general properties of \textit{ad hoc} asset purchase programmes, we intend to formulate an optimal path for the stock of government bonds held by the central bank, similar to the approach followed for standard monetary policy (Woodford, 2011a): we assume that policymakers seek to minimise an intertemporal loss function and have the enforcement
technology to commit to the optimal policy conduct from a "timeless perspective". More specifically, the objective function takes the form:

\[ \text{Loss}_t = \lambda_{\pi}(\pi_t)^2 + \lambda_Y(\hat{Y}_t - \hat{Y}_t)^2 + \lambda_R(\hat{R}_t)^2 + \lambda_{BCB}(\hat{B}_{CB,t})^2 + \beta E_t \text{Loss}_{t+1}, \]  

(32)

where \( \lambda_{\pi} \) is the weight on inflation volatility, \( \lambda_Y \) on the output gap volatility. We also introduced a penalty for each instrument: \( \lambda_R \) on interest rate volatility and \( \lambda_{BCB} \) on the variability in government debt held by the central bank. There are various theoretical and operational rationales for constraining the fluctuations of the standard and non-standard monetary policy instruments which we will not detail here. We only take for granted that operational monetary policy conduct faces implementation constraints in adjusting its bond portfolio or changing its key interest rates to a very large extent from one period to the other.

In the following, we consider as the benchmark optimal policy conduct, the case where both the short-term rate and the stock of government bonds held by the central bank are set optimally. To evaluate the stabilisation capacity of debt policy, we also consider the case in which only the short-term rate is set optimally. Along the section, configurations where the lower bound environment is relevant are contrasted with the unconstrained allocation. We start by reviewing the optimal asset purchase strategy in a specific crisis scenario that brings the economy to the lower bound. This first configuration links to the policy evaluation of the previous section and constitutes a relevant preamble to the analysis of optimal instrument combination through the business cycle. Indeed we investigate across a range of policymaker preference the optimal allocation in the absence of lower bound constraint before repeating the exercises when the constraint becomes occasionally binding. Within the confines of the model validity, interest rate policy and asset purchases feature strong strategic complementarities for both normal and crisis times. When constrained by the lower bound on the policy rate, optimal policy conduct displays: i) longer lower bound period and stronger "use of forward guidance" than in the estimated Taylor rule, ii) activist asset purchase policy and iii) a clear sequencing of the exit strategy, stopping first (and unwinding) the asset purchases and significantly later

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8The loss function approach, by contrast to the welfare-based optimal policy, enables to examine different menu of central bank preferences and for some specification actually approximates well the Ramsey allocation.

9The output gap is defined as the percentage deviation of output from its flexible price and wage equivalent.
lifting-off the policy rate. In terms of macroeconomic performance, optimal asset purchase strategies have the potential to fully offset the costs of lower bound constraint on the policy rate.

5.1 Optimal asset purchase strategy at the lower bound of interest rate

In order to generate a lower bound environment, and as opposed to section 4, we rely on an underlying scenario which brings the policy rate immediately to its lower bound but qualitative resembles the crisis scenarios of Figure 1. A combination of adverse financial shocks (risk shock and bank-specific shock) and adverse demand-side shocks (investment-specific shock and government expenditures shock) contract the economy. The reason why we depart from the previous crisis scenarios is that with our characterisation of the optimal policy conduct, the central bank does not switch to asset purchases as soon as the policy rate hits its lower bound. Instead, in the optimal allocation, there is always an active bond portfolio management and the probability of reaching the lower bound is strongly mitigated. Since we want first to study the optimal asset purchase strategy at the lower bound, we design a scenario in which the lower bound binds immediately and compare various policy conducts in this environment.

To derive the optimal policy in a lower bound environment, we introduce a non-negativity constraint on interest rate in the intertemporal maximisation programme of the policymaker. In computational terms, we again follow the approach of Holden (2016) to deal with the occasionally binding slackness condition on the interest rate constraint.

Regarding the weights of the loss function in equation (32), we normalise the penalty on inflation, $\lambda_\pi$ to 1 and set the penalty for both output gap, $\lambda_Y$, and interest rate volatility, $\lambda_R$, to 0.03. Those parameters are chosen such that, in absence of lower bound and APP, the volatility of inflation and interest rate under optimal policy is broadly the same as in the Taylor rule allocation. The penalty on the variance of central bank bond portfolio, $\lambda_{BCB}$, is set so that in the crisis scenario, the build up of asset purchased culminates at around 8% of GDP (similar to the initial calibration of the ECB’s APP). With the corresponding value for $\lambda_{BCB}$, the costs of such an asset purchase volatility in terms of unconditional loss function, turns out to be of a similar magnitude to the costs associated with interest rate volatility (using the estimated structural shocks and in the absence of the lower bound constraint).
Notice that we neglect here the inefficiency costs associated with asset purchases, as assumed by Gertler and Karadi (2011, 2013) in a very stylised manner. The inefficiency costs would in principle affect the effectiveness of asset purchases but we crucially lack microfoundations for it. Since our aim was to show the general properties of optimal asset purchase strategy, we do not present a sensitivity analysis of our results on inefficiency costs à la Gertler and Karadi (2011, 2013). Instead, the penalty introduced in the policymaker’s objective, indirectly controls for possible side effects of excessive reliance on this type of instrument.

In Figure 7 we present the responses of selected variables to the crisis shocks under three different monetary policy conduct: i) optimal instrument combination (black solid lines), ii) optimal interest rate policy only (blue dashed lines), and iii) the estimated Taylor rule without asset purchases (dashed red lines with dots). The last case is introduced to show how the economy behaves in the absence of optimal policy conduct based upon our estimation.

Indeed, under optimal policy conduct, the effects of the crisis scenario on output and
inflation are drastically milder than under the estimated Taylor rule. In this later case, the lower bound constraint is left much earlier. Optimal policy would actually keep interest rates at lower bound for almost twice longer. This illustrates the inefficiency of a policy rule described by equation (29) at the lower bound. The result is well-known in the literature on optimal monetary policy at the lower bound. In specific modelling frameworks, optimal monetary policy in the absence of the lower bound constraint could take the form of an interest rate feedback rule, to which the estimated Taylor rule used in this paper would not be drastically different (see Giannoni and Woodford (2003a,b)). However, once the lower bound constraint is introduced, the optimal management of expectations through a feedback rule on the "shadow" interest rate, \( \hat{R}_D \), as in equation (29), would not be the same as in the unconstrained case (which is assumed in equation (29)): the larger the constraint imposed by the lower bound in responding to the crisis scenario, the longer the optimal lower bound period and the higher the inflation expectations that the policymaker needs to feed. This intuition about optimal policy conduct at the lower bound is well-framed in the seminal work of Eggertsson and Woodford (2003) and can be interpreted in a loose sense, as requiring an intensive use of forward guidance.

Comparing the two optimal policy settings, the macroeconomic allocation is improved when the central bank also sets its government bonds purchases optimally. The optimal instrument combination dominates even if we introduced a penalty on the volatility of central bank balance sheet. As opposed to Woodford (2012), the propagation mechanism of asset purchases in our model works through bank portfolio rebalancing and the easing of credit conditions but does not entail a specific "signalling channel" through which it could directly support the forward guidance on the policy rate. Therefore, whereas the optimal asset purchase strategy appears consistent with a significant degree of forward guidance, it also yields an earlier lift-off date from the lower bound compared to optimal interest policy only.

Figure 8 zooms into the optimal policy allocations. We reproduce there the same cases as in Figure 7 (black lines for the optimal interest rate policy only and the blue dashed lines labelled case 1 for the optimal instrument combination). Besides we add another optimal instrument combination in which the penalty on central bank balance sheet volatility is reduced compared to the benchmark case (see dashed red lines with dots labelled case 2). The more activist asset purchase strategy leads to further stabilisation gains and implies an earlier "lift-
off" date for the policy rate, compared to both other cases. This shows that using forward guidance in conjunction with having a more intensive role for debt policy can be a powerful policy mix to stabilise the economy. As a result the economy would exit earlier from the lower bound environment. We can also relate this result to the findings from Section 4.5 on forward guidance on the policy rate. The stabilisation gains from the optimal asset purchase strategy are tangible relative to the optimal interest rate policy. However, these gains appear smaller than the improvements upon the estimated Taylor rule allocation, achieved through the optimal interest rate policy, or in other words, through forward guidance à la Eggertsson and Woodford (2003).

Finally Figure 8 also illustrates the optimal sequencing of instruments at the lower bound. With the specific calibration of the policymaker loss function, the central bank bond portfolio builds up over 8 quarters before the unwinding of positions starts. The time profile of the programme is not much affected by the more or less activist stance regarding asset purchases. The policy rate is kept at the lower bound for more than 14 quarters, and in any case much beyond the end of asset purchases. On this precise scenario, the lift-off date is relatively close to the point in time when the bond portfolio reaches back its initial size.

5.2 Optimal combination of instruments through the business cycle

Up to now, we have analysed the properties of optimal conventional and unconventional policy in a crisis situation yielding a lower bound environment. In this section we want to elaborate...
more on the achievable macroeconomic stabilisation from the class of optimal monetary policy conduct described previously. As a first step, we discard the lower bound constraint on the policy rate and exploit the business cycle regularities captured by the estimated DSGE model. Instead of investigating the optimal response to a specific crisis situation, we activate all structural disturbances in our model and use the estimated shock processes to derive policy efficiency curves. To construct those efficiency curves, we run a grid of weights on inflation in the policymaker loss function and we plot for every combination of weights, the theoretical

![Figure 9: Efficiency frontiers for different penalty on the volatility of asset purchases](image)

In order to quantify the stabilisation benefits provided by the optimal asset purchase strategy (in the absence of the lower bound constraint), we present four cases in Figure 9: i) optimal interest rate policy (black solid line), ii) optimal instrument combination with moderate asset purchases (blue dashed lines with dots), as in case 1 in Figure 8, iii) optimal instrument combination with intensive asset purchases (red dotted lines), as in case 2 in Figure
(iv) optimal instrument combination with more intensive asset purchases (dashed turquoise line), for which we reduced further the penalty on government debt in the loss function to obtain the case 3 in Figure 8. As can be seen, increasing the activism of central bank asset purchase strategy shifts the efficiency curves to the south west quadrant. Combining policy instruments enables the monetary authority to contain the volatility of both inflation and the output gap beyond what can possibly be achieved with interest rate policy only. This result is related to the fact that financial frictions in the banking sector create a wedge between long-term and short-term interest rates. Asset purchases are able to influence this wedge directly which is also tied to banks’ lending decisions.

Moreover, the overall efficiency curve gets steeper when both instruments are combined. This implies that the policy tradeoff between output and inflation stabilisation weakens. With activist asset purchase strategies (case 3) the sacrifice ratio of going from an extremely "dovish" policymaker to an extremely "hawkish" one represents less than 1 pp of output gap standard deviation. The comparable number in the absence of asset purchases (black line) would reach almost 3.5 pp of output gap standard deviation.

In order to see the benefits of combining policy instrument, we pick a point on the efficiency curve in the absence of asset purchases (point A) which yields the same inflation (and interest rate) volatility as under the estimated Taylor rule (point E). This precise point refers to the same weights in the loss function as the ones used for Figure 8. Now, allowing for asset purchases with the same penalty for central bank balance sheet volatility as in case 1 of Figure 8 brings the allocation to point B. It turns out that for the policy preferences at point A, the stabilisation gains of asset purchases are relatively even on both inflation and output gap. The same is true for going from point B to point C or from C to D, as the penalty on central bank balance sheet volatility is reduced and asset purchase strategies are more activist. Therefore, while the overall efficiency curves shift inwards but steepen with increasingly activist asset purchases, the stabilisation improvements for intermediate policy preferences appear to be more homothetic in the inflation/output gap space.
5.3 Sensitivity to portfolio rebalancing frictions

Since our model has a rich banking sector and given that the effectiveness of government bond purchases depends on the frictions within the banking segments, we review now how these frictions may actually affect the stabilisation performance of optimal asset purchase strategies. The central variable which controls the pricing of government bonds and the portfolio rebalancing in the banking sector is $\delta^{BG}$. Hence, we repeat the previous exercise and show efficiency frontiers for different values of $\delta^{BG}$. The results are presented in Figure 10. The black solid line correspond to the estimated value of the diversion parameter (at around 2). It can be contrasted with two other cases, in which the diversion parameter is lower (dashed blue line with dots) and larger (dotted red line).

![Figure 10: Efficiency frontiers for different degrees of financial frictions in banking sector](image)

Since the curves are very close to each other at the tails, we split them into three parts: the
middle part (Panel (a)), the left-hand side tail (Panel (b)), and the right-hand side tail (Panel (c)). It turns out that the efficiency frontiers for the two alternative values are above the one based on the estimated value of the diversion parameter. Thus, there is evidence of strong non-linearity in the relationship. Furthermore, the high diversion parameter curve is closer to the one based on the estimated value, at the left-hand-side tail, while the low diversion parameter curve is closer at the right-end-side tail. This means that the shape of the efficiency frontier changes slightly with different diversion parameters.

The non-linearity regarding the size of the diversion parameter is made more explicit in Figure 11, where we present the standard deviations of the output gap and inflation as a function in $\delta_{BG}$ and for moderate and intensive asset purchase strategies. As can be seen from the graphs, there seems to be a point at which the variability of both output gap and inflation is the lowest. In our specific case, this minimum roughly coincides with the estimated parameter for the diversion rate. For lower values of this parameter, the optimal debt policy becomes less effective for which reason the volatility of the economy is higher. For higher values, the economy becomes more volatile because there is feedback effect of debt policy on bank equity.

Figure 11: Impact of degrees of financial frictions in banking sector on macroeconomic volatility

Another important friction which affects the transmission of government bond purchases through the banking sector to the real economy is the imperfect lending rate pass-through in
the retail banking segment. If banks are unable to reset their return on assets (due to maturity
transformation, indexation schemes or other rigidities in the underwriting of loan contracts),
the portfolio rebalancing towards credit exposures is constrained and the associated easing
in financing conditions is muted. In the DSGE model, the main parameter underlying the
staggered lending rate setting and proxying for the retail banking frictions is $\xi_{RE}$. We zoom
again on the sensitivity of the efficiency curve to this parameter. This is shown in Panel (a) of
Figure 12 where the black line is based on the estimated parameter value. The case for nearly
full pass-through (dashed blue line with dots) corresponds to $\xi_{RE}$ close to zero, and is only
slightly below the black line. The case for very low pass-through (dotted red line) corresponds
to $\xi_{RE}$ close to one and is systematically above the black line. The dependency of the efficiency
curve on $\xi_{RE}$ seems monotonic but non-linear with marginal stabilisation costs being stronger,
the higher the rigidities in lending rate setting.

![Figure 12: Impact of staggered lending rate setting](image-url)
This is confirmed in Panel (b) and (c) of Figure 12 where we plot for given policy preferences, the variability of the economy as a function of $\xi_{\text{RE}}$. When the lending rate pass-through goes from moderate to high (for values of $\xi_{\text{RE}}$ between 0.6 and 0), standard deviations are lower as the impulse is transmitted to a large extent into the borrowing conditions of non-financial firms but the macroeconomic allocation is hardly sensitive to the parameter. For lower levels of pass-through (higher values of $\xi_{\text{RE}}$) the deterioration in macroeconomic volatilities increases rapidly as the banking system inefficiencies stand in the way of monetary policy actions.

Overall, this sensitivity analysis shows that the effectiveness of optimal policy conduct depends on the transmission of the impulses through the banking sector which is the reason for our modeling choice.

5.4 Asset purchases and the stabilisation costs of the lower bound on interest rate

In this last subsection, we extend the previous analysis on the efficiency curves to a lower bound environment. We aim at evaluating the stabilisation costs of the lower bound constraint and study whether active asset purchase strategies can meaningfully tame such distortions.

For the lower bound case, we need to rely on moments obtained from stochastic simulations compared to the theoretical moments derive in the absence of occasionally binding constraint. More specifically, for each point in the grid of policy preferences, we run simulations producing 500 time-series where we draw for every period in time and for every structural shock from a normal distribution with the corresponding estimated standard deviations. In Figure 13 we present the resulting efficiency curves (from a cubic smoothing approach). Figure 13 is basically the counterpart to Figure 9 except that the former bases upon simulated moments instead of theoretical moments.
In the absence of asset purchases, the lower bound restriction induces an outward shift in the efficiency curve (moving from the cyan dotted line to the red dashed line with dots): due to the lower bound environment, the achievable set of macroeconomic outcomes deteriorates as optimal monetary policy is not able to fully circumvent the constraint using forward guidance à la Eggertsson and Woodford (2003). As before, the activation of optimal asset purchase strategies provides an inward shift of the efficiency frontier. In the absence of the lower bound constrain, the efficiency frontier moves further to the southwest. Depending on the intensity of asset purchases, this gap can actually be fully closed. Therefore the stabilisation costs of the lower bound environments crucially depends on the operational constraints on activist asset purchase strategies, i.e. on the policy preferences regarding the volatility of central bank balance sheet.
These properties of the optimal asset purchase strategy can also be examined by looking at the distribution of short rates. Figure 14 shows in the top left chart the probability distribution of the short rate for the lower bound case while the right-hand side chart shows the same in the absence of the lower bound constraint. In the second row, the corresponding cumulative density functions are displayed. The red bars correspond to optimal policy with asset purchases, and the blue charts, to optimal policy without asset purchases. For both cases, with and without considering the effective lower bound constraint, the density distributions have smaller tails. In the absence of the lower bound constraint, this means that large policy rate changes (decreases and increases) occur with a lower probability mass if asset purchases are available as an additional policy measure. For the case of the lower bound constraint the probability of hitting the lower bound becomes even smaller. Asset purchases obviously reduce the volatility of the policy rate, as an additional policy tool is available to stabilise the economy. However,
the stabilisation gains crucially depend on the intensity of the purchase strategy.

6 Conclusion

This paper sheds some light on the effectiveness and suitability of central asset purchase programmes. In a lower bound environment, we first evaluate *ad hoc* programmes according to their implementation design and communication strategy. We start by assuming that the central bank wants to impose a specific path for its holdings of government debt. We demonstrate how the effects on the macroeconomy depend on the communication about the programme. Furthermore, we show that it is not only the period of holdings which matter for the success of the programme. Clearly communicated asset purchase strategy also helps to exit from the lower bound on interest rate earlier as it supports the economic recovery. While trying to mimic features of real world programmes in the first part, we also investigate central bank asset purchases from an optimal policy perspective. The consider the stabilisation benefits of optimally combining the two instruments, in normal times as well as when lower bound constraint on interest rates becomes binding.

Our results show that active bond portfolio management by a central bank can efficiently complement interest rate policy. This is particularly true once the lower bound is reached. However, the macroeconomic stabilisation gains crucially depends on the potential for the asset purchases to influence the term premium. In our model the term premium mainly results from frictions in the banking sector. Without these frictions the term premium disappears. In a lower bound environment asset purchases can be used to substitute the missing reaction of the short-term policy rate. Otherwise a policy which is able to reduce frictions in the banking sector might have similar effects than purchases of government bonds in our model. For example, Woodford (2016) investigates the joint conduct of unconventional monetary and macroprudential policy, which certainly constitutes an inspiring avenue for further research.
References


A Supplementary model description

A.1 Households behavior

The instantaneous household utility $U$ has the following functional form

$$U(X_1, X_2) = \frac{X_1^{1-\sigma_c}}{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_c} X_2^{1+\sigma_c}\right).$$

The first order conditions of the household problem with respect to consumption, labour, deposit are

$$\Lambda_t = U_t^{1-\sigma_c} - \beta \gamma^{-\sigma_c} \eta \mathbb{E}_t U_{t+1}^{1-\sigma_c}$$

$$\Lambda_t W_t^b = U_t^{1-\sigma_c}$$

$$\mathbb{E}_t \left[ \frac{\Xi_{t,t+1}}{\sigma_{t+1}} \right] = 1$$

where $\Lambda_t$ is the lagrange multiplier associated with the budget constraint and $\Xi_{t,t+1} = \beta \gamma^{-\sigma_c} \Lambda_{t+1}$ is the period $t$ stochastic discount factor of the households for nominal income streams at period $t+1$.

A.2 Labor supply and wage setting

Intermediate goods producers make use of a labor input $N^P_t$ produced by a segment of labor packers. Those labor packers operate in a competitive environment and aggregate a continuum of differentiated labor services $N_t(i)$, $i \in [0, 1]$ using a Kimball (1995) technology.

The Kimball aggregator is defined by

$$\int_0^1 H \left( \frac{N_t(i)}{N^P_t}, \theta_w, \psi_w \right) di = 1$$

where we consider the following functional form:

$$H \left( \frac{N_t(i)}{N^P_t} \right) = \frac{\theta_w}{(\theta_w(1+\psi_w) - 1)} \left[ \frac{N_t(i)}{N^P_t} \psi_w - \psi_w \right]^{\theta_w(1+\psi_w) - 1}$$

This function, where the parameter $\psi_w$ determines the curvature of the demand curve, has the advantage that it reduces to the standard Kimball aggregator under the restriction $\psi_w = 0$.

The differentiated labor services are produced by a continuum of unions which transform the homogeneous household labor supply. Each union is a monopoly supplier of a differentiated labour service and sets its wage on a staggered basis, paying households the nominal wage rate $W_t^*$. Every period, any union faces a constant probability $1 - \alpha_w$ of optimally adjusting its nominal wage, say $W_t^*(i)$, which will be the same for all suppliers of differentiated labor services.
We denote thereafter $w_t$ the aggregate real wage, expressed in effective terms, that intermediate producers pay for the labor input provided by the labor packers and $w^*_t$ the effective real wage claimed by re-optimizing unions. When they cannot re-optimize, wages are indexed on past inflation and steady state inflation according to the following indexation rule:

$$W_t(i) = \gamma \left[ \pi_{t-1}^{\xi w} \pi^* - \xi w \right] W_{t-1}(i)$$

with $\pi_t = \frac{P_t}{P_{t-1}}$ the gross rate of inflation.

Taking into account that they might not be able to choose their nominal wage optimally in a near future, $W^*_t(i)$ is chosen to maximize their intertemporal profit under the labor demand from labor packers.

Unions are subject to a time-varying tax rate $\tau_{w,t}$ which is affected by an i.i.d shock defined by $1 - \tau_{w,t} = (1 - \tau_{w}^*) \varepsilon_w t$.

### A.3 Capital producers

Using investment goods, a segment of perfectly competitive firms, owned by households, produce a stock of fixed capital. At the beginning of period $t$, those firms buy back the depreciated capital stocks $(1 - \delta)K_{t-1}$ at real prices (in terms of consumption goods) $Q_t$. Then they augment the various stocks using distributed goods and facing adjustment costs. The augmented stocks are sold back to entrepreneurs at the end of the period at the same prices. The decision problem of capital stock producers is given by

$$\max \{ K_t, I_t \} \mathbb{E}_t \sum_{k=0}^{\infty} \Xi_{t+k} \left( Q_t + k \left( K_t + k - \frac{(1 - \delta)K_t + k - 1 - \gamma}{\gamma} \right) - I_t + k \right)$$

subject to the constraints:

$$K_t = (1 - \delta)K_{t-1} / \gamma + \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t$$

$S$ is a non-negative adjustment cost function formulated in terms of the gross rate of change in investment and $\varepsilon^*_t$ is an efficiency shock to the technology of fixed capital accumulation. The functional form adopted is $S(x) = \varphi / 2 (x - \gamma)^2$.

### A.4 Final and intermediate goods producers

Final producers are perfectly competitive firms producing an aggregate final good $Y_t$, expressed in effective terms, that may be used for consumption and investment. This production is obtained using a continuum of differentiated intermediate goods $Y_t(z), z \in [0, 1]$ (also expressed in effective terms) with the Kimball (1995) technology. Here again, the Kimball aggregator is defined by

$$\int_0^1 G \left( \frac{Y_t(z)}{Y_t}, \theta_p, \psi \right) dz = 1$$
with
\[
G\left(\frac{Y_t(z)}{Y_t}\right) = \frac{\theta_p}{(\theta_p(1 + \psi) - 1)} \left[ (1 + \psi) \frac{Y_t(z)}{Y_t} - \psi \right]^{\frac{\theta_p(1 + \psi) - 1}{\theta_p(1 + \psi) - 1}} - \frac{\theta_p}{(\theta_p(1 + \psi) - 1)}.
\]

The representative final good producer maximizes profits \(P_t Y_t - \int_0^1 P_t(z) Y_t(z) dz\) subject to the production function, taking as given the final good price \(P_t\) and the prices of all intermediate goods. In the intermediate goods sector, firms \(z \in [0, 1]\) are monopolistic competitors and produce differentiated products by using a common Cobb-Douglas technology:
\[
Y_t(z) = \varepsilon a_t (u_t K_{t-1}(z))^{\frac{\alpha}{\gamma}} N_D^a(z)^{1-\alpha} - \Omega (36)
\]
where \(\varepsilon a_t\) is an exogenous productivity shock, \(\Omega > 0\) is a fixed cost and \(\gamma\) is the trend technological growth rate. A firm hires its capital, \(\tilde{K}_t(z) = u_t K_{t-1}(z)\), and labor, \(N_D^a(z)\), on a competitive market by minimizing its production cost. Due to our assumptions on the labor market and the rental rate of capital, the real marginal cost is identical across producers. We introduce a time varying tax on firm’s revenue is affected by an i.i.d shock defined by \(1 - \tau_{p,t} = (1 - \tau_\star) \varepsilon p_t\). In each period, a firm faces a constant (across time and firms) probability \(1 - \alpha_p\) of being able to re-optimize its nominal price, say \(P_t^* (z)\). If a firm cannot re-optimize its price, the nominal price evolves according to the rule \(P_t(z) = \pi^\xi_{p,t} (1 - \xi_p) P_{t-1}(z)\), ie the nominal price is indexed on past inflation and steady state inflation. In our model, all firms that can re-optimize their price at time \(t\) choose the same level, denoted \(p_t^*\) in real terms.

### A.5 Market clearing conditions

The market clearing condition on goods market is given by:
\[
Y_t = C_t + I_t + G^* + \varepsilon (u_t K_{t-1}(z) - \gamma) + \mu_t \int_0^{\omega} \omega dF_e(\omega) K_{t-1}(z) (37)
\]
\[
\Delta_{p,k,t} Y_t = \varepsilon a_t (u_t K_{t-1}(z) - \gamma) (N_D^a(z))^{1-\alpha} - \Omega (38)
\]
with \(\Delta_{p,k,t}\) is a price dispersion index whose dynamics is presented in the appendix. Equilibrium in the labor market implies that
\[
\Delta_{w,k,t} N_D^a = N_S^a (39)
\]
with \(N_D^a = \int_0^1 N_D^a(z) dz\) and \(N_S^a = \int_0^1 N_S^a(h) dh\). The dynamics of the wage dispersion index \(\Delta_{w,k,t}\) is also described in the appendix.

### B DSGE estimation

#### B.1 Data

Data for GDP, consumption, investment, employment, wages and GDP-deflator are taken from Fagan et al. (2001) and Eurostat. Employment numbers replace hours. Consequently, as in Smets and Wouters (2005), hours are linked to the number of people employed \(e_t^*\) with the
following dynamics:
\[ c^*_{t+1} = \beta E c^*_{t+1} + \frac{(1 - \beta \lambda_e)(1 - \lambda_e)}{\lambda_e} (l^* - c^*_t) \]

The three-month money market rate is the three-month Euribor taken from the ECB website and we use backdated series for the period prior to 1999 based on national data sources. Data on MFI loans are taken from the ECB website. Data prior to September 1997 have been backdated based on national sources. Meanwhile, data on retail bank loan and deposit rates are based on official ECB statistics from January 2003 onwards and on ECB internal estimates based on national sources in the period before. The lending rates refer to new business rates. For the period prior to January 2003 the euro area aggregate series have been weighted using corresponding loan volumes (outstanding amounts) by country.

For the estimation, the data are transformed in the following way. We take the quarterly growth rate of GDP, consumption, investment and loans, all expressed in real terms and divided by working age population. The employment variable is also divided by working age population. Real wages are measured with respect to the consumption deflator. Interest rates and spreads are measured quarterly. With the exception of loan growth and employment rate for which specific trend developments are not pinned down by the model, transformed data are not demeaned as the model features non-zero steady state values for such variables. A set of parameters are therefore estimated to ensure enough degrees of freedom to account for the mean values of the observed variables. Trend productivity growth \( \gamma \) captures the common mean of GDP, consumption, investment and real wage growth; \( \Upsilon \) is a level shift that we allow between the observed detrended employment rate and the model-consistent one; \( \pi \) is the steady state inflation rate which controls for the mean of the inflation rate; and we also estimate the preference rate \( \beta \).

B.2 Calibrated parameters and prior distributions

Some parameters are treated as fixed in the estimation. The depreciation rate of the capital stock \( \delta \) is set at 0.025 and the share of government spending in output at 18%. The steady state labor market markup is fixed at 1.5 and we chose curvature parameters of the Kimball aggregators of 10. We fix in the steady state calibration the ratio of banks’ holdings of government bonds to their loan book, \( \alpha_B = \kappa g_B \kappa l_B \), at 12%, in line with aggregate BSI statistics from the ECB. The total outstanding amount of sovereign debt in the steady state is assumed at 60% of annual GDP. In order to calibrate and choose the prior distribution for the parameters in the financial block of the model, the steady state level of lending rate spreads \( RLLE - RLE \) can be decomposed in three financial wedges.

- First the credit risk compensation corresponds to the spread between the lending rate applied by loan officers and the return on the overall loan portfolio for the retail bankers:
  \[ r_{\text{risk}} = 100 \frac{RLLE - RLK}{RLK}. \]
leverage, intermediation spreads is given by the banker’s parameter space. In the steady state, equation (26) which links bank leverage to \(\xi\) terms). We also estimate the Calvo lottery parameter related to retail lending rate setting, \(\chi\) is a free parameter in the estimation and its prior distribution has a mean of 40 bps (in annual parameter \(\chi\)).

Starting with the entrepreneurs, we target default frequencies for firms of 0.7% and a credit risk compensation on lending rate of 50 bps (in annual terms) which broadly corresponds to one third of the sample mean of the lending spreads. The external finance premium \(100 \times \frac{\text{LE}}{\text{R}} - 1\) is set at 200 bps (in annual terms). We also aim at a matching a credit to GDP ratio consistent with the loan data under consideration. Four parameters are assigned to those targets: the monitoring costs \(\mu\), the standard deviation of the idiosyncratic shock \(\sigma\), the limited seizability parameter \(\chi\), and the entrepreneurs survival probability \(\zeta\). Then, the competitive margin \(r_d\) is a free parameter in the estimation and its prior distribution has a mean of 40 bps (in annual terms). We also estimate the Calvo lottery parameter related to retail lending rate setting, \(\xi\), for which we choose a relatively uninformative prior distribution. Let us now consider the banker’s parameter space. In the steady state, equation (20) which links bank leverage to intermediation spreads is given by

\[
\lambda_0\tilde{\kappa}_B/\zeta_b = \beta\gamma^{-\alpha} \left( \frac{r_B}{100}\tilde{\kappa}_B + \frac{r_d}{100} + \tilde{R}_B \right) \left( \lambda_0\tilde{\kappa}_B + (1 - \zeta_b) \right).
\]

Assuming a fixed ratio of government bonds to loans in bank balance sheet, \(\alpha_B\), then this relation can be re-written as

\[
\beta\gamma^{-\alpha} \left( \frac{r_B}{100} + \frac{r_d}{100(1 + \delta\alpha_B)} \right) = \frac{\lambda_0\tilde{\kappa}_B - \zeta_b}{\tilde{\kappa}_B \left( \zeta_b + \lambda_0\tilde{\kappa}_B \frac{\zeta_b}{\zeta_b + \zeta_b} \right)}.
\]

For given values of \(\lambda_0\) and \(\zeta_b\), intermediation spreads are a non-monotonic function of bank leverage, \(f_{\lambda_0,\zeta_b}(\tilde{\kappa}_B)\). Moreover, steady state levels for the intermediation spreads and bank leverage can be consistent with multiple combinations of \(\lambda_0\) and \(\zeta_b\). Therefore, in order to reduce the parameter space in the estimation and bring back monotonicity in this steady state relationship, we restrict the steady state allocations for values of \(\tilde{\kappa}_B(\lambda_0, \zeta_b)\) which maximize \(f_{\lambda_0,\zeta_b}(\tilde{\kappa}_B)\). This is the case for

\[
\lambda_0\tilde{\kappa}_B = \zeta_b + \sqrt{\zeta_b}
\]

implying intermediation spreads of

\[
\beta\gamma^{-\alpha} \left( \frac{r_B}{100} + \frac{r_d}{100(1 + \delta\alpha_B)} \right) = \frac{(1 - \zeta_b)}{\tilde{\kappa}_B \left( \zeta_b + \zeta_b \right)}.
\]

Under such constraints, the intermediation spread \(\frac{r_B}{100} + \frac{r_d}{100(1 + \delta\alpha_B)}\) is a decreasing function of bank leverage \(\tilde{\kappa}_B\) which depends only on \(\zeta_b\). Moreover, bank leverage and the survival probability of bankers determine uniquely the diversion rate parameter \(\lambda_0\). Then, in our calibration strategy, we set first \(\tilde{\kappa}_B\) at 8 (i.e. “weighted” capital ratio of 12.5%). Then we estimate \(\zeta_b\), choosing a prior mean which implies a bank capital channel spread \(r_d\) of around 50 bps (in annual terms). This is consistent with a prior value for \(\lambda_0\) of around 0.3. Finally, the steady state value of initial transfers to new bankers, \(\Psi_B\), is endogenously set so that the
bank net worth accumulation holds (see equation (11)).

From the first order conditions of bankers decision problem (9) and (10), we see that the steady state level of sovereign spread is linked to $r_B$ by

$$\delta_b \frac{(R_G - R_D)}{\pi} = \frac{r_B}{100}.$$  \hspace{1cm} (43)

We estimate $\delta_b$ using a prior distribution of mean 2. We set the geometric-decay of the perpetual coupons on sovereign bond $\tau_g$ so that the duration of the securities is 10 years. The initial coupon level is adjusted to ensure that the steady state sovereign bond price $Q_B$ equals 1. Regarding households’ portfolio decisions, the adjustment cost parameter on the holding of sovereign securities, $\chi_H$, is left free in the estimation, choosing a prior distribution of mean 0.1. For the household first order condition on sovereign bond holdings to be consistent with the steady state sovereign spread and the share of bank holding of sovereign bonds, we let $T_BH$ clear the steady state relationship associated with equation (1).

### C Optimal stabilisation without asset purchases and in the absence of lower bound on interest rate

This section complements section 5.2 in the main text and presents in greater details in the derivation policy efficiency curves without asset purchase and neglecting the incidence of the lower bound on interest rate. We run a grid of weights on inflation in the policymaker loss function and we plot in Figure 15, for every combination of weights, the theoretical second moments for inflation on the y-axis and the model-consistent output gap on the x-axis. We then repeat the same grid with three different values for the penalty on interest rate volatility. The resulting curves represent efficiency frontiers reflecting the optimal tradeoff between inflation and output gap stabilisation. Furthermore, point $E$ shows the corresponding macroeconomic standard deviations under the estimated Taylor rule.

As expected, the estimated Taylor rule delivers a macroeconomic performance sizably inferior to the optimal policy allocation: point $E$ in Figure 15 lies well into the convex area delimited by the efficiency curves. The optimal policy implementing the same inflation (output gap) volatility as the estimated rule would lead to almost twice (three times) lower standard deviation for the output gap (inflation). The Figure 15 also shows that increasing the penalty on interest rate volatility deteriorates the performance of optimal policy but the stabilisation costs in terms of output and inflation are limited across a wide range of policy preferences. Indeed, for very "hawkish" policymakers (at the extreme right of the efficiency curve), the highest interest rate penalty (see red doted curve labelled Case 2) increases output and inflation standard deviations by less then 0.1 pp and 0.05 pp respectively compared to the benchmark case (see black curve). For "dovish" policymakers however (towards the extreme left of the efficiency curve), the stabilisation costs become significant, and particularly so for output gap volatility.
D Technical implementation for different information sets regarding ad hoc programmes

In this section we present the technicalities behind the investigation of different information sets for the agents, i.e. the communication strategy of the central bank, as done in section 4.3. The full communication of the programme we have already outlined in Equation (30). For the sake of completeness, we start from this equation

$$\log(\epsilon^{QE}_t) = \rho^{QE} \log(\epsilon^{QE}_{t-1}) + \sum_{i=0}^T \nu^{QE,i}_t$$  \hspace{1cm} (44)

As becomes clear by looking at the last part of the equation, the sum represents captures announced purchases which will occur at a later point in time. In order to investigate the communication strategy of the central bank, we impose the true path for the stock of government bonds purchased by the central bank. However, we assume that agents believe in a different policy rule. This means for the unexpected frontloading that they know the offloading path, i.e. the AR1 coefficient but only see the current purchases. Consequently, we impose the policy rule

$$\log(\epsilon^{QE}_t) = \rho^{QE} \log(\epsilon^{QE}_{t-1}) + \tilde{\nu}^{QE}_t$$  \hspace{1cm} (45)
where $\tilde{\nu}_{t}$ represents the hypothetical shocks which produce the same path as under the fully communicated programme. Hence, private agents see only news every period. The completely unexpected programme is implemented accordingly. In this case the AR-term only drops out.

$$\log(q_{t}^{QE}) = \tilde{\nu}_{t}^{QE}$$ (46)

and $\tilde{\nu}_{t}^{QE}$ are the perceived shocks. In the same vein, the two programmes with unexpected frontloading and unexpected offloading are implemented. In order to investigate the unexpected frontloading, we start from Equation (35) and impose AR coefficients which deviate from those in Equation (30). For the positive (negative) surprises during offloading, the AR1 coefficient that the agents see is smaller (larger) than the actual one.
### Table 3: Parameter Estimates 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A priori beliefs</th>
<th>A posteriori beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dist.</td>
<td>Mean</td>
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<tr>
<td>$\sigma_c$</td>
<td>Intertemp. elasticity of subst.</td>
<td>gamma</td>
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<td>Habit formation</td>
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<td>$\varphi$</td>
<td>Cap. utilization adj. cost</td>
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<td>$\omega_s$</td>
<td>Calvo lottery, price setting</td>
<td>beta</td>
</tr>
<tr>
<td>$\xi_s$</td>
<td>Indecation, price setting</td>
<td>beta</td>
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<tr>
<td>$\omega_w$</td>
<td>Calvo lottery, wage setting</td>
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<tr>
<td>$\psi$</td>
<td>Calvo lottery, lending rate</td>
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<tr>
<td>$\delta$</td>
<td>Diversion rate for sov. Bonds</td>
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<tr>
<td>$\chi$</td>
<td>Portfolio adj. cost</td>
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<td>$\zeta$</td>
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<td>Taylor rule coef. on d(output)</td>
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$[L_1, L_2]$ is the shortest interval covering eighty percent of the posterior distribution.
Table 4: Parameter Estimates 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A priori beliefs</th>
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<td>ρ_f</td>
<td>AR(1) Inv. Technology</td>
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<td>η_o</td>
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<td>AR(1) Wage markup</td>
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<tr>
<td>ρ_z</td>
<td>AR(1) Entrepr. risk</td>
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**Standard deviation**

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<tr>
<td>ε^T_i</td>
<td>Preference</td>
<td>unif</td>
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<td>ε^T_o</td>
<td>Gov. spending</td>
<td>unif</td>
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<tr>
<td>ε^T_f</td>
<td>Inv. Technology</td>
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<tr>
<td>ε^T_p</td>
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<td>unif</td>
<td>0.25</td>
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<tr>
<td>ε^T_r</td>
<td>Price markup</td>
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<tr>
<td>ε^T_w</td>
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<tr>
<td>ε^T_{e^*}</td>
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<tr>
<td>ε^T_{b^*}</td>
<td>Bankers net worth</td>
<td>unif</td>
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<tr>
<td>ε^T_{Rb}</td>
<td>Gov. bond valuation</td>
<td>unif</td>
<td>5</td>
</tr>
</tbody>
</table>

P(Y) = -48.716

[I_1, I_2] is the shortest interval covering eighty percent of the posterior distribution.
Figure 16: Impulse Response Functions associated to a shock on $\epsilon_t^{QE}$ (black lines for the mode and grey shaded areas for the 90% shortest density interval) and to a shock on $\epsilon_t^r$ (blue dashed lines with circles for the mode and blue dashed lines for the 90% shortest density interval).
Acknowledgements

We would like to thank referees and seminar participants for stimulating and helpful comments. The views expressed are solely our own and do not necessarily reflect those of the European Central Bank or the Eurosystem. The paper was written while Michael Kühl was visiting the ECB.

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