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## The response of asset prices to monetary policy shocks: stronger than thought

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to monetary policy shocks:  
stronger than thought

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## Abstract

Mainstream macroeconomic theory predicts a rapid response of asset prices to monetary policy shocks, which conventional empirical models are unable to reproduce. We argue that this is due to a deficient information set: Forward-looking economic agents observe vastly more information than the handful of variables included in standard VAR models. Thus, small-scale VARs are likely to suffer from *nonfundamentalness* and yield biased results. We tackle this problem by estimating a Structural Factor Model for a large euro area dataset. We find quicker and larger effects of monetary policy shocks, consistent with mainstream theory and the observed large swings in asset prices. Our results point to stronger financial stability consequences of an exogenous monetary policy tightening, also in the form of a quicker than expected unwinding of QE, than commonly thought.

*Keywords:* Asset Prices, Monetary Policy, Structural Factor Models, Nonfundamentalness.

*JEL classification:* C32, E43, E44, E52.

## Non-technical summary

Particularly in the aftermath of the global financial crisis and the current low interest rate environment, it is of utmost importance to properly assess the relationship between monetary policy and asset prices. In this paper we contribute to the empirical literature on this topic by estimating a Structural Factor Model (SFM). Compared to vector autoregressions (VARs), the most widely used model in the empirical literature, this factor model takes into account a significantly larger information set. Instead of a handful of macroeconomic variables that VARs are able to handle, our SFM incorporates more than a hundred monthly variables, covering real activity, prices, surveys, financial markets, as well as the US economy.

The advantages of using a large-scale model are twofold. First, in standard small-scale VARs only very few asset classes can be investigated at the same time. This is due to the “curse of dimensionality”, i.e. the number of parameters one has to estimate increases substantially by adding further variables. Given that samples are typically relatively small in macroeconometric settings, a too large VAR may yield inaccurate estimates. Using a factor model, on the other hand, allows us to investigate a wide range of asset prices in a unified framework. In particular, we study stock and house prices, various exchange rates, as well as corporate bond yields of different rating classes. Secondly, in a data-rich environment we overcome the problem of *nonfundamentalness*, caused by a deficient information set: If the empirical model incorporates less information than that used by economic agents (e.g. central banks, households and firms), the model’s results may be invalid. This issue is of particular relevance in small-scale VARs, since they can only handle a handful of macroeconomic variables, while economic agents arguably base their decisions on a much wider information set. In the

SFM, due to its ability to model a larger amount of information, nonfundamentalness is not an issue. As a consequence, the SFM is able to identify shocks which small-scale VARs are unable to identify.

To underpin our empirical analysis, we provide several examples of how nonfundamentalness may arise in theoretical models concerning asset prices, mainly owing to the role of expectations. In these cases, a small-scale VAR is likely to suffer from an omitted variables problem and yield invalid results.

Indeed, conventional VARs routinely find delayed hump-shaped responses of asset prices to monetary policy shocks. This is at odds with basic economic theory. If asset prices equal their expected discounted payoffs, and monetary policy has a bearing on interest rates, we should expect asset prices to respond quickly, and potentially drastically, to monetary policy surprises. In the SFM, this is precisely what we observe. Compared to the literature, our results point to stronger effects of monetary policy shocks on asset prices across the board. The peak effects are often reached at impact or shortly after the policy surprise. In other words, asset prices respond more and more quickly to monetary policy shocks than commonly thought.

Finally, we find that monetary policy is much more important in explaining asset price movements compared to available evidence. The SFM is able to account for the observed large swings in asset prices, which standard VARs are not able to explain.

Our results imply stronger financial stability consequences of monetary policy decisions compared to the literature. Overall, our findings call for increased vigilance on the repercussions that a monetary policy tightening in whichever form - including a quicker than expected unwinding of unconventional measures - could have on financial markets.

# 1 Introduction

There are at least two reasons behind the economic profession's renewed interest in the relationship between monetary policy and asset prices. First, until the global financial crisis, it was rather controversial whether central bankers should look at asset price dynamics over and above their impact on inflation. "Leaning against the wind" of rising asset prices by tightening the monetary policy stance was not an explicit objective of central banks. However, the 2008-2009 financial crisis and the economic crisis that followed, have shown that "mopping up the mess" after the burst of an asset price bubble may entail huge costs, also in terms of price stability (Brunnermeier et al., 2009). The second reason is that an environment of prolonged low interest rates poses a concrete risk that some asset classes may more easily become overvalued and vulnerable to abrupt correction. Hence, it becomes even more important to understand the relationship between monetary policy and asset prices, in order to assess the financial stability consequences of a monetary policy shock (Allen and Gale, 2004; Disyatat, 2010).

Against this background, we provide evidence on the response of a set of asset prices to monetary policy shocks in the euro area. Several papers exist that attempt to estimate the impact of monetary policy shocks on asset prices, mostly on US data. However, in almost all of these papers, small-scale models are used. We build on this stream of literature by significantly enlarging the information set, well beyond the standard handful of macrofinancial variables. Moreover, we extend the set of examined asset prices and study stock and house prices, exchange rates, as well as corporate bond yields, distinguishing between high yield and investment grade.

In fact, we model more than a hundred time series by means of a Structural Factor Model (SFM, see Forni et al., 2009). By doing so, we are able to identify shocks

which cannot be correctly identified by conventional small-scale VAR models, as they are *nonfundamental* with respect to the limited information set those models are able to incorporate. Nonfundamentalness of the shocks in a macroeconometric setting can be linked to the role of expectations in the associated theoretical model. If agents behave according to their expectations, and these expectations are based on a larger information set than the one used in the empirical model, then this latter won't be able to recover the structural shocks. Asset prices are the prototypical example of an economic variable which is determined based on expectations. Moreover, the central bank monitors a very large set of indicators and sets the monetary policy stance based on this wide information set. For both of these reasons, as we show, the empirical results will differ depending on whether the empirical model includes all of the relevant variables or not.

Compared to the literature and our benchmark VAR model, the SFM points to stronger effects of monetary policy shocks on asset prices across the board. The impulse responses we estimate generally exhibit their peak effect shortly after the shock or even at impact. In other words, asset prices respond more and more quickly to monetary policy shocks than commonly thought, which implies stronger financial stability consequences of monetary policy decisions. Particularly in a low interest rate environment, our findings call for increased vigilance on the repercussions that a monetary policy tightening in whichever form - including a quicker than expected unwinding of unconventional measures - could have on financial markets.

The paper is structured as follows. In the following Section we provide a brief overview of the relevant literature. Section 3 presents three asset pricing models where the monetary policy shock turns out to be nonfundamental in a standard VAR setting. This provides a theoretical motivation for the empirical investigations that follow. Section 4 describes the Structural Factor Model and outlines its estimation, Section 5 de-

scribes the dataset, and Section 6 discusses the model parametrization and the identification strategy. In Section 7 we present the estimation results while Section 8 concludes and discusses policy implications.

## 2 Literature overview

The effects of monetary policy on asset prices, especially stock prices, have been the subject of extensive empirical research. This section provides a brief overview of the relevant literature, grouping papers according to their focus (i.e. asset class) and/or the selected estimation approach (e.g. different identification strategies in a VAR setting).

Among the papers using a recursive identification scheme in a VAR framework, Li et al. (2010) find that US stocks drop by 1% on impact and by up to 8% a year and a half after an unanticipated 50bp rise in the policy rate. For Canada, the impact effect is virtually zero and the trough of 1.5% occurs after 4 months. Studying eight advanced economies in a similar recursive identification scheme, Neri (2004) finds considerable cross-country heterogeneity in stock price responses. The peak effects are reached after 2-12 months with drops of up to 3%. In a recent study, Galí and Gambetti (2015) find an even smaller and short-lived effect of monetary policy shocks on stock prices: the magnitude of the decline is less than 1% and fades away after just four months.

These findings stand in stark contrast with non-VAR studies, especially those exploiting higher frequency data. For example, Rigobon and Sack (2004) use an estimator based on heteroscedasticity that exploits the increase in the variance of policy shocks on Federal Open Market Committee (FOMC) meetings. They find a 50bp increase of the policy rate to result in a 3.5% drop in stock prices on impact. In a similar study for the euro area, Bohl et al. (2008) estimate the effect to be 4.2%. Bernanke and Kuttner



(2005) find somewhat smaller effects in an event-study analysis for the US, identifying unanticipated policy rate changes based on futures contracts prices.

Also VAR models identified via long-run restrictions, in particular assuming long-run neutrality of money, tend to estimate larger effects of monetary policy shocks on equity prices than recursively-identified VARs. Various authors have found substantial immediate effects on stocks: Rapach (2001) for the US, Cassola and Morana (2004) for the euro area, and Lastrapes (1998) for eight advanced economies. Lastly, Bjørnland and Leitemo (2009) combine short- and long-run restrictions and find that stock prices drop by 4-5% on impact after a 50bp increase in the US monetary policy rate.

Studying house prices in Norway, Sweden and the UK, Bjørnland and Jacobsen (2010) employ a similar combination of short-run and long-run restrictions in a VAR model. They find an immediate and strong reaction to monetary policy shocks: House prices fall contemporaneously in all three economies by 1–2%. Iacoviello (2005) studies the dependency of house prices on monetary policy by developing a DSGE model and comparing its shock responses to those estimated by a recursively identified VAR. While his model predicts the peak effect to take place on impact, the empirical VAR response is hump-shaped with a negligible impact effect. The same inertia in house prices is found by Goodhart and Hofmann (2008) and Calza et al. (2013) for a panel of 17 and 19 advanced economies, respectively. While both studies employ a similar recursive identification scheme, the latter estimates VARs country-by-country whereas the former study uses a pooled panel framework. Importantly, factor models have also been used to investigate the evolution of house prices. Del Negro and Otrok (2007) use a factor model to extract a pervasive component of US house price dynamics and within an otherwise conventional VAR, this component drops significantly in response to a monetary policy tightening, with the peak effect observed on impact. Lastly, in a comparable framework

to ours, Luciani (2015) finds that US house prices drop significantly on impact and by about 1% three years after a 50bp increase in the policy rate.

Regarding exchange rates, small-scale VAR models often exhibit the “delayed overshooting puzzle” (Eichenbaum and Evans, 1995; Grilli and Roubini, 1996). That is, exchange rates tend to react with a long delay to monetary policy shocks and are merely affected on impact. Faust et al. (2003) argue that this is due to the strict recursiveness assumption. By exploiting high-frequency data, they can attenuate but not fully solve the puzzle. The delayed exchange rate reaction is, of course, inconsistent with mainstream theory which predicts an instantaneous appreciation of the domestic currency. This is precisely what Forni and Gambetti (2010), employing a factor model similar to ours, find for the US.

Finally, and somewhat surprisingly, the interaction between monetary policy and corporate bonds has received rather little consideration in the VAR literature. While most authors focus on government bond yields, Beckworth et al. (2012), employing long-run restrictions, find that corporate bond spreads react significantly to monetary policy shocks. However, the effect is negligible on impact and peaks only after 1-2 years, depending on the sample period. Even more puzzling, Gertler and Karadi (2015) report that with a recursive identification scheme, corporate bond spreads slightly decline after a contractionary monetary policy surprise.<sup>1</sup> The authors conclude that conventional recursive VARs are inept to study the effects of monetary policy on financial variables. Instead, they identify monetary policy surprises in a high-frequency approach and use them as external instruments in a VAR. This way, they find corporate bond yields to increase significantly after a contractionary monetary shock with the peak effect oc-

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<sup>1</sup>More precisely, Gertler and Karadi (2015) study “excess premiums”, i.e. the spread between yields on corporate and government securities (of similar maturity) that is left after accounting for default risk.

curing on impact.

In this paper we stick to a recursive identification scheme and instead argue that most of the puzzling empirical results are due to nonfundamentalness of the structural shocks in small-scale VARs. That is, small-scale VARs are not able to incorporate all relevant information to correctly identify monetary policy shocks. In fact, by enlarging the information set in our Structural Factor Model, the above mentioned puzzles largely disappear.

### 3 Theoretical models

In this section we offer theoretical underpinnings to the empirical model that we estimate in the remainder of the paper. To do so, we present three theoretical models where the monetary policy shock turns out to be nonfundamental, i.e. not identifiable by means of a conventional small-scale VAR. The gist of the argument relies on the role of expectations. In a nutshell, when agents form their expectations based on information which is not captured by the empirical model, or when the expectation formation process is non-standard, the structural shocks are nonfundamental (in the context of that particular model), i.e. cannot be correctly identified. In other words, the empirical model suffers from an omitted variables problem. Hence, it yields invalid results.

Technically, nonfundamentalness is linked to the roots of the determinant of the matrix in the moving average (MA) representation. In particular, given a covariance stationary vector process  $x_t$  and a white noise vector  $u_t$ , the representation  $x_t = A(L)u_t$  is fundamental if the determinant of  $A(z)$  has no roots of modulus less than unity.<sup>2</sup> If at least one root is inside the unit disc, the MA representation is not invertible *in the*

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<sup>2</sup>If at least one of the roots is equal to one, the process is fundamental, but nonstationary.

*past*. In other words, past information is not sufficient to recover the structural shocks, no matter how many lags are included in the VAR. The representation is only invertible *in the future*, i.e. the econometrician would need to know the future value of the observables in order to identify the shocks.<sup>3</sup> Fundamental and nonfundamental representations may be observationally equivalent and typically the nonfundamental case is ruled out by assumption. However, even simple theoretical models can be associated with nonfundamental MA representations. While three simple examples are described in this section, Fernández-Villaverde et al. (2007) provide a general check for whether the shocks in an economic model are fundamental in a VAR framework.

### 3.1 Nonfundamentalness due to lagged shock effects

The first theoretical model we discuss is the textbook model for stock prices. In this model, the equilibrium value for stock prices  $p_t$  is equal to the discounted sum of expected dividends,  $d_t$ , as follows:

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t d_{t+j} \quad \text{with} \quad 0 < \beta < 1 \quad (1)$$

The dynamics for the dividends can be assumed as follows:

$$d_t = d_{t-1} + u_{t-k} + \eta_t \quad (2)$$

with  $u_t$  and  $\eta_t$  being disturbances. In particular,  $u_t$  is a shock that has only an effect on dividends with a  $k$ -period lag, though being observed at time  $t$  by agents. This implies that agents will adapt their behavior already at time  $t$ , as they anticipate the future effects

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<sup>3</sup>See Alessi et al. (2011) for a review of the literature on nonfundamentalness and identification in structural econometric models.

of  $u_t$  on dividends already at time  $t$ . In other words,  $u_t$  enters the information set on which agents form their expectations for  $d_{t+k}$ . However, the econometrician will only be able to see the impact of  $u_t$  at time  $t + k$ .

A type of shock having a lagged effect on the observables offers a wide range of interpretations. It could, for instance, be a technological innovation which takes time in order to translate into productivity gains. Alternatively, it could be a shock affecting government spending and taxation, which are both planned in advance with the budget law for the following year. Furthermore, it could be a shock related to the introduction of a new regulatory regime affecting particular sectors of the economy. A recent example would be the capital requirements regulation for financial institutions entering into force in a gradual manner (Brunnermeier et al., 2009). We focus on the case in which  $u_t$  is interpreted as a monetary policy shock. Indeed, especially in recent times with the activation of unconventional measures, announcements of future monetary policy have become more common than in the past. For example, when the ECB launched a series of targeted longer-term refinancing operations (TLTROs) in June 2014, it announced that these operations would be carried out over a window of two years. Arguably, the bulk of the effects unwind at the time each of the TLTROs takes place, is contingent on its take-up, and is reflected into short-term rates. However, the announcement of the TLTROs, together with information on their calendar, is a monetary policy shock in itself, which may not be fully captured by the short-term rate instantaneously but still affects the economy at large by influencing agents' expectations. Moreover, on the same occasion when it launched TLTROs, the ECB announced that it would "intensify preparatory work related to outright purchases of asset-backed securities (ABSs)". Again, this is a monetary policy shock linked to a nonstandard monetary policy measure, which already has an impact before being implemented, insofar agents already have an expectation on

it. As another example, since July 2013 the ECB has been providing forward guidance on the future path of policy rates (conditional on the inflation outlook). This induces expectations on the side of the markets, which will adjust accordingly. However, an econometrician will come to the wrong conclusions when trying to retrieve the underlying monetary policy shocks, if she only observes the actual level of policy rates and ignores the information coming from the ECB forward guidance.

In an attempt to address this problem, in the empirical analysis we replace the policy rate - which is constrained by the zero lower bound - with a shadow rate that exploits information on the whole term structure. This latter rate in principle incorporates the effects of unconventional policy measures. Though this rate is more appropriate than a constrained short-term rate, it is susceptible to debate if it fully captures the conditional character of the monetary policy measures. More generally, it may still be an imperfect measure of the monetary policy stance and it may fail to fully capture agents' expectations. Indeed, as we show, including the shadow rate in a small-scale SVAR is not enough to solve the issue of nonfundamentalness.

Going back to the model's equations, substituting the expression for  $E_t d_{t+j}$  into (2) yields:

$$p_t = \frac{\beta}{1-\beta} d_t + \frac{\beta}{1-\beta} [\beta^{k-1} u_t + \beta^{k-2} u_{t-1} + \dots + \beta u_{t-k} + u_{t-k+1}] \quad (3)$$

The structural moving average representation of this model is the following:

$$\begin{bmatrix} \Delta d_t \\ \Delta p_t \end{bmatrix} = \underbrace{\begin{bmatrix} L^k & 1 \\ \frac{\beta^k}{1-\beta} + \sum_{i=1}^{k-1} \beta^{k-1} L^i & \frac{\beta}{1-\beta} \end{bmatrix}}_{A(z)} \begin{bmatrix} u_t \\ \eta_t \end{bmatrix} \quad (4)$$

The determinant of matrix  $A(z)$  is given by the following expression:

$$\det(A) = -\frac{\beta^k}{1-\beta} - \sum_{i=1}^{k-1} \beta^{k-i} L^i + \frac{\beta}{1-\beta} L^k \quad (5)$$

Finding the zeros of this expression requires solving a  $k$ -th order equation. For the simple case where  $k = 2$ , the determinant vanishes for  $L = 1$  and  $L = -\beta$  (see Forni et al., 2014). Hence, one of the roots is inside the unit circle, given that  $\beta$  is a discount factor, i.e. in modulus smaller than one.

### 3.2 Nonfundamentalness due to unobserved variables

Another case in which nonfundamentalness can arise is owing to the presence of unobserved factors driving asset price dynamics. The following simple model by Hansen and Sargent (1991) exemplifies the issue. More generally than in the previous case, let's assume that a set of economic variables including asset prices,  $z$ , depend on the future path of a broad set of unobserved factors  $\omega_t$ :

$$z_t = \sum_{j=0}^{\infty} \beta^j E_t \omega_{t+j} \quad \text{with} \quad 0 < \beta < 1 \quad (6)$$

The relevant case for our exercise is when the unobserved factors are linked to monetary policy. For example, they might be related to the functioning of monetary policy transmission channels, e.g. linked to credit supply and banks' risk-taking. The strength of such channels plays a key role in determining the effectiveness of monetary policy, including its impact on asset prices, in particular in an environment characterized by persistently low interest rates

Assume the following simple dynamics for  $\omega_t$ :

$$\omega_t = u_t - \theta u_{t-1} \quad (7)$$

where  $u_t$  is the monetary policy shock. Substituting (7) into (6) yields:

$$z_t = (1 - \beta\theta)u_t - \theta u_{t-1} = A(L)u_t \quad (8)$$

The only root of  $A(z)$  is  $(1 - \beta\theta)/\theta$  which can be inside the unit circle. If this is the case, the information contained in  $z_t$  is not enough to recover the true structural shock  $u_t$ . Notice that this argument holds also in the context of standard monetary policy as we used to know it, and is complementary to the case of nonstandard monetary policy measures, previously discussed. In fact, in order to gauge the pass-through of monetary policy measures, central banks monitor literally hundreds of indicators, including on financial markets, macroeconomic developments, domestic and global economic dynamics, etc. If this wealth of information is not taken into account by the econometric model, it will not be able to correctly identify monetary policy shocks.

The well-known “price puzzle” is an obvious example. Conventional VAR models typically find that an unexpected interest rate rise increases inflation, a finding directly at odds with macroeconomic theory. Many authors have argued that this puzzling observation is due to a deficient information set used in the VAR. That is, central banks may have information about future inflation, so the response represents in fact reverse causality (monetary policy is tightened today because inflation is likely to increase tomorrow). Since the econometric model typically does not include measures of expected inflation, it suffers from nonfundamentalness and yields invalid results.<sup>4</sup>

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<sup>4</sup>By including commodity prices in the VAR (supposedly capturing information about future inflation),



### 3.3 Nonfundamentalness due to informational heterogeneity

Nonfundamentalness can also be linked to the existence of a heterogeneous beliefs equilibrium. Indeed, in order for a non-revealing equilibrium to exist, the model must be such that agents cannot infer private information in equilibrium, that is, they cannot identify structural shocks based on observations. In other words, agents can remain heterogeneously informed in equilibrium only if the model has a nonfundamental MA representation. An asset pricing model with persistent heterogeneous beliefs is developed by Kasa et al. (2014), of which what follows is a simplified version. Assume that fundamentals  $f_t$  are the sum of two orthogonal serially uncorrelated components:

$$f_t = \sum_{i=1}^m a_i(L)u_{it} \quad i = 1, 2.$$

and that the price  $p_t$  is a function of fundamentals  $f_t$  and is also influenced by noise,  $\epsilon_t$ . Heterogeneous symmetric information is modeled by assuming two types of agents. Each agent type observes the price  $p_t$ , the fundamentals  $f_t$  and just one shock  $u_{it}$ . The corresponding MA representation for type 1 traders is the following:

$$\begin{bmatrix} u_{1t} \\ f_t \\ p_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ a_1(L) & a_2(L) & 0 \\ \pi_1(L) & \pi_2(L) & \pi_3(L) \end{bmatrix}}_{A_1(z)} \begin{bmatrix} u_{1t} \\ u_{2t} \\ \epsilon_t \end{bmatrix},$$

where the polynomials  $\pi_1(z), \pi_2(z), \pi_3(z)$  are pricing functions. The determinant of matrix  $A_1(z)$  is equal to  $a_2(z)\pi_3(z)$ . For the symmetric case of type 2 agents, the determinant of matrix  $A_2(z)$  is equal to  $a_1(z)\pi_3(z)$ . If we assume for simplicity that the prize puzzle can be attenuated but not fully resolved (see Christiano et al., 1999).

$a_1(z) = a_2(z) = a(z)$ , requiring this MA representation to be nonfundamental implies that either  $a(z)$  or  $\pi_3(z)$  or both have at least one zero inside the unit circle. Given a non-fundamental representation, a non-revealing equilibrium exists because what agents do in equilibrium is to attempt to retrieve the structural shocks by estimating a VAR, hence assuming the corresponding fundamental representation. As a consequence, a problem of identification arises, as agents' information on one type of shock, on the price and on the fundamentals is not enough to retrieve both structural shocks  $u_{1t}$  and  $u_{2t}$ . As shown in Kasa et al. (2014), the characteristics of the heterogeneous beliefs equilibrium are such that agents overreact to public signals, i.e. partially-informative prices. This feature of the model accounts for the excess volatility empirically observed in asset prices and not explained by the linear present value model. Indeed, the higher-order beliefs ('forecasting the forecasts of others') implied by a non-revealing equilibrium generate persistence and remarkably higher values for the impulse response functions at short horizons, compared to the full information model. Finally, the authors show that by introducing nonfundamental representations it is possible to explain asset prices systematic violation of the standard variance bounds, which are consistent with the idea that prices should be smoother than expected discounted fundamentals.

## 4 The empirical model

### 4.1 The Structural Factor Model

Forni et al. (2009) show that by enlarging the space of observations one can solve the problem of nonfundamentality (see also Giannone and Reichlin, 2006). Indeed, non-

fundamentalness is not an issue for models which are able to handle very large panels of related time series. In particular, nonfundamentalness is nongeneric in the framework of dynamic factor models, i.e. it occurs with probability zero for  $N \rightarrow \infty$ , with  $N$  being the number of series included in the model.<sup>5</sup> We estimate the Structural Factor Model (SFM) by Forni et al. (2009), which in turn is a special case of the model in Forni and Lippi (2001) and Forni et al. (2005). We refer to these papers for a detailed description of the assumptions of the model, and limit ourselves to outlining the main features.

Denote by  $\mathbf{x}$  a panel of  $n$  stationary time series, where both the  $n$  and  $T$  dimensions are very large. In a factor model, each variable  $x_{it}$  is assumed to be the sum of two unobservable components: the common component  $\chi_{it}$  and the idiosyncratic component  $\xi_{it}$ . An important feature of this specific factor model and the closely related models by Stock and Watson (2002) and Bai (2003) is that the idiosyncratic components are allowed to be mildly cross-correlated (i.e. the factor model is *approximate*, as opposed to *exact*). The common component is assumed to be driven by  $q$  shocks  $\mathbf{u}_t = (u_{1t} \dots u_{qt})'$  which affect all variables in the panel, also referred to as *dynamic* common factors, with  $q \ll n$ . These are the structural shocks we aim at identifying. Formally:

$$\mathbf{x}_t = \boldsymbol{\chi}_t + \boldsymbol{\xi}_t = \mathbf{B}(L)\mathbf{u}_t + \boldsymbol{\xi}_t, \quad (9)$$

where  $\boldsymbol{\chi}_t = (\chi_{1t} \dots \chi_{nt})'$ ,  $\boldsymbol{\xi}_t = (\xi_{1t} \dots \xi_{nt})'$ , and  $\mathbf{B}(L)$  is a one-sided  $n \times q$  filter. Eq. 9 is called *dynamic representation* of the factor model. An alternative representation, which is called *static representation*, is the following:

$$\mathbf{x}_t = \boldsymbol{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t. \quad (10)$$

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<sup>5</sup>Lütkepohl (2012) also views nonfundamentalness as an omitted variables problem and discusses the conditions under which factor models can be a possible response.

where the  $r > q$  entries of  $\mathbf{F}_t$  are the *static* common factors, and  $\mathbf{\Lambda}$  is the  $n \times r$  matrix of factor loadings.

The link between the two representations is given by defining the  $r \times 1$  vector of the static common factors in terms of the shocks, as follows:

$$\mathbf{F}_t = \mathbf{N}(L)\mathbf{H}\mathbf{u}_t \quad (11)$$

where  $\mathbf{N}(L)$  is an  $r \times r$  matrix polynomial and  $\mathbf{H}$  is a maximum rank  $r \times q$  matrix.

Finally, it is assumed that  $\mathbf{N}(L)$  results from inversion of the VAR( $m$ )  $\mathbf{F}_t = (\mathbf{I}_r - \mathbf{A}L - \dots - \mathbf{A}_m L^m)^{-1} \boldsymbol{\epsilon}_t$ . For simplicity, we assume  $m = 1$ , so that  $\mathbf{N}(L) = (\mathbf{I}_r - \mathbf{A}L)^{-1}$ , where  $\mathbf{I}_r$  is the  $r$ -dimensional identity matrix, and  $\mathbf{A}$  is an  $r \times r$  matrix. Notice that  $\boldsymbol{\epsilon}_t = \mathbf{H}\mathbf{u}_t$ , i.e. the residuals of the VAR on the static factors have reduced rank  $q$ . More precisely,  $\boldsymbol{\epsilon}_t \in \overline{\text{span}}\{\mathbf{u}_t\}$ , i.e. the residuals belong to a  $q$ -dimensional linear space generated by the dynamic factors. Notice also that these latter, as well as the static common factors, are only identified up to a rotation.

## 4.2 The estimation procedure

The estimation of the SFM is based on Giannone et al. (2004) and Forni et al. (2009). We make use of the static representation (10) together with the VAR(1) specification of the static factors:

$$\mathbf{x}_t = \mathbf{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t, \quad (12)$$

$$\mathbf{F}_t = \mathbf{A}\mathbf{F}_{t-1} + \boldsymbol{\epsilon}_t, \quad \text{with } \boldsymbol{\epsilon}_t = \mathbf{H}\mathbf{u}_t. \quad (13)$$

This state-space representation is equivalent to the dynamic representation (9), with filters defined as

$$\mathbf{B}(L) = \mathbf{\Lambda}(\mathbf{I}_r - \mathbf{A}L)^{-1}\mathbf{H}. \quad (14)$$

Before estimating (12)-(13), the number of dynamic factors  $q$  and the number of static factors  $r$  have to be determined (see Section 6).

The estimation procedure is in four steps.

STEP 1 Given a consistent estimator of the covariance matrix  $\widehat{\mathbf{\Gamma}}_0^x$ , the static factors  $\mathbf{F}_t$  are consistently estimated as the  $r$  largest principal components. This yields also a consistent estimate of the loadings  $\mathbf{\Lambda}$ . We extract the principal components from the panel in levels, following Bai (2004) who shows that given a large-dimension factor model with nonstationary dynamic factors, the common component of I(1) time-series can be consistently estimated via principal components if the idiosyncratic components are stationary.<sup>6</sup> We test this assumption by three different panel unit root tests, namely Maddala and Wu (1999), Choi (2001) and Levin et al. (2002), which all clearly reject the unit root hypothesis.

STEP 2 A VAR is estimated on the estimated static factors, as in (13). The static factors come from the previous step, hence they are in levels. Though estimating a structural dynamic factor model in levels is rather uncommon in the literature, it is a legitimate approach in terms of consistency of the parameter estimates and validity of the impulse responses. In fact, if there are cointegration relationships in the panel, it is crucial to estimate the model in levels. As is well known from the VAR literature, a model in differences would be misspecified under these circumstances. Barigozzi et al. (2016) show that cointegration relationships are likely to be present in the context of

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<sup>6</sup>See also Peña and Poncela (2006).

factor models, hence they argue in favor of estimating a VAR on the static factors in levels in this step. The ability of taking into account long-run relationships is a strong argument in favor of estimating a factor model in levels if the focus is on asset prices. The reason being that the financial cycle has a much lower frequency than the business cycle, hence the focus needs to be on the medium-to-long term (see e.g. Drehmann and Tsatsaronis (2012) for a general characterization of the financial cycle and Stremmel (2015) specifically on the European financial cycle). In practice, differencing the data in our application would imply eliminating the boom-bust cycle in asset prices which started in 2000.

STEP 3 Since the estimated residuals  $\hat{\epsilon}_t$  have reduced rank, as they belong to the space spanned by the  $q$  dynamic factors, the principal components of the residuals can be used to obtain a consistent estimate of the dynamic factors.

STEP 4 The dynamic factors are identified only up to a static rotation. Hence, in order to interpret them as structural shocks, a specific rotation matrix needs to be selected. Formally, the identified structural shocks  $\mathbf{v}_t$  are identified as follows:

$$\mathbf{v}_t = \mathbf{R}'\mathbf{u}_t \quad (15)$$

with  $\mathbf{R}$  being a  $q \times q$  unitary matrix, i.e.  $\mathbf{R}\mathbf{R}' = \mathbf{I}_q$ .

The entries of the rotation matrix  $\mathbf{R}$  are restricted by means of standard techniques used in the Structural VAR literature, i.e. in our case a recursive Cholesky identification (see Section 6).

## 5 The data

Our dataset contains 127 macroeconomic series for the euro area, covering real activity measures, prices, forward-looking surveys, and importantly numerous financial sector variables. The time dimension of the sample ranges from April 2000 to June 2015, i.e. it covers 182 monthly values per series. A few series only available at a quarterly frequency have been interpolated with the Chow-Lin procedure, using the first 9 principal components of the monthly dataset as high-frequency regressors. Nominal series have been deflated by the HICP index, while GDP and its components refer to volume indices. As mentioned previously, most variables are kept in (log)-levels to avoid a loss of information. Table A1 in the Appendix lists all variables in the dataset along with the applied transformations.

To account for the fact that monetary policy has been subject to the zero lower bound in our sample period, we splice the EONIA rate (monthly average) with the shadow rate by Wu and Xia (2016). This series summarizes the stance of monetary policy based on information from the entire yield curve. More precisely, it is estimated as the policy rate that would generate the observed yield curve in the absence of a lower bound. The underlying model is an affine term structure model with three factors, where the short-term interest rate is the maximum of the shadow rate and a lower bound, which is set at 0.25%. This series is available from September 2004 onwards and closely resembles the EONIA rate prior to the global financial crisis. The two series substantially diverge in late 2008 with the implementation of the ECB unconventional policy measures and the zero lower bound becoming an increasingly binding constraint for conventional interest rate policy (see Figure A1 in the Appendix for a comparison of both series). The use of shadow rates in macroeconomic models, including factor models, is not novel in

the literature. In particular, Wu and Xia (2016) themselves put forward an application where their shadow rate is included in a Factor-Augmented VAR and used to identify monetary policy shocks. They also suggest their shadow rate could replace the short-term rate in any structural VAR. Other applications, also for the euro area, are in Chen and Zhu (2015) and Damjanović and Masten (2016).

Together with the short-term rate, other key variables in our study relate to asset prices. In particular, we focus on house prices and financial asset prices, namely stocks, corporate bonds - high-yield and investment grade - and currencies. Financial asset prices have frequently experienced large swings, often coinciding with the announcement of key monetary policy decisions. Figure A2 in the Appendix plots stock prices, various bond yields and exchange rates, indicating the timing of relevant monetary policy measures. While acknowledging that this very rough event-study does not permit to draw any conclusions on the relevance of monetary policy for asset price developments, some observations appear reasonable. For example, when the ECB increased the reference rate by 25 basis points in July 2008 amid concerns that inflation expectations could rise, the Eurostoxx went down by 9% compared to June, which is almost twice as much as its standard deviation. Of course, the negative stock market performance has to be interpreted in the context of the market uncertainty at that time, especially rising concerns of a global recession. However, this is precisely why the contemporaneous monetary tightening might have come as unexpected at least to some. A few months later, in October 2008, the ECB cut its policy rate by 50 basis points and introduced a fixed-rate full allotment policy in all its refinancing operations. In that month the Euro exchange rate went down by 7.3% against the US Dollar and the Chinese Yuan and 12.8%, against the Japanese Yen. Given a standard deviation of these exchange rates below 3%, these movements appear particularly large and one could partly link them to the contempora-



neous expansionary monetary policy decision, alongside the general instability caused by the Lehman bankruptcy one month earlier. As another example, when the Securities Markets Programme (SMP) was launched in May 2010, the exchange rate decreased by around 7% against the major currencies.

## 6 Model parametrization and identification strategy

Determining the number of factors is a crucial model selection step. In particular, the number of dynamic factors included in the model corresponds to the number of shocks driving economic and financial developments, and has therefore an important structural interpretation. For determining the number  $q$  of structural shocks, we apply the criteria available in the literature. However, it should be noticed that, to our knowledge, no criterion exists in the literature for determining the number of dynamic common factors in a nonstationary setting. The tests give different results. In particular, the test by Hallin and Liška (2007) in different specifications points to numbers between 3 and 6, the test by Bai and Ng (2007) yields 4 as a result in most of its specifications and the criterion by Amengual and Watson (2007) is clearly out of range, pointing to  $q = 9$ . Indeed, it is difficult to imagine that the primitive shocks, source of business cycle fluctuations, are so many. Finally, on our dataset the test by Onatski (2009) always points to  $q = q_{max}$ , where  $q_{max}$  is a pre-defined parameter indicating the maximum value allowed for  $q$ . In line with the literature and with the evidence from statistical criteria, we set the number of dynamic factors as  $q = 4$ .

As for the number of static common factors, it should be noted that also most of the available tests for determining  $r$  are designed for a stationary setting. Thus, they may not

work as expected on a nonstationary panel.<sup>7</sup> We apply the criterion suggested by Alessi et al. (2010), which points, in all its specifications, to a number of static factors equal to 7.<sup>8</sup> However, we should take into account that our non-stationary dataset displays features which this criterion is not designed to handle, in particular linear time trends and unit roots. Hence, to be on the safe side and sure that enough variance is captured, we set  $r = 9$  in our benchmark specification.<sup>9</sup>

The first principal component, indeed, corresponds to a linear time trend. Naturally, many variables in the data set load heavily on it.<sup>10</sup> Real activity measures also load heavily on the second factor, while surveys, stock prices and corporate bond yields mainly load on the third factor. Factors 4-6 are important for some US and further financial series, e.g. sovereign bond yields. Lastly, price and real activity measures for the US load heavily on factors 7-9. However, it is useful to recall that principal components, or static common factors, do not have an economic interpretation. Principal components are just statistical constructs, while economic meaning is attached only to the dynamic common factors, or structural shocks, identified by suitably rotating the VAR residuals.

As a robustness test, we estimate the model for various specifications of  $r$ . The results are shown in Figure A5 in the Appendix. This exercise indicates that the model is robust for  $r \geq 9$ . When  $r < 9$ , on the contrary, some unreasonable results emerge, e.g. the well-known “price puzzle”. This is consistent with the findings in Forni and

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<sup>7</sup>As shown by Corona et al. (2016), inducing stationarity by differencing the series, in order to apply a criterion designed for the stationary case, may not be a solution under some circumstances. Namely, if only the common factors are nonstationary but the idiosyncratic component are not, differencing the series may change the relative weight of the two components in terms of variance.

<sup>8</sup>This criterion is a refinement of the widely used criterion proposed by Bai and Ng (2002), which does not converge on our data.

<sup>9</sup>Table A2 in the Appendix displays the percentage of variance explained by the first 10 principal components for the key variables in the dataset.

<sup>10</sup>Figures A3 and A4 in the Appendix plot the static factors and the loadings of the 127 macro variables on those factors, respectively.

Gambetti (2010), who stress the importance of including a sufficient number of static factors.

To ensure comparability between our Structural Factor Model and the benchmark VAR, both models are estimated with a lag length of  $p = 1$ , as suggested by the BIC. The shock size is set to a 50bp rise in the short-term rate.

Following Forni and Gambetti (2010), we apply the same Cholesky identification to both models. The monetary policy shock is identified by imposing a standard recursive scheme on industrial production, the consumer price index, the monetary policy rate, and various asset prices (cf. Eichenbaum and Evans, 1995). Our ordering implies that monetary policy reacts contemporaneously (i.e. within the same month) to shocks in real activity and consumer prices, while the opposite is not true. Asset prices, on the other hand, are allowed to react immediately to monetary policy shocks. In particular, the benchmark VAR includes five asset prices: house and stock prices, high-yield and investment-grade corporate bonds, and the USD/EUR exchange rate.

Confidence intervals are based on the following two-step bootstrap procedure. First, we generate artificial common components by reshuffling the shocks and applying filters to them, given by the impulse responses. In the second step, we adopt a standard non-overlapping block bootstrap technique for the idiosyncratic parts. In particular, we partition the idiosyncratic component into 5-year blocks. We then add the bootstrapped idiosyncratic component to the bootstrapped common components to obtain simulated data sets. We perform 1000 bootstrap repetitions, applying the bias correction proposed by Kilian (1998). For each artificial sample we repeat the estimation and obtain non-structural impulse responses, which are then identified by imposing our identification assumptions.

## 7 Results

Let us now examine the responses of asset prices to a contractionary monetary policy shock. The difference in the responses between the benchmark VAR and the SFM is striking.

As shown in Figure 1, for stock prices the SFM suggests a rather rapid decline of roughly 3% on impact and a maximum decline of about 4% after four months. In the VAR, the stock price response peaks not until after one year, with the decline of only 2% being far from significant. House prices merely react at all to a contractionary shock in the VAR model. If anything, they slightly increase in the longer run. In the SFM, on the other hand, house prices drop on impact by about 1% and return to their previous level within 1-2 years. The last two rows of Figure 1 show the response of high-yield and investment-grade corporate bonds in the euro area. Again, we observe roughly the same pattern. The peak effect on both yield classes is 2-3 times bigger and reached much quicker in the SFM. In line with the financial accelerator and credit channel literature, the effect is multiple times higher for high-yield debt than for more credit-worthy borrowers in both frameworks. As for the exchange rate, the simple VAR model suggests a rather small and smooth response of the USD/EUR exchange rate, while in the SFM it spikes immediately with an appreciation of about 4%, four times stronger than in the VAR.<sup>11</sup>

A major advantage of the SFM approach is the fact that impulse responses can be estimated for any of the 127 series in the underlying dataset. This yields more informa-

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<sup>11</sup>While not the main focus of our paper, Figure A6 in the Appendix provides an analogous comparison for the two remaining variables in the VAR model, i.e. industrial production and consumer prices. As expected, both variables exhibit a negative temporary effect in the SFM. The VAR results, on the other hand, are hard to square with economic theory. They imply a rather permanent negative effect on the price level and, even more implausible, an expansionary effect on output. Note that adding more lags to the VAR model does not solve this issue.

tion and allows a more comprehensive check of the empirical plausibility of the model. Figure 2, for instance, depicts the impulse response functions of six additional exchange rates. All responses indicate a universal Euro appreciation with the peak effect occurring on impact and fading out rather quickly. The responses also exhibit some interesting heterogeneity. The appreciation is strongest relative to the Japanese Yen (4%, similar to the USD) and weakest for the Swiss Franc (roughly 1.5%). The latter finding might be explained by the exchange-rate peg maintained by the Swiss National Bank between 2011 and early 2015. Admittedly, the identification of the effects of ECB monetary policy on the euro/CHF exchange rate is made more challenging also owing to the fact that the Swiss National Bank announced the end of the peg in January 2015, i.e. in the same month when the ECB announced the public sector purchase programme. Furthermore, the effect on the British Pound exchange rate is by far the most persistent, turning insignificant not until a year after the shock.

Figure 3 plots the impulse response functions of a number of other important macro-financial variables. Long-term government bond yields, for instance, rise after a monetary policy shock, but the effect is smaller and more short-lived than for corporate bonds. The three month EURIBOR resembles the monetary policy rate, but with a more pronounced downturn after the initial spike. Notably, expected inflation also drops after a contractionary monetary policy shock. The effect is maximum after 8 months but is already significant on impact.<sup>12</sup> Lastly, two measures of financial stress, namely the composite indicator of systemic stress (CISS, see Hollo et al., 2012) and the volatility index of stock prices, also increase considerably after an unexpected monetary tightening. The peak effects are reached after 3 months and on impact, respectively.

To wrap up, the SFM impulse responses are - in contrast to those of the benchmark

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<sup>12</sup>Notice that the response of this series refers to the expected annual *growth rate* of the HICP.

VAR - largely consistent with basic economic theory. They are also in line with recent empirical evidence employing innovative identification approaches, cf. Section 2: First, the rapid drop in stock prices is remarkably close to the estimates of Rigobon and Sack (2004) and Bohl et al. (2008). The large immediate exchange rate appreciation is in line with evidence for the US provided by Forni and Gambetti (2010): the factor model solves the delayed overshooting puzzle within a recursive identification approach. As for corporate bonds, we confirm the finding in Gertler and Karadi (2015) that the peak effect of a policy tightening on corporate bonds occurs on impact.<sup>13</sup> The significant drop in house prices, lastly, is in line with the results of Del Negro and Otrok (2007) and Luciani (2015).

Another method to compare the importance of monetary policy for asset prices between the two empirical models is via forecast error variance decomposition. As shown in Table 1, VAR results indicate a substantial bearing of monetary policy on real activity and consumer prices, but very little relevance for asset prices. Regarding the latter, less than 10% of the variance is accounted for by monetary policy shocks across the board and at all horizons. The SFM, on the contrary, suggests a major role for monetary policy in affecting asset prices, particularly in the short-term. Starting with stock prices, the SFM indicates that the share of forecast error variance explained by monetary policy shocks at a six month horizon is larger than 16%, against less than 2% according to the VAR. The discrepancy is even more pronounced for house prices, where the figures are 35.6% and 1.6%, respectively. Regarding the two investigated corporate bond yields, the VAR suggests merely 3-5% of their variance is accounted for by monetary policy shocks over a four-year horizon, compared to 20-23% according to the SFM. At shorter

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<sup>13</sup>The magnitude of the effects is not directly comparable since we investigate yields, while Gertler and Karadi (2015) focus on “excess premiums”, cf. Footnote 1.

horizons, the figures differ depending on the bond issuers' creditworthiness: for high-yield bonds, the SFM suggests that 10% of the forecast error variance can be attributed to simultaneous surprises in the policy rate, whereas the fraction for investment-grade bonds is as high as 52%. As for the USD/EUR exchange rate, the SFM indicates that the percentage of variance explained by monetary policy shocks on impact is roughly six times larger (35%) than based on the VAR (6%). Again, these findings are consistent with Forni and Gambetti (2010). Finally, as for other exchange rates, results based on the SFM show some degree of heterogeneity. On impact, for example, the FEVD figures range from roughly 6% for the Swiss Franc up to almost 60% for the British Pound. Except for the Swiss Franc, the importance of policy rate shocks for all currencies reduces at longer horizons.

## 8 Conclusions

Particularly in the aftermath of the global financial crisis and the current low interest rate environment, it is of utmost importance to properly assess the relationship between monetary policy shocks and asset prices. We claim that analyses based on small-scale VAR models suffer from a serious issue, which goes under the name of *nonfundamentalness* of the shocks. Nonfundamentalness is linked to the undisputed fact that economic agents form their expectations based on an incomparably larger information set than the handful of variables typically included in VARs. As a result, these models are unlikely to correctly identify the structural shocks. Hence, they are unsuited to gauge the relevance of monetary policy measures in shaping asset price developments. To solve this problem, we estimate a Structural Factor Model for the euro area based on a large dataset of 127 variables.

The hump-shaped responses of asset prices we find in our benchmark small-scale VAR model are common in the literature, especially within a recursive identification framework. They are, however, at odds with both economic theory and the observed market volatility. Indeed, in standard macroeconomic models asset prices equal their expected discounted payoffs. That is, forward-looking agents discount the expected future cash flows associated with an asset to determine its price. The employed discount factor, in turn, is affected (at least in the short-run) by monetary policy due to its bearing on interest rates. Hence, we expect asset prices to respond quickly, and potentially drastically, to unexpected monetary policy shocks. In the Structural Factor Model, this is precisely what we observe for most variables. Moreover, we show that monetary policy is much more important in explaining asset price (forecast error) variance than commonly thought.

On top of accounting for the observed large swings in asset prices, which standard VARs are not able to explain, our results have concrete policy implications insofar as they call for an increased focus on the financial stability consequences of monetary policy shocks. Given the broader monetary policy concept we use in our empirical model, our results would also be relevant in the case of a quicker than expected phasing-out of the set of measures implemented by the ECB and referred to as quantitative easing, or QE.



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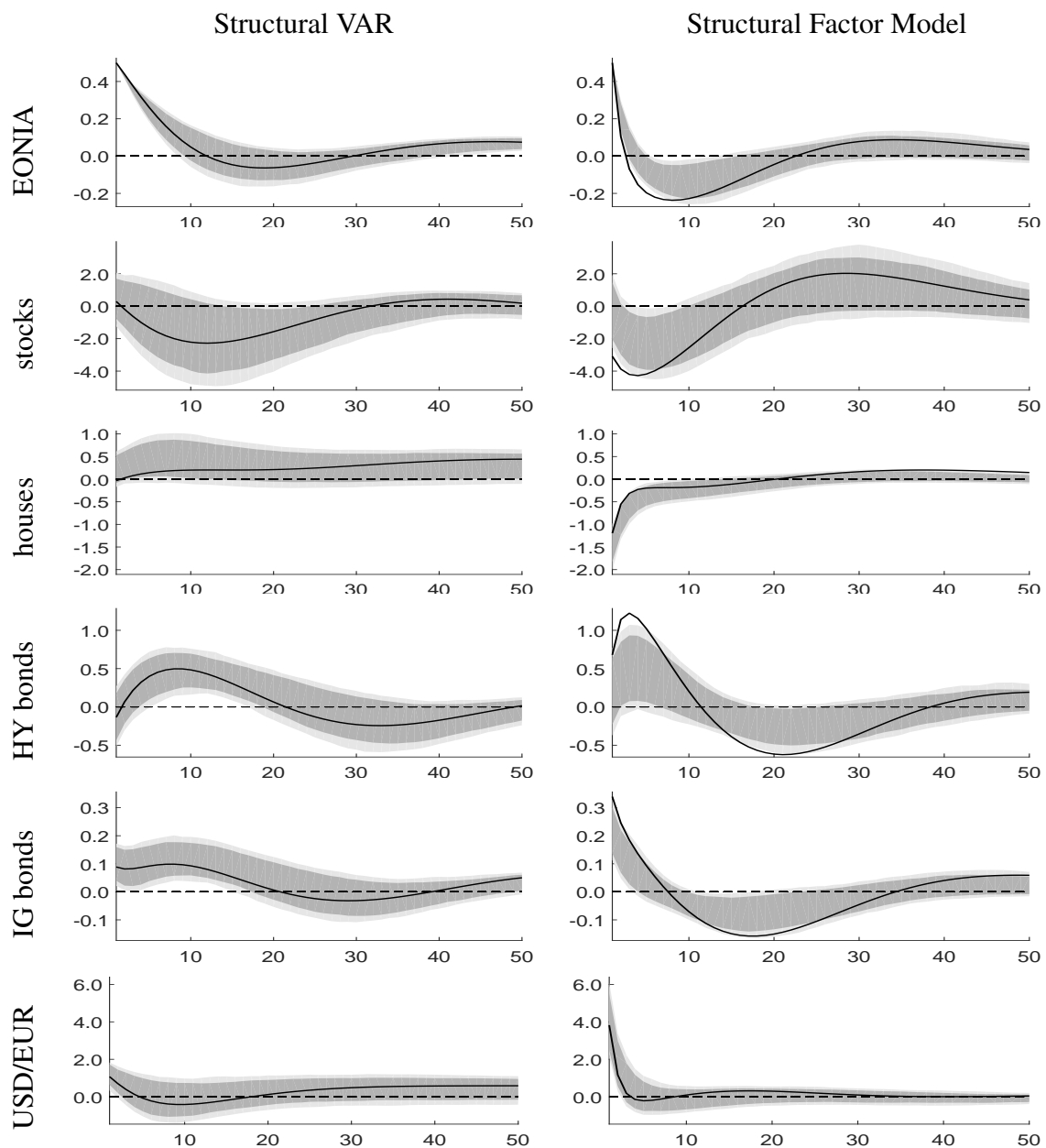


Figure 1: Comparison of VAR and SFM results.

Solid lines refer to point estimates, shaded areas to 80% and 90% confidence bands, respectively. All responses are in percentage terms (yields in percentage points) and the x-axis corresponds to months after the shock. HY: high yield corporate bond yields; IG: investment-grade corporate bond yields.

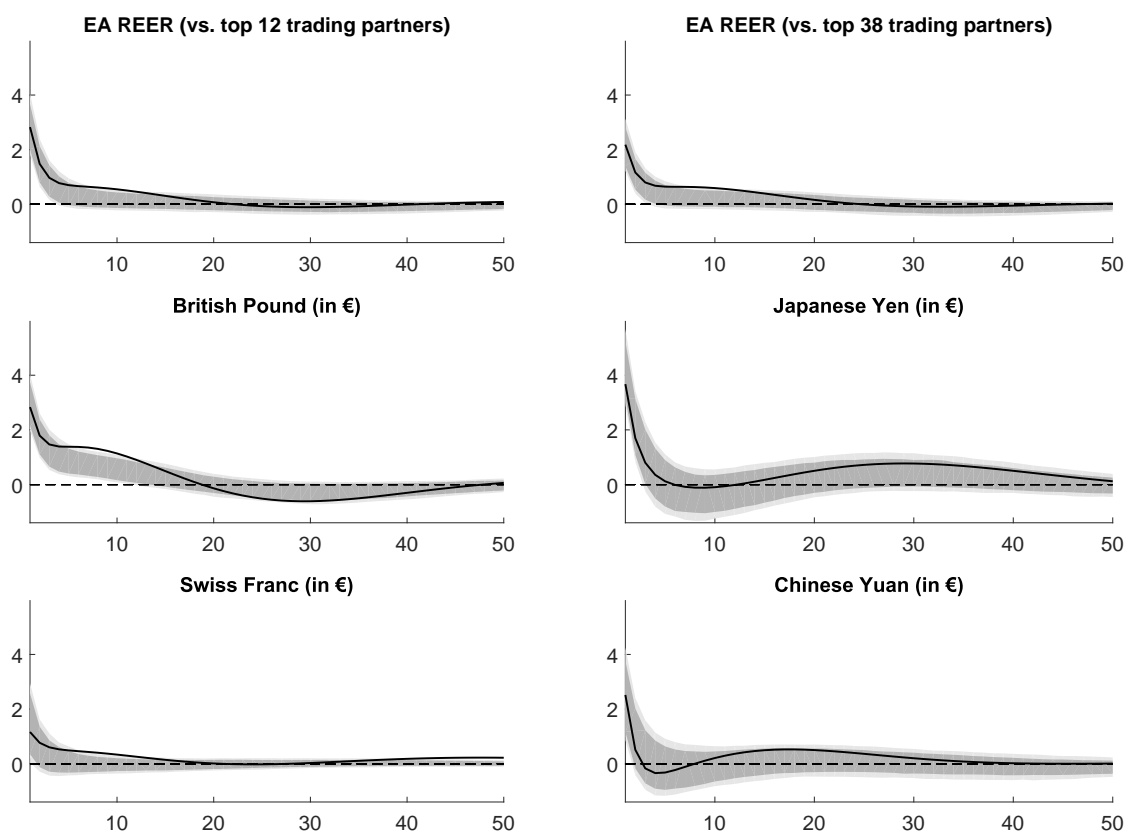


Figure 2: SFM results: Exchange rates

Solid lines refer to point estimates, shaded areas to 80% and 90% confidence bands, respectively. All responses are in percentage terms and the x-axis corresponds to months after the shock. REER: real effective exchange rate.

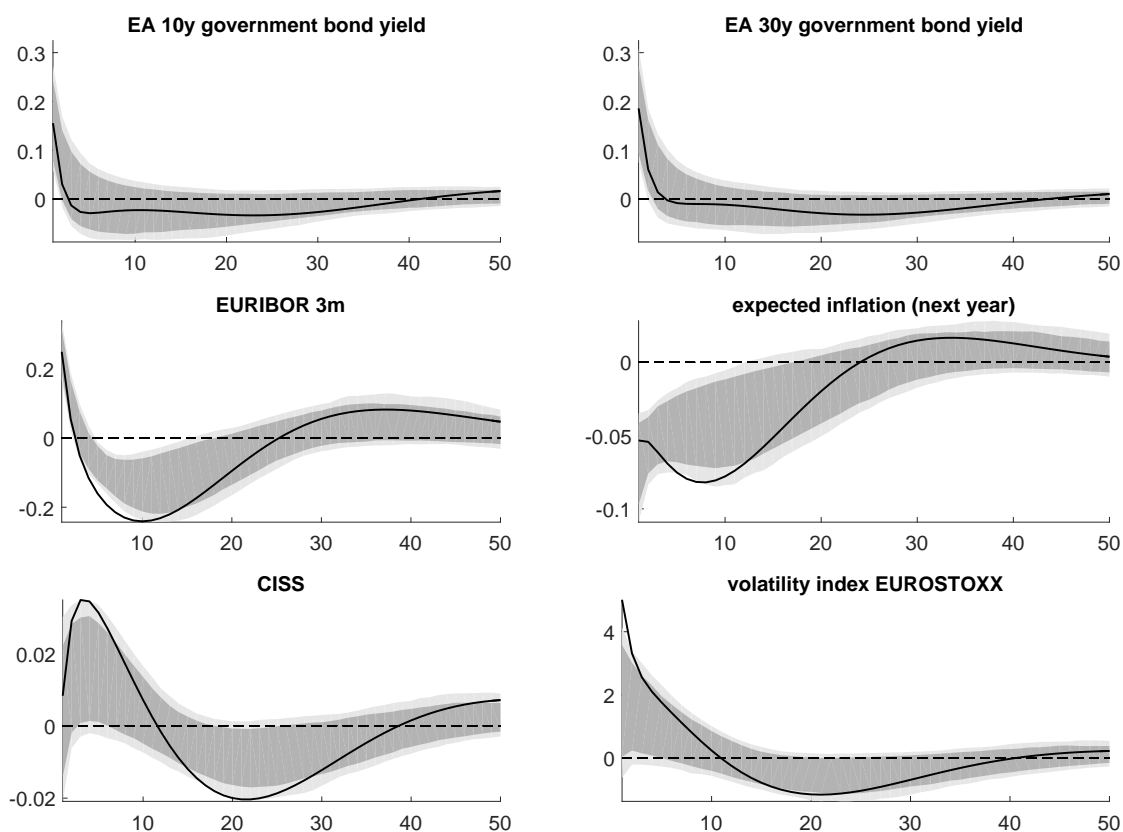


Figure 3: SFM results: Other selected macro-financial variables.

Solid lines refer to point estimates, shaded areas to 80% and 90% confidence bands, respectively. The first two rows of responses is in percentage points, the composite indicator of systemic stress (CISS) is normalized between 0 and 1 (Hollo et al., 2012) and the Eurostoxx volatility index roughly ranged between 10 and 60 in our sample. The x-axis corresponds to months after the shock.

Table 1: Forecast error variance decomposition

	0		6		12		48	
	SVAR	SFM	SVAR	SFM	SVAR	SFM	SVAR	SFM
IP	0.0	0.0	10.3	36.6	6.8	30.1	8.5	26.2
HICP	0.0	0.0	10.5	5.7	20.1	12.5	29.9	10.2
EONIA	98.7	76.0	69.2	49.0	43.2	38.2	23.4	27.6
EUROSTOXX	0.2	6.4	1.8	16.4	2.6	15.1	7.2	11.2
HOUSP	0.1	65.1	1.6	35.6	3.6	27.0	4.4	14.4
HYBONDY	0.6	9.8	4.7	21.9	8.4	21.2	5.0	20.7
IGBONDY	3.8	51.9	5.8	40.4	7.7	37.3	3.3	23.3
FXUS	5.8	35.1	2.2	13.9	2.4	12.0	4.7	8.7
SOVBONDY	-	0.2	-	7.8	-	9.6	-	9.6
REER12	-	43.4	-	31.9	-	29.4	-	19.7
REER38	-	47.9	-	31.7	-	27.9	-	15.8
FXUK	-	58.6	-	55.1	-	48.7	-	34.0
FXJP	-	18.8	-	12.4	-	11.7	-	11.6
FXCH	-	5.5	-	7.1	-	8.1	-	8.2
FXCN	-	22.4	-	6.3	-	5.5	-	4.5

All results in percentages. Months after the monetary policy shock on the columns.



# Appendix

Table A1: Data description

#	variable name	description	source	tcode
<i>real activity</i>				
1	IP	real industrial production	ECB	3
2	IP2	-excl. construction and energy	ECB	3
3	IP CONS	-construction	ECB	3
4	IP MIG1	-goods	ECB	3
5	IP MIG2	-capital goods	ECB	3
6	IP MIG3	-durable consumer goods	ECB	3
7	IP MIG4	-non-durable consumer goods	ECB	3
8	IP CONSUM	-consumer goods	ECB	3
9	IP ENERGY	-energy	ECB	3
10	OTR1	new passenger cars	ECB	3
11	OTR2	manufacturing new orders	ECB	3
12	OTR3	manufacturing turnover index	ECB	3
13	OTR5	-retail trade	ECB	3
14	OTR6	-food	ECB	3
15	OTR7	-non-food	ECB	3
16	OTR8	-textiles	ECB	3
17	OTR9	-equipment	ECB	3
18	BUILD1	building permits all	Eurostat	3
19	BUILD2	-one-dwelling	Eurostat	3
20	BUILD3	-two- or more dwelling	Eurostat	3
21	BUILD4	construction input prices	ECB	3
22	UNEMP1	unemployment rate total	Eurostat	0
23	UNEMP2	-25 and over	Eurostat	0
24	UNEMP3	-under 25	Eurostat	0
25	TRADE1	total exports extra EA 16	Eurostat	3
26	TRADE2	total exports intra EA 16	Eurostat	3
27	TRADE3	total imports extra EA 16	Eurostat	3
28	TRADE4	total imports intra EA 16	Eurostat	3
29	TRADE5	consumption goods imports extra EA 16	Eurostat	3
30	TRADE6	capital goods imports extra EA 16	Eurostat	3
31	TRADE7	capital goods exports extra EA 16	Eurostat	3
32	CA	real current account	ECB	1
<i>prices</i>				
33	HICP	harmonised index of consumer prices	ECB	3
34	HICP CORE	-core	ECB	3
35	HICP FOOD	-food	ECB	3
36	HICP GOODS	-goods	ECB	3
37	HICP SERVICE	-services	ECB	3
38	HICP RENT	-rent	ECB	3
39	INFL FC CY	EA HICP inflation forecast - current year	Consensus Econ.	0
40	INFL FC NY	EA HICP inflation forecast - next year	Consensus Econ.	0
41	PPI	producer price index	ECB	3

42	RAWMPRICE1	world raw material price index	HWWA	3
43	RAWMPRICE2	-excl. energy	HWWA	3
44	COMMPRICE1	Commodity Price index - Food	ECB	3
45	COMMPRICE2	-non-food	ECB	3
46	COMMPRICE3	-total non-energy	ECB	3
47	GOLDP	gold price	BIS	3
48	OILP	brent oil price	BIS	3
<i>surveys</i>				
49	SUR1	economic sentiment survey	Eurostat	0
50	SUR2	industrial confidence survey	Eurostat	0
51	SUR3	consumer confidence survey	Eurostat	0
52	SUR4	construction confidence survey	Eurostat	0
53	SUR5	purchasing manager employment survey	Eurostat	0
<i>US data</i>				
54	USIND	US real industrial production	BIS	2
55	USUNEMP	US unemployment rate	BIS	1
56	USEMP	US employment index	BIS	2
57	USTRADE	US trade volume	BIS	2
58	USPROD	US production expectations	BIS	1
59	USCONSEXP	US consumer expectations	BIS	1
60	USCPI	US consumer prices	BIS	2
61	USPPI	US producer prices	BIS	2
62	USM2	US real M2	BIS	2
63	USTBILLS3M	US 3 month T-Bill rate	BIS	0
64	USTREAS10Y	US 10 year treasury rate	BIS	0
<i>financial</i>				
65	EONIA*	EONIA rate	ECB	0
66	EURIBOR3M	Euribor 3-month rate	ECB	0
67	M1	real M1	ECB	3
68	M2	real M2	ECB	3
69	M3	real M3	ECB	3
70	LOANS	real loan volume	ECB	3
71	LOANSADJ	-adjusted for securitisation	ECB	3
72	LOANSNFC	-to non-financial corporations	ECB	3
73	LOANSHH	-to households	ECB	3
74	LOANSHOUS	-for house purchase	ECB	3
75	IR30Y	30 year government benchmark bond yield	ECB	0
76	IR10Y	10 year government benchmark bond yield	ECB	0
77	IR10YGDP	-GDP weighted	ECB	0
78	IR7Y	-7 year	ECB	0
79	IR5Y	-5 year	ECB	0
80	IR2Y	-2 year	ECB	0
81	BUND2Y	2 year German government bond yield	ECB	0
82	BUND10Y	-10 year	ECB	0
83	IRCBSPREAD	spread 7-10 year gov. and corp. bond	ECB	0
84	IRNFC	interest rate new loans to NFC up to EUR 1M	ECB	0
85	IRHH	interest rate new mortgage loans to HH	ECB	0
86	REER12	EA real effective exchange rate vs EER-12	ECB	3
87	REER38	EA real effective exchange rate vs EER-38	ECB	3

88	FXUS	exchange rate US dollar / Euro	ECB	3
89	FXUK	exchange rate UK pound / Euro	ECB	3
90	FXJP	exchange rate Japanese yen / Euro	ECB	3
91	FXCH	exchange rate Swiss franc / Euro	ECB	3
92	FXCN	exchange rate Chinese yuan / Euro	ECB	3
93	CISS	Composite Indicator of Systemic Stress (CISS)	ECB	0
94	VOLAT	EURO STOXX 50 Volatility Index	ECB	0
95	STOXXYIELD	Euro Stoxx Yield	ECB	0
96	EUROSTOXX	real Dow Jones Euro Stoxx Price Index	ECB	3
97	STOXXFIN	-Financials	ECB	3
98	STOXXIND	-Industrials	ECB	3
99	STOXXBM	-Basic Materials	ECB	3
100	STOXXCG	-Consumer goods	ECB	3
101	STOXXCS	-Consumer services	ECB	3
102	STOXXTEC	-Technology	ECB	3
103	STOXXHC	-Health Care	ECB	3
104	STOXXOIL	-Oil and Gas	ECB	3
105	STOXXTEL	-Telecommunication	ECB	3
106	STOXXUTI	-Utilities	ECB	3
107	SOVBONDY	EA sovereign bond yield	Merill Lynch	0
108	IGBONDY	EA investment-grade corporate bond yield	Merill Lynch	0
109	HYBONDY	EA high-yield corporate bond yield	Merill Lynch	0
110	NFCYIELDS	EA non-financial corporations 1-10 year yield	Merill Lynch	0
111	NFCYIELDL	-non-financial corporations above 10 years	Merill Lynch	0
112	FCYIELDS	-financial corporations 1-10 year	Merill Lynch	0
113	FCYIELDL	-financial corporations above 10 years	Merill Lynch	0
<i>quarterly data</i>				
114	CAPUTIL	EA capacity utilisation survey	ECB	0
115	GDP	GDP	ECB	3
116	VA	value added	ECB	3
117	CONS P	consumption expenditure	ECB	3
118	CONS GV	-government	ECB	3
119	INVEST	gross fixed capital formation	ECB	3
120	EMPL	total employment in persons	ECB	3
121	EMPL2	-employees	ECB	3
122	EMPL3	-self employed	ECB	3
123	HOURS	total hours worked	ECB	3
124	PROD	labour productivity	ECB	3
125	ULC	unit labour costs	ECB	3
126	COMP	compensation per employee	ECB	3
127	HOUSP	real EA residential property price index	ECB	3

Note: tcode refers to the applied transformation code (0: levels, 1: first difference, 2: first log-difference, 3: log-levels). The time horizon of the sample is April 2000 - June 2015. \*The EONIA rate (monthly average) has been sliced with the shadow rate by Wu and Xia (2016) from September 2004 onwards.

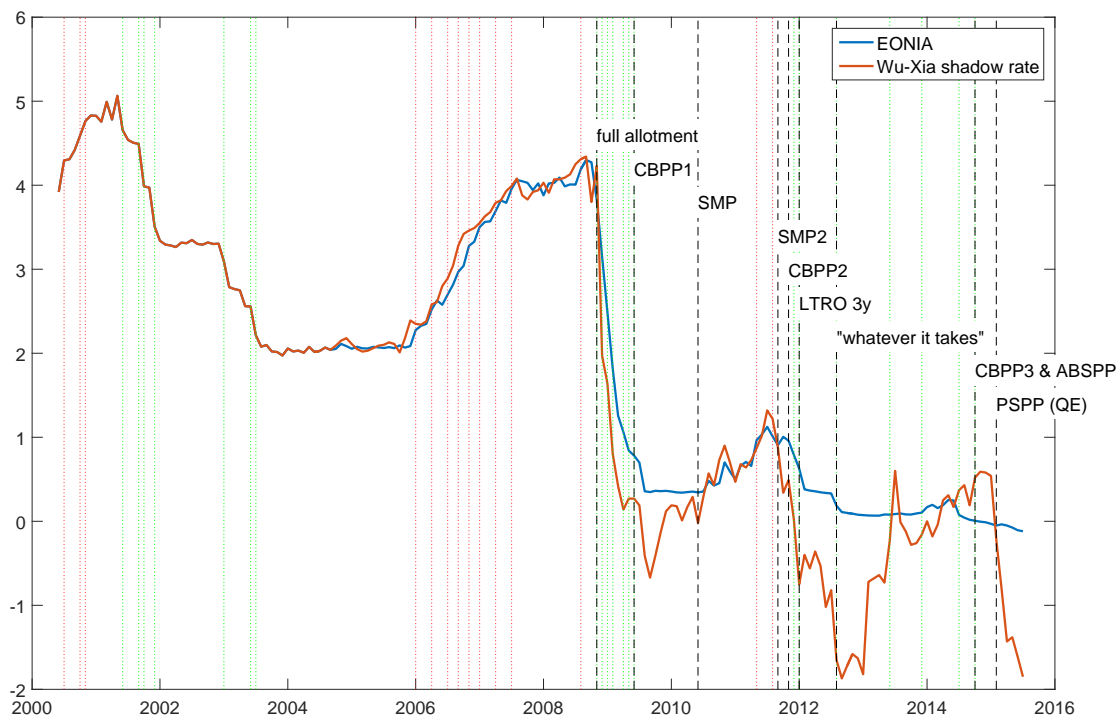


Figure A1: EONIA and shadow rate.

The vertical green (red) dotted lines indicate decreases (increases) in ECB's main refinancing operation (MRO) rate. The black dashed lines indicate 9 important (unconventional) monetary policy decisions. Besides the introduction of the fixed-rate full allotment policy and Mario Draghi's famous speech in July 2012, they include the announcement of various programmes: Covered Bond Purchase Programme (CBPP), Securities Markets Programme (SMP), Long Term Refinancing Operations (LTRO), Asset-Backed Securities Purchase Programme (ABSPP), Public Sector Purchase Programme (PSPP).

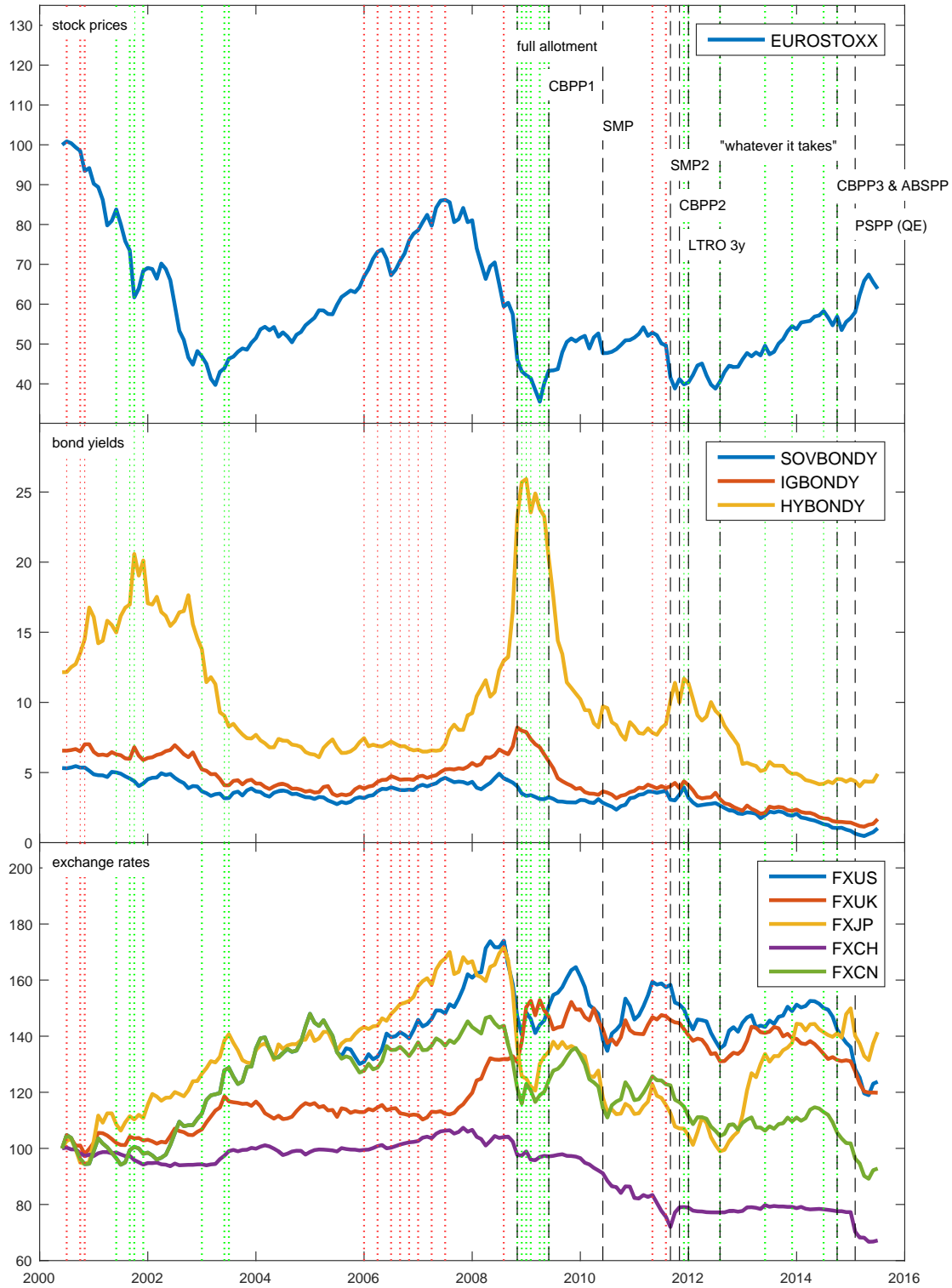


Figure A2: Financial asset prices

See Figure A1 for a description of the vertical lines. Foreign currencies (in Euro) and real stock prices rebased to 100 at sample start. See Table A1 for the exact variable definitions.

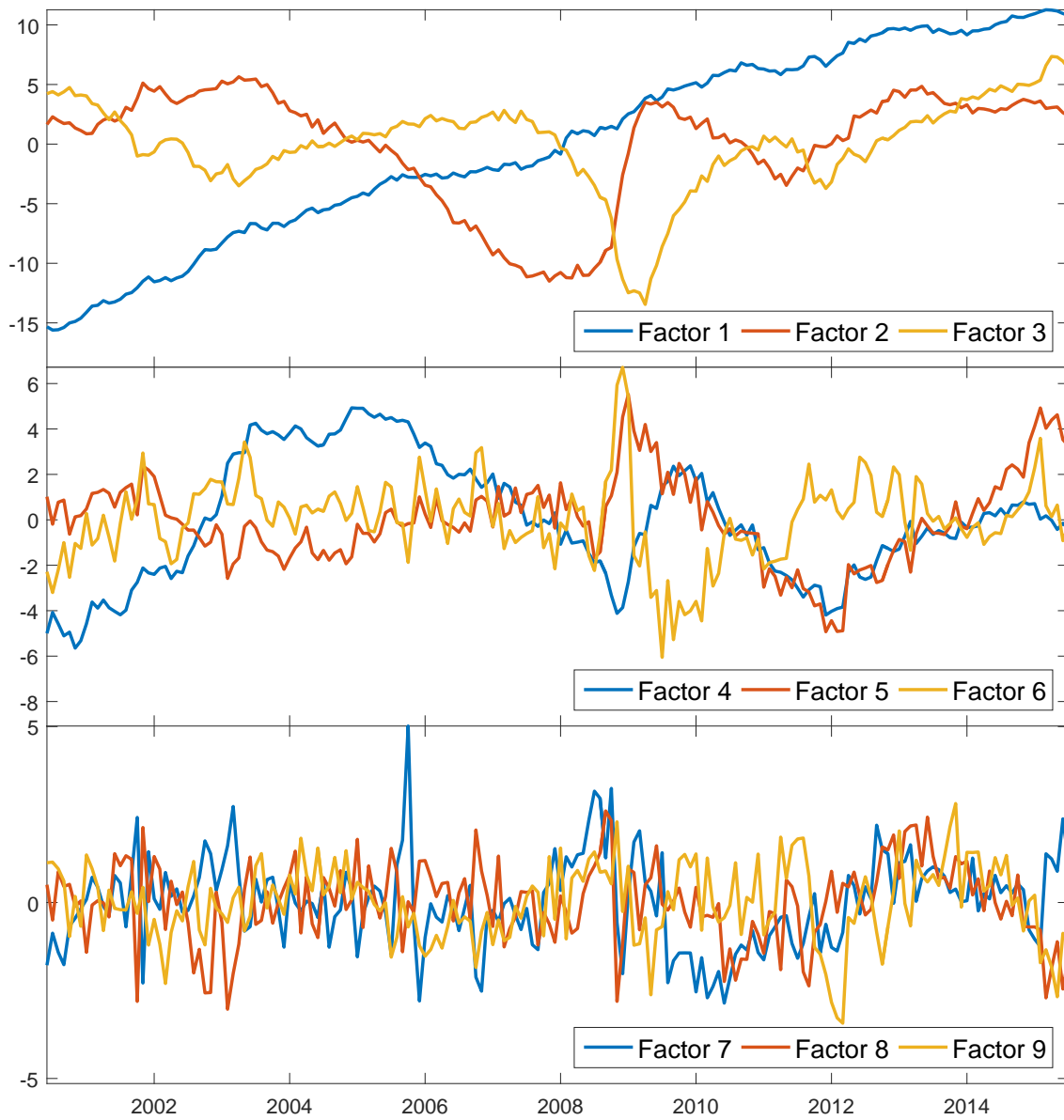


Figure A3: Static factors

Static factors are extracted as the first 9 principal components of the (demeaned and standardized) dataset, see Section 5.

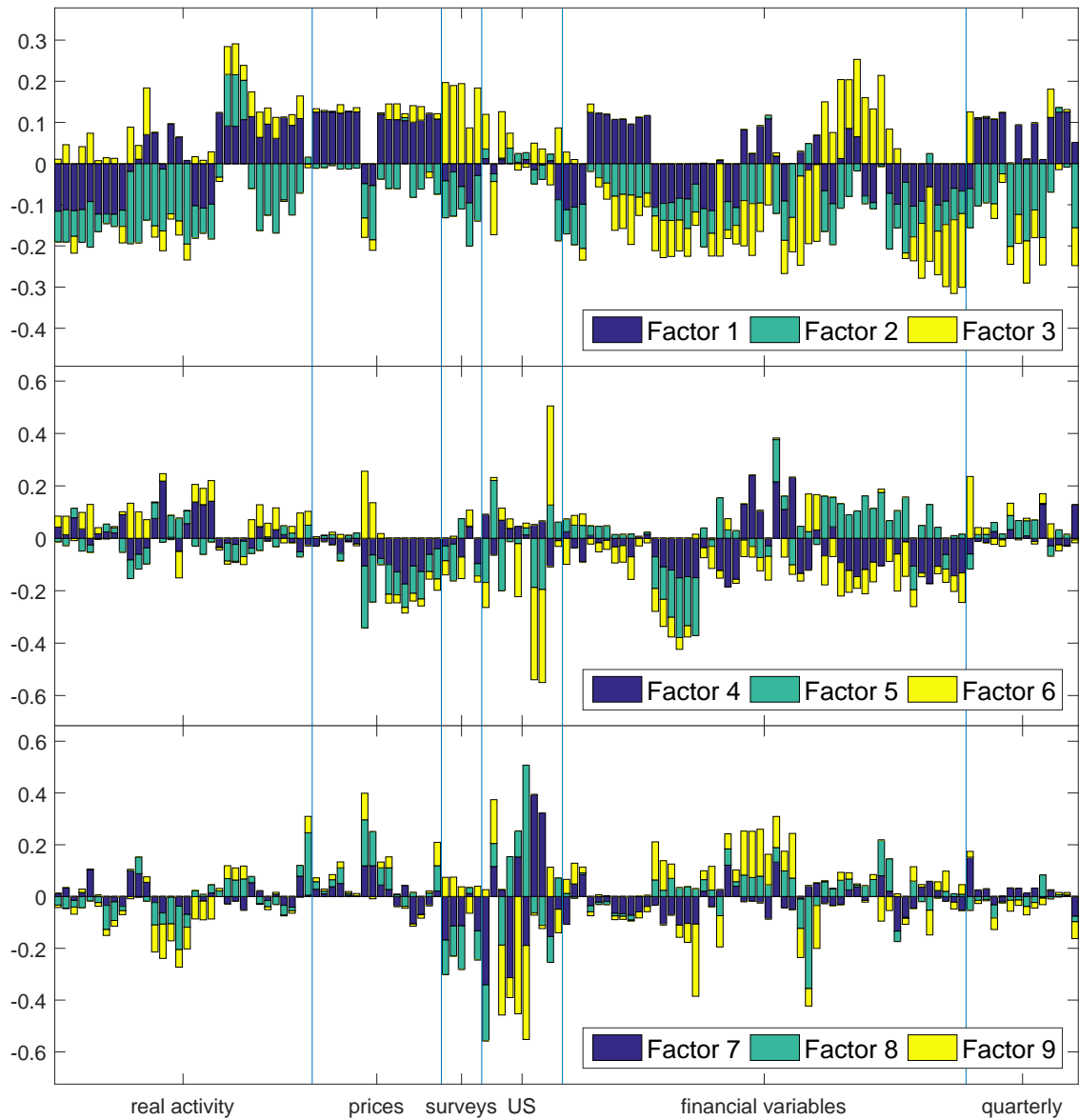


Figure A4: Loadings of variables on static factors

Loadings of standardized variables on the first 9 principal components. The x-axis refers to the variable number, see table A1. Vertical lines separate variable groups: real activity (variable 1–32), prices (33–48), surveys (49–53), US data (54–63), financial variables (64–113), and quarterly series (114–127).

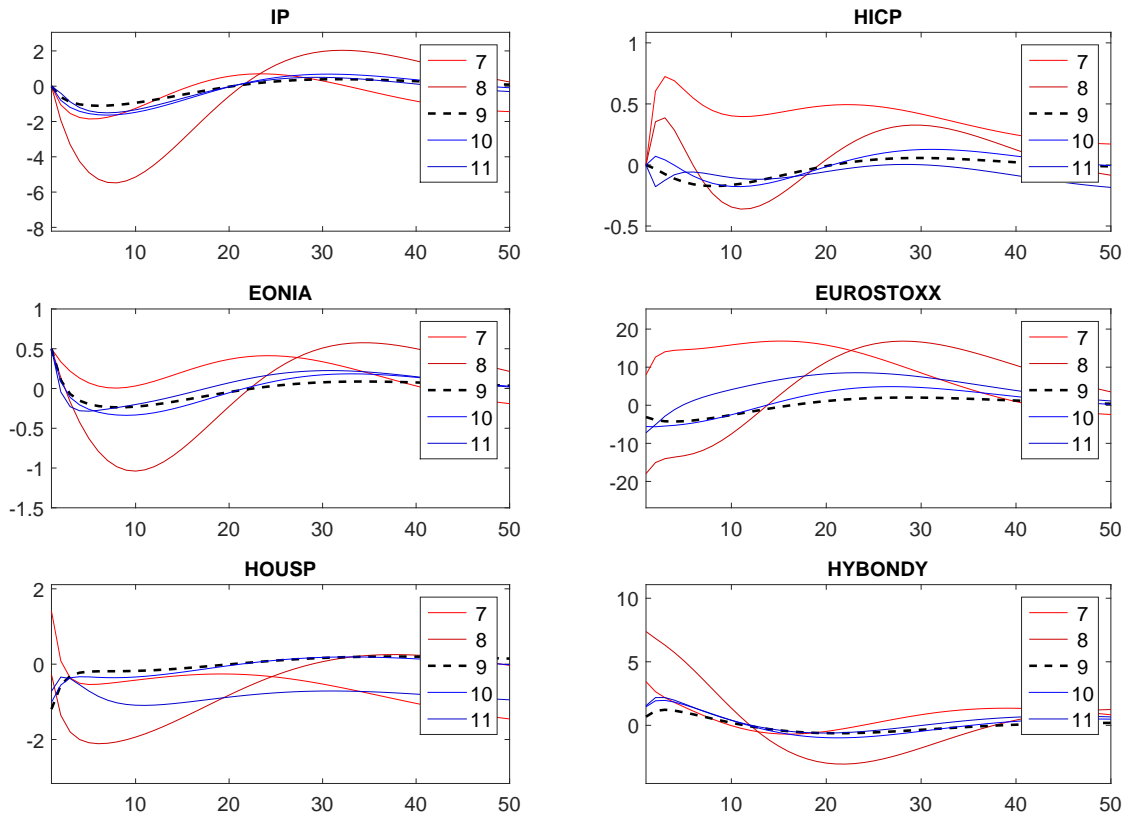


Figure A5: Robustness check: IRFs for different numbers of static factors

Results are obtained by re-estimating the model described in Section 6 (i.e. with  $q = 4$  dynamic factors) for different values of  $r$ . Our benchmark specification for the number of static factors is  $r = 9$ .



Table A2: Percentage of variance explained by the first 10 principal components

	Number of principal components									
	1	2	3	4	5	6	7	8	9	10
IP [1]	84.3	97.2	97.3	98.7	98.7	99.3	99.3	99.5	99.5	99.5
HICP [33]	98.3	98.6	98.7	99.3	99.3	99.3	99.5	99.6	99.6	99.7
EONIA [65]	70.2	89.6	89.8	90.7	91.7	92.1	92.5	92.5	93.2	93.8
EUROSTOXX [96]	27.5	50.2	81.9	85.1	94.6	98.4	98.6	99.0	99.0	99.0
SOVBONDY [107]	67.3	80.1	84.9	92.9	95.9	97.2	97.5	98.0	98.4	98.4
IGBONDY [108]	52.5	59.2	84.3	96.7	97.6	97.7	97.9	97.9	97.9	98.0
HYBONDY [109]	20.3	21.7	67.5	88.5	94.6	94.6	95.2	95.5	96.6	96.6
REER12 [86]	43.3	61.9	79.0	91.3	91.3	92.6	92.6	93.5	96.9	97.1
REER38 [87]	3.9	25.7	48.0	89.5	89.9	90.4	90.5	91.2	94.9	95.1
FXUS [88]	52.9	74.1	80.8	88.7	90.7	91.5	91.6	92.4	96.3	96.4
FXUK [89]	74.4	74.6	88.8	89.4	90.0	92.6	93.8	94.0	95.7	95.7
FXJP [90]	2.0	36.0	36.1	69.1	78.5	78.5	81.3	81.7	83.5	86.5
FXCH [91]	52.1	73.1	82.2	90.9	91.8	93.5	93.8	95.0	95.7	95.8
FXCN [92]	0.0	39.4	49.3	87.7	91.4	91.8	92.3	92.9	96.4	96.4
HOUSP [127]	16.7	72.9	84.7	96.6	96.6	96.6	97.5	97.6	98.1	98.4

Note: numbers in square brackets correspond to the series in Table A1.

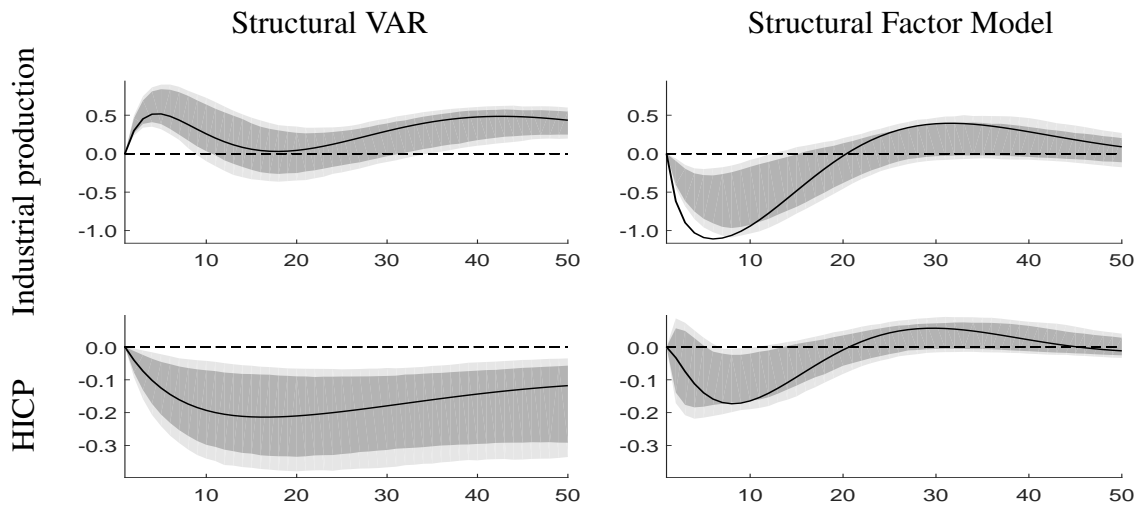


Figure A6: Comparison of VAR and SFM results

Solid lines refer to point estimates, shaded areas to 80% and 90% confidence bands, respectively. All responses are in percentage terms and the x-axis corresponds to months after the shock.

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