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Working Paper

Search-based endogenous asset liquidity and the macroeconomy

ECB Working Paper, No. 1917

Provided in Cooperation with:
European Central Bank (ECB)

Suggested Citation: Cui, Wei; Radde, Sören (2016) : Search-based endogenous asset liquidity and the macroeconomy, ECB Working Paper, No. 1917, ISBN 978-92-899-2165-7, European Central Bank (ECB), Frankfurt a. M., https://doi.org/10.2866/07962

This Version is available at:
https://hdl.handle.net/10419/154350

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EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

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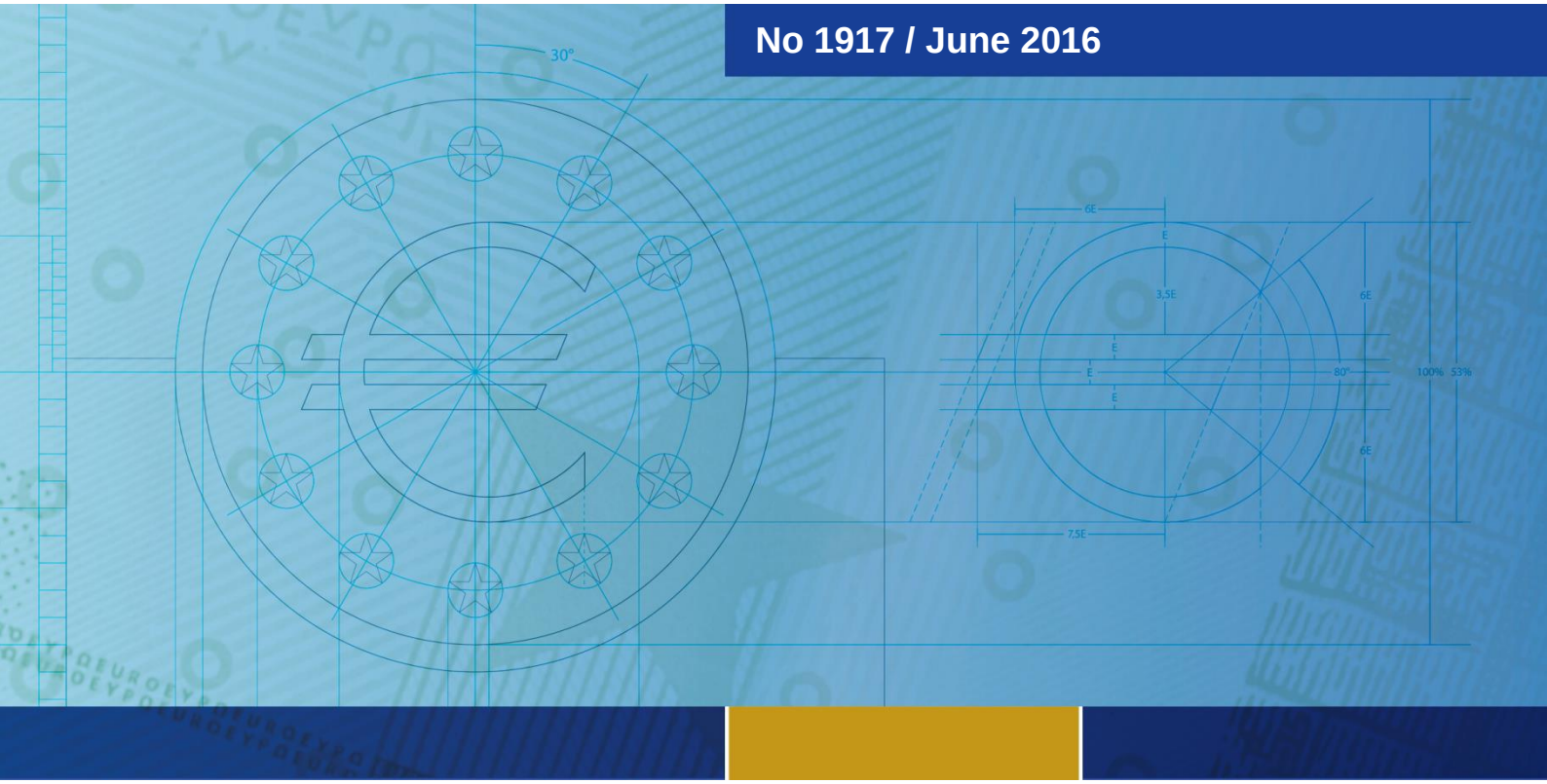
Search-based endogenous asset liquidity and the macroeconomy

No 1917 / June 2016



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Abstract

We endogenize asset liquidity in a dynamic general equilibrium model with search frictions on asset markets. In the model, asset liquidity is tantamount to the ease of issuance and resaleability of private financial claims, which is driven by investors' participation on the search market. Limited market liquidity of private claims creates a role for liquid assets, such as government bonds or fiat money, to ease financing constraints. We show that endogenising liquidity is essential to generate positive co-movement between asset (re)saleability and asset prices. When the capacity of the asset market to channel funds to entrepreneurs deteriorates, investment falls while the hedging value of liquid assets increases, driving up liquidity premia. Our model, thus, demonstrates that shocks to the cost of financial intermediation can be an important source of flight-to-liquidity dynamics and macroeconomic fluctuations, matching key business cycle characteristics of the U.S. economy.

Keywords: endogenous asset liquidity; liquidity premium; asset search markets; financing constraints; financial shocks

classification: E22; E44; G11

Non-technical summary

This paper explores the feedback effects between asset liquidity and the real economy by developing a tractable macroeconomic model featuring *endogenous* asset liquidity. Asset market liquidity captures the ease with which financial assets can be issued, traded or used for re-financing purposes. Empirical evidence points to pro-cyclical variation in the liquidity of a wide range of financial assets. The experience of the 2007-2009 financial crisis has further reinforced the view that asset market liquidity deteriorates during economic downturns and may, in fact, strongly amplify recessionary dynamics.

Illiquid asset markets adversely affect macroeconomic dynamics by tightening the financing constraints of firms. At the same time, liquidity frictions on private asset markets motivate demand for publicly provided liquidity. Indeed, firms tend to rebalance their portfolios towards liquid assets when asset markets become illiquid - a phenomenon known as "flight to liquidity".

To capture these rich interactions between financial markets and macroeconomic dynamics, we endogenise asset liquidity in a dynamic general equilibrium model by introducing search frictions on asset markets. Asset liquidity is determined by the participation decisions of buyers and sellers on financial markets. Shocks to the participation costs drive away demand for private financial claims, thus eroding their liquidity. As a result, financing constraints tighten endogenously, thereby amplifying the impact of the financial shocks. Illiquidity of private financial claims, in turn, creates a role for liquid public assets, such as fiat money or government bonds, which provide insurance against financing constraints as they can be readily used for financing purposes at any time. Financial shocks strongly affect the hedging value of such liquid assets and give rise to flight-to-liquidity dynamics. Asset market frictions spill over to the real economy, slow down capital accumulation, and dampen real economic activity.

Importantly, our model is able to match the dynamics of both the physical and the price dimension of asset liquidity. The physical dimension captures turnover on asset markets, i.e. the fraction of an asset portfolio, which can be converted into consumption goods in a given period. We refer to this dimension as *asset saleability*. The price dimension relates to the costs incurred during asset transactions, reflected in bid-ask spreads, and the price impact of trades. The search market structure links these dimensions of asset liquidity, which jointly give rise to liquidity premia.

Our key contribution is to show that endogenising asset liquidity is essential to generate positive comovement between asset saleability and asset prices and to capture its amplifying effects on business cycle fluctuations. In particular, financial sector shocks, such as exogenous increases in participation costs, affect supply and demand on financial markets simultane-

ously. This feature enables a calibrated version of our model to capture counter-cyclical liquidity premia and asset saleability, while preserving pro-cyclical asset price dynamics as observed in the data for the U.S. economy. We, thus, avoid common asset pricing anomalies plaguing macroeconomic models with financial shocks operating directly on the marketability or collateral value of assets. Aggregate productivity shocks, on the other hand, do not generate counter-cyclical liquidity premia, which suggests that this variable could serve as a discriminant between financial sector and real shocks.

Finally, we map out the conditions for the existence of private and public financial asset markets. We show that public liquidity is only demanded if participation costs in financial markets are sufficiently large, such that private financial assets do not provide sufficient liquidity on their own. Conversely, private asset markets break down for large, but finite values of participation costs. This result highlights the inherent fragility of the financial sector.

1 Introduction

Asset market liquidity captures the ease with which financial assets can be traded without strongly affecting their prices. Empirical evidence points to pro-cyclical variation in the liquidity of a wide range of financial assets.[1] The 2007-2009 financial crisis has further reinforced the view that asset market liquidity deteriorates during economic downturns and may, in fact, strongly amplify recessionary dynamics.[2]

Asset illiquidity adversely affects macroeconomic dynamics by limiting the amount of funding that can be channelled to firms with financing constraints. At the same time, illiquid asset markets motivate demand for liquid assets, such as money or government bonds. Indeed, firms tend to rebalance their portfolios towards liquid assets when asset markets become illiquid - a phenomenon known as "flight to liquidity". The idea that liquidity hoarding reflects a desire to hedge against asset market illiquidity and associated financing constraints harks back to Keynes' speculative motive for cash balances (Keynes, 1936) and Tobin's theory of risk-based liquidity preferences (Tobin, 1958, 1969).

To explore the feedback effects between asset liquidity and the real economy, this paper proposes a tractable macroeconomic model featuring *endogenous* asset liquidity. Asset liquidity is determined by the market participation decisions of buyers and sellers on a search market for privately issued financial assets, where intermediaries implement a costly matching process. Illiquidity of private financial claims creates a role for liquid assets, such as fiat money, which provide insurance against financing constraints. Privately issued assets thus carry a liquidity premium.

As our key contribution, we show that endogenizing asset liquidity is essential to generate positive co-movement between asset saleability and asset prices. This feature allows us to capture the amplifying effect of variation in asset market liquidity on business cycle fluctuations. Importantly, shocks to the cost of asset market participation affect asset supply and demand simultaneously in our endogenous asset liquidity framework. More specifically, higher participation costs reduce asset supply and physical investment, thus pushing up the

[1] Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005) and Naes, Skjeltorp, and Odegaard (2011) assert that market liquidity is pro-cyclical and highly correlated across asset classes such as bonds and stocks in the US.

[2] Dick-Nielsen, Feldhütter, and Lando (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the sub-prime crisis. The liquidity component of spreads of all but AAA rated bonds increased and turnover rates declined, making refinancing more difficult. Commercial paper, which is largely traded on a search market, experienced pronounced illiquidity reported by Anderson and Gascon (2009). In addition, money market mutual funds, the main investors in the commercial paper market, shifted to highly liquid and secure government securities. Finally, Gorton and Metrick (2012) show that the repo market has registered strongly increasing haircuts during the crisis. The macro impact of the liquidity freeze during the 2007-2009 financial crisis is studied, for instance, by Radde (2015) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

marginal product of capital and exerting upward pressure on asset prices; at the same time, they drive demand away from the asset search market and raise the liquidity premium, thus exerting downward pressure on asset prices. We show that the demand channel dominates the supply channel under mild conditions. Therefore, adverse financial shocks erode both asset saleability and prices. As a result, firms' financing constraints tighten endogenously, thereby amplifying the initial shocks. Our framework can thus jointly capture the tightening of financing constraints and the drop of asset prices as observed in the data.

Consider an economy where both money and privately issued financial claims circulate. The latter are backed by the cash flow from physical capital, which is owned by households and rented to final goods producers. All household members are endowed with a portfolio of liquid assets (money) and private claims.[3] During each period, household members are temporarily separated and face idiosyncratic investment risks. Some become workers, others entrepreneurs. Only the latter have access to investment opportunities for capital goods creation.

Entrepreneurs can finance investment using their cash balances. In addition, they can tap into private asset markets by issuing new financial claims on their investment projects and liquidating their existing portfolio of private assets. Private claims (both new and old) are only partially liquid. They are traded on a search market where intermediaries offer costly matching services for buy and sell orders. Only a fraction of quoted orders is successfully matched each period. This fraction is endogenously determined by the participation intensity on either side of the market. For instance, the more buy orders are posted relative to sell orders, the easier it is to match a sell order. Intermediaries determine the transaction price in successful matches by maximizing the total match surplus, similar to the bargaining process in the labour search literature (Mortensen and Pissarides, 1994; Shimer, 2005). As the respective match surplus of buyers and sellers depends on the ease with which private claims can be traded, the transaction price also depends on the relative asset supply and demand conditions. Finally, the intermediation cost on the search market drives a wedge between this transaction price and the effective purchase and sale prices.

In this search market structure, the liquidity of a private asset is characterised by three dimensions: i) the fraction of an asset portfolio, which can be converted into consumption goods in a given period, i.e. its *saleability*; ii) the *cost* incurred during the conversion; and iii) the *price impact* of trading the asset. Money, on the other hand, is traded on a frictionless spot market and can be converted into consumption goods instantly and costlessly, with minimal price impact. Private assets need to pay a liquidity premium over publicly issued money in order to compensate investors for carrying liquidity risks.

[3]Note that we do not distinguish between equity and debt instruments.

As market- and bank-based financial intermediation both share the essential feature of matching savers and borrowers, our framework admits both interpretations of the intermediation process as in De-Fiore and Uhlig (2011). On the one hand, the search and matching framework echoes features of over-the-counter (OTC) markets, in which a large fraction of corporate bonds, asset-backed securities, and private equity is traded. On these markets participation costs can arise from information acquisition, brokerage, and settlement services offered by dealers and market makers, as well as trading delays and costs related to IPOs or the reallocation of capital across firms.[4] On the other hand, our framework can be seen as a reduced-form approach towards modelling the costly matching process between savers (investors) and firms through financial intermediaries.

In order to assess the dynamic properties of the model against the data, we consider two types of *persistent* exogenous shocks: an aggregate productivity shock and a shock to the participation costs in the asset market, which we interpret as an "intermediation cost shock". The latter captures any generic disruption in the financial sector that increases the cost of providing intermediation services.

Negative aggregate productivity (TFP) shocks decrease the return to capital, make investment into capital goods less attractive, and hence crowd out investors from the search market. Negative intermediation cost shocks make investment into liquid assets more attractive as a hedge against future financing constraints. This reduces investors' incentives to post costly buy orders on the search market.

In either case, the fall in demand on the asset market exceeds that of supply, such that sell orders have a lower chance of being matched with a buy order. Hence, the saleability of financial claims drops, which implies that entrepreneurs need to retain a larger equity stake in new investment projects. At the same time, the asset price falls as the demand effect dominates the supply effect. Lower saleability and asset prices jointly tighten entrepreneurs' financing constraints, such that less resources are transferred to entrepreneurs in the aggregate. Real investment thus falls and economic activity contracts.

While both shocks generate pro-cyclical asset saleability and prices, only intermediation cost shocks induce a persistent flight to liquidity, manifested in a higher liquidity premium. Negative TFP shocks depress the expected return on capital, thereby exerting downward pressure on the profitability of future investment projects. Therefore, investors have a weak incentive to hedge against future financing constraints. Adverse intermediation cost shocks, however, do not affect the quality of investment projects as such. Therefore, investors strongly value the hedging service provided by money and rebalance their asset portfolios accordingly. More active portfolio rebalancing increases asset price volatility.

[4]See, e.g., Dick-Nielsen, Feldhütter, and Lando (2012), Bao, Pan, and Wang (2011), Anderson and Gascon (2009) and Eisfeldt and Rampini (2006)

As a result of these dynamic features, intermediation cost shocks are able to match the counter-cyclical liquidity premium and pro-cyclical, but volatile asset prices in addition to the dynamics of macroeconomic variables. Productivity shocks, however, generate a strongly pro-cyclical liquidity premium, and pro-cyclical but insufficiently volatile asset prices. Liquidity premia thus emerge as a potential discriminant between financial sector and productivity shocks.

Importantly, our endogenous asset liquidity framework avoids asset pricing anomalies plaguing macroeconomic models with financial shocks operating directly on the saleability or pledgeability of assets. As pointed out by Shi (2015), for instance, exogenous shocks to asset liquidity constraints act as negative supply shocks and trigger persistent asset price booms in recessions. In a similar vein, Justiniano, Primiceri, and Tambalotti (2015) argue that a slackening of leverage constraints affecting mortgage demand without a simultaneous expansion of mortgage supply leads to a rise in interest rates, thus generating a fall rather than a boom in real estate prices. By modeling asset liquidity as an endogenous phenomenon, instead, we reconcile declining asset saleability with falling asset prices without recourse to aggregate productivity shocks (e.g. Shi, 2015), capital-specific productivity shocks (e.g. Gertler and Karadi, 2011), or simultaneous demand and supply shocks (e.g. Justiniano, Primiceri, and Tambalotti, 2015).

Finally, we map out the conditions for the existence of private and public financial assets. The equilibrium in our model spans different types depending on the kind of financial claims that circulate and the amount of risk-sharing between agents through financial markets. We show that private and public financial claims only coexist for intermediate values of intermediation costs. If these costs drop below a certain threshold, private claims provide sufficient liquidity, such that public liquidity is not valued (non-monetary equilibrium). On the other hand, if participation becomes too costly, private asset markets break down and only public liquidity circulates (pure monetary equilibrium). This break-down of financial intermediation for finite values of participation costs highlights that our asset search framework captures the inherent fragility of the financial sector. As polar cases, the model also nests an RBC economy with frictionless asset markets and full risk-sharing, as well as an autarky economy in which financial markets cease to exist and risk-sharing collapses.

Related Literature. Our model is closely related to Kiyotaki and Moore (2012) (henceforth KM) and Shi (2015), who propose models with *exogenous* differences in the market liquidity between private claims and government-issued assets to study the impact of liquidity shocks on macroeconomic dynamics. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) extend the KM framework with a zero lower bound on nominal interest rates to simulate

"unconventional" monetary policy in response to an exogenous liquidity crisis.[5]

The search literature provides a natural theory of endogenous asset liquidity following the pioneering work of Duffie, Gârleanu, and Pedersen (2005), who apply search-theory to model trading frictions on OTC markets in a partial equilibrium setting. Lagos and Rocheteau (2009) show that asset demand in an OTC market depends not only on traders' current asset valuations, but crucially also on their expected valuation over the holding period of an asset and the anticipation of future trading frictions. These elements are present in our asset-search framework.

Search frictions have been used to study the features of markets for a wide range of financial assets, such as asset-backed securities, corporate bonds, federal funds, private equity and housing, amongst others (Duffie, Gârleanu, and Pedersen, 2007; Ashcraft and Duffie, 2007; Feldhutter, 2011; Wheaton, 1990; Ungerer, 2012). Rocheteau and Weill (2011) provides an extensive survey on search theory and asset market liquidity. As shown in this strand of literature, search frictions are well-suited to explain a host of empirical measures of asset market liquidity (e.g., bid-ask spreads and trading delays). Further, work by Den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) has emphasized the role of search and matching frictions in credit markets and their impact on aggregate dynamics.[6]

Nevertheless, the joint behaviour of asset prices and asset saleability is not explored in a general equilibrium setting in the above lines of research, such that mutual feedback effects are not considered. An alternative approach to endogenising asset liquidity focuses on information frictions, such as adverse selection models in Eisfeldt (2004) and Guerrieri and Shimer (2012). While providing a theory of endogenous asset liquidity, these studies do not consider the feedback effects of fluctuations in liquidity on production and employment. A notable exception is Kurlat (2013), who extends KM with endogenous resaleability through adverse selection but neglects the role of liquid assets. In Eisfeldt and Rampini (2009), firms need to accumulate liquid funds in order to finance investment. While the supply of liquid assets affects investment, secondary markets for asset sales are shut off as an alternative means of financing. In contrast to these contributions, we jointly model endogenous liquidity on primary and secondary asset markets, the role of liquid assets as the lubricant of investment financing, and feedback effects between asset liquidity and business cycles. In this sense, we complement the studies of cyclical capital reallocation, such as Eisfeldt and

[5]More generally, Kara and Sin (2013) show that market liquidity frictions induce a trade-off between output and inflation stabilization off the zero lower bound that can be attenuated by quantitative easing measures.

[6]Further, Kurmann and Petrosky-Nadeau (2006) study search frictions associated with physical capital in a macroeconomic setting. As shown in Beaubrun-Diant and Tripier (2013), search frictions also help explain salient business cycle features of bank lending relationships.

Rampini (2006) and Cui (2013).

Our framework also differs along important dimensions from search-theoretic models of money, such as Lagos and Wright (2005) and Rocheteau and Wright (2005). In this literature, money has a transaction function in anonymous search markets. Recent extensions include privately created liquid assets such as claims to capital Lagos and Rocheteau (2008); Lagos, Rocheteau, and Weill (2009) or bank-deposits (Williamson, 2012) as media of exchange. Our framework rather emphasizes the role of financial assets - both public and private - as stores of value. That is, money and private claims are used for financing purposes. Moreover, we focus on *endogenous variation* in asset liquidity and the associated premium, because private claims are subject to search frictions themselves, rather than serving to overcome such frictions on other markets. These differences notwithstanding, a common tenet is that liquid assets play an important role in economic transactions by relaxing deep financial frictions.

By studying intermediation cost shocks which affect asset market liquidity, we complement the literature on financial shocks. Recent contributions by Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014), and Jaccard (2013) identify financial shocks as an important source of business cycles. Our approach shows how such shocks may be endogenously amplified through the interlinkages between financial markets and the real economy.

2 The Model

Time is discrete and infinite ($t = 0, 1, 2, ...$). The economy has three sectors: final goods producers, households, and financial intermediaries. Following Shi (2015), there is a continuum of households (with measure one) and each household has a continuum of members of two types: entrepreneurs and workers.[7] Each period is divided into four sub-periods:

- *The household's decision period.* Aggregate shocks to productivity (A_t) and intermediation costs (κ_t) are realized. Types are still unknown and all members *equally divide* the household's assets. Households hold (physical) capital stock, equity claims issued against capital stock by other households, and fully liquid assets (money). Each household instructs its members on their type-contingent decisions.

- *The production period.* Each member receives a status draw, becoming an entrepreneur with probability χ and a worker otherwise. The type-draw is independent across members and over time. By the law of large numbers, each household thus consists of a

[7] The representative household with temporarily separated agents has been introduced in Lucas (1990) and applied to the KM framework in Shi (2015) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

fraction χ of entrepreneurs and a fraction $(1-\chi)$ of workers. An entrepreneur has investment projects but no labour endowment, while a worker has a unit of labour endowment but no investment project. Both groups are temporarily separated during each period and there is no consumption risk insurance among them. Firms rent capital and labour to produce consumption goods.

- *The investment period.* Entrepreneurs use their liquid assets and seek further external funding to finance *scalable* investment projects, which can transform 1 unit of consumption goods into 1 unit of capital stock. Entrepreneurs obtain external funding in terms of consumption goods by selling claims to the cash-flow from their investment projects to workers, who accumulate financial claims on behalf of their households. The asset market, on which such claims are traded, is characterized by search frictions. Financial intermediaries implement a costly matching process in the asset market and determine the transaction price via a bargaining process.

- *The consumption period.* After investment, agents of both types consume and then return to their households with their assets.

Asset liquidity. Our asset liquidity notion has three dimensions: i) the speed at which an asset can be converted into consumption goods; ii) the cost incurred during the conversion; iii) the price impact of trading the asset. Liquid assets are traded on a frictionless spot market and can be converted into consumption goods instantly and costlessly with minimal price impact.

To abstract from government policies, we model liquid assets as non-interest bearing money in *fixed supply* with $\bar{B}$. In contrast to such public liquidity, privately created financial assets are only partially convertible in each period due to the costly search-and-matching market structure. Moreover, as we will show in Section 3, the search market structure also makes the price of private financial assets sensitive to the transaction volumes. Therefore, we refer to private financial claims as *illiquid assets* or *partially liquid assets.*

2.1 A Representative Household

Note that, variables related to entrepreneurs are denoted with superscript "i", which stands for *investment*, while variables related to workers are denoted with superscript "n" for *no investment.* Let c_t^i denote the consumption of an individual entrepreneur. Let c_t^n and n_t denote the consumption and hours worked of an individual worker. The total goods consumed by entrepreneurs and workers in each household are $C_t^i = \chi c_t^i$ and $C_t^n = (1-\chi)c_t^n$, while the total labour supply is $N_t = (1-\chi)n_t$.

Preferences. The household aggregates the utility of consumption and leisure from all its members by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \{\chi u(c_t^i) + (1-\chi)u(c_t^n) - (1-\chi)h(n_t)\}, \quad \beta \in (0,1)$$

where the expectation is taken over aggregate shocks (A_t, κ_t), $u(.)$ is a standard strictly increasing and concave utility function of consumption, and $h(.)$ captures the dis-utility derived from labour supply n_t.

Balance sheet. Households can invest into nominal and fully liquid assets (money). Physical capital (K_t^h), earning a rental return r_t, is owned by households and rented to final goods producers. There is a claim to the future return of every unit of capital. For example, the owner of one unit of assets issued at time $t-1$ is entitiled to payoffs r_t, $(1-\delta)r_{t+1}$, $(1-\delta)^2 r_{t+2}$,... For expositional simplicity, we follow Kiyotaki and Moore (2012) and normalize such claims by the capital stock, such that they depreciate at the same rate δ, but earn a return r_{t+s} at any date $t+s$ ($\forall s \geq 0$). If a sell offer (ask quote) is successfully matched with a buy offer (bid quote), a private financial claim can be sold at an endogenous price q_t as will be explained in Section 2.2.

Hence, at the onset of period t, households own a portfolio of liquid assets, financial claims on other households' return on capital (S_t^O), and own physical capital (K_t^h). These assets are financed by net worth and private financial claims issued to outside investors (S_t^I), backed by a fraction of an households' own physical capital. This financing structure gives rise to the beginning-of-period balance sheet in Table 1:

Table 1: Household's Balance Sheet

Assets		Liabilities	
liquid assets	B_t/P_t	financial claims issued	$q_t S_t^I$
financial claims on other households' capital	$q_t S_t^O$		
capital stock	$q_t K_t^h$	net worth	$q_t S_t + B_t/P_t$

Note that all existing claims (S_t^O) to capital are traded on the search market and are valued at price q_t; similarly, the fraction of the capital stock on which no financial claims has been written yet (i.e., $K_t^h - S_t^I$) would also be offered on the search market and valued at q_t. As a result, besides liquid assets B_t, we only need to keep track of net private financial

claims S_t, defined as

$$\begin{aligned} S_t &\equiv \text{financial claims on other households' capital} + \text{unissued capital stock} \\ &= S_t^O + (K_t^h - S_t^I) \end{aligned}$$

Asset accumulation. S_t^j and B_t^j denote net private financial claims and money held by entrepreneurs ($j = i$) or workers ($j = n$). Let S_{t+1}^j and B_{t+1}^j denote asset position at the end of t. Since all financial assets are equally divided, the fraction of private claims held by entrepreneurs and workers corresponds to their respective population shares, i.e., $S_t^i = \chi S_t$ and $S_t^n = (1-\chi)S_t$. A similar division applies to liquid assets, i.e., $B_t^i = \chi B_t$ and $B_t^n = (1-\chi)B_t$. Net private financial claims evolve according to

$$S_{t+1}^j = (1-\delta)S_t^j + I_t^j - M_t^j, \tag{1}$$

where I_t^j is physical investment, and M_t^j corresponds to the quantity of private claims sold by group j. When M_t^j is negative, it implies that j group members are buying rather than selling private financial assets.

2.1.1 Workers' Constraints

The household delegates purchases of private financial claims to workers, because they do not have investment opportunities ($I_t^n = 0$) and earn a wage rate w_t. Therefore, workers post bid quotes of size V_t through financial intermediaries to acquire new or old private claims at a unit cost κ_t. On the search market, each bid is matched with an ask quote by financial intermediaries with a probability $f_t \in [0,1]$ (which is endogenous), such that an individual buyer expects to purchase an amount $M_t^n = -f_t V_t$. Workers' flow-of-funds constraint in terms of consumption goods reads

$$C_t^n + \kappa_t V_t + q_t f_t V_t + \frac{B_{t+1}^n}{P_t} = w_t N_t + r_t S_t^n + \frac{B_t^n}{P_t}, \tag{2}$$

where labour income, the return on private financial claims, and money are used to finance consumption, search costs, and the new acquisition of private claims and money. To simplify, we define the bid (or effective buy) price per unit of private claims as

$$q_t^n \equiv q_t + \frac{\kappa_t}{f_t}, \tag{3}$$

where q_t captures the transaction price and $\frac{\kappa_t}{f_t}$ represents search costs per transaction (scaled by the probability of encountering a matching ask quote f_t). By using (1), $M_t^n = -f_t V_t$,

$S_t^n = (1-\chi)\, S_t$, and $B_t^n = (1-\chi)\, B_t$, we rewrite the flow-of-funds constraint (2) as

$$C_t^n + q_t^n S_{t+1}^n + \frac{B_{t+1}^n}{P_t} \quad = \quad w_t N_t + [r + (1-\delta) q_t^n]\,(1-\chi)\, S_t + \frac{(1-\chi)\, B_t}{P_t}. \tag{4}$$

Workers do not have an extra financing constraint on S_{t+1}^n as long as they have enough resources (i.e., (4) is satisfied). In addition, workers are restricted to hold non-negative amounts of money,

$$B_{t+1}^n \geq 0. \tag{5}$$

2.1.2 Entrepreneurs' Constraints

In order to finance new investment ($I_t^i > 0$), entrepreneurs can use return from claims on capital and liquid assets; they can also choose an ask size U_t, i.e., the amount of private financial claims to be quoted for sale, also at a unit cost κ_t. These assets include net private claims $(1-\delta)S_t^i$, plus claims on new investment I_t^i. The ask size is bounded from above by entrepreneurs' existing private financial asset holdings and the volume of new investment, i.e., $U_t \leq (1-\delta)\, S_t^i + I_t^i$. Ask quotes are matched with bid quotes with the - endogenously determined - probability $\phi_t \in [0,1]$. Therefore, entrepreneurs expect to sell $M_t^i = \phi_t U_t$ units of financial claims. The flow-of-funds constraint can then be written as

$$C_t^i + I_t^i + \kappa_t U_t + \frac{B_{t+1}^i}{P_t} = r_t S_t^i + q_t \phi_t U_t + \frac{B_t^i}{P_t}, \tag{6}$$

where the returns on private claims and money are used to finance consumption, search costs, physical investment, and end-of-period's money holdings. Symmetrically to the bid price, we define the ask (or effective sell) price of a unit of private financial assets as

$$q_t^i \equiv q_t - \frac{\kappa_t}{\phi_t}. \tag{7}$$

When $\kappa_t > 0$, the ask price is below the transaction price. Hence, entrepreneurs not only face constraints regarding the quantity of private claims that can be issued and resold, they also have to sell at a discount due to the intermediation cost κ_t/ϕ_t when liquidating financial claims. Then, by inserting $M_t^i = \phi_t U_t$ to (1) and using q_t^i, we rewrite the flow-of-funds constraint (6) as

$$C_t^i + I_t^i + q_t^i \left[S_{t+1}^i - I_t^i - (1-\delta) S_t^i\right] + \frac{B_{t+1}^i}{P_t} \quad = \quad r_t S_t^i + \frac{B_t^i}{P_t}. \tag{8}$$

It is instructive to substitute out new investment by defining the fraction of total assets that entrepreneurs choose to quote for sale $0 \leq e_t \leq 1$. That is, $U_t = e_t[(1-\delta)S_t^i + I_t^i]$.

Following equation (1), we then express the evolution of entrepreneurs' private asset holdings as

$$S_{t+1}^i = (1 - e_t\phi_t)\left[(1-\delta)S_t^i + I_t^i\right] \tag{9}$$

By using (9), we can express $I_t^i = \frac{S_{t+1}^i - (1-e_t\phi_t)(1-\delta)S_t^i}{1-e_t\phi_t}$ and rewrite the flow-of-funds constraint (8) to

$$C_t^i + q_t^r S_{t+1}^i + \frac{B_{t+1}^i}{P_t} = r_t\chi S_t + (1-\delta)\chi S_t + \frac{\chi B_t}{P_t}, \tag{10}$$

$$\text{where} \quad q_t^r \equiv \frac{1 - e_t\phi_t q_t^i}{1 - e_t\phi_t}. \tag{11}$$

where we have used the fact that $S_t^i = \chi S_t$ and $B_t^i = \chi B_t$. The left-hand side (LHS) of (10) captures entrepreneurs' spending on consumption and holdings of private claims and money at the end of t, while the right-hand side (RHS) represents entrepreneurial (total) *net-worth* including rental income from private financial claims, the value of existing financial claims, and the real value of money.

Note that the LHS involves private assets S_{t+1}^i's replacement cost q_t^r : for every unit of claims, entrepreneurs need to make a "down-payment" $(1 - e_t\phi_t q_t^i)$ and retain a fraction $(1 - \phi_t)$ as inside equity claims.

Entrepreneurs face two financing constraints, because of $e_t \leq 1$ and the non-negativity of money holdings:

$$S_{t+1}^i \geq (1-\phi_t)\left[(1-\delta)S_t^i + I_t^i\right] \quad \text{and} \quad B_{t+1}^i \geq 0. \tag{12}$$

To further understand the financing constraints, notice that (10) involves gross investment. New investment can be backed out from (9) and (10). Formally, as all investment projects are carried out only by entrepreneurs, aggregate investment I_t is

$$I_t = I_t^i = \frac{\left[r_t + e_t\phi_t q_t^i(1-\delta)\right] S_t^i + \frac{B_t^i}{P_t} - C_t^i - \frac{B_{t+1}^i}{P_t}}{1 - e_t\phi_t q_t^i} \leq \frac{\left[r_t + \phi_t q_t^i(1-\delta)\right] S_t^i + \frac{B_t^i}{P_t} - C_t^i}{1 - \phi_t q_t^i}. \tag{13}$$

To invest in new capital stock, entrepreneurs' liquid net-worth- including the saleable private claims $e_t\phi_t q_t^i(1-\delta)$ - net of consumption and newly purchased liquid assets can be leveraged at $(1 - e_t\phi_t q_t^i)^{-1}$. If both financing constraints bind with equality, (13) determines the upper bound on investment.

2.1.3 The Representative Household's Problem

Let $J(S_t, B_t; \Gamma_t)$ be the value of the representative household with net private financial claims S_t, money holdings B_t, and some aggregate state variables Γ_t, which are taken as

given.[8] Since at the end of period t, workers and entrepreneurs reunite to share their stocks of private claims and money, we have

$$S_{t+1} = S^i_{t+1} + S^n_{t+1} \quad \text{and} \quad B_{t+1} = B^i_{t+1} + B^n_{t+1}. \tag{14}$$

and the value $J(S_t, B_t; \Gamma_t)$ satisfies the following Bellman equation

Problem 1:

$$J(S_t, B_t; \Gamma_t) = \max_{\{e_t, N_t, C^i_t, C^n_t, S^i_{t+1}, S^n_{t+1} B^i_{t+1}, B^n_{t+1}\}} \left\{ \chi u\left(\frac{C^i_t}{\chi}\right) + (1-\chi)\, u\left(\frac{C^n_t}{1-\chi}\right) \right.$$
$$\left. - (1-\chi) h\left(\frac{N_t}{1-\chi}\right) + \beta \mathbb{E}\left[J(S_{t+1}, B_{t+1}; \Gamma_{t+1}) | \Gamma_t\right] \right\}$$

subject to (4), (5), (9), (10), (12), *and* (14).

2.2 Financial Intermediation

Financial intermediaries. Intermediaries collect bid quotes V_t from workers and ask quotes U_t from entrepreneurs. Then, they implement the matching technology against a participation - or intermediation - cost of κ per unit of the quoted quantities paid by buyers and sellers. Only a fraction of bid and ask quotes are successfully matched. This costly matching process captures financial market frictions in a generic way.

Participation costs and trading frictions in financial markets, for instance, may arise from many sources, such as brokerage and settlement services offered by dealers and market makers on OTC markets, as well as costs and trading delays related to IPOs and capital reallocation across firms (mergers and acquisitions). At the same time, financial intermediation through the banking sector is both costly and time consuming as a result of screening and monitoring activities. As our model does not distinguish between these different types of financial intermediation and the associated frictions, the intermediation process could be regarded as either market- or bank-based, with financial intermediaries being interpreted as dealers or banks. For a detailed discussion of these two types of agents and their impact on macroeconomic dynamics refer to De-Fiore and Uhlig (2011).

Search and matching. The technology operated by financial intermediaries takes the form of a matching function $M_t = M(U_t, V_t)$. M is concave and homogenous of degree 1 in

[8] Once we proceed to the equilibrium definition, the vector of aggregate state variables will be given by $\Gamma \equiv \left(K_t, \bar{B}; A_t, \kappa_t\right)$, where K is the total capital stock, $\bar{B}$ is the total amount of money circulated. The exogenous stochastic processes for A and κ are specified in the numerical examples in Section 5.

(U, V) space, with continuous derivatives. Let $\theta_t = V_t/U_t$ denote asset market tightness from buyers' perspective. Then,

$$\phi_t = \frac{M(U_t, V_t)}{U_t} = M(1, \theta_t) \quad \text{and} \quad f_t = \frac{M(U_{t,}, V_t)}{V_t} = M(\frac{1}{\theta_t}, 1)$$

are the probability that one unit of the quoted ask size can be sold and the probability that one unit of the quoted bid size can be purchased. Recall that ϕ_t also represents the fraction of financial assets that can be sold *ex post* in a given period. Therefore, we refer to ϕ_t as *asset saleability.*Without loss of generality, we specify the matching function as

$$M(U, V) = \xi U^{\eta} V^{1-\eta}, \tag{15}$$

where ξ captures matching efficiency and $\eta \in [0, 1]$ is the elasticity w.r.t. ask quotes. Then, as asset market tightness θ_t increases, it becomes easier for the sellers to find potential buyers (ϕ_t increases), whereas buyers have more difficulty in finding appropriate investment opportunities (f_t decreases). The opposite is true, when θ_t goes to zero.

2.3 Asset Pricing

The nominal price of *liquid* assets is unity. Their real price $1/P_t$ is determined through the spot market. For simplicity, we refer to the price of *illiquid* assets as 'the' asset price.

The transaction price of private financial claims is determined by a bargaining process, which sellers and buyers delegate to financial intermediaries. Therefore, once a unit of assets offered for sale is matched to a buy quote, intermediaries offer a price q_t to both parties. This price is chosen with a view to maximizing the total surplus of the trade by bargaining on behalf of each side. As the amount of matched assets M_t^j is predetermined at the point of bargaining, buyers and sellers interact at the margin. In other words, the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Transaction surpluses. Denote by J_t^n and J_t^i the transaction surplus of individual workers (buyers) and entrepreneurs (sellers) from the point of view of the household at time t. A buyer's surplus amounts to

$$J^n(S_t, B_t; \Gamma_t) = -u'\left(\frac{C_t^n}{1-\chi}\right) q_t + \beta \mathbb{E}\left[J_S(S_{t+1}, B_{t+1}; \Gamma_{t+1}) | \Gamma_t\right]. \tag{16}$$

Intuitively, a buyer's surplus consists of the resources sacrificed today to acquire an additional unit of private assets and the value of this additional unit of asset holdings to its household

tomorrow.[9]

Similarly, a seller's surplus is the marginal value to the household of an additional match for entrepreneurs

$$J^i(S_t, B_t; \Gamma_t) = u'\left(\frac{C_t^i}{\chi}\right)\left(q_t - \frac{1}{\phi_t}\right) + \left(\frac{1}{\phi_t} - 1\right)\beta\mathbb{E}\left[J_S(S_{t+1}, B_{t+1}; \Gamma_{t+1})|\Gamma_t\right]. \tag{17}$$

A seller earns the contemporary surplus $\left(q_t - \phi_t^{-1}\right)$ plus a continuation value from a successful match. Recall that $\phi_t U_t = M_t$, and the contemporary surplus reflects the fact that sellers obtain q_t additional consumption goods at the margin, while they commit $\partial U_t/\partial M_t = \phi_t^{-1}$ investment to find an additional match. Therefore, for each match, the contemporary net gain is $\left(q_t - \phi_t^{-1}\right)$. Second, the evolution of entrepreneurs' asset position can be expressed as the difference between offered and sold assets, i.e., $S_{t+1}^i = U_t - M_t = \left(\phi_t^{-1} - 1\right) M_t$. Hence, entrepreneurs retain a fraction $\left(\phi_t^{-1} - 1\right)$ for each successfully matched unit of offered assets as inside equity, which is brought back to the household. Therefore, the continuation value of a match consists of the marginal value of future assets to the household multiplied by $\left(\phi_t^{-1} - 1\right)$.

Bargaining. Note that all members within the groups of buyers and sellers are homogeneous, such that the type-specific valuations are identical in all matched pairs. We consider the case in which the transaction price q_t is determined by surplus division between buyers and sellers. That is, intermediaries set a price q_t to maximize

$$\max_{q_t}\{(J_t^i)^\omega (J_t^n)^{1-\omega}\} \tag{18}$$

where $\omega \in (0, 1)$ is the fraction of the surplus that goes to sellers. This set-up is similar to bilateral (generalized) Nash bargaining between buyers and sellers over the match surplus. In the bilateral bargaining case, ω is the bargaining power of sellers.[10]

2.4 Equilibrium

The model is closed by the production side. Competitive firms rent aggregate capital stock K_t and hire aggregate labour N_t from households to produce output (consumption goods)

[9] Note that search market participation costs are already sunk at the bargaining stage. However, search costs are not ignored since households take them into account when determining optimal asset posting decisions by workers and entrepreneurs.

[10] In this sense, our price setting is similar to the wage determining process in Ravn (2008) and Ebell (2011), where individual workers come to bargain on behalf of their respective households.

according to a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha},$$

where $\alpha \in (0,1)$ and A_t measures exogenous aggregate productivity. The profit-maximizing rental rate and wage rate are thus

$$r_t = \alpha A_t \left(\frac{K_t}{N_t}\right)^{\alpha-1}, \quad w_t = (1-\alpha) A_t \left(\frac{K_t}{N_t}\right)^{\alpha}. \tag{19}$$

The aggregate state variable Γ_t in the household problem is thus equivalent to $(K_t, \bar{B}; A_t, \kappa_t)$, and the recursive competitive equilibrium can then be defined as follows:

Definition 1:
*The **recursive competitive equilibrium** is a mapping $K_t \to K_{t+1}$, with associated consumption, investment, labour, and portfolio choices $\{C_t^i, C_t^n, N_t, e_t, I_t, S_{t+1}, B_{t+1}\}$, asset market features $\{\theta_t, \phi_t, f_t\}$, and a collection of prices $\{P_t, q_t^i, q_t^n, q_t^r, w_t, r_t\}$, given exogenous evolutions of aggregate productivity A_t, intermediation costs κ_t, and positive fixed money supply $\bar{B}$, such that*

1. *given prices, the policy functions solve the representative household's decision problem (Problem 1); aggregate investment is determined by equation (13);*
2. *final goods producers' optimality conditions in (19) hold;*
3. *market clearing conditions hold, i.e.,*
   (a) *the capital market clears: $K_{t+1} = (1-\delta) K_t + I_t$ and $S_t = K_t$;*
   (b) *the search market "clears": $\phi_t = M(1, \theta_t)$, $f_t = M(\theta_t^{-1}, 1)$ and the asset price q_t solves (18), with the effective prices defined in (3), (7) and (11);*
   (c) *the market for liquid assets clears: $B_{t+1} = B_t = \bar{B}$.*

To verify that Walras' Law is satisfied, notice that the entrepreneurs' and workers' budget constraints (4) and (10), together with (14) and the equilibrium clearing condition 3.(a), imply

$$C_t + I_t + \kappa_t (V_t + U_t) = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{20}$$

which is the aggregate resource constraint. For accounting purposes, gross investment is defined as $I_t + \kappa_t (V_t + U_t)$, of which only real investment I_t adds to the capital stock at time $t+1$.

3 Equilibrium Characterization

The equilibrium of the economy described in the previous section admits different types, which can be distinguished by the activity of financial markets and the kind of financial assets that circulate. One polar case is autarky, i.e., an equilibrium in which neither private claims nor money exist. In this equilibrium, both financial intermediation via the search market and the money market shut down, and entrepreneurs finance investment fully with inside funding.[11] We restrict our attention to the more realistic case of a non-autarky economy in which at least one type of financial claims exists.

The non-autarky equilibrium features three cases. First, when search costs are prohibitively high for agents to participate in private asset markets, such markets collapse and only public liquidity circulates. Second, if intermediation costs are sufficiently small, such that the return on private claims dominates the return on public liquidity, only private claims circulate. Finally, both private and public liquidity may coexist for intermediate levels of search costs.

For ease of exposition, we adopt a guess-and-verify strategy by first illustrating the equilibrium under co-existence of private claims and money. Then, we discuss under which parameter restrictions this type of equilibrium exists. That is, we assume $\kappa > 0$ and that the economy features both private and public liquidity and subsequently verify the co-existence.

3.1 The Representative Household's Optimal Decisions

A necessary condition for *private* claims to exist is that the replacement cost does not exceed one, $q_t^r \leq 1$. Otherwise, entrepreneurs would only use internal funding for investment as they can transform consumption goods into capital goods one-to-one. A necessary condition for *public* claims to exist is that the household's optimality condition with respect to money holdings be satisfied.

The assumption of nonzero search costs and the necessary condition for the existence of private claims, $q_t^r \leq 1$, imply by definition that $q_t^n > q_t > q_t^i \geq 1$. This condition ensures that the ask price for private claims q_t^i on the search market (weakly) exceeds entrepreneurs' internal cost of investment, such that the issuance of financial claims against new investment yields non-negative profits. Therefore, the representative household will prompt entrepreneurs to spend whatever net worth they are not consuming on creating new financial claims. Accordingly, they sell as many private financial assets as possible and divest

[11] Such a complete breakdown of financial transactions on the private asset market may become self-fulfilling. For instance, when one party of the market does not participate, the other party would expect this inaction and stay out of the search market, validating the initial non-participation decision of their counterparts.

their entire stock of money holdings, i.e., $e_t = 1$ (or $U_t = (1-\delta)\chi S_t + I_t$) and $B^i_{t+1} = 0$. That is, entrepreneurs' financing constraints (12) are binding and investment is bounded from above in (13).

In contrast, workers cannot create new financial claims as they lack investment projects. Moreover, they would incur losses if they sold their existing stock of private financial assets. Therefore, the accumulation of financial assets - including private claims - on behalf of the household is delegated to workers and below conditions determine the optimal portfolio choice. The workers' financing constraint (5) is thus slack.

Under these conditions, we can simplify the household's decision problem by aggregating both types' budget constraints to a household-wide budget constraint. Let ρ_t be defined as the ratio between the effective purchasing price q^n_t and the effective replacement cost q^r_t , i.e.,

$$\rho_t \equiv \frac{q^n_t}{q^r_t}. \tag{21}$$

We sum over the type-specific budget constraints (4) and (10), multiplying the latter by ρ_t; we further eliminate S^i_{t+1}, S^n_{t+1}, B^i_{t+1}, and B^n_{t+1} by using $e_t = 1$, $B^i_{t+1} = 0$, and (14). This yields

$$\begin{aligned} \rho_t C^i_t + C^n_t + q^n_t S_{t+1} + \frac{B_{t+1}}{P_t} &= w_t N_t + [\chi\rho_t + (1-\chi)]\, r_t S_t \\ &+ [\chi\rho_t + (1-\chi) q^n_t]\,(1-\delta) S_t + [\chi\rho_t + (1-\chi)]\,\frac{B_t}{P_t}. \end{aligned} \tag{22}$$

The household then maximizes $J(S_t, B_t; \Gamma_t)$ subject to (22) by choosing only labour supply N_t, consumption C^i_t and C^n_t, total private financial claims S_{t+1}, and liquid assets B_{t+1}.[12]

Labour choice. The first-order condition for labour from this optimization problem is

$$u'\left(\frac{C^n_t}{1-\chi}\right) w_t = \mu, \tag{23}$$

which is a standard intra-period optimality condition. It requires that the marginal gain of extra consumption goods from earning wages equal the marginal dis-utility from working.

Risk sharing. The allocation of consumption between entrepreneurs and workers satisfies

$$u'\left(\frac{C^i_t}{\chi}\right) = \rho_t u'\left(\frac{C^n_t}{1-\chi}\right). \tag{24}$$

Notice that ρ_t is inversely related to risk-sharing among household members and measures

[12] Once we know N_t, C^i_t, C^n_t, S_{t+1}, and B_{t+1}, the choice of S^i_{t+1}, S^n_{t+1}, B^i_{t+1}, and B^n_{t+1} are straightforward. We use $e_t = 1$ and C^i_t to obtain I_t from (13), and S^i_{t+1} from (9). Then, $S^n_{t+1} = S_{t+1} - S^i_{t+1}$. Finally, since $B^i_{t+1} = 0$, we know that $B^n_{t+1} = B_{t+1}$.

the impact of financing frictions on consumption risk sharing. To see this, suppose that idiosyncratic risks can be fully insured, as in a standard RBC model. In this case, entrepreneurs are not financially constrained and can implement the first-best investment schedule, such that the market price of private claims equals its internal replacement cost. This would imply $q_t = q_t^i = q_t^r = q_t^n = 1$. In such an unconstrained economy, entrepreneurs do not need to restrain themselves and are able to implementthe same consumption level as workers. Therefore, full insurance implies $\rho_t = 1$.

In contrast, in an economy where idiosyncratic labour income and investment risks are not insurable and the search market structure imposes further financing frictions, entrepreneurs cannot finance the first-best investment schedule. The market price of private assets remains above its replacement cost and $\rho_t > 1$. Because of the concavity of $u(.)$, we have $C_t^i/\chi < C_t^n/(1-\chi)$, i.e., entrepreneurs consume less than workers in order to expand investment.[13]

Portfolio choice. We now turn to the asset pricing formula for private assets. The first-order condition for S_{t+1} is

$$q_t^n u'\left(\frac{C_t^n}{1-\chi}\right) = \beta \mathbb{E}\left[J_S(S_{t+1}, B_{t+1}; \Gamma_{t+1})|\Gamma_t\right] \tag{25}$$

where J_S denotes the partial derivative of J w.r.t. S. The envelope condition implies that J_S is

$$J_S(S_t, B_t; \Gamma_t) \quad = \quad u'\left(\frac{C_t^n}{1-\chi}\right)\left[\chi\rho_t\left(r_t + 1 - \delta\right) + (1-\chi)\left(r_t + (1-\delta)q_t^n\right)\right]. \tag{26}$$

Then, we use (25) and (26) to obtain the asset pricing formula (Euler equation) for private claims

$$\mathbb{E}\left[\frac{\beta u'\left(\frac{C_{t+1}^n}{1-\chi}\right)}{u'\left(\frac{C_t^n}{1-\chi}\right)} r_{t+1}^e|\Gamma_t\right] = 1, \tag{27}$$

where the second term in the expectations operator captures the return on private claims from the perspective of the household:

$$r_{t+1}^e \equiv \chi\left(\rho_{t+1} r_{t+1}^{ni}\right) + (1-\chi) r_{t+1}^{nn}, \quad \text{with } r_{t+1}^{ni} \equiv \frac{r_{t+1} + (1-\delta)}{q_t^n},\ r_{t+1}^{nn} \equiv \frac{r_{t+1} + (1-\delta)q_{t+1}^n}{q_t^n}.$$

r_{t+1}^{ni} and r_{t+1}^{nn} are the returns from an individual worker's perspective. Recall that an entrepreneur's marginal utility of consumption is ρ_{t+1} times that of a worker. If a period-t

[13]Note that the absence of search costs, i.e. $\kappa = 0$, is not a sufficient condition for full consumption risk insurance, because labour income and wealth still differ across types, again leading to un-insurable investment risks. For a more detailed discussion see section 3.5.

worker becomes an entrepreneur at time $t+1$ (which happens with probability χ), the household's return to holding a unit of private assets is given by $\rho_{t+1} r_{t+1}^{ni}$, since workers value each unit of next-period resources in the hands of entrepreneurs at ρ_{t+1}. If the worker does not change type at time $t+1$ (which happens with probability $1-\chi$), the return to private claims is r_{t+1}^{nn}. Therefore, the return from the perspective of the household is a weighted sum of $\rho_{t+1} r_{t+1}^{ni}$ and r_{t+1}^{nn}.

Following similar steps, we can derive another asset pricing formula for money. Note that the return from money is the inverse of inflation, where inflation is defined as

$$\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}.$$

The optimality condition for money holdings B_{t+1}^{n} is then

$$u'\left(\frac{C_t^n}{1-\chi}\right)\frac{1}{P_t} = \beta \mathbb{E}\left[J_B\left(S_{t+1}, B_{t+1}; \Gamma_{t+1}\right)|\Gamma_t\right],$$

where J_B denotes the partial derivative of J w.r.t. B. The return on money from the household's point of view is simply $\left(\chi\rho_{t+1} + 1 - \chi\right)/\Pi_{t+1}$, where the return accruing to a future entrepreneur is again valued at ρ_{t+1}. Therefore, the asset pricing formula for money reads

$$\mathbb{E}\left[\frac{\beta u'\left(\frac{C_{t+1}^n}{1-\chi}\right)}{u'\left(\frac{C_t^n}{1-\chi}\right)}\frac{\left(\chi\rho_{t+1} + 1 - \chi\right)}{\Pi_{t+1}}|\Gamma_t\right] = 1. \tag{28}$$

In sum, equations (23), (24), (27), and (28) are the household's optimality conditions.

3.2 The Liquidity Premium

The above asset pricing formulae imply that private claims carry a liquidity premium, which compensates investors for impediments to transactions of these assets. For simplicity, we illustrate the liquidity premium by focusing onsteady state values, which will be denoted without time subscripts.

Notice that the optimality condition for money holdings (28) implies $\left(\chi\rho + 1 - \chi\right)\Pi^{-1} = \beta^{-1}$ with $\beta \in (0,1)$. Suppose that consumption risk can be fully insured. Then $\rho = 1$ and $\Pi^{-1} = \beta^{-1}$. However, since $\Pi = 1$ in the steady state with money in fixed supply, this condition cannot be satisfied. Therefore, money would not be valued in an unconstrained economy.

Further, given $\Pi = 1$ in the steady state with money being valued, we must have

$$\rho = \rho^* \equiv \chi^{-1}[\beta^{-1} - (1-\chi)] > 1$$

where ρ^* is a *parameter* that denotes the degree of risk-sharing in the steady state of an economy in which money is valued. As shown in (24), with $\rho > 1$ individual entrepreneurs are financing constrained and consumes less than workers. As a result, the real interest rate on liquid assets $\Pi^{-1} = 1$ is lower than the rate of time preference β^{-1} in such a constrained economy.[14] By providing a liquidity service, money mitigates such financing constraints and is, therefore, valued by agents in the steady state.[15]

Conversely, privately issued assets carry a liquidity premium, which amounts to the difference between the return from holding private claims and the return from holding money:

$$\Delta_t^{LP} \equiv \mathbb{E}\left[\chi r_{t+1}^{ni} + (1-\chi) r_{t+1}^{nn} | \Gamma_t\right] - \mathbb{E}\left[\Pi_{t+1}^{-1} | \Gamma_t\right].$$

As argued above, the liquidity premium $\Delta_t^{LP} > 0$, if two types of assets circulate and $\rho_t > 1$. This fact is best demonstrated in the steady state:

Proposition 1:
Suppose that the economy is in the steady state and that both private claims and money exist. Then, $r^{nn} > 1$ and money provides a liquidity service in the neighbourhood around the steady state. The steady state liquidity premium amounts to

$$\Delta^{LP} = \left[1 - (\rho^*)^{-1}\right](r^{nn} - 1)(1-\chi) > 0.$$

Proof. See Appendix B.1. □

3.3 The Asset Price and the Dimensions of Asset Liquidity

The asset price. Financial intermediaries determine the asset price to maximize the total match surplus of buyers and sellers. The sufficient and necessary first-order condition

[14] Although we focus on fiat money, such that $P_t = P$ in the steady state (and $\Pi^{-1} = 1 < \beta^{-1}$), similar results obtain in an economy where the government issues interest-bearing securities.

[15] One might believe that the optimal policy would consist in reducing this liquidity premium by paying a nominal interest rate R, such that the real interest rate $\bar{R}$ equals the rate of time preference β^{-1}. However, this is not necessarily true when government expenditure G is strictly positive and directly enters the household's utility. In this case, a benevolent government might still prefer a low real interest rate to finance its expenditures. While the exploration of optimal monetary and fiscal policies is beyond the scope of this paper, we refer the reader to Cui (2015).

associated with the bargaining problem (18) is

$$\omega J_t^n(S_t, B_t; \Gamma_t) = (1-\omega) J_t^i(S_t, B_t; \Gamma_t). \tag{29}$$

By using the household's optimality condition for asset holdings (25) and the risk-sharing condition (24), we can derive an analytical solution for the asset price:

Proposition 2:
Suppose that private claims exist. The bargaining solution for the asset price simplifies to

$$\rho_t = \frac{\omega}{1-\omega}\theta_t. \tag{30}$$

Alternatively, (30) *can be solved for the asset price*

$$q_t = \frac{\rho_t\left(1+\frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1+(\rho_t-1)\phi_t}. \tag{31}$$

Proof. See Appendix B.2. □

Proposition 2 is our main analytical result linking the asset price with search costs and asset saleability.[16] Importantly, the equilibrium on the market for private financial assets is not simply determined by a market clearing condition and the Euler equation for private claims, as it requires the asset price and asset market tightness (or asset saleability) to be pinned down simultaneously. The bargaining solution (30) solves this issue by establishing a relationship between asset saleability and the asset price.[17]

Market Participation. (30) is, in fact, a participation condition, which is similar to the entry conditions commonly found in the asset search literature (e.g., Rocheteau and Weill (2011) and Vayanos and Wang (2007)). To be specific, if the Euler equation for private assets determines the asset price, then demand and supply conditions as captured by asset market tightness θ_t need to be such that (30) is satisfied in order to induce individual agents to participate in the market. This participation decision can be illustrated by rewriting (30) as

$$\frac{(1-\omega)q_t^n M_t}{\omega q_t^r M_t} = \theta_t = \frac{\kappa_t V_t}{\kappa_t U_t},$$

where the LHS captures the ratio of the valuation of asset transactions by buyers and sellers,

[16] Although we do not solve explicitly for asset market tightness θ_t, ask size V_t, and bid size U_t, these could be easily backed out from (30) with the laws of motion of S_t^i and S_t^n.

[17] As a comparison, in a traditional asset pricing model, the Euler equation of the investors will determine the asset price, given their consumption profiles. In a production economy, we need further conditions to determine consumption paths. But in either case, assets have full liquidity and $\phi_t = 1$.

weighted by their respective bargaining weights; the RHS is the ratio of participation costs of buyers ($\kappa_t V_t$) and sellers ($\kappa_t U_t$). Buyers and sellers enter the market until this equilibrium condition is satisfied, i.e., they increase their ask and bid sizes until the ratio of gains and costs from participation are equal on either side of the market.[18]

Asset liquidity. As mentioned in Section 2, asset liquidity has three dimensions in our model: asset saleability, intermediation costs, and the price impact of asset transactions. All three dimensions interact and jointly affect the liquidity of private financial assets. The following arguments illustrates these inter-linkages.

First, consider the steady state. Suppose both private and public financial claims exist. Then, the asset pricing formula of money uniquely pins down the measure of consumption risk sharing to $\rho^* = \chi^{-1}[\beta^{-1} - (1-\chi)] > 1$ and (30) implies that search market tightness is $\theta = (1-\omega)\omega^{-1}\rho^*$. Since both asset saleability $\phi = M(1,\theta)$ and purchase probability $f = M(\theta^{-1}, 1)$ are functions of asset market tightness θ only, (30) also implies that that the relationship between the asset price q and intermediation costs κ can be directly determined. The formal result is stated in the following Corollary, an implication of Proposition 2:

Corollary 1:
Suppose that the economy is in the steady state and that both private claims and money co-exist. Then, asset market tightness satisfies $\theta = \frac{1-\omega}{\omega}\rho^*$. If the equilibrium after a permanent increase of intermediation costs κ still features the co-existence, then this increase of κ

1. *does not affect asset saleability ϕ or the purchase probability f and, hence, increases the bid-ask spread $\Delta^s \equiv q^n - q^i = \kappa(\phi^{-1} + f^{-1})$;*
2. *decreases the ask price q^i, i.e., $\frac{\partial q^i}{\partial \kappa} < 0$;*
3. *decreases the asset price q, i.e., $\frac{\partial q}{\partial \kappa} < 0$, if and only if $M(1, \frac{1-\omega}{\omega}\rho^*) < 1-\omega$.*

Proof. See Appendix B.3. □

Intuitively, an increase in κ implies that entrepreneurs have to spend more resources to engage in private asset transactions, such that their financing constraints tighten. As a result the supply of private claims on the search market and aggregate investment fall, while the marginal product of capital (MPK) increases, pushing up the asset price q_t. At the same

[18] One can interpret V_t as the amount of capital inflow into the asset market, which is akin to the concept of "funding liquidity" in Brunnermeier and Pedersen (2009). As $\theta_t = \xi^{\frac{1}{\eta-1}}\phi_t^{\frac{1}{1-\eta}}$ is also related to market liquidity ϕ_t, our framework thus provide the two-way interaction between "funding liquidity" and "market liquidity" in an otherwise standard general equilibrium setting.

time, demand for private claims will fall, provided that money circulates, as higher search costs drive buyers out of the private asset market into the money market, pushing up the liquidity premium and putting downward pressure on q.

The latter effect dominates the former if the demand side is more sensitive to changes in κ than the supply side. That is, when the market is relatively tight $M(1,\theta) < 1-\omega$. Note that sellers will never be able to recover the increased intermediation costs fully, such that the ask price q^i always decreases with κ. Since the bid-ask spreads Δ^s does not depend on the asset price, it always increases with intermediation costs.[19]

Second, in the presence of aggregate shocks, asset saleability fluctuates with the asset price q_t, linking the quantitative and price dimensions of asset liquidity. In the case of aggregate productivity shocks, for instance, the asset price co-moves positively with asset saleability, as long as the latter is sufficiently small.

Corollary 2:
Suppose that both private claims and money exist. In the absence of intermediation cost shocks, q_t then correlates positively with asset saleability ϕ_t, i.e., $\frac{\partial q_t}{\partial \phi_t} > 0$ if

$$\phi_t^{\frac{1}{1-\eta}} \leq \frac{1-\omega}{2\omega(1-\eta)} \xi^{\frac{1}{1-\eta}} \left[1 - 2\eta + \frac{\eta}{\phi_t}\right]. \tag{32}$$

When $\eta = \frac{1}{2}$, the above sufficient condition simplifies to

$$\phi_t \leq \xi^{\frac{2}{3}} \left[\frac{1-\omega}{2\omega}\right]^{\frac{1}{3}}.$$

Further, if the economy starts with the steady state, $\phi_t = \phi = M(1,\theta) = M\left(1, \frac{(1-\omega)\rho^*}{\omega}\right)$, then the sufficient condition is $M\left(1, \frac{(1-\omega)\rho^*}{\omega}\right) \leq \xi^{\frac{2}{3}} \left[\frac{1-\omega}{2\omega}\right]^{\frac{1}{3}}$.

Proof. See Appendix B.4. □

Again, this results reflects the impact of variation in asset market features on supply relative to demand. On the one hand, a drop in asset saleability tightens entrepreneurs'financing constraints, which reduces the supply of private claims and aggregate investment, but raises the MPK and the asset price q_t. On the other hand, persistently lower asset saleability implies that private assets are poorer instruments to insure against future idiosyncratic investment

[19] The analytical result presented in Corollary 2 is specific to aggregate productivity shocks with a constant $\kappa_t = \kappa$. We cannot obtain general results when κ_t is stochastic, as ϕ_t depends on κ_t and other macro variables. However, as shown by the numerical simulations in Section 5, the positive co-movement between q_t and ϕ_t is preserved as long as intermediation cost shocks dampen asset demand relative to asset supply.

risks. Therefore, demand falls, given the existence of the money market, as investors rebalance from private to public liquidity. This, again, raises the liquidity premium of private claims, thus pushing down q_t. Proposition 2 shows that the demand (or liquidity-premium) effect dominates the supply (or MPK) effect as long as asset saleability is sufficiently small.

Our model can thus generate simultaneous decreases in the asset price and asset saleability, thereby endogenously tightening financing constraints.

In sum, the cost, the quantity, and the price aspects of asset liquidity are linked in our model through the participation decisions of sellers and buyers on the asset search market and jointly give rise to the liquidity premium Δ_t^{LP}.

This result highlights the importance of modelling asset saleability as an *endogenous* market outcome, rather than an exogenous constraint. In Shi (2015) and Kiyotaki and Moore (2012), asset saleability ϕ is an exogenous parameter and constrains entrepreneurs, such that asset demand is only a (fixed) clearing factor. When ϕ falls, the asset supply schedule shifts to the left, while demand is not directly affected. Therefore, a drop of ϕ pushes up the asset price q in these models. In our model, a drop of ϕ reflects a simultaneous left-shift of both asset supply *and* demand, such that the asset price can drop.

Remark: The drop of both ϕ_t and q_t reduces aggregate investment

$$I_t = \frac{\left[r_t + \phi_t q_t^i (1-\delta)\right] S_t^i + \frac{B_t^i}{P_t} - C_t^i}{1 - \phi_t q_t^i}$$

via two effects. First, it reduces the saleable part of existing assets, thus shrinking the numerator. Second, it tightens the financing constraints and restricts entrepreneurs' ability to leverage, thus increasing the denominator.

3.4 The Existence of Private and Public Liquidity

After having characterized the equilibrium in which both private and public liquidity coexist, we now discuss the conditions for the existence of the different types of non-autarky equilibria. We focus on the respective steady states.

Types of non-autarky equilibria. The different types of non-autarky equilibria are parametrized by intermediation costs κ. Intuitively, one would conjecture the existence of two thresholds for the steady state level of intermediation costs, which separate these equilibria and characterize the existence of private and public liquidity.

To see this, we again start with the coexistence of private claims and money, such that $\rho = \rho^*$ as implied by the asset pricing formula for bonds. From (30), we know that search intensity $\theta = \frac{1-\omega}{\omega}\rho^*$ in the private asset market is uniquely determined. Also, asset saleability $\phi = M(1, \theta)$ is known. On the one hand, private liquidity (private financial claims) will only

be created if intermediation costs are not too large. To see this, recall from Section 3.1 that the replacement cost needs to satisfy $q^r \leq 1$ for private claims to exist, as entrepreneurs would otherwise only resort to internal financing. The definition of q^r implies that the ask price is bounded from below by unity, i.e., $q^i = q - \frac{\kappa}{\phi} \geq 1$. Moreover, the bargaining price q is bounded from below by zero and must be bounded from above as total resources of buyers are limited; similarly, asset saleability is bounded from below by zero and by unity from above. Therefore, a threshold $\kappa = \kappa_2$ must exist such that $q^i = 1$. Any value of intermediation costs in excess of this threshold would push the ask price q^i to below unity, and entrepreneurs would prefer to self-finance.

On the other hand, for public liquidity (money) to exist, intermediation costs cannot be too small. Otherwise, money would not be valued, because private claims would provide sufficient liquidity by themselves and dominate the return of money (which is $\Pi^{-1} = 1$). In order to verify the existence of public liquidity, consider the real value of liquidity L_t defined as

$$L_t \equiv \frac{B_t}{P_{t-1}}.$$

Given exogenous parameters, we check that money is held in equilibrium, i.e., $L(\kappa) \geq 0$. It turns out that for any level of intermediation costs below a threshold $\tilde{\kappa}_1$, the real value of liquid claims would become negative, and public liquidity would, hence, not be held in equilibrium. Finally, if $\tilde{\kappa}_1 \leq 0$, a non-monetary equilibrium does not exist with positive intermediation costs $\kappa > 0$.

These arguments are formally stated in the following proposition.

Proposition 3:

Suppose that the economy is in the steady state. Then, there are three types of non-autarky equilibria, depending on the level of intermediation costs κ:

1. *Co-existence. Both types of financial assets exist if and only if κ satisfies*

$$\kappa \in [\kappa_1, \kappa_2] \tag{33}$$

where κ_2 and κ_1 are defined as

$$\kappa_2 \equiv \frac{\rho^* - 1}{\frac{1-\omega}{\omega}\rho^* + 1} M\left(1, \frac{1-\omega}{\omega}\rho^*\right), \quad \kappa_1 \equiv \max\{0, \tilde{\kappa}_1\}, \quad \tilde{\kappa}_1 = H(\chi, \beta, \delta, \alpha, \xi, \eta, \omega)$$

and H is some non-linear function specified in appendix B.5. In this equilibrium, asset prices satisfy $q^n > q > q^i \geq 1 \geq q^r$ and the degree of risk-sharing is given by $\rho = \rho^* > 1$;

2. *Non-monetary equilibrium. For $\kappa \in [0, \kappa_1)$, public liquidity is not valued, such that only private liquidity exists. In this case, we have $q^n > q > q^i \geq 1 \geq q^r$, but risk-sharing improves to $1 < \rho < \rho^*$. Moreover, risk sharing deteriorates with κ, i.e., $\frac{\partial \rho}{\partial \kappa} > 0$, and asset saleability and the asset price also increase in intermediation costs, i.e., $\frac{\partial \phi}{\partial \kappa} > 0$ and $\frac{\partial q}{\partial \kappa} > 0$;*

3. *Pure monetary equilibrium. For $\kappa \in (\kappa_2, +\infty)$, private liquidity is not issued, such that only public liquidity exists. In this case, $\rho = \rho^* > 1$.*

Proof. See Appendix B.5. □

When $\kappa > \kappa_2$, entrepreneurs are strongly financing constrained, but the benefits of outside financing cannot compensate the costs of transacting private financial assets anymore. The upper bound κ_2 in condition (33) depends on the parameters of the matching function, and parameters ω, β, and χ. They determine the value of trading a unit of private claims and the willingness to participate in the private asset market. For example, the more impatient market participants are, i.e., the lower β, the more strongly their financing constraints will bind, i.e., the higher ρ^*. As both $\frac{\rho^*-1}{\gamma\rho^*+1}$ and $M(1, \gamma\rho^*)$ are increasing functions of ρ^*, the above condition implies that agents are willing to bear higher search costs before ceasing to issue private financial claims. That is, the threshold κ_2 moves up with more impatient participants who are more financing constrained.

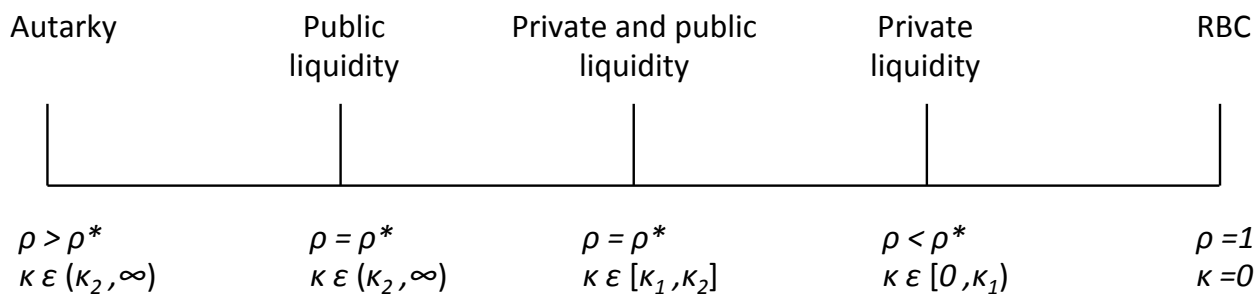
Importantly, there is a dis-continuity at the point κ_2. Although the replacement cost $q^r = 1$ whether entrepreneurs participate in the financial market or not, more wealth will be accumulated if private claims circulate. Once private claims cease to be traded, only low-yielding money provides liquidity, and the economy becomes less efficient. We will illustrate the dis-continuity with numerical examples in Section 4.

When $\kappa < \kappa_1$, financial frictions are less severe. This feature is reflected in the improved degree of risk-sharing $\rho < \rho^*$. In such a non-monetary equilibrium, intermediation costs act like capital-adjustment costs. An increase of κ within the region $[0, \kappa_1)$ impairs risk-sharing, such that ρ increases. The asset price q increases, reflecting the tighter financing constraints, while asset saleability ϕ needs to increase in order to encourage sellers' participation in the search market. That is, the MPK channel or the asset supply channel dominates, as there is no liquidity premium.

3.5 A continuum of equilibria

The level of intermediation costs κ and the degree of risk-sharing ρ jointly characterize a continuum of equilibria as shown in Figure 1.

Figure 1: **A Continuum of Equilibria** The different types of equilibria are differentiated by the level of intermediation costs κ and the degree of risk-sharing ρ. The latter measures the difference between the marginal utility of consumption of an entrepreneur relative to a worker.



The two polar cases in these dimensions are the equilibrium of a basic RBC model and the autarky equilibrium. In the basic RBC model, consumption risk is fully shared, such that $\rho = 1$. In contrast, an autarky economy features no risk-sharing due to the absence of insurance or private/public asset markets, such that $\rho > \rho^*$.

If we allow limited risk-sharing through the ciruclation of private claims and/or money to provide, we have a model that is between the RBC and the autarky economy. Between these polar cases exists a continuum of non-autarky equilibria as described in proposition (3), which are differentiated by the prevailing degree of intermediation costs.

Moreover, all equilibria, in which money is valued, exhibit the same degree of risk-sharing in the steady state. Nonetheless, aggregate demand, i.e., investment and consumption, are higher in the intermediate region $\kappa \in [\kappa_1, \kappa_2]$, where private claims and money co-exist, than in the region $\kappa \in (\kappa_2, +\infty)$ with purely monetary equilibria. This is, because the joint circulation of public *and* private liquidity facilitates the transfer of resources to entrepreneurs with investment opportunities. In other words, active asset markets serve as a lubricant for economic activity.

Remark: The absence of intermediation costs, $\kappa = 0$, is not sufficient for the steady state asset price to be one. It only implies $q^n = q^i$. Entrepreneurs will still be financing constrained as there are uninsured labour income risks. Money may or may not be valued depending whether $\kappa_1 \geq 0$ or not. However, if $\kappa = 0$ *and* labour income risks can be insured, we have $q_t^i = q_t^n = q_t^r = \rho_t = 1$ (see Appendix A.5).[20]

[20]In this case, if we denote total consumption as $C_t = C_t^i + C_t^n$, the representative household's budget constraint becomes $C_t + S_{t+1} = w_t N_t + [r_t + (1-\delta)] S_t$, which resembles the budget constraint in a basic RBC model. In fact, since the amount of private financial claims is normalized by the capital stock, we have $S_t = K_t$ in equilibrium, such that our model resembles a basic RBC framework.

4 Calibration

In order to illustrate the analytical results of our model, we calibrate the model to the US economy using data on macroeconomic aggregates and financial markets. In Section 5, we use this calibrated version of the model to evaluate its dynamic features in response to aggregate productivity and intermediation cost shocks. We choose a conventional CRRA utility function of consumption and a linear function for the dis-utility of labour:[21]

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \quad h(n) = \mu n.$$

4.1 Targets

We calibrate the steady state of the model to match several long-run characteristics of the US economy. The parameters capturing the discount factor, the coefficient of relative risk aversion, and the depreciation rate of capital (β, σ, and δ), are set exogenously to standard values in the literature. The capital share of output α and the weight of leisure in the period utility function μ are set to target the investment-to-GDP ratio and working hours (Table 2). Note that GDP in the model corresponds to the sum of real private consumption (C_t in the model) and real private investment in the data ($I_t + \kappa_t U_t + \kappa_t V_t$ in the model). Using this definition, we obtain an investment-to-GDP ratio of about 20% based on quarterly data from 1971Q1 to 2014Q4 from the FRED data set.

Table 2: Steady state calibration

Parameter		Baseline Value	Target/Source
Preferences and Production Technology			
Household discount factor	β	0.9850	Exogenous
Relative risk aversion	σ	2	Exogenous
Utility weight on leisure	μ	2.6904	Working time: 33%
Mass of entrepreneurs	χ	0.0540	Doms and Dunne (1998)
Depreciation rate of capital	δ	0.0250	Exogenous
Capital share of output	α	0.3750	Investment-to-GDP ratio: 20%
Search and Matching			
Supply sensitivity of matching	η	0.5000	Exogenous
Matching efficiency	ξ	0.2695	Saleability $\phi = 0.3000$
Bargaining weight	ω	0.5085	Tobins $q = 1.1500$
Search costs	κ	0.0216	Liquidity/GDP = 30.1%

There are four less conventional parameters related to the asset search-market $\{\xi, \eta, \kappa, \omega\}$

[21] This dis-utility function facilitates the steady state solution. The main results are robust to a more complicated specification. See the discussion in the calculation of steady state values in the appendix.

and one parameter χ that is related to idiosyncratic investment risks. ξ and η are not independent due to the constant returns to scale matching technology on the search market. Without loss of generality, we set $\eta = 0.5$ and calibrate ξ.

We are then left with four independent parameters $\{\xi, \kappa, \omega, \chi\}$, which we calibrate to match four targets. The population share of entrepreneurs in the model χ, can be interpreted as the fraction of firms, which adjust capital in each period. According to Doms and Dunne (1998), this fraction is about 20% annually in the US, which translates to a value of $\chi = 0.054$ at quarterly frequency (a similar value to the one in Shi (2015)).

The remaining three parameters are calibrated to jointly match three long-run targets: The asset price q captures Tobin's q, which ranges from 1.1 to 1.21 in the U.S. economy according to COMPUSTAT data. We target a value of $q = 1.15$. Steady state asset saleability ϕ is targeted at 0.30, which corresponds to the ratio of funds raised in the market to fixed investment in the U.S. flow-of-funds data.[22] Finally, as $\kappa = 0$ is likely to generate an equilibrium without the existence of fully liquid publicly issued assets (e.g. money), we calibrate κ such that the ratio of liquid assets to GDP, i.e. the real value of public liquidity divided by GDP L/Y, is 30.1% in the steady state. In the data, this number is identified as the ratio of the total amount of money-like assets, such as cash, checkable deposits and short-term Treasury bills, from the flow-of-funds data divided by GDP.

These calibration targets imply that intermediation costs are only about 2% of total GDP, and the annualized liquidity premium is about 106 basis points in line with previous empirical studies (e.g., Chen, Lesmond, and Wei, 2007).

4.2 The Long-run Impact of Intermediation Costs

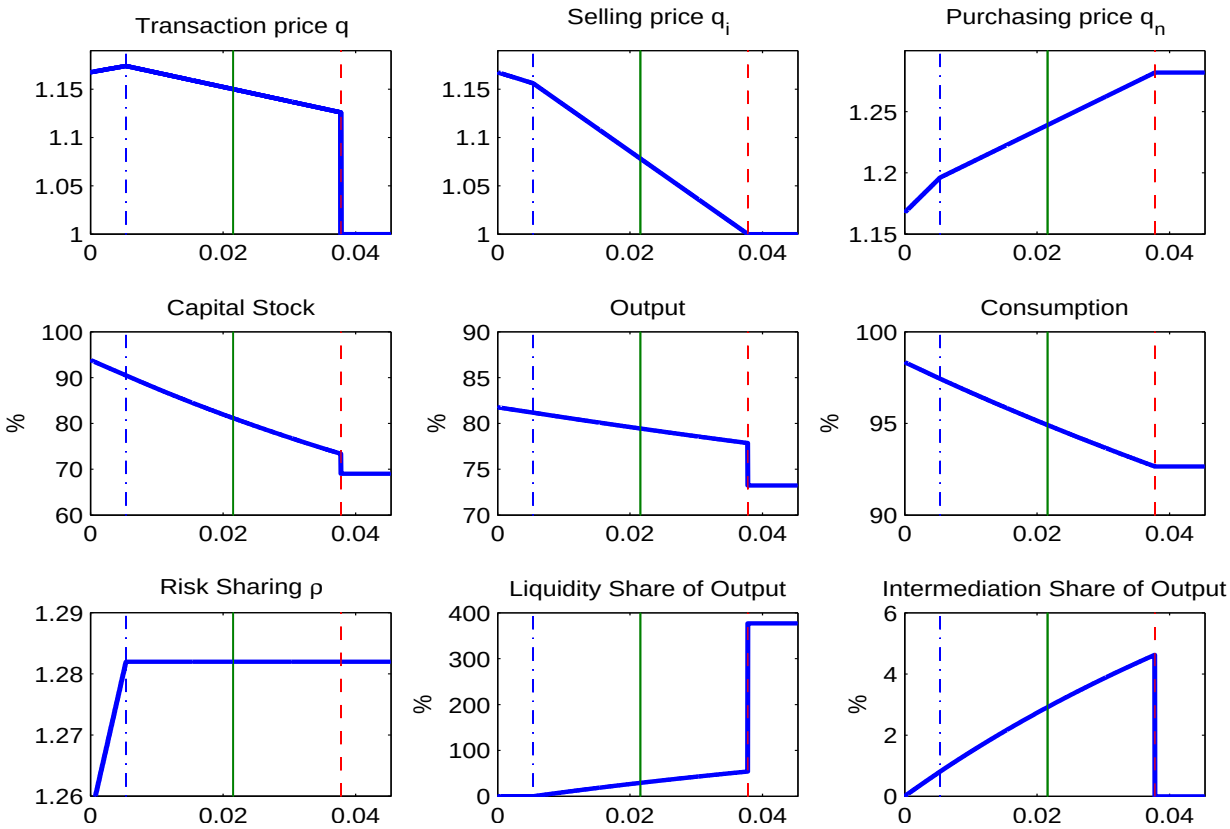
With the above parameters' choice, the conditions set out in Corollary 1 and Proposition 3 are satisfied. Using this calibration, we can verify that the model, indeed, exhibits the steady state properties of the three types of non-autarky equilibria discussed in Section 3. To that end, we trace steady state intermediation costs κ over the positive domain to illustrate the comparative statics of asset search market features and macroeconomic variables (see Figure 2). The two critical thresholds for intermediation costs, which separate the non-autarky equilibria, are

$$\kappa_1 = 0.0054 \quad \text{and} \quad \kappa_2 = 0.0378.$$

Non-monetary equilibrium. When $\kappa \in [0, 0.0054)$, only private claims exist as they provide sufficient liquidity to dominate money. Recall that the absence of intermediation costs ($\kappa = 0$) is not equivalent to a basic RBC economy, as investment opportunities and labour

[22] Nezafat and Slavik (2010) use the US flow-of-funds data for non-financial firms to estimate the stochastic process of ϕ.

Figure 2: **Comparative Statics**. Consumption, investment, and output are expressed as percentages of the corresponding quantities in a frictionless model, i.e., a basic RBC model (for details see Appendix A.4). The liquidity share of output is defined as L/Y and the intermediation-cost-to-output ratio as $\kappa(U+V)/Y$. The bold vertical line indicates the calibrated level of intermediation costs κ, while the dash-dotted line represents the lower threshold κ_1 and the dashed line the upper threshold κ_2.



income risk are still not fully insurable. As a result, the steady state capital stock of a model with search frictions characterizing financial markets is 7% lower than the RBC level even if $\kappa = 0$. Given the Cobb-Douglas production technology, lower capital accumulation reduces the marginal product of labour, such that demand for labour drops. Lower factor input depresses output to about 82% of that in a basic RBC model, while consumption drops by 2 percentage points.

As κ increases from 0 to κ_1, the risk-sharing coefficient ρ rises from 1.26 to $\rho^* = 1.282$. That is, the risk-sharing capacity of the economy deteriorates increasingly in intermediation costs as transferring funds via the search market becomes more costly. However, in this region intermediation costs are still small enough for money *not* to be valued, such that the liquidity share of output L/Y is zero. Asset saleability ϕ increases with ρ in order to encourage sellers' participation in the search market as captured by (30). Asset prices q

and q^n increase with search costs, reflecting the tighter financing constraints. Thus, search costs act like investment adjustment costs that consume resources, in response to which investment, consumption, and production fall.

Private and public liquidity. When $\kappa \in [0.0054, 0.0378]$, the liquidity of private claims - as captured by the combination of steady state asset saleability and transaction costs - falls sufficiently, such that money is valued and circulates in addition to private claims. The hedging value of money increases as intermediation costs rise. The liquidity share of output thus increases monotonically with search costs from 0% to 54%. In contrast to the region $[0, \kappa_1)$, the equilibrium asset price q now decreases in intermediation costs, as demand for private claims falls more strongly than supply, reflecting the theoretical results in Corollary 1. Meanwhile, the ask price q_i decreases to 1 when $\kappa = \kappa_2$, at which point entrepreneurs become indifferent between participating in the financial market and self-financing.

As intermediation costs increase from κ_1 to κ_2, capital stock, output and consumption drop, respectively, by about 20, 8, and 6 percentage points compared to a frictionless economy. The under-accumulation of capital and the associated fall in production and consumption, are driven by two effects: first, agents rebalance their portfolios towards money as intermediation costs increase. However, money delivers a smaller return than capital. Second, higher trading costs imply a larger resource loss per transaction of private claims. Both effects reduce entrepreneurs' net worth, thus limiting their capacity to create new capital.

Note that the macroeconomic impact is brought about by a mere increase of intermediation costs from about 1% to 4.8% of output. The strong impact of intermediation costs on real allocations points to sizeable amplification.

Purely monetary equilibrium. When $\kappa \in (0.0378, +\infty)$, only money circulates and ρ is still equal to ρ^*. As the private asset market are shut down, entrepreneurs cannot finance investment projects by issuing or re-selling claims, i.e., $\phi = 0$. Therefore, financing constraints tighten abruptly when intermediation costs cross the threshold κ_2, inducing a downward jump in investment and the steady state capital stock. Output declines by another 4.5% of the RBC level, as a result. In the absence of private assets, money is the only available means for risk-sharing purposes, such that demand for money soars and the liquidity share of output increases to about 380%. Once private asset markets are inactive, intermediation costs are irrelevant for real allocations, such that the long-run equilibrium becomes invariant to them.

The dis-continuity at $\kappa = \kappa_2 = 0.0378$ reflects our previous discussion. The shutting down of private asset market leads to much less capital accumulation and, therefore, reduces consumption and output permanently. Our calibration illustrates that even comparatively small increases in intermediation costs can lead to a complete shut-down of asset markets.

This finding highlights the inherent fragility of financial markets with endogenous participation similar to Gorton and Ordonez (2013).

5 Equilibrium Responses to Shocks

This section uses numerical tools to illustrate the model's dynamics after exogenous shocks. We consider two types of exogenous disturbances: standard total factor productivity (TFP) shocks and shocks to the intermediation capacity of financial markets, which are modelled as temporary changes of participation costs on the asset search market.

5.1 TFP Shocks and Intermediation Cost Shocks

We consider a standard AR(1) process for aggregate productivity, i.e.,

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A, \quad 0 < \rho_A < 1$$

with i.i.d. $\epsilon_t^A \sim N(0, \sigma_A^2)$. We further introduce a shock to the cost of financial intermediation, which in our framework corresponds to a change in the participation costs. We let

$$\log(1+\kappa_t) = \rho_\kappa \log(1+\kappa_{t-1}) + (1-\rho_\kappa)\log(1+\kappa) + \epsilon_t^\kappa, \quad 0 < \rho_\kappa < 1$$

with i.i.d. $\epsilon_t^\kappa \sim N(0, \sigma_\kappa^2)$. Rather than affecting the production frontier of the economy, this shock only impairs the capacity of the financial sector to intermediate funds between workers and entrepreneurs. Both in a market and a banking context, such an increase in intermediation costs may, for example, be triggered by rising uncertainty about counter-party risk. Such shocks unfold their effects through the endogenous response of asset saleability and prices, which affect entrepreneurs' financing constraints, investment, and production.

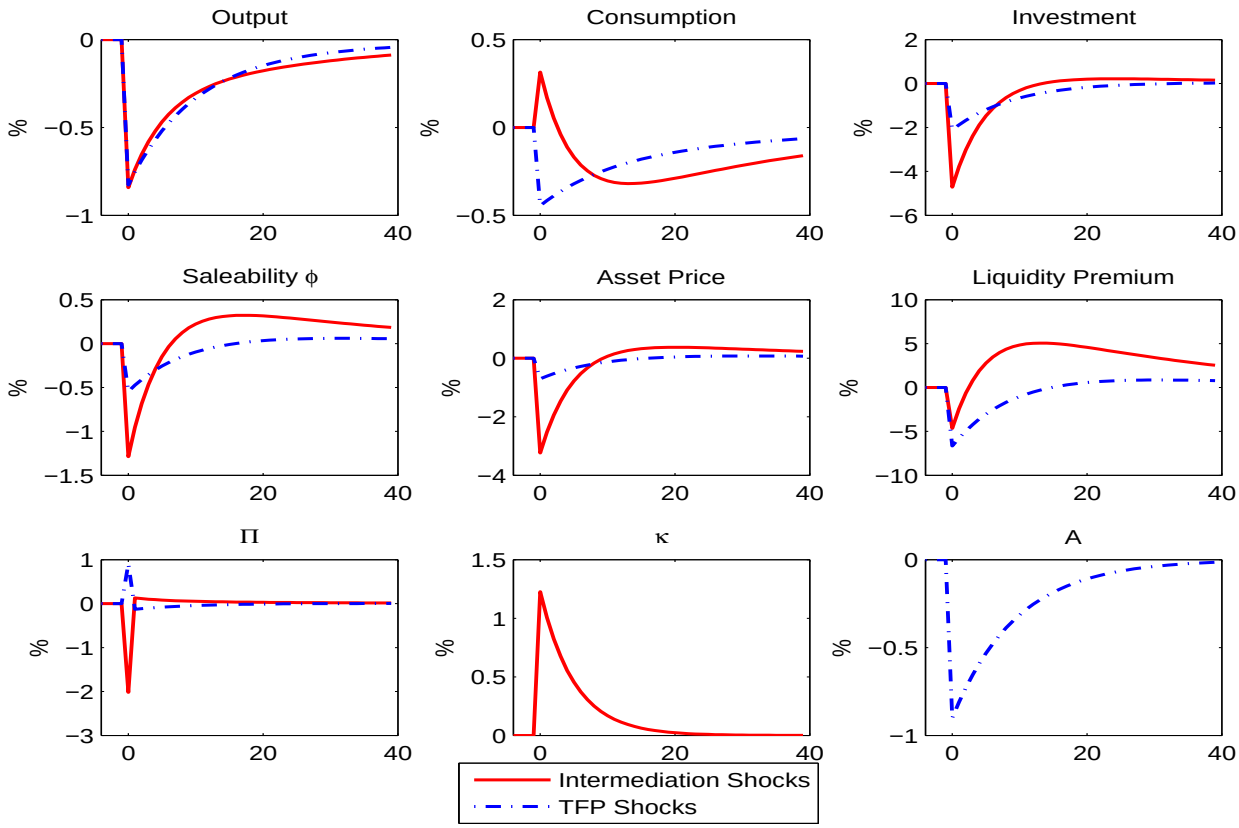
To compare productivity and intermediation cost shocks, the persistence and standard deviation of the underlying shock processes target the volatility (0.02) and first order correlation (0.91) of GDP's cyclical components (HP filtered with a smoothing coefficient of 1600). When using only productivity shocks, we have

$$\rho_A = 0.90, \ \sigma_A = 0.008.$$

When focusing on shocks to intermediation costs only, the exercise yields

$$\rho_\kappa = 0.82, \ \sigma_\kappa = 0.012.$$

Figure 3: Impulse responses after a standard deviation shock to aggregate productivity or intermediation search costs at time 0. Units of variables are percentage changes from their steady state levels.



We use these parameters in the subsequent numerical simulations. By design, both shocks will generate very similar aggregate output dynamics. We focus on the differences in the paths of other variables.

Negative aggregate productivity shocks. Suppose an adverse productivity shock hits the economy at time 0 (see A_t in Figure 3). This shock depresses the marginal product of capital and its value to the household. Search for investment into entrepreneurs becomes less attractive and the amount of purchase orders from workers drops. The demand-driven fall is reflected in the endogenous drop in asset saleability ϕ, which amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the asset price, falls - though only modestly - in line with our analytical result in Proposition 2. Both effects render private financial asset less liquid, thus tightening entrepreneurs' financing constraints. As a result, investment falls; consumption also falls because fewer resources are produced at the lower level of aggregate productivity.

In principle, money's liquidity service becomes more valuable to households when private

claims' liquidity declines. However, in the case of a persistent TFP shock, lower expected returns to capital make *future* investment less attractive. This effect weakens the incentive to hedge against asset illiquidity for future investment. The former effect has a positive impact on the liquidity premium, while the latter has a negative impact. Which effect dominates depends on the calibration and is thus an empirical question.

In our calibration, the decline in the profitability of investment projects is sufficient for the liquidity premium to drop. Therefore, the demand for liquid assets falls, which is reflected in the decrease of their price $1/P_t$ on impact and, conversely, a surge in inflation $\Pi_t = P_t/P_{t-1}$. To the extent that total factor productivity reverts back to the steady state, while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the liquidity premium.

Remark: The liquidity of financial assets is endogenously generated through the search market, and the impact of productivity shocks on investment is amplified by the tightening of entrepreneurs' financing constraints. In the absence of search frictions, these liquidity effects would not occur after adverse shocks.

Intermediation shocks. Suppose an increase of intermediation costs hits the economy at time 0 (see the dynamics of κ in Figure 3). The output dynamics in this scenario are, by construction, similar to those of the productivity shock.

Note that higher search costs bind resources. Both the substitution and income effects induce households to adjust their portfolios. Realizing that search market participation is more costly now and later, households seek to reduce their exposure to private financial claims, such that demand on the search market falls. On the supply side, financing-constrained entrepreneurs would still like to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Therefore, asset demand on the search market shrinks relative to supply, reducing the likelihood for sellers' quotes to be matched with buyers' quotes and depressing asset saleability.

The sharp drop in asset liquidity tightens entrepreneurs' financing constraints substantially. But the liquidity premium dominates the increase of the MPK. hence, q_t falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about six times stronger than that of output.

As saving via the financial market becomes more expensive with higher intermediation costs, workers reduce their labour supply and consume slightly more after the initial shock. Entrepreneurs, on the other hand, have to cut back consumption significantly in view of tightly binding financing constraints. Given the small population share of entrepreneurs, aggregate consumption increases slightly initially, while output falls on impact because of

the drop of labour hours. However, lower investment into the capital stock soon reduces the marginal product of labour and the wage rate. As labour income of workers falls, consumption persistently drops below the steady state.

While the intermediation cost shock depresses the demand for and the liquidity of private assets, it substantially increases the hedging value of money. To see this, note that future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by expanding their holdings of public liquidity. The additional demand increases the price of money, $1/P_t$, on impact, such that inflation $\Pi_t = P_t/P_{t-1}$ drops. Therefore, the liquidity premium initially falls due to the "flight to liquidity". However, once the real value of liquid assets has adjusted, the higher valuation of money relative to private assets leads to a persistent rise in the liquidity premium.

The faster accumulation of public liquidity relaxes future financing constraints, as entrepreneurs can finance more out of their stock of liquid assets and buyers have more resources to buy private claims. Both effects improve liquidity conditions on the private asset market. That is why both the asset price and asset saleability overshoot above the steady state levels after about 3 years.

5.2 Business Cycle Statistics

The equilibrium dynamics confirm two key results: (1) In order to reconcile declining asset saleability with falling asset prices, the former must be an endogenous phenomenon. In other words, ϕ_t must be a consequence, rather than a cause of economic disturbances. (2) Both standard productivity and intermediation cost shocks affect the hedging value of liquid assets. However, only the latter generate a negative co-movement between the liquidity premium and aggregate output. In light of these findings, we further compare the data with the model's predictions for the cyclical behaviour of macroeconomic and financial variables.

We use the Wilshire 5000 price full cap index from 1971Q1-2014Q4 as a proxy for q, as it covers a vast universe of traded stocks. An aggregate measure of asset liquidity, on the other hand, is more difficult to construct due to the various dimensions of asset liquidity and associated measurement problems. For instance, transaction costs and trading delays depend on many factors such as the size of a trade relative to market depth, its timing and the market structure.

In order to condense these characteristics of asset markets in a single indicator, we follow Naes, Skjeltorp, and Odegaard (2011), in choosing a simple and popular proxy for the illiquidity of private claims and money-like government issued assets suggested by Amihud (2002). This measure can be easily obtained from quarterly, monthly, or even daily data and

is constructed as an illiquidity ratio (ILR) as follows:

$$ILR_{i,T} = \frac{\sum_{t=1}^{T} \frac{|R_{i,t}|}{VOL_{i,t}}}{D_T}$$

where D_T is the number of trading days within a time window T, $|R_{i,t}|$ is the absolute return on day t for an asset i, and $VOL_{i,t}$ is the trading volume (in units of currency) on date t.

The ILR captures the price impact per volume unit of trades for a security i , thus combining the price and volume dimensions of asset liquidity. Liquid assets are traded in deep markets characterized by large transaction volumes and low price volatility. Therefore, liquid assets are associated with a low ILR, while the opposite is true for illiquid assets.

Notice that our model suggests the different liquidity properties of privately and publicly issued assets are collapsed in to the liquidity premium. As we do not directly observe the liquidity premium in the data, we construct an *illiquidity difference measure* as the empirical counterpart for the model-implied premium. This measure is computed as the difference between the illiquidity ratio for private claims ILR_T^P and the corresponding ratio for money-like assets ILR_T^M, i.e.

$$ILR_T^D = ILR_T^P - ILR_T^M.$$

To calculate the illiquidity difference measure, we use data on stock prices, returns, and trading volume. We use quarterly averages of monthly data obtained from the Center for Research in Security Prices (CRSP). Furthermore, we obtain an aggregate illiquidity ratio ILR_T^P as the equally weighted average of cross-sectional $ILR_{i,T}$ measures. Note that we restrict attention to stocks listed at the NYSE to keep the sample as homogenous as possible. We use a similar strategy to compute the illiquidity ratio ILR_T^M for 3-month Treasury bills.

We de-trend all time series with the HP filter (with a smoothing coefficient of 1600). Asset prices tend to fall in recessions, while our illiquidity difference measure increases, possibly reflecting portfolio rebalancing towards liquid assets (Figure 4). Not surprisingly, the illiquidity difference measure correlates negatively with GDP (-0.67), while asset prices correlate positively with GDP (0.51).

Some key business cycle statistics of the model in comparison to the data are reported in Table 3, where only TFP shocks are considered. Similar to a basic RBC model, consumption and investment volatility, the correlation of macroeconomic variables with GDP, and first-order autocorrelations are roughly in line with the data. However, the liquidity premium and the asset price move too little in the model. Besides, the model-implied positive correlation between the liquidity premium and GDP (0.78) is at odds with the data (-0.67). All these

Figure 4: **Cyclical components of illiquidity difference measure, asset prices, and GDP.** All series are cyclical components of HP filtered original series times 100. The shaded areas are NBER dated recessions.

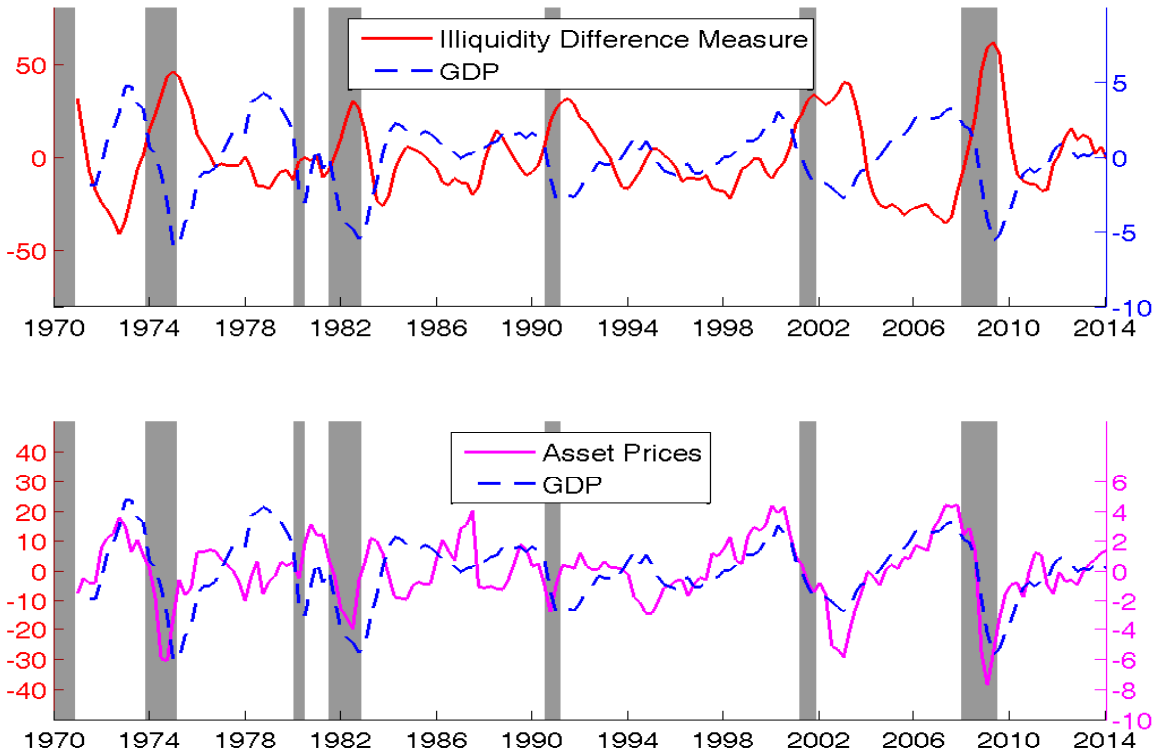


Table 3: **Business cycle statistics with only TFP shocks**

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.66	0.93	0.98	0.89	0.94
Investment	3.21	2.26	0.95	0.98	0.86	0.89
Liquidity premium	9.31	6.62	-0.67	0.78	0.92	0.87
Asset Price	4.88	0.70	0.51	0.82	0.81	0.87

statistics confirm the results gleaned from the impulse responses to TFP shocks.

As a comparison, Table 4 show the relevant statistics associated with intermediation cost shocks as the only exogenous disturbance. Compared to the previous case, the volatility of both the liquidity premium and the asset price increase substantially. In addition, the volatility of investment is closer to the data, while consumption becomes more volatile.

Importantly, the model with shocks to intermediation costs successfully generates coun-

Table 4: **Business cycle statistics with only intermediation cost shocks**

	Relative volatility $\frac{\sigma_x}{\sigma_y}$		Correlation $\rho(x, y)$		1st auto-correlation	
Variable x	Data	Model	Data	Model	Data	Model
Output	0.02	0.02	1.00	1.00	0.91	0.91
Consumption	0.58	0.82	0.93	0.48	0.89	0.97
Investment	3.21	3.86	0.95	0.78	0.86	0.80
Liquidity premium	9.31	13.03	-0.67	-0.50	0.92	0.97
Asset Price	4.88	2.64	0.51	0.58	0.81	0.79

tercyclical movements in the liquidity premium (correlation with GDP: -0.50), mimicking the deterioration of private assets' liquidity relative to publicly issued assets typically observed in recessions. As a result, the liquidity premium can serve as a discriminant between the sources of recessions. In addition, the asset price is more volatile, and its correlation with GDP (0.58) is closer to the data (0.51). This correlation is substantially smaller than that in the model with only TFP shocks (0.82) due to the overshooting of the asset price after search cost shocks (Figure 3).

5.3 Discussion

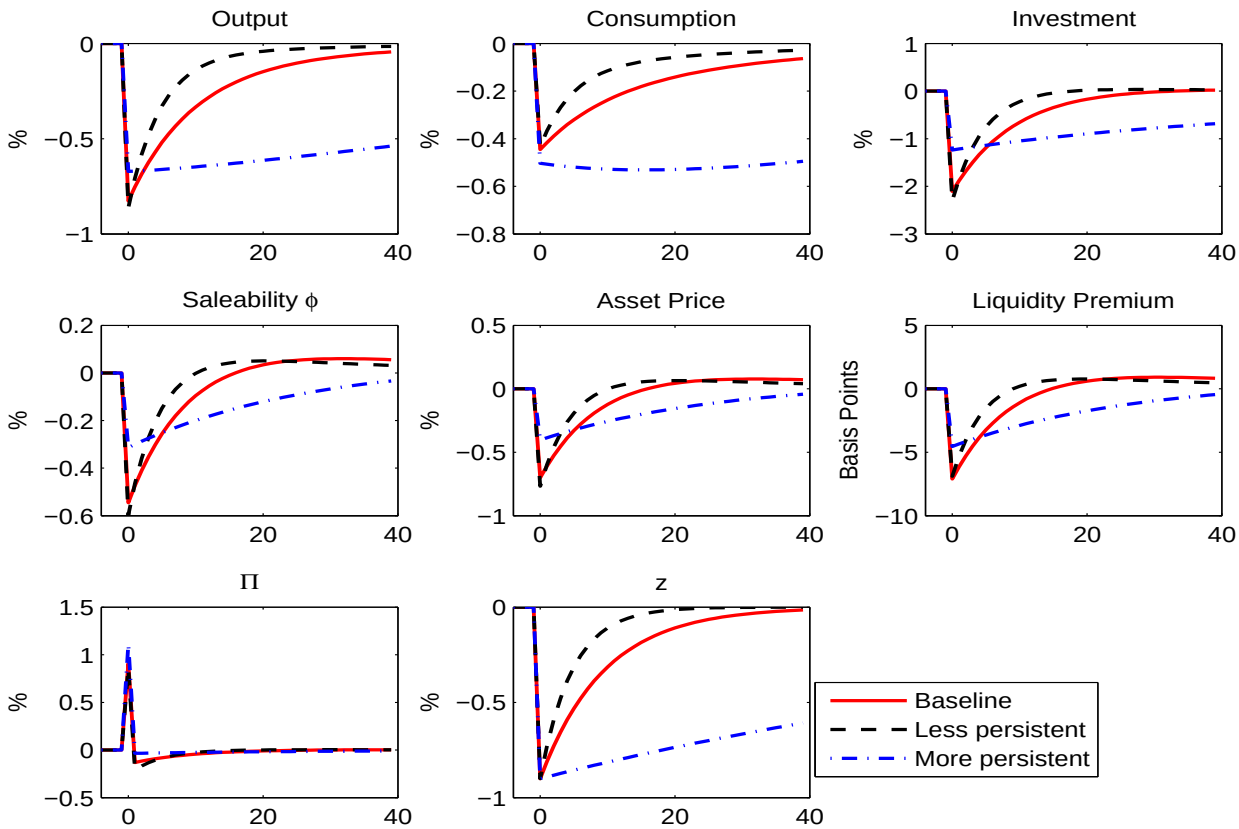
After having illustrated the dynamic properties of our model, we discuss a number of key determinants of these dynamics in greater detail in this section.

Hedging value of money. While intermediation cost shocks increase the hedging value of money as shown in Section 5.1, TFP shocks may have an ambiguous effect on the incentive to hold cash reserves. Persistently low productivity diminishes the return on capital, such that investment becomes less profitable and the willingness to hedge idiosyncratic investment risks shrinks. At the same time, low productivity depresses entrepreneurs' net worth, such that financing constraints become more binding. This effect should raise the hedging motive and the willingness to hold money. In the baseline experiment displayed in Figure 3, the first effect clearly dominates the second effect, such that the liquidity premium contracts strongly.

However, as both effects crucially depend on the persistence of the low-productivity spell, we illustrate the sensitivity of the net effect by varying the persistence of the productivity shock, by plus and minus 10 percent compared to the baseline calibration. After changing the persistence of shocks, the equilibrium dynamics are similar to the baseline simulation (Figure 5). But different degrees of persistence alter the magnitude and the speed of the adjustment of macroeconomic and financial variables.

When the productivity process is more persistent than in the baseline scenario, agents

Figure 5: Impulse responses after a standard deviation shock to aggregate productivity at time 0. The units of liquidity premium are annualized changes in basis points. Units of other variables are percentage changes from their steady state levels.



anticipate that their net worth will be persistently lower, such that financing constraints will remain tight for longer. As a result, the hedging value of money rises, such that the liquidity premium contracts less in the first few quarters after the initial shock. In fact, households curtail their consumption compared to the baseline scenario, in order to accumulate money holdings and buffer investment on impact. Thereafter, higher cash reserves help entrepreneurs finance investment and workers purchase private claims, which prevents output, investment, saleability, and the asset price from dropping as much as in the baseline. Naturally, the economy takes longer to revert back to the steady state from these less compressed levels due to the high persistence of the aggregate productivity shock.

Endogenous asset saleability. Kiyotaki and Moore (2012) and Shi (2015) consider aggregate liquidity shocks in the form of an exogenous and persistent reduction of asset saleability ϕ. This shock mechanically depresses the supply of private assets on financial markets by tightening entrepreneurs' financing constraints. Demand for private claims, on the other hand, is hardly affected by such a shock as the return on capital goods does not fall. As

supply contracts relative to supply on the asset market, such adverse aggregate liquidity shocks have the unrealistic feature of generating asset price booms. In other words, tighter financing constraints implies a higher value of Tobin's q.

In contrast, our endogenous asset liquidity framework demonstrates that financial shocks need to strongly affect the demand side of the asset market in order to overturn this anomaly in the reaction of the asset prices. In our model environment, higher intermediation costs directly deter buyers from participating in the asset search market. As a result the average size of their bid quotes declines. This slump in asset demand is amplified by the persistence of intermediation cost shocks. This is because buyers who perceive financial markets to be illiquid for an extended period anticipate that holding additional private financial assets may constrain their own funding ability in the future, thus becoming even less inclined to buy them. These demand side effects only occur with endogenous asset saleability.[23] This is why we consider *shocks to the financial sector*, instead of shocks to *the financing constraints*.

Investment-specific technological shocks. While Shi (2015) suggests that aggregate productivity shocks are necessary to overturn the asset price anomaly generated by exogenous liquidity shocks, our simulations show that pure financial shocks are sufficient, provided that asset liquidity is modelled endogenously. The financial shocks considered here are different from productivity shocks, since they affect investment via financing constraints rather than directly reducing the production frontier of the economy.

To see this, recall the goods market clearing condition (20)

$$C_t + I_t + \kappa_t (V_t + U_t) = Y_t.$$

Aggregate productivity shocks directly affect the RHS and then affect consumption and investment on the LHS, while intermediation cost shocks directly affect the investment-related costs $\kappa_t (V_t + U_t)$ on the LHS and then affect labour supply and output on the RHS. Such cost shocks may thus be interpreted as a particular form of non-linear investment-specific technology shocks (see, e.g., Greenwood, Hercowitz, and Krusell (1997), Fisher (2006) and Primiceri, Justiniano, and Tambalotti (2010)), whose impact is amplified by their effect on endogenous market participation.

[23] An alternative financial market disturbance in our framework is a shock to the matching function itself. More specifically, we shock the matching efficiency ξ in order to check whether an efficiency problem occurring in the financial sector could induce co-movment between the asset price and asset saleability. An adverse efficiency shock, capturing, for example, a contagious bank run, impairs the intermediation capacity of the financial sector. Nevertheless, such a shock triggers an increase in the asset price as the supply reaction dominates. The detailed simulation results of the matching efficiency shock are available upon request from the authors.

6 Conclusion

We endogenise asset liquidity in a macroeconomic model with search frictions. Endogenous fluctuation of asset liquidity are triggered by shocks that affect asset demand and supply on the search market either directly (intermediation cost shocks), or indirectly (productivity shocks). By tightening entrepreneurs' financing constraints, these shocks feed into investment, consumption and output. Our model is able to capture several dimensions of asset liquidity. In particular, we show that asset prices are linked to market pairticipation by both sellers and buyers and can co-move with asset saleability. The *endogenous* nature of asset liquidity is key to match this positive correlation, as *exogenous* liquidity shocks would act as negative supply shocks on the asset market and lead to asset price booms in recessions.

We also show that the liquidity service provided by intrinsically worthless liquid assets, is higher when financing constraints bind tightly. As a result, shocks to the cost of financial intermediation increase the hedging value of liquid assets, enabling our model to replicate the flight-to-liquidity dynamics measured by a countercyclical liquidity premium, thus matching U.S. business cycle features.

Our model admits different types of equilibria, which are distinguished by the types of financial assets that circulate and the amount of riks sharing via financial markets. One polar case is autarky, i.e., an equilibrium in which neither private claims nor money exist, financial intermediation shuts down and entrepreneurs finance investment fully with inside funding. The non-autarky case spans three types of equilibria. In the first, only public liquidity circulates. Intuitively, this is the case when search costs are prohibitively high for agents to participate in private asset markets. If intermediation costs are sufficiently small, on the other hand, private claims provide sufficient liquidity, such that they dominate public liquidity. In the limit, as private financial asset markets become entirely frictionless and as all types of risk are shared, the model also nests a RBC economy. Finally, both private and public liquidity may coexist for intermediate levels of intermediation costs.

While it is straightforward to interpret our asset search framework as a model of market-based financial intermediation, it can also be seen as a short-cut to modelling bank-based intermediation: Financial intermediaries help channel funds from investors to suitable creditors in need of outside funding, which resembles a matching process. Adding further texture by explicitly accounting for intermediaries' balance sheets would open interesting interactions between liquidity cycles and financial sector leverage and maturity transformation.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, government demand may crowd out private demand due to congestion externalities in an endogenous liquidity framework. Therefore, future research could focus

on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

References

Amihud, Y. (2002): "Illiquidity and stock returns: cross-section and time-series effects," *Journal of financial markets*, 5(1), 31–56.

Anderson, R. G., and C. S. Gascon (2009): "The Commercial Paper Market, the Fed, and the 2007-2009 Financial Crisis," *Federal Reserve Bank of St. Louis Review*, 91(6), 589–612.

Ashcraft, A. B., and D. Duffie (2007): "Systemic Illiquidity in the Federal Funds Market," *AEA Papers and Proceedings*, 97(2), 221–25.

Bao, J., J. Pan, and J. Wang (2011): "The Illiquidity of Corporate Bonds," *Journal of Finance*, 66(3), 911–946.

Beaubrun-Diant, K. E., and F. Tripier (2013): "Search Frictions, Credit Market Liquidity, and Net Interest Margin Cyclicality," working paper, CEPII.

Brunnermeier, M. K., and L. H. Pedersen (2009): "Market liquidity and funding liquidity," *Review of Financial studies*, 22(6), 2201–2238.

Chen, L., D. A. Lesmond, and J. Wei (2007): "Corporate yield spreads and bond liquidity," *The Journal of Finance*, 62(1), 119–149.

Chordia, T., R. Roll, and A. Subrahmanyam (2001): "Market Liquidity and Trading Activity," *The Journal of Finance*, 56(2), 501–530.

Chordia, T., A. Sarkar, and A. Subrahmanyam (2005): "An Empirical Analysis of Stock and Bond Market Liquidity," *Review of Financial Studies*, 18(1), 85–129.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104(1), 27–65.

Cui, W. (2013): "Delayed Capital Reallocation," working paper, University College London.

——— (2015): "Optimal Policy with Liquidity Frictions," working paper, University College London.

De-Fiore, F., and H. Uhlig (2011): "Bank Finance Versus Bond Finance," *Journal of Money, Credit and Banking*, 43(7), 1399–1421.

Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2011): "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities," *Federal Reserve Bank of New York Staff Report*, 520.

Den Haan, W. J., G. Ramey, and J. Watson (2003): "Liquidity flows and fragility of business enterprises," *Journal of Monetary Economics*, 50(6), 1215–1241.

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2012): "Corporate bond liquidity before and after the onset of the subprime crisis," *Journal of Financial Economics*, 103(3), 471–492.

Doms, M., and T. Dunne (1998): "Capital Adjustment Patters in Manufacturing Plants," *Review of Economic Dynamics*, 1, 409–429.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005): "Over-the-counter markets," *Econometrica*, 73(6), 1815–1847.

——— (2007): "Valuation in Over-the-Counter Markets," *Review of Financial Studies*, 20(6), 1865–1900.

Ebell, M. (2011): "On the cyclicality of unemployment: Resurrecting the participation margin," *Labour Economics*, 18(6), 822–836.

Eisfeldt, A. L. (2004): "Endogenous Liquidity in Asset Markets," *Journal of Finance*, 59(1), 1–30.

Eisfeldt, A. L., and A. A. Rampini (2006): "Capital reallocation and liquidity," *Journal of Monetary Economics*, 53, 369–399.

Eisfeldt, A. L., and A. A. Rampini (2009): "Financing Shortfalls and the Value of Aggregate Liquidty," working paper, UCLA and Duke University.

Feldhutter, P. (2011): "The Same Bond at Different Prices: Identifying Search Frictions and Selling Pressures," *Review of Financial Studies*, 25(4), 1155–1206.

Fisher, J. (2006): "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks," *Journal of Political Economy*, 114(3), 413–451.

Gertler, M., and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58(1), 17–34.

Gorton, G., and A. Metrick (2012): "Securitized banking and the run on repo," *Journal of Financial Economics*, 104(3), 425–451.

Gorton, G. B., and G. Ordonez (2013): "The supply and demand for safe assets," Discussion paper, National Bureau of Economic Research.

Greenwood, J., Z. Hercowitz, and P. Krusell (1997): "Long-run implications of investment-specific technological change," *The American Economic Review*, pp. 342–362.

Guerrieri, V., and R. Shimer (2012): "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *NBER Working Paper*, 17876.

Huberman, G., and D. Halka (2001): "Systematic Liquidity," *Journal of Financial Research*, 24(2), 161–178.

Jaccard, I. (2013): "Liquidity Constraints, Risk Premia, and the Macroeconomic Effects of Liquidity Shocks," ECB working paper, 1525.

Jermann, U., and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), 238–271.

Justiniano, A., G. Primiceri, and A. Tambalotti (2015): "Credit Supply and the Housing Boom," working paper, Northwestern University.

Kara, E., and J. Sin (2013): "Liquidity, Quantitative Easing and Optimal Monetary Policy," working paper, University of Bristol.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*. Macmillan Cambridge University Press.

Kiyotaki, N., and J. Moore (2012): "Liquidity, Business Cycles, and Monetary Policy," *NBER Working Paper*, 17934.

Kurlat, P. (2013): "Lemons Markets and the Transmission of Aggregate Shocks," *American Economic Review*, 103(4), 1463–1489.

Kurmann, A., and N. Petrosky-Nadeau (2006): "Credit Market Frictions with Costly Capital Reallocation as a Propagation Mechanism," SED 2006 Meeting Papers 365.

Lagos, R., and G. Rocheteau (2008): "Money and capital as competing media of exchange," *Journal of Economic Theory*, 142(1), 247–258.

——— (2009): "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 77(2), 403–426.

Lagos, R., G. Rocheteau, and P.-O. Weill (2009): "Crises and Liquidity in Over-the-Counter Markets," .

Lagos, R., and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463–484.

Lucas, R. E. (1990): "Liquidity and interest rates," *Journal of Economic Theory*, 50(2), 237–264.

Mortensen, D. T., and C. A. Pissarides (1994): "Job creation and job destruction in the theory of unemployment," *The Review of Economic Studies*, 61(3), 397–415.

Naes, R., J. A. Skjeltorp, and B. A. Odegaard (2011): "Stock Market Liquidity and the Business Cycle," *Journal of Finance*, 66, 138–176.

Nezafat, M., and C. Slavik (2010): "Asset Prices and Business Cycles with Financial Shocks," Social Science and Research Network working paper 1571754.

Petrosky-Nadeau, N., and E. Wasmer (2013): "The cyclical volatility of labor markets under frictional financial markets," *American Economic Journal: Macroeconomics*, 5(1), 1–31.

Primiceri, G., A. Justiniano, and A. Tambalotti (2010): "Investment Shocks and Business Cycles," *Journal of Monetary Economics*, 57(2), 132–145.

Radde, S. (2015): "Flight to Liquidity and the Great Recession," *Journal of Banking and Finance*, 54, 192–207.

Ravn, M. O. (2008): "The Consumption-Tightness Puzzle," *NBER International Seminar on Macroeconomics 2006*, pp. 9–63.

Rocheteau, G., and P.-O. Weill (2011): "Liquidity in frictional asset markets," *Journal of Money, Credit and Banking*, 43(s2), 261–282.

Rocheteau, G., and R. Wright (2005): "Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium," *Econometrica*, 73(1), 175–202.

Shi, S. (2015): "Liquidity, Assets and Business Cycles," *Journal of Monetary Economics*, 70, 116–132.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

Tobin, J. (1958): "Liquidity Preferences as Behaviour Towards Risk," *Review of Economic Studies*, 25(2), 65–86.

——— (1969): "A General Equilibrium Approach to Monetary Theory," *Journal of Money, Credit and Banking*, 1(1), 15–29.

Ungerer, C. (2012): "Monetary Policy , Hot Housing Markets And Household Debt," working paper, LSE.

Vayanos, D., and T. Wang (2007): "Search and endogenous concentration of liquidity in asset markets," *Journal of Economic Theory*, 136(1), 66–104.

Wasmer, E., and P. Weil (2004): "The Macroeconomics of Labor and Credit Market Imperfections," *American Economic Review*, 94(4), 944–963.

Wheaton, W. C. (1990): "Vacancy , Search , and Prices in a Housing Market Matching Model," *Journal of Political Economy*, 98(6), 1270–1292.

Williamson, S. D. (2012): "Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach," *American Economic Review*, 102(6), 2570–2605.

Appendices

A Equilibrium Conditions

A.1 Recursive Competitive Equilibrium

Suppose we have the both existence of equity and money. Using the fact that total consumption $C_t = C_t^n + C_t^i$ and (24), we know $C_t^n = \rho_t^n C$ and $C_t^i = \rho_t^i C$, where

$$\rho_t^n \equiv \frac{1-\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}, \quad \rho_t^i \equiv \frac{(u')^{-1}(\rho_t)\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}.$$

$(u')^{-1}$ is the inverse function of marginal utility of consumption. When we use the CRRA utility $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, $\rho_t^n \equiv \frac{1-\chi}{1-\chi+\rho_t^{-1/\sigma}\chi}$, $\rho_t^i \equiv \frac{\chi\rho_t^{-1/\sigma}}{1-\chi+\rho_t^{-1/\sigma}\chi}$. We further define the real liquidity as $L_t = \frac{B_t}{P_{t-1}}$. Given the aggregate state variables (K_t, A_t, κ_t), we solve the equilibrium system

$$\left(K_{t+1}, L_{t+1}, C_t, I_t, N_t, \rho_t, \rho_t^i, \rho_t^n, \phi_t, f_t, q_t, q_t^i, q_t^n, r_t, w_t, \Pi_t\right)$$

together with the exogenous laws of motion of (A_t,κ_t). To solve for these 16 endogenous variables, we use the following 16 equilibrium conditions:

1. The representative household's optimality conditions:

$$u'\left(\frac{\rho_t^n C_t}{1-\chi}\right) w_t = h'(\frac{N_t}{1-\chi}), \quad \rho_t^n \equiv \frac{1-\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}, \quad \rho_t^i \equiv \frac{(u')^{-1}(\rho_t)\chi}{1-\chi+(u')^{-1}(\rho_t)\chi}.$$

$$1 = \beta\mathbb{E}_t\left[\frac{u'(\rho_{t+1}^n C_{t+1})}{u'(\rho_t^n C_t)}\left[\chi\rho_{t+1}+1-\chi\right]\frac{1}{\Pi_{t+1}}\right] \tag{34}$$

$$1 = \beta\mathbb{E}_t\left[\frac{u'(\rho_{t+1}^n C_{t+1})}{u'(\rho_t^n C_t)}\frac{\left(\chi\rho_{t+1}+1-\chi\right)r_{t+1}+(1-\delta)\left(\chi\rho_{t+1}+(1-\chi)\,q_{t+1}^n\right)}{q_t^n}\right] \tag{35}$$

$$I_t = \frac{\left[\left(r_t+(1-\delta)\phi_t q_t^i\right)\chi K_t + \chi L_t\right] - \rho_t^i C_t}{1-\phi_t q_t^i} \tag{36}$$

2. Final goods producers:

$$r_t = \alpha A_t\left(\frac{K_t}{N_t}\right)^{\alpha-1}, \quad w_t = (1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha} \tag{37}$$

3. Market clearing:

   (a) The household's budget constraint:

$$\begin{aligned}\left(\rho_t\rho_t^i+\rho_t^n\right)C_t + L_{t+1} + q_t^n K_{t+1} &= w_t N_t + \left[\chi\rho_t+(1-\chi)\right]\frac{L_t}{\Pi_t} + \left[\chi\rho_t+(1-\chi)\right]r_t K_t \\ &+ \left[\chi\rho_t+(1-\chi)q_t^n\right](1-\delta)K_t \end{aligned} \tag{38}$$

   (b) Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t$

   (c) Given the matching function (note: $\gamma \equiv \frac{\omega}{1-\omega}$,)

$$\phi_t = M(1, \frac{\rho_t}{\gamma}), \quad f_t = M(\frac{\gamma}{\rho_t}, 1)$$

When we specify $M(U_t, V_t) = \xi U_t^{\eta} V_t^{1-\eta}$, we have $\phi_t = \xi \left(\gamma^{-1}\rho_t\right)^{1-\eta}$, $\quad f_t = \xi^{\frac{1}{1-\eta}} \phi_t^{\frac{\eta}{\eta-1}}$

(d) Asset Prices

$$q_t = \frac{\rho_t \left(1 + \frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1 + (\rho_t - 1)\,\phi_t}, \quad q_t^i = q - \frac{\kappa_t}{\phi_t}, \quad q_t^n = q + \frac{\kappa_t}{f_t}$$

(e) Liquid assets in fixed supply (note: $L_t = \frac{B_t}{P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$): $L_{t+1} = \frac{L_t}{\Pi_t}$.

A.2 The Steady State

The following illustrates one particular approach of computing the steady state values of 16 variables when both private claims and money are valued. For ease of exposition, we directly use $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and $h(n) = \mu n$. In fact, no numerical solver is necessary because of the design of the model. Again, we use the variable itself without the time subscript to denote the steady state.

First, notice that market clearing for liquid assets implies that $\Pi = 1$. Next, we use (34) to obtain

$$\rho = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right] \tag{39}$$

and therefore

$$\rho^n \equiv \frac{1-\chi}{1-\chi+\rho^{-1/\sigma}\chi}, \quad \rho^i \equiv \frac{\chi\rho^{-1/\sigma}}{1-\chi+\rho^{-1/\sigma}\chi} \tag{40}$$

With ρ, we know that

$$\phi = M(1, \frac{\rho}{\gamma}), \quad f_t = M(\frac{\gamma}{\rho}, 1) \tag{41}$$

Again, when we specify $M(U_t, V_t) = \xi U_t^{\eta} V_t^{1-\eta}$ we have $\phi = \xi\left(\gamma^{-1}\rho\right)^{1-\eta}$ and $f = \xi^{\frac{1}{1-\eta}}\phi^{\frac{\eta}{\eta-1}}$. Next, we can compute asset prices

$$q = \frac{\rho\left(1+\frac{\kappa}{\omega}\right) - \frac{\kappa}{f}}{1+(\rho-1)\,\phi}, \quad q^i = q - \frac{\kappa}{\phi}, \quad q^n = q + \frac{\kappa}{f} \tag{42}$$

From (35) and (37), we have

$$r = \frac{\frac{q^n}{\beta} - (1-\delta)\left(\chi\rho + (1-\chi)\,q^n\right)}{\chi\rho + 1 - \chi} \tag{43}$$

$$w = (1-\alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \quad C = \left(\frac{w}{\mu}\right)^{1/\sigma}\frac{1-\chi}{\rho^n} \tag{44}$$

Now, we need to solve real liquidity value L and capital stock K. One can simplify (36) and (38) to be

$$\rho^i C + dK = \chi L$$

$$(\rho\rho^i + \rho^n)C = \left[\frac{(1-\alpha)r}{\alpha} + q^n\left(\beta^{-1}-1\right)\right]K + \chi(\rho-1)L$$

where we use $I = \delta K$ and $d = \delta(1-\phi q^i) - \chi(r + (1-\delta)\phi q^i)$. Then, we can solve real liquidity and capital stock as

$$L = \frac{\left(\rho\rho^i + \rho^n\right)d + \rho^i\left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1}-1)\right]}{\chi\left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1}-1) + (\rho-1)d\right]}C \tag{45}$$

$$K = \frac{1}{\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1}-1) + (\rho-1)d}C \tag{46}$$

Finally, we express labor supply N and (physical) investment as a function of K

$$N = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}}K, \quad I = \delta K \tag{47}$$

Finally, the similar steps still go through with other types of utility function $u(.)$ and $h(.)$. A different $u(.)$ only changes the computation of ρ^i and ρ^n. A different $h(.)$ (especially a non-linear $h(.)$) modifies (44) to

$$C = (u')^{-1}\left(\frac{h'\left(\frac{N}{1-\chi}\right)}{w}\right)\frac{1-\chi}{\rho^n}$$

Then, we need to guess a value of N and check whether the guess is correct by following (45), (46), and (47). For these reasons, the following proofs do not rely on a particular choice of $u(.)$ and $h(.)$ either.

A.3 Two Special Cases

There are two special cases: one when money exists, and the other when money does not exist. The following briefly illustrates the numerical computation procedure for steady state equilibrium.

If money does not exist, (34) does not hold and $L_{t+1} = 0$; When computing the steady state equilibrium, we follow the previous steps but by picking a particular ρ that gives us $L = 0$.

If only money exist, (35) does not hold and (36) is changed to

$$I_t = r\chi K_t + \chi L_t - \rho^i C_t \tag{48}$$

and we should replace 3.(d) by $q_t^i = q_t = 1$, $q^n = \rho$, and do not solve 3.(c). Therefore, L and K still solve (45) and (46), but we replace $d = \delta - \chi r$.

A.4 A Basic RBC Model

We briefly describe the corresponding basic RBC model relative to our model. As is well known, one can solve the planner's solution to a RBC model. The planner maximizes

$$V(K_t; A_t) = \max_{C_t, N_t, K_{t+1}} \{\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \mu N_t + \beta \mathbb{E}\left[V(K_{t+1}; A_{t+1})|A_t\right]$$

$$s.t. \quad C_t + K_{t+1} = A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t \tag{49}$$

where $\mathbb{E}$ is conditioning on aggregate productivity shocks A_t, as only A_t will be the aggregate shocks. The optimality conditions of labor supply and capital accumulation are

$$(C_t)^{-\sigma}(1-\alpha)A_t\left(K_t/N_t\right)^{\alpha} = \mu, \tag{50}$$

$$1 = \beta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left[\alpha A_t\left(K_t/N_t\right)^{\alpha-1} + 1 - \delta\right]\right]. \tag{51}$$

Therefore, equations (49), (50), and (51) solve (K_{t+1}, C_t, N_t) given the state variables (K_t, A_t). Calculating the steady state values is straightforward. We substitute out capital-labor ratio K/N in (50) and (51), and obtain

$$\frac{K}{N} = \left(\frac{\mu C^{\sigma}}{1-\alpha}\right)^{\frac{1}{\alpha}}, \quad C = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\sigma}}\left[\frac{\beta^{-1} - (1-\delta)}{\alpha}\right]^{\frac{\alpha}{(\alpha-1)\sigma}}.$$

Therefore, from the social resources constraint (49) and by using the capital labor-ratio, we obtain capital and labor as

$$K = \left[\frac{\beta^{-1} - (1-\delta)}{\alpha} - \delta\right]C, \quad N = \left[\frac{\beta^{-1} - (1-\delta)}{\alpha} - \delta\right]C/\left(\frac{\mu C^{\sigma}}{1-\alpha}\right)^{\frac{1}{\alpha}}.$$

A.5 Alternative Modeling: Consumption Sharing

Suppose we further assume that the household members share consumption after trading, we will not have ρ_t to measure the degree of consumption risk-sharing. A buyer's surplus amounts to

$$J_t^n = -u'(C_t)q_t + \beta \mathbb{E}_t \left[J_{S,t+1}(S_{t+1}, B_{t+1}) \right].$$

The sellers' surplus is the marginal value to the household of an additional match for entrepreneurs

$$J_t^i = u'(C_t) \left(q_t - \frac{1}{e_t \phi_t} \right) + \beta \left(\frac{1}{e_t \phi_t} - 1 \right) \mathbb{E}_t \left[J_{S,t+1}(S_{t+1}, B_{t+1}) \right].$$

Then, intermediaries maximize joint surplus $(J_t^i)^\omega (J_t^n)^{1-\omega}$ by picking a specific price q_t, which gives rise to (by following similar derivation in the proof of Proposition 2)

$$q_t = 1 + \left[\frac{2\omega - 1}{1 - \omega} \phi_t - 1 \right] \frac{\kappa}{f_t}$$

Importantly, when $\kappa \to 0$, we see that $q_t \to 1$. This result implies that when household members share consumption goods together, $\kappa \to 0$ implies that the economy looks as if it is a basic RBC economy.

B Proofs

B.1 Proposition 1

We first rewrite two asset pricing formulae in steady state as

$$\rho \chi r^{ni} + (1 - \chi) r^{nn} = \beta^{-1}, \quad \rho \chi \frac{1}{\Pi} + (1 - \chi) \frac{1}{\Pi} = \beta^{-1} \tag{52}$$

Since $\Pi = 1$, one knows that $\rho = \rho^* = \left[\beta^{-1} - (1 - \chi) \right] / \chi > 1$. We will keep writing $1/\Pi$ to denote the return from money. Notice that $r^{nn} = \frac{r + (1-\delta) q^n}{q^n} > \frac{r + (1-\delta)}{q^n} = r^{ni}$ since $q^n > 1$. Then, $r^{nn} > 1$; otherwise, the two asset pricing formulae in (52) cannot simultaneous hold. Further, rearranging (52), we have

$$\chi r^{ni} = \frac{\beta^{-1} - (1 - \chi) r^{nn}}{\rho}, \quad \chi \frac{1}{\Pi} = \frac{\beta^{-1} - (1 - \chi) \frac{1}{\Pi}}{\rho}$$

and the liquidity premium Δ^{LP} can be expressed as $\Delta^{LP} = \chi r^{ni} + (1-\chi) r^{nn} - \frac{1}{\Pi} = \left(1 - \rho^{-1}\right) (r^{nn} - \frac{1}{\Pi})(1-\chi)$. Since $\rho > 1$ and $r^{nn} > 1$, we know that $\Delta^{LP} > 0$. $\square$

B.2 Proposition 2

We first simplify the bargaining solution to (29)

$$\frac{\omega}{\rho \left(q_t - \frac{1}{\phi_t} \right) + \frac{1 - \phi_t}{\phi_t} q_t^n} = \frac{1 - \omega}{q_t^n - q},$$

by using the first-order condition (25) and the risk-sharing condition (24) from the household. Then

$$\omega \frac{\kappa_t}{f_t} = (1 - \omega) \left[\rho_t \left(q_t - \frac{1}{\phi_t} \right) + \frac{1 - \phi_t q_t^i}{\phi_t} \frac{(1 - \phi_t) q_t^n}{1 - \phi_t q_t^i} \right].$$

Using the definition $\rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^n}$, we further simplify the above identity to (30)

$$\omega \frac{\kappa_t}{f_t} = (1-\omega)\,\rho_t\left(q_t - q_t^i\right) \iff \rho_t = \frac{\omega}{1-\omega}\frac{\phi_t}{f_t}.$$

Using the definition $\rho_t \equiv \frac{q_t^n}{q_t^r} = \frac{(1-\phi_t)q_t^n}{1-\phi_t q_t^i} = \frac{(1-\phi_t)\left(q_t+\frac{\kappa_t}{f_t}\right)}{1-\phi_t q_t+\kappa_t}$, we can express q as

$$q_t = \frac{\rho_t\left(1+\kappa_t\right) - \left(1-\phi_t\right)\frac{\kappa_t}{f_t}}{1+\left(\rho_t-1\right)\phi_t} = \frac{\rho_t\left(1+\frac{\kappa_t}{\omega}\right) - \frac{\kappa_t}{f_t}}{1+\left(\rho_t-1\right)\phi_t}, \tag{53}$$

where the second equality uses (30) again. □

B.3 Corollary 1

Firstly, when both private claims and money exist, $\rho = \rho^*$ and $\theta = \frac{1-\omega}{\omega}\rho$. Then, ρ, $\phi(\theta)$, and $f(\theta)$ are functions of parameters that are independent of search costs κ. We know that the spread $q^n - q^i = \kappa\left(\phi^{-1} + f^{-1}\right)$ increases with search costs.

Second, notice that $q^i = q - \frac{\kappa}{\phi} = \frac{\rho+\kappa\left[\frac{\rho}{\omega}-\frac{1}{f}-\frac{1}{\phi}-(\rho-1)\right]}{1+(\rho-1)\phi}$ and is a decreasing function of κ because $\frac{\rho}{\omega} - \frac{1}{f} - \frac{1}{\phi} - (\rho-1) \leq 0$. To see this, $\frac{\rho}{\omega} - \frac{1}{f} - \frac{1}{\phi} - (\rho-1) \leq 0$ is equivalent to

$$\frac{\rho(1-\omega)}{\omega} - \frac{1}{f} \leq \frac{1-\phi}{\phi} \iff \frac{1}{f}(\phi-1) \leq \frac{1-\phi}{\phi},$$

where we use the relationship $\rho = \frac{\omega}{1-\omega}\frac{\phi}{f}$. The inequality is then satisfied for any $\phi \in [0,1]$.

Finally, we use $q = \frac{\rho(1+\frac{\kappa}{\omega})-\frac{\kappa}{f}}{1+(\rho-1)\phi}$, and $\frac{\partial q}{\partial \kappa} > 0$ is equivalent to

$$\frac{\rho}{\omega} - \frac{1}{f} < 0 \iff \phi < 1-\omega \iff M(1, \frac{1-\omega}{\omega}\rho^*) < 1-\omega$$

where we use $\rho = \frac{\omega}{1-\omega}\frac{\phi}{f}$ again in the first equivalence result and $\phi = M(1,\theta)$ in the second.

B.4 Corollary 2

Using $\rho_t = \frac{\omega}{1-\omega}\theta_t$, $\theta_t = \phi_t/f_t$, and $\phi_t = M(1,\theta_t)$, we can express$\rho_t = \frac{\omega M^{-1}(\phi_t)}{(1-\omega)}$ and $f_t = \frac{\phi_t}{M^{-1}(\phi_t)}$. Therefore, q_t in (53) can be written as

$$q_t = \frac{\frac{\omega M^{-1}(\phi_t)(1+\frac{\kappa_t}{\omega})}{(1-\omega)} - \frac{\kappa_t M^{-1}(\phi_t)}{\phi_t}}{1+\left[\frac{\omega M^{-1}(\phi_t)}{(1-\omega)} - 1\right]\phi_t} = \frac{M^{-1}(\phi_t)\left[\left(\omega+\kappa_t\right)\phi_t - (1-\omega)\kappa_t\right]}{\omega\phi_t^2 M^{-1}(\phi_t) + (1-\omega)\phi_t(1-\phi_t)} = \frac{(\omega+\kappa_t)\phi_t - (1-\omega)\kappa_t}{\omega\phi_t^2 + (1-\omega)f_t(1-\phi_t)}$$

Fixing κ_t, we know that a sufficient condition for q_t to positively co-move with ϕ_t is that the denominator is a decreasing function of ϕ_t (since the numerator is an incrasing function of ϕ_t). Call the denominator $g(\phi_t)$, then

$$\begin{aligned}\frac{dg(\phi_t)}{d\phi_t} &= 2\omega\phi_t - \frac{(1-\omega)\eta}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{1}{\eta-1}}(1-\phi_t) - (1-\omega)\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}} \\ &= 2\omega\phi_t - \frac{(1-\omega)\eta}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{1}{\eta-1}} - \frac{(1-\omega)\left(1-2\eta\right)}{1-\eta}\xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}\end{aligned}$$

where we use the fact that $f_t = \xi^{\frac{1}{1-\eta}}\phi_t^{\frac{\eta}{\eta-1}}$. Therefore, $\frac{dg(\phi_t)}{d\phi_t} \leq 0$ is equivalent to

$$\phi_t^{\frac{1}{1-\eta}} \leq \frac{1-\omega}{2\omega(1-\eta)}\xi^{\frac{1}{1-\eta}}\left[1-2\eta+\frac{\eta}{\phi_t}\right]$$

as in the main text. When $\eta = \frac{1}{2}$, the above in equality can be rewritten as

$$\phi_t \leq \xi^{\frac{2}{3}} \left[\frac{1-\omega}{2\omega} \right]^{\frac{1}{3}}, \qquad \square$$

B.5 Proposition 3

We use the guess-and-verify strategy. Suppose the private claims and money co-exist. Then, all the equilibrium conditions from (39)-(47) are satisfied.

First, we search for the threshold κ_2 that yields $q^i \geq 1$ when $\kappa \leq \kappa_2$. Using the asset price derived in Proposition 2, the selling price $q_t^i = q_t - \frac{\kappa_t}{\phi}$, and $\rho_t = \frac{\omega}{1-\omega}\frac{\phi_t}{f_t}$, we know that

$$q_t^i = \frac{\rho_t(1+\frac{\kappa_t}{\omega}) - \frac{\kappa_t}{f_t} - \frac{\kappa_t}{\phi_t} - (\rho_t - 1)\kappa_t}{1+(\rho_t-1)\phi_t} == \frac{\rho_t + \kappa_t\left(\phi_t - 1\right)\left(\frac{1}{\phi_t} + \frac{1}{f_t}\right)}{1+(\rho_t-1)\phi_t}.$$

Therefore, $q_t^i \geq 1$ is equivalent to

$$(1-\phi_t)\left(\rho_t - 1 - \frac{\kappa_t}{f_t} - \frac{\kappa_t}{\phi_t}\right) \geq 0 \iff \rho_t - 1 \geq \frac{\kappa_t}{f_t} + \frac{\kappa_t}{\phi_t} = \frac{\kappa_t\left(\theta+1\right)}{M(1,\theta_t)}$$

where we use the fact that $\phi_t \in [0,1]$ together with the definition of f_t and ϕ_t. By using the relationship $\rho_t = \frac{\omega}{1-\omega}\theta_t$, we can simplify the above condition to

$$\kappa_t \leq \frac{\rho_t - 1}{\gamma\rho_t + 1} M\left(1, \gamma\rho_t\right)$$

where $\gamma \equiv (1-\omega)/\omega$. Since ρ is bounded above by $\rho^* = [\beta^{-1} - (1-\chi)]/\chi$ (again, ρ^* is pinned down from the asset pricing formula of money in steady state), and $M(1,\gamma\rho)$ and $\frac{\rho-1}{\gamma\rho+1}$ are increasing functions of ρ, we know that the threshold $\kappa_2 = \frac{\rho^*-1}{\gamma\rho^*+1} M\left(1, \gamma\rho^*\right)$.

Next, we calculate the threshold κ_1 as a function H of other parameters. Notice with the existence of meony, the asset pricing formula of money holds. Therefore, $\rho = \rho^* = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right]$ and (40)-(47) hold. Importantly, ϕ and f are not functions of κ. Further, we have calculated q^i and can also calculate q^n:

$$q^i = \frac{\rho + \kappa\left(\phi - 1\right)\left(\frac{1}{\phi} + \frac{1}{f}\right)}{1+(\rho-1)\phi}, \quad q^n = q + \frac{\kappa}{f} = \frac{\rho + \kappa\left[\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f}\right]}{1+(\rho-1)\phi}$$

Therefore, we can express

$$q^i = c_{i1} + \kappa c_{i2}, \quad q^n = c_{n1} + \kappa c_{n2},$$

where

$$c_{i1} = \frac{\rho}{1+(\rho-1)\phi}, \quad c_{i2} = -\frac{(1-\phi)\left(\frac{1}{\phi}+\frac{1}{f}\right)}{\phi\left[1+(\rho-1)\phi\right]}, \quad c_{n1} = \frac{\rho}{1+(\rho-1)\phi}, \quad c_{n2} = \frac{\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f}}{1+(\rho-1)\phi},$$

By inspecting the coefficients, we know that except $c_{i2} < 0$, others are strictly positive. For similar reasons, we can express r from (43)

$$r = c_{r1} + \kappa c_{r2}$$

where

$$c_{r1} = \frac{\left[1-\beta(1-\delta)(1-\chi)\right]\rho}{1+(\rho-1)\phi} - \beta(1-\delta)\chi\rho, \quad c_{r2} = \frac{\left[1-\beta(1-\delta)(1-\chi)\right]\left(\frac{\rho}{\omega} + \frac{(\rho-1)\phi}{f}\right)}{1+(\rho-1)\phi} > 0$$

As we have already used the Euler equation for money, money exists if $L \geq 0$. Since $K > 0$, from (45) and (46), we know that the denominator of the two equation has to be positive. Then, $L \geq 0$ iff

$\left(\rho\rho^i + \rho^n\right) d + \rho^i \left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1)\right] \geq 0$. That is,

$$\delta(1 - \phi q^i) + \frac{\rho^i \left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1)\right]}{(\rho\rho^i + \rho^n)} \geq \chi(r + (1-\delta)\phi q^i)$$

If we plug in the expression of q^i, q^n, and r, we have

$$\left[\frac{\rho^i}{(\rho\rho^i + \rho^n)} \frac{(1-\alpha)}{\alpha} - \chi\right] r + \frac{\rho^i \left(\beta^{-1} - 1\right) q^n}{(\rho\rho^i + \rho^n)} - \phi\left[\delta + \chi(1-\delta)\right] q^i + \delta \geq 0$$

Let $\zeta_r = \left[\frac{\rho^i}{(\rho\rho^i+\rho^n)} \frac{(1-\alpha)}{\alpha} - \chi\right]$, $\zeta_n = \frac{\rho^i}{(\rho\rho^i+\rho^n)} \left(\beta^{-1} - 1\right)$, and $\zeta_i = \phi\left[\delta + \chi(1-\delta)\right]$, then

$$\kappa\left(\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2}\right) \geq -\delta - \zeta_r c_{r1} - \zeta_n c_{n1} + \zeta_i c_{i1}$$

Notice that $\zeta_r = \chi\left[\frac{1}{\chi\rho+(1-\chi)/\rho^\sigma} \frac{1-\alpha}{\alpha} - 1\right] > \chi\left[\beta^{-1}\frac{1-\alpha}{\alpha} - 1\right] > 0$, $\zeta_n > 0$, and $\zeta_i > 0$, then $\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2} > 0$. This implies that we have $\tilde{\kappa}_1$ as

$$\kappa \geq \frac{-\delta - \zeta_r c_{r1} - \zeta_n c_{n1} + \zeta_i c_{i1}}{\zeta_r c_{r2} + \zeta_n c_{n2} - \zeta_i c_{i2}} = H(\chi, \beta, \delta, \alpha, \xi, \eta, \omega) \equiv \tilde{\kappa}_1$$

Finally, with some tedious algebra, one can show that $\frac{dL}{d\kappa} > 0$ by using the expression (45) together with $\rho = \chi^{-1}\left[\beta^{-1} - (1-\chi)\right]$ and all other equilibrium conditions. That is, $L(\kappa)$ is an increasing function of κ. Therefore, when private claims and money co-exist, liquidity value increases with search costs κ. Liquidity value L only cross zero once at $\kappa = \tilde{\kappa}_1$. When $\kappa < \tilde{\kappa}_1$, $L(\kappa) < 0$ which means that the assumption of co-existence of private claims and money is not verified.

When $\kappa \in [0, \kappa_1)$, we know that money does not circulate. Therefore, the asset pricing formula for money will not be satisfied in order to pin down ρ. Instead, $L = 0$, and from (45) we know that ρ solves the following equation

$$\left(\rho\rho^i + \rho^n\right) d + \rho^i \left[\frac{(1-\alpha)r}{\alpha} + q^n(\beta^{-1} - 1)\right] = 0$$

where all equilibrium variables can be expressed as a function of ρ if we use equations (40)-(47). The implicity function theorem implies that $\frac{d\rho}{d\kappa} > 0$ (where the derivation is tedious and available upon requests). Then, ϕ is also an increasing function of κ. Using the relationship (42) again, one knows that q and q^n are all increasing function of κ.[24] □

[24]Note that to illustrate κ's impact on ρ , ϕ, q, and q^n, we provide a numerical example after we calibrated the model.

Acknowledgements
The views expressed in this paper are those of the authors and do not necessarily reflect those of the European Central Bank (ECB). We gratefully acknowledge insightful discussions with Frank Heinemann and Nobuhiro Kiyotaki. We also greatly benefited from discussions with Marco Bassetto, Tobias Broer, Mariacristina De Nardi, Patrick Kehoe, Ricardo Lagos, Iourii Manovskii, Guido Menzio, Guillermo Ordonez, Fabien Postel-Vinay, Morten Ravn, Victor Rios-Rull, Thomas Sargent, Robert Shimer, Shouyong Shi, Ted Temzelides, and Pierre-Olivier Weill. We thank seminar participants at Princeton, Humboldt, UCL, DIW Berlin, Southampton, Konstanz, Aarhus, CEMFI, Mannheim, ECB, Geneva Graduate Institute, EPFL/UNIL, Upenn, Edingburgh, the 2013 7th Nordic Macro Symposium, the 2013 Vienna Macro Workshop, ESSIM 2015, Barcelona Summer Workshop 2015, and the 2013 and 2015 SED Annual Meetings, for their comments. Finally, we thank DIW Berlin and the Centre for Macroeconomics for providing financial support for this project. All remaining errors are our own.

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ISSN	1725-2806 (online)
ISBN	978-92-899-2165-7
DOI	10.2866/07962
EU catalogue No	QB-AR-16-034-EN-N