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Governments' payment discipline: the macroeconomic impact of public payment delays and arrears

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Governments' payment discipline:
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Note: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.
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Abstract

This paper considers the impact of changes in governments’ payment discipline on the private sector. We argue that increased delays in public payments can affect private sector liquidity and profits and hence ultimately economic growth. We test this prediction empirically for European Union countries using two complementary approaches. First, we use annual panel data, including a newly constructed proxy for government arrears. Using panel data techniques, including methods that allow for endogeneity, we find that payment delays and to some extent estimated arrears lead to a higher likelihood of bankruptcy, lower profits, and lower economic growth. While this approach allows a broad set of variables to be included, it restricts the number of time periods. We therefore complement it with a Bayesian VAR approach on quarterly data for selected countries faced with significant payment delays. With this second approach, we also find that the likelihood of bankruptcies rises when the governments increase the average payment period.

JEL Classification Numbers: E6, H6, H8
Keywords: Public Payment Delays, Government Arrears, Accounts Payable, Government Spending.
Non-Technical Summary

The issue of government arrears has gained prominence during the European sovereign debt crisis. Particularly in EU/IMF programme countries – both in and outside the euro area – but also in other fiscally vulnerable economies, such as Italy and Spain, the identified amounts were considerable and measures to reduce the stock of arrears featured prominently in government strategies and as programme targets.

To the best of our knowledge, this is the first paper to systematically address and econometrically estimate the economic impact of public spending delays and arrears. Our key contributions are twofold. First, we put forward a statistical measure that can serve as a proxy for arrears. Second, we use this and other measures of payment delays to gauge the short-term effect on some key economic variables.

Our empirical results from annual panel data across the EU-27 countries suggest that payment delays may reduce profits, increase the likelihood of bankruptcies and, ultimately, slow economic growth. While the exact size of such effects is hard to estimate from the available data, results are significant in most specifications. On average for the European Union sample, the total amount of outstanding payments (including trade credits) does not appear to play a role, suggesting that predictable and regularly cleared payment delays are not necessarily a problem. Hence, our results for the EU sample confirm the intuitive reasoning that only payment delays that are substantial in the amount and the time outstanding can put a significant strain on private sector liquidity or growth.

The results from Bayesian VARs performed on available quarterly data for Spain, Italy and Portugal show that an increase in the average duration of government payments leads to an increase in the likelihood of private sector defaults, with weaker evidence on the direct growth channel. The overall BVAR results for this subset of countries suggest that public payment delays affect the economy mostly through a liquidity channel.

Our analysis and results have several implications for policy makers. Based on the findings in this paper it appears that delaying payments to deal with a funding issue or a debt limit is a costly way of achieving these aims. Quite to the contrary, efforts to accelerate payments and reduce existing stocks of arrears could be helpful in boosting the economy in the shorter run and would typically not increase deficits if all spending was properly captured when it accrued. Having established that there is an inverse relationship between public payment delays and overall economic performance or growth, the first policy recommendation is to closely monitor the amount of arrears and payment practices in a given country to foster economic performance. That being said, it becomes clear that an immediate second policy implication would be to address the prevailing measurement
issues associated with variables such as "other accounts payable" and install a comprehensive and frequent measurement and accounting system for public payment practices. Third, given their impact on economic performance, delayed payments or arrears could be included in economic and fiscal surveillance. This would also entail monitoring whether the 2011 EU Directive on combating late payment in commercial transactions is effectively applied at the country level to reduce government payment delays and pay surcharges for infringement.
1 Introduction

The issue of government arrears has gained prominence during the European sovereign debt crisis. Particularly in EU/IMF program countries—both in and outside the euro area—but also in other fiscally vulnerable economies, such as Italy and Spain, the identified amounts were considerable, and measures to reduce the stock of arrears featured prominently in government strategies and as program targets. At the same time, the European Commission took initiatives at the EU level to reduce payment delays, such as the 2011 Directive on combating late payment in commercial transactions, which also covers transactions between private undertakings and public authorities.\(^1\)

To the best of our knowledge, this is the first paper to systematically address and econometrically estimate the economic impact of public spending delays and arrears. The existing literature on arrears is mostly concerned with the measurement of spending arrears in different systems of national accounts, especially in developing countries (Diamond & Schiller (1993)), and ways to reduce the stock of arrears, for example through restructuring and/or securitization (Ramos (1998)). Flynn & Pessoa (2014) contains a recent overview of the issues arising in preventing and managing government arrears. The likely macroeconomic effects are only discussed via intuition and example, but are not modelled in a theoretical setup or estimated using econometric techniques.\(^2\) Our key contributions therefore are (i) to put forward a statistical measure that can serve as a proxy for arrears, and (ii) to use this and other measures of payment delays to gauge the effect on some key economic variables, such as growth and profitability.

Like private agents, governments have some discretion on when to pay their bills and other obligations. The outstanding payments of governments are, however, different in various respects from trade credit among private sector agents. First, within the private sector, paying a bill shifts liquidity across firms, but does not affect aggregate private sector liquidity. Second, given the size of the government, particularly in European countries, its payment policies are important to a large base of suppliers. Third, the government is at the same time a debtor and a creditor, but in a very distinct way, as most of the funds owed to the government are taxes, i.e., unrequited payments, whose payment terms are set by the government.

\(^1\)The directive, which entered into force in March 2013, imposes a maximum delay for new government payments of 30 days (60 days for a limited set of exceptions, such as in the health sector) and an 8 percent surcharge for infringement.

\(^2\)Bank of Italy (2013) estimated the impact of the Italian initiative to clear arrears on growth to be close to unity if payments are used to finance investment, roughly 0.3 if used for firms’ wage arrears and close to zero if kept for precautionary saving. Overall they estimate a positive impact on the economic growth rate of between 0.5 and 0.7 percentage points. They do not provide a description of how exactly these numbers were estimated or which econometric model or rationale was used to obtain them.
The discretion governments have in choosing when to pay may be foreseen already in contracts that include explicit or customary trade credit, but it can also go beyond that if governments miss due dates and fall into arrears. Payment traditions and expectations vary across countries and sectors, but as long as the situation is static the impact should be limited. If a government has a tradition of taking a long time to pay bills, then suppliers will price the cost of such credit into the goods supplied. There could still be some impact, though, as firms with extreme credit constraints may then not be able to do business with the government.

In times of economic crisis, however, payment delays often change in unexpected ways. Most obviously, a government facing a funding constraint could delay payments. This could affect bills for goods supplied, thus increasing trade credits, or it could affect wages and pensions. Typically, delaying payments for interest and amortization on public debt occurs only as a last resort to avoid official defaults. Even governments with full access to cheap financing delay payments sometimes. Depending on the accounting framework used, this can lead to lower public debt or deficit figures. Whether debt turns out to be lower depends on whether trade credit and arrears are counted as government debt. Under the Excessive Deficit Procedure (EDP) definition, this is not the case, but EDP submissions include separate reporting of accounts payable.\(^3\) The deficit in selected years would also be reduced if measured on cash basis, but not normally if an accrual definition is used (although in practice, some transactions may be missed until payment takes place).

Governments could also decide to accelerate payments to suppliers or previously accumulated arrears, at some stage in a crisis, in particular to support a liquidity-constrained private sector. In this spirit, for instance, the Italian government announced in April 2012 a major program (EUR 40 billion) to clear arrears over two years. This program was later augmented (reaching EUR 66 billion), and by October 2014 already EUR 32.5 billion had been paid out. Similarly, Spain announced in May 2012 a mechanism in the form of a government guaranteed syndicated loan worth EUR 30 billion by which the central government helps regional and local governments clear their arrears. Payment delays may also have purely administrative reasons.\(^4\)

Changes in payment lags can be expected to have implications on the macroe-

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\(^3\)In the recent update of the public accounting standards (ESA-2010, enforced as of September 2014), the EU governments did not take the opportunity to include the obligation to count trade credits (and arrears) under government debt.

\(^4\)To address these, the Italian government, for example, introduced compulsory electronic invoices for central government administrations in mid-2014 and plans to extend them to local governments by spring 2015. Moreover, to increase transparency the related data will be published on the web.
economic situation through various channels:

- Corporate profits can be affected, because unexpected delays change the present discounted value of payments. If no or a low interest rate applies, this reduces suppliers profitability.

- The size of the corporate sector can be affected if liquidity-constrained firms, in particular small and medium-sized enterprises (SMEs), go bankrupt. This will also have knock-on effects on creditors of such firms. Various second-round effects are also likely, e.g., a higher bankruptcy rate could increase the cost of capital even to firms with access to credit; the cost of future orders of goods and services to the government could rise, as suppliers built the anticipated financing costs, including the uncertainty, into offers.

- Business investment can be affected in liquidity-constrained firms. These may not only be those directly dependent on government payments, but also their own suppliers as payment delays trickle on. Aggregate demand, and finally output and growth, could thus be negatively impacted.

To lay the foundation of our analysis, Section II discusses the various forms of payment delays and the extent to which they form arrears. It also describes the available data and explains the construction of our measures of arrears and delayed payments. Section III provides an analysis of the impact of payment delays on profits, bankruptcies and growth, using dynamic panel data techniques. Section IV complements the previous analysis by using a Bayesian VAR on quarterly data for Italy, Spain and Portugal. Section V brings together the findings and concludes.

2 Definitions and Data Availability

2.1 Official data

According to the IMF Government Finance Statistics Manual (IMF (2001)), “arrears arise when an obligatory payment is not made by its due-for-payment date.” The term arrear should not be confused with general unpaid government bills or other obligations. A true arrear only occurs if a bill is not paid by the due date, whether this is based on a contractual agreement, commercial law or custom (e.g., 60 days after the invoice date). A government may therefore have large amounts of unpaid bills without falling into arrears. Conversely, it is also possible that overall unpaid bills are small, but some of them are in arrears, maybe because of some administrative glitch. Nevertheless, an increase in unpaid bills could be indicative of potential arrears. Arrears may also occur in expenditure categories where there are no bills, such as pensions, transfers or wages. In that case, the definition is less
clear, especially as the government could define the payment terms. However, a payment that occurs much later than the month to which it refers would probably be seen as an arrear.

Public accounts typically do not track true arrears, except following ad hoc audits to identify them (as sometimes required under IMF programs). Alternative sources from international datasets do not report fiscal arrears either. For example, in the IMF’s Government Finance Statistics, public payment arrears are a memorandum item that member countries are free to report, but rarely do. Instead, depending on the public accounting system in place, there could be data on spending commitments, payment orders and actual payments (check or transfer). Differences between these stages can provide indications of the development of payment lags.

- The difference between commitments and payment orders can reflect late supply by private parties or delays by the government in issuing payment orders.

- The difference between payment orders and actual payments (accounts payable) is necessarily due to government procedures. An increase in this figure could, however, still take place without the government breaching due dates.  

- Finally, if checks are used, there is a float as a result of uncashed checks. This would not lead to arrears, as companies would consider a debt cleared on receipt of a check, unless the check bounces.

An unusual increase in any of these measures would indicate a potential problem, but would not be proof for the presence of arrears. Conversely, small or stable differences are not proof for the absence of arrears either, as these aggregated figures could hide individual payments with excessive delays. Moreover, if only some steps are observed, arrears can be missed. For example, if only accounts payable are known, arrears could occur because of the delayed issue of payment orders (or more generally recognition of liabilities). Finally, irregular payments, made without recording a commitment could still be potentially legally valid, but would not be known until regularized.

While it may not be possible to cleanly identify arrears in a legal sense, from an economic point of view, it may be more important to identify payment delays that go beyond what is expected by suppliers. Accounts payable, possibly as

5Typically only the amount is known, but even if the delay is known, it is not possible to be certain about the absence of arrears. If the delay is shorter than the payment term, there could still be an arrear if the payment order was issued later than the invoice date. For a delay that exceeds the payment term, however, arrears are very likely to be present, as payment orders are unlikely to be issued in advance of invoice dates.
a share of total spending, would be a proxy for the average payment duration, even if an imperfect one as governments may delay or avoid recognizing valid liabilities. In this paper we mainly use Eurostat’s Sector Accounts data on accounts payable (ESA-1995 code AF.7) as a basis for further data construction. For a few countries, we also have direct estimates of arrears that allow us to make comparisons.

In an accrual accounting system, such as ESA-1995, the timing of payments should not affect reported spending (with a few exceptions), as spending is registered at the time of good supply or service provision. If payment is not made at the same time—be it an arrear or a delay within permissible payment terms—then it shows up under the category “other accounts payable” (AF.7) in the national accounts. This category comprises any financial liabilities “which are created as a counterpart of a financial or a non-financial transaction in cases where there is a timing difference between this transaction and the corresponding payment. It includes trade credits and advances and any other receivables and payables. Trade credits and advances are financial assets/liabilities arising from the direct extension of credit by suppliers and buyers for goods and services transactions and advance payments for work that is in progress or to be undertaken and associated with such transactions” (Eurostat (1996)). The variable AF.7 is generally available for different parts of the public sector. It is important to use data at the general government level, because liabilities between different government levels can be substantial, but are not part of the outstanding payments from the public to the private sector. We therefore consistently use the variable AF.7 at the general government level on a consolidated basis. See developments by country over the period 2005-2012 for the AF.7-to-GDP ratios in Figure 1. In principle, there is a further breakdown into two sub-categories: trade credits and advances (AF.71), and other accounts payable excluding trade credit and advances (AF.79). In practice, however, the breakdown into the two subcategories suffers from rather severe measurement and reporting issues (European Commission (2012)), or are unavailable altogether, e.g., for Greece. Moreover, AF.7 is regularly reported to be only very small relative to GDP, even for countries where recent ad hoc audits have revealed substantial spending arrears, as for instance in Spain.

2.2 A proxy for fiscal arrears

As the exact amount of payments in arrears is not available from ESA-1995 national accounts data, we put forward a method to construct a proxy for the amount of payments in arrears. We do this by combining the national accounts data on

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6In ESA-2010, the code changes to AF.8, while AF.7 refers to financial derivatives and employee stock options.
Figure 1: Accounts Payable (AF.7) in EU Countries (percent of GDP).

Source: Eurostat.

accounts payable with survey data from a private credit management company (Intrum Justitia) on payment durations. This combination of information allows us to estimate the share of accounts payable that are within or beyond the due-for-payment date. The accuracy of the resulting estimate depends on the validity of our assumptions, which are described below, but also on the quality of the data. Specifically for the survey data, we should keep in mind that the likelihood of
responses to the survey may not be independent of being paid on time or any other characteristics of the respondents. Moreover, there could be differences in opinion between payor and payee, e.g., the government may not agree on some transactions that businesses consider overdue, like those for which the payment is withheld for incomplete delivery.

To illustrate how we construct our proxy, first suppose we had full information. In this ideal situation, we could, on a given day, retrieve the full payment record of the public sector (ESA-1995 sector code S.13) from the national accounts. That is, on a given day of a fiscal year $\tau$ and for every invoice $i$, we would have information on: (i) the amount $e_i$ to be paid, (ii) the contractual payment period $\bar{T}_i$ and (iii) the payment duration $T_i$. We then say that invoice $i$ is in arrears, if $T_i > \bar{T}_i$. For example, if the contractual payment period $\bar{T}_i$ is 30 days and we are 45 days behind the invoice date, the payment has been in arrears for 15 days. For any date $\tau$, one could then immediately determine the amount of payments in arrears, but also construct the full duration distribution $F_T(c) = \Pr[T \leq c]$ of public payments. Hence, $1 - F_T(\bar{T})$ represents the share of payments beyond the due-for-payment date. The duration distribution of payments can therefore be used, e.g., to compute the amount of arrears.

In our less ideal case, the ESA-1995 accounts only provide the total amount of other accounts payable (AF.7) for each country. In order to estimate the share of AF.7 that is in arrears, we first reconstruct the duration distribution of public payments. One might argue that it is more adequate to calculate a proxy for arrears not as a share of AF.7, but rather AF.71, as this has the advantage of avoiding biases due to certain liabilities that fall under AF.79, such as pending tax settlements, but which are not our main interest. At the same time, this would then also exclude delayed payments of salaries, in which we are interested. Moreover, the data quality of the subcategories is less reliable than that of the aggregate (see above). We therefore prefer to use exclusively the aggregate figure.

Because administrative data on the duration of public payments are not available, we use survey data on the average payment duration and the average contractual payment period of public authorities. These data are provided by Intrum Justitia, a private credit management firm, which conducts an annual written survey among several thousand firms in 27 countries. The results from this survey are published in an annual European Payment Index Report (Intrum Justitia (2013) and previous editions). Among several other payment statistics, the survey reports (i) the average annual payment duration and (ii) the average annual contractual payment period. Both numbers are further disaggregated into consumer, business-to-business and public sector debtors. We have plotted the reported data for the public sector from the 2013 report in Figure 2. \(^7\)

\(^7\)Given the entry into force of the EU Directive on Combating Late Payments, it is likely that
In order to estimate the duration distribution we assume that the duration distribution of public payments is exponential, i.e., its c.d.f. is given by

\[
F_T(t) = \begin{cases} 
1 - \exp(-\lambda t) & \text{for } t \geq 0 \\
0 & \text{for } x < 0
\end{cases},
\]

where \( \lambda > 0 \) is the parameter of the distribution and is often called the *rate* or *intensity* of the distribution. The duration \( T \) decreases in \( \lambda \) in the sense of first order stochastic dominance, i.e., higher values for \( T \) become uniformly less probable. The exponential distribution is often used to model time-to-event data, such as waiting times, queuing times or the time until default in credit risk modelling. This is therefore comparable to invoices that remain outstanding until paid, but it requires the assumption that the size of invoices is independent of the duration distribution, i.e., that the government does not systematically delay payments of particularly large invoices. One of its key features that motivates its use in our case is the fact that we may estimate the key parameter \( \lambda \) via simple methods of moments (MM). Let the reported average payment duration for country \( j \) be denoted by \( \bar{T}_j \). Under weak regularity conditions, the sample average provides a consistent estimator for the mean duration of payments, and hence we would delays have improved subsequently in various countries.
estimate $\lambda_j$ in the following way

$$E[T_j] = \lambda_j^{-1} \Rightarrow \hat{T}_j = \hat{\lambda}_j^{-1} \Rightarrow \hat{\lambda} = \hat{T}_j^{-1}.$$  \hfill (2.1)

This immediately leads to the estimated duration distribution

$$\hat{F}_T(t) = \begin{cases} 1 - \exp(-\hat{\lambda}t) & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}. \hfill (2.2)$$

Hence, with information on the average payment duration, an exponential distribution of payment durations is fully identified.\(^8\) If we do not allow for any grace period, the estimated share of payments in arrears equals (Figure 3):

$$\text{Other accounts payable in arrears} = \text{AF}.7 \times (1 - \hat{F}_T(\bar{T})) \hfill (2.3)$$

In the existing literature on the measurement of arrears, there is no general consensus which value to take for $\bar{T}$. An exact notion of payment arrears would consider any payment for which $T > \bar{T}$ to be an arrear. In practice, however, this strict notion of arrears is often loosened to allow for the fact that the exact limit may vary for each bill and this precise information is not available.

In a similar vein, the IMF’s Compilation Guide on Financial Soundness Indicators (IMF (2006), section 4.84) defines loans to be in arrears once “payments of

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\(^8\)More flexible distributions that seem pertinent for our use, e.g., a Gamma distribution, feature two parameters and hence need more information than only the sample average to be identified.
principal and interest are past due by three months (90 days) or more” and goes on to note that “the 90-day criterion is the time period that is most widely used by countries to determine whether a loan is nonperforming.” Since trade credit granted by the private sector to the public sector is a form of a loan, this criterion is equally applicable and provides another way to define an “acceptable grace period.”

We follow this approach and set $\bar{T}_j$ equal 90 days or the contractual payment period—whatever is longer—in order to consider the possibility that some variation in terms between different bills is allowed for.

Hence, in a first step we use (2.1) to estimate $\hat{\lambda}$ and thus $\hat{F}_T(\cdot)$ using the average reported payment duration in the European Payment Index survey for a given year. In the second step, we compute $1 - \hat{F}_T(c)$ where

$$c = \max\{90, \bar{T}_j\}.$$ 

In the final step, we take the share $1 - \hat{F}_T(c)$ and calculate the total amount of payments in arrears using (2.3).

To make figures comparable across countries, we plotted our estimates as a share of GDP in Figure 4. We also included available administrative data on actual payment arrears, as obtained for example during a financial assistance program. There are several features worth mentioning. First, several European countries e.g., Finland, Denmark, Sweden and Bulgaria tend to have relatively large AF.7-to-GDP ratios. While this may be indicative of payment arrears, especially Scandinavian countries are known to roll over their debt in a timely manner and should have only very little payment in arrears, if any. Our measure incorporates this explicitly via the average payment duration in these countries. As a result, our estimate of arrears for these countries is attenuated by their high payment discipline. Second, the individual time series for the different countries show fairly little variation over time and thus appear to be very persistent. Third, the time series variation is higher for countries with relatively high arrears-to-GDP ratios, being the highest in Greece and Spain. Fourth, in terms of matching official numbers, our estimates come surprisingly close in most cases, but may still deviate substantially in individual country-years, as for example the estimate for 2012 arrears in Greece. This deviation in some cases, however, is also very likely to stem from conflicting definitions of what is subsumed under the term payment arrears. For example, official figures from Bulgaria do not comprise outstanding hospital bills from state-owned hospitals.
3 The Aggregate Effects of Payment Arrears—Evidence from Panel Regressions

In a first step we estimate the macroeconomic impact of government delayed payments in a panel setting, exploiting both the country and time variation in data. In line with the theoretical insights on the potential channels through which delayed payments may affect the economy, we investigate the short-term impact on real GDP growth, on profitability as proxied by the economy-wide gross operating surplus, and on liquidity as proxied by probability of default (using Moody’s...
Given the large potential for endogeneity of government delayed payments and arrears, we use lagged variables and, additionally, use the system GMM (Blundell & Bond (1998)) estimator for dynamic panel models. This is particularly suitable for the regressions with variables constructed based on the European Payment Index dataset, which has a rather short time dimension (maximum $T = 7$, i.e. the period 2006-2012) and larger cross-section dimension (the number of EU countries with sufficient observations to be kept in the regressions being 24). We also correct for heteroskedasticity and autocorrelation that may be present in the error structure by using the consistent estimator.

Our macroeconomic data are taken from the European Commission’s AMECO database, except for GDP in purchasing power parity-adjusted terms, which is taken from the World Development Indicators.

### 3.1 Growth regressions

In this subsection, we investigate the short-term impact of government payment delays on real GDP growth using three measures. First, we use a broad measure of delays, constructed as an interaction term between the variable ”other accounts payable” of the general government (AF.7) as a share of GDP and the surveyed number of days public contracts are in delay, as available from Intrum Justitia (2013) (Table 1). Second, we employ our estimated measure of arrears overdue more than 90 days (or the legal limit if greater) as a share of GDP (Table 2). Third, we consider simply the total amount of accounts payable as a share of GDP. As this final variable is mostly not statistically significant when the GMM estimator is used, results are not shown.

In Table 1, we show the estimation results for various regressions starting with the simplest one, in which we only include delayed government payments in addition to country and year fixed-effects and two lags of the dependent variable (using only the first lagged GDP growth does not eliminate auto-correlation as indicated by the rejection of the AR(2) test null hypothesis). In the next columns (2) to (9), one potentially relevant variable is added at a time, as follows (by category): (i) fiscal variables: we first control for a base effect of our variable of interest.

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9The distance to default measures the number of standard deviations it takes a shock to be large enough to render a firm’s asset value lower than the value of the firm’s debt. The country average is weighted by firm assets (for details see http://www.moodysanalytics.com/).

10The results remain robust if the difference GMM (Arellano & Bond (1991)) estimator is used instead. The same holds if the forward orthogonal transformation is used instead of differencing.

11Checks performed with other measures (estimated arrears overdue more than 30 or 60 days) showed less robust results.
by adding the government spending-to-GDP ratio (column 2) in order to capture
the possibility of higher delayed payments accumulating only as a result of higher
total spending. We then aim to capture the short-term impact of discretionary
fiscal policy on the economy, as proxied by the change in the structural primary
balance ratio (column 3); (ii) financial channel: the credit to the private sector
as captured by the GDP share of loans to private entities (column 4); (iii) the
position in the business cycle as captured by: the output gap (column 5) or the
unemployment rate (column 6); (iv) basic determinants of growth in a conditional
convergence model, that is the labour force growth rate (column 7), the saving
(investment) ratio to GDP (column 8) and the initial level of GDP per capita
(column 9). Column 10 includes all the three variables of the convergence growth
model together with our variable of interest. Overall, the results presented in Ta-
bble 1 show pretty robust evidence that delayed payments have a negative impact
on growth. The impact is also economically significant, as coefficients between
-0.005 and -0.009 mean that a one standard deviation change in delayed payments
reduces the growth rate by 0.8 to 1.5 percentage points.

The findings with estimated arrears (Table 2) are more variable, but a significant
result is obtained in many of the specifications, and particularly in those that
control for the economic cycle. The insignificant results with total accounts payable
support the idea that amounts that are rolled-over regularly, including trade credits
within the contractual payment period, may not be a problem.
Table 1: Panel Regressions of Real GDP Growth on Payment Delays.

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<td>-0.161*</td>
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Notes: All explanatory variables lagged by one year except the change in the structural primary balance and the growth rate of the labour force. Accounts payable, expenditure (general government), private credit, and savings rate are in percent of GDP; per capita GDP in thousands of 2011 PPP USD. All regressions are estimated with System GMM and use the second to fifth lag, collapsed, as instruments. Regressions include time and country fixed effects. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.
Table 2: Panel Regressions of Real GDP Growth on Estimated Arrears.

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Notes: All explanatory variables lagged by one year except the change in the structural primary balance and the growth rate of the labour force. Accounts payable, expenditure (general government), private credit, and savings rate are in percent of GDP, per capita GDP in thousands of 2011 PPP USD. All regressions are estimated with System GMM and use the second to fifth lag, collapsed, as instruments. Regressions include time and country fixed effects. Robust standard errors in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1.
3.2 Impact on profit growth

We also investigate the impact of delayed payments (Table 3), estimated arrears and accounts payable on profit growth, using the economy-wide gross operating surplus as an indicator of profits. We find a statistically significant, robust impact only in the case of delayed payments, which are associated with a reduction in the growth rate of the operating surplus. This relationship holds across various specifications, including when controlling for the economic cycle, such as by adding the unemployment rate or output gap. A one-standard deviation increase in delayed payments reduces profit growth by 1.5 to 3.4 percentage points. Results for the other two variables of interest are, however, mostly not significant and therefore not reported.

3.3 Impact on likelihood of bankruptcy

Finally, we consider the impact of payment delays and arrears on the likelihood of bankruptcies using the distance-to-default measure. This can also be considered a proxy for the degree of liquidity constraints persisting in an economy. The distance-to-default variable is available for fewer countries. Moreover, it is less persistent and the likelihood of endogeneity of the public payment arrears is lower. We therefore present specifications with one or no lagged dependent variable, and we also use the fixed effects estimator with Newey standard errors in some specifications. We find that delayed payments (Table 4) and estimated arrears (Table 5)—but again not total accounts payable—reduce the distance to default. That is, the larger such delayed payments, the smaller the distance to default or the higher the probability of default among private companies though only publicly listed companies are hereby captured.
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**Notes:** All explanatory variables lagged by one year except the labor force growth rate. Accounts payable, spending, private credit and savings rate are in percent of GDP. All regressions are estimated with System GMM and use the second to fifth lag, collapsed, as instruments. Regressions include time and country fixed effects. Robust standard errors in parentheses.

*** p < 0.01, ** p < 0.05, * p < 0.1.
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<tr>
<td><strong>Private credit</strong></td>
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<td>-0.00604</td>
<td>-0.00345</td>
<td>-8.21e-05</td>
<td>-0.0116*</td>
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<tr>
<td></td>
<td>(0.000961)</td>
<td>(0.000795)</td>
<td>(0.00690)</td>
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</tr>
<tr>
<td><strong>Unemployment rate</strong></td>
<td>0.00933</td>
<td>-0.000604</td>
<td>-0.0345</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.0245)</td>
<td>(0.0291)</td>
<td>(0.0198)</td>
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<td></td>
</tr>
<tr>
<td><strong>Growth</strong></td>
<td></td>
<td>-0.0303*</td>
<td></td>
<td>-0.0110</td>
<td>-0.0412*</td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.0162)</td>
<td></td>
<td>(0.0147)</td>
<td>(0.0214)</td>
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Notes: All explanatory variables lagged by one year. Accounts payable, spending, private credit are in % of GDP. All regressions are estimated with System GMM (using the second to fifth lag, collapsed, as instruments), except models 2 and 8, in which the fixed effect estimator is used. Regressions include time and country fixed effects. Robust standard errors in parentheses (models 2 and 8 use Newey standard errors).

*** p<0.01, ** p<0.05, * p<0.1.
## Table 5: Panel Regressions of the Distance to Default on Estimated Arrears.

<table>
<thead>
<tr>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td>Distance to default_{t-1}</td>
<td>0.767***</td>
<td>0.846***</td>
<td>0.884***</td>
<td>0.766***</td>
<td>0.890***</td>
<td>0.904***</td>
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<td></td>
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<td>(0.0734)</td>
<td>(0.0683)</td>
<td>(0.0795)</td>
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<tr>
<td>Estimated arrears</td>
<td>-0.0959***</td>
<td>-0.672***</td>
<td>-0.0574</td>
<td>-0.0689</td>
<td>-0.102**</td>
<td>-0.118***</td>
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<td>(0.0285)</td>
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<td>Expenditure ratio</td>
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<td></td>
<td>(0.0152)</td>
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<tr>
<td>Private credit</td>
<td>-0.000586</td>
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<td></td>
<td>0.00713</td>
<td></td>
<td>0.00301</td>
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<td>(0.0147)</td>
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<td>(0.0175)</td>
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<tr>
<td>Unemployment rate</td>
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<td>0.00713</td>
<td></td>
<td>0.00301</td>
<td>-0.0261</td>
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<td>(0.0147)</td>
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<tr>
<td>Growth</td>
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<td>-0.0249</td>
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<td></td>
<td>(0.0143)</td>
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<td>Observations</td>
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<tr>
<td>AR(1) p</td>
<td>0.0178</td>
<td>0.0125</td>
<td>0.0247</td>
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<td>0.0205</td>
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<tr>
<td>AR(2) p</td>
<td>0.417</td>
<td>0.441</td>
<td>0.453</td>
<td>0.384</td>
<td>0.603</td>
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</tr>
<tr>
<td>Hansen p</td>
<td>0.306</td>
<td>0.590</td>
<td>0.430</td>
<td>0.610</td>
<td>0.485</td>
<td>0.999</td>
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</tbody>
</table>

*Notes: All explanatory variables lagged by one year. Accounts payable, spending, private credit are in percent of GDP. All regressions are estimated with System GMM (using the second to fifth lag, collapsed, as instruments), except models 2 and 8, in which the fixed effect estimator is used. Regressions include time and country fixed effects. Robust standard errors in parentheses (models 2 and 8 use Newey standard errors).

*** p<0.01, ** p<0.05, * p<0.1.
4 The Aggregate Effect of Payment Delays—Evidence from Bayesian VARs

As noted in the previous section, the approach using annual panel data has its pros and cons. One of the major shortcomings is the difficulty of dealing with endogeneity. While we used System-GMM to address this, it could be argued that a more systematic approach would be to move to a system of equations that takes each variable to be endogenous with respect to one another. This simultaneous equations framework is accommodated in a structural Bayesian VAR. Contrary to classical reduced-form VARs which identify shocks using a recursive identification scheme, we allow for a less restrictive identification scheme and move toward non-recursive identification as in Waggoner & Zha (2003).

Bayesian VARs seem a natural alternative to the single equation framework we considered in the previous sections. First, they provide a well-established way to take into account the complex interdependencies among the variables under consideration and thus control for their mutual feedback. Second, by imposing prior restrictions on the parameters in the model we are able to address (i) the proliferation of the parameter space and (ii) the relatively small sample size, which makes it likely that an unrestricted VAR would mistake much of the sample variation to be systematic instead of unsystematic. Using prior restrictions we are able to provide conservative estimates of cross-variable effects, because we ”shrink” them toward a zero prior mean (Koop & Korobilis (2010)). Third, the cross-variable effects from a shock in variable \( j \) to variable \( i \), may be easily gauged by computing the dynamic multipliers

\[
\frac{\partial y_{i,t+k}}{\partial \epsilon_{j,t}} , \ k = 0, \ldots
\]

which at the same time control for shocks to the other variables in the system.

The change in methodology requires various changes to our approach. First, we are more restricted in our choice of control variables. Given that we need longer time series, we move to quarterly data, for which many variables are not available. In any case, given the inclusion of many more lagged variables, we cannot overburden the equations with excessive explanatory variables. Hence we undertake a number of simplifications. First, instead of our measures of arrears or delayed payments, we now simply focus on accounts payable. Their movement over time, especially at the quarterly level, should be indicative of underlying payment delay or arrear issues. Second, instead of dividing AF.7 by GDP and separately controlling for the share of government expenditure in GDP, we now use directly the ratio of AF.7 to total expenditure. This saves one variable (more if lags are counted), but still allows to control for the purely mechanical positive
relationship between expenditure and the amount of outstanding payments. After all, it seems natural to assume that AF.7 rises when spending increases. If the general government rolls over these additional obligations with the same efficiency, our measure of payment efficiency should not be affected. This, however, could be the case with the AF.7-to-GDP ratio. This way we also control for expenditure shocks. Much like the debt-to-GDP ratio, the AF.7-to-expenditure is in units of time and measures how many quarters on average the general government needs to pay its obligations, for every euro it committed to pay. The smaller this ratio, the more efficient the general government is in a given quarter in paying its obligations.

4.1 Data

We use a similar, but reduced, set of variables as in the single equation regression analysis. First, we include the standard set of macroeconomic variables, i.e., quarterly real GDP (seasonally adjusted, national currency) in log-levels, inflation as measured by the GDP deflator (2005=100), the 3-month Euribor money market rate and the AF.7-to-Expenditure ratio. In particular, we take the AF.7 as a ratio of total expenditure, i.e., including wages and transfers, as AF.7, unlike the less often available AF.71 (trade credit), also includes accounts payable in these categories. The liquidity channel through which we suspect the AF.7-to-Expenditure ratio to affect the private sector is proxied by the distance-to-default measure that was used earlier, too.

The sample ranges are unbalanced across countries, but mostly go from 1999Q3 until 2012Q4. We discard countries from our analysis for which (i) the data are not available before 2002Q1, (ii) an entire series contains only missing values or (iii) one or more series contain gaps. This leaves 16 countries in our sample. Further, the empirical analysis on quarterly data will be selectively performed for Italy, Spain, and Portugal, as explained below.

4.2 Structural identification

In this subsection we estimate a structural VAR, i.e., a model that is not generically identified using a Cholesky ordering among variables. Instead, we will follow the approach put forward by Sims (1986) and Waggoner & Zha (2003) and identify shocks directly via restrictions on the contemporaneous impact matrix. This approach is more flexible than recursive identification, because (i) it allows for non-recursive causation and (ii) restrictions can interpreted as representing behavioural equations in the sense of simultaneous equations models (SEMs). The first point plays an important role in our case, because we can implement the restriction that shocks to the AF.7-to-Expenditure ratio do not directly enter the
equation for GDP and DTD, without having to put both to the top of the vector \( y_t \) as in a Cholesky ordering.

Our point of departure is the standard structural BVAR model, i.e., let \( y_t \) be an \( n \)-dimensional random vector, following the \textit{structural} VAR model

\[
y_t = c + \sum_{i=1}^{p} y_{t-i} A_i + \epsilon_t', t = 1, \ldots, T,
\]

(4.2)

where \( A_k \in \mathbb{R}^{n \times n} \) are matrices of parameters, \( c \) is an intercept and \( \epsilon_t \in \mathbb{R}^n \) denotes the vector of \textit{structural} shocks or disturbances in the system. We assume that \( \epsilon_t \) is the standard zero-mean spherical disturbance.

Letting \( x_t' = [y_t, \ldots, y_{t-p}, 1] \) and

\[
Y = [y_t', \ldots, y_1']' ; X = [x_t', \ldots, x_1']' ; E = [\epsilon_t', \ldots, \epsilon_1']' ; F = [A_1, \ldots, A_p, c]'
\]

we may write the whole system more compactly as

\[
Y \in \mathbb{R}^{T \times n} A_0 \in \mathbb{R}^{n \times n} = X \in \mathbb{R}^{T \times k} F \in \mathbb{R}^{k \times n} + E \in \mathbb{R}^{T \times n},
\]

(4.3)

where \( k = np + 1 \). In this form, it becomes apparent that the structural VAR may be viewed as a system of linear simultaneous equations with endogenous variables \( Y \) and exogenous (or predetermined) variables \( X \). The system is identified imposing exclusion restrictions on the matrix \( A_0 \).

The key behavioural assumption will be that shocks to the AF.7-to-Expenditure ratio do \textit{not} directly affect GDP and DTD contemporaneously. We base this assumption on the European Payment Index Report and the average payment duration of countries. Note that for countries where the average payment duration is at least one quarter (90 days), the private sector is very likely to anticipate no payment within the same quarter. That is, for any invoice dated in a given quarter, payment is expected not before the next quarter. If this holds true, then any shock to public payment durations will not by itself affect GDP or the DTD immediately, but either (i) only indirectly via affecting other variables in the system or (ii) only with a lag. We are thus not assuming there is no contemporaneous effect, but merely preclude it is a direct effect. A shock to the average payment duration, for example, may have an immediate direct effect on interest rates, due to the effect it has on credit demand, which in turn can have an effect on GDP within the same quarter.

However, the European Payment Index report shows that this assumption is only warranted for three countries in our sample. At the same time, the three countries—Italy, Spain and Portugal—which exhibit an average payment duration of at least 90 days are those that have been in the focus in terms of their payment
discipline (Figure 5).\textsuperscript{12}

**Figure 5:** Average Reported Payment Duration by the Public Sector (number of days).

![Figure 5: Average Reported Payment Duration by the Public Sector (number of days).](image)


We implement the identification scheme through the matrix $A_0$. In our case, it will be given as

$$
\begin{bmatrix}
\text{GDP}_t \\
\pi_t \\
\text{AF.7-ratio}_t \\
\text{DTD}_t \\
i_t
\end{bmatrix}^\prime
\begin{bmatrix}
a_{11} & 0 & a_{31} & a_{41} & a_{51} \\
0 & a_{22} & a_{32} & a_{42} & a_{52} \\
0 & 0 & a_{33} & 0 & a_{53} \\
0 & 0 & 0 & a_{44} & a_{54} \\
0 & 0 & 0 & 0 & a_{45} & a_{55}
\end{bmatrix}
= c + \sum_{i=1}^{p} y'_{t-i} A_i + \epsilon'_t \tag{4.4}
$$

where AF.7-ratio means the AF.7-to-expenditure ratio. The first column of $A_0$ represents the assumption that any contemporaneous shocks to aggregate growth are pure TFP shocks and that any feedback from the other endogenous variables affects GDP only with a lag. Hence, $\epsilon_{1,t}$ may be viewed as the TFP shock. The second column states that prices are sticky in the short run. The third column serves to identify the shock from the AF.7-to-expenditure ratio, in particular to set it apart from the TFP shock. It states that shocks to GDP affect the average payment duration in the public sector, but not vice versa. In principle, this scheme stems from the observation that for the three countries under consideration, the average payment delay is 90 days or at least very close to 90 days. Thus, private suppliers are thought to anticipate this average delay and to adjust their businesses

\textsuperscript{12}For Greece the average payment duration also exceeds 90 days, but the country drops out owing to insufficient data, according to our criteria data section.
accordingly. Only once an entire quarter goes and payments still do not arrive, private suppliers realize that they had underestimated the public payment delay. Column four says that DTD is affected directly and immediately by all variables, but the AF.7-ratio. Finally, column five states that the interest rate as a fast-moving variable reacts to all shocks immediately.

We set the following hyperparameters for the model: $\lambda_0 = 0.5$, $\lambda_1 = 0.1$, $\lambda_3 = 2$ and $\lambda_4 = 1$.\textsuperscript{13}

Further details on the prior and the posterior simulation via Gibbs sampling can be found in in the Mathematical Appendix.

4.3 Empirical results

The impulse responses that derive from the structural model—summarised in Table 6 together with the associated cumulative responses—are depicted in Figure 6. We restrict ourselves to report only impulse responses of interest, i.e., the impulse response of the DTD, GDP and the short-term interest rate to a 10 percent expenditure shock. The solid black lines show the median impulse response drawn from 3,000 Monte Carlo draws from equation (6.9). Additionally, we have plotted classical pointwise 68th percentile error bands.

The model yields fairly rich dynamics in terms of the impulse responses. For the three countries under consideration, we find that private sector solvency as measured by the distance to default contracts as the average payment period of the general government increases. Signs for the responses of interest are as expected. For all three countries we find that an increase in the AF.7-to-Expenditure ratio results in a negative shock to the distance to default in the private sector. The cumulative response of the distance to default to a shock in the AF.7-to-expenditure ratio is sizeable after just 4 quarters, e.g., for Spain the annual response is such that the median distance to default is roughly 0.8 standard deviations smaller. For the direct impact on aggregate growth we find almost no significant impact. Only in Portugal, the response is significantly negative, albeit small in the short run. The response of the interest rates to an increase in public payment delays is ambiguous. While for example the initial response is positive in all countries, the pattern quickly reverses for Italy and Portugal and interest rates make up for the initial increase. For Italy, this renders the cumulative response even negative over the course of a year. For the other two countries, the annual response is significantly positive, economically sizeable and persistent.

\textsuperscript{13}Please refer to the Mathematical Appendix for a detailed discussion of the prior hyperparameters. We use the R package MSBVAR by Brandt (2014) for estimation. We also did a prior specification search, but the marginal likelihood criterion suggested only very little shrinkage. We believe that given the small sample size, it is appropriate to be more conservative than is suggested by the prior search.
Table 6: Quarterly Structural Responses.

<table>
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<th>Country</th>
<th>Variable</th>
<th>Impulse response</th>
<th>Cumulative response</th>
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<tr>
<td></td>
<td></td>
<td>No. of quarters ahead</td>
<td>annual response</td>
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<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
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<tr>
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<td></td>
<td>i</td>
<td>0.05</td>
<td>-0.06</td>
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<tr>
<td>ESP</td>
<td>GDP</td>
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</tr>
<tr>
<td></td>
<td>i</td>
<td>0.22</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Source: Authors’ calculation

The overall results for the subset of countries in this section suggest that public payment delays affect the economy through a liquidity channel. While in aggregate terms, growth is not immediately affected (and we would arguably not expect it to do so significantly), the resilience of private sector entities—here publicly listed firms—is negatively affected. Moreover, the amount of liquidity absorbed by the central government also affects interest rates in the very short term. The three-month Euribor rate reacts with a mild increase over the first few quarters.
**Figure 6:** Structural Impulse Responses of Selected Variables to a 10 Percent of Expenditure Shock to AF.7 (denoted as "Credit" in the figure).

Source: Authors’ calculation.
5 Conclusion

This paper has considered the impact of the government’s payment discipline on the private sector. The overall conclusion is that government decisions on the speed of effecting payments have important repercussions for the economy. Interestingly, the crucial aspect appears to be the total amount of outstanding payments and their average delay, rather than whether or not payments are arrears in a legal or accounting sense.

Our empirical results from panel data have shown that payment delays appear to reduce profits, increase the likelihood of bankruptcies, and even reduce economic growth. While the exact size of the impact is hard to pin down from the available data, results are significant in most specifications. Findings using estimated arrears are qualitatively similar, but are less often significant. This could either be interpreted as meaning that whether a payment is in arrear in a formal sense is less important than the size and average delay of payments, or it could be due to our estimation. If data on actual arrears were available, this aspect could be investigated further. Finally, on average for the European Union sample, the total amount of accounts payable does not appear to play a role, suggesting that predictable and regularly cleared payment delays are not necessarily a problem.

Our results from Bayesian VARs performed on available quarterly data for Spain, Italy and Portugal, show that an increase in the average payment duration leads to an increase in the likelihood of private sector defaults, with weaker evidence on the direct growth channel.

Our analysis and results have several implications for policy makers. Based on the findings in this paper it appears that delaying payments to deal with a funding issue or a debt limit is a costly way of achieving these aims. Quite to the contrary, efforts to accelerate payments and reduce existing stocks of arrears could be a helpful way of boosting the economy in the short run. Moreover, they would typically not increase deficits as long as all spending was properly accounted when it accrued.14 Having established that there is an inverse relationship between public payment delays and overall economic performance or growth, the first policy recommendation is to closely monitor the amount of arrears and payment practices in a given country to foster economic performance. A second policy implication would be to address the prevailing measurement issues associated with variables such as other accounts payable and install a comprehensive and frequent measurement and accounting system for public payment practices. Ideally, this would aim to record the entire payment history for each individual invoice, i.e., the outstand-

14In countries where the issue is lack of funding, speeding up of payments would have to be weighed against other spending, as this would also have a positive impact on the economy. The benefits of other spending may be reduced, though, if suppliers cannot be sure about when they will be paid.
ing amount, the invoice date, the contractual payment period, and the payment duration for both the central and the local/regional levels of the general government on a consolidated and unconsolidated basis. Third, given their impact on economic performance, delayed payments or arrears could be included in economic surveillance. For that purpose, it would also be useful to improve the availability of data on accounts payable and arrears, as well as on their breakdown.
6 Mathematical Appendix

We follow Sims and Zha (1998) and Waggoner and Zha (2003) in estimating the model. Toward that end, note that the (conditional) likelihood function of the data is given as

\[ p(y_1, \ldots, y_T \mid A) \propto |A_0| \exp \left\{ -\frac{1}{2} E'E \right\} \]

Conditional on \( A_0 \) the above likelihood is quadratic in \( F \) and thus together with an appropriate prior \( F \mid A_0 \) is matricvariate normal. The posterior for \( A_0 \) however turns out to be non-standard and requires further processing. The exclusion restrictions we impose on each of the columns of \( A_0 \), may be represented by the restriction matrices \( Q_i \) of rank \( q_i \).

\[ Q_i a_i = 0 \quad (6.1) \]

Elements of \( F \) may be restricted in a similar way via a matrix \( R_i \) that has rank \( r_i \). As has been demonstrated by Waggoner and Zha (2003), \( a_i \) and \( f_i \) will satisfy the above restrictions, iff there exists a \( n \times q_i \) matrix \( U_i \) and \( n \times r_i \) matrix \( V_i \), such that

\[ a_i = U_i b_i \quad (6.2) \]
\[ f_i = V_i g_i \quad (6.3) \]

The matrix \( U_i \) may be found via a singular value decomposition, that takes \( U_i \) to be the matrix of right-singular vectors that lie in the Null space of \( \text{diag}(a_i) \). The set of parameters given by \( b_i \) and denotes \( g_i \) is the set of parameters that is free to estimate.

Our prior on \( (a_i, f_i) \) is of the form

\[ p(A_0)p(F \mid A_0) \quad (6.4) \]

where

\[ a_i \sim N(0, S_i) ; \quad S_i = \text{diag} \left( \frac{\lambda_0^2}{\sigma_i^2} \right) \quad (6.5) \]
\[ f_i \mid a_i \sim N(P_i a_i, H_i) ; \quad P_i = [I_n, 0_{n(p-1)+1 \times n}] ; \quad H_i = \begin{bmatrix} (\frac{\lambda_0 \lambda_k}{\lambda_0^2 \sigma_i})^2 I_k & 0_{k-1} \\ 0_{1 \times k-1} & \lambda_0^2 \lambda_k^2 \end{bmatrix} \quad (6.6) \]

This prior on \( (a_i, f_i) \) is then mapped to a prior on \( (b_i, g_i) \) (we refer the reader to
Waggoner and Zha, 2003 for any details):

\[ b_i \sim N(\mathbf{0}, \tilde{S}_i) \ ; \ \tilde{S}_i = (U_i' \tilde{S}_i U_i)^{-1} \]  

(6.7)

\[ g_i \mid b_i \sim N(\tilde{P}_i b_i, \tilde{H}_i) \ ; \ \tilde{P}_i = P_i U_i \ ; \ \tilde{H}_i = H_i \]  

(6.8)

Combining this prior with the likelihood, the posterior is found to be

\[ p(b_i, g_i \mid Y) = p(b_1, \ldots, b_n \mid Y) \prod_{i=1}^{n} p(g_i \mid b_i, Y) \]  

(6.9)

where

\[ p(g_i \mid b_i, Y) = N(\tilde{P}_i b_i, \tilde{H}_i) \ ; \ \tilde{P}_i = H_i \left( V_i' X' X V_i + \tilde{H}_i^{-1} \tilde{P}_i \right) \]  

(6.10)

\[ \tilde{H}_i = \left( V_i' X' X V_i + \tilde{H}_i^{-1} \right)^{-1} \]  

(6.11)

and

\[ p(b_1, \ldots, b_n \mid Y) = ||U_1 b_1, \ldots, U_n b_n||' \exp \left\{ -\frac{T}{2} \sum_{i=1}^{n} b_i' S_i b_i \right\} \]  

(6.12)

with

\[ S_i = \left( \frac{1}{T} \left[ U_i' Y' Y U_i + \tilde{S}_i + \tilde{P}_i' \tilde{H}_i^{-1} \tilde{P}_i - P_i' H_i^{-1} P_i \right] \right)^{-1} \]  

(6.13)

In order to estimate the model, we use the Waggoner-Zha Gibbs sampler, because there is no straightforward way to sample from (6.9). Especially the fact that \( b_i \) appears in the determinant of (6.12) makes the posterior of \( b_i \) non-normal. Waggoner and Zha (2003) show that alternatively one may sample from

\[ p(b_1, \ldots, b_{q_i} \mid b_{-i}, Y) \propto |\beta_1|' \exp \left\{ -\frac{T}{2} \sum_{i=1}^{q_i} \beta_i^2 \right\} \]  

(6.14)

where

\[ b_i = T_i \sum_{i=1}^{q_i} \beta_i w_i \]  

(6.15)
denotes the set of $b_j$ such that $j \neq i$,

$$T_i T'_i = S_i \quad (6.16)$$

and $w_1, \ldots, w_{q_i}$ form an orthonormal basis of $\mathbb{R}^{q_i}$.\(^{15}\)

We use the orthogonalization approach of Waggoner and Zha (2003) to devise the following Gibbs sampler:

1. Choose a starting value $A_0^{(0)}$ satisfying (6.1).\(^{16}\)

2. Draw $A_0^{(s)}$ conditional on $(F^{(s-1)}, Y)$: for $i = 1, \ldots, n$ draw $\beta_1, \ldots, \beta_{q_i}$ from (6.14) conditional on $b_1^{(s)}, \ldots, b_{i-1}^{(s)}, b_i^{(s-1)}, \ldots, b_n^{(s)}$, and let $b_i^{(s)}$ be defined by (6.15) and take $a_i = U_i b_i^{(s)}$.

3. Draw $F^{(s)}$ conditional on $(A_0^{(s)}, Y)$ from (6.10).

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\(^{15}\)Use for example the Gram-Schmidt method to find them.

\(^{16}\)We take the posterior mode of the marginal posterior of $A_0$, which we find by numerical maximization (Nelder-Mead then BFGS).
References


