

Colliard, Jean-Edouard

**Working Paper**

## Catching falling knives: speculating on market overreaction

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## WORKING PAPER SERIES

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### CATCHING FALLING KNIVES SPECULATING ON MARKET OVERREACTION

Jean-Edouard Colliard



In 2013 all ECB publications feature a motif taken from the €5 banknote.

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**Jean-Edouard Colliard**

European Central Bank; e-mail: [jean-edouard.colliard@ecb.europa.eu](mailto:jean-edouard.colliard@ecb.europa.eu)

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<b>Address</b>	Kaiserstrasse 29, 60311 Frankfurt am Main, Germany
<b>Postal address</b>	Postfach 16 03 19, 60066 Frankfurt am Main, Germany
<b>Telephone</b>	+49 69 1344 0
<b>Internet</b>	<a href="http://www.ecb.europa.eu">http://www.ecb.europa.eu</a>
<b>Fax</b>	+49 69 1344 6000

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## **Abstract**

Market participants often invest in order to acquire information that pertains to the market itself (e.g. order flow) rather than to fundamentals. This enables them to infer more information from past trades. I show that agents trading on such information, typically high-frequency traders, decrease the likelihood of short-lived mispricings by trading against price pressure. In the long-run however, such countervailing speculation amounts to signal-jamming, slowing down price discovery. These traders insure the market against short-run crashes by “catching falling knives”. Higher adverse selection and slower convergence form the “premium” paid by other market participants.

*Journal of Economic Literature* Classification Number: D82, G0, G12, G14.

*Keywords:* market crashes, speculation, supply information, high-frequency trading.

## Non-technical summary

Market participants have invested a lot in the acquisition of information over the past decades, using in particular the new tools offered by computerization. There is widespread agreement that markets have become more efficient as a result, in the sense that it has become more difficult to forecast price movements using past market data. It is not clear however that this has led to greater price informativeness. [Bai, Philippon, and Savov \(2012\)](#) for instance show that returns have not become better predictors of earnings. This is consistent with the idea that a large fraction of information acquisition focuses on the market itself (prices, order flow) and not on fundamentals.

This paper builds a model where “supply-informed traders” have private information on a non-fundamental component, namely whether the market is under pressure coming from fire sales or not. I study the behavior of these traders and their impact on price-discovery and financial stability. Accounts of famous market crashes often involve uncertainty on a non-fundamental parameter: the extent of portfolio insurance for the 1987 crash, the trigger of the price drop during the “flash crash” of May 6, 2010, how crowded the long/short equity fund category was during the “quant event” of August 2007... all seem to have played an important role during these stress episodes. Non-fundamental uncertainty generates market crashes in the model: as uninformed agents do not know the extent to which observed trades are affected by uninformed fire sales, they mistake those for informative sales and tend to overreact in the short-run.

The contribution of the paper is to study the impact of supply-informed traders in such a setup. When the amount of fire sales is high, they know that the market is overreacting to sales and thus tend to buy. This contrarian behavior makes it less likely that quick price drops are observed, thus crashes are less likely. Interestingly, this same behavior makes the market less efficient in the long-run: with a huge flow of sell orders, the market would learn quickly that fire sales are important and would take into account that sales are not very informative. Because supply-informed traders are buying and reduce the trade imbalance, it becomes actually more difficult to learn the amount of fire sales. I show that the same phenomenon happens when a trading halt is triggered after large price movements.

Supply-informed traders are “catching falling knives”: they buy when price drops are likely due to overreaction and the probability of a reversal is high. I propose to see this activity as a form of insurance against non-fundamental crashes: their occurrence is reduced, but there is a premium to pay in the form of higher spreads (due to increased adverse selection) and slower price convergence in the long-run.

Many agents on financial markets acquire information not related to the fundamentals of assets, but to the market itself, in particular information about prices and order flow. High-frequency trading relies heavily on such data, and this paper claims that they should be analyzed not as having fundamental information, but as supply-informed traders. This gives new predictions about their impact on the market, the shape of their profit, and which types of assets they can be expected to focus on.

Finally, the paper has two main policy implications:

Firstly, even if high-frequency trading is suspected to increase volatility, it may also have a stabilizing function in extreme events. Indeed, something remarkable about the flash crash is how quickly the market rebounded and prices came back to normal. Information about the supply and the quick realization that the sudden price drop didn’t have the features of an event driven by fundamental negative information may have been crucial to allow a quick stabilization. This is an additional element to take into account when deciding on policies aimed at curbing high-frequency trading, for instance financial transactions taxes.

Second, it is often advocated that more use should be made of “circuit-breakers” (automatic closure of the market after extreme price movements) to avoid flash crashes. This is useful in the model if it gives time to traders to learn whether something else than negative news may explain the observed sales. If the break is too short to allow for this acquisition of supply information but only breaks the feedback loop giving rise to the sales (typically, a glitch in a selling algorithm), it will stop the crash but also prevent a quick recovery: uninformed participants cannot learn anymore whether past sales were informative or not. A circuit-breaker should thus either intervene soon to prevent a crash in the first place, or last long enough for market participants to acquire information about the source of the crash.

# 1 Introduction

Computerization has enabled market participants to quickly access and process data on prices and order flows, and to base their trading on this information. Between 1996 and 2002, the impact on prices of pressure coming from order imbalances has dramatically declined due to countervailing trades by “astute traders” (Chordia, Roll and Subrahmanyam (2002, 2005)). However, Bai, Philippon, and Savov (2012) show that returns have not necessarily become better predictors of earnings, in line with the idea that efforts in information acquisition have focused on non-fundamental information. Modeling informed agents as agents with private *fundamental* information may then be misleading to analyze current market developments. I propose a simple model with a role for traders endowed with private information about the composition of orders and show how their impact on the market differs from the one exerted by traditional informed traders.

Non-fundamental information is particularly relevant in periods of financial turbulence: some agents have to sell large amounts due to liquidity constraints, and other investors face both fundamental uncertainty and uncertainty about whether sales are informative or not. Accounts of market crashes typically involve this second source of uncertainty: the extent of portfolio insurance for the 1987 crash (Gennotte and Leland (1990)), speculation about whether the “flash crash” of May 2010 was due to fundamental or non fundamental causes<sup>1</sup>, unawareness of long/short equity fund managers of how crowded this category was during the “quant event” of August 2007 (Khandani and Lo (2007)). Agents able to identify whether sales are informative or not could be important for market stability in such contexts. I focus in my model on this particular type of agents with non fundamental information which, following Gennotte and Leland (1990), I call “supply-informed” traders.

I show that supply-informed traders correct short-run mispricings: they behave as contrarians when the market overreacts, and as positive feedback traders when the market underreacts to observed trades. This behavior however jams the signals market-makers receive about non-fundamental information, so that the long-run impact of these speculators

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<sup>1</sup>See explanations 1-6 in Easley, Lopez de Prado, and O’Hara (2011), pp. 2-3, while Kirilenko *et al.* (2011) deem it possible that some “Fundamental Buyers could not distinguish between macroeconomic fundamentals and market-specific liquidity events”.

is negative. They can also be excluded from trading in the long-run if too many traders have fundamental information. Moreover, their profit has fat tails: they are often prevented from trading by high spreads, but with a small probability a sudden crash appears and they try to “catch a falling knife”. On average they make a profit because they know when the crash is more likely to be driven by uninformed sales, but when this is actually not the case they make a loss.

The model is deliberately abstract so as to apply to different types of agents. Supply-informed traders can be seen as liquidity-taking high-frequency traders who use data on the order flow and base their trading on information on liquidity rather than on fundamentals. At a different frequency, they can be hedge funds relying on quantitative techniques to detect a stock’s undervaluation due to liquidity events, then buy stocks they suspect to be affected by fire sales, and sell similar stocks that are not (see [Cella, Ellul, and Giannetti \(2011\)](#)). They more generally correspond to any “astute traders” ([Chordia, Roll, and Subrahmanyam \(2002\)](#)) with a better ability to recognize uninformative price pressure.

To understand their behavior, I build an extension of [Glosten and Milgrom \(1985\)](#) with “positive feedback traders”<sup>2</sup>, the number of which is uncertain, and supply-informed traders who privately know this number (e.g. due to their quick processing of order flow information) but not the asset’s value. The supply-informed traders’ impact is different from traditional value-informed traders, so that the latter category cannot be thought of as a theoretical shortcut to encompass these two different types of private information.

In order to give some intuition, assume a situation in which price drops trigger more fire sales than expected by market-makers. Since they overestimate the probability of an informed sale<sup>3</sup> and thus believe sales to be more informative than they really are, market-makers update prices downwards too much after each sale. Supply-informed traders understand this is over-reaction, buy and behave as contrarians ([Proposition 3](#)). The proportion of sales thus decreases, which reduces the likelihood of a price drop driven by fire sales. For the same reason, it becomes more difficult for other agents to understand that the market

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<sup>2</sup>Meaning here traders who mechanically sell after price drops.

<sup>3</sup>As will be apparent below, uninformed agents know ex ante the true probability that positive feedback trading is high. But they don’t know which state of the world realizes and thus always under- or over-estimate positive feedback ex post.



is affected by positive feedback sales. This signal-jamming effect slows down the update of market-makers about positive feedback in the long-run, and thus price discovery (Proposition 5). Both effects are illustrated by Fig. 7 (to be detailed later), which shows how increasing the number of supply-informed traders affects the average price path when the asset has a high value and positive feedback is high.

When the asset’s value is high supply-informed traders make a gain, otherwise they make losses by trying to “catch a falling knife”. If their informational advantage is low, supply-informed traders are eventually excluded from the market by too high spreads (Lemma 2 and Corollary 1), thus they make most of their profit at the beginning of the trading period, when market-makers are uncertain about the amount of positive feedback trading. Finally, supply-informed traders’ profit is on average positive but exhibits fat tails (Numerical result 2).

Identifying supply-informed trading with some high-frequency trading strategies (liquidity-taking and based on order-flow information, for instance “sniffing”) yields the implication that HFT activity should increase in periods of non-fundamental uncertainty (Remark 4). Results on supply-informed traders’ profit may also help explain why some traders who usually provide liquidity exit the market during extreme events. The contrarian behavior of supply-informed traders implies that this type of informed trading is not always captured by measures such as the PIN (Remark 1), although it increases the spread (Remark 3).

I also show that sales can contain a positive signal on the asset’s value in this framework as they can lead market-makers to understand that many past sales were probably uninformative (Remark 2). Finally, a stylized circuit-breaker is introduced in the model and shown to have an effect similar to supply-informed traders, in particular as it can slow down long-run price discovery (Numerical result 3).

The remainder of this paper is organized as follows: the end of this section relates the paper to previous works, section 2 derives the equilibrium, section 3 studies market liquidity, section 4 the supply-informed traders’ impact on short term and long term price discovery as well as their profit, section 5 concludes. References, figures and proofs are in the Appendix.

**Related literature:** this work bridges a gap between the literature on “non-fundamental uncertainty” and the literature on speculation. In the former, a dimension of uncertainty other than the value of the asset is introduced to investigate an anomaly: excess volatility in [Easley and O’Hara \(1992\)](#), herding behavior in [Avery and Zemsky \(1998\)](#)<sup>4</sup> or, closer to this paper, non-fundamental crashes or bubbles in [Genotte and Leland \(1990\)](#), [Jacklin, Kleidon, and Pfleiderer \(1992\)](#) or [Romer \(1993\)](#). I contribute to this literature by studying how speculation based on supply information only can affect these anomalies.

[Genotte and Leland \(1990\)](#) introduce “supply-informed” traders, but in too small a number to prevent a crash from happening; since then the literature has underestimated this possibility and identified supply-informed traders with a few agents having a very precise information about supply due to market-making activity. Since many (in particular high-frequency) traders now base their strategy on data about prices or order flow but not on fundamentals<sup>5</sup>, I reconsider this problem in a market with many supply-informed traders having less precise information. [Ganguli and Yang \(2009\)](#) make a similar assumption, but their focus on complementarity and multiplicity of equilibria is quite different. In [Dumitrescu \(2005\)](#) supply-informed trading can have a big impact, but the focus is on liquidity and the supply-informed trader is a monopolist. Moreover both papers use a static framework.

The literature on speculation (seminal works here include [Hart and Kreps \(1986\)](#) and [Stein \(1987\)](#)) focuses on speculators with “fundamental” information or trying to use public information to infer the value of an asset. In [De Long \*et al.\* \(1990b\)](#) they are implicitly informed about both positive feedback and the asset’s fundamental value.

In contrast, this paper studies traders with information about the order flow but not about fundamentals. Moreover, the interaction between two types of informed traders can be important, as supply-informed trades can mistakenly be interpreted as carrying information about fundamentals. In my dynamic framework, uninformed liquidity providers face a situation where both uncertainty and adverse selection are two-dimensional, as in the static framework of [Genotte and Leland \(1990\)](#). [Hong and Rady \(2002\)](#) is to my knowledge the only example featuring a dynamic market with two-dimensional adverse selection but

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<sup>4</sup>[Park and Sabourian \(2011\)](#) show that multi-dimensional uncertainty is actually not the fundamental cause of herding.

<sup>5</sup>Although “price watching” can also give a signal about fundamentals, see [Cespa and Foucault \(2012\)](#).

market-makers are the agents informed about liquidity, thus no type of agent has to learn about both parameters. In [Brunnermeier and Pedersen \(2005\)](#), predatory traders have information about distressed market participants but fundamental information is symmetric. In [Jacklin, Kleidon, and Pfleiderer \(1992\)](#) supply information is imperfect but symmetric.

[Menkveld and Yueshen \(2013\)](#) also point at a signal-jamming effect of high-frequency traders, seen as “middlemen”. The mechanism is quite different however. A key specificity of my paper is that supply-informed traders can hinder updating precisely about the parameter they privately know, which is in sharp contrast with traders with value information who always reveal their private information by trading.

A last theoretical strand of the literature that is important for the model is concerned with possible microfoundations of positive feedback trading. [Scharfstein and Stein \(1990\)](#) and [Dasgupta and Prat \(2006\)](#) show fund managers’ incentives to mimic other managers for reputational motives, a behavior which can lead to positive feedback (and herding). In [De Long \*et al.\* \(1990b\)](#) “destabilizing speculation” can reinforce the effects of positive feedback trading instead of attenuating them. [Chowdhry and Nanda \(1998\)](#) study the effect of margin calls, which trigger uninformed sales. In [Brunnermeier and Pedersen \(2009\)](#) margin calls endogenously drive prices down, which increases margin requirements, so that “margin spirals” occur. I take as given a simplified form of positive feedback behavior, that may be partly rationalized by one of the papers above, and study how supply-informed agents anticipating this behavior help or hinder the aggregation of information. I complement this literature by showing that trading on supply information attenuates the short-term effects of positive feedback trading, but can have a negative long-run impact.

Finally, as the framework of this paper is motivated by troubled period on financial markets, it is linked to papers investigating recent events such as the “flash crash” of May 2010 ([Easley, Lopez de Prado, and O’Hara \(2011\)](#), or [Kirilenko \*et al.\* \(2011\)](#) whose “opportunistic traders” behave very much as supply-informed traders) or the “quant event” of August 2007 ([Khandani and Lo \(2011\)](#)), as well as to empirical papers documenting the performance of different strategies that could be identified with supply-informed trading, which will be discussed in section [4.3](#).

## 2 Framework and equilibrium

### 2.1 Traders and trading mechanism

An asset gives a final payoff  $v$ ,  $v = 1$  with publicly known prior probability  $\pi$  and  $v = 0$  otherwise. There are  $3\hat{x}_N$  **uninformed traders** in two groups:

-*Noise traders* sell, buy, or hold, each with probability  $1/3$ , regardless of the price in the current period or of past history. They can be seen as trading for exogenous liquidity motives and their behavior micro-founded as in [Glosten and Milgrom \(1985\)](#).

-*Positive feedback traders* have to sell after a sell order and represent a proportion  $\alpha F < 1$  of uninformed traders. There is uncertainty about positive feedback: with prior probability  $\lambda$  we have  $F = F^+ = 1$ , and with prior probability  $1 - \lambda$  we have  $F = F^- = 0$ . After a purchase or no trade these traders behave as noise traders, if  $F = F^-$  there is thus no positive feedback. We then have two groups of **informed traders**:

-*Value-informed traders*, in number  $\hat{x}_I$ , know the value  $v$  of the asset.

-*Supply-informed traders*, in number  $\hat{x}_S$ , know  $F$ . They hold a different expectation than market-makers about the asset's value and buy, sell or hold depending on prices. They are called  $F^+$  (resp.  $F^-$ ) traders when they know positive feedback is high (resp. low).

Under these assumptions the total number of traders is  $X = \hat{x}_S + \hat{x}_I + 3\hat{x}_N$ . Denote  $x_S, x_I$  and  $x_N$  respectively the  $\hat{x}$ s normalized by  $X$ , thus giving the probability given a trade to face a trader from each category. Let  $t$  be an index for the number of trades, and  $\tau$  an index for real time. Assuming traders arrive according to a Poisson process as in [Easley, Kiefer, O'Hara, and Paperman \(1996\)](#), in  $\tau$  seconds we expect  $t = \tau \times X$  trades and it is easy to convert real time to trade time and conversely. I normalize  $\hat{x}_I + 3\hat{x}_N = 1$  so that when there is no supply-informed trader there is perfect correspondence between the two indices, and  $X = 1 + \hat{x}_S$ . This approach takes into account that increasing  $\hat{x}_S$  has an impact on the probability to face a supply-informed trader, but also increases the total trading intensity and thus the average number of trades in a given time interval<sup>6</sup>. Denote by  $T_t$  the trade that takes place in period  $t$ , with  $T_t = 1$  a purchase,  $-1$  a sale, and  $0$  no trade. The following

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<sup>6</sup>Notice that traders are assumed to be small here, so that trading intensities are not the choice variables of large traders.

table sums up the partition of traders in  $t + 1$  depending on  $T_t$ :

Types		Proportion, $T_t = -1$	Proportion, $T_t \neq -1$
Uninformed	Noise	$3(1 - \alpha F)x_N$	$3x_N$
	Positive feedback	$3\alpha Fx_N$	0
Informed	Value-informed	$x_I$	$x_I$
	Supply-informed	$x_S$	$x_S$

Positive feedback traders are close to “noise traders”, but their behavior is completely determined by the previous trade.<sup>7</sup> They could be funds facing funding constraints and withdrawals, which according to [Ben-David, Franzoni, and Moussawi \(2010\)](#) accounted for 78% of equity sell-offs by hedge funds around the Lehman collapse. Other interpretations were developed in the literature review. Most of them imply a more complicated behavior, that would typically depend not only on the last trade but also on prices. To keep the model simple, I abstract from such behaviors the positive feedback component, take it as given and see what it implies for the aggregation of information. A key feature is that market-makers do not know if some of the negative price pressure they observe comes from positive feedback traders or not, typically leading to market crashes due to overreaction. The model can be extended to the symmetric case of positive feedback after purchases, or after both purchases and sales, which allows for studying bubbles.

**Trading mechanism:** in each period  $t$ , perfectly competitive market-makers quote bid and ask prices  $B_t, A_t$ . A trader is randomly selected and buys, sells or holds. Since the asset can only take the values 0 or 1, value-informed traders always buy or sell<sup>8</sup>. Note that each trader is “small” and cannot expect to trade again in the future.

Having kept track of trades before time  $t$ , market-makers know what value of the asset supply-informed traders would have inferred from these trades, depending on  $F$ . Denote by  $E_t^+, E_t^-$  the expected value of the asset at the beginning of period  $t$ , respectively from the point of view of  $F^+$  and  $F^-$  supply-informed traders. From their prior probability  $\lambda_1$  and previous trades, market-makers in period  $t$  have inferred the probability  $\lambda_t$  that  $F = F^+$ . Finally, all traders know the last order and thus the direction of positive feedback.

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<sup>7</sup>I still need noise traders however, otherwise all uncertainty would quickly disappear.

<sup>8</sup>This excludes informational cascades, as value-informed traders always trade based on their perfectly accurate signal.

Market-makers observe the order flow, which allows them to learn both about the asset's value and about positive feedback. Supply-informed traders have an even more precise information (here, perfect) about the latter, that may come from inside sources (liquidity needs of constrained institutions) or from access to/quick processing of data on order flow (high-frequency trading). Market-makers can also be interpreted as a modeling device to mirror uninformed liquidity providers more generally.

Denoting  $I_t$  the public information available in period  $t$  we have<sup>9</sup>:

$$\begin{aligned} E_t^+ &= \Pr(v = 1 | I_t, F = F^+) = \mathbb{E}(v | I_t, F = F^+) \\ E_t^- &= \Pr(v = 1 | I_t, F = F^-) = \mathbb{E}(v | I_t, F = F^-) \\ \lambda_t &= \Pr(F = F^+ | I_t) = \mathbb{E}(F | I_t) \end{aligned}$$

The analysis will use these variables as the evolution of  $E_t^+$  and  $E_t^-$  over time is given by simple Bayesian learning with one-dimensional uncertainty. Fig. 1 shows the probability tree of a given period if there was a sale in the previous one. Using  $E_t^+$  and  $E_t^-$  it is straightforward to compute the price  $p_t$  in period  $t$ :

$$p_t = \Pr(v = 1 | I_t) = \lambda_{t+1} E_{t+1}^+ + (1 - \lambda_{t+1}) E_{t+1}^- \quad (1)$$

[Insert Fig. 1 here.]

## 2.2 Prices, orders and updates

**Equilibrium:** I first show the existence of a unique equilibrium and derive how traders and market-makers update their beliefs from one period to the next. At the beginning of each period, the direction of positive feedback and value-informed traders' orders are already determined and do not depend on quotes. When  $F = F^j$  supply-informed traders buy in period  $t$  if  $A_t < E_t^j$ , sell if  $B_t > E_t^j$ , hold otherwise. The direction in which they want to trade is denoted  $O_t^+$  if  $F = F^+$  and  $O_t^-$  if  $F = F^-$ . For brevity I also use  $O_t^F, E_t^F$  to denote the trading direction and the expectation of  $v$  conditional on the realized value of  $\tilde{F}$ , while

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<sup>9</sup>All notations used in the paper can be found for reference in the Appendix A.1

$O_t^{1-F}, E_t^{1-F}$  are conditioned on the other value.  $A_t$  and  $B_t$  bring zero expected profit to market-makers:

$$A_t = \Pr(v = 1 | I_t, O_t^+, O_t^-, T_{t-1}, T_t = 1) \quad (2)$$

$$B_t = \Pr(v = 1 | I_t, O_t^+, O_t^-, T_{t-1}, T_t = -1) \quad (3)$$

The quotes  $A_t, B_t$  depend on the behavior  $O_t^+, O_t^-$  market-makers expect from supply-informed traders. This behavior depends on the quotes. In equilibrium  $O_t^+, O_t^-, A_t, B_t$  must be consistent with each other, that is:

$$\text{For } j \in \{-, +\}, O_t^j = 1 \Leftrightarrow E_t^j > A_t, O_t^j = 0 \Leftrightarrow A_t > E_t^j > B_t, O_t^j = -1 \Leftrightarrow E_t^j < B_t \quad (4)$$

This problem has a unique solution:

**Proposition 1.** *For a given vector  $(E_t^+, E_t^-, T_{t-1}, \lambda_t)$  there is a unique vector  $(O_t^+, O_t^-, A_t, B_t)$  such that (2), (3) and (4) are satisfied. Moreover  $O_t^+ = O_t^- \Rightarrow O_t^+ = O_t^- = 0$ .*

See the Appendix A.2.1 for the expression of bid and ask prices and the proof. The intuition is the following: assume a period in which  $E_t^+ > A_t > E_t^-$ . Market-makers make zero profit in expectation on a sale, knowing  $F^+$  traders buy. With another ask price  $A' \in [E_t^-, E_t^+]$ , profit would be different from zero. If  $A'$  is higher than  $E_t^+$ , adverse selection is lower since no supply-informed trader can buy anymore, and the ask price is higher. Thus market-makers' profit is strictly positive. Conversely it would be strictly negative with  $A' < E_t^-$ . Hence if  $A_t$  is an equilibrium price it is unique. Finally, since a price is a weighted average of supply-informed traders' expectations (equation (1)) market-makers cannot expect both types of supply-informed traders on the same side: if both types were selling, market-makers would make a loss at the current bid.

**Updates:** Supply-informed traders update their beliefs after each trade, thus  $E_{t+1}^+ = \Pr(v = 1 | T_t, F = F^+, E_t^+)$ ,  $E_{t+1}^- = \Pr(v = 1 | T_t, F = F^-, E_t^-)$ . Knowing the direction of the previous trade it is straightforward to update these beliefs using Bayes' law but there are 18 cases to consider (trade direction in  $t + 1 \times$  trade direction in  $t \times$  value of  $F$ ). The

Appendix A.2.2 gives compact formulas for the updating rule.

Notice in particular that how supply-informed traders update their beliefs about  $v$  when there is a sale in  $t + 1$  depends on  $F$ . A sale is less informative if some positive feedback traders sell in  $t + 1$ , that is if  $F = F^+$ . A further difference is that if  $O_{t+1}^+ \neq O_{t+1}^-$  the behavior they expect from other supply-informed traders depends on  $F$ .

How the updating differs depending on the information about positive feedback trading can be best understood with an example. Assume there has just been a sale. If  $F = F^-$ , supply-informed traders know positive feedback is lower than what market-makers think, thus they infer from a further sale a higher probability that the asset is not valuable. Conversely, if  $F = F^+$  they tend to be more optimistic about  $v$  after a sale than market-makers. Depending on their information and on history, supply-informed traders may be more optimistic or more pessimistic than market-makers and may thus buy, sell, or stay inactive. This complex behavior captures the uncertainty prevailing on financial markets during critical times and the difficulty to know who just sold or bought, and why.

Assume for instance 3 sales and then 3 purchases in a row, no trade in period 0, with  $\alpha = 0.5$ , 10% of value-informed traders and no supply-informed traders. I plot  $E_t^+$  and  $E_t^-$  in each period on Fig. 2. After the first sale, supply-informed traders draw the same inference if  $F = F^+$  or  $F = F^-$ , as in both cases there was surely no positive feedback. In the next two periods positive feedback traders are expected to sell. If  $F = F^-$  supply-informed traders know that there are no feedback traders on the market and that in each period there is a 0.3 probability to observe a sale if  $v = 1$ , against 0.4 if  $v = 0$ , giving a likelihood ratio of 0.3/0.4. Thus they consider a sale as a quite informative signal and update  $E^-$  quickly downwards. If  $F = F^+$ , supply-informed traders think that the second sale may come from a positive feedback trader, giving a likelihood ratio of 0.6/0.7, closer to one, and they update  $E^+$  downwards less strongly. In period 4 the first purchase is observed. As there was a sale before it cannot come from a positive feedback trader and is a strong signal for  $F^+$  traders that  $v$  may be high (ratio of 0.25/0.15). Further purchases are less informative because there are more noise traders (ratio of 0.4/0.3). After 3 purchases and 3 sales  $F^+$  traders have a higher  $E^+$  than at the start, whereas  $F^-$  traders who believe there is no positive feedback have the same  $E^-$ .



Finally, a trade also gives information about the extent of positive feedback trading. Market-makers infer information about  $F$  from an observed trade for three reasons: observing two sales in a row is more likely when positive feedback is high; if  $F^+$  (resp.  $F^-$ ) traders are expected to buy/sell, seeing a purchase/sale is more likely when  $F = F^+$  (resp.  $F = F^-$ ); if  $E_t^+ > E_t^-$ , a purchase (resp. sale) is more likely when  $F = F^+$  (resp.  $F^-$ ) because there is a higher probability that value-informed traders buy (resp. sell).

[Insert Fig. 2 here.]

**The process step by step:** The market starts in period 1; all prior beliefs and which trade took place in period 0 are given.  $E_1^+ = E_1^- = \pi$ ,  $\lambda_1 = \lambda$  and  $T_0 = 0$ . Now consider  $E_t^+, E_t^-, \lambda_t, T_{t-1}$  are known:

1. As shown in Proposition 1, the previous trade  $T_{t-1}$  plus the beliefs of market-makers and supply-informed traders uniquely determine the direction of supply-informed trading and the bid and ask prices:  $(E_t^+, E_t^-, T_{t-1}, \lambda_t) \rightarrow (O_t^+, O_t^-, A_t, B_t)$ .
2. The direction of positive feedback and supply-informed trading, plus the realizations of  $\tilde{v}$  and  $\tilde{F}$ , give the true probabilities that a buy order, a sell order or no order is received. The random draw of the next trader with these probabilities gives  $T_t$ . The true probability to observe  $T$  is  $\Pr(T_t = T | I_t, F, v)$ .
3. The realized trade  $T_t$ , supply-informed traders' previous expectations and the direction of positive feedback and supply-informed trading give supply-informed traders the necessary information to update their beliefs according to the formulas of section 2.2:  $T_t, E_t^+, O_t^+, T_{t-1} \rightarrow E_{t+1}^+$ ,  $T_t, E_t^-, O_t^-, T_{t-1} \rightarrow E_{t+1}^-$ .
4. Finally market-makers can update their beliefs:  $T_t, T_{t-1}, O_t^+, O_t^-, E_t^+, E_t^-, \lambda_t \rightarrow \lambda_{t+1}$ .

Knowing the vector  $(E_t^+, E_t^-, \lambda_t, T_{t-1})$  is enough to compute the other variables of interest in period  $t$ , the values  $(E_{t+1}^+, E_{t+1}^-, \lambda_{t+1}, T_t)$  may take and the probability distribution on these values. Given  $E_1^+, E_1^-, \lambda_1, T_0$  and the realizations of  $\tilde{v}$  and  $\tilde{F}$ , the above transitions define a stochastic process. Denoting  $S_t = (E_t^+, E_t^-, \lambda_t, T_{t-1})$  we have the following proposition:

**Proposition 2.** *The stochastic process defined above is a one-step Markov chain:*

$$\forall t \geq 1, \forall \{\hat{S}_i\}_{1 \leq i \leq t}, \Pr(S_t = \hat{S}_t | S_i = \hat{S}_i, 1 \leq i \leq t-1, F, v) = \Pr(S_t = \hat{S}_t | S_{t-1} = \hat{S}_{t-1}, F, v)$$

### 2.3 Dynamics and stationarity

Trades by the supply-informed do not depend on the last trade only, which prevents us from directly applying standard Markov tools to study the original process. The end of this section gives conditions under which this problem can be bypassed. First, we can still define the transition matrix from one period to another between three states corresponding in this order to a purchase, no trade and a sale. Define first the following matrix, giving the impact of noise and positive feedback traders on the market:

$$M_N(F) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 - \alpha F & 1 - \alpha F & 1 + 2\alpha F \end{pmatrix}$$

The behavior of value-informed traders can be characterized by the  $3 \times 3$  matrix  $M_V(v)$  whose lines are  $(v, 0, 1 - v)$ : value-informed traders buy if  $v = 1$  and sell if  $v = 0$ . Similarly, supply-informed traders' behavior is given by  $M_S(O_t^F)$  whose lines are  $(\mathbf{1}_{O_t^F=1}, \mathbf{1}_{O_t^F=0}, \mathbf{1}_{O_t^F=-1})$ . For given values of  $v$  and  $F$ , the transition between periods  $t$  and  $t + 1$  obeys the following matrix:

$$M(v, F, O_t^F) = x_N M_N(F) + x_I M_V(v) + x_S M_S(O_t^F) \quad (5)$$

Transition probabilities are not constant over time due to supply-informed traders changing their behavior. This problem disappears if  $x_S \rightarrow 0$ , or if after enough time this behavior is constant and  $M_S(O_t^F)$  does not depend on  $t$ . Then trades follow a homogeneous Markov chain with three states under either one of the two following assumptions:

**Assumption 1.**  $x_S \rightarrow 0$ .

**Assumption 2.** *Both types of supply-informed traders' behavior is constant in the long-run:*

$$\lim_{t \rightarrow +\infty} \Pr(\forall t' \geq t, O_{t'}^j = O^j | I_0, F^j, v) = 1, j \in \{+, -\}.$$

Numerical simulations suggest that Assumption 2 is always satisfied, and that actually

traders' behavior becomes constant quite quickly. This is difficult to show analytically however, as one would need to exclude complicated behaviors by supply-informed traders that could lead to updates sustaining them. For this reason I will more often use Assumption 1 which bears directly on a parameter of the model.

$E_t^F$ ,  $\lambda_t$  and  $p_t$  are bounded personal martingales. Standard arguments show these beliefs converge to the truth. As section 4 will show however,  $\lambda_t$  and  $p_t$  can nonetheless diverge from the truth *on average* for a limited number of periods, because these beliefs are linked to  $E_t^{1-F}$ . As  $E_t^{1-F}$  is a belief on  $v$  conditional on the wrong value of  $\tilde{F}$  it may not even converge. Under Assumptions 1 or 2 however, the stationary measures associated to  $M(v, F, O^F)$  give us the average number of purchases, holds, and sales  $\mu_T(v, F, O^j)$ , from which we deduce (see the Appendix A.2.2 for the proof):

**Lemma 1.**  $|E_t^F - v|$ ,  $|p_t - v|$  and  $|\lambda_t - F|$  converge in probability to 0.

*Under Assumptions 1 or 2,  $E_t^{1-F}$  converges either to  $v$  or to  $1 - v$  depending on the parameters. The stationary expected updates of  $E_t^+$ ,  $E_t^-$ ,  $\lambda_t$ ,  $p_t$  can be computed analytically.*

### 3 Supply-informed traders and liquidity

This section is devoted to studying the pattern of trading by the supply-informed, and the impact they have on market liquidity, mainly on trade imbalances and on the bid-ask spread.

#### 3.1 Long-run behavior of supply-informed traders

**Contrarian behavior and activity:** Notice first that supply-informed traders' update does not depend on  $F$  when the previous period saw a purchase or a hold, because no positive feedback is expected. After a sale however,  $F^+$  traders update their beliefs less downwards than  $F^-$  traders if there is a further sale because they know a sale was likelier even with  $v = 1$ . Conversely, they update their beliefs more upwards than  $F^-$  traders if there is a purchase instead, because they know there are not so many uninformed traders who wanted to buy. As a consequence, we should intuitively have  $E_t^+ > E_t^-$  in all periods. This is not necessarily the case however. Assume for instance in  $t$  we have  $E_t^+ > E_t^-$  and  $F^+$  traders

buy. Then if there is a purchase  $F^+$  traders know that it may come from another  $F^+$  trader, hence their behavior makes purchases noisier. The next proposition gives a condition for this effect to be dominated (the exact proof is in the Appendix A.2.3):

**Proposition 3.** *When  $x_S < \alpha x_N$ , we have  $E_t^+ \geq E_t^-$  in any period.  $F^+$  traders thus trade against positive feedback, while  $F^-$  traders trade with it.*

When  $F = F^+$  the market overreacts to sales, so that supply-informed traders adopt a contrarian behavior and either hold or buy (remember we cannot have both types buying or selling, see Proposition 1). This behavior attenuates the effects of positive feedback. Conversely, when  $F = F^-$  the market underreacts to sales because market-makers think they are noisier than they actually are. Supply-informed traders then hold or sell, trading in the same direction as positive feedback. The condition  $x_S < \alpha x_N$  means that there are not enough supply-informed traders for their effect to be more important than positive feedback traders. Otherwise the supply-informed traders' private knowledge of their own trading direction would be more important than their knowledge of positive feedback trading.

Proposition 3 tells us in which direction supply-informed traders trade, depending on  $F$ , if they are not excluded by too high spreads. They may trade for some periods, then be excluded and trade again. The next result gives conditions under which they are excluded or on the contrary active in the long-run. These conditions depend on the beliefs about  $v$  market-makers would have if they knew the true  $F$  ( $E_t^F$ ), compared to the beliefs they would have conditional on believing in the wrong  $F$  ( $E_t^{1-F}$ ). These are also the beliefs both types of supply-informed traders would have<sup>10</sup>. A difficulty is that these beliefs depend on the behavior of supply-informed traders, which is why Assumptions 1 or 2 are needed:

**Lemma 2.** *If  $x_S < \alpha x_N$ , under Assumption 1 or 2:*

*If  $F$  traders trade in the long-run in direction  $O^F \in \{1, -1\}$ , the stationary updates of all beliefs defined in Lemma 1 satisfy  $|E_t^F - v| > \max(|\lambda_t - F|, |E_t^{1-F} - v|)$  for  $t$  high enough.*

*If  $F$  traders do not trade in the long-run, the stationary updates of all beliefs defined in Lemma 1 satisfy  $|E_t^F - v| < \max(|\lambda_t - F|, |E_t^{1-F} - v|)$  for  $t$  high enough.*

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<sup>10</sup>A difference with the “differences of opinions” literature is that both types of supply-informed traders are never present at the same time.  $F^+$  and  $F^-$  traders otherwise share common features with the “unresponsive” and “responsive” traders of Harris and Raviv (1993).

*These necessary conditions are also sufficient under Assumption 1.*

See the Appendix A.2.3 for the proof of a slightly stronger result. The intuition is simple: if supply-informed traders are active forever, then their informational advantage over market-makers has to increase over time. They must learn more quickly about the asset than market-makers learn about positive feedback, and also more quickly than market-makers would learn about the asset if they were completely misled about positive feedback. Conversely, if they are inactive forever, their informational advantage has to decrease. Notice that both the spread and the supply-informed traders' informational advantage converge to zero, what matters are the relative speeds at which both happen. The lemma implies the following:

**Corollary 1.** *Under Assumption 1 or 2 and  $x_S < \alpha x_N$ , supply-informed traders are inactive in the long-run if either  $v = 1, F = F^-$  or  $v = 0, F = F^+$ .*

The corollary follows from Proposition 3 and Lemma 2. For instance when  $v = 1$  and  $F = F^-$  supply-informed traders always update their beliefs more downwards than market-makers and actually have an informational disadvantage, as they are more misled about the asset than market-makers. The lemma implies that they will stop trading at some point. Supply-informed traders have an informational advantage over market-makers on average over all realizations of  $(\tilde{v}, \tilde{F})$ , but not for all realizations. When they have a disadvantage they stop trading, so that they make losses for a finite number of periods only.

For the other cases  $v = 1, F = F^+$  and  $v = 0, F = F^-$ , Figure 3 plots regions of parameters  $(\alpha, \hat{x}_N, \hat{x}_S)$  where the necessary condition for supply-informed traders to be inactive in the long-run cannot be satisfied. When  $v = 0, F = F^-$  at least some long-run activity is more likely when  $\alpha$  is larger,  $\hat{x}_N$  is larger and  $\hat{x}_S$  is lower. Under such parameters the supply-informed traders' information is more valuable and adverse selection is lower, so that the supply-informed are more likely to overcome the spread. When  $v = 1, F = F^+$ , the effect of  $\hat{x}_N$  is no longer monotonic, as inactivity becomes possible for high values of  $\hat{x}_N$ : a high  $\hat{x}_N$  makes it easier for market-makers to learn the value of  $F$  by looking at trades, this reducing the informational advantage of the supply-informed.

[Insert Fig. 3 here.]

**Contrarian behavior and order imbalance:** Knowing supply-informed traders' constant behavior in the long-run, we can compute the stationary measures  $\mu_T(v, F, O^j)$ .  $|\mu_1(v, F, O^j) - \mu_{-1}(v, F, O^j)|$  is then a measure of order imbalance in the long-run. A widely used measure of informed trading on financial markets is the PIN (Easley *et al.* (1996), Easley *et al.* (2008)), which interprets order imbalance as coming from informed traders. Due to positive feedback sales, the order imbalance is positive in my model even without any informed trading. When  $x_I$  is not too large, there are more sales than purchases on average when  $F = F^+$  even if  $v = 1$  and informed traders are actually contributing negatively to order imbalance. When  $x_I$  is larger, value-informed traders are contributing positively to order imbalance. Supply-informed traders trade in the same direction as value-informed traders in the long-run, or do not trade (Corollary 1), hence they also contribute positively to order imbalance in this case:

**Remark 1.** *Under Assumption 1 or 2 and  $x_S < \alpha x_N$ , supply-informed traders contribute positively to order imbalance if  $(x_I + x_S)(1 - \alpha x_N) \geq \alpha x_N$ , but not necessarily otherwise.*

The exact proof can be found in the Appendix A.2.4. This remark means that when the order imbalance is a meaningful measure of trading based on value information, then it is also positively affected by trading on supply information if there are not too many supply-informed traders. This is due to supply-informed traders having the same strategy as value-informed traders in the long-run, or not trading anymore. Interestingly, when Assumptions 1 and 2 are not valid, this result may not hold. It is possible in particular that  $F^+$  traders always buy and  $F^-$  traders always sell, and examples can be constructed in which supply-informed traders reduce order imbalance.

### 3.2 Supply informed traders and the bid-ask spread

As usual in Glosten-Milgrom type models, the spread is a useful measure of adverse selection. A specificity here is that adverse selection is two-dimensional, so that trades bring information about both  $v$  and  $F$ . To understand the implications on market-makers' reaction to

observed trades, consider the covariance of  $\tilde{v}$  and  $\tilde{F}$  conditional on history:

$$\text{Cov}(\tilde{v}, \tilde{F} | I_t) = \lambda_t(1 - \lambda_t)(E_t^+ - E_t^-) \quad (6)$$

$\tilde{v}$  and  $\tilde{F}$  are independent random variables, and accordingly for  $t = 0$  we have  $E_1^+ = E_1^-$  and the covariance is null. As time goes on however, we have  $E_t^+ > E_t^-$  (Proposition 3) and the covariance is positive. After observing a sequence of sales for instance, the event  $v = 1 \cap F = 1$  is much more likely than  $v = 1 \cap F = 0$ , because only high positive feedback can explain many sales when the asset value is high. As  $t$  becomes large  $\lambda_t$  goes to 1 or 0 and the covariance tends to 0 again in the limit.

This property of the model is not specific to positive feedback trading but is more generally valid in Glosten-Milgrom frameworks with multi-dimensional uncertainty. An implication is that a sale makes it more likely that  $F = F^+$ , and thus contains some indirect positive information about  $v$ . To see this, assume after some period  $\hat{t}$  we shut down trading by value and supply-informed. Then  $E_t^+$  and  $E_t^-$  are not updated anymore and remain equal to some constants  $E^+, E^-$  but  $\lambda_t$  is still updated. We can rewrite the spread in  $t > \hat{t}$  as:

$$\begin{aligned} A_t - B_t &= \Pr(v = 1 | T_t = 1, I_{t-1}) - \Pr(v = 1 | T_t = -1, I_{t-1}) \\ &= [\Pr(F = F^+ | T_t = 1, I_{t-1}) - \Pr(F = F^+ | T_t = -1, I_{t-1})](E^+ - E^-) \end{aligned} \quad (7)$$

This reexpression gives us the following (see the Appendix A.2.5 for the detailed proof):

**Remark 2.** *If informed trading is shut down in  $\hat{t}$ , for  $t > \hat{t}$  the spread  $A_t - B_t$  is still non-zero after each sale. Moreover  $B_t > B_{t-1}$ : the bid goes up after a sale.*

Market-makers know that if there are two sales in a row it is a signal that  $F = F^+$ , and thus more weight should be given to  $E^+$  when setting the price. Thus we have a non-zero spread even when adverse selection disappears from the market. The spread does not come from adverse selection only, as trades here are informative *per se*. This implies that a sale can contain some positive signal. In the extreme case where the adverse selection component of the spread is zero, a sale is even a more positive signal than a purchase. In this model it also implies a negative spread, which is not a logical contradiction as traders cannot buy

and sell back to market-makers in the same period. The “spread” in the model is only the information component of the actual spread, the former can sometimes be negative here.

Finally, the supply-informed traders’ impact on the spread is similar to the value-informed traders’: in a given period they are a source of adverse selection since traders of type  $F^j$  trade at the ask (bid) only if  $E_t^j > A_t$  ( $E_t^j < B_t$ ). This gives us the following (see the Appendix A.2.5 for the proof):

**Remark 3.** *Holding  $E_t^+, E_t^-, \lambda_t$  constant, an increase in  $\hat{x}_S$  increases  $A_t - B_t$ .*

### 3.3 Discussion

**Supply-informed traders’ behavior:** Proposition 3 characterizes how supply-informed traders behave depending on their information. When  $F = F^+$  they know the market is overreacting to sales, while if  $F = F^-$  they know it is under-reacting. In the former case they become more optimistic about  $v$  than the market and follow contrarian strategies after sales, while in the latter they are more pessimistic and behave as the momentum traders in Hong and Stein (1999). For how long supply-informed traders have a contrarian or positive feedback impact on the market depends on how long they are able to trade. Lemma 2 gives necessary conditions for supply-informed traders to trade in the long-run: their informational advantage compared to market-makers must grow over time; if they never trade it must decrease over time.

The spread compensates market-makers for two adverse selection problems: the risks to trade with a value-informed trader or with a supply-informed trader. If the spread required to compensate the first risk is enough to exclude supply-informed traders, then they won’t trade. In other words, if supply-informed traders’ private information becomes not relevant enough compared to value-informed traders’ private information, a low spread will be enough to exclude the former from trading. Thus supply-informed traders’ activity in the long-run depends on a comparison between the speeds at which market-makers learn about  $v$  and about  $F$ . In particular, Corollary 1 shows that under some conditions supply-informed traders are necessarily inactive in the long run for two given realizations of  $(\tilde{v}, \tilde{F})$ , because their update is necessarily lagging behind those of market-makers.



Thus, in some situations, market participants with supply information are actively trading only at the beginning of the trading-period, and eventually become inactive. They help to correct public beliefs when these are sufficiently wrong but may not have this role in the long-run. This also implies that trading volume is larger when market-makers still have imprecise information about positive feedback. As Figure 3 shows, supply-informed activity and volume are also higher when price pressure is higher, which can be compared with [Malinova and Park \(2011\)](#). Supply-informed traders are also less active when value-informed trading is more important, so that they will stop trading in a contrarian manner and thus providing liquidity when adverse selection becomes too high.

When they are not too many on the market, supply-informed traders are either inactive in the long-run or behave as value-informed traders. If a measure based on trade imbalance such as the PIN is increasing in value-informed trading (which is not warranted in this model), then it is also increasing in supply-informed trading. When there are many supply-informed traders however, this result does not necessarily hold. This is particularly interesting if some high-frequency trading strategies are akin to supply-informed trading: while such strategies do incorporate private information, they may be completely missed by the PIN measure.

**Supply information and the spread:** The spread in this model does not come only from adverse selection, but also from the information given by the last trade on how all past trades should be interpreted. For this second source of information, a sale can be a positive signal about the asset’s value. This is not as counter-intuitive as it may seem. Consider for instance the “flash crash” of the Dow Jones on May 6, 2010. If the Dow Jones loses 5% in two minutes, it is a bad signal about the long-term value of this index. But if it loses 4 additional points in the next three minutes and there is still no news that could justify such a large drop, it is also a strong signal that the market does not function properly. It may imply that the first drop of 5% was probably not entirely driven by information either. Thus the latter sales can actually be interpreted as a positive signal.

Remark 2 shows that when the market is very noisy and market-makers still do not know whether past trades were informative or not, the spread can be high even if adverse selection is low. Conversely, if given the past pattern of trades a sale would be a good signal,

the spread can be low but adverse selection high. In extreme cases, spreads (rather their informational component) become negative in the Glosten-Milgrom framework. To avoid this it is enough to assume there are enough value-informed traders, or that the spread also contains an order-processing cost.

This does not imply that adding supply-informed traders reduces the spread (Remark 3): in a given period, more supply-informed traders may be a source of information about positive feedback, but also of adverse selection. Of course they may also speed up the convergence of prices, so that their dynamic impact on the spread is negative. This is not always the case however, as shown in the next section.

## 4 Supply-informed traders and market stability

In this section I analyze further the role of supply-informed traders by looking at the impact of increasing their number on the price-discovery process and the likelihood of crashes.

### 4.1 Short-run crashes and long-term convergence

Lemma 1 shows that in the long-run private information about the value of the asset and about positive feedback trading will be fully disclosed. In a one-dimensional environment, market-makers would additionally come closer to the truth on the asset's value in each period on average. This is not necessarily the case in a multi-dimensional environment: market-makers can simultaneously get closer to the truth about  $F$  but further from the truth about  $v$ . Conditionally on a realization of  $(\tilde{v}, \tilde{F})$ , prices can diverge from the truth *on average* in the short-run, which means that a pattern such as a crash and then a recovery (or a bubble and then a correction) is likely.

**Proposition 4.** *If  $x_I$  and  $x_S$  are small,  $\alpha$  and  $|\lambda_1 - F|$  high enough, there exists  $t_1 > 1$  such that  $E(|p_t - v| \mid I_0, F, v)$  is increasing in  $t$  for  $t \in [1, t_1]$  if  $(v, F)$  equals  $(1, 1)$  or  $(0, 0)$ . Increasing  $\hat{x}_S$  reduces the expected divergence  $|p_t - v|$  between two trades.*

Consider for instance the case  $v = 1, F = F^+$ . If market-makers are far from the truth about positive feedback trading at the beginning ( $\lambda$  small), their beliefs are close to those

of  $F^-$  traders. In particular, they do not take enough into account that observed sales can come from positive feedback traders. If positive feedback is high enough, this leads them to update their belief downwards on average at the beginning, even if  $v = 1$ . See the Appendix A.2.6 for the complete proof of this first point. For the second one, with a low  $\lambda_t$  the  $F^-$  traders will be excluded from trade by high spreads (see equations (13) and (14)) while  $F^+$  traders buy. Thus, increasing  $\hat{x}_S$  makes a purchase more likely to occur, without affecting the update of  $E_t^-$  (since they wrongly assume supply-informed do not trade), as a result  $E_t^-$  diverges less from  $v$  in expectation and so does  $p_t$ .

As we know that in the end prices will converge to the asset's value, this proposition means that for some parameters expected prices will exhibit reversal, conditionally on  $(v, F)$ . This happens when market-makers' update on  $\lambda_t$  leads them to converge to the beliefs of  $F^+$  traders. This anomaly is quite general and is already present in the “rational crashes” literature, for instance in Jacklin, Kleidon, and Pfleiderer (1992).

Fig. 4<sup>11</sup> illustrates Proposition 4:  $E^-$  beliefs get more and more misled by positive feedback sales and are updated downwards on average, while correct  $E^+$  beliefs are updated upwards. If at the beginning market-makers give more weight to  $E^-$  (low prior probability  $\lambda$ ) the average prices go down at first, then over time market-makers learn about  $F$  and go back to the supply-informed traders' expectations.

Do supply-informed traders also help the aggregation of information in the long-run? Two effects need to be taken into account: increasing the number of supply-informed traders increases the probability that a given trade comes from a supply-informed trader, but it also increases total trading intensity, and thus the average number of trades in a given time span. As explained in section 2.1, a real time of  $\tau$  seconds correspond to  $(1 + \hat{x}_S)\tau$  trades on average. I prove in the Appendix A.2.7 that  $E(|p_\tau - v| | I_0, F, v)$  has a simple equivalent when  $\tau \rightarrow +\infty$  of the form  $Ke^{-\beta\tau}$ . Studying how  $\beta$  varies when  $\hat{x}_S$  increases gives the following:

**Proposition 5.** *If  $x_I$  and  $x_S$  are small, for  $(v, F) = (1, 1)$  increasing the number of supply-informed traders slows down the update of  $\lambda_\tau$  and price discovery in the long-run, as measured in real time.*

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<sup>11</sup>In all figures empirical averages are based on 10.000 simulations, except when confidence intervals are given, in which case the model is run 40.000 times. The parameters used are those of the turbulence scenario defined in section 4.2. The variable “Price” refers to the last price at which a trade took place.

See the Appendix A.2.7 for the proof. When supply-informed traders keep an informational advantage in the long-run, prices become equivalent to  $\lambda_\tau$ , and hence the speed of convergence is determined by market-makers' updating about positive feedback. This updating is actually made slower by supply-informed traders themselves. After a sale there is an excellent opportunity to acquire information about  $F$ : if  $F = F^+$  a further sale is likely since positive feedback is high, if  $F = F^-$  it is not. But if  $F = F^+$  supply-informed traders buy, while if  $F = F^-$  they sell. If  $\hat{x}_S$  increases, the likelihood ratios  $\Pr(T = -1|F = F^+)/\Pr(T = -1|F = F^-)$  and  $\Pr(T = 1|F = F^-)/\Pr(T = 1|F = F^+)$  thus decrease: additional supply-informed traders jam the signal due to their contrarian behavior. Figure 5 shows how  $\beta$  varies with  $\hat{x}_S$  and  $\alpha$  for  $\hat{x}_N = 0.3$ ,  $O^+ = 1$  and  $O^- = -1$ .

This effect can be compared to what Smith and Sorensen (2000) call “confounded learning”: when there are many positive feedback traders, supply-informed traders buy because they know there is overreaction, but by doing so they make it more difficult for market-makers to infer information about positive feedback trading. In a more general model, it may be possible that with positive probability the market reaches a state where market-makers cannot infer anything about positive feedback trading from observed transactions.

[Insert Fig. 4 and 5 here.]

## 4.2 Supply-informed activity and profit in turbulent times

**Parameters:** To go further, I run simulations of the model to estimate the unconditional expectation of  $|p_\tau - v|$  as a function of  $\hat{x}_S$ . The results from the simulations are given using real time  $\tau$ . I assume  $\hat{x}_N = 0.3$ ,  $\hat{x}_I = 0.1$  so that there are nine times more noise traders than value-informed traders.  $\hat{x}_S$  is set to zero at first and then allowed to increase until 0.2, so that there are at most twice more supply-informed traders than value-informed traders. I always assume  $E_1^+ = E_1^- = \pi = 0.5$ . I consider two main scenarios:

-In the baseline scenario,  $\lambda = 0.5$  so that market-makers never start too misled about  $F$  and  $\alpha = 0.1$  so that positive feedback is moderate. This case is quite close to a standard Glosten-Milgrom framework and is meant to reflect a market in normal times.

-In the turbulence scenario,  $\alpha = 0.8$  so that there are many positive feedback sales when

$F = F^+$ . Moreover  $\lambda = 0.05$  (to compare with the 0.9999 probability that no informational event occurs in Avery and Zemsky (1998)). This set of parameters represents a market which is rarely affected by large shocks. From time to time such a large shock happens and market-makers largely underestimate positive feedback sales at the beginning.

**Supply-informed traders and prices:** Fig. 6 shows the empirical average  $|p_\tau - v|$  in  $\tau = 100$  as a function of  $\hat{x}_S$  in the turbulence scenario, with confidence intervals at 95%. The effect of  $\hat{x}_S$  on long-run convergence seems non-monotonic. Notice in particular that convergence is slower for  $\hat{x}_S = 0.15$  than for zero supply-informed trading, although we have added more supply-informed traders than there are value-informed traders, and the number of trades in 100 periods has increased from 100 to 115. The same picture emerges when considering convergence after a shorter time, but prices become noisier.

I then look at average prices over time, but conditionally on  $v = 1, F = F^+$  in the turbulence scenario, see Fig. 7. Prices move away from fundamental value on average in the 10 first seconds, which means that many “rational crashes” are happening in the simulations. Average prices first go downwards as expected from Proposition 4, then bounce back and converge to  $v$ . This anomaly however is less and less pronounced when  $\hat{x}_S$  increases: supply-informed traders act as support-buyers and help avoiding crashes. But more supply-informed traders imply less price-discovery in the long-run, as expected from Proposition 5, because they jam the updating process of market-makers about  $\lambda_\tau$ .

Finally, having more supply-informed traders does more than reducing the average discrepancy between  $v$  and  $p_\tau$ . On Fig. 8 I plot the empirical CDF of the random variable  $\max_{\tau \in [1, 100]} |v - p_\tau|$  when  $v = 1$  and  $F = F^+$  for several values of  $\hat{x}_S$ . This variable is the largest deviation from fundamental value observed in a given simulation, a crash. Observe that 50% of the time the price drops from 0.5 at the beginning to 0.3 or lower, even if  $v = 1$  (the median of the distribution is above 0.7). Around 10% of the time it actually drops to 0.1! Moreover, the CDFs are ordered in the sense of first-order stochastic dominance. Adding supply-informed traders thus seems to reduce the probability of a “crash” of any magnitude:

**Numerical result 1.** *In the turbulence scenario, conditionally on  $v = 1, F = F^+$  and for any  $u$ ,  $\Pr(\max_{\tau \in [1, 100]} |v - p_\tau| \leq u)$  increases in the number of supply-informed traders.*

[Insert Fig. 6, 7 and 8 here.]

**Supply-informed traders' profit:** finally, I study the profit supply-informed traders make on their activity. Profits should be proportional to the number of trades, not time periods, hence I use trade time. Given a history  $\{A_t, B_t, T_t, O_t^+, O_t^-, \lambda_t, E_t^+, E_t^-\}_{t=1\dots 100}$ , ex-post profits are given by:

$$\Pi = \sum_{t=1}^{100} \mathbf{1}_{O_t^F=1}(v - A_t) + \mathbf{1}_{O_t^F=-1}(B_t - v) \quad (8)$$

I compute the unconditional average of this measure in both scenarios for several values of  $\hat{x}_S$ , and also averages conditional on specific realizations of  $(\tilde{v}, \tilde{F})$  for the turbulence scenario. Fig. 9 and 10 show my estimates, as well as the 95% confidence level interval for the estimates of unconditional expected profit. In the turbulence scenario supply-informed traders earn a lot when  $F = F^+, v = 1$ , that is in the case where they prevent the formation of a crash, but this happens with a low probability. Moreover, sometimes they buy because they think the market is overreacting to sales, whereas the asset's value is actually low, then they make large losses. When  $F = F^-$  their informational advantage is small and supply-informed traders are most of the time excluded by too high spreads. Note that the signs of average profit estimates conditional on  $(v, F)$  are consistent with Proposition 3. Profit is lower in the baseline scenario, except when  $\hat{x}_S$  is very small. The supply-informed traders' informational advantage is lower in this case and they are excluded more quickly. Finally, Fig. 11 shows the quantile function of realized profits in the turbulence scenario: most of the time supply-informed traders make a profit close to zero, but the first and last deciles show important losses and profits.

[Insert Fig. 9, 10 and 11 here.]

**Numerical result 2.** *In the baseline scenario supply-informed traders's profit is not statistically different from 0 except for the smallest values of  $\hat{x}_S$ . For  $\hat{x}_S \geq 0.05$  profit is higher in the turbulence scenario. Moreover:*

1. *Supply-informed traders make a profit on average, but with a high variance.*
2. *They trade more when positive feedback is important, mostly remain inactive otherwise.*

3. *The profit distribution is fat-tailed: they make a high profit 2.5% of the time, moderate losses 2.5% of the time, and close to zero profit otherwise.*

These simulations give us a clearer picture of what traders with supply information do: most of the time they wait for the market to be misled enough about the amount of positive feedback trading, in which case they have a short-lived profit opportunity. Once in a while positive feedback is important and widely underestimated by the market, then they know the value of the asset is underestimated and buy for long periods, generating high profits if the asset's value was indeed high, and losses otherwise. This pattern implies a risky activity, that can generate high losses but even higher profits.

Assume now that supply information can be acquired at some cost  $c$ . A marginal agent considering acquiring supply information and becoming a supply-informed trader would expect to get a profit proportional to the expectation of equation (8). Numerical results show that this expectation is decreasing in  $\hat{x}_S$ , thus there is an equilibrium number of supply-informed traders such that marginal profit is equal to  $c$ . Identifying supply-informed traders to some HFT strategies gives the following:

**Remark 4.** *All else equal, higher HFT activity should be caused by: an increase in noise trading, more uncertainty about whether the market is overreacting or underreacting to trades, more positive feedback trading.*

Indeed, when  $\hat{x}_I$  increases information is revealed faster, supply information is outdated more quickly and supply-informed traders are sooner excluded from trading. When  $\lambda$  is more extreme and  $\alpha$  is high, uncertainty about positive feedback is higher and supply information is more valuable. Finally, a higher asymmetry between sales and purchases implies higher mispricings and thus profits for supply-informed traders.

**Circuit-breakers:** I introduce circuit-breakers in the model to illustrate the signal-jamming effect, as their impact here is similar to the supply-informed traders<sup>12</sup>. Assume  $F = F^+$ ,  $v = 1$ , and in  $t = 5$  there is a trading halt if the price falls below a given threshold ( $p = 0.35$  in

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<sup>12</sup>See [www.sec.gov/investor/alerts/circuitbreakersbulletin.htm](http://www.sec.gov/investor/alerts/circuitbreakersbulletin.htm) for a wrap-up of the different measures implemented in the U.S. as an answer to the “flash crash”, and Draus and Van Achter (2012) for a recent theoretical study of circuit-breakers.

the figure below). I assume that the trading halt breaks the positive feedback spiral: it is common knowledge that *from  $t = 6$  onwards* positive feedback is low,  $F = F^-$ . This does not give any information about past trades: market-makers still do not know whether they were due to positive feedback or to negative fundamental information. With the circuit-breaker, prices on average go up from  $t = 6$  onwards, but slowly because the negative information acquired before has a lasting impact. Without the circuit-breaker, prices continue to fall on average, but then a V-shaped recovery occurs: market-makers learn by observing new sales that the first ones were not so informative, and the informational impact of the first sales is washed away. Fig. 12 illustrates this phenomenon: I plot the average trajectory of prices conditional on the circuit-breaker being triggered, as well as the counter-factual trajectory starting at  $t = 6$  had the circuit-breaker not been implemented. This example implies that the length of the trading halt matters:

**Numerical result 3.** *A trading halt long enough to break the positive feedback is likely to stop the crash. If it is not long enough for uninformed traders to learn the cause of the price drop, then it also prevents a quick rebound from happening.*

[Insert Fig. 12 here.]

### 4.3 Discussion

The trading behavior and the profit of supply-informed traders in this model have various empirical counterparts depending on how one interprets “supply information”. Most of the time (ie. when  $F = F^-$ ) these traders follow momentum strategies, which give a positive return because conditionally on  $F = F^-$  prices under-react to observed trades. When  $F = F^+$ , supply-informed traders are supplying liquidity by trading in the direction opposite to price pressure. Coval and Stafford (2007) show that investors short selling stocks likely to be affected by flow-induced selling by mutual funds and buying ahead of forced purchases earn an average abnormal return over 10%. The turbulence parameters can be seen as a stylized representation of the crisis episodes studied by Cella, Ellul, and Giannetti (2011)<sup>13</sup>,

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<sup>13</sup>Mainly the aftermath of Lehman Brothers’ bankruptcy in 2008, but also October 1987, the Russian default of 1998, the quant event of 2007, the bailout of Bear Sterns in 2008.



who show that stocks mainly held by institutions with a short trading horizon experience larger price drops than the others, and then experience larger reversals. A long/short equity strategy consisting in buying stocks held by short horizon institutions and short selling similar stocks with more long term investors is an example of what is defined in this paper as supply-informed trading.

An interesting empirical counterpart to supply-informed traders are liquidity-taking high-frequency traders (as supply-informed traders pay the spread in this model). [Baron, Brogaard, and Kirilenko \(2012\)](#) show that they make more profit than other HFTs, but the distribution is also very dispersed, in line with Numerical result 2. [Brogaard, Hendershott, and Riordan \(2011\)](#) also show that HFTs typically use marketable orders to trade in the direction opposite to transitory pricing errors, which is in line with the model. Consistent with Proposition 4, [Hasbrouck and Saar \(2012\)](#) find that HFT activity had an important positive impact on liquidity in a crisis time (June 2008), while [Nagel \(2011\)](#) shows that reversal strategies (corresponding to the case  $v = 1, F = F^+$ ) fare better in times of high volatility, as expected from Remark 4.

Numerical result 2 illustrates clearly the risk of a trading strategy based on catching falling knives. In practice this risk is magnified by the possibility of a slow reversal: the supply-informed trader can report a trading loss for a long time if too many noise trades go in the wrong direction (“noise trader risk” as in [De Long \*et al.\* \(1990a\)](#)), or if other supply-informed traders are slow to arbitrage the mispricing (“synchronization risk” in [Abreu and Brunnermeier \(2002\)](#)). Supply-informed traders could thus trade less than they do in this model, or become positive feedback traders themselves if they hit funding constraints. Papers such as [Easley, Lopez de Prado, and O’Hara \(2011\)](#) suggest that natural liquidity providers actually became liquidity consumers during the flash crash of 2010. The result on profit shows that due to the inherent risks of trading on supply information this problem is actually likely. Moreover, as “mini flash crashes” seem to become more common ([Golub, Keane, and Poon \(2012\)](#)), a key aspect of “supply information” that HFTs may have is learning quickly whether such an event is driven by news or by a glitch.

The risk of actually catching a falling knife suggests that it would be profitable for supply-informed traders to have some value information as well. Endogenizing the choice of both

types of information would be difficult in this model however. [Ganguli and Yang \(2009\)](#) study in a static framework the choice to acquire both value and supply information, but only as a bundle. Whether we can expect some traders to specialize in acquiring mainly supply information thus remains an open question.

It would be interesting for future research to see how allowing supply-informed traders to time their trades would interact with two-dimensional adverse selection and the spread. Selling strategically to depress prices further before buying as in [Brunnermeier and Pedersen \(2005\)](#) requires market power from the trader, while this paper assumes competitive traders. Selling at the beginning of the period and trying to buy at the trough is another possible strategy, but it should be heavily discouraged by the spread. There is promising research on the role of “timing” in Glosten-Milgrom type models ([Back and Baruch \(2004\)](#), [Malinova and Park \(2009\)](#)), but a tractable framework that could be applied in complex settings is still lacking. This section shows that, even in the optimistic case where supply-informed traders always buy stocks they see as undervalued and sell stocks they consider to be overvalued, their positive short-run impact also implies a negative impact on long term price discovery. Interestingly, supply-informed traders in the model act as a smooth version of the circuit-breaker studied in Numerical result 3: by preventing crashes they also prevent quick rebounds from happening. [Menkveld and Yueshen \(2013\)](#) show a similar signal-jamming when trades between “middlemen” can be mistaken for informed trades. Here the cause is different as informed trades are making it more difficult to learn precisely the type of information that these informed traders have.

## 5 Conclusion

Market participants have massively invested in recent years to obtain more information, or more quickly, about other elements than fundamentals. This paper offers a simple extension of the Glosten-Milgrom model allowing for non-fundamental uncertainty, namely on the intensity of positive feedback trading, in a stylized but also flexible way, and for traders with private information about this intensity. These supply-informed traders can be identified with various high-frequency trading strategies relying on a lot of data about order flow

rather than about fundamentals. At a different frequency, various funds may have supply information, typically about the financial situation of large players who could exert price pressure on the market if financially distressed.

The welfare implications of such an investment in non fundamental information are ambiguous: when uncertainty about positive feedback is important, supply-informed traders slow down price discovery in the long-run because they jam the market-makers' update on the intensity of positive feedback, and add a second dimension to adverse selection, which widens the spread. Their only, but possibly crucial, clear positive effect on the market is that they help prevent large deviations due to underestimation of positive feedback. Such agents can be seen as providing the market with an insurance against crashes caused by misinterpretation of past trades, the premium to pay being larger spreads and slower long-run price-discovery.

Interestingly, in markets such that non-fundamental crashes can be expected to happen, profits from supply-informed trading can be large, but they also have a high variance, the risks associated with being wrong being important ("catching a falling knife"), and are positively correlated with the asset's value. The short term volatility implied by supply uncertainty is thus attenuated by supply informed traders but not eliminated, as the number of market participants acquiring such information are limited by the risks associated with supply informed trading.

# A Appendix

## A.1 Notations

$v$	value of the asset, equals 0 or 1
$x_N$	$3x_N$ is the proportion of uninformed traders
$x_I$	proportion of value-informed traders
$x_S$	proportion of supply-informed traders
$\alpha F$	is the proportion of positive feedback traders among uninformed traders
$F^j$ , $j \in \{+, -\}$	two possible values of $F$
$T_t$	direction of the trade in period $t$
$A_t$	ask price in period $t$
$B_t$	bid price in period $t$
$p_t$	price of the transaction in period $t$
$E_t^j$	with $j \in \{+, -\}$ denotes the expected value of the asset conditional on $F = F^j$
$E_t^j$	with $j \in \{F, 1 - F\}$ is the expected value of the asset conditional on the true/wrong $F$
$\lambda_t$	probability market-makers assign to $F = F^+$
$O_t^j$	with $j \in \{+, -\}$ , direction of trade of $F^+, F^-$ supply-informed traders
$O_t^F$	direction of trade of the realized type of supply-informed traders
$H_t^j$	with $j \in \{+, -\}$ is equal to $\ln\left(\frac{E_t^j}{1-E_t^j}\right)$
$\Lambda_t$	is equal to $\ln\left(\frac{\lambda_t}{1-\lambda_t}\right)$

## A.2 Proofs

### A.2.1 Proof of Proposition 1

Supply-informed traders with the highest expectation cannot sell while the others do nothing or buy. Let us show that the two types of supply-informed traders cannot both buy or both sell in the same period. Remember that if there is a sale (resp. a purchase) in period  $t$ , we have  $B_t$  (resp.  $A_t$ ) =  $\lambda_{t+1}E_{t+1}^+ + (1 - \lambda_{t+1})E_{t+1}^-$ . If both types of supply-informed traders sell we have  $E_t^+ < B_t, E_t^- < B_t$ , but if there is a sale in period  $t$  we also have  $E_{t+1}^+ < E_t^+, E_{t+1}^- < E_t^-$ , so  $B_t < \lambda_{t+1}E_{t+1}^+ + (1 - \lambda_{t+1})E_{t+1}^-$ , which means that either  $E_t^+$  or  $E_t^-$  is larger than  $B_t$ , and thus that one type at least does not sell. Similarly both types cannot

buy.

If  $E_t^+ > E_t^-$  there are thus only four possible values for  $(O_t^+, O_t^-)$ :  $(1, 0), (1, -1), (0, 0), (0, -1)$ , and conversely if  $E_t^- > E_t^+$  the four possible values are  $(0, 1), (0, 0), (-1, 0), (-1, 1)$ . I now use Bayes' law to compute the bid and ask prices, assuming the orders submitted by supply-informed traders to be known. All probabilities are conditional on  $I_t$ :

$$A_t = \frac{\Pr(v = 1 \cap T_t = 1)}{\Pr(v = 1 \cap T_t = 1) + \Pr(v = 0 \cap T_t = 1)}$$

I use the following notations with  $j \in \{+, -\}, T \in \{-1, 0, 1\}$ :

$$\begin{aligned} u_{t,T}^j &= \Pr(T_t = T | I_t, v = 1, F = F^j) = x_N + \mathbf{1}_{O_t^j=T} x_S + x_I \mathbf{1}_{T=1} - \alpha F^j \mathbf{1}_{T_{t-1}=-1} x_N (1 - 3 \times \mathbf{1}_{T=-1}) \quad (9) \\ v_{t,T}^j &= \Pr(T_t = T | I_t, v = 0, \alpha = \alpha^j) = u_{t,T}^j + x_I (\mathbf{1}_{T=-1} - \mathbf{1}_{T=1}) \quad (10) \end{aligned}$$

Now we can reexpress the bid and the ask prices:

$$A_t = \frac{\lambda_t E_t^+ u_{t,1}^+ + (1 - \lambda_t) E_t^- u_{t,1}^-}{\lambda_t (u_{t,1}^+ - (1 - E_t^+) x_I) + (1 - \lambda_t) (u_{t,1}^- - (1 - E_t^-) x_I)} \quad (11)$$

$$B_t = \frac{\lambda_t E_t^+ u_{t,-1}^+ + (1 - \lambda_t) E_t^- u_{t,-1}^-}{\lambda_t (u_{t,-1}^+ + (1 - E_t^+) x_I) + (1 - \lambda_t) (u_{t,-1}^- + (1 - E_t^-) x_I)} \quad (12)$$

Denoting  $\Omega_t = x_I (1 - \lambda_t E_t^+ - (1 - \lambda_t) E_t^-)$  the probability of trading with a value-informed trader and the asset value being low, direct calculation shows these necessary and sufficient conditions:

$$E_t^j < B_t \Leftrightarrow \Omega_t < |\lambda_t - F^j| \left( \frac{E_t^{-j}}{E_t^j} - 1 \right) u_{t,-1}^{-j} \quad (13)$$

$$E_t^j > A_t \Leftrightarrow \Omega_t < |\lambda_t - F^j| \left( 1 - \frac{E_t^{-j}}{E_t^j} \right) u_{t,1}^{-j} \quad (14)$$

$$(15)$$

Finally, assuming  $E_t^+ > E_t^-$  (the same result holds by symmetry when this is not the case)

and denoting  $K_t = \lambda_t \left( \frac{E_t^+}{E_t^-} - 1 \right) u_{t,-1}^+, L_t = (1 - \lambda_t) \left( 1 - \frac{E_t^-}{E_t^+} \right) u_{t,-1}^-$  we have

$$\begin{aligned} (O_t^+, O_t^-) = (1, 0) &\Leftrightarrow E_t^+ > A_t > E_t^- > B_t \Leftrightarrow K_t < \Omega_t < L_t \\ (O_t^+, O_t^-) = (1, -1) &\Leftrightarrow E_t^+ > A_t > B_t > E_t^- \Leftrightarrow \Omega_t < \min(K_t, L_t) \\ (O_t^+, O_t^-) = (0, 0) &\Leftrightarrow A_t > E_t^+ > E_t^- > B_t \Leftrightarrow \Omega_t > \max(K_t, L_t) \\ (O_t^+, O_t^-) = (0, -1) &\Leftrightarrow A_t > E_t^+ > B_t > E_t^- \Leftrightarrow L_t < \Omega_t < K_t \end{aligned}$$

Thus for any values taken by  $\Omega_t, K_t, L_t$  there is one and only one possible vector  $(O_t^+, O_t^-)$ .

### A.2.2 Supply-informed traders and market-makers' updates

Denote  $H_t^j = \ln \left( \frac{E_t^j}{1 - E_t^j} \right)$  for  $j \in \{+, -\}$  and  $\Lambda_t = \ln \left( \frac{\lambda_t}{1 - \lambda_t} \right)$ . For supply-informed traders' beliefs, we have by Bayes' law:

$$H_{t+1}^j = H_t^j + \Delta H^j(T_{t-1}, T_t, O_t^j) \quad (16)$$

with  $\Delta H^j(T_{t-1}, T_t, O_t^j) = \ln u_{t,T_t}^j - \ln v_{t,T_t}^j$ . Notice that neither  $u_{t,T_t}^j$  nor  $v_{t,T_t}^j$  depend directly on time, and the update can be expressed in terms of  $T_{t-1}, T_t, O_t^j$  only. For market-makers' beliefs on positive feedback trading we can write similarly:

$$\Lambda_{t+1} = \Lambda_t + \Delta \Lambda(T_{t-1}, T_t, O_t^+, O_t^-, E_t^+, E_t^-) \quad (17)$$

with  $\Delta \Lambda(T_{t-1}, T_t, O_t^+, O_t^-, E_t^+, E_t^-) = \ln y_{t,T_t} - \ln z_{t,T_t}$  and

$$y_{t,T} = \Pr(T_t = T | I_t, F = F^+) = E_t^+ u_{t,T}^+ + (1 - E_t^+) v_{t,T}^+ \quad (18)$$

$$z_{t,T} = \Pr(T_t = T | I_t, F = F^-) = E_t^- u_{t,T}^- + (1 - E_t^-) v_{t,T}^- \quad (19)$$

Consider now Lemma 1. The first part follows from standard arguments. Under Assumptions 1 or 2, the average proportion of periods with  $T_t = T$  for  $T \in \{-1, 0, 1\}$  is given by stationary measures  $\mu_T(v, F, O^F)$  associated to the transition matrix  $M(v, F, O^F)$ . We

can then define the stationary expected update of  $H_t^j$  as:

$$\Delta H_\infty^j = \sum_{T=\{1,0,-1\}} \mu_T(v, F, O^F) \sum_{T'=\{1,0,-1\}} \Pr(T'|T, v, F, O^F) \Delta H^j(T, T', O^j)$$

where  $\Pr(T'|T, v, F, O^F)$  is given by the relevant cell in the transition matrix  $M(v, F, O^F)$ . This gives us  $\Delta H_\infty^F$  and  $\Delta H_\infty^{1-F}$ . The set of parameters for which the latter is by chance equal to zero has null measure and we can neglect it, so that the average update is strictly positive or strictly negative, thus  $E_t^{1-F}$  converges to  $v$  or  $1 - v$ . For market-makers' beliefs about  $F$ , notice that the observation of a given trade  $T_t$  may give rise to different updates depending on the values of  $E_t^+$  and  $E_t^-$  (see equations (18) and (19)). But as these beliefs converge to  $v$  or  $1 - v$ , in the limit the update of  $\lambda_t$  after a given trade  $T_t$  will not depend on  $t$ , so that again the average update in each period will converge to a stationary limit  $\Delta \Lambda_\infty$ .

### A.2.3 Proof of Proposition 3 and Lemma 2

Consider first Proposition 3. In the first period  $T_0 = 0$  and  $E_1^+ = E_1^-$ .  $E_t^+$  will remain equal to  $E_t^-$  until the first sale. After the first sale we have  $u_i^+ \neq u_i^-$  and  $v_i^+ \neq v_i^-$  for  $i \in \{-1, 1\}$  and no type of supply-informed wants to trade, thus it is easy to compute that  $u_1^+/v_1^+ > u_1^-/v_1^-$  and  $u_{-1}^+/v_{-1}^+ > u_{-1}^-/v_{-1}^-$ . If there is a purchase or a sale, in the next round we have  $E_t^+ > E_t^-$ . Otherwise both beliefs are still equal and we can consider the next sale. Now, assume up to a given period  $t'$  we have  $E_t^+ > E_t^-$ . If in  $t' - 1$  there was no sale then both types update similarly and we have  $E_{t'+1}^+ > E_{t'+1}^-$ . Assume there was a sale, and we have a purchase in  $t'$ . A sufficient condition to have  $E_{t'+1}^+ > E_{t'+1}^-$  is that  $u_1^+/v_1^+ > u_1^-/v_1^-$ . We already know this is the case if  $F^+$  traders do not buy ( $F^-$  traders cannot buy since  $E_{t'}^+ > E_{t'}^-$ ). If they do buy, then we need:

$$\frac{u_1^+}{v_1^+} > \frac{u_1^-}{v_1^-} \Leftrightarrow \frac{x_N(1 - \alpha) + x_I + x_S}{x_N(1 - \alpha) + x_S} > \frac{x_N + x_I}{x_N} \Leftrightarrow \alpha x_N > x_S$$

If in  $t'$  we have a sale instead we need  $u_{-1}^+/v_{-1}^+ > u_{-1}^-/v_{-1}^-$ .  $F^+$  traders cannot sell due to the condition  $E_{t'}^+ > E_{t'}^-$ . We already know the required condition holds if  $F^-$  traders do not

sell either. If they do sell, then we need:

$$\frac{u_{-1}^+}{v_{-1}^+} > \frac{u_{-1}^-}{v_{-1}^-} \Leftrightarrow \frac{x_N(1+2\alpha) + x_I}{x_N(1+2\alpha)} > \frac{x_N + x_I + x_S}{x_N + x_S} \Leftrightarrow 2\alpha x_N > x_S$$

Thus  $\alpha x_N > x_S$  is a sufficient condition to always have  $E_t^+$  updated more upwards than  $E_t^-$ , which shows that we have  $E_t^+ \geq E_t^-$  for any  $t$ .

I now turn to Lemma 2. Under Assumptions 1 or 2, by Lemma 1 we can define  $\Delta H_\infty^+, \Delta H_\infty^-$  and  $\Delta \Lambda_\infty$ . In the long run  $E_t^+, E_t^-$  and  $\lambda_t$  thus behave like  $e^{t\Delta H_\infty^+}/(1 + e^{t\Delta H_\infty^+})$ ,  $e^{t\Delta H_\infty^-}/(1 + e^{t\Delta H_\infty^-})$  and  $e^{t\Delta \Lambda_\infty}/(1 + e^{t\Delta \Lambda_\infty})$  on average. Consider the case  $v = 1, F = F^+$ . By Proposition 3 we have  $\Delta H_\infty^+ > \Delta H_\infty^-$  and if supply-informed traders are active they have to buy, thus equation (14) has to be satisfied, which can be written for  $F^+$  traders as:

$$\frac{(1-\lambda)(E^+ - E^-)}{E^+(1 - E^+)} = \frac{1 + e^{H^+}}{1 + e^{H^-}} \times \frac{1}{1 + e^\Lambda} \times \left( \frac{e^{H^+} - e^{H^-}}{e^{H^+}} \right) > \frac{x_I}{u_1^- - E^+ x_I}$$

The dominant term in the numerator of the left-hand side is  $e^{2H^+}$ , while the dominant term in the denominator is  $e^{H^+ + \Lambda + \max(0, H^-)}$ . Thus if  $\Delta H_\infty^+ > \Delta \Lambda_\infty + \max(\Delta H_\infty^-, 0)$  the left-hand side goes to infinity and is larger than the right-hand side which goes to a positive constant, if the opposite then the left-hand side goes to zero and is smaller than the right-hand side. The condition  $\Delta H_\infty^+ > \Delta \Lambda_\infty + \max(\Delta H_\infty^-, 0)$  implies that  $|E_t^+ - v| > \max(|\lambda_t - F|, |E_t^- - v|)$ , which is thus a necessary condition for long-run activity. Conversely, if we start with limits corresponding to the case where  $F^+$  traders are inactive and have  $|E_t^+ - v| < \max(|\lambda_t - F|, |E_t^- - v|)$  then necessarily  $\Delta H_\infty^+ < \Delta \Lambda_\infty + \max(\Delta H_\infty^-, 0)$ , which is a sufficient condition for inactivity. All other cases are dealt with symmetrically. Finally, under Assumption 1 both types of traders must have a constant behaviour in the long-run, so that they are either always inactive or always active and buying if  $F = F^+$  or selling if  $F = F^-$ . The two cases of the lemma partition the possible outcomes and the necessary conditions are thus also sufficient.



#### A.2.4 Proof of Remark 1

Under the assumptions of the remark, when  $(v, F) = (1, 0)$  or  $(0, 1)$  supply-informed traders do not trade in the long-run and thus do not add to trade imbalance. When  $(v, F) = (0, 0)$  either they are inactive or they sell. Since there is no positive feedback in this case we know there are more sales than purchases due to value-informed traders, hence supply-informed traders add to order imbalance. The last case to consider is  $(v, F) = (1, 1)$ , in which case supply-informed traders either are inactive in the long-run and do not affect order imbalance, or are active and buy. In the latter case, the transition matrix in the long-run is  $M(1, 1, 1)$ . We can compute the associated stationary measures  $\mu_1$  and  $\mu_{-1}$  as:

$$\mu_1(1, 1, 1) = \frac{1 - 2x_N(1 + \alpha) + 3\alpha x_N^2}{1 - 2\alpha x_N}, \quad \mu_{-1}(1, 1, 1) = \frac{x_N}{1 - 2\alpha x_N}$$

As supply-informed traders are buying, they add to order imbalance in this case if and only if  $\mu_1(1, 1, 1) \geq \mu_{-1}(1, 1, 1)$ , which gives us the condition mentioned in the remark.

#### A.2.5 Proof of Remarks 2 and 3

I first prove Remark 2. Starting with equation (7), since  $E^+ > E^-$  the spread has the sign of  $\Pr(F = F^+ | T_t = 1, I_{t-1}) - \Pr(F = F^+ | T_t = -1, I_{t-1})$ . Using the results of section A.2.2, this difference is positive if and only if  $y_{t,1}/z_{t,1} \geq y_{t,-1}/z_{t,-1}$ . Since  $x_I = 0$ , we have  $u_{t,T}^j = v_{t,T}^j$  for any  $T$  and  $j$  and the inequality simplifies to  $u_{t,1}/v_{t,1} \geq u_{t,-1}/v_{t,-1}$ . There is equality and thus a null spread if the previous trade was a purchase or a hold, otherwise the inequality does not hold and the spread is negative. To see that this implies the bid price going up after a sale, write:

$$\begin{aligned} p_{t-1} &= \Pr(F = F^+ | I_{t-1})E^+ + \Pr(F = F^- | I_{t-1})E^- \\ B_t &= \Pr(F = F^+ | I_{t-1}, T_t = -1)E^+ + \Pr(F = F^- | I_{t-1}, T_t = -1)E^- \end{aligned}$$

so that  $B_t - p_{t-1} = (E^+ - E^-)[\Pr(F = F^+ | I_{t-1}, T_t = -1) - \Pr(F = F^+ | I_{t-1})]$ . As  $y_{t,-1}/z_{t,-1} > 1$  we get the desired result.

For Remark 3, define  $\hat{u}_{t,T}^j = (1 + \hat{x}_S)u_{t,T}^j$  and  $\hat{v}_{t,T}^j = (1 + \hat{x}_S)v_{t,T}^j$  so that the different probabilities can be expressed in terms of the  $\hat{x}_S$ . The ask price is given by equation (11) and we have:

$$\begin{aligned}\Pr(v = 1 \cap T = 1) &= \lambda E^+ \frac{\hat{u}_1^+}{1 + \hat{x}_S} + (1 - \lambda) E^- \frac{\hat{u}_1^-}{1 + \hat{x}_S} \\ \Pr(v = 0 \cap T = 1) &= \lambda(1 - E^+) \frac{\hat{v}_1^+}{1 + \hat{x}_S} + (1 - \lambda)(1 - E^-) \frac{\hat{v}_1^-}{1 + \hat{x}_S}\end{aligned}$$

Remember that as  $E^+ > E^-$  the  $F^-$  traders cannot buy. Differentiating with respect to  $\hat{x}_S$ :

$$\begin{aligned}\frac{\partial \Pr(v = 1 \cap T = 1)}{\partial \hat{x}_S} &= \frac{1}{1 + \hat{x}_S} \left( \lambda E^+ \left( O_1^+ - \frac{\hat{u}_1^+}{1 + \hat{x}_S} \right) - (1 - \lambda) E^- \frac{\hat{u}_1^-}{1 + \hat{x}_S} \right) \\ &= (\lambda E^+ O_1^+ - \Pr(v = 1 \cap T = 1)) / (1 + \hat{x}_S) \\ \frac{\partial \Pr(v = 0 \cap T = 1)}{\partial \hat{x}_S} &= \frac{1}{1 + \hat{x}_S} \left( \lambda(1 - E^+) \left( O_1^+ - \frac{\hat{v}_1^+}{1 + \hat{x}_S} \right) - (1 - \lambda) E^- \frac{\hat{v}_1^-}{1 + \hat{x}_S} \right) \\ &= (\lambda(1 - E^+) O_1^+ - \Pr(v = 0 \cap T = 1)) / (1 + \hat{x}_S)\end{aligned}$$

After some rearrangements, this gives:

$$\frac{\partial A}{\partial \hat{x}_S} = \frac{\lambda O_1^+}{(1 + \hat{x}_S)P(T = 1)}(E^+ - A)$$

If  $F^+$  traders do not buy the effect is zero. Otherwise  $E^+ > A$  and thus the effect is positive. The reasoning for the bid is similar, and bid is negatively affected if and only if  $F^-$  traders sell. In all cases the impact of  $\hat{x}_S$  on the spread is non-negative.

#### A.2.6 Proof of Proposition 4

Consider the case  $v = 1, F = 1$ . As explained in the text, for a low enough  $\lambda_1$  prices  $p_t$  can be made arbitrarily close to  $E_t^-$  for a finite number of periods. It is thus enough to show that  $E_t^-$  goes down on average for a finite number of periods. If  $x_I$  and  $x_S$  are small and  $\alpha$  high, we can make the proportion of sales among observed trades arbitrarily close to 1. It is thus enough to show that  $E_t^-$  goes down in expectation when  $T_{t-1} = -1$ . Using equation

(16), this is equivalent to:

$$u_1^+ \ln \left( \frac{u_1^-}{v_1^-} \right) + u_{-1}^+ \ln \left( \frac{u_{-1}^-}{v_{-1}^-} \right) < 0 \Leftrightarrow \ln \left( \frac{x_I + x_N}{x_N} \right) (x_I - 3\alpha x_N) < 0$$

which is true under the assumptions of the proposition. The case  $v = 0, F = 0$  is more direct. We have to show that  $E_t^+$  moves upwards, which is equivalent to:

$$u_1^- \ln \left( \frac{u_1^+}{v_1^+} \right) + u_{-1}^- \ln \left( \frac{u_{-1}^+}{v_{-1}^+} \right) > 0 \Leftrightarrow (x_I + x_N) \ln \left( \frac{x_I + (1 - \alpha)x_N}{(1 - \alpha)x_N} \right) + x_N \ln \left( \frac{(1 + 2\alpha)x_N}{(1 + 2\alpha)x_N + x_I} \right) > 0$$

which is true for  $\alpha$  close enough to one as the first log goes to  $+\infty$ .

### A.2.7 Proof of Proposition 5

Under the assumptions of the proposition, dropping time subscripts and rewriting equation (11) or (12) gives:

$$1 - p = \frac{\lambda(1 - E^+)v^+ + (1 - \lambda)(1 - E^-)v^-}{\lambda(v^+ \pm E^+x_I) + (1 - \lambda)(v^- \pm E^-x_I)}$$

where for the ask price  $\pm$  is a  $+$  and  $(v^+, v^-) = (v_1^+, v_1^-)$  and for the bid price  $\pm$  is a  $-$  and  $(v^+, v^-) = (v_{-1}^+, v_{-1}^-)$ . Using the definition of  $H^+, H^-, \Lambda$  this gives:

$$1 - p = \frac{e^\Lambda(1 + e^{H^-})v^+ + (1 + e^{H^+})v^-}{(1 + e^{H^+})(1 + e^{H^-})(e^\Lambda v^+ + v^-) \pm x_I(e^\Lambda e^{H^+}(1 + e^{H^-}) + e^{H^-}(1 + e^{H^+}))}$$

Since  $H_t^+, H_t^-, \Lambda$  behave as  $t \times \Delta H_\infty^+, t \times \Delta H_\infty^-, t \times \Delta \Lambda_\infty$ ,  $1 - p$  is equivalent when  $t$  goes to infinity to the ratio of the dominant terms in the numerator and denominator. When  $F = F^+$  both  $H^+$  and  $\Lambda$  are positive in the long run and  $H^-$  is negative since  $E^-$  is assumed to converge to zero. If  $F^+$  traders are active then according to the proof in A.2.3 we have  $H^+ > \Lambda + \max(0, H^-) = \Lambda$  so that the dominant term in the numerator is  $e^{H^+}$  and the dominant term in the denominator is  $e^{\Lambda + H^+}$ , hence the ratio is equivalent to  $\frac{v^+}{v^+ \pm x_I} e^{-\Lambda}$ . For  $x_I$  small enough this is close to  $e^{-t\Delta\Lambda_\infty}$ . This reasoning takes  $v^+, v^-$  as given but they will take different values after sales, purchases and no trade. For a small enough  $x_I$  this does not affect the equivalent. We can then deduce that the equivalent in real time of  $|p_\tau - v|$  is given by  $e^{-\tau(1 + \hat{x}_S)\Delta\Lambda_\infty}$ .

It remains to show that increasing  $\hat{x}_S$  negatively affects  $(1 + \hat{x}_S)\Delta\Lambda_\infty$ . This quantity is an average, weighted by the stationary measures of each state, of the update of  $\Lambda_t$  when the trade in  $t - 1$  was a purchase, a sale or a hold.

$$\begin{aligned}(1 + \hat{x}_S)E(\Delta\Lambda_t|I_t) &= (1 + \hat{x}_S) \left( u_1^+ \ln \left( \frac{u_1^+}{u_1^-} \right) + u_0^+ \ln \left( \frac{u_0^+}{u_0^-} \right) + u_{-1}^+ \ln \left( \frac{u_{-1}^+}{u_{-1}^-} \right) \right) \\ &= \hat{u}_1^+ \ln \left( \frac{\hat{u}_1^+}{\hat{u}_1^-} \right) + \hat{u}_0^+ \ln \left( \frac{\hat{u}_0^+}{\hat{u}_0^-} \right) + u_{-1}^+ \ln \left( \frac{\hat{u}_{-1}^+}{\hat{u}_{-1}^-} \right)\end{aligned}$$

where the  $\hat{u}$ s are defined by  $\hat{u}_1^+ = \hat{x}_N + \hat{x}_I + \hat{x}_S - \alpha\hat{x}_N\mathbf{1}_{T_{t-1}=-1}$ ,  $\hat{u}_1^- = \hat{x}_N + \hat{x}_I$ ,  $\hat{u}_{-1}^+ = \hat{x}_N + 2\alpha\hat{x}_N\mathbf{1}_{T_{t-1}=-1}$  and  $\hat{u}_{-1}^- = \hat{x}_N + \hat{x}_S$ . If  $T_{t-1} = -1$ , then the derivative of  $(1 + \hat{x}_S)E(\Delta\Lambda_t|I_t)$  with respect to  $\hat{x}_S$  can be expressed as:

$$\left[ 1 + \ln \left( \frac{\hat{u}_1^+}{\hat{u}_1^-} \right) - \frac{\hat{u}_1^+}{\hat{u}_1^-} \right] + \frac{\hat{u}_1^+}{\hat{u}_1^-} - \frac{\hat{u}_{-1}^+}{\hat{u}_{-1}^-}$$

the first term in brackets is negative as  $1 + \ln x - x$  is negative. For  $\hat{x}_S$  small it is easy to show that the second term is negative as well. When  $T_{t-1} \neq -1$  the derivative is:

$$1 - \frac{\hat{x}_N}{\hat{x}_N + \hat{x}_S} + \ln \left( \frac{\hat{x}_S + 1 - 2\hat{x}_N}{1 - 2\hat{x}_N} \right)$$

this is positive but can be made arbitrarily small as  $\hat{x}_S$  tends to zero. Thus the total effect of  $\hat{x}_S$  on  $(1 + \hat{x}_S)\Delta\Lambda_\infty$  is strictly negative when  $\hat{x}_S$  is small enough.

### A.3 Figures

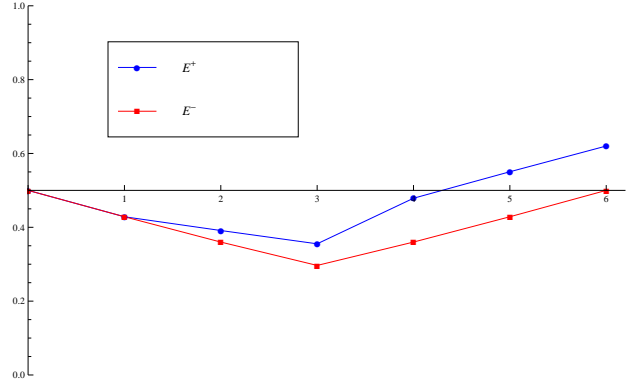


Figure 2: Supply informed traders' expectations. 3 sales, 3 purchases.

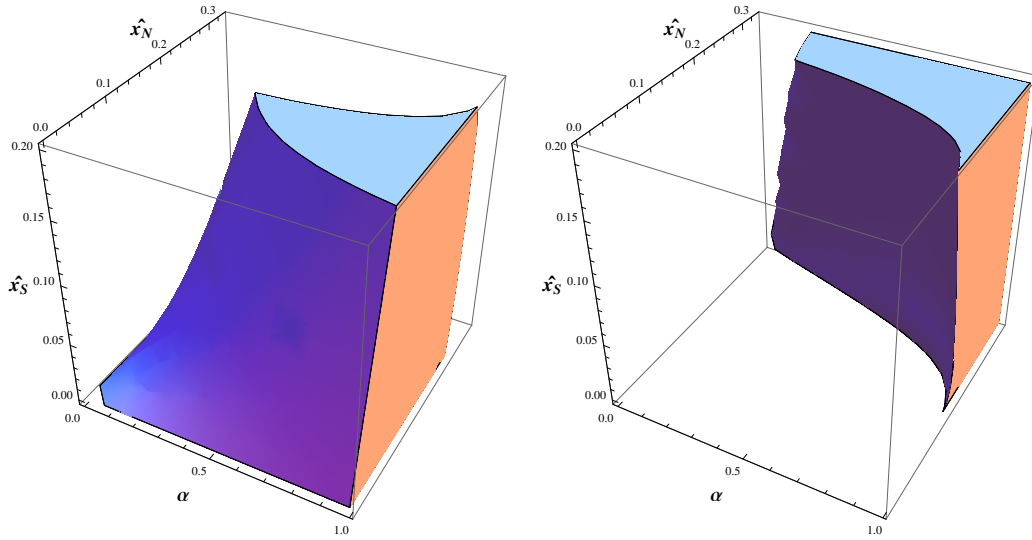


Figure 3: Parameters for which the supply-informed cannot be inactive in the long-run.

$v = 1, F = F^+$  (left) and  $v = 0, F = F^-$  (right).

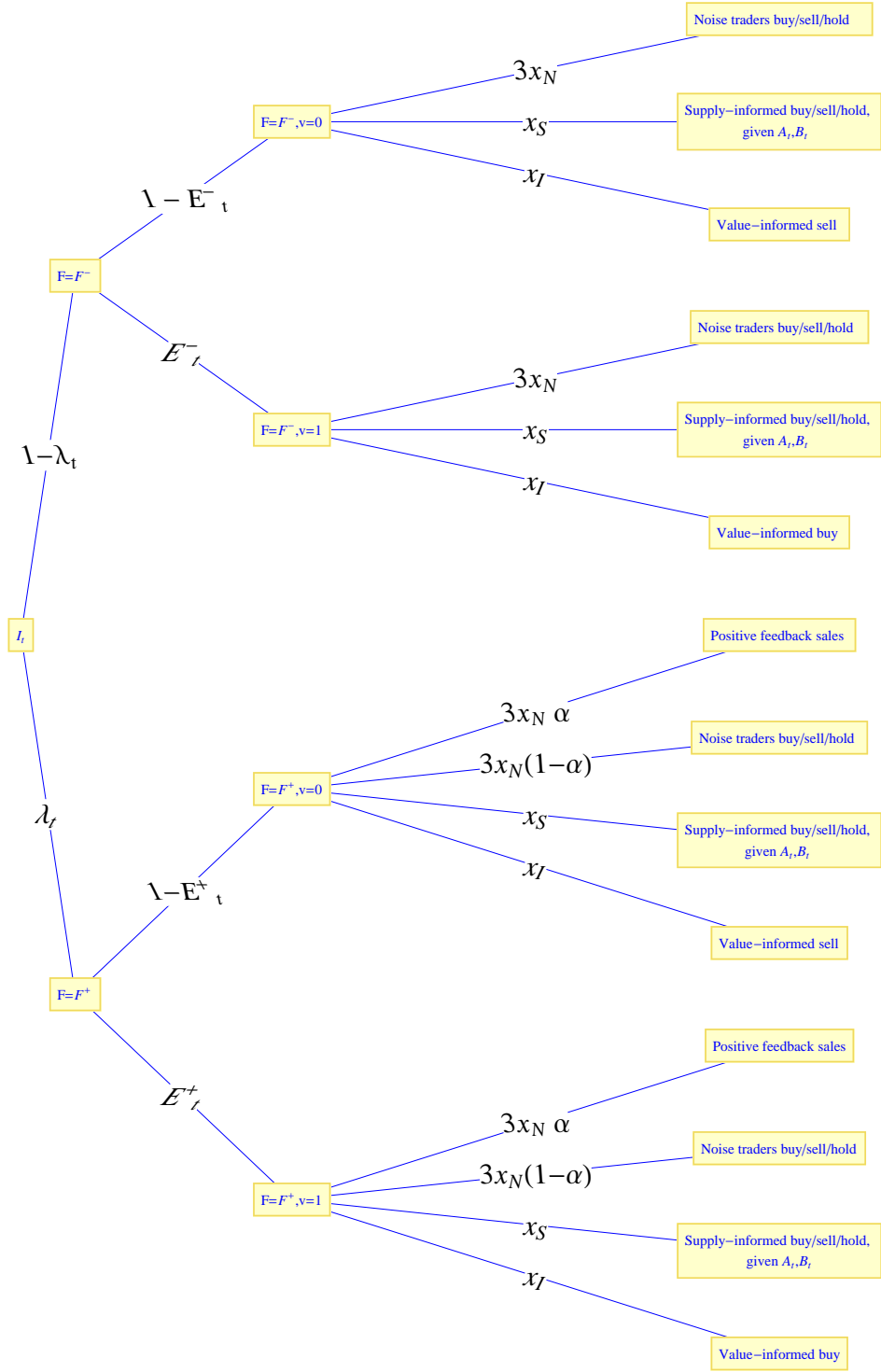


Figure 1: Probability tree in period  $t$ , after a sale.

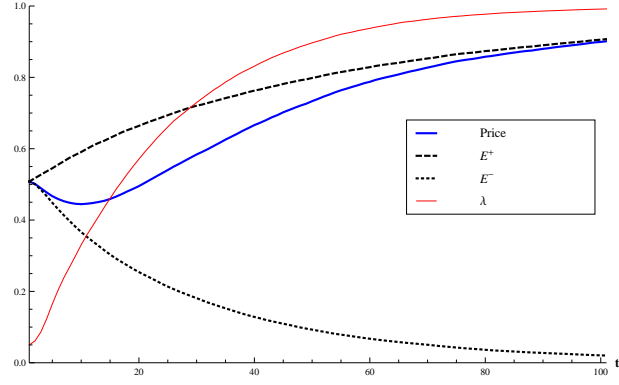


Figure 4: Beliefs and prices. Turbulence case, conditional on  $v = 1, F = F^+$ .

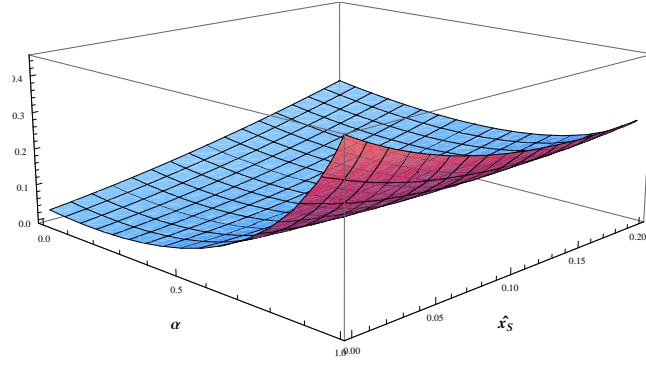


Figure 5: Stationary speed of convergence  $\beta$  as a function of  $\hat{x}_S$  and  $\alpha$ , for  $\hat{x}_N = 0.3$ .

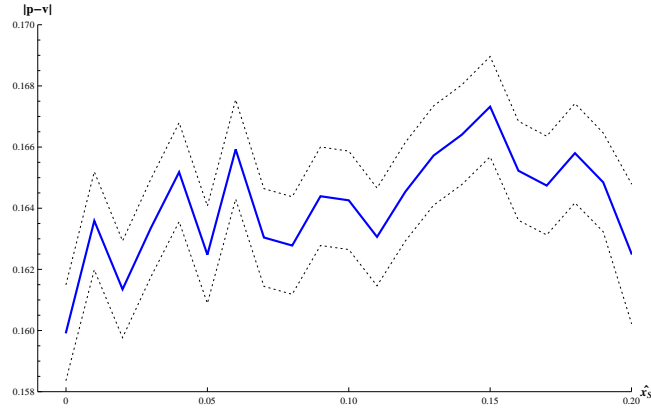


Figure 6:  $|p_\tau - v|$  after 100 periods as a function of  $\hat{x}_S$ . Turbulence case.

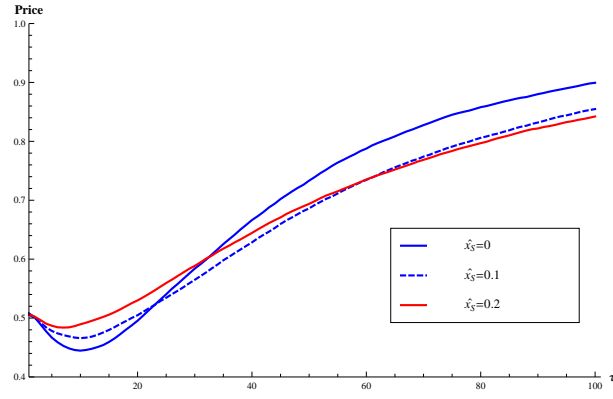


Figure 7: Price as a function of time for different  $\hat{x}_S$ . Turbulence case, conditional on  $v = 1, F = F^+$ .

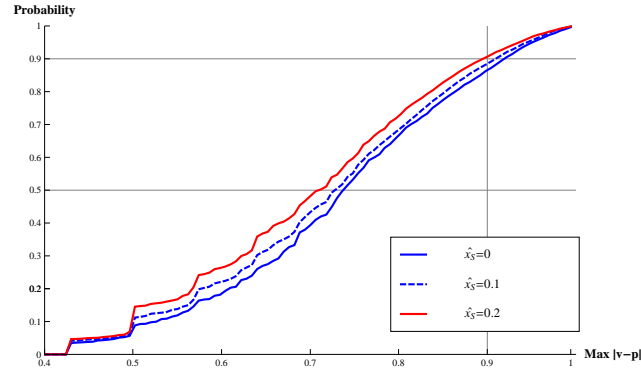


Figure 8: Empirical CDFs of the largest  $|v - p_t|$  observed, for different  $\hat{x}_S$ . Turbulence case, conditional on  $v = 1, F = F^+$ .

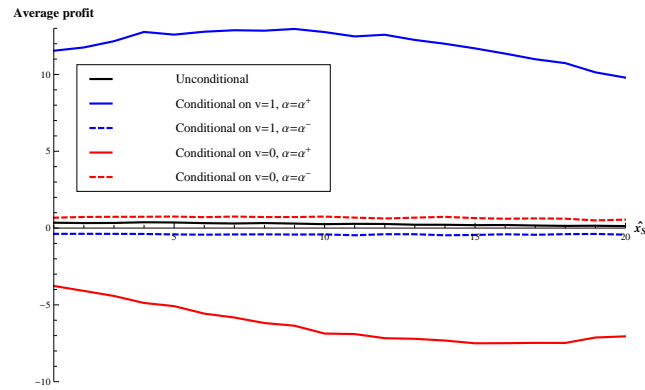


Figure 9: Supply-informed traders' average profits over 100 periods. Turbulence case, conditional and unconditional.



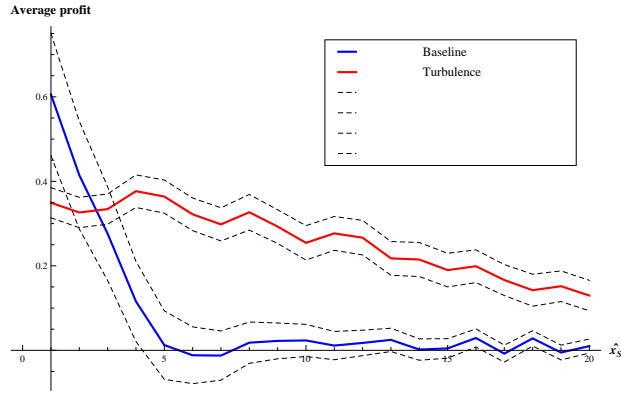


Figure 10: Supply-informed traders' average profits over 100 periods. Baseline and turbulence cases, unconditional.

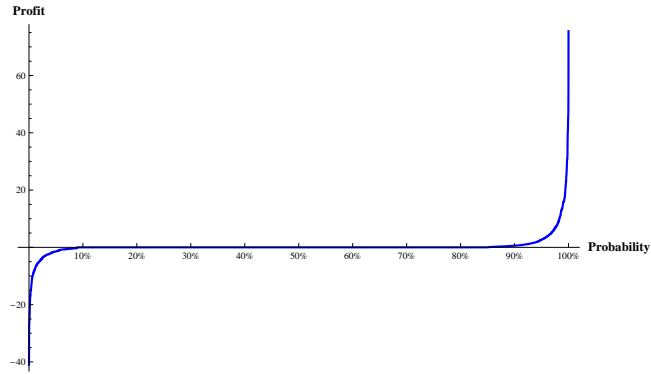


Figure 11: Quantile function of supply-informed traders' profits over 100 periods. Turbulence case, unconditional.

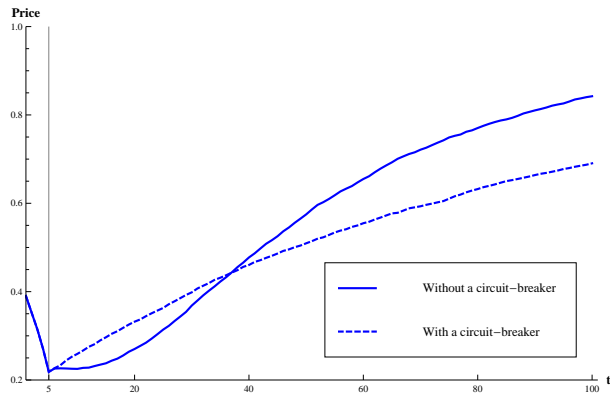


Figure 12: Average prices conditional on a circuit-breaker being triggered at  $t = 5$ , and counterfactual trajectory without a circuit-breaker.

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