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ASSET PRICING AND HOUSING SUPPLY IN A PRODUCTION ECONOMY

by Ivan Jaccard

NOTE: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.
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Abstract

We develop a representative agent model of a production economy in order to explain the joint dynamics of house prices and equity returns. In a model generating costly business cycle fluctuations, we find that restrictions on housing supply have important implications for asset pricing. Together with habit formation in the composite of consumption and leisure, building restrictions provide an explanation for the high volatility of house prices and contribute to the resolution of asset pricing puzzles.

*JEL classification:* E2, E3, G1.

*Keywords:* House Prices, Cost of Business Cycle, Adjustment Costs, Housing Returns
Non-Technical Summary

Real estate is by far the largest component of household total wealth, yet very few studies have attempted to explain the joint dynamics of house prices and financial returns in models able to capture the main business cycle regularities.

Constructing a unified framework that could be used to simultaneously study business cycle, housing market, and asset pricing facts is not as straightforward as it might at first seem. Most of the literature studies financial returns and housing market variables separately. And while many macroeconomic models have proven successful at reproducing business cycle regularities, resolving asset pricing puzzles in richer frameworks remains a formidable challenge.

This study finds that modeling housing supply, and in particular the frictions that affect the aggregate supply of housing, significantly improves the ability of an otherwise standard dynamic general equilibrium model to simultaneously explain asset pricing and business cycle facts. Together with a particular preference specification, building restrictions provide an explanation for the high volatility of house prices observed in the data and contribute to the resolution of asset pricing puzzles.

As illustrated by the high volatility of residential investment or hours worked in the construction sector observed at business cycle frequency, the housing market provides an important margin of adjustment. In our economy, rigidities in the housing sector diminish the flexibility of this adjustment margin and alter the economy’s capacity to absorb shocks. This mechanism works through the impact of the elasticity of housing supply on the dynamics of aggregate variables. The effects of tighter restrictions on housing supply spill over to the other sectors of the economy by amplifying the impact of aggregate shocks. And since supply constraints reduce the potential for inter-temporal smoothing provided by the residential investment margin, the risk premia demanded by investors are higher in an economy with a more rigid housing market. In a model calibrated to U.S. data, our main finding is that the quantitative magnitude of this general equilibrium effect is quite large.

Our second main finding is that the macroeconomic and the asset pricing implications of housing supply restrictions are exacerbated by economic uncertainty. While real estate is an effective hedge against shocks when the housing market is sufficiently flexible, rigidities that reduce the elasticity of housing supply amplify the impact of non-linearities, and increase the welfare cost of business cycle fluctuations.
1 Introduction

Real estate is by far the largest component of household total wealth, yet very few studies have attempted to explain the joint dynamics of house prices and financial returns in a DSGE model able to match the equity risk premium (Mehra and Prescott 1985). The objective of the present article is to fill this gap.

Compared to the existing asset pricing literature (Piazzesi, Schneider and Tuzel 2007, Flavine and Yamashita 2008, Stokey 2009), the main difference is that equity and housing returns are studied in a model with housing production (Davis and Heathcote 2005). Explaining asset pricing facts in models featuring production and an endogenous labor supply decision gives rise to well-known complications (Jermann 1998, Boldrin, Christiano and Fisher 2001, Danthine and Donaldson 2002, Guvenen 2009, Jaccard 2009a). At the same time, modeling in a unified framework economic choices that are usually studied in isolation increases the number of dimensions on which the theory can be tested.

Our main finding is that supply restrictions which reduce the elasticity of housing supply have important implications for asset pricing. As illustrated by the high volatility of residential investment and of housing market variables in general, the housing sector provides an important margin of adjustment. Frictions that increase the extent to which housing supply is constrained can therefore increase risk premia by reducing the potential for inter-temporal smoothing provided by the housing sector. In a model calibrated to U.S. data, our main finding is that the quantitative magnitude of this general equilibrium effect is quite large.

The impact of building restrictions on risk premia appears to crucially depend on the model’s ability to generate costly business cycle fluctuations (Lucas 2003). Interestingly, in our production economy, the effects of supply restrictions on the equity premium are significant only in versions of the model that are able to produce realistic amounts of risk. Without our specification of habit formation for instance (Jaccard 2009a), the equity premium becomes implausibly small, the cost of business cycle fluctuations is negligible, and the asset pricing implications of housing and of supply restrictions are insignificant (Davis and Martin 2009, Stokey 2009).

The effects of building restrictions on the equity premium are considerably amplified by economic uncertainty. This can be explained by the fact that, in our economy, housing provides a hedge against business cycle fluctuations. In a model where the housing stock is introduced in the utility function (Davis and Heathcote 2005), investing in housing provides a stable source of consumption that responds gradually to shocks. Our results suggest that this specific characteristic is of par-
ticular importance in a model generating costly business cycle fluctuations. Supply restrictions prevent agents from exploiting the hedging property of housing and their impact on the equity premium can be sizeable.

Building restrictions are modeled by introducing adjustment costs on housing supply into an otherwise standard macro-housing model. The objective is to capture the impact of regulation (Glaeser, Gyourko, and Saks 2005b) or land (Davis and Heathcote 2007) in constraining large-scale development. The relevance of supply frictions in the housing market is supported by a large amount of microeconomic evidence. In the United States, many empirical studies find that housing supply is constrained and that these building restrictions affect the dynamics of housing market variables (Glaeser and Gyourko 2003, Green, Malpezzi, and Mayo 2005, Quigley and Raphael 2005, Saks 2008). Glaeser, Gyourko, and Saks (2005a, 2005b), for instance, show that rising house prices have been accompanied by reductions in residential developments. Overall, regulation appears to be an important factor constraining the supply of housing.

To evaluate the asset pricing implications of housing supply, the second crucial assumption is that habits are formed over the composite of consumption and leisure, where consumption is an aggregate of housing and non-housing expenditures. This specification of habit formation, which introduces only one additional parameter to calibrate, aims at capturing the idea that agents get hooked to a certain mix of consumption, housing, and leisure reflecting their standards of living. Augmenting general equilibrium macro-housing models with this particular specification considerably improves their asset pricing predictions (Jaccard 2009b).

As far as the dynamics of house prices is concerned, in a model calibrated to match the volatility of residential investment and the equity premium, we find that building restrictions lead to a substantial increase in the volatility of house prices. This result seems in line with the empirical facts reported by Glaeser, Gyourko, and Saiz (2008), who emphasize the importance of housing supply elasticities in explaining house price dynamics. Our analysis also suggests that, while housing supply regulation is essential to generate empirically plausible house price dynamics, demand factors are likely to play an equally important role. Even with very high supply restrictions, it would be considerably more difficult to explain the high volatility of house prices without habit formation.

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1 In Davis and Heathcote (2005) introducing land in the production function of new homes improves the model’s ability to explain the dynamics of residential investment. Land is a fixed factor in the production of new homes that acts like an adjustment cost.

2 Van Nieuwerburgh and Weill (2010) also attribute the increase in house price dispersion to a combination of supply and demand factors. According to their findings, while housing supply regulation is important, the increase in wage dispersion is an essential part of the explanation.
Section 2 presents a set of stylized facts for the United States. We present the environment in section 3 and discuss the asset pricing implications in section 4. We describe the calibration in section 5 and present and discuss the results in section 6. Section 7 concludes.

2 Data description

In Table 1, we present empirical facts describing the volatility and the cyclicity of the business cycle and of the asset pricing variables under study. These statistics were computed using quarterly data. All the variables are expressed in logs, and the cyclical component was extracted using a HP-filter.

Following the literature on the equity premium puzzle, Table 2 reports the mean and standard deviation of equity and housing returns. The financial statistics presented in Table 2 are expressed in annualized percentage terms.

In Table 1, the volatility of total output is denoted $\sigma_{y_T}$, and the relative standard deviation of variable $x_i$ with respect to output is denoted $\sigma_{x_i}/\sigma_{y_T}$. The correlation of variable $x_i$ with respect to output is denoted $\rho(x_{it}, y_{Tt})$, while $\rho(x_{it}, x_{it-1})$ denotes the first-order autocorrelation of variable $x_i$. Market consumption, which in the data corresponds to real consumption of services and of non-durables goods, is denoted $c$. Business investment, which corresponds to non-residential investment, is denoted $i_T$ and residential investment is denoted $y_H$. Total hours worked are denoted $n_T$ and correspond to a measure of total employment. Wages in the business sector are denoted $w_B$ and correspond to a measure of real compensation per hour in the nonfarm business sector. Finally, employment and real wages in the housing sector are respectively denoted $n_H$ and $w_H$, and they correspond to a measure of total employment and earnings in the construction sector.\(^3\) Real wages have been deflated using CPI inflation.

House prices, equity prices and dividends are denoted $p_H, p_E$ and $d$. The Case-Shiller index is used as a proxy for house prices, and equity prices and earnings are taken from the online database of Robert Shiller (Shiller 2005). Compared to other house price indices, the advantage of the Case-Shiller index is that it includes transaction prices which are based on a wider range of mortgage contracts.\(^4\)

\(^3\)The series hours worked in the construction sector, $n_H$, is not available before 1964.
\(^4\)To be able to compare the volatility and standard deviation of house prices and equity prices, the sample is restricted to the period 1987–2010, since the Case-Shiller index is not available before 1987. Over the restricted sample 1987–2010, the standard deviation of equity prices, $\sigma_{p_E}$, is 11.6, and the volatility of output, $\sigma_{y_T}$, is 1.12. Over the whole sample 1871–2010, the standard deviation of equity prices, $\sigma_{p_E}$, is 12.1.
Residential rents are denoted, \( z_H \), and are proxied using the housing component of the CPI index.

<table>
<thead>
<tr>
<th>Total output (HP-filter 1947-2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{y_T} ) &amp; ( \rho(y_{Tt}, y_{Tt-1}) ) &amp; ( \rho(y_{Tt}, y_{Tt-4}) )</td>
</tr>
<tr>
<td>( y_T )</td>
</tr>
</tbody>
</table>

Business cycle statistics (HP-filter 1947-2010)

| \( \sigma_{x_i}/\sigma_{y_T} \) & \( \rho(x_{iT}, y_{Tt}) \) & \( \rho(x_{iT}, x_{iT-1}) \) |
|----------------|----------------|----------------|
| \( \text{c} \) | 0.49 | 0.79 | 0.84 |
| \( \text{i}_T \) | 2.38 | 0.71 | 0.87 |
| \( y_H \) | 5.90 | 0.53 | 0.88 |
| \( n_T \) | 1.03 | 0.88 | 0.89 |
| \( n_H \) | 2.86 | 0.77 | 0.90 |
| \( w_B \) | 0.57 | 0.18 | 0.66 |
| \( w_H \) | 0.78 | -0.13 | 0.68 |

<table>
<thead>
<tr>
<th>Asset prices (HP-filter 1987-2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{x_i}/\sigma_{y_T} ) &amp; ( \rho(x_{iT}, y_{Tt}) ) &amp; ( \rho(x_{iT}, x_{iT-1}) )</td>
</tr>
<tr>
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<td>( d )</td>
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<td>( p_E )</td>
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<td>( p_H )</td>
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<tr>
<td>( z_H )</td>
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</tbody>
</table>

Co-movement (HP-filter)

| \( \rho(y_{Ht}, i_{Tt}) \) & \( \rho(n_{Ht}, n_{Bt}) \) & \( \rho(p_{Ht}, p_{Et}) \) |
|----------------|----------------|----------------|
| 0.28 | 0.87 | 0.40 |

Table 1: Volatility and correlation

In Table 2, the equity premium and the housing risk premium are respectively denoted \( E(r_E - r_f) \) and \( E(r_H - r_f) \), where \( r_f \) is the real risk-free rate. The volatility of equity returns, of housing returns, and of the risk-free rate are respectively denoted \( \sigma(r_E) \), \( \sigma(r_H) \), and \( \sigma(r_f) \). Finally, the first-order autocorrelation of equity returns, housing returns, and the risk-free rate is denoted \( \rho(r_{Et}, r_{Et-1}) \), \( \rho(r_{Ht}, r_{Ht-1}) \), and \( \rho(r_{ft}, r_{ft-1}) \). These financial statistics are taken from the study of Piazzesi, Schneider, and Tuzel (2007).
3 The environment

The representative firm is composed of two sectors that use capital and labor as factors of production. The business sector produces a standard final output good, which can be divided between consumption and investment, while new homes are produced by a housing sector. The housing sector generates revenue from renting the stock of new homes to the representative household. The economy is subject to a single source of exogenous disturbances that take the form of random shocks to total factor productivity. The specifications of preferences and technology are compatible with balanced growth. The deterministic growth rate at which the economy is growing, along the balanced growth path, is denoted $\gamma$.

3.1 The firm

In each period, the representative firm has to decide how much labor to hire in each sector, how much to invest, and how to allocate capital across the two sectors. Managers maximize the value of the firm which is equal to the present discounted value of all current and future expected cash flows:

$$E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{t+k}}{\lambda_t} d_{t+k}$$

with $\beta^k \frac{\lambda_{t+k}}{\lambda_t}$ being the discount factor of the representative agent, who is the owner of the firm, and where dividends are given by:

$$d_t = A_t k_{B_{t+1}}^{1-\alpha} + z_{H_t} h_t + p_{CB_t} b_{t+1} - w_t (n_{B_t} + n_{H_t}) - i_t - b_{t-j} - T_t$$
As far as the business sector is concerned, the capital stock used to produce the final output good is $k_Bt$, $n_Bt$ is the quantity of labor input, $w_t$ is the wage rate, and the stochastic total factor productivity level is denoted $A_t$. The real estate activity of the representative firm generates a revenue from renting the existing housing stock to the household. The housing stock and the rental rate are denoted $h_t$ and $z_{Ht}$. The quantity of labor input needed to produce new homes is denoted $n_{Ht}$. Capital accumulation is financed via retained earnings, and non-residential investment is denoted $i_Tt$. Production, rental income, labor costs, and investment determine the component of dividends related to operating profits.

In practice, borrowing is an important source of financing, and the majority of firms finance part of their activity through debt. To capture the impact of leverage on the dynamics of dividends, borrowing is introduced by assuming that, at time $t$, firms can issue $j$-periods corporate discount bonds, $b_{t+1}$, that pay a fixed revenue in $j$ periods. The price of the corporate discount bond is denoted $p_{CBt}$. The capital structure of the firm is chosen by managers who can use debt to reduce the tax bill of the firm. To keep the analysis simple, we assume that the government levies a tax, which is denoted $T_t$, and that the total amount of tax is composed of two components:

$$T_t = g_t - \tau(b_{t+1})$$

The first component, $g_t$, is independent of the firm’s capital structure and is set by the government. The tax advantage provided by debt is captured by introducing a component that varies with the level of debt. This tax rebate, which we denote $\tau(b_{t+1})$, increases with $b_{t+1}$ so that $\tau'(b_{t+1}) > 0$ and where $\tau''(b_{t+1}) < 0$.

**Production of new homes**

The evolution of the housing stock that can be rented to the household depends on the amount of new homes produced each period. The production function of new homes has the standard Cobb-Douglas characterization:

$$y_{Ht} = A_t k_{Ht}^{\phi} n_{Ht}^{1-\phi}$$

where $k_{Ht}$ is the capital stock. Compared to Davis and Heathcote (2005), the impact of land on the dynamics of residential investment is captured by introducing an adjustment cost and the law of motion characterizing the evolution of the housing stock is given by:

$$\gamma_{t+1} = (1 - \delta_H)h_t + \phi_{H} \left( \frac{y_{Ht}}{h_t} \right) h_t$$

Footnote 5: In Davis and Heathcote (2005) land is a fixed factor in the production of new homes that acts like an adjustment cost on residential investment.
where $\delta_H$ is the depreciation rate of the housing stock, and where $\phi_H(\frac{y_H}{h})$ is the adjustment cost function. To keep the analysis as general as possible, the type of adjustment costs used by Jermann (1998) in the context of asset pricing models is adopted.\(^6\) The adjustment cost, which is a function of the new homes to housing stock ratio, $\frac{y_H}{h}$, is denoted $\phi_H()$. Concavity of the function $\phi_H()$ captures the idea that changing the housing stock rapidly is more costly than changing it slowly.$^7$

Housing supply regulation can be summarized by a single elasticity parameter, $\varepsilon_H$, capturing the curvature of the adjustment cost function:$^8$

$$\varepsilon_H = \frac{\phi''_H(\frac{y_H}{h}) \frac{y_H}{h}}{\phi'_H(\frac{y_H}{h})}$$

While our specification is aimed at capturing the impact of land in constraining new homes production (Davis and Heathcote 2005), it would also be consistent with the facts reported by Glaeser, Gyourko, and Saks (2005a, 2005b) suggesting that these constraints could also be the result of housing supply regulation.$^9$

The above specification, which increases the cost of large projects as measured by changes in $\frac{y_{Ht}}{h_t}$, aims at capturing the fact that housing supply regulation makes large-scale developments more costly to implement.

**Capital accumulation and its allocation across sectors**

Capital accumulation, which is determined by the firm’s investment policy and the intraperiod allocation of total capital across sectors, is the result of two distinct decisions. In addition to the amount devoted to capital accumulation, managers have to decide how to allocate the total stock of capital, $k_{Tt}$, between the housing and the business sectors, where:

$$k_{Tt} = k_{Bt} + k_{Ht}$$

\(^{6}\)Lucas and Prescott (1971), Hayashi (1982), and Baxter and Crucini (1993) also study models with similar types of adjustment costs.

\(^{7}\)Near the steady state, we have that:

$$\phi_H\left(\frac{y_H}{h}\right) > 0, \ \phi'_H\left(\frac{y_H}{h}\right) > 0 \ \text{and} \ \phi''_H\left(\frac{y_H}{h}\right) < 0$$

To reduce the number of free parameters, $\phi_H()$ is parametrized so as to make the steady state of the model with and without adjustment costs similar.

\(^{8}\)The case $1/\varepsilon_H = \infty$ corresponds to a model without adjustment costs, while the case $1/\varepsilon_H = 0$ corresponds to a specification with infinite adjustment costs.

\(^{9}\)Housing supply regulation could explain the high volatility of house prices observed in cities like Phoenix where the availability of land is a priori not an issue.
As shown by Jermann (1998), production economy models with investment and habit formation cannot generate plausible asset pricing predictions without capital adjustment costs. While capital adjustment costs also play a key role in our study, as will be shown later, a model with capital adjustment costs but without housing supply restrictions would fail on several key dimensions.

To keep the analysis as simple as possible, we assume that the accumulation of capital is subject to the same type of adjustment costs as housing. The firm’s capital stock obeys an intertemporal accumulation equation that is given by:

\[ \gamma k_{T_t+1} = (1 - \delta_K)k_{T_t} + \phi_I \left( \frac{i_{T_t}}{k_{T_t}} \right) k_{T_t} \]

where the cost of adjusting the capital stock depends on the elasticity parameter, \( \varepsilon_I \):

\[ \varepsilon_I = \frac{\phi''_I \left( \frac{i_{T_t}}{k_{T_t}} \right) i_{T_t}}{\phi'_I \left( \frac{i_{T_t}}{k_{T_t}} \right)} \]

and where \( \delta_K \) is the depreciation rate of capital.

As shown by Tuzel (2009), for instance, introducing asymmetric adjustment costs generally contributes to increasing risk premiums. This result suggests that assuming costly reversibility could help to further reduce the potential for intertemporal smoothing. The mechanism under study should therefore be robust to a more realistic specification of adjustment costs.

**Firm’s net worth**

As will be discussed in the next section, we assume that the introduction of a tax rebate creates an agency problem between households and the firm. The agency problem will be resolved by using the firm’s net worth as a proxy for its financial health. Given that the firm owns the capital and the housing stocks, the market value of its net worth is given by:

\[ nw_{t+1} = q_{T_t}k_{T_t+1} + p_{H_t}h_{t+1} \]

where \( q_{T_t} \) denotes Tobin’s Q.

### 3.2 Households

In this economy, utility is derived from consuming a market consumption good, \( c_t \), from enjoying leisure, \( l_t \), and from the housing stock that has been accumulated
over time, $h_t$. As far as preferences are concerned, the key assumption is that habits are formed over the mix of total consumption and leisure (Jaccard 2009a). The reference level or habit stock is denoted, $x_t$, and lifetime utility is given by:

$$U = E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[ \frac{1}{1-\sigma} \left( c^\kappa h_{t+k}^{1-\kappa} (l_{t+k}^\psi + \chi) - x_{t+k} \right)^{1-\sigma} \right] \right\}$$

(8)

Net utility is given by the difference between the composite good, $c^\kappa h_t^{1-\kappa} (l_t^\psi + \chi)$, and the reference level, $x_t$. The modified discount factor and the coefficient of relative risk aversion are respectively denoted $\beta^*$ and $\sigma$. As in Constantinides 1990, the evolution of the habit stock is governed by a law of motion that allows for memory effects:

$$\gamma x_{t+1} = mx_t + (1-m)c_t^\kappa h_{t}^{1-\kappa} (l_{t}^\psi + \chi)$$

(9)

where $m$ captures the rate at which the habit stock depreciates. The labor supply parameters $\psi$ and $\chi$ determine the steady state allocation of time and the Frisch elasticity of labor supply. To restrict the number of degrees of freedom, we assume that the parameter measuring the impact of $c_t^\kappa h_t^{1-\kappa} (l_t^\psi + \chi)$ on the habit stock is given by $1 - m$. Compared to a macro-housing model (Davis and Heathcote 2005), this specification of internal habit formation therefore adds only one free parameter.

The representative household faces the following sequential budget constraint:

$$w_t n_{Bt} + w_t n_{Ht} + s_t d_t + b_{t-j} + TR_t = z_{Ht} h_t + c_t + p_{Et} (s_{t+1} - s_t) + p_{CBt} b_{t+1}$$

(10)

where equity prices are denoted $p_{Et}$, $s_t$ is equity holding, and $TR_t$ is a transfer received from the government. As far as the allocation of time is concerned, households decide how to divide their time endowment between leisure activities, hours worked in the business sector, and hours worked in the housing sector. Normalizing the total time endowment to 1, we have that:

$$n_T + l_t = 1$$

where:

$$n_T = n_{Bt} + n_{Ht}$$

(11)

and where $n_T$, $n_B$ and $n_{Ht}$ respectively denote the total number of hours worked, hours worked in the market sector, and hours worked in the housing sector.

Households decide how many corporate bonds to purchase from the representative firm. New bond purchases are denoted $p_{CBt} b_{t+1}$. To pin down the capital

---

10 where $\beta^* = \beta \gamma^{1-\sigma}$
structure of the firm, we assume that the tax advantage provided by debt creates an agency problem between lenders and borrowers. Households are not able to directly assess the solvency of the firm, but the firm’s net worth is observable. Because of imperfect monitoring, agents are never willing to hold an amount of corporate debt that exceeds a fraction $\xi$ of the firm’s net worth:

$$b_{t+1} \leq \xi nw_{t+1}$$

(12)

where $\xi$ is the leverage ratio,$^{11}$ and $nw_{t+1}$ is taken as given by the household.

### 3.3 Market equilibrium

An equilibrium is a set of prices $\{\lambda_t, \psi_t, \sigma_t, z_{Ht}, p_{Et}, qT, p_{Ht}, w_{Br}, w_{Ht}, p_{CBr}\}$ for all possible states and for all $t \geq 0$ such that, when households and firms maximize utility and profit, taking these prices as given, all markets clear, and the government budget constraint is satisfied:

$$T_t = TR_t$$

Market clearing for the consumption/investment goods market implies that all produced goods are either consumed or invested:

$$y_{Br} = c_t + iT_t$$

Labor supply equals labor demand, the quantity of housing stock produced equals the quantity rented by the households, and the quantity of corporate debt issued by the firm is equal to the amount demanded by the household. Finally, financial market equilibrium requires that the investors hold all outstanding equity shares.

### 4 Asset pricing implications

The dynamics of equity prices can be characterized by deriving the first-order conditions of the household problem. The standard intertemporal arbitrage equation, where the cost of buying the asset today, and tomorrow’s expected future gains have to be equalized, can be derived from the maximization problem:$^{12}$

$$p_{Et} = \beta^* E_t \frac{T_{t+1}}{\lambda_t} [d_{t+1} + p_{Et+1}]$$

(13)

$^{11}$In the growing economy the constraint is $\Gamma_t b_{t+1} \leq \tilde{\xi} \tilde{nw}_{t+1}$ where the deterministic growth rate of the economy is given by $\gamma = \Gamma_{t+1}/\Gamma_t$. Given that the level of debt is stationary, in the detrended economy we have $b_{t+1} \leq \tilde{\xi} \tilde{nw}_{t+1}$ where the leverage ratio $\xi = \tilde{\xi} / \gamma$ has been adjusted for growth.

$^{12}$See equation (30) in the appendix
Equity returns are given by the standard definition:

\[ r_{Et+1} = \frac{p_{Et+1} + d_{t+1}}{p_{Et}} \]

House prices, which can be derived from the first-order conditions of the firms, can be characterized by a similar inter-temporal arbitrage equation linking prices to fundamentals: \(^{13}\)

\[ p_{Ht} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta_H + \omega_{t+1}) p_{Ht+1} + z_{Ht+1} \right] \]

where:

\[ \omega_{t+1} = \phi_H \left( \frac{y_{Ht+1}}{h_{t+1}} \right) - \phi_H' \left( \frac{y_{Ht+1}}{h_{t+1}} \right) \frac{y_{Ht+1}}{h_{t+1}} \]

and where \( \beta = \bar{\beta} \gamma^{-\sigma} \). Compared to equity prices, the fact that the housing stock depreciates and that the accumulation of housing is subject to building restrictions creates a wedge that shows up in the asset pricing formula. Building restrictions and physical depreciation affect the dynamics of house prices through the capital gain component of the valuation. The term \( \omega_{t+1} \) captures the impact of regulation on the accumulation of the housing stock. Compared to a financial asset, the capital gain component also has to be adjusted for the fact that the housing stock depreciates at rate \( \delta_H \).

The pay-off from increasing the stock of housing is determined by the rental rate, \( z_{Ht} \), which is given by the following ratio of marginal utilities:

\[ z_{Ht} = \frac{U_h(c_t, h_t, l_t)}{U_c(c_t, h_t, l_t)} \]

where \( U_c \) and \( U_h \) respectively denote the marginal utility of market consumption and of the housing stock. With the specification of preferences that have been adopted, this expression reduces to:

\[ z_{Ht} = \frac{(1 - \kappa) c_t}{\kappa h_t} \]

Most of the empirical literature studying the determinants of housing returns usually abstracts from the direct impact of building restrictions on the return of housing investments (Piazzesi, Schneider, and Tuzel 2007, Flavin and Yamashita 2002). To ensure consistency between the model implications and the empirical facts, we adopt a standard definition and define housing returns, \( r_{Ht+1} \), as:

\[ r_{Ht+1} = \frac{(1 - \delta_H)p_{Ht+1} + z_{Ht+1}}{p_{Ht}} \]

\(^{13}\)See equation (19) in the appendix
The equilibrium level of corporate debt is pinned down by the following equilibrium condition, which can be derived using equations (20) and (29) in the appendix:

$$\frac{\sigma_t}{\lambda_t} = \tau'(b_{t+1})$$

where $\sigma_t$ is the Lagrange multiplier associated to the enforcement constraint (12). The assumption that debt has tax advantage ensures that the constraint is always binding.

5 Parameter selection

The parameter selection is carried out in two steps. A first set of parameters is chosen based on National Income Account data, following the standard in the business cycle literature. A second set of parameters, for which a priori knowledge is weak, is chosen to maximize the model’s ability to replicate a set of stylized facts of interest, namely the equity premium and the volatility of investment.

Preference parameters

With internal habit formation, the steady state coefficient of relative risk aversion is independent of the habit parameter $m$ and is exactly equal to the curvature parameter $\sigma$ (Jaccard 2009a). To ensure that the conclusion of this study does not rely on an implausible curvature coefficient, we set $\sigma$ to 1. Following King and Rebelo (1999), the subjective discount factor $\beta$ is set to 0.984, which is a standard value used in the literature.

Market sector and growth rate

The quarterly trend growth rate $\gamma$ is set to 1.005, and the constant capital share in the Cobb-Douglas production function, $\alpha$, is 0.36. These are standard values used in the literature. Following Davis and Heathcote (2005), the depreciation rate of business capital, $\delta_K$, is set to 0.0136.

Housing sector

Following the estimated value reported by Davis and Heathcote (2005), the depreciation rate of the housing stock, $\delta_H$, is set to 0.0035. Finally, the residential capital
share, $\varphi$, is set to 0.30 to ensure that the steady state investment to output ratio, $y_H/y_B$, is equal to 4.7%, as in Davis and Heathcote (2005).\(^{14}\)

**Housing and non-housing consumption share**

With Cobb-Douglas preferences, the expenditure share of non-housing consumption is given by the utility weight, $\kappa$:

$$\kappa = \frac{c}{z_H h + c}$$

The empirical evidence presented in Piazzesi, Schneider, and Tuzel (2007) can therefore be used to calibrate $\kappa$ using data on the expenditure share of housing and non-housing consumption. Following their empirical findings, we set $\kappa$ to 0.826, which implies that housing consumption represents about seventeen percent of total consumption.

**Hours worked**

The introduction of endogenous labor supply involves the calibration of two additional parameters, $\psi$ and $\chi$. First, evidence from the 2008 time of use survey is used to calibrate $\chi$. In 2008, households spent on average 3.75 hours on work-related activities. Assuming that the available time for leisure and working activities is 16 hours per day, this implies a steady state value for $n_T$ of about 0.23. This first restriction pins down $\chi$. The second elasticity parameter, $\psi$, determines the elasticity of labor supply. To our knowledge empirical evidence regarding the elasticity of labor supply in the construction sector is not available. Given this lack of a priori knowledge, we choose a value for $\psi$ that implies an elasticity of labor supply in the total number of hours worked, $n_T$, of about 1. Values for the Frisch elasticity of labor supply that are used in the literature usually range from 1 to 4 (King and Rebelo 1999, Uhlig 2007).

Given this calibration strategy, the structure of the model implies that the steady state allocation of time $n_B/n_H$ is determined endogenously.\(^{15}\) Given values

\(^{14}\)Given that the structure of our model is not as rich as in Davis and Heathcote (2005), we need to set the residential capital share to a slightly higher value in order to match the same steady state ratio.

\(^{15}\)In the steady state, we have that:

$$\frac{n_B}{n_H} = \frac{y_B (1 - \alpha)}{c (1 - \varphi)} \frac{1 - \beta (1 - \delta_H)}{\beta (1 - \kappa)} \frac{\kappa}{y_H}$$
for \( c/y_B \) and \( h/y_H \), which depends on the structural parameters discussed above, this calibration implies that \( n_H = 0.0225, n_B = 0.2094, \) and \( n_T = 0.2259 \). The model therefore predicts a steady state share of hours worked in the housing sector, \( n_H/n_T \), of about 5.9\%, which is close to the value found in the data.\(^{16}\)

**Productivity shock**

Following the literature on the real business cycle, we assume that technology shocks are the only source of business cycle fluctuations. Total factor productivity, \( A_t \), has the usual autoregressive characterization:

\[
\log A_t = \rho \log A_{t-1} + \epsilon_t
\]

Compared to Davis and Heathcote (2005) and Greenwood and Hercowitz (1991), the number of degrees of freedom is therefore reduced by considering an economy where business fluctuations are entirely driven by one single exogenous shock. Following the business cycle literature, the persistence parameter \( \rho \) is set to 0.979, and the innovation standard deviation \( \sigma_\epsilon \) is set to 0.0072.

As shown by Chari, Kehoe, and McGrattan (2007), fluctuations caused by technology shocks can be reinterpreted as variations in the efficiency wedge. The equivalence results presented in their study allow us to interpret technology shocks in a broader sense. Variations in the efficiency wedge can capture financial frictions, leading to an inefficient allocation of input across firms. Variations in the efficiency wedge can also reflect underlying frictions affecting the allocation of factor inputs (Restuccia and Rogerson 2008).

**Leverage**

To calibrate the leverage ratio, \( \xi = b/nw \), we use the measure of net leverage proposed by Bates, Kahle and Stulz (2009). The impact of cash holdings on firms’ dividend policies has been documented by many studies (Fama and French 2001). A potential concern is that using a measure of leverage that does not take into account cash holdings may overstate the impact of leverage on the dynamics of dividends. For instance, according to the evidence reported by Bates, Kahle and Stulz (2009), in 2006, while the commonly used measure of leverage was on average 0.22, the measure of net leverage was -0.01. We take this potential issue into account by choosing a conservative value for leverage and set \( \xi \) to 0.02, which is roughly the average value for net leverage over the period 1996-2006.

\(^{16}\)In the data, the ratio \( n_H/n_T \) is equal to 5.5\%. 

Next, following Jermann (1998), we assume that firms can issue \( j \)-period discount bonds. To keep the financial policy as simple as possible, we assume that firms only issue bonds of one single maturity and normalize the bond constant coupon so that in the non-stochastic steady state, \( p_{CB} = 1 \).\(^{17}\) Given available data on corporate debt, which suggests that on average corporate bond maturity ranged from 7.2 to 13.7 years between 1996 and 2010, we set the maturity \( j \) to 10 years.\(^{18}\)

**Adjustment costs, building restrictions, and habit formation**

The three remaining parameters to select are the housing supply regulation parameter \( \varepsilon_H \), the capital adjustment cost parameter \( \varepsilon_I \), and the habit parameter \( m \). This second set of parameters is picked to maximize the model’s ability to match the equity premium, the volatility of business investment, and the volatility of residential investment. The parameter search has been restricted to the following range of values:

\[
m = [0 : 1], \quad \varepsilon_H = [0.0 : 6.25], \quad \varepsilon_I = [0.0 : 6.25]
\]

As regards the adjustment costs and the housing supply regulation parameters, the model without frictions corresponds to the case \( \varepsilon_I = \varepsilon_H = 0 \), and the model reduces to the case without habit formation when \( m \) is set to 1.

### 6 Results and discussion

The model’s ability to match the equity premium and the volatility of investment is maximized at the following values:

\[
m = 0.7, \quad \varepsilon_H = 1.59, \quad \varepsilon_I = 4.0
\]

The model is solved using perturbation methods and by taking a second-order approximation to the policy function (Adjemian, Juillard, Mihoubi, Perendia, and Villemot 2009). All model’s implications are computed using theoretical moments. The results reported in Table 3, where the moments that are targeted are emphasized in bold, confirm that business cycle fluctuations cannot be entirely explained by a model with only one shock and a low Frisch elasticity of labor supply. Despite the low volatility of output, the model is still able to generate a 6.19% equity premium. This illustrates that the mechanism under study considerably amplifies the impact of business cycle fluctuations on risk premia.

\(^{17}\)This is to ensure that the models with and without leverage have the same non-stochastic steady state.

Term premium

As shown by Jermann (1998), risk premia can be decomposed into a common and an asset specific component. The term premium is the component that is common to every asset and that depends on the yield curve. The payout uncertainty premium is asset specific and is determined by the covariance between the dividend paid by the asset and marginal utility. Given that the stochastic discount factor is known, the term premium can be computed by pricing the following long term bond:

\[ p_{Ct} = \beta^* E_t \frac{\lambda_{t+1}}{\lambda_t} [1 + p_{Ct+1}] \]

The return on the asset can be defined as:

\[ r_{Ct, t+1} = \frac{p_{Ct+1} + 1}{p_{Ct}} \]

where, for simplicity, the constant coupon paid by the asset has been normalized to one. The risk premium on this console bond, \( E(r_C - r_f) \), corresponds to the term premium and is entirely determined by the term structure of the interest rates.

The term premium generated by the benchmark model being 3.78%, the cyclical behavior of dividends therefore accounts for 6.19 - 3.78 = 2.41% of the total equity premium. Compared to the benchmark calibration, removing leverage would make dividends considerably smoother and countercyclical. This cyclical behavior of dividends would make equity less risky than a console bond, an implication that would be difficult to reconcile with the empirical facts (Jermann 1998, Abel 1999).

In the standard neoclassical model, dividends can be countercyclical because their dynamics is dominated by the response of investment. Dividends being financed via retained earnings, managers will find it optimal to increase investment and pay lower dividends to shareholders during boom periods. With leverage, the fact that managers can issue debt allows them to increase investment and at the same time to pay higher dividends to shareholders.

Introducing a maturity \( j \) larger than 1 increases the persistence of dividends by postponing the reimbursement of a loan received at period \( t \). Lengthening the maturity helps to raise the payout uncertainty premium without generating excessive dividend volatility (Jermann 1998).
Table 3: Output and business cycle statistics. The theoretical moments of the variable reported in Table 3 are expressed in logs and the cyclical component was extracted using a HP-filter. $\sigma_{x_i/y_T}$ and $\rho(x_i, y_T)$ respectively denote the relative standard deviation of variable $x_i$ with respect to output and the correlation of variable $x_i$ with output. $\rho(x_i, x_i-k)$ is the $k$ order autocorrelation of variable $x_i$. $y_T$ is total output, $c$ is consumption, $i_T$ is business investment, $y_H$ is residential investment, $n_T$ is total hours worked, and $n_H$ is hours worked in the housing sector. $w_B$ and $w_H$ are the wage rates in the business and in the housing sector. Dividends, equity prices, house prices and residential rents are respectively denoted $d$, $p_E$, $p_H$ and $z_H$. 

<table>
<thead>
<tr>
<th>Total output (HP-filter)</th>
<th>$\sigma_{y_T}$</th>
<th>$\rho(y_{Tt}, y_{Tt-1})$</th>
<th>$\rho(y_{Tt}, y_{Tt-4})$</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>$y_T$</td>
<td>1.68</td>
<td>1.08</td>
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<table>
<thead>
<tr>
<th>Business cycle statistics (HP-filter)</th>
<th>$\sigma_{x_i/y_T}$</th>
<th>$\rho(x_{it}, y_{Tt})$</th>
<th>$\rho(x_{it}, x_{it-1})$</th>
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<tr>
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<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>$c$</td>
<td>0.49</td>
<td>0.64</td>
<td>0.79</td>
</tr>
<tr>
<td>$i_T$</td>
<td><strong>2.38</strong></td>
<td><strong>2.44</strong></td>
<td>0.71</td>
</tr>
<tr>
<td>$y_H$</td>
<td><strong>5.90</strong></td>
<td><strong>5.90</strong></td>
<td>0.53</td>
</tr>
<tr>
<td>$n_T$</td>
<td>1.03</td>
<td>0.62</td>
<td>0.88</td>
</tr>
<tr>
<td>$n_H$</td>
<td>2.86</td>
<td>5.21</td>
<td>0.77</td>
</tr>
<tr>
<td>$w_B$</td>
<td>0.57</td>
<td>0.67</td>
<td>0.18</td>
</tr>
<tr>
<td>$w_H$</td>
<td>0.78</td>
<td>0.67</td>
<td>-0.13</td>
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</table>

<table>
<thead>
<tr>
<th>Co-movement (HP-filter)</th>
<th>$\rho(y_{Ht}, i_{Tt})$</th>
<th>$\rho(n_{Ht}, n_{Bt})$</th>
<th>$\rho(p_{Ht}, p_{Et})$</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>$y_H$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.87</td>
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</table>

<table>
<thead>
<tr>
<th>Asset prices (HP-filter)</th>
<th>$\sigma_{x_i/y_T}$</th>
<th>$\rho(x_{it}, y_{Tt})$</th>
<th>$\rho(x_{it}, x_{it-1})$</th>
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<td>Model</td>
</tr>
<tr>
<td>$d$</td>
<td>29.8</td>
<td>26.4</td>
<td>0.69</td>
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<td>$p_E$</td>
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<td>0.65</td>
</tr>
<tr>
<td>$p_H$</td>
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<td>9.40</td>
<td>0.56</td>
</tr>
<tr>
<td>$z_H$</td>
<td>0.39</td>
<td>0.66</td>
<td>-0.31</td>
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</table>
Mean in annualized (% p.a.)

<table>
<thead>
<tr>
<th></th>
<th>$E(r_E - r_f)$</th>
<th>$E(r_f)$</th>
<th>$E(r_H - r_f)$</th>
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</thead>
<tbody>
<tr>
<td>Data Model</td>
<td>6.19</td>
<td>6.19</td>
<td>0.75</td>
</tr>
<tr>
<td>Data Model</td>
<td>3.79</td>
<td>3.79</td>
<td>1.77</td>
</tr>
<tr>
<td>Data Model</td>
<td>1.77</td>
<td>1.77</td>
<td>3.63</td>
</tr>
</tbody>
</table>
| Standard deviation (% p.a.)
|                  | $\sigma(r_E)$ | $\sigma(r_H)$ | $\sigma(r_f)$ |
| Data Model       | 16.56          | 26.75    | 2.73           |
| Data Model       | 15.88          | 3.68     | 6.25           |
| Autocorrelation
|                  | $\rho(r_{Et}, r_{Et-1})$ | $\rho(r_{Ht}, r_{Ht-1})$ | $\rho(r_{ft}, r_{ft-1})$ |
| Data Model       | -0.06          | -0.01    | 0.48           |
| Data Model       | 0.0            | 0.73     | 0.97           |

Table 4: Asset returns. The financial moments reported in Table 4 were computed using theoretical moments. The results are expressed in annualized percent. $r_E$ denote equity returns, $r_f$ is the risk-free rate and housing returns are denoted $r_H$. $E()$, $\sigma()$, and $\rho()$ respectively denote the unconditional mean, standard deviation, and first-order autocorrelation of the variable under study.

**Equity and housing risk premium**

While the housing risk premium predicted by the model is too large, it is still possible to conclude that housing is significantly less risky than equity.\(^{19}\) The spread between the equity and the housing risk premium is essentially due to the difference in the volatility of dividends and rents generated by the model. Residential rents, the dynamics of which are determined by the consumption to housing services ratio, react slowly to shocks, whereas leverage makes dividends considerably more volatile. The spread reflects that the unfavorable cyclical property of dividends, which are very volatile and pro-cyclical, has to be compensated by a higher risk premium.

The fact that the supply of housing is endogenously determined contributes to decreasing the housing risk premium. Figure 1, which shows the impulse response of rents to a positive technology shock, illustrates this point. The increase in

\(^{19}\)In the non-stochastic steady state, the model implied risk premia are equal to zero. Time-variation in the equity and the housing risk premiums can only be obtained by taking a third-order approximation to the policy function. See Jaccard (2009a) for a discussion of the cyclicality of the equity risk premium implied by this preference specification.
residential rents is short-lived because the supply of new homes increases gradually in response to a positive shock. This supply effect, which puts downward pressure on residential rents, reduces the cyclicality of the pay-off and therefore lowers the payout uncertainty component of the housing risk premium.

As shown in Table 4, while the model is able to explain the very low autocorrelation of equity returns, it is not possible to simultaneously explain why the autocorrelation of housing returns observed in the data is so high. The fact that the model also overstates the volatility of housing returns is another indication of potential model misspecification.

Wages

Not surprisingly, the rather extreme assumption of perfect mobility of labor across the two sectors, implying that $w_B = w_H$, is rejected by the data. Wages in the construction sector are more volatile and less correlated with output than wages in the business sector. The model also fails to capture the low correlation between wages in the two sectors and output. As far as the cyclicality of wages is concerned, as shown by Christiano and Eichenbaum (1992), introducing government spending
shocks usually helps to overcome this problem, which is typical of real business cycle models.

6.1 Quantitative implications of building restrictions

Table 5 shows the sensitivity of the results to a change in the housing supply coefficient, $\varepsilon_H$, which captures the tightness of housing supply regulation. Compared to the results reported in Table 3 and 4, all other parameters are kept constant.

House prices

Increasing $\varepsilon_H$ reduces the elasticity of housing supply and generates a dramatic increase in the relative standard deviation of house prices, $\sigma_{\rho H}/\sigma_{yT}$. The adjustment in quantities occurs in the labor market and leads to an equally dramatic reduction in the relative standard deviation of hours worked, $\sigma_{nH}/\sigma_{yT}$. Housing supply regulation amplifies the response of prices and reduces the response of quantities by acting as an adjustment costs on hours worked.

The large quantitative impact of housing supply restrictions on house prices very much depends on the closed economy assumption. As shown by van Nieuwerburgh and Weill (2010), the impact of regulation is smaller in a model with perfect mobility across cities, because agents can choose to move out in response to a tightening in regulation. While building restrictions generate a reallocation of resources across sectors, agents cannot completely escape regulation in our economy.

Whereas our results suggest that housing supply regulation could have a major impact on the volatility of house prices, it is important to stress, however, that this effect is considerably amplified by the introduction of habit formation. Table 5, which also reports the sensitivity of the results to a change in $\varepsilon_H$, in the case $m = 1$, illustrates this point. In a model without habit, while reducing the elasticity of housing supply still contributes to increasing the volatility of house prices, the quantitative impact is substantially smaller.

Even with implausibly large housing supply adjustment costs, the model without habit formation could only explain less than half of the observed house price volatility. This result suggests that it would be difficult to find a plausible explanation for the high volatility of house prices without combining housing supply restrictions with habit formation.

The introduction of habit formation increases the volatility of house prices because it affects agents’ saving and investment choices (Jermann 1998). In good times, for instance, the large decline in marginal utility induced by habit formation, which reflects a strong desire to postpone consumption, increases the demand for
investment. Similarly, this effect leads to an increase in the demand for new homes since residential investment can be used to transfer wealth from periods with high housing consumption to periods with low housing consumption. As illustrated in Table 5, when combined with building restrictions, this effect gives rise to fluctuations in house prices that can be very large.

**Equity and housing risk premiums**

The effect of a change in $\varepsilon_H$ on the equity premium, $E(r_E - r_f)$, is quite striking. With housing services accounting for 17% of total consumption, this large quantitative impact illustrates that introducing housing into the utility function has key asset pricing implications. Housing increases the potential for consumption risk diversification and could in principle generate a significant reduction in the equity premium. Therefore, the extent to which housing contributes to the resolution of asset pricing puzzles very much depends on the degree of housing supply restrictions.

The impact of building restrictions on the equity premium works via its effect on the stochastic discount factor. Tighter regulation increases the cost of adjusting the housing stock and generates a decline in the volatility of residential investment, $\sigma_{yH}/\sigma_{yT}$. The key is that this reduction in the volatility of residential investment makes consumption of housing services smoothing more difficult to achieve. The reduction in household’s tolerance to these variations, which is induced by habit formation, makes marginal utility more volatile and increases the uncertainty of future pay-offs. The resulting increase in risk premia reflects that investors need to be compensated by this rise in uncertainty to accept to hold equity. The housing risk premium, $E(r_H - r_f)$, is more sensitive to changes in $\varepsilon_H$ because building restrictions not only affect the stochastic discount factor but also have a direct impact on the dynamics of house prices.

A reduction in the elasticity of housing supply makes house prices more volatile, which increases the capital gain component of housing returns. Combined with the indirect effect on the stochastic discount factor that is common to every asset, this direct effect on house prices makes the housing risk premium very sensitive to changes in the regulatory environment.

As illustrated by the sensitivity analysis reported in Table 5, the impact of building restrictions on risk premiums very much relies on the presence of habit formation. This result illustrates that it is the combination of habit formation and building restrictions that matters for the determination of risk premia.
### Model with habit formation, $m = 0.7$

<table>
<thead>
<tr>
<th>Asset Pricing</th>
<th>Data</th>
<th>$\varepsilon_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{PH}/\sigma_{YT}$</td>
<td>3.78</td>
<td>0.05 4.45</td>
</tr>
<tr>
<td>$\sigma_{PE}/\sigma_{YT}$</td>
<td>10.4</td>
<td>6.67 11.1</td>
</tr>
<tr>
<td>$E(r_E - r_f)$</td>
<td>6.19</td>
<td>0.54 1.93</td>
</tr>
<tr>
<td>$E(r_H - r_f)$</td>
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<td>0.0 0.76</td>
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<table>
<thead>
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<tbody>
<tr>
<td>$\sigma_{YT}$</td>
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<td>0.68 0.83</td>
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<tr>
<td>$\sigma_{IT}/\sigma_{YT}$</td>
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<td>0.92 1.65</td>
</tr>
<tr>
<td>$\sigma_{YH}/\sigma_{YT}$</td>
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<td>20.6 13.5</td>
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<tr>
<td>$\sigma_{nH}/\sigma_{YT}$</td>
<td>2.64</td>
<td>19.4 12.6</td>
</tr>
<tr>
<td>Welfare Cost</td>
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<td>1.0 1.93</td>
</tr>
</tbody>
</table>

### Model without habit formation, $m = 1$

<table>
<thead>
<tr>
<th>Asset Pricing</th>
<th>Data</th>
<th>$\varepsilon_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{PH}/\sigma_{YT}$</td>
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<td>0 0.02</td>
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<table>
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<tbody>
<tr>
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<tr>
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<td>-</td>
<td>0.21 0.21</td>
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</table>

Table 5: Sensitivity analysis. The benchmark calibration corresponds to the column $\varepsilon_H = 1.59$, in the case $m = 0.7$. When $\varepsilon_H = 0$, the model reduces to a case without building restrictions while $\varepsilon_H = 3.5$ corresponds to a case with very high building restrictions. The lower part of Table 5 shows the sensitivity of the results to changes in $\varepsilon_H$ when, compared to the benchmark case, the habit formation channel is completely switched off by setting $m = 1$. 
Finally, the large quantitative impact of a change in $\varepsilon_H$ on the equity premium also depends on the degree of capital adjustment costs. The presence of high capital adjustment costs reduces the potential for intertemporal smoothing provided by the investment margin (Jermann 1998). Without capital adjustment costs, the quantitative impact of housing supply restrictions on the equity premium would be considerably smaller.

Output

As shown by the impact of $\varepsilon_H$ on $\sigma_{yt}$, building restrictions increase the volatility of output and amplify business cycle fluctuations. This effect works through the impact of building restrictions on the allocation of labor across sectors. While both sectors are equally penalized by capital adjustment costs, the construction sector is less affected by the distortion because of its higher labor intensity. In good times, this difference in labor intensity gives rise to an increase in hours worked that is larger in the construction sector than in the final output good sector.

Building restrictions reduce new homes production and act as an adjustment cost on hours worked in the construction sector. By reducing the comparative advantage provided by high labor intensity, this reallocation of labor over the business cycle increases the volatility of output, and at the same time reduces the volatility of residential investment.

The mechanism is similar to the case studied by van Nieuwerburgh and Weill (2010), where a tightening in housing supply regulation induces workers to move out. In our case, this is the perfect mobility between sectors that enables workers to escape housing supply regulation by increasing hours worked in the business sectors.

6.2 Risk premia, housing, and the welfare cost of uncertainty

The model’s ability to explain the equity premium and the mean risk-free rate in an environment with endogenous labor supply essentially relies on the assumption that habits are formed over a mix of consumption and leisure (Jaccard 2009a). Introducing this particular type of habit formation decreases the volatility of $c_t^h l_t^{1-K}(l_t^v + \chi)$, and at the same time increases the volatility of marginal utility, which allows the model to generate the large fluctuations in the stochastic discount factor. These fluctuations are necessary to resolve asset pricing anomalies (Cochrane and Hansen 1992).

The key is that this increase in the volatility of marginal utility is achieved by inducing a strong willingness to smooth fluctuations in $c_t^h l_t^{1-K}(l_t^v + \chi)$, as op-
Figure 2: Consumption and the composite good. Impulse response of consumption, $c_t$, and of the composite good, $c_t^x h_t^{1-\kappa} v(l_t)$, in percentage deviation from steady state to a one standard deviation technology shock. The impulse responses are simulated using the benchmark calibration.

posed to fluctuations in $c_t$. This assumption aims at capturing the idea that agents get hooked to a certain mix of consumption, housing, and leisure reflecting their standards of living. With habit formation, abrupt changes in lifestyles, as measured by changes in the composite good, are very costly and lead agents to choose total consumption and leisure so as to maintain the smoothest possible path for $c_t^x h_t^{1-\kappa} (l_t^p + \chi)$.

This point is illustrated in Figure 2, which shows the impulse response of consumption and of the composite good to a positive technology shock. While this mechanism makes fluctuations in the composite good very costly, it does not lead to excess consumption smoothing. On impact, agents compensate the increase in consumption and in housing services by reducing leisure to prevent their habit stock from rising too quickly. This specification of habit in the composite good therefore enables us to generate the volatility in marginal utility which is needed to resolve asset pricing puzzles, without generating excessive consumption smoothing.
This mechanism, which makes business cycle fluctuations very costly, also increases the welfare cost of uncertainty. To illustrate this point, following Lucas (2003), the cost of uncertainty is evaluated by comparing the stochastic and the deterministic economy. The welfare cost of uncertainty is measured by comparing the mean level of consumption in the stochastic case, $E(c)$, with consumption evaluated at the deterministic steady state, $\bar{c}$. The difference $E(c_t) - \bar{c}$ can be interpreted as the risk compensation that is required to make agents indifferent between a deterministic economy and an economy subject to business cycle fluctuations. The risk compensation, which is measured in annual percentage of agents’ consumption, $(E(c_t) - \bar{c})/E(\bar{c})$, obtained under the benchmark calibration is equal to 4.86%.

The welfare cost of uncertainty obtained under different levels of building restrictions is also reported in Table 5. In the model with habit formation, reducing the elasticity of housing supply raises both the equity premium and the welfare cost of uncertainty. Without habit formation, however, the cost of business cycle fluctuations is negligible and the impact of supply restrictions on the equity premium is insignificant.

This confirms that the asset pricing implications of housing very much depend on the level of uncertainty generated by the underlying economy. If the cost of business cycle fluctuations is negligible, restricting the supply of housing has very little impact because agents do not value the particular cyclical properties of housing in an economic environment that is stable. But as shown in Table 5, when the welfare cost of uncertainty is higher, the effects of building restrictions on risk premia can be sizeable.

### 6.3 Co-movement and lead-lag correlation

As discussed by Boldrin, Christiano, and Fisher (2001), explaining the strong positive co-movement between hours worked in the business sector and output is a challenge for models with adjustment costs. While the correlation $\rho(n_B, y_T)$ is higher than 0.8 in the data, standard models with high capital adjustment costs usually generate a negative correlation, which is at odds with the facts.

The model’s ability to explain the positive co-movement observed in the data essentially relies on the introduction of habit formation in the mix of consumption and leisure. In standard models, the cause of the problem is that the presence of adjustment costs induces a strong negative wealth effect, which reduces the incentive to supply labor in good times. In boom periods, this effect dominates the positive substitution effect induced by the increase in real wages and generates leftward shifts in labor supply. This labor supply effect, which is responsible for the
counter cyclical variations in hours worked obtained in these models, is therefore the source of the problem.

The key is that our specification of habit formation enables us to offset the effect of capital adjustment costs on labor supply by reducing the wealth elasticity of labor supply (Jaccard 2009a). Compared to the benchmark calibration, removing habit formation by setting \( m \) to 1 would compromise the model’s ability to generate this positive co-movement.\(^{20}\) The effect on labor supply is therefore similar to the effect obtained using the type of preferences proposed by Greenwood, Hercowitz, and Huffman (1988) and Jaimovich and Rebelo (2009). When it comes to the resolution of asset pricing puzzles in models with housing, however, adopting a specification based on habit formation is of the essence (Jaccard 2009b).

As documented by Fisher (2007), and as shown by Figure 3 in the appendix, the fact that residential investment leads business investment over the cycle, that is \( \text{corr}(i_{Bt}, y_{Ht} - k) > \text{corr}(i_{Bt-k}, y_{Ht}) \), is an important empirical regularity typical of the housing market. As illustrated by the right panel of Figure 3, the fact that the model can only partially account for this robust empirical regularity seems to falsify the specification of capital adjustment costs that has been adopted. When it comes to alternative specifications that could potentially help to fix this problem, the findings presented by Gomme, Kydland, and Rupert (2001) suggest that introducing time-to-build into the analysis may provide a solution to this problem.

As shown by Figure 3, the fact that house prices lead the cycle, that is \( \text{corr}(y_{Tt}, p_{Ht-k}) > \text{corr}(y_{Tt-k}, p_{Ht}) \), is another well-documented empirical regularity. House prices have leading indicator properties that are often used in forecasting. While the exact magnitude cannot be reproduced, as illustrated by the left panel of Figure 3, it is encouraging to see that the model predictions are broadly consistent with this other important empirical regularity.

\section{Conclusion}

The recent episode of financial distress has revived the debate over whether monetary policy should react to fluctuations in house prices. While this debate has been ongoing for many years, the answer to this question still very much depends on central banks’ ability to distinguish between “excessive” and “fundamentally driven” movements in house prices.

Our analysis suggests that ignoring factors that affect the elasticity of housing supply, such as housing supply regulation or the impact of land in constraining the production of new homes, may lead central banks to overstate the potential

\(^{20}\)Without habit, the correlation between \( n_T \) and output is negative.
for house price misalignments. As we have shown in section 4, in a model with housing supply constraints, the value of a house can deviate from the standard infinite discounted sum formula. Moreover, a model with building restrictions and habit formation can generate “fundamentally driven” fluctuations in house prices that can be very large.

Finally, the equilibrium value of a house is closely linked to the model’s financial market implications. Versions that failed to generate a plausible equity premium also generated smaller house price fluctuations. This result illustrates the importance of using well-specified stochastic discount factors when trying to detect house prices misalignments.
Appendix

Figure 3: Residential and business investment lead-lag correlation. The left panel reports the observed cross correlogram between residential and business investment for the period 1947–2010. The right panel shows the corresponding model implications. The series have been expressed in logs and HP-filtered. For each lag considered ($k = 1$ to $4$), the left (blue) bar shows the correlation between residential investment in $t - k$ and business investment while the right (gray) bar shows the correlation between residential investment in $t + k$ and business investment in $t$.

Figure 4: House prices and output lead-lag correlation. The left panel reports the observed cross correlogram between house prices and output for the period 1987–2010. The right panel shows the corresponding model implications. The series have been expressed in logs and HP-filtered. For each lag considered ($k = 1$ to $4$), the left (blue) bar shows the correlation between house prices in $t - k$ and output in $t$ while the right (gray) bar shows the correlation between house prices in $t + k$ and output in $t$. 
Data appendix

<table>
<thead>
<tr>
<th>Variable</th>
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<tr>
<td>$y_T$</td>
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</tr>
<tr>
<td>$c$</td>
<td>BEA, Table 3 (1947-2010), Nondurable goods+services</td>
</tr>
<tr>
<td>$i_T$</td>
<td>BEA, Table 3 (1947-2010), Private nonresidential investment+Nondefense investment+state and local investment</td>
</tr>
<tr>
<td>$i_H$</td>
<td>BEA, Table 3 (1947-2010), Residential investment</td>
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<td>BLS, Table B-10 (1947-2009), All employee hours</td>
</tr>
<tr>
<td>$n_H$</td>
<td>BLS, Table B-10 (1964-2010), Hours construction</td>
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<tr>
<td>$n_B$</td>
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<tr>
<td>$w_H$</td>
<td>St. Louis Fed, AHECONS (1947-2009), deflated with CPI inflation</td>
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<tr>
<td>$p_{E_t}$</td>
<td>R. Shiller, Real equity prices (1871-2009)</td>
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<tr>
<td>$d_t$</td>
<td>R. Shiller, Real earnings (1871-2009)</td>
</tr>
<tr>
<td>$p_{H_t}$</td>
<td>R. Shiller, case-Shiller 15 index (1987-2009)</td>
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<td>$z_H$</td>
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<td>Piazzesi, Schneider, and Tuzel (2007)</td>
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<td>$E(r_H - r_f), \sigma(r_H)$</td>
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<td>$E(r_f), \sigma(r_f)$</td>
<td>Piazzesi, Schneider, and Tuzel (2007)</td>
</tr>
</tbody>
</table>

Technical appendix

The firm

Managers maximize the value of the firm by solving the following dynamic optimization program:

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} [A_t k_{B_t} n_{B_t}^{1-\alpha} + z_{H_t} h_t + p_{CB_t} b_t + 1 + \tau(b_t+1)] ight.$$  
$$- w_{B_t} n_{B_t} - w_{H_t} n_{H_t} - i_T - b_t - g_t$$

$$+ p_{H_t} \left( (1 - \delta_H) h_t + \phi_H \left( \frac{A_t (k_{T_t} - k_{B_t})^{\varphi} n_{H_t}^{1-\varphi}}{h_t} \right) h_t - \gamma h_{t+1} \right)$$
\[+ q_{T_t} \left( (1 - \delta_K)k_{T_t} + \phi_I \left( \frac{i_{T_t}}{k_{T_t}} \right) k_{T_t} - \gamma k_{T_{t+1}} \right) \right] \} \]

First-order conditions:

\( n_{B_t} \):

\[w_{B_t} = (1 - \alpha) \frac{y_{B_t}}{n_{B_t}} \]

(14)

\( n_{H_t} \):

\[w_{H_t} = p_{H_t} \phi'_H \left( \frac{y_{H_t}}{h_t} \right) (1 - \varphi) \frac{y_{H_t}}{n_{H_t}} \]

(15)

\( k_{B_t} \):

\[\alpha \frac{y_{B_t}}{k_{B_t}} = p_{H_t} \phi'_H \left( \frac{y_{H_t}}{h_t} \right) \varphi - \frac{y_{H_t}}{k_{T_t} - k_{B_t}} \]

(16)

\( i_{T_t} \):

\[1 = q_{T_t} \phi_I \left( \frac{i_{T_t}}{k_{T_t}} \right) \]

(17)

\( k_{T_{t+1}} \):

\[q_{T_t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{T_{t+1}} \left[ (1 - \delta_K) + \phi_I \left( \frac{i_{T_{t+1}}}{k_{T_{t+1}}} \right) \right. \]

\[- \phi_I \left( \frac{i_{T_{t+1}}}{k_{T_{t+1}}} \right) \frac{i_{T_{t+1}}}{k_{T_{t+1}}} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{H_{t+1}} \phi'_H \left( \frac{y_{H_{t+1}}}{h_{t+1}} \right) \varphi - \frac{y_{H_{t+1}}}{k_{T_{t+1}} - k_{B_{t+1}}} \]

(18)

\( h_{t+1} \):

\[\lambda_t p_{H_t} = \beta E_t \lambda_{t+1} \left[ (1 - \delta_H) + \phi_H \left( \frac{y_{H_{t+1}}}{h_{t+1}} \right) - \phi'_H \left( \frac{y_{H_{t+1}}}{h_{t+1}} \right) \frac{y_{H_{t+1}}}{h_{t+1}} \right] p_{H_{t+1}} + z_{H_{t+1}} \]

(19)

\( b_{t+1} \):

\[p_{CB_t} + \tau'(b_{t+1}) = \beta^{j\ast} E_t \frac{\lambda_{t+j}}{\lambda_t} \]

(20)
\[ \lambda_t : \]
\[ d_t = A_t k_t^{\alpha} n_{t+1}^{1-\alpha} + z_{Ht} h_t + \lambda_{t+1} \]
\[ -w_{Bt} n_{t} - w_{Ht} n_{Ht} - i_{Tt} - b_t - T_t \]  (21)

\[ q_{Tt} : \]
\[ (1 - \delta_T) k_{Tt} + \phi_I \left( \frac{i_{Tt}}{k_{Tt}} \right) k_{Tt} - \gamma k_{Tt+1} = 0 \]  (22)

\[ p_{Ht} : \]
\[ (1 - \delta_H) h_t + \phi_H \left( \frac{y_{Ht}}{h_t} \right) h_t - \gamma h_{t+1} = 0 \]  (23)

**Households**

\[ L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^{st} \left[ c_t^{1-\kappa} h_t^{1-\kappa} v(l_t) - x_t \right]^{1-\sigma} \right. \]
\[ + \sum_{t=0}^{\infty} \beta^{st} \lambda_t \left[ w_{Bt} n_{Bt} + w_{Ht} n_{Ht} + s_t d_t + b_{t-1} + TR_t \right. \]
\[ -z_{Ht} h_t - c_t - p_{E} [s_{t+1} - s_t] - p_{CBt} b_{t+1} + \sum_{t=0}^{\infty} \beta^{st} \delta_t \left[ \xi nw_{t+1} - b_t \right] \]
\[ \left. + \sum_{t=0}^{\infty} \beta^{st} \psi_t \left[ mx_t + (1-m) \left[ c_t^{1-\kappa} h_t^{1-\kappa} v(l_t) - \gamma x_{t+1} \right] \right] \right\} \]

To simplify the notation, we define:

\[ u_t = c_t^{1-\kappa} h_t^{1-\kappa} v(l_t) - x_t \]

First-order conditions:

\[ c_t : \]
\[ [\kappa c_t^{1-\kappa} h_t^{1-\kappa} v(l_t)] u_t^{-\sigma} + [\kappa c_t^{1-\kappa} h_t^{1-\kappa} v(l_t)] (1-m) \psi_t = \lambda_t \]  (24)
\( n_{Bl} : \)
\[
\left[ c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] u_t^{-\sigma} + \left[ c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] (1 - m) \psi_t = \lambda_t w_{Bl} \tag{25}
\]

\( n_{Ht} : \)
\[
\left[ c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] u_t^{-\sigma} + \left[ c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] (1 - m) \psi_t = \lambda_t w_{Ht} \tag{26}
\]

\( h_t : \)
\[
z_{Ht} \lambda_t = \left[ (1 - \kappa)c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] u_t^{-\sigma} + \left[ (1 - \kappa)c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] (1 - m) \psi_t \tag{27}
\]

\( x_{t+1} : \)
\[
\psi_t = m\beta E_t \psi_{t+1} - \beta E_t u_{t+1}^{-\sigma} \tag{28}
\]

\( b_{t+1} : \)
\[
p_{CBt} + \frac{\sigma_t}{\lambda_t} = \beta_j \frac{\lambda_{t+j}}{\lambda_t} \tag{29}
\]

\( s_{t+1} : \)
\[
p_{Et} = \beta^x E_t \frac{\lambda_{t+1}}{\lambda_t} [d_{t+1} + p_{Et+1}] \tag{30}
\]

\( \lambda_t : \)
\[
w_{Bl} n_{Bl} + w_{Ht} n_{Ht} + s_t d_t + b_t - j + TR_t = z_{Ht} h_t + c_t + p_{Et} [s_{t+1} - s_t] + p_{CBt} b_{t+1} \tag{31}
\]

\( \psi_t : \)
\[
mx_t + (1 - m) \left[ c_t^{\kappa} h_t^{1-\kappa} v'(l_t) \right] - \gamma x_{t+1} = 0 \tag{32}
\]

\( \sigma_t : \)
\[
\xi nw_{t+1} - b_{t+1} = 0 \tag{33}
\]

**Adjustment costs and utility**

Following Jermann (1998), the following adjustment costs specification are chosen:

\[
\phi_H \left( \frac{y_{Ht}}{h_t} \right) = \frac{\theta_H^1}{1 - \epsilon_H} \left( \frac{y_{Ht}}{h_t} \right)^{1-\epsilon_H} + \theta_H^2
\]
\[ \phi_I \left( \frac{i_T}{k_T} \right) = \frac{\theta_I}{1 - \varepsilon_I} \left( \frac{i_T}{k_T} \right)^{1-\varepsilon_I} + \theta_2 \]

where \( \theta_I^H, \theta_H^I, \theta_I^1, \) and \( \theta_I^2 \) are calibrated such that models with and without adjustment costs have the same steady state. In the steady state, this implies that:

\[ \phi_H \left( \frac{y_H}{h} \right) = \frac{y_H}{h} \]

\[ \phi_I \left( \frac{i_T}{k_T} \right) = \frac{i_T}{k_T} \]

\[ \phi'_H \left( \frac{y_H}{h} \right) = \phi'_I \left( \frac{i_T}{k_T} \right) = 1 \]

Following Uhlig (2007b), the following specification of \( v(l) \) is chosen:

\[ v(l) = l^\nu + \chi \]

where \( \chi \) is pinned down by the steady state of the model (King and Rebelo 1999, Uhlig 2007b).
References


