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Trend-cycle decomposition of output and euro area inflation forecasts: a real-time approach based on model combination

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EUROPEAN CENTRAL BANK

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WORKING PAPER SERIES

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TREND-CYCLE DECOMPOSITION OF OUTPUT AND EURO AREA INFLATION FORECASTS

A REAL-TIME APPROACH BASED ON MODEL COMBINATION ¹

by Pierre Guérin², Laurent Maurin³,
and Matthias Mohr³

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Abstract

The paper focuses on the estimation of the euro area output gap. We construct model-averaged measures of the output gap in order to cope with both model uncertainty and parameter instability that are inherent to trend-cycle decomposition models of GDP.

We first estimate nine models of trend-cycle decomposition of euro area GDP, both univariate and multivariate, some of them allowing for changes in the slope of trend GDP and/or its error variance using Markov-switching specifications, or including a Phillips curve. We then pool the estimates using three weighting schemes.

We compute both ex-post and real-time estimates to check the stability of the estimates to GDP revisions. We finally run a forecasting experiment to evaluate the predictive power of the output gap for inflation in the euro area.

We find evidence of changes in trend growth around the recessions. We also find support for model averaging techniques in order to improve the reliability of the potential output estimates in real time. Our measures help forecasting inflation over most of our evaluation sample (2001-2010) but fail dramatically over the last recession.

Keywords: Trend-cycle decomposition, Phillips curve, Unobserved components model, Kalman Filter, Markov-switching, Auxiliary information, Model averaging, Inflation forecast, Real-time analysis.

JEL Classification Code: C53, E32, E37.

Non-technical summary

The estimation of potential output is of primary importance for policy makers since it represents the maximum level of output not associated with inflationary pressures. The output gap - i.e. the difference between the actual level of output and the potential output - conveniently summarizes the transitory state of the economy by determining whether the economy operates below or above its sustainable level.

The outbreak of the financial crisis and the following economic recession opened a sizeable negative output gap. However, the standard measures of the output gap are usually associated with a considerable level of uncertainty due to both model uncertainty and parameter instability. Model uncertainty means that model selection is a tricky issue since the level of the output gap is not observed and parameter instability means that parameter estimates can be sensitive to the estimation window chosen.

Therefore, in this paper we estimate several trend-cycle decomposition models of the output gap. This class of model decomposes the output in between a trend (i.e. the potential output) and a cycle (i.e. the output gap) using the Kalman filter. In particular, we estimate nine different models of the output gap: univariate and multivariate, linear and non-linear. We model non linearities in the trend equation of output with parameter changes governed by a Markov chain. This allows us to investigate whether strong economic downturns affect the trend of potential output. In this way, we can also estimate the probabilities of changes in the slope of potential output. We also use two classes of multivariate models: (i) a bivariate model with an equation for an indicator well correlated to the economic activity and (ii) a bivariate model with a Phillips curve since inflation is - in theory - linked to the size of the output gap.

To cope with both model uncertainty and parameter instability that are inherent to trend-cycle decomposition models of the output gap, we construct model-averaged measures of the output gap. We also investigate the impact of revisions on the estimates of the output gap and run a pseudo real-time estimation exercise. We find that our model-averaged measures reduce the uncertainty surrounding the estimates of the output gap with respect to their individual estimates counterparts.

We finally run a forecasting experiment to assess the predictive power of our output gap measures for forecasting inflation. We use two different evaluation samples: 2001Q1-2007Q4 and 2001Q1-2010Q4 to study the impact of the last recession on our results. We also use both ex-post and real-time estimates of the output gap and test statistically whether the forecasts based on our output gap measures outperform a standard autoregressive model for inflation. We find that the predictive power of the real-time estimates of the output gap for inflation is limited, whereas the ex-post estimates of the output gap marginally improve the forecasting performance with respect to their real-time counterparts. In addition, we find that the performance of the output gap for predicting inflation considerably failed over the last recession.

Overall, we find evidence of changes in trend growth around the recessions. We also find support for model averaging techniques in order to improve the reliability of the potential output estimates in real time.

1 Introduction

The estimation of potential output is of primary importance for policy makers since it represents the maximum level of output not associated with inflationary pressures. The output gap - i.e. the difference between the actual level of output and the potential output - conveniently summarizes the transitory state of the economy by determining whether the economy operates below or above its sustainable level.

Unobserved components (UC) models are often used to measure potential output since they are specifically designed to deal with latent (i.e. unobserved) variables. Univariate trend-cycle decomposition model of real GDP can be traced back to Watson (1986) and Clark (1987). These studies, which focus on the US economy, find that the cyclical part of output closely matches the US recessions identified by the NBER. Indeed, they allocate most of the variation of output to the cycle and leave the trend mostly unchanged over time. Conversely, the Beveridge and Nelson (1981) (BN) decomposition of GDP attributes most of its variability to its trend, whereas its cyclical component remains small, noisy and does not match the NBER business cycle dating of economic activity.

Morley et al. (2003) explain the discrepancy between the BN and UC decompositions by the fact that it is usually assumed in the literature that there is no correlation between the shocks to the trend and the cycle. The authors find that relaxing this restriction makes the UC decomposition of GDP identical to the BN decomposition. Moreover, they report a negative and significant correlation between the shocks to the trend and to the cycle.

Conversely, Perron and Wada (2009) emphasize the importance of allowing for a change in the slope of the trend. They model the shocks to the trend and cycle as a mixture of two normal distributions that permits to capture endogenously changes in the slope of trend GDP. In doing so, they identify a structural break in the slope of the trend of US real GDP around 1973:Q1 and obtain a cycle component of GDP that is consistent with the NBER dating of the economic activity. In this paper, we extend this approach and propose to capture changes in the slope of trend GDP with regime switches in the slope and the variance of the error.

The Markov-switching model of Hamilton (1989) is appealing since it makes the probability of parameter changes dependent on past realizations, whereas assuming that the errors of the state follow a mixture of normal distributions (i.e. the approach followed by Perron and Wada (2009)) implies that the probabilities that the errors are drawn from one regime to the other are independent from past realizations. In this respect, adopting a Markov-switching specification implies that, unlike Perron and Wada (2009), we allow for a change in trend growth to last several quarters while remaining short-lived, and to happen more than once.

To cope with model uncertainty inherent to trend-cycle decomposition and Markov-switching models, we also incorporate additional information to improve the estimation of the output gap. First, we consider the use of an auxiliary indicator - the rate of capacity utilisation - to help identifying the transitory component of GDP. Given the high correlation between this indicator and the business cycle component of economic activity, we can expect

that it improves the estimation of the output gap. Second, we add a Phillips curve to the trend cycle decomposition model of GDP. The use of a Phillips curve for estimating the output gap has been first advocated by Kuttner (1994). The author appends a Phillips curve to a univariate trend-cycle decomposition of GDP and finds that this bivariate model helps to better estimate the output gap.

The estimation of the output gap is characterized by both model uncertainty and parameter instability. Model uncertainty means that model selection is a tricky issue since we do not observe the true level of the output gap, while parameter instability refers to the idea that parameter estimates can be sensitive to the estimation window chosen. As a consequence, the output gap estimates are surrounded by a large uncertainty. One solution consists in reporting predictive densities of the output gap (see e.g. Garratt et al. (2009)). Another solution is to compute model-averaged measures of the output gap in order to reduce model uncertainty (see e.g. Morley and Piger (2009)).

Another issue pointed out by Orphanides and Van Norden (2002) is the unreliability of the estimates of output gap in real-time. However, Marcellino and Musso (2010) find that the use of real-time data is less problematic to estimate the euro area output gap.

We estimate nine models of the euro area output gap: linear, non-linear, univariate and bivariate models. We then report model-averaged measures of the output gap with their single model counterparts and show that the differences across estimates are sizeable. We find some evidence of regime changes in the slope of the trend of the euro area GDP for few periods, around 1974 and since 2008. We then run a pseudo out-of-sample forecasting experiment to forecast the level and the change in inflation using both ex-post and real-time estimates of the output gap. We find that our output gap measures help forecasting inflation over most of the sample but fail dramatically since the last recession. We also find support for model averaging techniques in order to improve the reliability of the potential output estimates in real time.

The paper is organized as follows. Section 2 presents the univariate and multivariate models of trend/cycle decomposition of GDP with and without regime switching. Section 3 discusses the estimation method and reports the empirical results for the euro area. In this section, we also discuss the estimation of a univariate time-varying Phillips curve. The estimation of the output gap in real-time and its forecasting performance for predicting inflation in the euro area is analysed in Section 4. Section 5 concludes. Four appendices complete the paper.

2 Trend-cycle decomposition of output

In this section, we present the models used to decompose GDP in between trend and cycle. We start with the univariate model, discuss the inclusion of Markov-switching parameters and then present the bivariate models.

Watson (1986) provides a starting point to decompose the level of output y_t into a trend n_t and a cycle z_t :

$$y_t = n_t + z_t \quad (1)$$

The trend n_t is modeled as a random walk with drift and the cyclical component z_t is modeled as an AR(2) process:

$$n_t = \mu + n_{t-1} + \epsilon_t^n \quad (2)$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_t^z \quad (3)$$

The disturbances ϵ_t^n and ϵ_t^z are assumed to be normally distributed, i.i.d, with mean-zero and are not correlated. The trend component n_t is interpreted as the level of potential output, while the cycle z_t is interpreted as the output gap. This model is relatively standard in the literature and can be cast in state-space form, with a state vector of dimension 3 (see Appendix A for the measurement and state equations).

2.1 Extension to regime changes in the slope of the trend

To extend the standard model, we consider regime changes in the intercept of the trend component μ and in the variance of the shock ϵ_t^n using regime switches governed by a Markov chain. This allows trend growth to be regime dependent. The general Markov-switching model we consider is:

$$y_t = n_t + z_t \quad (4)$$

$$n_t = \mu(S_t) + n_{t-1} + \epsilon_t^n(S_t) \quad (5)$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_t^z \quad (6)$$

where $\epsilon_t^n | S_t \sim NID(0, \sigma_n^2(S_t))$ and $\epsilon_t^z \sim NID(0, \sigma_z^2)$

The regime generating process is an ergodic Markov chain with a finite number of states $S_t = \{1, \dots, M\}$ defined by the following transition probabilities:¹

$$p_{ij} = Pr(S_{t+1} = j | S_t = i) \quad (7)$$

$$\sum_{j=1}^M p_{ij} = 1 \forall i, j \in \{1, \dots, M\} \quad (8)$$

Regime changes in the intercept μ of the trend component can occur following a decline in productivity due to unemployment hysteresis or stronger scrapping of capital during recessions associated with a restructuring of the economy. Similarly, changes in the variance of shocks to trend GDP can be attributed to stronger shocks affecting the economy during recessions. For example Cogley and Sargent (2005), Sims and Zha (2006) and Fernández-Villaverde et al. (2010) emphasize the importance of allowing the variance of the shocks to vary. The prior view is that low growth is associated with large negative shocks. In the set of models estimated below, we consider both changes in the slope together with changes in the variance of the shocks.²

¹See Hamilton (1989) for more details.

²Models with only switches in the variance of the innovations have also been estimated. They are not retained in the paper as likelihood ratio tests do not favor them.

As the level of potential output n_t and the output gap z_t are not observed, the model has to be cast in state-space form before being estimated with the Kalman filter. The inclusion of regime changes in some parameters of the model complicates the estimation since there is an additional latent variable S_t . However, Kim and Nelson (1999b, chapter 5) show how to estimate state-space models with regime switching, i.e. how to combine the Kalman and Hamilton filters in a tractable way. Further details about the estimation are provided in Section 3.1, while Appendix B reports the equations for the Kalman and Hamilton filters.

It is important to note that we only include regime changes in some parameters of the trend equation of GDP since we want to capture possible changes in the level of potential output. Conversely, Kim and Nelson (1999a) include regime switches in the intercept of the cycle equation of GDP and Sinclair (2009) extends their specification by allowing for a correlation between the errors in the trend and the cycle.

In the empirical application, the linear model given by equations (1) to (3) is labeled as MODEL UC-1, the Markov-switching model with only a switch in the intercept of the trend component of GDP is labeled as MODEL UC-2 and the Markov-switching model with a switch in both the drift of the trend component of GDP and its shock variance is labeled as MODEL UC-3.

2.2 Extension to use auxiliary information

Beside the three univariate models described above, we also consider the use of an auxiliary indicator to better estimate the output gap. In the empirical application, the indicator is the rate of capacity utilisation which is often used as a proxy for the cyclical component of GDP. Indeed, if one considers the output gap as the transitory component of GDP, appending an indicator well correlated to the economic activity should provide relevant information for estimating the output gap.

The measurement and transition equations for the bivariate model with GDP and the auxiliary indicator are then respectively given by:

$$\begin{bmatrix} y_t \\ aux_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} n_t \\ z_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_t^{aux} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} n_t \\ z_t \\ z_{t-1} \end{bmatrix} = \begin{bmatrix} \mu(S_t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ z_{t-1} \\ z_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t^n(S_t) \\ \epsilon_t^z \\ 0 \end{bmatrix} \quad (10)$$

where $\epsilon_t^{aux} \sim NID(0, \sigma_{aux}^2)$, $\epsilon_t^n(S_t) \sim NID(0, \sigma_n^2(S_t))$, $\epsilon_t^z \sim NID(0, \sigma_z^2)$ and $\epsilon_t^\kappa \sim NID(0, \sigma_\kappa^2)$.

We consider linear bivariate models (labeled as MODEL MUC-1 (auxiliary)). For the non-linear bivariate models we estimate, we include regime changes in the parameters of the trend equation of GDP in the same way as the univariate modeling: (i) switch in the slope of the trend only (labeled as MODEL MUC-2 (auxiliary)) or (ii) switch in both the slope of the trend and its shock variance (labeled as MODEL MUC-3 (auxiliary)).

2.3 Extension to incorporate a time-varying Phillips curve

We follow Kuttner (1994) and add an equation for inflation along with the trend-cycle decomposition of GDP. We can indeed expect gains by adding an inflation equation to our model since - in theory - inflation is linked to the level of the output gap. Although it is indeed sometimes found that inflation can help to estimate the transitory component of output (see e.g. Kuttner (1994) and Proietti et al. (2007)), there is no clear agreement in the literature. For instance, based on US data, Orphanides and Van Norden (2002) find that multivariate models do not outperform their univariate counterparts.

An additional problem with the Phillips curve specification relates to the well known fact that over the forty years covered in our empirical analysis, the inflation regime has changed. To account for this, we use a time-varying version of the Phillips curve, which is then incorporated in the model of trend-cycle decomposition of GDP. We consider a time-varying Phillips-curve of the form:

$$\pi_t = \kappa_t + \sum_{j=1}^J \lambda_{\pi,j} \pi_{t-j} + \sum_{j=0}^J \lambda_{z,j} z_{t-j} + \sum_{j=1}^J \lambda_{EXR,j} EXR_{t-j} + \sum_{j=1}^J \lambda_{OIL,j} OIL_{t-j} + \epsilon_t^\pi \quad (11)$$

$$\kappa_t = \kappa_{t-1} + \epsilon_t^\kappa \quad (12)$$

where $\epsilon_t^\pi \sim NID(0, \sigma_\pi^2)$, $\epsilon_t^\kappa \sim NID(0, \sigma_\kappa^2)$ and π_t , z_t , EXR_t and OIL_t are the inflation rate, the cyclical component of output,³ the nominal effective exchange rate and the price of oil respectively. The intercept κ_t is modeled as a random walk without drift in order to capture changes in the trend of inflation and can be interpreted as the level of medium term inflation. The other parameters of the model (λ 's, σ_κ^2 and σ_π^2) are kept constant.

Again, as the parameter κ is not constant over time, equations (11) and (12) have to be estimated via maximum likelihood using the Kalman filter. The state-space representation of this model is given by:

$$\pi_t = \kappa_t + \lambda x_t + \epsilon_t^\pi \quad (13)$$

$$\kappa_t = \kappa_{t-1} + \epsilon_t^\kappa \quad (14)$$

where x_t is a matrix of observables and λ its corresponding vector of coefficients.

The measurement and transition equations for the bivariate model of GDP and inflation are instead respectively given by:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \lambda_z & 0 & 1 \end{bmatrix} \begin{bmatrix} n_t \\ z_t \\ z_{t-1} \\ \kappa_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{x}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_t^\pi \end{bmatrix} \quad (15)$$

³We use here the HP filtered cycle as a proxy for the cyclical component of output.

where \mathbf{x}_t is a matrix of explanatory variables and λ its corresponding vector of coefficients.

$$\begin{bmatrix} n_t \\ z_t \\ z_{t-1} \\ \kappa_t \end{bmatrix} = \begin{bmatrix} \mu(S_t) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ z_{t-1} \\ z_{t-2} \\ \kappa_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^n(S_t) \\ \epsilon_t^z \\ 0 \\ \epsilon_t^\kappa \end{bmatrix} \quad (16)$$

where $\epsilon_t^{aux} \sim NID(0, \sigma_{aux}^2)$, $\epsilon_t^n(S_t) \sim NID(0, \sigma_n^2(S_t))$, $\epsilon_t^z \sim NID(0, \sigma_z^2)$ and $\epsilon_t^\kappa \sim NID(0, \sigma_\kappa^2)$.

We consider linear bivariate models labeled as MODEL MUC-1 (inflation). For the non-linear bivariate models we estimate, we include regime changes in the parameters of the trend equation of GDP in the same way as the univariate modeling: (i) switch in the slope of the trend only (labeled as MODEL MUC-2 (inflation)) or (ii) switch in both the slope of the trend and its shock variance (labeled as MODEL MUC-3 (inflation)).

3 In-sample estimates for the euro area

The estimation of the nine models described above is carried out using quarterly data for the euro area as a whole over the period 1970Q1-2010Q4 (i.e. 164 observations). Real GDP is taken from Eurostat and backcasted with the AWM database before 1995Q1. The auxiliary indicator is the rate of capacity utilisation published by the European Commission. It is available for the euro area since 1985Q1 and backcasted with country data before. It is demeaned prior to the estimation. Regarding the variables entering the Phillips curve, the harmonised index of consumer prices (HICP) is taken from Eurostat, the oil price in US dollars from TWI, while the euro US dollar exchange rate and the euro nominal effective exchange rate against its 16 main competitors are taken from BIS data.

All the models are estimated with maximum likelihood. The computations are carried out with the optimization library OPTMUM of GAUSS 9.0.0. selecting the BFGS algorithm. Denoting ω the parameters of the model to be estimated, the algorithm we use is described by the following steps:

- STEP 1: Give initial values to all parameters of the model ω^0 and to the expectation of the state vector and its variance.
- STEP 2: If there is regime switching in at least one parameter of the model, implement the filtering procedure of Kim and Nelson (1999b) for state-space models with regime switching using in the first iteration ω^0 and in the following iterations ω^j . If there is no regime switching, one needs to implement the standard Kalman filter using in the first iteration the initial values for the state vector and its variance, and in the next iterations their updated versions. At the end of Step 2, we thus obtain estimates of the filtered probabilities (if there is regime switching), the state vector and the log-likelihood function.



- STEP 3: Maximize the log-likelihood function to obtain an updated version of the parameters ω^j .
- STEP 4: Iterate over STEPS 2 to 3 until the algorithm has converged.

Hamilton (1994) pointed out that this algorithm is a special case of the EM algorithm: the expectation (E) step is step 2 and the maximization (M) step is step 3. Note that the expectation step aims at formulating guesses about the latent variables (i.e. the unobserved components and the regime probabilities) given the data and the initial or updated values of the parameters, while the maximization step yields the values of the parameters that maximize the log-likelihood over the iterations.

3.1 Time-varying Phillips curve

We first estimate a univariate time-varying Phillips curve without a trend-cycle decomposition model of GDP since model selection would raise difficulties in the multivariate framework.⁴ The specification of the Phillips curve that best fits the data is chosen in a univariate context, using a basic HP filter as a measure of the output gap, before being included and re-estimated jointly with the output gap in the multivariate framework.

We estimate equations (11)-(12) by maximizing the log-likelihood function via the EM algorithm as described in the previous subsection. Inflation is 100 times the quarterly change in consumer prices (HICP) and the output gap is the cycle extracted from the Hodrick-Prescott filter. The exchange rate and oil prices in euro terms are 100 times the quarterly change of their logarithm. For selecting the right number of lags in equation (11), we proceed sequentially: we first estimate a model with four lags for each of the explanatory variables and delete the least significant variables until all coefficients are significant at least at the 10% level.

Table 1 in Appendix D reports the maximum likelihood parameter estimates and their standard errors. First, applying the above criterion to determine the number of lags on each explanatory variable, we select a model with no lagged inflation. We see two explanations for this result: (i) the time-varying parameter can capture part of the significance of lagged inflation (ii) confirmation of the purely forward looking New Keynesian Phillips Curve, which states that current inflation only depends on expected inflation and current marginal cost.⁵ Besides, this result is in line with Hondroyannis et al. (2009), who use time-varying parameter models on data for Germany, France, Italy and the United Kingdom and also favor specifications that exclude lagged inflation. Second, the coefficient entering before the output gap is highly significant and positive: a one percentage point increase in the

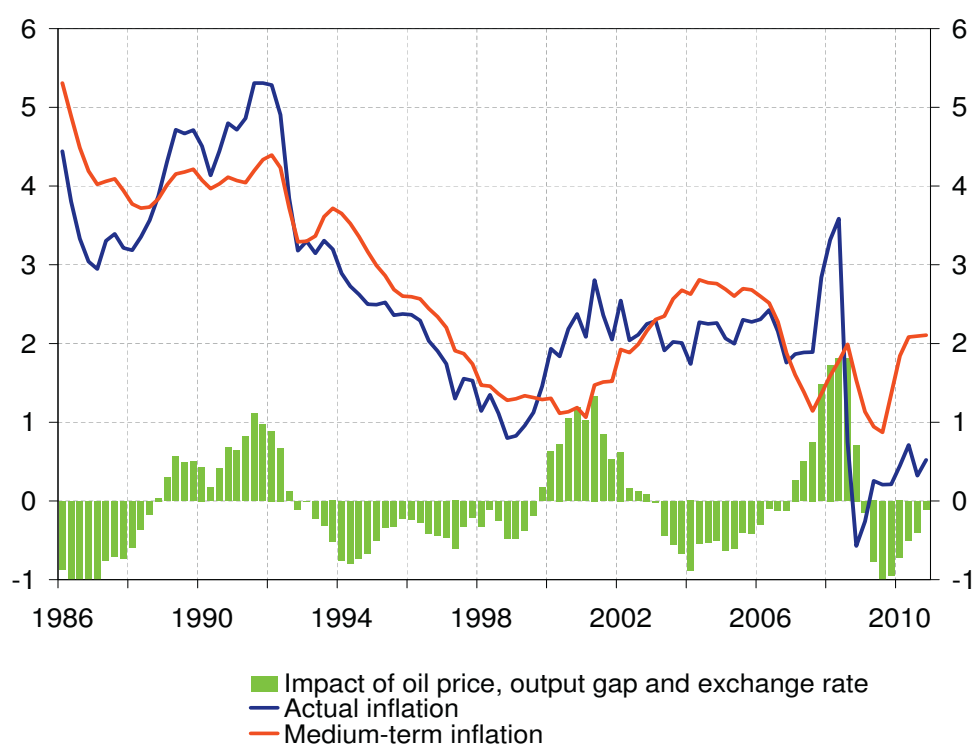
⁴We experimented problems of convergence of our algorithm when we carried out model selection within the bivariate framework with regime switching.

⁵In the estimation of the New Keynesian Phillips curve, current marginal costs are often approximated by the output gap (see e.g. Rudd and Whelan (2007)). However, some argue that unit labor costs should be used as the driving variable in the Phillips curve (see e.g. Galí and Gertler (1999)). This debate is beyond the scope of this paper.

output gap pushes up inflation by 0.15%. This is the lower bound of the estimates reported in the literature. Third, the coefficients on lagged exchange rate and oil price, taken in euros, are both significant and have the expected signs. An appreciation of the euro has a negative impact on inflation, with a 10% appreciation diminishing inflation by about 0.2%. Finally, an increase in oil price has a positive impact on inflation.

Figure 1 shows the time-varying parameter of the Phillips curve, which is interpreted as the level of medium term inflation with actual inflation and the difference between actual and expected inflation. For example, over the most recent period, oil price, the exchange rate and the output gap are estimated to have contributed to annual inflation by almost 2 p.p. at the end of 2008 and around -1 p.p. at the end of 2009.

Figure 1: MODEL DECOMPOSITION OF EURO AREA INFLATION (ANNUAL GROWTH, %)



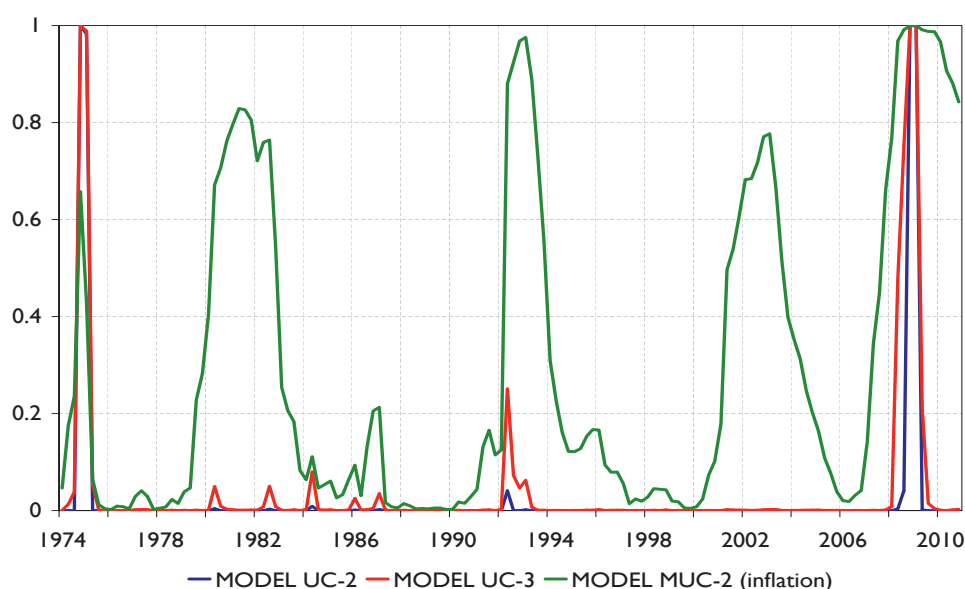
Note: The moving constant corresponds to the time-varying parameter of the univariate Phillips curve (see equation (11)). The difference reflect the cumulated impact of exchange rate, oil price and output gap.

3.2 Univariate and bivariate trend-cycle decomposition of euro area real GDP

We estimate the univariate and bivariate trend-cycle decomposition models of the euro area GDP described in Section 2: the three univariate models and the six bivariate models.⁶ Tables 2, 3 and 4 in Appendix D report the maximum likelihood estimates.

Table 2 in Appendix D shows the results for the univariate models of the trend-cycle decomposition of GDP. First, the regime switching intercepts are highly significant in the two regimes. In addition, both regime switching models increase the log likelihood by about 15 with respect to the linear model.⁷ This points out the relevance of parameter switching in the trend equation of GDP and provides evidence for possible decreases in trend output growth during recessions. Indeed, Figure 2 shows that the probability for a negative intercept for potential output peaks for few periods around 1974 and 2009.

Figure 2: SMOOTHED PROBABILITIES OF BEING IN THE FIRST REGIME



Note: MODEL UC-2 is the model with a switch in the slope of the trend only. MODEL UC-3 is the model with in the slope of the trend and its error variance. MODEL MUC-2 (inflation) is a bivariate model with an equation for inflation and a switch in the slope of the trend of GDP only.

Table 3 in Appendix D reports the results for the models using the demeaned rate of capacity utilisation as an auxiliary indicator to better estimate the output gap. The

⁶In all the models, stationarity constraints on the parameters ϕ_1 and ϕ_2 and positive definiteness constraints on the variance parameters of the innovations were imposed. Standard deviations were computed from the inverse of the outer product estimate of the Hessian.

⁷However, the improvement in the log-likelihood cannot be tested. A standard likelihood ratio test cannot be implemented since (i) the transition probabilities are not identified and (ii) the scores of the log likelihood are identically equal to zero under the null hypothesis of no regime switching.

coefficients for the auxiliary indicator are highly significant, which shows its relevance to estimate the output gap.

The results for the bivariate models using inflation as an extra variable in the system are reported in Table 4 of Appendix D, while the resulting medium-term inflation is represented in Figure D. In the equation for inflation, we include one lag for the output gap, the exchange rate and the oil price following the results obtained in the previous subsection. As a robustness check, we also include one lag for inflation even if it is not significant in the univariate analysis (the state-space representation of the model is given by equations (15) and (16)). The parameter estimates for the coefficients of the exchange rate, oil price and output gap are similar across all specifications and consistent with the results obtained in the univariate analysis (see Table 1 in Appendix D). The coefficient for lagged inflation is not significant at the 5% level, in line with the results obtained in the univariate analysis (except for the model MUC-3 (inflation)).

Figure 2 shows the probability of a low trend for GDP growth. A high probability of this regime is associated with all the recession episodes recorded in the euro area over the estimation period: 1982-1984, 1992-1993 and 2008-2009.⁸ However, there is less evidence for regime switching for the models using inflation since the increase in the log-likelihood for the regime switching models is modest with respect to the linear model (see the last row of Table 4).

3.3 Comparison of estimated output gaps and model-averaged measures

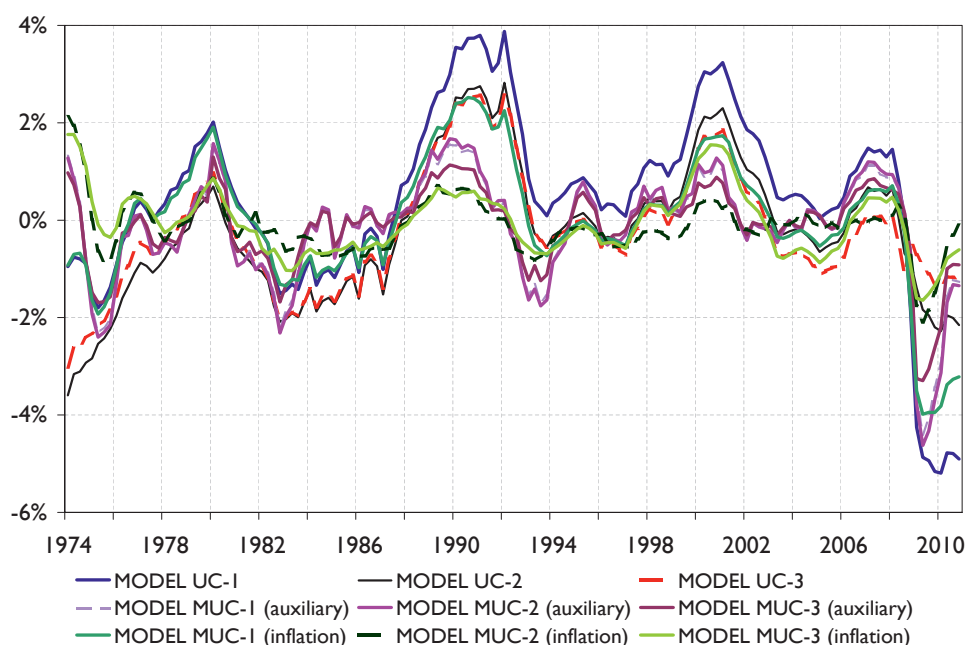
Figure 3 shows the output gaps estimated for the nine models under scrutiny. There are important differences between the estimates of the output gap, with a range max-min between the estimates of the output gap reaching high levels during the two important recessions identified in the sample: 4% in the beginning of 1992 and 5% in the beginning of 2010.

The output gaps estimated from the univariate models differ depending on whether there is regime switching or not. In particular, the output gap estimated from a linear univariate model captures well the expansions and recessions experienced by the euro area. However, the univariate regime switching models estimate a smaller negative output gap for the last recession, which suggests that the last recession also affected the level of potential output. The models with inflation tend to yield smoother estimates of the output gap and therefore allocate more variation to the trend of output. Conversely, the models with the rate of capacity utilisation as an auxiliary indicator are very close to each other. They closely match the evolution of the euro area economic activity, and therefore allocate little variation to the level of potential output.

In the forecasting literature, it is often found that combining forecasts from different models allows to improve the forecasts from individual models (see e.g. Drechsel and

⁸See Figure A in Annex D for a plot of the nine estimates of euro area potential output over the entire estimation period.

Figure 3: ESTIMATES OF EURO AREA OUTPUT GAP DERIVED FROM THE UNOBSERVED COMPONENTS MODELS ESTIMATED



Note: MODEL UC-1 is the linear univariate trend-cycle decomposition of GDP, MODEL UC-2 is the univariate trend-cycle decomposition of GDP with a switch in the slope of the trend only and MODEL UC-3 is the univariate trend-cycle decomposition of GDP with a switch in both the slope of the trend of GDP and its error variance. MODEL MUC-1 (auxiliary) is the linear bivariate model with the trend-cycle decomposition of GDP and an equation for capacity utilisation, MODEL MUC-2 (auxiliary) is the bivariate model with capacity utilisation and a switch in the slope of the trend of GDP and MODEL MUC-3 (auxiliary) is the bivariate model with capacity utilisation and a switch in the slope of the trend of GDP and its error variance. MODEL MUC-1 (inflation) is the linear bivariate model with the trend-cycle decomposition of GDP and an equation for inflation, MODEL MUC-2 (inflation) is the bivariate model with inflation and a switch in the slope of the trend of GDP and MODEL MUC-3 (inflation) is the bivariate model with inflation and a switch in the slope of the trend of GDP and its error variance.

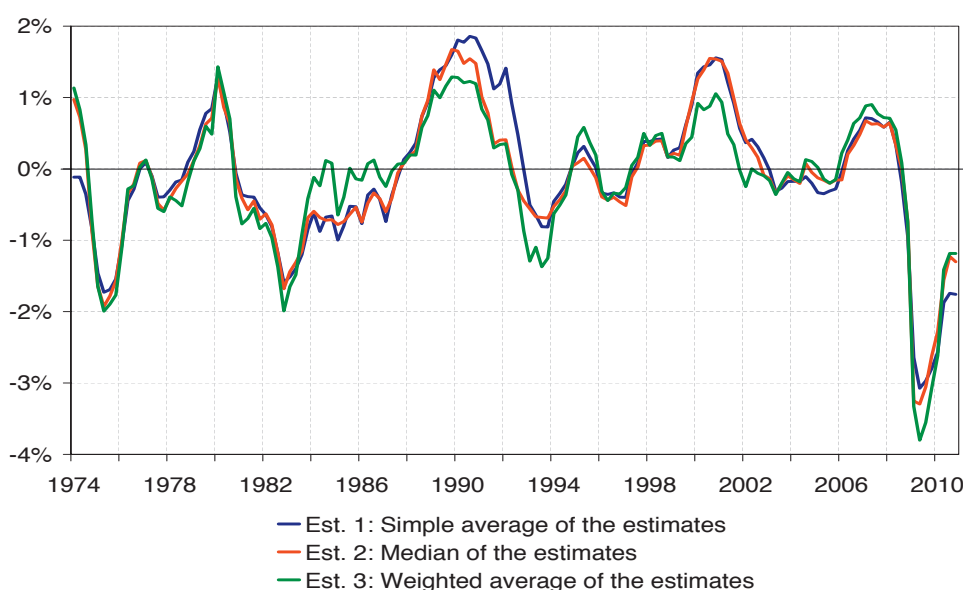
Maurin (2011)). This is particularly relevant for estimating the output gap since the model estimates are characterized by both model uncertainty and parameter instability. Therefore, we compute three different model-averaged measures of the output gap: (i) one measure obtained as the simple arithmetic average over each of the nine models, labeled EST. 1, (ii) one measure obtained as the median estimate over each of the nine models, labeled EST. 2, (iii) the last measure takes into account the uncertainty in the estimation of the output gap and is labeled EST. 3. In particular, the latter measure gives higher weights $w_t(l)$ to the models with smaller variances attached to the estimated output gaps:

$$w_t(l) = \frac{[V_t(z_t^{(l)})]^{-1}}{\sum_{l=1}^L [V_t(z_t^{(l)})]^{-1}} \quad (17)$$

where $w_t(l)$ are the weights given to model l at time t , $z_t^{(l)}$ is the output gap from model l at time t and $V_t(z_t^{(l)})$ its corresponding variance estimated from the Kalman filter. In this way, the weights are time-varying, positive and sum to one.

Figure 4 plots the three model-averaged measures. The model-averaged measure with time-varying weights (labelled as "EST. 3") is more cyclical since it gives more weights to the models using the demeaned rate of capacity utilisation, which yield more precise estimates of the output gap (i.e. with a smaller variance). In particular, focussing on the most recent period, the amplitude of the model-averaged output gap estimates is largely reduced compared to the one of the initial nine estimates (from between -0.5 and -4.8 p.p. to between -1.2 p.p. and -1.8 p.p. at the end of 2010). The three model-averaged measures are overall fairly close unlike the estimates from the individual models, which shows the relevance of combining individual model estimates to obtain more reliable estimates of the output gap.

Figure 4: MODEL AVERAGED MEASURES OF THE NINE ESTIMATES



4 Do real-time estimates of the output gap improve inflation forecasts?

In this section, we assess the usefulness of our different output gap measures to predict inflation and also investigate the importance of data revisions for the predictive power of

the output gap. We first compute the real-time estimates of the estimated output gap measures. We then evaluate the usefulness of our different output gap measures to predict inflation and also investigate the importance of data revisions for the predictive power of the output gap for inflation. The Clark and McCracken (2009a) test of equal forecast accuracy is then implemented to compare the forecasting performances.

4.1 Real-time estimates of the output gap

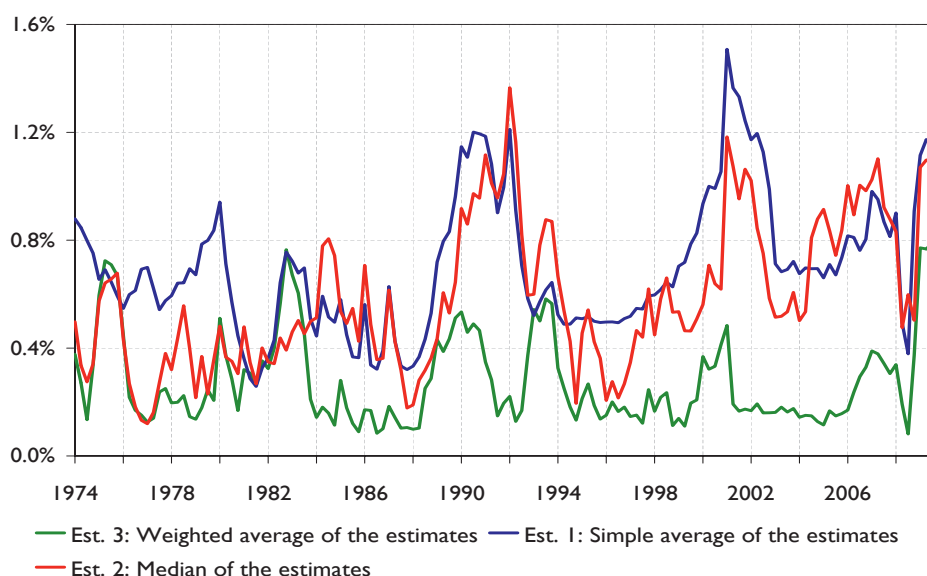
It has long been advocated that there are severe differences between the real-time estimates of the output gap and their final vintages counterparts (see e.g. Orphanides and Van Norden (2002)). We use the first releases of GDP to construct the real-time measures of the euro area output gap. This estimate is published around 60 days after the end of the reference quarter. The first estimation sample goes from $t=1970Q1$ to $t=2001Q1$. The sample is then recursively expanded until we reach the end of the estimation sample $T=2010Q4$. We therefore obtain 40 different vintage series for the output gap, each of them being associated with a different date for its final observation (i.e. from $T=2001Q1$ to $T=2010Q4$). We run the pseudo real-time estimation exercise for the univariate trend-cycle decomposition models of GDP (i.e. MODEL UC-1, MODEL UC-2, MODEL UC-3) and for the bivariate model with capacity utilisation⁹ as an auxiliary indicator (i.e. MODEL MUC-1 (auxiliary), MODEL MUC-2 (auxiliary), MODEL MUC-3 (auxiliary)).¹⁰ We also combine the individual estimates of the output gap in the three model-averaged measures detailed above.

Figure 5 plots the range of revisions for each of the three different model-averaged measures, while Figures A, B and C in Appendix D plot all the individual measures. In line with Orphanides and Van Norden (2002), we indeed find that the estimation of the output gap in real-time is associated with a large uncertainty. Figure 5 also shows that the equal weights and the median measures are associated with large revisions as the path of the output gap is changing significantly across the different vintages used (see also Figures A and B in the appendix), this is particularly acute prior to important economic downturns. Focussing on the measure based on equal weights, the maximum revision change amounts to more than 1 p.p. around 1990-1992, around 1.5 p.p. around 2001-2002. At the end of 2010, 1 year after the first estimate, the estimates of the output gap for the end of 2009 has already change by more than 1 p.p. Conversely, the measure that gives time-varying weights depending on the uncertainty associated with the output gap is considerably less affected by GDP data revisions. This comes from the fact that this measure gives heavy weights to the output gaps estimated with the demeaned rate of capacity utilisation, which have a smaller variance than the output gaps estimated from univariate models.

⁹The rate of capacity utilisation is not revised over time.

¹⁰The bivariate models with an equation for inflation are not included since we encountered problems of convergence of the algorithm in the real-time exercise.

Figure 5: ESTIMATES OF OUTPUT GAP IN REAL TIME: RANGE OF REVISIONS, (MAX-MIN)



4.2 Inflation forecasts

The predictive ability of the output gap for forecasting inflation is contrasted as it seems that the relation between inflation and output gap has weakened since the mid-1980s. Atkeson and Ohanian (2001) find that Phillips curve forecasts do not outperform simple univariate benchmarks. Stock and Watson (2008) extensively study Phillips curve forecasts using different sample periods, inflation series and benchmarks. They find that the Phillips curve predictive abilities are rather episodic and depend upon the evaluation sample chosen. Orphanides and van Norden (2005) also find that the forecasting performance of the output gap is unstable over time and point out the discrepancies between inflation forecasts based on real-time estimates of the output gap and their ex-post counterparts. However, the output gap - as a measure of economic slackness - is conceptually an intuitive predictive variable for inflation. Indeed, the triangle model of Gordon (1997) states that inflation depends on lagged inflation, the unemployment rate and supply shock variables.

The present forecasting exercise aims at assessing the predictive power of the output gap for forecasting inflation using the real-time estimates of the output gap as well as the ex-post estimates obtained from the last vintage of data available to us (T=2010Q4). The inflation forecasts are computed for horizon varying from 1 quarter ahead to 2 years ahead. For each horizon, the Clark and McCracken (2009a) test of equal forecast accuracy is implemented to compare the forecasting performances.¹¹

¹¹The test of equal forecast accuracy with real-time and revised data is described in detail in Appendix C.

We first consider the specification described in Orphanides and van Norden (2005) and forecast the level of inflation:

$$\pi_{t+h}^{(h)} = \alpha + \sum_{k=1}^P \beta_k \pi_{t-k} + \sum_{j=1}^J \gamma_j x_{t-j,\tau}^{(l)} + \epsilon_{PC,t+h} \quad (18)$$

The benchmark model is an AR(p) model for the level of inflation:

$$\pi_{t+h}^{(h)} = \alpha + \sum_{k=1}^P \beta_k \pi_{t-k}^{(1)} + \epsilon_{AR,t+h} \quad (19)$$

We also follow Stock and Watson (1999), Clark and McCracken (2009a) and use a Phillips curve for forecasting the change in inflation:

$$\pi_{t+h}^{(h)} - \pi_t = \alpha + \sum_{k=1}^P \beta_k \Delta \pi_{t-k} + \sum_{j=1}^J \gamma_j \Delta x_{t-j,\tau}^{(l)} + \epsilon_{PC,t+h} \quad (20)$$

The benchmark model is instead defined as:

$$\pi_{t+h}^{(h)} - \pi_t = \alpha + \sum_{k=1}^P \beta_k \Delta \pi_{t-k} + \epsilon_{AR,t+h} \quad (21)$$

where: $\pi_{t+h}^{(h)} = (\frac{400}{h}) \ln(\frac{p_{t+h}}{p_t})$, $\pi_t = 400 \ln(\frac{p_t}{p_{t-1}})$, and $x_{t,\tau}^{(l)}$ is a real-time measure of the output gap from model l at time t using data for GDP from the data vintage τ and $\Delta x_{t,\tau}^{(l)}$ is its quarterly difference.

The design of the pseudo-out-of-sample forecasting exercise is the following. The forecasts are computed with the direct method and the maximum lag lengths P and J are chosen with the SIC (maximum lag of 8) using the first estimation sample of the recursive forecasting exercise.¹² We do not select recursively the number of lags since the Clark and McCracken (2009a) test of equal predictive accuracy with real-time data requires the number of parameters to be constant within each forecasting experiment. The real-time measures of the output gap are obtained from the previous subsection. We estimate equations (18)-(21) with OLS and compute for a given model i with forecast error $\hat{u}_{i,t+\tau}$ the mean squared forecast error (MSE_i):

$$MSE_i = (P - \tau + 1)^{-1} \sum_{t=R}^T \hat{u}_{i,t+\tau}^2$$

where R is the initial forecast origin and $(P - \tau + 1)$ is the number of forecast errors. The actual value for inflation is taken from the last vintage of data available to us (i.e.

¹²Selecting the lag length recursively or using the AIC rather than the SIC does not change qualitatively the results.

T=2011Q1). We compute forecasts one-quarter-ahead ($h = 1$), two-quarter-ahead ($h = 2$), one-year-ahead ($h = 4$) and two-year-ahead ($h = 8$).

A few additional comments are required. Note first that we do not consider additional explanatory variables in the Phillips curve equations (19) and (21) since we explicitly focus our analysis on the predictive power of the output gap for inflation and consider inflation excluding energy. Second, we only use real-time data for our output gap measures since we want to concentrate our analysis on the importance of data revisions to GDP for the predictive power of the output gap for inflation. Data revisions to HICP excluding energy - our measure of inflation here - are usually very small and are unlikely to affect our results. In this respect, we follow Orphanides and Van Norden (2005). Third, we consider two evaluation samples for our forecasting exercise 2001-2007:Q4 and 2001-2010:Q4 in order to assess the impact of the last recession on our results. Finally, we do not use the model with a time-varying constant in the forecasting exercise since it brings an additional source of uncertainty in the model that could cloud the interpretation of the results on the forecasting performance of the output gap.

Table A reports the forecasting results for the forecasts of the level of inflation. The benchmark model yields better forecasts than the Phillips curve specifications based on the real-time estimates of the output gap for forecasting the level of inflation for one-quarter-ahead forecasts (see Panel A of Table A). Conversely, the models with the real-time estimates of the output gap improve the forecasting performance of the benchmark model for forecasting horizons $h = \{2, 4, 8\}$ (except for the UC-3, MUC-1, MUC-2, MUC-3 and Est.3 models with $h = 2$). However, the improvement in forecasting performance seems to be only of marginal importance since the increase in MSE is always lower than 20% and often inferior to 10%. Besides, the Clark and McCracken (2009a) test of equal predictive ability with real-time data cannot reject the null hypothesis of equal forecast accuracy except for the UC-1 and UC-2 models at the forecasting horizon $h = 4$ and $h = 2$ respectively. Interestingly, excluding the 2008-2010 period from the evaluation sample improves the forecasting performance for all models for one-quarter-ahead forecasts. In addition, nearly all models with forecast horizons $h = 8$ statistically improve the forecasts with respect to the benchmark model (except for the Est. 3 model) (see Panel C of Table A).

Besides, using the ex-post estimates worsens the one-quarter-ahead forecasts and do not clearly improve forecasts for forecast horizon $h > 1$ with respect to the forecasts that use the real-time estimates of the output gap (see Panel B and D of Table A). However, the improvement in forecasting performance is often significant when using the ex-post estimates of the output gap for forecast horizons $h = 2$ and $h = 8$. The p-values are computed from the Clark and McCracken (2005) test of equal forecast accuracy, which is described in Appendix C.

Table 5 in the appendix reports the forecasting results for the change in inflation. First, none of the models with the real-time estimates of the output gap as a predictor can outperform the benchmark model for forecasting the change in inflation (see Panel A of Table 5). This is particularly acute for one-quarter-ahead predictions for the output gap models using capacity utilisation as an extra indicator. The reason for this poor forecasting

Table A: Forecast comparison exercise: results for inflation (HICP excluding energy)

<i>Model</i>	UC-1	UC-2	UC-3	MUC-1 (auxiliary)	MUC-2 (auxiliary)	MUC-3 (auxiliary)	Est. 1	Est. 2	Est. 3
<i>Panel A. Real-time output gap series, 2001-2010</i>									
h=1	1.777	1.292	1.088	2.662	2.520	2.350	2.234	1.559	2.551
h=2	0.844	0.864 ^(b)	1.035	1.227	1.185	1.145	1.170	0.833	0.933
h=4	0.814 ^(a)	0.862	0.993	0.978	0.969	0.975	0.970	0.906	0.955
h=8	0.947	0.928	0.959	0.983	0.983	0.984	0.984	0.966	1.000
<i>Panel B. Ex-post output gap series, 2001-2010</i>									
h=1	1.781	1.420	1.427	2.953	2.881	2.725	2.638	1.909	2.660
h=2	0.804 ^(c)	0.923 ^(b)	0.925 ^(b)	1.104	1.065	1.065	1.078	0.820 ^(c)	0.891 ^(b)
h=4	0.797 ^(b)	0.936	0.939	0.970	0.962	0.968	0.959	0.884 ^(b)	0.900 ^(b)
h=8	0.932 ^(a)	0.936 ^(a)	0.936	0.979	0.977	0.980	0.978	0.953	0.967
<i>Panel C. Real-time output gap series, 2001-2007</i>									
h=1	1.247	1.551	1.081	1.199	1.202	1.234	1.237	1.206	1.478
h=2	0.829	1.018	1.017	0.738	0.727	0.717	0.721	0.817	0.774
h=4	0.903	1.001	0.998	0.923	0.919	0.923	0.928	0.957	0.967
h=8	0.850 ^(c)	0.804 ^(c)	0.823 ^(c)	0.829 ^(c)	0.829 ^(c)	0.829 ^(c)	0.831 ^(c)	0.861 ^(c)	0.905
<i>Panel D. Ex-post output gap series, 2001-2007</i>									
h=1	1.474	1.472	1.477	1.688	1.765	1.602	1.693	1.692	1.609
h=2	0.829 ^(c)	0.828 ^(c)	0.826 ^(c)	0.556 ^(c)	0.532 ^(c)	0.564 ^(c)	0.549 ^(c)	0.770 ^(c)	0.771 ^(c)
h=4	0.925 ^(a)	0.904 ^(a)	0.903 ^(a)	0.918	0.917	0.921	0.916	0.939 ^(a)	0.935 ^(a)
h=8	0.801 ^(a)	0.789 ^(b)	0.788 ^(b)	0.830 ^(a)	0.831 ^(a)	0.830 ^(a)	0.832 ^(a)	0.830 ^(a)	0.842 ^(a)

Note: Ratio of the mean squared forecast error between the forecasts obtained from a Phillips curve equation with a real-time measure of the output gap as a proxy for the activity-based measure and a benchmark model given by an $AR(p)$. Est. 1, Est. 2 and Est. 3 are the model averaged measures detailed in the text. The superscripts ^a, ^b and ^c indicate that the test of equal forecast accuracy rejects respectively the null hypothesis of equal forecast accuracy at significance levels of 10%, 5% and 1% level. Appendix C details the Clark and McCracken (2009a) test for real-time data and the Clark and McCracken (2005) test for equal forecast accuracy with revised data.

performance is that these models estimate a very negative output gap for the 2008-2010 period, which translates into very low or negative forecasts for the change in inflation. Indeed, if we exclude the 2008-2010 period from the evaluation sample, the one- and two-quarter-ahead forecasts do improve with respect to the full evaluation sample 2001-2010, although they do not beat the autoregressive benchmark or not significantly (see Panel C

of Table 5)¹³. Second, using the ex-post rather than the real-time estimates of the output gap does not clearly improve the forecasts for the change in inflation (see Panel B and Panel D of Table 5).

The evidence on the importance of the output gap for predicting inflation is therefore contrasted. Indeed, the real-time measures of the output gap do improve the forecasts for the level of inflation for forecasting horizons $h = \{2, 4, 8\}$ but this improvement is mostly statistically insignificant. Besides, the forecasts for the change in inflation based on the real-time estimates of the output gap are always outperformed by a standard autoregressive benchmark for the change in inflation when using the full evaluation sample. The use of the ex-post estimates of the output gap does not clearly improve forecasts for forecasting both the change and the level of inflation with respect to the forecasts based on the real-time output gap estimates.

5 Concluding remarks

This paper estimates various trend-cycle decomposition models of the euro area GDP using state-space models. We consider univariate and multivariate models as well as linear and non linear models. Non linearities are modelled with regime changes in the intercept of the trend equation and/or in the variance of its innovation. Multivariate models consider alternatively inflation and the demeaned rate of capacity utilisation as additional variables in the system to better estimate the output gap. The univariate non linear specifications point out evidence for regime changes in the slope of the trend equation for GDP for few periods around 1974 and 2009. Besides, the demeaned rate of capacity utilisation proves to be useful for obtaining more reliable estimates of the output gap by reducing the uncertainty. With this model, we also find some evidence of regime changes in the slope of the trend of the euro area GDP for few periods, around 1974 and since 2008. We also conduct a real-time analysis for computing real-time estimates of the output gap and found that model averaging techniques improve the reliability of the potential output estimates in real time. Indeed, our model-averaged estimates of the output gap decrease the uncertainty surrounding the output gap estimates and soften the impact of data revisions.

We then run a pseudo out-of-sample forecasting experiment to forecast the level and the change in inflation using both ex-post and real-time estimates of the output gap. We find that our output gap measures help forecasting inflation over most of the sample but fail dramatically since the last recession.

One possible avenue for further research on this topic would be to exploit the regime changes in the variance-covariance matrix of the innovations of the measurement and transition equations in order to obtain identification of more complicated trend-cycle decomposition models of the output gap. This could be done along the lines of Rigobon (2003) and Lanne et al. (2010). Alternatively, it would also be interesting to estimate models with mixed-frequency data to provide monthly estimates of the output gap and evaluate whether this provides more relevant information, in terms of accuracy and/or timeliness.

¹³Using the level of the output gap rather than the change in the output gap in equation (20) worsens the forecasting results.

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Appendix A: State-space representation of the original model of trend-cycle decomposition of output

$$\begin{bmatrix} y_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_t \\ z_t \\ z_{t-1} \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} n_t \\ z_t \\ z_{t-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ z_{t-1} \\ z_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t^n \\ \epsilon_t^z \\ 0 \end{bmatrix} \quad (23)$$

Appendix B: Kalman filter, Hamilton filter and Kim and Nelson filtering procedure

The equations of the basic Kalman filter can be found in a standard time series textbook such as Luetkepohl (2005). The general representation for a state-space model with regime switching in both measurement and transition equations is given by:

$$\begin{aligned} y_t &= H(S_t)\beta_t + A(S_t)z_t + e_t \\ \beta_t &= \tilde{\mu}(S_t) + F(S_t)\beta_{t-1} + G(S_t)v_t \end{aligned}$$

where: $e_t \sim N(0, R(S_t))$, $v_t \sim N(0, Q(S_t))$, and e_t and v_t are not correlated.

Kim and Nelson (1999b) show how to combine the Kalman and Hamilton filters in a tractable way, the equations of the Kim and Nelson (1999b) filtering procedure for state-space models with regime switching are:

$$\begin{aligned} \beta_{t|t-1}^{(i,j)} &= \tilde{\mu}_j + F_j \beta_{t-1|t-1}^i \\ P_{t|t-1}^{i,j} &= F_j P_{t-1|t-1}^i F_j' + G Q_j G_j' \\ \eta_{t|t-1}^{(i,j)} &= y_t - H_j \beta_{t|t-1}^{(i,j)} - A_j z_t \\ f_{t|t-1}^{(i,j)} &= H_j P_{t|t-1}^{(i,j)} H_j' + R_j \\ \beta_{t|t}^{(i,j)} &= \beta_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} H_j' [f_{t|t-1}^{(i,j)}]^{-1} \eta_{t|t-1}^{(i,j)} \end{aligned}$$

$$P_{t|t}^{(i,j)} = (I - P_{t|t-1}^{(i,j)} H_j' [f_{t|t-1}^{(i,j)}]^{-1} H_j) P_{t|t-1}^{(i,j)}$$

When there is regime switching, it is also necessary to introduce approximations at the end of the Kalman and Hamilton filters to avoid the proliferation of cases to be considered:

$$\beta_{t|t}^j = \frac{\sum_{i=1}^M Pr[S_{t-1} = i, S_t = j | \Psi_t] \beta_{t|t}^{(i,j)}}{Pr[S_t = j | \Psi_t]}$$

$$P_{t|t}^j = \frac{\sum_{i=1}^M Pr[S_{t-1} = i, S_t = j | \Psi_t] \{P_{t|t}^{(i,j)} + (\beta_{t|t}^j - \beta_{t|t}^{(i,j)})(\beta_{t|t}^j - \beta_{t|t}^{(i,j)})'\}}{Pr[S_t = j | \Psi_t]}$$

Appendix C: Clark and Mc Craken tests for comparing forecasting performance

We first detail the test of equal forecast accuracy with real-time data.

Denote P the number of forecasts, R the sample size at the initial forecast origin, T the full sample size, τ the forecast horizon, $\hat{u}_{2,t+\tau}$ the forecast error in model 2, \bar{d} the squared forecast loss differential between model 1 and model 2, k_1 the number of parameters in the benchmark model (i.e. model 1), k_{22} the number of excess parameters in model 2.

The Clark and McCracken (2009a) test statistic S for comparing predictive accuracy for nested models with real-time data is given by:

$$S = \frac{P^{\frac{1}{2}} \bar{d}}{\sqrt{\Omega}}$$

Under the null hypothesis of equal predictive accuracy:

$$S \xrightarrow{A} N(0, 1)$$

This differs from Clark and McCracken (2005), where simulated critical values are required in the tests of equal predictive accuracy for nested models. The use of real-time data instead strongly changes the asymptotic for these tests and allows to use standard normal tables for inference as long as we can obtain an asymptotically valid long run variance for Ω . A consistent asymptotic long run variance of the scaled forecasting loss differential Ω is:

$$\Omega = 2(1 - \pi^{-1} \ln(1 + \pi)) \mathbf{F}(-\mathbf{J}\mathbf{B}_1\mathbf{J}' + \mathbf{B}_2) \mathbf{S}_{hh}(-\mathbf{J}\mathbf{B}_1\mathbf{J}' + \mathbf{B}_2) \mathbf{F}'$$

where:

$$\pi = \frac{P}{R}$$

$$\mathbf{J}' = (\mathbf{I}_{k_1 \times k_1}, \mathbf{0}_{k_1 \times k_{22}})$$

$$\hat{\mathbf{B}}_i = (T^{-1} \sum_{s=1}^{T-\tau} \mathbf{x}_{i,s} \mathbf{x}'_{i,s})^{-1}$$

$$\hat{\mathbf{F}} = 2[P^{-1} \sum_{t=R}^T \hat{u}_{2,t+\tau} \mathbf{x}'_{2,t}]$$

The long run variance $\hat{\mathbf{S}}_{hh}$ is obtained by weighting the relevant leads and lags of Γ_{hh} following Newey and West's (1987) HAC estimator with a bandwidth of 2τ , where

$$\hat{\Gamma}_{hh}(j) = E \mathbf{h}_{t+\tau} \mathbf{h}'_{t+\tau-j}$$

and

$$\mathbf{h}_{t+\tau} = (y_{s+\tau} - \mathbf{x}_{2,s} \boldsymbol{\beta}_{2,T}) \mathbf{x}_{2,s}$$

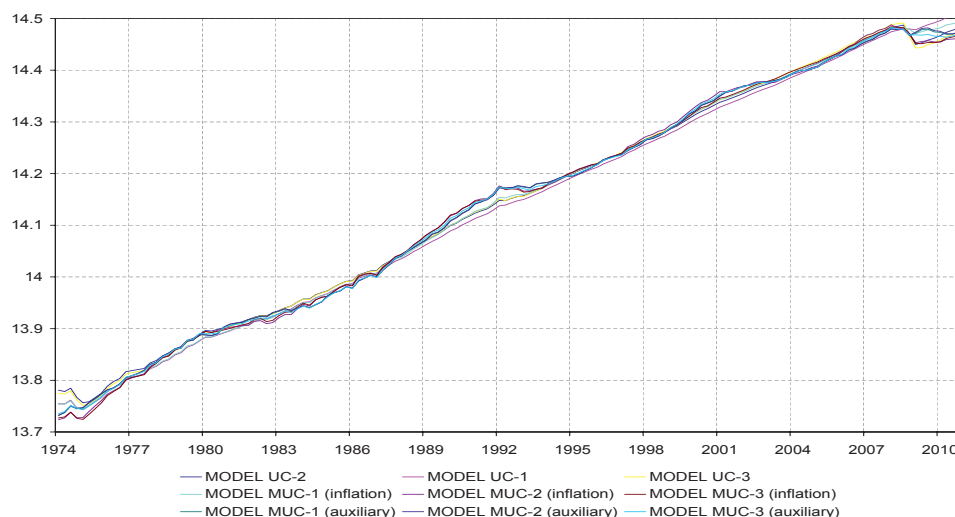
We also compute the Clark and McCracken (2009a) MSE-F test statistic for equal predictive ability of two nested models with revised data as follows:

$$MSE - F = \frac{\sum_{t=R}^{T-\tau} \hat{d}_{t+\tau}}{MSE_2}$$

where $\hat{d}_{t+\tau}$ is the difference between the squared forecast errors $\hat{d}_{t+\tau} = \hat{u}_{1,t+\tau}^2 - \hat{u}_{2,t+\tau}^2$, and MSE_2 is the mean squared forecast error of model 2. We implement the novel bootstrapping procedure described in Clark and McCracken (2009b) and compute the p-values from 1000 replications.

Appendix D: Estimation results

Figure A: EURO AREA POTENTIAL OUTPUT FROM THE NINE ESTIMATES, 1974Q1-2010Q4



Note: Logarithm - MODEL MUC-1 (inflation) is the linear bivariate model with the trend-cycle decomposition of GDP and an equation for inflation, MODEL MUC-2 (inflation) is the bivariate model with inflation and a switch in the slope of the trend of GDP and MODEL MUC-3 (inflation) is the bivariate model with inflation and a switch in the slope of the trend of GDP and its error variance.

Figure B: EURO AREA OUTPUT GAP, REAL-TIME DATA, MODEL-AVERAGED MEASURES WITH EQUAL WEIGHTS, 1974Q1-2010Q4

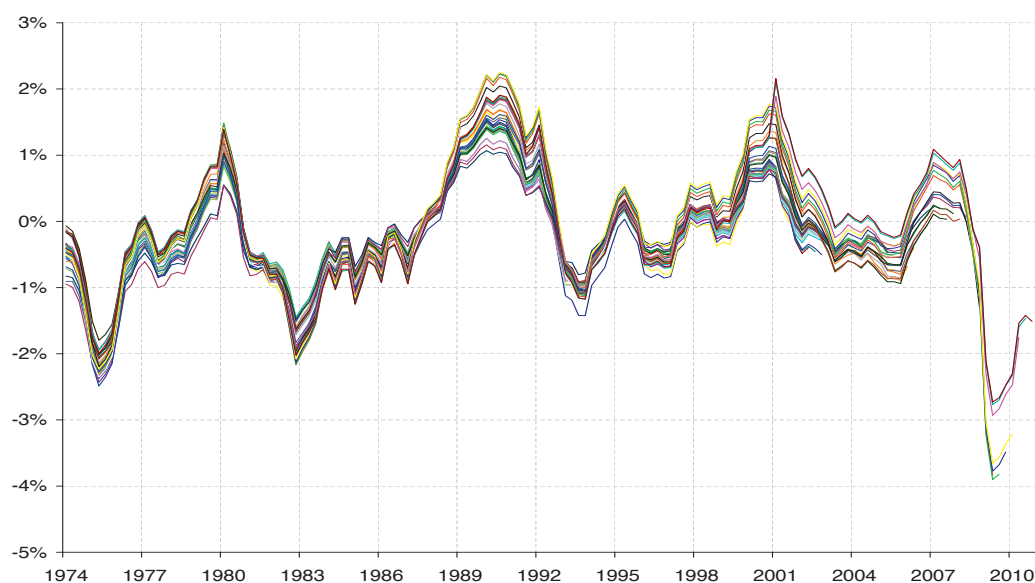


Figure C: EURO AREA OUTPUT GAP, REAL-TIME DATA, MODEL-AVERAGED MEASURES COMPUTED AS THE MEDIAN OF THE INDIVIDUAL OUTPUT GAPS, 1974Q1-2010Q4

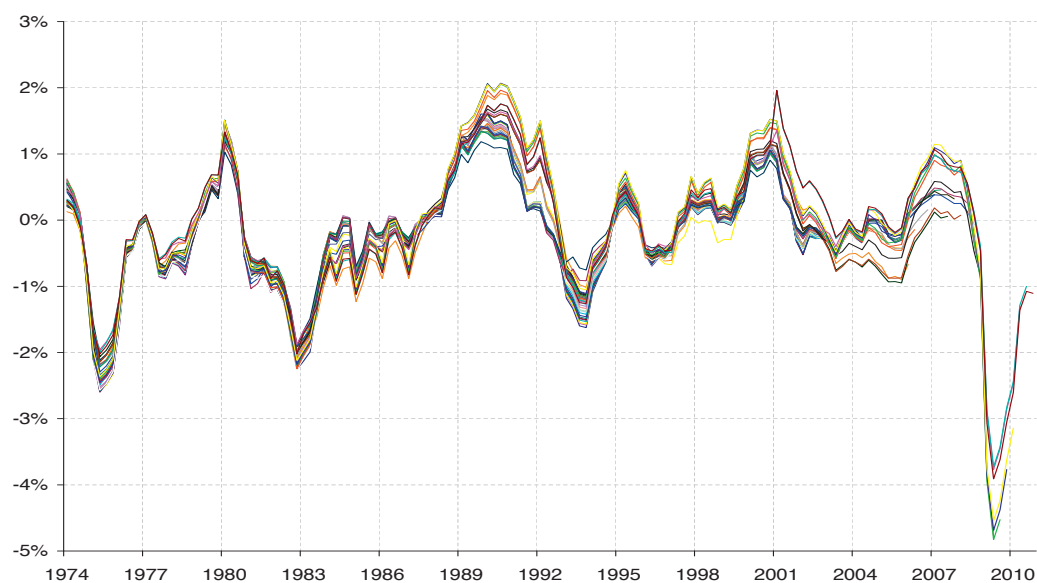


Figure D: EURO AREA OUTPUT GAP, REAL-TIME DATA, MODEL-AVERAGED MEASURES WITH TIME-VARYING WEIGHTS, 1974Q1-2010Q4

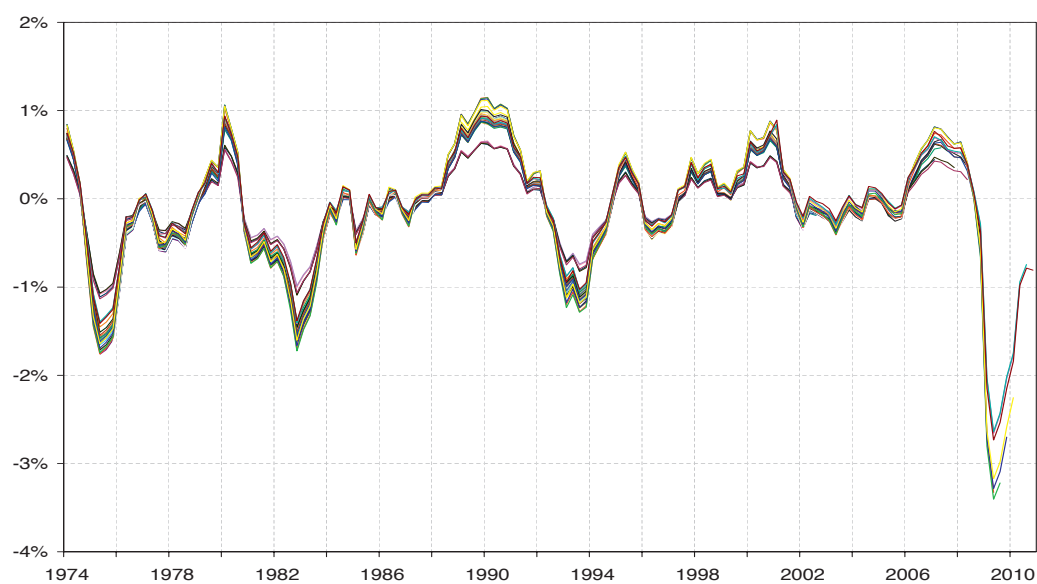
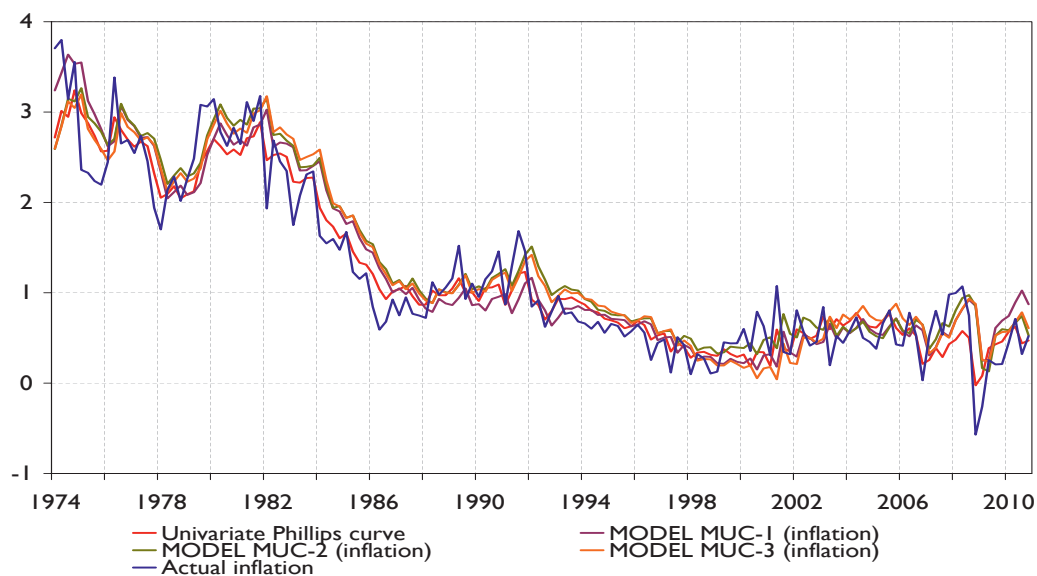


Figure E: ACTUAL INFLATION AND MEDIUM-TERM INFLATION FROM UC AND MUC MODELS



Note: See footnote Chart A in the same appendix.

Table 1: Time-varying Phillips curve

z_t	0.149*** [0.033]
EXR_{t-1}	-0.017* [0.009]
OIL_{t-1}	2.838×10^{-3} * [1.500×10^{-3}]
σ_π^2	0.227*** [0.019]
σ_μ^2	0.146*** [0.022]

Note: Maximum likelihood estimates of the time-varying Phillips curve (see Equations 11 and 12). We imposed positive definiteness constraints on the variance parameters of the innovations. The measure for the output gap z_t is the cycle computed by the HP filter. ***, ** and * indicate significance at 1%, 5% and 10%. Standard deviations are reported in brackets and are computed from the inverse of the outer product estimate of the Hessian. $\text{Log}(L)$, the value of the log likelihood function is -40.1.

Table 2: Univariate model

	Model UC-1	Model UC-2	Model UC-3
p_{11}	-	0.491*** [0.038]	0.568** [0.098]
p_{22}	-	0.986*** [0.010]	0.982*** [0.014]
ϕ_1	1.236*** [0.032]	1.194*** [0.116]	1.205*** [0.026]
ϕ_2	-0.292*** [0.046]	-0.237*** [0.114]	-0.251*** [0.037]
μ_1	$5.147 * 10^{-3}$ *** [$0.227 * 10^{-3}$]	$-17.0 * 10^{-3}$ *** [$3.977 * 10^{-3}$]	$-12.2 * 10^{-3}$ ** [$5.219 * 10^{-3}$]
μ_2	-	$5.301 * 10^{-3}$ *** [$0.261 * 10^{-3}$]	$5.495 * 10^{-3}$ *** [$0.287 * 10^{-3}$]
$\sigma^{n,1}$	$1.5 * 10^{-3}$ [$1.964 * 10^{-3}$]	$1.5 * 10^{-3}$ [$3.021 * 10^{-3}$]	$6.749 * 10^{-3}$ * [$3.630 * 10^{-3}$]
$\sigma^{n,2}$	-	-	$1.5 * 10^{-3}$ [$2.221 * 10^{-3}$]
σ^z	$5.502 * 10^{-3}$ *** [$0.615 * 10^{-3}$]	$4.490 * 10^{-3}$ *** [$0.107 * 10^{-3}$]	$4.345 * 10^{-3}$ *** [$0.848 * 10^{-3}$]
$P(S_t = 1)$	-	0.026	0.041
Log(L)	608.041	624.010	624.732

Note: Maximum likelihood estimates for the three univariate unobserved components models of trend-cycle decomposition of log GDP (see Equation 22 and 23 in Appendix A). Model UC-1 is the linear model described by equations (1) to (3). Model UC-2 is a model with a switch in the drift of the trend equation for the level of GDP. Model UC-3 is a model with switches in the drift of the trend equation and in the variance of the innovation for the trend component of GDP. $P(S_t = 1)$ is the unconditional probability of being in the first regime. $\text{Log}(L)$ is the value of the log likelihood function. Standard deviations are reported in brackets. ***, ** and * indicate significance at 1%, 5% and 10%.

Table 3: Bivariate model (GDP and rate of capacity utilisation)

	Model MUC-1(auxiliary)	Model MUC-2(auxiliary)	Model MUC-3(auxiliary)
p_{11}	-	0.981*** [0.038]	0.823*** [0.151]
p_{22}	-	0.994*** [0.007]	0.958*** [0.022]
ϕ_1	1.371*** [0.028]	1.372*** [0.028]	1.349*** [0.023]
ϕ_2	-0.509*** [0.048]	-0.527*** [0.048]	-0.472*** [0.040]
μ_1	$5.698 * 10^{-3}$ *** [$0.374 * 10^{-3}$]	$11.9 * 10^{-3}$ *** [$1.101 * 10^{-3}$]	$0.693 * 10^{-3}$ [$1.539 * 10^{-3}$]
μ_2	-	$5.010 * 10^{-3}$ *** [$0.351 * 10^{-3}$]	$6.764 * 10^{-3}$ *** [$0.482 * 10^{-3}$]
α_1	0.962*** [0.269]	0.913*** [0.238]	0.842** [0.364]
α_2	1.840*** [0.305]	1.737*** [0.254]	2.753*** [0.574]
σ^{aux}	$2.856 * 10^{-3}$ *** [$0.356 * 10^{-3}$]	$2.936 * 10^{-3}$ *** [$0.337 * 10^{-3}$]	$2.504 * 10^{-3}$ *** [$0.444 * 10^{-3}$]
$\sigma^{n,1}$	$4.753 * 10^{-3}$ *** [$0.287 * 10^{-3}$]	$4.143 * 10^{-3}$ *** [$0.269 * 10^{-3}$]	$1.562 * 10^{-3}$ *** [$0.256 * 10^{-3}$]
$\sigma^{n,2}$	-	-	$4.469 * 10^{-3}$ *** [$0.290 * 10^{-3}$]
σ^z	$3.444 * 10^{-3}$ *** [$0.381 * 10^{-3}$]	$3.610 * 10^{-3}$ *** [$0.364 * 10^{-3}$]	$4.347 * 10^{-3}$ *** [$0.910 * 10^{-3}$]
$P(S_t = 1)$	-	0.229	0.192
Log(L)	1315.089	1327.591	1344.208

Note: Maximum likelihood estimates for the three bivariate unobserved components models of trend-cycle decomposition of log GDP (see Equations 9 and 10). Model MUC-1 (auxiliary) is a model without regime switching. Model MUC-2 (auxiliary) is a model with a switch in the slope of the trend. Model MUC-3 (auxiliary) is a model with switches in the slope of the trend equation and in the variance of its error. $P(S_t = 1)$ is the unconditional probability of being in the first regime. $Log(L)$ is the value of the log likelihood function. Standard deviations are reported in brackets. ***, ** and * indicate significance at 1%, 5% and 10%.

Table 4: Bivariate model (GDP and inflation)

	Model MUC-1(inflation)	Model MUC-2(inflation)	Model MUC-3(inflation)
p_{11}	-	0.885*** [0.074]	0.974*** [0.022]
p_{22}	-	0.948*** [0.032]	0.989*** [0.012]
ϕ_1	1.285*** [0.044]	1.406*** [0.063]	1.351*** [0.052]
ϕ_2	-0.367*** [0.069]	-0.571*** [0.114]	-0.475*** [0.089]
μ_1	$5.085 * 10^{-3}$ *** [$0.277 * 10^{-3}$]	$0.91610 * 10^{-3}$ [$1.542 * 10^{-3}$]	$5.265 * 10^{-3}$ *** [$0.275 * 10^{-3}$]
μ_2	-	$7.332 * 10^{-3}$ *** [$0.661 * 10^{-3}$]	$5.567 * 10^{-3}$ *** [$0.648 * 10^{-3}$]
λ^π	-0.044 [0.388]	-0.086 [0.001]	-0.069** [0.034]
λ^z	0.157* [0.081]	0.259*** [0.071]	0.289*** [0.074]
λ^{EXR}	-0.017* [0.010]	-0.017* [0.007]	-0.015* [0.009]
λ^{OIL}	$3.018 * 10^{-3}$ * [$1.706 * 10^{-3}$]	$2.751 * 10^{-3}$ * [$1.470 * 10^{-3}$]	$2.928 * 10^{-3}$ * [$1.509 * 10^{-3}$]
σ^π	$2.181 * 10^{-3}$ *** [$0.621 * 10^{-3}$]	$2.088 * 10^{-3}$ *** [$0.180 * 10^{-3}$]	$2.094 * 10^{-3}$ *** [$0.209 * 10^{-3}$]
$\sigma^{n,1}$	$2.961 * 10^{-3}$ ** [$0.126 * 10^{-3}$]	$3.792 * 10^{-3}$ *** [$0.568 * 10^{-3}$]	$6.324 * 10^{-3}$ *** [$0.611 * 10^{-3}$]
$\sigma^{n,2}$	-	-	$1.5 * 10^{-3}$ * [$0.818 * 10^{-3}$]
σ^z	$4.708 * 10^{-3}$ *** [$0.833 * 10^{-3}$]	$3.209 * 10^{-3}$ *** [$0.772 * 10^{-3}$]	$2.830 * 10^{-3}$ *** [$0.515 * 10^{-3}$]
σ_μ	$1.5 * 10^{-3}$ * [$0.874 * 10^{-3}$]	$1.5 * 10^{-3}$ *** [$0.270 * 10^{-3}$]	$1.519 * 10^{-3}$ *** [$0.311 * 10^{-3}$]
$P(S_t = 1)$	-	0.311	0.292
Log(L)	1446.312	1446.861	1451.821

Note: Maximum likelihood estimates for the three univariate unobserved components models of trend-cycle decomposition of log GDP (see Equations 15 and 16). Model MUC-1 is a model without regime switching, Model MUC-2 is with a switch in the slope of the trend, and Model MUC-3 incorporates switches in the slope of the trend and in the variance of its innovation. $P(S_t = 1)$ is the unconditional probability of being in the first regime. $Log(L)$ is the value of the log likelihood function. Standard deviations are reported in brackets. ***, ** and * indicate significance at 1%, 5% and 10%.

Table 5: Forecast comparison exercise: results for the change in inflation (HICP excluding energy)

<i>Model</i>	UC-1	UC-2	UC-3	MUC-1 (auxiliary)	MUC-2 (auxiliary)	MUC-3 (auxiliary)	Est. 1	Est. 2	Est. 3
<i>Panel A. Real-time output gap series, 2001-2010</i>									
h=1	1.632	1.291	1.046	3.518	3.215	3.036	3.104	1.784	2.983
h=2	1.842	1.420	1.144	2.286	2.173	2.058	2.111	1.533	2.130
h=4	1.523	1.256	1.210	1.447	1.558	1.514	1.544	1.378	1.540
h=8	1.442	1.093	1.115	1.430	1.421	1.438	1.420	1.301	1.339
<i>Panel B. Ex-post output gap series, 2001-2010</i>									
h=1	1.757	1.310	1.318	3.785	3.634	3.398	3.545	2.110	2.654
h=2	1.376	1.311	1.316	2.280	2.159	2.159	2.276	1.396	1.807
h=4	1.331	1.141	1.145	1.686	1.717	1.593	1.691	1.442	1.558
h=8	1.268	1.252	1.257	1.421	1.398	1.409	1.404	1.372	1.346
<i>Panel C. Real-time output gap series, 2001-2007</i>									
h=1	1.244	1.481	1.061	1.451	1.444	1.494	1.459	1.205	1.578
h=2	1.417	1.587	1.094	0.859	0.875	0.886	0.882	1.107	1.031
h=4	1.122	1.377	1.058	1.219	1.293	1.289	1.283	1.095	1.216
h=8	2.280	1.080	1.043	1.630	1.651	1.632	1.638	1.647	1.503
<i>Panel D. Ex-post output gap series, 2001-2007</i>									
h=1	1.537	1.318	1.323	2.103	2.208	2.014	2.147	1.901	1.818
h=2	1.066	1.033	1.034	0.941	0.958	1.047	1.126	0.982	1.031
h=4	0.928	0.958	0.962	1.615	1.657	1.478	1.598	1.262	1.315
h=8	1.673	1.762	1.774	1.659	1.677	1.631	1.675	1.777	1.567

Note: Ratio of the mean squared forecast error between the forecasts obtained from a Phillips curve equation with a real-time measure of the output gap as a proxy for the activity-based measure and a benchmark model given by an $AR(p)$. Est. 1, Est. 2 and Est. 3 are the model averaged measures detailed in the text. The superscripts ^a, ^b and ^c indicate that the test of equal forecast accuracy rejects respectively the null hypothesis of equal forecast accuracy at significance levels of 10%, 5% and 1% level. Appendix C details the Clark and McCracken (2009a) test for real-time data and the Clark and McCracken (2005) test for equal forecast accuracy with revised data.

