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The term structure of risk premia: new evidence from the financial crisis

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THE TERM STRUCTURE OF RISK PREMIA

NEW EVIDENCE FROM THE FINANCIAL CRISIS

by Tobias Berg
In 2010 all ECB publications feature a motif taken from the €500 banknote.

THE TERM STRUCTURE OF RISK PREMIA
NEW EVIDENCE FROM THE FINANCIAL CRISIS

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NOTE: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.


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Abstract

This study calibrates the term structure of risk premia before and during the 2007/2008 financial crisis using a new calibration approach based on credit default swaps. The risk premium term structure was flat before the crisis and downward sloping during the crisis. The instantaneous risk premium increased significantly during the crisis, whereas the long-run mean of the risk premium process was of the same magnitude before and during the crisis. These findings suggest that (marginal) investors have become more risk averse during the crisis. Investors were, however, well aware that risk premia will revert back to normal levels in the long run.

Keywords: credit risk, risk premia, equity premium, mean reversion, structural models of default

JEL Classification: G12, G13
Non-technical summary:

Risk premia are an important determinant of asset prices and asset returns. During the 2007/2008 financial crisis many market observers attributed the significant and world-wide decline in asset prices not only on fundamentals but also on an increase in the required compensation for risk. This ‘gut-feeling’ raises two important questions that this study tries to answer: First, are we able to measure the alleged increase in risk premia based on a sound methodological framework? Second, is this increase in risk premia a temporary increase or does it constitute a permanent shift in investor’s behaviour?

The study uses credit default swaps (CDS) for several maturities (3-, 5-, 7- and 10-year) to extract the risk premium term structure from market prices. The underlying idea is quite simple: The CDS premium is split into an expected loss component and into a risk premium component. The risk premium component itself can be transformed into a widely used measure of risk aversion: the Sharpe ratio. Based on more than 150,000 observations from April 2004 - March 2009 for Europe and the U.S. the study draws the following conclusions: First, short-term risk premia have increased significantly during the financial crisis. Second, long-term risk premia remained almost unchanged during the financial crisis.

These results indicate that a significant part of the world-wide decline in asset prices is indeed attributable to an increase in risk premia. This increase does, however, not constitute a structural and permanent shift in investor’s behaviour but merely constitutes a short-term decline in the demand for risky assets.
1 Introduction

Practitioners and academics usually claim that risk premia must have increased significantly during the 2007/2008 financial crisis to justify the returns and valuations seen in the market. This raises a set of questions: How can this ‘gut feeling’ by market participants be scrutinized in a solid methodological framework? And: Is the change in risk premia expected to be a permanent shift or do market participants expect risk premia to revert back to normal levels once the crisis is over? If risk premia are indeed volatile and mean reverting a risk premium term structure emerges. E.g. during times when (marginal) investors demand above average risk premia - such as gut feeling suggests for the 2007/2008 financial crisis - short duration assets would be expected to have a larger risk premium than longer duration assets.

This paper analyzes the risk premium term structure before and during the 2007/2008 financial crisis and estimates the corresponding parameters of the instantaneous risk premium process (long-run mean, mean reversion speed, volatility). Throughout this paper the risk premium is measured as the market Sharpe ratio, i.e. the excess return of the market portfolio per unit of standard deviation. Of course, a measure such as the equity premium could also be used. The equity premium does, however, have the drawback that it combines risk aversion (‘Sharpe ratio’) and the quantity of risk (‘volatility’). The target of this paper is not to state that the quantity of risk has increased during the 2007/2008 financial crisis. Instead, the focus is on the excess return per unit of risk measured via the market Sharpe ratio.¹

In a survey of Welch (2000), 20% of the participants offered monotonically increasing and 50% monotonically decreasing equity premium term structures. While this may be a good indication of the survey participants perception in this

¹One could therefore also more precisely refer to the term structure of risk aversion. Our results do, however, hold for both the term structure of risk aversion as well as the term structure of risk premia and we therefore prefer to refer to the latter.
specific year, reliance on surveys or expert estimates is not satisfactory from an academic perspective.

Strongly time-varying risk premia are also part of pricing kernels that have been suggested to solve the equity premium puzzle such as Campbell/Cochrane (1999). This pricing kernel is also used by Chen et al. (2009) to analyze the effect on credit spreads. However, these studies only provide indirect evidence for time-variation in risk premia, i.e. they provide indications that time-varying Sharpe ratios can help to explain realized moments (averages, volatility) of historical returns. In contrast, this paper tries to explicitly measure time-variation in risk premia by analyzing risk premium term structures.

Unfortunately, risk premium processes are inherently hard to measure. One part of the literature (e.g. Poterba/Summers (1988), Cambell/Viceira (1999)) has tried to use historical returns to estimate mean reversion in stock prices. Based on an AR(1)-specification, Campbell/Viceira (1999) find indeed that there is mean reversion in historical equity returns. Their results are, however, only significant when looking at a time series of more than 100 years. Using shorter time horizons does not yield significant coefficients anymore. This is not surprising since there are two layers of noise in this methodology: First, the equity premium process is not deterministic insofar as it includes a stochastic error term. In addition, the equity premium is not equal to the realized returns.

The return predictability literature has used the identity that dividend/price-ratios (d/p-ratios) have to predict either dividend growth or future returns. Since d/p-ratios only weakly predict dividend growth this has been attributed to changes in expected returns (see Fama/French (1988), Cochrane (1992)). D/p-ratios are time-varying and mean-reverting, therefore the same should be true for expected (excess) returns. Other authors have used term spreads, T-bill rates, earnings and macro variables to predict excess returns (Fama/French (1989),

\[ \text{2Or price/dividend-ratios must be allowed to grow explosively (bubble).} \]
Lamount (1998), Lettau/Ludvigson (2001)). However, these studies are only able to find indirect indications for time-varying expected returns. No direct link has yet been established.

This paper takes a new approach and uses credit markets to identify time-varying risk premia. This approach offers two distinctive advantages: First, implicit risk premia can be used instead of realized returns. We thereby make use of a new approach proposed by Berg/Kaserer (2009) which allows to estimate the Sharpe ratio of the underlying firm’s asset value process directly from CDS spreads. Second, in contrast to equities, credit instruments are available for a variety of distinct maturities. In particular for credit default swaps (CDS), standard maturities (usually 3, 5, 7, 10 years) have been established. Therefore, Sharpe ratios can be extracted from the credit markets for each maturity separately. This in turn yields a term structure of Sharpe ratios. Based on a time series of Sharpe ratio term structures the parameters of the instantaneous Sharpe ratio process can be estimated with a very high accuracy. Indeed, the methodology proposed in this paper results in reliable estimations for the parameters of the Sharpe ratio process with sample periods as small as 12 months.

This study uses more than 150,000 CDS spreads from the constituents of the main CDS indices in Europe (iTraxx Europe) and the U.S. (CDX.NA.IG) from April 2004 - March 2009. The resulting term structure of risk premia has been flat before the 2007/2008 financial crisis and downward sloping during the 2007/2008 financial crisis. Calibrating these results to a CIR process for the instantaneous Sharpe ratio shows that the long-run mean is of the same magnitude before and during the 2007/2008 financial crisis. In contrast, the instantaneous Sharpe ratio has increased significantly during the 2007/2008 financial crisis.

\[\text{It is well known from the interest rate literature that adding cross sections (i.e. different maturities) greatly enhances the estimation accuracy for the underlying process. Especially in situations where short-term interest rates are far above or below the long-run mean, adding information from further maturities significantly decreases the resulting standard errors for the estimation of the long-run mean and mean-reversion speed.}\]
These results convey the idea that (marginal) investors have become more risk averse during the crisis. Investors were, however, well aware that risk premia will revert back to normal levels again. As a result short-term risk premia increased more than long-term risk premia. The slope of the risk premium term structure (measured as 10-yr Sharpe ratio minus 3-year Sharpe ratio) has been approximately zero before the 2007/2008 financial crisis and has become negative during the 2007/2008 financial crisis.

A higher risk aversion during the 2007/2008 financial crisis seems to be very reasonable given the excessive fear of market participants about what is going on in the markets. Indeed, we do not see this result in itself as a breakthrough. Rather, the methodology used is - to our best knowledge - the first one that is able to extract this kind of risk premium term structure out of current market prices. The scope of the methodology is not limited to the pure measurement of time-varying risk premia but could also be fruitful in areas such as return predictability, tactical asset allocation and company valuation.

The paper is organized as follows. Section 2 presents the theoretical framework. Section 3 describes our data sources. Section 4 provides the empirical results for the estimation of the term structure of Sharpe ratios and the parameters of the instantaneous Sharpe ratio process. Robustness tests are shown in section 5. Section 6 concludes.

2 Model setup

This section first links asset valuation to debt valuation (subsection 2.1) and then introduces a process for the instantaneous Sharpe ratio (subsection 2.2) in order to derive an estimator for the Sharpe ratio term structure (subsection 2.3) (based on Berg/Kaserer (2009)) and for the parameters of the Sharpe ratio process (subsection 2.4) (based on a Kalman Filter approach).
2.1 Asset value process and default mechanism

The basic idea is to use structural models of default to link credit to equity valuations. This approach goes back to Merton (1974) and has already been used in Berg/Kaserer (2009) to estimate Sharpe ratios and equity premia from CDS spreads. The following two input factors have to be specified:

- The dynamics of the asset value process (including the dynamics of the asset Sharpe ratio).
- The default mechanism which - in addition to the asset value process - determines the default time.

**Asset value process:** The real world asset value process $V_t$ is modeled as a diffusion with a risk-free rate $r$, a payout ratio $\delta$, a mean-reverting market Sharpe ratio $\theta$, and an asset volatility $\sigma_V$.

\[
\text{Asset value process: } dV_t = \left(\theta_V V_t \cdot \sigma_V + r - \delta - (\sigma_V^2/2)\right)V_t dt + \sigma_V V_t dW_V,
\]

\[
\text{CAPM condition: } \theta = \rho \theta_t, \quad \text{with } \rho = Corr(r_{M,t}, r_{V,t})
\]

\[
\text{Sharpe ratio process: } \theta_t = \mu^\theta(\theta_t)dt + \sigma^\theta(\theta_t)dW^\theta.
\]

Equation (2) assumes that the continuous time CAPM holds, i.e. that the company Sharpe ratio ($\theta$) is the product of the asset/market-correlation ($\rho$) and the market Sharpe ratio ($\theta$). Here, $r_{M,t}$ and $r_{V,t}$ denote the stochastic variables for the return on the market portfolio and the asset return. The last equation-formulates the process for the instantaneous Sharpe ratio. This process will be specified in more detail in section 2.2.

The formulas above can be easily reformulated in terms of the equity premium $\pi_t$ instead of the market Sharpe ratio $\theta_t$. The Sharpe ratio was chosen because it is a purer measure of risk aversion (whereas the equity premium measures both risk aversion as well as the quantity of risk).
Default mechanism: A simple Merton model is assumed, i.e. default can only happen at the end of maturity if the asset value is lower than the default barrier. Therefore, the actual \( PD^P \) and risk-neutral \( PD^Q \) default probability can be calculated as

\[
PD^P = P[V_T^P < L] \quad (4)
\]
\[
PD^Q = Q[V_T^Q < L] \quad (5)
\]

The choice of such a specific default mechanism may seem to be a rather hard restriction. Fortunately, our results are robust with respect to the use of other models, e.g. first-passage time mechanism or strategic default models. This issue will be discussed in more detail in subsection 2.3.

2.2 A process for the instantaneous Sharpe ratio

The instantaneous Sharpe ratio \( \theta_t \) is modeled as a mean-reverting CIR process, i.e.

Process: \( d\theta_t = \kappa(\bar{\theta} - \theta_t)dt + \sigma \sqrt{\theta_t}dW^\theta_t \) \quad (6)
\[
\theta_t \quad (7)
\]

The expectation of \( \theta_s \) can be determined as

\[
E[\theta_s] = \theta(t)e^{-\kappa(s-t)} + \bar{\theta} \left( 1 - e^{-\kappa(s-t)} \right) \quad (8)
\]

and therefore the real-world average of the Sharpe ratios between \( s \) and \( t \) equals

Average: \( \Theta(t, \tau) := \frac{1}{\tau}E^P \left[ \int_t^{t+\tau} \theta(s)ds \right] = \bar{\theta} + \frac{1}{\tau}(\theta_t - \bar{\theta}) \frac{1 - e^{-\kappa\tau}}{\kappa} \quad (9) \)

Please note that we have defined the average Sharpe ratio \( \Theta(t, \tau) \) as a real world arithmetic average rather than a risk-neutral average of the respective discount
rates.\textsuperscript{4} The reason behind that definition will become clearer in the next subsections.

The choice of a specific process for the instantaneous Sharpe ratio process is by no means trivial. Several other candidates (e.g. Vasicek-process) would certainly also qualify. The CIR process was chosen for several reasons: First, it is able to capture mean reversion and volatility. Second, its parameters can be easily interpreted ($\bar{\theta}$: long-run mean, $\kappa$: mean reversion speed, $\sigma$: volatility). Third, it is analytically tractable. Fourth, its volatility is dependent on the current state of the process which best suits the real-world data, cf. section 4.

There are several possible underlying reasons why Sharpe ratios may be time-varying. One reason may be a time-varying risk aversion of the marginal investor. But even under the assumption of constant relative risk aversion time-varying expected returns may emerge if the volatility of consumption is time-varying. This paper does not aim to explain the drivers of time-varying expected returns, rather it takes the indications of several academic studies (see Fama/French (1988) and Cochrance (1992) for an overview) to analyze if expected time-varying Sharpe ratios can be validated based on current asset prices.

2.3 Estimating Sharpe ratios from CDS spreads

This subsection derives a formula to estimate both the Sharpe ratio of the underlying firm’s asset value process and the market Sharpe ratio directly from CDS spreads and estimates for the real world default probability. Similar to Berg/Kaserer (2009) the following estimator for the market Sharpe ratio $SR_M$ can be derived in a Merton framework with constant Sharpe ratios:\textsuperscript{5}

$$SR_M = \frac{\Phi^{-1}(PD^Q(t, \tau)) - \Phi^{-1}(PD^P(t, \tau))}{\sqrt{\tau}} \cdot \frac{1}{\rho_{V,M}},$$

\textsuperscript{4}I.e. $E^P\left[\int_t^{t+\tau} \theta(s)ds\right]$ instead of the usual ‘spot-rate’ definition $lnE^Q\left[e^{-\int_t^{t+\tau} \theta(s)ds}\right]$.

\textsuperscript{5}The Sharpe ratio of the underlying firm’s asset value process can be estimated by omitting the $\rho_{V,M}$-term.
where $PD^Q$ and $PD^P$ denote the cumulative risk-neutral and actual default probability, $\tau$ denotes the maturity and and $\rho_{V,M}$ denotes the asset/market correlation. Details are provided in appendix B.

Although this estimator is derived in a simple Merton framework, Berg/Kaserer (2009) have shown that the estimator is still robust in first-passage-time frameworks, strategic default frameworks and frameworks with unobservable asset values (Duffie/Lando (2001) model). Huang/Huang (2003) also show that - given a certain actual default probability - the risk-neutral default probability is almost the same for the main structural models of default in the literature. They analyze the Longstaff/Schwartz (1995) model with stochastic interest rates, the Leland/Toft (1996) model with endogeneous default, strategic default models of Anderson/Sundaresan/Tychon (1996) and Mella-Barral/Perraudin (1997) and a model with mean-reverting leverage ratios. Their analysis also includes a model with time-varying asset risk premium.

Appendix A shows that the Sharpe ratio estimator above is approximately true for a model with time-varying Sharpe ratios if the constant Sharpe ratio $SR_M$ is substituted by its real world arithmetic average $\Theta(t, \tau)$ as defined in (9):

$$\Theta(t, \tau) \approx \frac{\Phi^{-1}(PD^Q(t, \tau)) - \Phi^{-1}(PD^P(t, \tau))}{\sqrt{\tau}} \frac{1}{\rho_{V,M}}$$

The cumulative risk-neutral default probability $PD^Q$ can be derived from CDS spreads $s$ for a maturity $\tau$ via the relationship (cf. Duffie/Singleton (2003))

$$PD^Q = 1 - e^{-s/LGD \cdot \tau},$$

where $LGD$ denotes the loss given default. The respective calibration issues are discussed in the empirical part.
2.4 Estimation of the parameters of the instantaneous Sharpe ratio process

Unfortunately, the Sharpe ratio process $\theta_t$ can not be observed directly. Instead, only the average expected Sharpe ratios $\Theta(t_i, \tau_j)$ can be measured via (11) and this measurement may be subject to noise. Therefore, the Kalman filter methodology is needed in order to estimate the parameters of the Sharpe ratio process. The application of this methodology is similar - but not equal - to the literature on interest rate processes.\(^6\) In our case, the transition and measurement equations can be derived based on equation (6) and (9), i.e.

$$\begin{align*}
\theta_{t_i} &= F \theta_{t_{i-1}} + C + \epsilon_{t_i} \\
\Theta(t_i, \tau) &= H \theta_{t_i} + A + \nu_{t_i}
\end{align*}$$

with

$$\begin{align*}
F &= e^{-\kappa \Delta t} \\
C &= (1 - e^{-\kappa \Delta t}) \cdot \bar{\theta} \\
\epsilon_{t_i} &\sim N \left(0, \frac{\sigma^2}{2 \kappa} \left(1 - e^{2 \kappa \Delta t}\right)\right) \\
\epsilon_{t_i} &\sim \text{noncentral } \chi^2 \text{ with } Var[\epsilon_{t_i} | \theta_{t_{i-1}}] = \theta_{t_{i-1}} \frac{\sigma^2}{\kappa} \left(e^{-\kappa \Delta t} - e^{-2 \kappa \Delta t}\right) + \bar{\theta} \frac{\sigma^2}{2 \kappa} \left(1 - e^{-2 \kappa \Delta t}\right)^2 \\
(H)_{j} &= \frac{1}{\tau_j} \frac{1 - e^{-\kappa \tau_j}}{\kappa} \\
(A)_{j} &= \left(1 - \frac{1}{\tau_j} \frac{1 - e^{-\kappa \tau_j}}{\kappa}\right) \cdot \bar{\theta} \\
\nu_{t_i} &\sim N \left(0, R\right),
\end{align*}$$

where $\tau$ denotes a vector of all available maturities. $R$ is unknown and estimated within the Kalman filter methodology. The error term $\epsilon_{t_i}$ has a non-central $\chi^2$ distribution. We will approximate this by a normal distribution, cf. Bolder\(^6\).

\(^6\)Cf. Bolder (2001) for a good overview about the Kalman filter approach and interest rate modeling.
The corresponding log-likelihood function is therefore given by

\[
l(\kappa, \bar{\theta}, \sigma, R) = -\frac{n N \ln(2\pi)}{2} - \frac{1}{2} \sum_{i=1}^{N} \ln(\det(var_i)) + err_i var_i^{-1} err_i \quad (15)
\]

\[err_i = \Theta(t_i) - E[\Theta(t_i) | F_{t_i-1}], \quad var_i = Var[\Theta(t_i) | F_{t_i-1}] .\]

Here, \(\Theta(t_i)\) denotes a row vector where each row represents one maturity. The conditional expectations and variances are determined based on (14).

The parameter estimation proceeds in two steps:

- First, the Sharpe ratios \(\Theta(t_i, \tau_j)\) are estimated based on (11) for all available maturities \(\tau_j\) and for all available dates \(t_i\).
- Second, the parameters of the Sharpe ratio process are estimated based on (13)-(15).

### 3 Data sources and descriptive statistics

The sample consists of weekly observations of 3-, 5-, 7- and 10-year CDS spreads from on-the-run companies of the main CDS indices in Europe and the U.S. from April 2004 until March 2009. The iTraxx Europe index was used for Europe and CDX.NA.IG index was used for the U.S. Both indices consist of 125 members and are rolled over every six month (end of March and end of September). The first series of the iTraxx was launched on Mar, 20th, 2004, the first series of the CDX.NA.IG was launched on Nov, 20th, 2003. In order to obtain the same sample period for both Europe and the U.S. our sample starts for both regions in April 2004.

For each observation, the average market Sharpe ratio was estimated based on (11). The following input parameters were needed: a) CDS spreads and b) loss given default (to estimate the risk-neutral default probability based on (12)),

\[(2001) \text{ and Duan/Simunato (1995).} \]
c) real world cumulative default probabilities and d) correlation between asset returns and market returns.

The CDS spreads for 3-, 5-, 7- and 10-year maturity were taken from CMA (Credit Markets Association) via Datastream. Mid spreads were used for the analysis. Together with each CDS spread CMA provides a veracity score which indicates if the spread is based on an actual trade, a firm bid or other sources (e.g. indicative bid, bond spread derived). A date/company-combination was only included in our sample if either trades or firm bids have been reported for all maturities (3,-5,-7-, 10-year) in that respective week. The loss given default was set to 60% based on Moody’s (2007) and robustness tests were performed.

The actual cumulative default probability was determined based on expected default frequencies (EDFs) from Moody’s KMV. Moody’s KMV provides EDFs from 1- to 10-year maturities based on the distance-to-default measure. To calibrate the distance-to-default to default probabilities KMV uses its proprietary database of historical default events. Therefore, there is no problem of any circular arguments, since the level of default probabilities does not rely on any Sharpe ratio or drift assumptions taken by Moody’s KMV. Robustness tests based on a hybrid hazard-rate model have been conducted.

Correlations between asset returns and the market return were proxied by the rolling 2-year median industry correlation between the corresponding equity returns and the market return. The data was taken from Datastream. Using equity instead of asset correlations is justified as equity is a deep-in-the-money call on the company’s assets. Therefore, delta is approximately one and gamma

---

7 Median industry correlations were taken for robustness reasons. Since the correlation enters our formula in the denominator, estimation errors result in an upward biased estimator for the market Sharpe ratio. Industry medians have lower standard errors than a company-by-company estimation and also allow to include companies without a 2-year equity price history. At each date, we estimated the 2-year weekly correlation between the performance index of the respective stocks in each industry and the major index in each region (Europe: Stoxx600, U.S.: S&P 500).
approximately zero and correlations are (almost) the same.

Finally, the sample period was split into two sub-periods: 'Before Crisis' (April 2004-June 2007) and 'During Crisis' (July 2007-March 2009). Of course there is no single starting date of the 2007/2008 financial crisis\(^8\), therefore our division of the sample period is - to a certain extent - arbitrary.\(^9\) Already in Feb. 2007, HSBC announced losses of $10bn related to subprime mortgages. In April 2007, New Century Financial, one of the biggest mortgage lenders in the U.S. declared bankruptcy. The crisis accelerated in June and July 2007 when Bear Stearns had to inject $3.2 bn to bail out two of its hedge fonds and when Moody’s and Standard & Poor’s downgraded more than 250 subprime RMBS. The Dow Jones index peaked as late as in October 2007. However, our main conclusions do also hold when choosing Q2 2007 or Q4 2007 as a starting point for the 2007/2008 financial crisis. It is not the target to show that certain risk premia changes happened \textit{exactly} at the beginning of the crisis. Rather it should be demonstrated that the implied risk premia have gradually changed throughout the turmoil.

All in all, the study uses 23,532 observations for each maturity for Europe (15,103 before the crisis, 8,429 observations during the crisis) and 22,819 observations for each maturity for the U.S. (14,416 before, 8,403 during the crisis). The descriptive statistics for the input parameters are shown in table 1.

\(^8\)Although our data sample ends in March 2009 and the 'During Crisis' subperiod spans from July 2007 until March 2009, we will still use the term '2007/2008 financial crisis' as the years 2007 and 2008 are most closely related to the financial crisis.

Table 1: **Descriptive statistics for input parameters**

The sample consists of the intersection of the KMV database, the iTraxx on-the-run companies (Europe)/CDX.NA.JG on-the-run companies (U.S.) and the CMA CDS database (via Datas-tream) from April 2004 to March 2009. \( CDS3/CDS5/CDS7/CDS10 \) denote 3-/5-/7-/10-year CDS spreads in bp. \( EDF3/EDF5/EDF7/EDF10 \) denote 3-/5-/7-/10-year cumulative expected default frequencies from Moodys KMV. \( \rho \) denotes the equity/market correlation. Median industry correlations have been used based on rolling 2-year weekly returns. The corresponding market returns are based on the return index of the Stoxx600 (Europe) and S&P 500 (U.S.). \( \sigma_M \) denotes the implied market volatility based on maturities from 18-23 months from the VIX term structure (mid prices) of the VStoxx sub-index and the CBOE. Averages are calculated as unweighted averages over all observations.

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Panel B: US

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<td>CDS10</td>
<td>14,416</td>
<td>79.74</td>
</tr>
<tr>
<td>EDF3</td>
<td>14,416</td>
<td>0.54%</td>
</tr>
<tr>
<td>EDF5</td>
<td>14,416</td>
<td>1.21%</td>
</tr>
<tr>
<td>EDF7</td>
<td>14,416</td>
<td>1.94%</td>
</tr>
<tr>
<td>EDF10</td>
<td>14,416</td>
<td>3.01%</td>
</tr>
<tr>
<td>( \rho )</td>
<td>14,416</td>
<td>0.51</td>
</tr>
<tr>
<td>( \sigma_M )</td>
<td>14,416</td>
<td>16.08%</td>
</tr>
</tbody>
</table>
4 Results

4.1 Risk premium term structure

The Sharpe ratio estimates based on 3-, 5-, 7- and 10-year CDS spreads are shown in figure 1 (Europe) and figure 2 (U.S.). Both Europe and the U.S. show a very similar pattern of CDS-implied Sharpe ratios over the whole sample period. In addition, for both Europe and the U.S. CDS-implied Sharpe ratios before the 2007/2008 financial crisis are very similar for all maturities. In fact, the implied Sharpe ratio estimates for different maturities rarely deviate by more than five percentage points before the 2007/2008 financial crisis. The overall level of the Sharpe ratio estimates is slightly higher for the U.S. (predominantly between 20-50% before the crisis) than for Europe (predominantly between 10-40% before the crisis) as also mentioned in Berg/Kaserer (2009). This is consistent with both historical experience as well as with standard portfolio theory.

The time series of CDS-implied Sharpe ratios shows two significant changes: The first occurs around April 2005, when CDS-implied Sharpe ratios increase by approximately 20PP (from 20% to 40% for Europe and from 30% to 50% for the U.S.). This is likely to be due to the downgrades of Ford and General Motors which resulted in an overall repricing in the CDS markets. However, it can be observed, that CDS-implied Sharpe ratios increase for all maturities simultaneously and by the same amount. Therefore, both before and after April 2005, 3-/5-/7- and 10-year CDS-implied Sharpe ratios are almost the same for a given

---

10. Median Sharpe ratios for each date are depicted to decrease the influence of outliers. Mean estimates are, however, very similar but slightly higher. Detailed results and descriptive statistics can be found in appendix B and an in-depth discussion and comparison with other Sharpe ratio and equity premium estimates can be found in Berg/Kaserer (2009).

11. Under the assumption that the U.S. market has a higher correlation with the global market portfolio than the European market, the European Sharpe ratios are smaller than U.S. Sharpe ratios based on CAPM considerations.

12. Ford and General Motors were downgraded to junk status on May, 5th, 2005 by S&P but this is likely to have been anticipated beforehand in the bond and CDS markets due to a negative outlook status. E.g. Hull et al. (2004) find that spread changes lead rating announcements by up to 90 days.
The second significant change occurs at the beginning of the financial crisis in July 2007. The change differs, however, significantly from the one observed in April 2005: For both Europe as well as the U.S. implied Sharpe ratios generally increase during the 2007/2008 financial crisis. The increase is, however, much more pronounced for short-term maturities (3-year, 5-year) than for longer maturities (7-year, 10-year). As a result, the term structure of risk premia changes from a flat term structure before the 2007/2008 financial crisis to an inverse term structure during the 2007/2008 financial crisis. This term structure of Sharpe ratios is depicted in figure 3. In addition, the behaviour of the risk premium term structure (flat before the crisis and inverse during the crisis) is very persistent as can be seen from figure 1 (Europe) and figure 2 (U.S.). Indeed, the inverse nature of the risk premium term structure was prevalent in any week without exception from mid 2007 until the end of our sample period.

The economic interpretation is straightforward: Assume that the risk aversion of the marginal investors is mean-reverting. Then the instantaneous Sharpe ratio will also be mean-reverting. If the instantaneous Sharpe ratio equals the long-run mean, a flat risk premium term structure will be observed. In contrast, if the current instantaneous Sharpe ratio is high, the expected Sharpe ratio will be a decreasing function of the maturity. Therefore, an inverse risk premium term structure emerges.

Figure 4 shows the slope of the risk premium term structure - defined as the 10-year CDS-implied Sharpe ratio minus the 3-year CDS-implied Sharpe ratio. Similar to the interest rate literature, this slope determines the difference between long-run and short run risk premia. The resulting slope is close to zero before the 2007/2008 financial crisis. Then, it drops significantly at the end of the second quarter of 2007 with local minima around March/April 2008 (after the Bear Stearns rescue) and October/November 2008 (after the Lehman default). It
Figure 1: CDS-implied Sharpe ratio for Europe (index: iTraxx Europe) based on 3-/5-/7- and 10-year CDS spreads and EDFs form 04/2004-03/2009.

stays negative until the end of our sample period (March 2009).

### 4.2 Instantaneous Sharpe ratio process

Based on the estimates of the Sharpe ratio term structure from subsection 4.1 the parameters for the instantaneous Sharpe ratio process have been estimated as described in subsection 2.4. The resulting parameter estimates for the long-run mean Sharpe ratio $\bar{\theta}$, the mean reversion speed $\kappa$ and the Sharpe ratio volatility $\sigma_\theta$ are shown for both Europe and the U.S. in table 2. The Kalman methodology has been applied for the total sample period (April 2004-March 2009) as well as for the periods 'Before Crisis' (April 2004-June 2007) and 'During Crisis' (July 2007-March 2009). Based on the observations from the last subsection, we also split the results for the 'Before Crisis' period into two subperiods ('Before GM/Ford' denoting the period from April 2004 - March 2005 and 'After GM/Ford' denoting the period from April 2005-June 2007). In addition to the parameter estimates,
Figure 2: CDS-implied Sharpe ratio for the U.S. (index: CDX.NA.IG) based on 3-/5-/7- and 10-year CDS spreads and EDFs form 04/2004-03/2009.

the average of the (filtered) instantaneous Sharpe ratio \( \bar{\Theta}_t \) is also depicted in the second column of table 2.

For Europe, table 2 reveals that the long-run mean estimate \( \bar{\Theta} \) based on the ‘During Crisis’ subperiod is larger than in the ‘Before Crisis’ subperiod (42.0% vs. 30.0% with standard errors of 1.0% and 2.0%). However, if the period before the GM and Ford downgrades is excluded, long-run mean estimates are not significantly different at a 5% level before and during the crisis (42.0% vs. 37.9% with standard errors of 1.0% and 1.9%). In contrast to the long-run mean estimates, the estimates for the instantaneous Sharpe ratio \( \theta_t \) are an order of magnitude higher for the ‘During Crisis’ subperiod than for the ‘Before Crisis’ subperiod (186.7% vs. 40.0%). I.e., while long-run means have not significantly changed during the financial crisis, short-term risk premia have increased significantly. The results for the U.S. mainly support the evidence from the European sample with both long-run means and instantaneous Sharpe ratios in a similar order of
A special attention should be devoted to the standard errors. All standard errors are rather small. These small standard errors seem to be especially surprising for the 'During Crisis' period - where parameter estimates are only based on 91 weekly observation. There are two (interlinked) reasons: First, this study uses cross-sectional information, e.g. maturities of 3-, 5-, 7- and 10-years. This cross-sectional information is very stable during the 2007/2008 financial crisis: The risk premium curve is inverse for every single week from mid 2007 on and the slope of the risk premium term structure is similar during the whole 'During Crisis' subperiod. Second, it is a well known fact from interest rate modeling that, if information for several maturities is available, long-run means and mean reversion parameters can be estimated with higher accuracy when instantaneous rates are far above or below their long-run mean parameters. Indeed, if the parameters are estimated only based on one of the maturities, standard errors are
In particular the fact that the long-run mean is of a similar magnitude before and during the crisis seems to be an intuitive, but interesting finding. This finding probably does not come as a surprise to most researchers. However, this methodology is - to our best knowledge - the first one that is able to extract these risk premium term structures together with estimates for the Sharpe ratio process out of current asset prices with a satisfying accuracy. The main advantage of this approach is the feature, that it is able to estimate a whole term structure of risk premia for each date in our sample. Adding this cross-sectional information in addition to the pure time series evolvement of the implied or realized risk premia renders the estimates much more precise.

It should be noted, however, that the estimates for the mean-reversion speed and the volatility of the Sharpe ratio process show a significant difference between the pre-crisis and crisis period and that the standard errors around the mean are in some cases economically significant. However, a fair judgement of this approach and the results presented in this paper can only be compared rela-
tive to the performance of alternative approaches. As indicated in the introduction, the risk premium process is inherently hard to measure. Campbell/Viceira (1999) use historical U.S. equity returns of more than 100 years (1890-1993) to estimate a process for the equity premium. They derive an average of 4.165% with a standard error of 1.3% (approximately 30% of the mean) and a mean-reversion parameter of 0.202 with a standard error of 0.062 (approximately 30% of the mean). Using shorter time periods, Campbell/Viceira are not able to derive statistically significant results anymore. Given the results from theoretical research (cf. Poterba/Summers (1988) and Cochrane (2005)) and the fact that Campbell/Viceira use realized equity returns, this should not be surprising. In contrast, our results for the U.S. based on only 5 years of observation (April 2004 - March 2009) have a standard error of 1.1% compared to a mean of 34.4% (approximately 3% of the mean) for the long-run mean estimate and a standard error of 0.8% compared to a mean of 13.5% (approximately 6% of the mean) for the mean-reversion estimate. Therefore - despite the drawback described above - we see the results and their accuracy as a step forward in the estimation of the term structure of risk aversion.

5 Robustness tests

5.1 General remarks on robustness

The question of robustness arises for almost every empirical study. In the context of the 2007/2008 financial crisis it does, however, gain special importance. Many economic parameters have seen extraordinary levels during the 2007/2008 financial crisis and may possibly distort our results. This may include the influence of certain subsamples, subperiods or a bias in the measurement of any of the input parameters.

Looking at certain subsamples - especially exclusion of financials - or certain subperiods during the 2007/2008 financial crisis does not change the main result of a downward sloping risk premium term structure during the 2007/2008 financial
Table 2:
Parameter estimates for instantaneous Sharpe ratio process

This table shows the parameter estimates for the instantaneous Sharpe ratio process based on the 3-, 5-, 7- and 10-year Sharpe ratio estimates from subsection 4.1 and the Kalman filter methodology as described in subsection 2.4. Total denotes the total period from 04/2004-03/2009, Before Crisis denotes the period from 04/2004-06/2007, Before GM/Ford denotes the period from 04/2004-03/2005 before the Ford and General Motors were downgraded to junk stats, After GM/Ford denotes the period from 04/2005-06/2007 and During Crisis denotes the period from 07/2007-03/2009. $\bar{\theta}_t$ denotes the average of the instantaneous Sharpe ratio, $\bar{\theta}$ the long-run mean Sharpe ratio, $\kappa$ the mean reversion, $\sigma_\theta$ the Sharpe ratio volatility and $R$ the standard deviation of the error term. $SE$ denotes the standard errors for the respective parameter estimates. */** denotes significance at the 10%/1% level.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\bar{\theta}_t$</th>
<th>$\bar{\theta}$</th>
<th>$\kappa$</th>
<th>$\sigma_\theta$</th>
<th>$R$</th>
<th>$SE$</th>
<th>$\bar{\theta}$</th>
<th>$\kappa$</th>
<th>$\sigma_\theta$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Europe</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (Apr 04 - Mar 09)</td>
<td>64.1%</td>
<td>34.3%**</td>
<td>16.0%**</td>
<td>37.6%**</td>
<td>7.0%**</td>
<td>0.7%</td>
<td>0.6%</td>
<td>2.7%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>Before Crisis (Apr 04 - Jun 07)</td>
<td>40.8%</td>
<td>30.0%**</td>
<td>3.4%**</td>
<td>25.0%**</td>
<td>2.9%**</td>
<td>2.0%</td>
<td>0.4%</td>
<td>2.4%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>Before GM/Ford (Apr 04 - Mar 05)</td>
<td>16.4%</td>
<td>20.8%**</td>
<td>48.9%**</td>
<td>68.6%**</td>
<td>5.3%**</td>
<td>0.7%</td>
<td>6.5%</td>
<td>14.4%</td>
<td>0.3%</td>
<td></td>
</tr>
<tr>
<td>After GM/Ford (Apr 05 - Jun 07)</td>
<td>50.5%</td>
<td>37.9%**</td>
<td>6.1%**</td>
<td>20.2%**</td>
<td>2.4%**</td>
<td>1.9%</td>
<td>1.1%</td>
<td>2.8%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>During Crisis (Jul 07 - Sep 08)</td>
<td>186.4%</td>
<td>42.0%**</td>
<td>77.3%**</td>
<td>63.8%**</td>
<td>9.5%**</td>
<td>1.0%</td>
<td>2.6%</td>
<td>8.3%</td>
<td>0.4%</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: U.S.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (Apr 04 - Sep 08)</td>
<td>69.9%</td>
<td>34.4%**</td>
<td>13.5%**</td>
<td>34.2%**</td>
<td>7.6%**</td>
<td>1.1%</td>
<td>0.8%</td>
<td>2.6%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>Before Crisis (Apr 04 - Jun 07)</td>
<td>54.4%</td>
<td>20.1%**</td>
<td>4.2%**</td>
<td>20.1%**</td>
<td>4.4%**</td>
<td>4.5%</td>
<td>0.6%</td>
<td>2.2%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>Before GM/Ford (Apr 04 - Mar 05)</td>
<td>50.8%</td>
<td>29.8%**</td>
<td>107.8%**</td>
<td>81.5%**</td>
<td>6.6%**</td>
<td>0.9%</td>
<td>30.9%</td>
<td>27.6%</td>
<td>0.3%</td>
<td></td>
</tr>
<tr>
<td>After GM/Ford (Apr 05 - Jun 07)</td>
<td>60.9%</td>
<td>35.3%**</td>
<td>2.9%</td>
<td>13.0%**</td>
<td>4.3%**</td>
<td>21.7%</td>
<td>2.5%</td>
<td>2.4%</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>During Crisis (Jul 07 - Sep 08)</td>
<td>151.1%</td>
<td>39.8%**</td>
<td>65.2%**</td>
<td>52.7%**</td>
<td>14.5%**</td>
<td>1.6%</td>
<td>2.8%</td>
<td>7.2%</td>
<td>0.6%</td>
<td></td>
</tr>
</tbody>
</table>
crisis. The sensitivity with respect to the input parameters is analyzed in the next step. First, a simple target value search is conducted. This target value search looks at each parameter separately and determines the value of this parameter as to yield a flat risk premium term structure during the crisis. Of course, this procedure is not able to gauge errors in several input parameters at the same time. In addition, it is not able to explain why any of the input parameters may have been biased during the crisis. Therefore, in a second step, a different proxy for the most crucial input parameter - the real world default probability - is used. In addition, this study controls for parameters which have seen extraordinary levels during the 2007/2008 financial crisis via a regression approach. As a final check, market microstructure effects on CDS liquidity are analyzed.

5.2 Target search procedure

The target search proceeds in two steps. First, any of the input parameters (e.g. the default probability) is chosen. Second, the value for this parameter which would yield a flat risk premium term structure for the 'During Crisis' subperiod (i.e. which yields the same 3-year CDS-implied Sharpe ratio than the 10-year CDS-implied Sharpe ratio) is implicitly determined. This study takes a very conservative position: It is assumed, that the parameter for the 10-year calculations remain unchanged while only the input parameters for the 3-year calculations are changed. E.g., the 3-year PD-estimates are increased until the same Sharpe ratio than with the original 10-year PD-estimates is obtained. Of course, if 3-year PD-estimates are downward biased one would expect the same for 10-year PD estimates. Therefore, the results provide a lower level of the change that is necessary to yield a flat term structure.

Table 3 depicts the value of the target search. The 'During Crisis' period is

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13 It should be noted that financials have a lower average CDS-implied market Sharpe ratio than non-financials. The slope of the risk premium term structure is, however, almost the same for financials and non-financials. Detailed results are available in appendix B.
split into three subperiods to give a more detailed picture of the results. In the following, we will focus on the average values for the ‘During Subprime’ period. The resulting values for the LGD are larger than 100% and therefore economically impossible for both Europe and the U.S. A correlation of 0.99 (Europe) and 0.88 (U.S.) also seems unrealistic, the values for some of the subperiod are even larger than one and therefore statistically impossible. A 58% (Europe) / 53% (U.S.) lower CDS spread is more than 10 times the bid/ask spread and cannot be explained by market microstructure issues alone. The 3-year EDF would have to be at least 178% (Europe) and 134% (U.S.) higher than it actually was to yield a flat term structure of risk premia during the crisis. For some subperiods in Europe it would have to increase by even more than 200%. The resulting PDs are 2.26% for Europe and 5.19% for the U.S. with the highest values in the post Lehman period of more than 6% (Europe) and 10% (U.S.) respectively. Please note that our sample consists only of investment grade obligors since the CDS indices used include only investment grade obligors. The maximum 3-year cumulative PD for investment grade obligors from 1970-2006 based on Moody’s (2007) was 1.22%. The maximum 1-year default probability for the investment grade obligors in the U.S. from 1920-2007 was 1.557% (in 1938). It would need more than three years in a row as bad as the worst year from 1920-2007 to yield a cumulative PD as high as 5.19 % or even three years twice as bad as the worst year from 1920-2007 to yield a cumulative PD of more than 10%. These values seem to be unreasonable given historical experience.

However, the PD estimate is probably the single most crucial input parameter of our procedure. Therefore we will perform two other robustness tests: First, an alternative measures for the PD estimate is applied, then a robustness test based on a linear regression of CDS spreads on certain parameters with very high/low levels during the 2007/2008 financial crisis is performed.

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14 Cf. Moody’s (2007), Exhibit 33. The study is a worldwide study but is dominated by U.S. and European data. The average default rate for Europe is usually lower than for the U.S., although Moody’s does not provide a disaggregation on this level.
### Table 3: Robustness: Target search procedure

This table shows the parameters that have to be used for the 3-year Sharpe ratio estimate in order to yield the same Sharpe ratio than for the 10-year Sharpe ratio estimate. The parameters for the 10-year Sharpe ratio estimate are left unchanged. Column 'Total During Crisis' shows the results for the period from 07/2007-03/2009, column 'Before Bear Stearns' shows the results from 07/2007-03/2008, column 'Before Lehman' shows the results from 04/2008-09/2008 and column 'After Lehman' from 10/2008-03/2009. PD denotes the average real world default probability, LGD denotes the loss given default, s the average CDS spread and ρ the average asset/market correlation. Base Case is the average of the respective parameters without adjustments, %Change is the change in percentage relative to the base case and Target the value of the respective parameter that is needed for a flat risk premium term structure. For the sample decomposition please see table 1. Averages are calculated as unweighted averages over all observations.

| Parameter | Panel A: Europe | | Panel B: US |
|-----------|----------------|----------------|
|           | Total During | Pre Bear | Pre | Post | Total During | Pre Bear | Pre | Post |
|           | Crisis | Stearns Lehman | Lehman | Lehman | Crisis | Stearns Lehman | Lehman | Lehman |
| EDF3      | 07/07-03/09 | 07/07-03/09 | 04/08-09/08 | 10/09-03/10 | 07/07-03/09 | 07/07-04/08-10/09 |
| Base      | 0.81% | 0.29% | 0.51% | 1.76% | 2.21% | 0.71% | 1.71% | 4.78% |
| %Change   | 178% | 1.19 | 1.94 | 2.4180917 | 134% | 117% | 176% | 122% |
| Target    | 2.26% | 0.65% | 1.50% | 6.03% | 5.19% | 1.53% | 4.73% | 10.62% |
| LGD       | 60% | 60% | 60% | 60% | 60% | 60% | 60% | 60% |
| %Change   | 139% | 94% | 149% | 195% | 112% | 90% | 143% | 114% |
| Target    | 143% | 117% | 149% | 177% | 127% | 114% | 146% | 128% |
| s         | 112.52 | 49.37 | 85.28 | 219.16 | 185.69 | 105.30 | 160.75 | 320.72 |
| %Change   | -58% | -48% | -60% | -66% | -53% | -47% | -59% | -53% |
| Target    | 47.10 | 25.47 | 34.31 | 74.34 | 87.80 | 55.35 | 66.20 | 150.00 |
| ρ         | 0.60 | 0.57 | 0.60 | 0.62 | 0.55 | 0.52 | 0.53 | 0.62 |
| %Change   | 65% | 43% | 64% | 99% | 60% | 44% | 72% | 83% |
| Target    | 0.99 | 0.82 | 0.99 | 1.24 | 0.88 | 0.75 | 0.91 | 1.14 |
5.3 Different PD estimates

PD estimates can be categorized into three approaches: Agencies ratings (Moody’s, Fitch, S&P), estimates derived from structural models of default (such as KMV EDFs) and hazard rate models. Agencies ratings are (partly) expert-based and are ‘through-the-cycle’ ratings, therefore they are not suited for this study. Hazard rate models similar to Shumway (2001) are especially popular for internal rating models of major banks. Recently, Fitch also launched its new equity-implied ratings (EIR). Fitch uses a hazard rate approach but also includes the distance-to-default-measure as one of the covariates. In addition, financial ratios, market performance and macro variables are used for default prediction.\footnote{For details see FitchRatings (2007).}

The resulting term structure of risk premia using Fitch EIR as a proxy for the real world default probability is shown in figure 5 together with the estimates based on Moody’s KMV for the same sample.\footnote{Fitch EIRs were only available to us until October 2008, therefore figure 5 only shows the comparison between the two estimates from April 2004 until October 2008.} Financials had to be excluded as they are not covered by Fitch EIR. In addition, estimates for the cumulative default probability are only available for up to 5 years maturity. Therefore the term structure of risk premia was calculated as (5-year CDS-implied Sharpe ratio) minus (3-year CDS-implied Sharpe ratio). Figure 5 shows a very similar pattern for estimates based on either Moody’s KMV or Fitch EIR. Indeed, the correlation coefficient between the two slopes are between 0.8 and 0.9. In both cases, the slope of the term structure is positive directly before the financial crisis (first half of 2007) and becomes negative during the financial crisis. Our results are therefore not due to the specific distance-to-default-based specification of Moody’s KMV but also holds if a larger amount of covariates is included for default prediction.

5.4 Regression analysis

The following regression controls for the influence of several financial parameters that have seen extraordinary levels during the 2007/2008 financial crisis. These
parameters may effect any of our input parameters, especially the estimate for the real world default probability. We control for the effect of the following parameters:

- **Implied Volatility (IV):** Volatility may not be captured correctly in the KMV EDF estimates since these are based on historical volatilities. Implied volatility is measured based on short-term at-the-money option prices from Bloomberg.

- **Skewness (SKEW):** KMV uses a volatility-based measure in their distance-to-default. A low skewness might therefore underestimate the default probability. Skewness is measured as the 180-days historical return skewness based on Datastream return data.

- **Information uncertainty (UNCERT):** Based on Duffie/Lando (2001), higher
information uncertainty leads to higher (short-term) default probabilities. Information uncertainty may have been larger during the 2007/2008 financial crisis, therefore leading to an underestimation of the true PDs. Information uncertainty is measured as the coefficient of variation of the 12-months ahead I/B/E/S earnings forecasts.\textsuperscript{19}

Unfortunately, a direct adjustment of the PD estimates is not possible. Instead an indirect approach is necessary. In the first step, the impact of these parameters (together with the EDF) on CDS spreads is measured for each maturity $\tau$ via a regression, i.e.\textsuperscript{20}

$$
\ln(CDS\text{spread}_\tau) = \beta_0 + \beta_1 \ln(EDF_\tau) + \beta_2 \ln(IV) + \beta_3 \text{SKEW} + \beta_4 \ln(UNCERT) + \epsilon.
$$

In a second step the CDS-spread for the ‘During Crisis’ subperiod are adjusted based on the assumption, that $IV$, $\text{SKEW}$ and $UNCERT$ would take average values of the ‘Before Crisis’ subperiod. Based on these adjusted (lower) CDS spreads, the adjusted 3-, 5-, 7- and 10-year Sharpe ratio estimates were determined. It should be noted that this is a very conservative robustness test: Some of these parameters may (and are likely to) be positively correlated with risk aversion. Subtracting the part of the CDS spread which is due to the high level of these parameters during the 2007/2008 financial crisis then also leads to a subtraction of a ‘risk premium part’ and not only to a subtraction of the ‘PD part’.

Indeed, the results from the regression are as expected: Especially $IV$ and $UNCERT$ have an economically significant impact on CDS spreads, cf. table 4. In addition, the influence is larger for shorter CDS maturities. The effects are qualitatively the same for Europe and the U.S. although they are more pronounced for the U.S. Adjusting the CDS spreads based on these regression results yields lower Sharpe ratio estimates with a slightly smaller downward sloping

\textsuperscript{19}The coefficient of variation for I/B/E/S forecasts for the next fiscal year end and the year after the next fiscal year end is interpolated in order to receive a constant maturity 12-month ahead coefficient of variation.

\textsuperscript{20}Berndt et.al. (2005) use a similar regression of CDS spreads on EDFs. They also show that a log/log formulation performs better than a linear regression.
Table 4: **Robustness: Regression analysis**

This table depicts the results of the regression \( \ln(CDS - spread_\tau) = \beta_0 + \beta_1 \ln(EDF_\tau) + \beta_2 \ln(IV_\tau) + \beta_3 \text{SKEW} + \beta_4 \ln(UNCERT_\tau) + \epsilon. \) \( \ln(EDF) \) denotes the logarithm of the expected default frequency for the respective maturity, \( \ln(IV) \) denotes the logarithm of the short-term option implied volatility, \( \text{SKEW} \) denotes the 180-days historical skewness in equity returns, \( \ln(UNCERT) \) denotes the 12-months ahead coefficient of variation of the I/B/E/S earnings forecasts. T-statistics are in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: Europe</th>
<th>Panel B: US</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Maturity</td>
<td>Maturity</td>
</tr>
<tr>
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<td>5</td>
</tr>
<tr>
<td>( \ln(EDF) )</td>
<td>0.19</td>
<td>0.22</td>
</tr>
<tr>
<td>( \ln(IV) )</td>
<td>0.50</td>
<td>0.49</td>
</tr>
<tr>
<td>(14.97)</td>
<td>(15.46)</td>
<td>(15.41)</td>
</tr>
<tr>
<td>( \text{SKEW} )</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>(4.60)</td>
<td>(4.41)</td>
<td>(4.77)</td>
</tr>
<tr>
<td>( \ln(UNCERT) )</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>(5.38)</td>
<td>(4.76)</td>
<td>(4.66)</td>
</tr>
</tbody>
</table>

However, the general statement of an inverse term structure of risk premia during the 2007/2008 financial crisis does not change based on the adjusted CDS spreads (cf. figure 6).

### 5.5 Liquidity, Market microstructure effects

In contrast to bonds, CDS are unfunded exposures without fixed supply and without large upfront payments. Both theoretical considerations as well as empirical studies indicate that liquidity has only a minor effect on CDS spreads (Longstaff et al. (2005), Ericsson et al. (2006) and Bühler/Trapp (2008)). However, in an OTC market like the CDS market, market microstructure liquidity effects may drive valuations either below or beyond the ‘fair’ market values due to supply/demand effects. These effects may be especially preeminent in turmoil periods such as the 2007/2008 financial crisis.
Figure 6: Term structure of risk premia before (04/2004-06/2007) and during (07/2007-03/2009) the 2007/2008 financial crisis for Europe (index: iTraxx Europe) and the U.S. (index: CDX.NA.IG) before and after adjustments as described in table 4. x-axis: maturity, y-axis: CDS-implied market Sharpe ratio based on (11).

This study uses two measures to capture the potential effect of market microstructure liquidity effects: First, the bid/ask spread. Second, the veracity score as provided by CMA. The veracity score is a trade indicator. A veracity score of '1' denotes that a trade has taken place, a veracity score of '2' denotes a firm quote, lower veracity scores indicate even lower liquidity, e.g. indicative quotes or bond-derived CDS spreads. Figure 7 plots these measures for all maturities for both the 'Before crisis' and 'During crisis' subperiod. As expected, both liquidity measures indicate that the 5-year maturity is the most liquid one (lowest bid/ask spread, lowest average veracity score). If market microstructure effects dominate our results we would expect U-shaped CDS-implied risk premia which are lowest for the most liquid - 5-year - maturity. In addition, there are even signs that trades during turmoil periods tend to concentrate in the most liquid maturity, i.e. 5-year CDS. Again, this result indicates that market microstructure effects are not the cause for the downward sloping risk premium curve.
6 Conclusion

This paper analyzes the term structure of risk premia before and during the 2007/2008 financial crisis. The analysis was based on a new approach proposed by Berg/Kaserer (2009). We show that the term structure of risk premia was flat before the 2007/2008 financial crisis and inverse during the crisis. These results are probably not surprising given the sentiment of market participants during the financial turmoil. However, the approach is - to our best knowledge - the first approach which is suited to monitor such a behavior of risk premia for different time horizons. Indeed, the standard errors of our estimates are small enough to see a significant difference in risk premium term structures before and during the 2007/2008 financial crisis.

Certainly, the 2007/2008 financial crisis is an unprecedented event in history. Therefore, quantitative research in such a period will always have its limits. It is possible that quantitative measures alone are not able to capture the dynamics
of certain parameters. E.g. market participants may have a ‘gut feeling’ about future default probabilities that are neither captured by agencies ratings nor by any quantitative procedure. This paper analyzes alternative sources for the main input parameters and tries to control for the parameters that can most reasonably be linked to the ‘abnormal’ situation during the crisis (implied volatility, skewness, information uncertainty). Of course, in these turbulent times room for misinterpretation remains. However, based on all evidence available, the main statements are quite robust. It would take extraordinary adjustments for any of the input parameters to come up with a risk premium term structure which is not inverse during the 2007/2008 financial crisis. Therefore we are confident that the main results correctly mirror the situation during the financial turmoil.

The resulting slope of the risk premium term structure may be useful for several applications: First, practitioners might use it as a simple turmoil indicator. E.g. if the slope is a lot smaller than zero there are two possible interpretations: Either standard methodology to estimate default probabilities or correlations does not work or short-term risk premia are indeed far above long-run levels. Both interpretations are likely to indicate a turmoil situation. This turmoil indicator may also be useful to assess if and when a turmoil has ended. Second, it can be used in asset allocation decisions as well as for asset pricing applications (e.g. company valuation).
A Estimating Sharpe ratios from CDS Spreads

A.1 Merton framework

This section briefly reviews the procedure to estimate Sharpe ratios from CDS spreads. It is mainly based on the (theoretical) results from Berg/Kaserer (2009). The asset value \( V_t \) is modelled as a geometric Brownian motion with volatility \( \sigma \) and drift \( \mu = \mu_V \) (actual drift) and \( r \) (risk-neutral drift) respectively, i.e.

\[
dV^P_t = \mu V_t dt + \sigma V_t dB^P_t \quad \text{and} \quad dV^Q_t = r V_t dt + \sigma V_t dB^Q_t,
\]

where \( B_t \) denotes a standard Wiener process. The company’s debt consists of a single zero-bond and default occurs if the asset value of the company falls below the nominal value \( N \) of the zero bond at maturity of the bond.

In this framework, the real world default probability \( PD^P(t, \tau) \) between \( t \) and \( T = t + \tau \) can be calculated as:

\[
PD^P(t, \tau) = P \left[ V_T < N \right] = \Phi \left[ \frac{\ln \frac{N}{V_t} - (\mu - \frac{\tau}{2} \sigma^2) \cdot \tau}{\sigma \cdot \sqrt{\tau}} \right]. \tag{16}
\]

Here, \( \Phi \) denotes the cumulative standard normal distribution function. The default probability under the risk-neutral measure \( Q \) can be determined accordingly as

\[
PD^Q(t, \tau) = Q \left[ V_T < N \right] = \Phi \left[ \frac{\ln \frac{N}{V_t} - (r - \frac{\tau}{2} \sigma^2) \cdot \tau}{\sigma \cdot \sqrt{\tau}} \right]. \tag{17}
\]

Combining (16) and (17) yields a formula for the asset Sharpe ratio \( SR_V \):

\[
SR_V := \frac{\mu - r}{\sigma} = \frac{\Phi^{-1}(PD^Q(t, \tau)) - \Phi^{-1}(PD^P(t, \tau))}{\sqrt{\tau}}. \tag{18}
\]

Using the CAPM relationship between the Sharpe ratio of the company’s assets \( (SR_V) \), the market Sharpe ratio \( (SR_M) \) and the correlation between asset
returns and market returns \( (\rho_{V,M}) \)

\[
SR_V = SR_M \cdot \rho_{V,M}
\]

yields an estimator for the market Sharpe ratio:

\[
\hat{\gamma}_{SR,M\text{,Merton}} := \frac{\Phi^{-1}(PD^Q(t,\tau)) - \Phi^{-1}(PD^P(t,\tau))}{\sqrt{\tau}} \frac{1}{\rho_{V,M}} \tag{19}
\]

An interesting feature of this estimator is its robustness with respect to model changes. The estimator uses the difference between (the inverse of the cumulative normal distribution of the) risk-neutral and actual default probability. This difference is barely effected by the choice of the specific structural model of default. Huang/Huang (2003) have first demonstrated this robustness based on different models, including a model with jumps in the asset value process and a time-varying equity premium. Berg/Kaserer (2009) analyze a first-passage time framework and a model with unobservable asset values based on Duffie/Lando (2001) to show this robustness.

A.2 Time-varying risk premia

If Sharpe ratios are time-varying then the derivation of the Merton framework does not hold anymore. However, it can be shown that the formula from the Merton framework still approximates the average expected Sharpe ratio if Sharpe ratios follow a mean-reverting process (e.g. CIR process) with reasonable parameters. For ease of notation \( \sigma_V = \sigma, \rho_{V,M} = \rho \) and \( W^V_t = W_t \) is used. The real world default probability can be derived as

\[
P[V_{t+\tau} < L] = P\left[ V_t \cdot e^{\int_t^{t+\tau} \sigma \theta_s ds + \int_t^{t+\tau} r_s - \frac{1}{2} \sigma^2 ds + \sigma W^V} < L \right] \\
= P\left[ \sigma W_t + \rho \sigma \int_t^{t+\tau} \theta_s ds < \ln \left( \frac{L}{V_t} \right) - (r - 0.5\sigma^2)\tau \right] \\
\approx \Phi \left[ \frac{\ln \left( \frac{L}{V_t} \right) - \sigma \rho E^P \left[ \int_t^{t+\tau} \theta_s ds \right] - (r - 0.5\sigma^2)\tau}{\sigma \sqrt{\tau}} \right]
\]
In the last row the approximation

\[
\begin{align*}
Var \left( \sigma W_\tau + \rho \sigma \int_t^{t+\tau} \theta_s ds \right) \\
= Var(\sigma W_\tau) + \rho^2 \sigma^2 Var \left( \int_t^{t+\tau} \theta_s ds \right) + 2Cov \left( \sigma W_\tau, \int_t^{t+\tau} \theta_s ds \right) \\
\approx Var(\sigma W_\tau)
\end{align*}
\]  

(20)  

(21)  

(22)

is used to substitute the integral by its expected value \( E^P \left[ \int_t^{t+\tau} \theta_s ds \right] \). The approximation is justified since \( \theta_s \) is mean-reverting and the volatility of \( \rho \sigma \int_t^{t+\tau} \theta_s ds \) is 'a lot' smaller than the volatility of \( \sigma W_\tau \). In addition, the covariance term will usually be negative if it is assumed that negative equity returns go hand in hand with an increase in risk aversion.

Accordingly, the risk-neutral default probability can be calculated as

\[
Q[V_{t+\tau} < L] = \Phi \left[ \frac{ln(L/V_t) - (r - 0.5\sigma^2)\tau}{\sigma \sqrt{\tau}} \right]
\]

so that

\[
\frac{\Phi^{-1}(PDQ(t, \tau)) - \Phi^{-1}(PD^P(t, \tau))}{\frac{1}{\rho}} \approx \frac{\sigma E^P \left[ \int_t^{t+\tau} \theta_s ds \right]}{\tau \sigma \rho}
\]

\[
= \Theta(t, \tau)
\]

The same result was shown by Huang/Huang (2003) based on a specific calibration of a mean-reverting model.
B  Details on the estimation of CDS-implied Sharpe ratios

Table 5 provides details on the Sharpe ratio estimation from CDS spreads. For a detailed discussion of the general characteristics and a comparison to other Sharpe ratio estimates we refer to an in-depth analysis in Berg/Kaserer (2009). It should, however, be noted that the average Sharpe ratio of 30-50% before the crisis is slightly lower than comparable historical equity premium estimates and slightly higher than equity-implied Sharpe ratio estimates, the cross-sectional variation is also in line with results from other studies.\footnote{Cf. e.g. Claus/Thomas (2001), Fama/French (2002) and Dimson et al. (2006) for comparisons with historical Sharpe ratios/equity premia and Gebhardt et al. (2001), Fama/French (2002) and Botosan/Plumlee (2005) for equity-implied Sharpe ratios/equity premia.}

Figure 8 provides a time-series plot of the 25th and 75th-percentile of the 5-year CDS-implied Sharpe ratios. It should be noted, that we have assumed a fixed recovery rate which is not dependent on the industry sector or other firm characteristics. Although the literature on recovery estimates is still evolving, there is significant evidence of cross-sectional variation in recovery rates between different industry sectors, cf. Fitch (2005). Our estimator is convex in the recovery rate, but the convexity is not very pronounced. Therefore, the average Sharpe ratio estimate should not be significantly biased. However, some part of the cross-sectional variation of our Sharpe ratio estimates is likely due to this constant recovery rate assumption and we expect a smaller variation if industry-specific recovery rates are applied.\footnote{In case that this methodology is applied to analyze cross-sectional variations in the cost of capital we see a more sophisticated modeling of the cross-sectional variations in the recovery rate as one of the key challenges.}

Finally, we report the results for the term structure of risk premia separately for financials and non-financials. The results are shown in figure 9. It can be seen that - for both the U.S. as well as Europe - the resulting slope of the risk premium term structure is similar for financials and non-financials.
Table 5: Descriptive statistics for Sharpe ratio estimation

The sample consists of the intersection of the KMV database, the iTraxx on-the-run companies (Europe)/CDX.NA.IG on-the-run companies (U.S.) and the CMA CDS database (via Datastream) from April 2004 to March 2009. $SR_{3/5/7/10}$ denote 3-/5-/7-/10-year CDS-implied Sharpe ratios.

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<tr>
<td></td>
<td>N</td>
<td>Mean</td>
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<tr>
<td>Panel A: Europe</td>
<td></td>
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<tr>
<td>SR 3</td>
<td>15,103</td>
<td>33.28%</td>
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<td>SR 5</td>
<td>15,103</td>
<td>32.69%</td>
</tr>
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<td>SR 7</td>
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<td>SR 10</td>
<td>15,103</td>
<td>32.26%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>SR 3</td>
<td>14,416</td>
<td>51.88%</td>
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<td>SR 5</td>
<td>14,416</td>
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<td>SR 7</td>
<td>14,416</td>
<td>52.10%</td>
</tr>
<tr>
<td>SR 10</td>
<td>14,416</td>
<td>50.31%</td>
</tr>
</tbody>
</table>
Figure 8: 25th and 75th percentile of CDS-implied Sharpe ratios based on 5-year CDS spreads and EDFs. Picture on left-hand side: Europe, picture on right-hand side: U.S. Solid line: Median estimate, dotted lines: 25th and 75th percentile.

Figure 9: Slope of risk premium term structure measured as 10-year CDS-implied Sharpe ratio minus 3-year CDS-implied Sharpe ratio for financials and non-financial companies. Picture on left-hand side: U.S., picture on right-hand side: Europe. Solid line: Estimate based on financial companies, dotted line: Estimates based on non-financial companies.
References


DISCRETIONARY FISCAL POLICIES OVER THE CYCLE
NEW EVIDENCE BASED ON THE ESCB DISAGGREGATED APPROACH

by Luca Agnello and Jacopo Cimadomo