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Can non-linear real shocks explain the persistence of PPP exchange rate disequilibria?

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Can non-linear real shocks explain the persistence of PPP exchange rate disequilibria?

by Tuomas A. Peltonen,
Adina Popescu
and Michael Sager
CAN NON-LINEAR REAL SHOCKS EXPLAIN THE PERSISTENCE OF PPP EXCHANGE RATE DISEQUILIBRIA? ¹

by Tuomas A. Peltonen², Adina Popescu² and Michael Sager³

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Abstract

A core stylized fact of the empirical exchange rate literature is that half-life deviations of equilibrium real exchange rates from levels implied by Purchasing Power Parity (PPP) are very persistent. Empirical efforts to explain this persistence typically proceed along two distinct paths, resorting either to the presence of real shocks such as productivity differentials that drive equilibrium exchange rates away from levels implied by PPP, or the presence of non-linearities in the adjustment process around PPP. By contrast, we combine these two explanations in the context of an innovative panel estimation methodology. We conclude that both explanations are relevant to the behavior of exchange rates and that resulting half-lives are much shorter than estimated using linear PPP and more consistent with the observed volatility of nominal and real exchange rates.

Keywords: EPSTAR, exchange rate, PPP, Balassa-Samuelson, productivity

JEL Classification: F31, C23, L6-L9
Non-technical summary

This paper reappraises the persistence of real exchange rate deviations from levels implied by Purchasing Power Parity (PPP) by employing a non-linear productivity-augmented PPP specification, within the context of an innovative panel estimation technique.

The existing empirical literature addresses this persistence in one of two ways. First, by modeling the real exchange rate as a non-linear process that oscillates around a PPP-implied level such that the tendency to mean-revert towards this equilibrium will be faster in the presence of larger deviations (Michael, Nobay and Peel, 1997; O’Connell and Wei, 1997; Taylor and Peel, 2000; Taylor, Peel and Sarno, 2001; Sarno and Taylor, 2002; Leon and Najarian, 2005; Lothian and Taylor, 2007). This non-linearity may reflect the presence in the foreign exchange market of investor heterogeneity, for instance due to differences in the transaction costs faced by the various investor groups or the speed with which each group learns of an emerging disequilibrium. Investor heterogeneity means that the perceived profits from arbitraging fundamental disequilibria will vary at any given time between market participants.

Second, by incorporating into the basic linear specification the impact of real economy shocks that drive the exchange rate and its equilibrium value away from PPP. These shocks will typically be supply related (Beveridge and Nelson, 1991), for instance productivity shocks in either the domestic or foreign economy. Intuitively, this augmentation assumes that the real exchange rate mean-reverts around a stochastic trend equilibrium determined by intra- and inter-country productivity differentials rather than the constant equilibrium level implicit in PPP. It also assumes, therefore, that PPP is misspecified. Relevant in this respect are the results of Coakley, Flood, Fuertes and Taylor (2005), who find in favor of generalized Relative PPP.

The main contribution of this paper is to combine both these augmentations to linear PPP using an innovative non-linear panel estimation procedure. Accordingly, our null hypothesis is that concurrent introduction of both augmentations - real shocks and a non-linear functional form - within a panel framework can robustly reduce the half-life persistence of real exchange rate deviations compared with linear PPP.

We report three important findings that extend the existing literature. First, a substantial reduction in half-life persistence, to approximately one half the level found using linear PPP. Second, that estimated half-lives are generally shortened by adoption of both augmentations rather than employing either in isolation, indicating that functional form and the incidence of productivity shocks are both important in
determining the speed at which real exchange rates revert back towards equilibrium following a shock. Our results suggest, however, that adoption of a non-linear functional form is the most important augmentation. And third, that the sign of the relationship between OECD real exchange rates and productivity shocks contradicts the prediction of the Balassa-Samuelson hypothesis, and instead is consistent both with a rapid assimilation of technological advances that bears down on non-traded sector price levels, and a growing proliferation of pricing-to-market (PTM) strategies in the traded sector that contradict the Law of the One Price (LOOP).
1 Introduction

This paper reconsiders the persistence of real exchange rate disequilibria relative to levels implied by Purchasing Power Parity (PPP). Employing a non-linear productivity-augmented PPP specification, within the context of an innovative panel estimation technique, we conclude that reversion of real exchange rates to time-varying equilibria occurs non-linearly and at speeds more consistent with the volatility of nominal and real exchange rates than reported in the existing linear PPP literature. We also report robust evidence of a negative correlation between productivity differentials and real exchange rates that contradicts the mainstay Balassa-Samuelson hypothesis.

The existing empirical literature addresses the persistence of real exchange deviations implied by PPP by augmenting the linear model in one of two ways. First, by modeling the real exchange rate as a non-linear process that oscillates around a PPP-implied level such that the tendency to mean-revert towards this equilibrium will be faster in the presence of larger deviations (Michael, Nobay and Peel, 1997; O’Connell and Wei, 1997; Taylor and Peel, 2000; Taylor, Peel and Sarno, 2001; Sarno and Taylor, 2002; Leon and Najarian, 2005; Lothian and Taylor, 2007). This non-linearity may reflect the presence in the foreign exchange market of investor heterogeneity, for instance due to differences in the transaction costs faced by the various investor groups—for instance, mutual funds, hedge funds, inter-dealer brokers and corporations—or the speed with which each group learns of an emerging disequilibrium (Sager and Taylor, 2006). Investor heterogeneity means that the perceived profits from arbitraging fundamental disequilibria will vary at any given time between market participants.

Second, by incorporating into the basic linear specification the impact of real economy shocks that drive the exchange rate and its equilibrium value away from PPP. These shocks will typically be supply related (Beveridge and Nelson, 1991), for instance productivity shocks in either the domestic or foreign economy. Intuitively, this augmentation assumes that the real exchange rate mean-reverts around a stochastic trend equilibrium determined by intra- and inter-country productivity differentials rather than the constant equilibrium level implicit in PPP. It also assumes, therefore, that PPP is misspecified. Relevant in this respect are the results of Coakley, Flood, Fuertes and Taylor (2005), who find in favor of generalized Relative PPP. Relevant in this respect are the results of Coakley, Flood, Fuertes and Taylor

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1 Although these augmentations are not mutually exclusive, the literature generally treats them as such. Two exceptions are Sager (2006) and Lothian and Taylor (2007).
Equation (1) allows for the possibility of a unit root in $\varepsilon_t$ reflecting, *inter alia*, the existence of other variables that cointegrate with the real exchange rate.

The main contribution of this paper is to *combine* both these augmentations to linear PPP using an innovative non-linear panel estimation procedure. Accordingly, our null hypothesis is that concurrent introduction of both augmentations to linear PPP—real shocks and a non-linear functional form—within a panel framework can robustly reduce the half-life persistence of real exchange rate deviations from implied equilibrium levels compared to the persistence of traditional linear PPP disequilibria. Empirical evidence presented below validates this hypothesis, with estimated half-lives of large disequilibria approximately one half as persistent as those generated by a traditional linear PPP analysis, and more consistent with the observed volatility of nominal and real exchange rates. We are aware of only two other contributions—Sager (2006) and Lothian and Taylor (2007)—that attempt to combine these augmentations to linear PPP. Both contributions also find in favor of a substantial reduction in estimated half life deviations compared with linear PPP. Unlike our study, however, neither explicitly considers non-traded and traded sector productivity data as suggested by the Balassa-Samuelson hypothesis (henceforth Balassa-Samuelson; Balassa, 1964; Samuelson, 1964), but rather traded sector data alone. In addition, whereas Lothian and Taylor (2007) is a long-span univariate study of two major exchange rates, and Sager (2006) a relatively short span univariate investigation of four major exchange rates, in this paper we employ an innovative panel estimation technique and 23 OECD exchange rates. Overall, we consider our results to be complementary to Sager (2006) and, particularly, Lothian and Taylor (2007), and an important extension to the existing literature.

The remainder of the paper is organized as follows. In Section 2 we describe our estimation methodology, and in Section 3 we discuss details of our dataset. We present and discuss estimation results in Section 4, and Section 5 concludes.

## 2 Estimation Methodology

Our empirical analysis is based on four model specifications: linear PPP, which is the benchmark against which we assess the performance of the other three specifications; linear PPP augmented by intra- and inter-country productivity differentials; non-linear PPP; and then non-linear PPP augmented by intra- and inter-country
productivity differentials.

Linear PPP has been widely analyzed in the literature, using both panel and univariate estimation methods. For comprehensive surveys of methods and associated results, see Froot and Rogoff (1994), Taylor (1995), Sarno and Taylor (2002), Taylor (2003) and Taylor and Taylor (2004). In general, using appropriately powerful tests early long-span univariate empirical studies conclude in favor of PPP (Abuaf and Jorion, 1990; Froot and Rogoff, 1994; Lothian and Taylor, 1996; Taylor, 2002). By contrast, the latest long-span evidence, reported by Lothian and Taylor (2007), is more equivocal towards PPP and presents evidence of persistent disequilibria for one of the two exchange rates examined—sterling-dollar—that is inconsistent with PPP and instead fits with the predictions of Balassa-Samuelson.\(^2\)

In order to generate our linear PPP benchmark results, we adopt a fixed effects panel estimation methodology incorporating White serial correlation and heteroskedasticity robust standard errors. The linear PPP model is simply an autoregression of the real exchange rate on lags of itself,

\[
q_{it} = \sum_{j=1}^{p} \beta_j q_{it-j} + u_{it}
\]  

where \(q_{it}\) is the real exchange rate for country \(i\) at time \(t\), \(p\) is the number of lags and the error term \(u_{it} = \eta_i + \epsilon_{it}\), where \(\eta_i\) is the country-specific effect and \(\epsilon_{it} \sim (0, \Sigma_i)\).

2.1 Augmentations to Linear PPP

2.1.1 Productivity Augmented Linear PPP

The main contribution of this paper is to augment linear PPP with a combination of both real shocks that drive the equilibrium exchange rate away from the level implied by PPP, and a non-linear functional form. Consistent with the conclusions of Beveridge and Nelson (1991), real shocks are often supply related, for instance to productivity differentials between the Home and Foreign economies. This is the approach that we adopt here. A significant empirical literature also considers the impact of shocks to

\(^2\)The paper by Lothian and Taylor (2007) is related to our paper. However, there are two main differences. First, whereas Lothian and Taylor (2007) model two exchange rates versus the US dollar over a long data span (1820-1998 for French franc and 1820-2001 for the UK pound sterling), our paper uses data for 23 OECD economies over the period 1980-2003. Second, Lothian and Taylor (2007) proxy productivity differentials across the three countries (France, the UK and the US) using real GDP per capita data. Instead, our paper uses disaggregated sectoral data (two digit NACE) for 23 countries in constructing productivity variables for traded and non-traded sectors. Given the significant differences in data and methodologies between the two papers, our paper is an important complement to Lothian and Taylor (2007).
shorter-term demand-related variables such as the Terms of Trade—defined as the ratio of export to import prices, both expressed in domestic currency terms—General Government Final Consumption Expenditure and Net Foreign Assets (NFAs). Another strand of the literature considers deviations from PPP due to real interest rate differentials (for instance, Baxter, 1994, and Clarida and Gali, 1994). As we wish to concentrate our analysis on an examination of the behavior of equilibrium exchange rates, consistent with Beveridge and Nelson (1991) we augment the basic linear PPP equation only with sector-based productivity differentials.

Assessments of the impact of productivity shocks upon real exchange rate equilibria usually afford a prominent role to Balassa-Samuelson. Although a critique of this hypothesis is not our primary objective, we consider the relationship between real exchange rates and productivity differentials in this context. The standard Balassa-Samuelson equation expresses the real exchange rate as a function of productivity differentials, $\zeta_{it}$, between the traded and non-traded sectors in the Home economy relative to the same ratio in the Foreign country,

$$q_{it} = \alpha + \beta \zeta_{it} + u_{it}$$ (3)

where, from above, $q_{it}$ is the log real exchange rate of country $i$ and,

$$\zeta_{it} = \gamma(a_{it}^T - a_{it}^N) - \gamma^*(a_{it}^{T*} - a_{it}^{N*}).$$ (4)

$\gamma$ is equal to the the share of traded output in the consumption expenditure of the Home economy and $a^T$ and $a^N$ are the log of traded and nontraded productivity levels, also in the Home country; the * superscript denotes the corresponding variables in the US, our Foreign economy. Following Obstfeld and Rogoff (1996), it is common to assume that that the weight of nontraded goods and services is equivalent in the Home and Foreign country price levels—so that $\gamma = \gamma^*$. Similarly, production functions for traded and nontraded goods and services are assumed identical for both Home and Foreign countries; that is, $\theta^N = \theta^T$ and $\theta^{N*} = \theta^{T*}$. As Mihaljek and Klau (2008) argue, these are both relatively restrictive assumptions.

Arguably, shocks to the Terms of Trade could be classified as either demand- or supply related (for instance, DeLoach, 2001).

We also recognize, but do not consider, the possibility that the persistence of exchange rate deviations from PPP may also reflect a number of important statistical issues, including the low power of conventional unit root tests (Lothian and Taylor, 1996), the relatively short data span and data measurement error.

Although a wide range of productivity measures have been used in the empirical exchange rate literature, most are relatively blunt proxies and do not incorporate sector-based series (for instance, Lothian and Taylor, 2007), in contradiction of Balassa-Samuelson and in contrast to our approach here. This is an important shortcoming of much of the existing literature.
For recent discussions of the core assumptions and implications of Balassa-Samuelson, see Lothian and Taylor (2007) and Peltonen and Sager (2009). We present here only a summary of its main implications. The mechanics of Balassa-Samuelson are triggered by a rise in the level of traded sector labor productivity in the Home economy. With real wage levels equal to their marginal product and equivalent across both sectors of each economy, this shock implies an increase in wages in the Home economy. As the Law of the One Price (LOOP) is assumed to hold in the traded sector of both economies, price levels in this sector remain unchanged. But absent a commensurate and concurrent rise in the level of domestic non-traded sector productivity, rising wages push up the average price level in this sector as firms act to maintain prices equal to marginal costs. This in turn raises aggregate domestic price levels and, assuming the nominal exchange rate is sticky, appreciates the real value of the domestic currency relative to its PPP-implied level. By implication, currencies of relatively more productive countries may trade above levels implied by PPP for extended periods.

2.1.2 Non-Linear Mean Reversion (EPSTAR)

In parallel to studies that attempt to reduce half-life estimates of shocks to linear PPP by augmenting the basic specification with real variables such as productivity differentials, another strand of the literature introduces a non-linear functional form into the traditional linear PPP framework. To this end, our EPSTAR model extends the univariate ESTAR specification of Lothian and Taylor (2007) to a panel framework. To our knowledge, this is the first application of a non-linear PPP model in a panel framework. Our specification is similar to the panel smooth transition autoregression (STAR) model of Fok, van Dijk and Franses (2005). The general form of the fully heterogenous panel specification can be written as

$$y_{it} = \sum_{j=1}^{p} \beta_{ij} y_{lt-j} + \Phi(z_{it-d_i}, \theta_i) \sum_{j=1}^{p} \beta_{ij} y_{lt-j} + u_{it}$$

(5)

where $y_{it}$ is the dependent variable, $\Phi(z_{it-d_i}, \theta_i)$ is a continuous and smooth transition function, with $\Phi$ bounded by 0 and 1, $p$ is the lag length, and the value of $d$ is chosen to maximize the probability of rejection of the null hypothesis of linearity. The error term $u_{it}$ is equal to $\eta_i + \epsilon_{it}$, where $\eta_i$ is the country-specific effect and $\epsilon_{it} \sim (0, \Sigma_i)$. The STAR model framework encompasses a continuum of regimes, with researchers typically assuming two extreme regimes within this continuum. The parameter $\theta$ governs the speed of transition between regimes embedded in the model and takes a value $0 \leq \theta \leq \infty$; lower values of $\theta$ imply slower adjustment between regimes.
One extreme regime, called the Inner Regime, prevails when \( y_{it} \) are close to their equilibrium value—that is when \( \Phi(z_{it-d}, \theta) \to 0 \)—such that \( y_{it} \) will be a function of the AR process given by,

\[
y_{it} = \sum_{j=1}^{p} \beta_{ij} y_{it} + u_{it} \tag{3a}
\]

In the Inner Regime, \( y_{it} \) are often characterized as exhibiting unit root, or even explosive behavior, implying that \( \beta_{ij} \geq 1 \). The second extreme regime—the Outer Regime—prevails when \( \Phi(q_{it-d}, \theta) \to 1 \) so that \( y_{it} \) exhibit large disequilibria and are determined by the alternative, non-linear AR process,

\[
y_{it} = \sum_{j=1}^{p} (\beta_{ij} + \hat{\beta}_{ij}) y_{it} + u_{it} \tag{3b}
\]

in which the speed of mean reversion is given by \( \beta_{ij} + \hat{\beta}_{ij} < 1 \). Consequently, the tendency to revert back to equilibrium in our panel STAR framework is a positive function of the magnitude of disequilibria, as is the case also in univariate STAR models (O’Connell and Wei, 1997; van Dijk, Teräsvirta and Franses, 2002; Taylor et al., 2001; Sarno and Taylor, 2002). As discussed above, this characteristic is consistent with the existence in the foreign exchange market of substantial investor heterogeneity (Sager and Taylor, 2006).

Following Granger and Teräsvirta (1993), Teräsvirta (1994) and Jansen and Teräsvirta (1996), empirical characterization of the adjustment function associated with STAR models normally concentrates upon the exponential and logistic functions. Under the exponential function, \( \Phi(q_{it-d}, \theta) \) is U-shaped and symmetric, such that the rate at which the real exchange rate reverts back to its equilibrium value following a shock will be equivalent for large positive and negative disequilibria of similar size. By contrast, the logistic function is asymmetric, implying that large positive and negative disequilibria are arbitraged at different speeds. Although the logistic function is intuitive in the context of real economic variables, \textit{a priori} there is no good reason to believe that undervaluations of real exchange rates relative to equilibrium are on average arbitraged away at different speeds than overvaluations. Accordingly, the exponential function is our preferred functional form, although we do test both forms as a robustness check on our results. Thus, the transition function in our application can be written,

\[
\Phi(y_{it-d_i}, \theta_i) = 1 - \exp(-\theta_i y_{it-d_i})^2 \tag{6}
\]

where \( \theta_i \) and \( d_i \) are defined above.

Estimating the fully heterogenous model (5) is infeasible due to the large number of parameters to be estimated with the limited data sample available to us (for a
full description of our database, see the following section). Accordingly, we make the
model specification more parsimonious by a partial pooling of parameters that allows
only for country fixed effects \( \eta_i \) and by imposing homogeneity on the estimated \( \beta_s \),
the lag length \( p \), the delay parameter \( d \) and on \( \theta \), the slope of transition function.\(^6\)
These simplifying assumptions mean that the speed of adjustment of real exchange
rates towards equilibria in the wake of a shock will be homogenous across all country
members in our panel. Consequently, the model specification becomes,

\[
y_{it} = \alpha + \sum_{j=1}^{p} \beta_j y_{it-j} + \Phi(y_{it-d}, \theta) \sum_{j=1}^{p} \beta_j y_{it-j} + u_{it}.
\]  

(7)

where, in our EPSTAR specification, \( y_{it} \) are defined as the absolute deviation of real
exchange rates from their equilibrium value,

\[
y_{it} = q_{it} - \mu_i
\]

(8)

with \( q_{it} \) equal to the log real exchange rate of country \( i \) versus the US dollar, and
\( \mu_i \) its the long-run mean. Substituting (8) into (5) gives our simplified EPSTAR
estimation specification as,

\[
q_{it} - \mu_i = \alpha + \sum_{j=1}^{p} \beta_j (q_{it-j} - \mu_i) + \Phi(q_{it-d} - \mu_i, \theta) \sum_{j=1}^{p} \beta_j (q_{it-j} - \mu_i) + u_{it}
\]

(9)

In order to estimate this equation we need to determine the values of \( p \) and \( d \). We
select \( p \) based on country-specific partial autocorrelation functions. Consistent with
intuition, we find little evidence of serial correlation beyond the first-order for all
countries in the sample and accordingly set \( p = 1 \) for all panel members, as per our
simplifying homogeneity assumption above. We select \( d \) using the testing procedure
detailed by Granger and Teräsvirta (1993) and Teräsvirta (1994), on the basis of
Lagrange Multiplier-type linearity tests, where \( d \) is chosen to maximize the rejection
of the null hypothesis of linearity, \( H_0: \theta = 0 \), in favour of the null of a STAR-type
non-linearity. This procedure begins by replacement of the nonlinear function \( \Phi(\cdot) \)
with a Taylor expansion to give the auxiliary regression,

\[
q_{it} - \mu_i = \alpha + \beta_1 (q_{it-1} - \mu_i) + \beta_1 (q_{it-1} - \mu_i) (q_{it-d} - \mu_i) + \beta_2 (q_{it-1} - \mu_i) (q_{it-d} - \mu_i)^2 + u_{it}
\]

(10)

\(^6\)To the best of our knowledge, appropriate tests for heterogeneity, cross-sectional dependence
and cross-sectional heteroskedasticity have not yet been developed for non-linear panel models of
the type that we estimate. We leave this development for future research. In the absence of such
rigorous theory, our approach will be to check empirical estimates for evidence of robustness to
various subsamples of our dataset, and also to additional controls.
where \( d = 1, \ldots, 5 \). Testing for \( H_0 : \theta = 0 \) or \( H_0 : \beta_j = 0 \) in the original nonlinear model (7) is equivalent to a Langrange Multiplier test of \( H_0 : \beta_1 = \beta_2 = 0 \) in the auxiliary regression (10).\(^7\) Based upon estimated p-values, we select \( d = 2 \) (see the first column of Table 6). We also estimate a third order Taylor expansion of (10) to formally select between exponential and logistic transition functions; consistent with economic intuition we find that mean reversion for our panel of exchange rates is symmetric and, as noted above, adopt an exponential transition function in the empirical analysis that follows. With the selected values of \( p \) and \( d \), our estimation equation simplifies to

\[
q_{it} - \mu_i = \beta_1(q_{it-1} - \mu_i) + \beta_1[1 - \exp(-\theta(q_{it-2} - \mu_i)^2)](q_{it-1} - \mu_i) + u_{it}. \tag{11}
\]

The EPSTAR model is estimated using non-linear least squares (NLLS), and with starting parameter values based on those used in the existing univariate estimation literature. Experimentation with different starting values yielded identical results, indicating that the likelihood function converges to a global maximum. Cross-country heterogeneity is taken into account during estimation by removing country fixed effects and performing the specified regression on the demeaned series. Additionally, we allow for cross-section heteroskedasticity as well as serial correlation and thus report serial correlation and heteroskedasticity robust standard errors in results tables.

### 2.1.3 Productivity Augmented Non-Linear PPP

**BS-EPSTAR** Estimation of univariate non-linear PPP ESTAR models has proliferated in recent years; van Dijk et al. (2002) provide an excellent survey of results. By contrast, to our knowledge this is the first paper to estimate a non-linear Balassa-Samuelson model using a variant of the panel STAR estimation methodology of Fok et al. (2005). Our non-linear productivity-augmented PPP model is specified as,

\[
q_{it} - \zeta_{it} = \alpha + \sum_{j=1}^{p} \beta_j(q_{it-\tau - j} - \zeta_{it-\tau - j}) + \Phi(q_{it-\tau - d}, \zeta_{it-\tau - d}, \theta) \sum_{j=1}^{p} \beta_j(q_{it-\tau - j} - \zeta_{it-\tau - j}) + u_{it}. \tag{12}
\]

The transition function \( \Phi(q_{it-\tau - d}, \zeta_{it-\tau - d}, \theta) \) is defined as,

\[
\Phi(q_{it-\tau - d}, \zeta_{it-\tau - d}, \theta) = 1 - \exp \left(-\theta(q_{it-\tau - d} - \zeta_{it-\tau - d})^2 \right) \tag{13}
\]

where \( \zeta_{it} \) is the time-varying productivity term given by (4) above. With \( \gamma = \gamma^* \), our specification becomes,

\[
\zeta_{it} = \gamma[(a_{it}^T - a_{it}^{T*}) - (a_{it}^N - a_{it}^{N*})] = \gamma \nu_{it} \tag{14}
\]

\(^7\)As discussed in Luukkonen, Saikkonen, and Teräsvirta (1988), tests of either alternative null hypothesis in the original non-linear specification are nonstandard because under the null the model contains unidentified nuisance parameters.
Combining (12), (13) and (14), and determining appropriate values for \( p \) and \( d \) as above, gives our BS-EPSTAR estimation equation as,

\[
q_{it} - \gamma v_{it} = \alpha + \beta_1 (q_{it-1} - \gamma v_{it-1}) + \beta_1 [1 - \exp(-\theta(q_{it-2} - \gamma v_{it-2})^2)](q_{it-1} - \gamma v_{it-1}) + u_{it}.
\]

As for the EPSTAR model above, we estimate the BS-EPSTAR specification using NLLS with serial correlation and heteroskedasticity robust standard errors.

### 3 Data Description

Our empirical analysis is based upon annual data over the sample period 1980 to 2003 for 23 OECD countries.\(^8\) As detailed in Table 1, our panel consists of 14 EU Member States, as well as Australia, Canada, Iceland, Japan, Mexico, New Zealand, Norway, South Korea and the United States, which is our numéraire currency. In addition to performing our empirical analysis on the panel as a whole, we also examine real exchange rate behavior for two country sub-groups, again both versus the US dollar: first, European Economic and Monetary Union (EMU) and Exchange Rate Mechanism (ERM) countries; and second, all countries except the commodity currencies.\(^9\) This will allow us to compare the behavior of exchange rates involving the currencies of these country groups, and make inferences about commodity exchange rates as well. Again, Table 1 provides a complete listing of the country composition of these two sub-groups.

Real exchange rates are expressed in terms of the US dollar, calculated using GDP deflators and defined as the dollar price of domestic currency, so that an increase of the real exchange rate is equivalent to an appreciation of the domestic currency. GDP deflators and nominal US dollar exchange rates were obtained from the OECD Reference Series database, with the exception of Euro Area currencies, for which the data source is Bloomberg.

Sector labor productivity variables were obtained from the OECD STAN Indicators 2005 database, with the labor productivity index (OECD STAN code \( LPDTY \)) for country \( c \) at time \( t \) and for sector \( i \) calculated as value added at constant prices

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\(^8\)Seven OECD countries—the Czech Republic, Hungary, Ireland, Poland, Slovakia, Switzerland and Turkey—are excluded from the analysis due to insufficient data.

\(^9\)Commodity currencies are defined as the Australian and Canadian dollars, Mexican peso, Norwegian krone and New Zealand dollar.
(\textit{VALUK}) divided by the total number persons engaged \((\textit{EMPN}),^{10}\)

\[ LPDTY_{t,i}^C = \frac{VALUK_{t,i}^C}{EMPN_{t,i}^C} \quad (16) \]

Table 2 lists the division of sectors into traded and non-traded used in our empirical analysis.\(^{11}\) One advantage of using the OECD STAN database is that the researcher is able to construct traded and non-traded sector aggregates from the component data, rather than using proxy definitions such as "manufacturing" or "services" (e.g. Peltonen and Sager, 2009). Labor productivity variables for composite traded and non-traded sectors are weighted using the relevant sector weights in total nominal value added \((\textit{VALU}),\)

\[ VASH_{t,i}^C = \frac{VALU_{t,i}^C}{VALU_{t,\text{total}}^C} \quad (17) \]

Productivity and real exchange rate data are plotted in Figure 2 and descriptive statistics for these series are presented in Tables 3.1 and 3.2. These suggest that our panel encompasses a relatively homogeneous set of countries; as our panel constituents are all OECD members, and a large subset are EMU members, this is consistent with our prior expectation.\(^{12}\)

We construct dummy variables to control for the introduction of EMU and for currency crises \((\text{CRISIS})\) that periodically afflict some of the real exchange rates in our panel. Our EMU dummy is set equal to one from 1999 onwards and to zero previously for the twelve EU Member States that joined EMU in that year; for Greece, this dummy equals one in 2001-03, and zero before, reflecting this country’s delayed EMU entry. Periods of currency crises are defined as years when the real value of the domestic currency changes by more than 2 country-specific standard deviations—positive or negative—in year-on-year terms.

4 Empirical Results

We begin in standard fashion by testing the order of integration of real exchange rate and productivity series, using the panel unit root tests of Levin, Lin and Chui (2002)\(^{10}\)

\(^{10}\)In the case of Mexico and the United Kingdom, the number of employees was used instead, due to data limitations.

\(^{11}\)Results reported below are robust to various alternative sector definitions.

\(^{12}\)We recognize the risk that our country panel may be too homogeneous. We are grateful to an anonymous referee for this observation. Unfortunately, data limitations prevent us from exploring this possibility further.
and Im, Peseran and Shin (2003). Results indicate that all series are stationary in levels.\(^{13}\)

### 4.1 Purchasing Power Parity

Estimation results for linear PPP are presented in Table 4. As discussed, we run three different regressions for this specification, using the whole of our panel, as well as sub-panels that include only member countries of EMU/EMS and then all countries except commodity currencies. From these regressions, the model appears to be well-specified. Estimated AR(1) coefficients are statistically significant.\(^{14}\) Furthermore, estimated coefficients imply an average half-life deviation of real exchange rates from PPP-implied equilibrium levels across our panel as a whole of four years, which is consistent with the existing literature (Froot and Rogoff, 1994; Rogoff, 1996). This result applies to both sub-panels (EMU/EMS and ex-commodity currencies) as well, suggesting that it is not a function simply of exchange rate regime.

As a first step, therefore, we have confirmed the existence of one of the key puzzles in empirical exchange rate research: the speed of real exchange rate mean reversion to PPP-based equilibria is too slow to be consistent with the relatively high volatility of nominal and real exchange rates. This is particularly the case if monetary and financial shocks are the principle sources of this exchange rate volatility (Obstfeld and Rogoff, 2000). The remainder of the paper will evaluate whether the introduction of non-linear adjustment together with sectoral productivity shocks are able to reduce this estimated half-life deviation, in a robust manner, to more realistic levels.

### 4.2 Linear Balassa-Samuelson

Table 5 reports the results of our linear Balassa-Samuelson analysis. Most important is the finding that although there is a significant correlation between real exchange rates and productivity differentials, the sign of this correlation is negative. This result is robust across both our sub-panels, as well as the overall panel. It contradicts the theoretical prediction of Balassa-Samuelson, as well as much of the existing empirical literature. It is, however, consistent with one of the key conclusions of Peltonen and Sager (2009).\(^{15}\)

\(^{13}\)To conserve space, these results are not reported but are available on request.

\(^{14}\)A lag length of one is chosen based on the examination of the partial autocorrelation function for each country in our panel.

\(^{15}\)Our sample of countries is similar to the Advanced country sub-panel of Peltonen and Sager (2009), suggesting that this may partly be responsible for this finding. As Balassa-Samuelson is most applicable to Emerging Market economies, we recognize that it is also a potential limitation of
From a theoretical perspective, there are a number possible explanations for this important finding. We highlight two. First, in the context of relatively fast traded sector productivity in the Home economy, our finding is consistent with the presence of pricing-to-market (PTM) strategies that contradict the Law Of One Price (LOOP) assumption central to Balassa-Samuelson (Krugman, 1987; Marston, 1990; Bergin and Feenstra, 2000; Bussière and Peltonen, 2008). Consistent with the New Open Macroeconomic modeling framework of Beningo and Thoenissen (2002) and also Lee and Tang (2007), PTM strategies are particularly likely in the presence of a low elasticity of substitution between domestic and foreign traded sector output and a bias of domestic consumers towards home-produced goods (Obstfeld and Rogoff, 2000). In the event of a positive productivity shock in the domestic traded goods sector, both characteristics require domestic producers to lower prices in order to encourage consumers in the foreign economy to absorb the associated increase in domestic traded sector output. This effect will dominate any positive impact of the traded sector productivity shock on the prices of non-traded goods and services that is the mainstay of Balassa-Samuelson (that is, the appreciation of the internal real exchange rate in the terminology of Beningo and Thoenissen, 2002). The theoretical framework of Stockman (1987), incorporating a model of imperfect competition, and the empirical results of IMF (2002) appear consistent with our findings and the theoretical predictions of Beningo and Thoenissen (2002) and Lee and Tang (2007).

Second, our reported contradiction of Balassa-Samuelson may reflect relatively rapid productivity growth in the non-traded sector. This possibility is explicitly ruled out by Balassa-Samuelson, but is consistent with recent data trends reported by Peltonen and Sager (2009). In line with the findings of Beningo and Thoenissen (2002), this productivity growth may reflect assimilation of technological advances in the non-traded sector of the Home economy due to so-called leapfrogging.\textsuperscript{16} It may also reflect improvements in business organization and corporate governance, increased foreign direct investment, or deregulation. All these innovations can drive down price levels in the non-traded sector relative to the traded sector of the Home economy, as well as the Foreign economy. The result is a depreciation of the real exchange rate.\textsuperscript{17}

\textsuperscript{16}Leapfrogging refers to the practice in EMEs of bypassing intermediate technologies by replacing old-fashioned systems with state-of-the-art technologies.

\textsuperscript{17}We leave to future research an assessment of whether the negative relationship between real exchange rates and productivity differentials is driven by the traded or non-traded sector, or productivity innovations in the domestic or foreign economies.
4.3 EPSTAR

A second common approach to the PPP half-life puzzle is to adopt a non-linear functional form, consistent with equation (3) above. In order for this approach to have validity, it is of course necessary to demonstrate the existence of non-linearity in exchange rates included in our panel. The results of these tests are reported in Table 6. As discussed above, we find in favor of significant non-linearity, and determine that an exponential STAR (EPSTAR) model is the appropriate functional form with which to augment our basic linear PPP regression.

Table 7 reports the results of our EPSTAR regressions. We estimate the model for the same three panels as above. The nonlinear specification is better than linear PPP in terms of overall goodness of fit—compare $R^2$ and Schwartz Information Criterion statistics, for instance—except for the EMU/EMS sub-panel. All the estimated coefficients are statistically significant. In the Inner regime, the estimated beta coefficients exceed unity for all three sub-panels, indicating that the exchange rate is explosive for values close to its mean value. This characteristic is not present in the Outer Regime, with the sum of $\beta_1$ and $\beta_1$ less than unity.

Estimated $\theta$ coefficients are statistically significant for the whole panel and the sub-panel excluding commodity currencies; the EMU/EMS sub-panel is borderline significant at traditional significance levels. As discussed above, this result indicates that the speed at which OECD real exchange rates revert back towards equilibrium is dependent upon the magnitude of the initial disequilibrium, with larger disequilibria consistent with more rapid reversion.

4.4 BP-EPSTAR

That we conclude the nonlinear EPSTAR specification improves the fit of the linear PPP equation is not particularly surprising, given the number of existing studies that report this finding, albeit on the basis of univariate estimation (Taylor et al., 2001, and van Dijk et al., 2002). We now look to augment this specification further by also

\footnote{Significance levels for the transition parameter $\widehat{\theta}$ are computed using Monte Carlo methods, as suggested by Lothian and Taylor (2007). This reflects the fact that under the null hypothesis $H_0 : \theta = 0$, the adjusted real exchange rate $q_{it} - \mu_t$ has a unit root. So testing this null versus $H_A : \theta \neq 0$ amounts to testing for the presence of a unit root against the alternative of no unit root and nonlinearity. As standard test statistics cannot be used, the suggested procedure is to first compute the empirical significance level of the parameter $\theta$ under the null hypothesis of a unit root from 10,000 random walks initialized at zero (from which we retain the last 500 observations). For each simulation, we estimate ESTAR and BS-EPSTAR equations, and calculate the percentage of simulated t-ratios larger in absolute value than the t-ratios estimated using our dataset. This percentage is retained as the empirical marginal significance level of the parameter $\theta$.}
incorporating intra- and inter-country productivity differentials, as discussed above. To our knowledge, only Sager (2006) and Lothian and Taylor (2007) have similarly estimated a non-linear productivity-augmented PPP specification. But whereas both these studies estimate univariate models, we continue to estimate a panel specification using the innovative estimation methodology of Fok, van Dijk and Franses (2005).

From (15) above, our BS-EPSTAR estimation equation is,

$$q_{it} - \gamma \nu_{it} = \alpha + \beta_1 (q_{it-1} - \gamma \nu_{it-1}) + \beta_1 [1 - \exp(-\theta(q_{it-2} - \gamma \nu_{it-2})^2)](q_{it-1} - \gamma \nu_{it-1}) + u_{it}$$

(18)

Results for the BS-EPSTAR specification are reported in Table 8. We continue to find evidence of a significant, negative correlation between real exchange rates and productivity differentials that contradicts the predictions of Balassa-Samuelson. The estimated value of the relevant coefficient, $\gamma$, indicates that on average real exchange rates depreciate by 7.4% with every 10% increase in the ratio of traded to non-traded sector productivity in the domestic economy relative to the United States.

The whole panel BS-EPSTAR specification generates an adjusted $R^2$ of 0.67. This represents an improvement in model fit compared with the linear (adjusted $R^2$ of 0.61) and non-linear (0.65) PPP models, and a larger improvement versus the linear Balassa-Samuelson specification (0.22).

Both these results—the negative correlation between productivity and real exchange rates, and the improvement in model fit using both augmentations to linear PPP—are robust across both sub-panels used in our analysis. Furthermore, the speed of mean reversion, as given by the estimated $\theta$ parameter, is larger for the BP-EPSTAR specification than the simple EPSTAR non-linear augmentation to linear PPP.

In summary, a key hypothesis of our analysis has been validated. The speed of reversion of real exchange rates back towards equilibrium is faster once explicit allowance has been made for both intra- and inter-country productivity differentials that amend the implicit path of exchange rate equilibria compared with traditional, linear PPP, and a non-linear functional form, in the context of a panel estimation.

In addition, although we report a significant correlation between productivity differentials and real exchange rates, the sign of this correlation contradicts the central prediction of Balassa-Samuelson. A recent theoretical literature has developed that accommodates this finding, for instance on the basis of significant consumer home bias in favor of domestic traded sector goods that has encouraged proliferation of pricing-to-market (PTM) strategies reported in the literature, or rapid technological innovation concentrated on the non-traded sector of the Home economy. As our study is the first to analyze the relationship between productivity and real exchange rates
by explicitly incorporating data for the non-traded sector within a non-linear panel framework, we believe our finding to be robust.

4.4.1 Half-Life Calculation

The computation of the average speed of mean reversion is straightforward in the case of panel linear PPP, and is based on impulse response functions constructed using the moving average representation of the model. This calculation is more demanding for non-linear specifications such as ours. This reflects the fact that the shape of the impulse response function depends on the history of the system at the time the shock occurs, the size of the shock and the distribution of future innovations. Therefore, the half-life must be calculated using simulation methods in a four-step procedure, following Gallant et al. (1993) and Lothian and Taylor (2007):

- First, compute the forecasts of the model for T periods ahead, where T=5. The forecasts are computed conditional on two scenarios. First, that the real exchange rate starts the forecast period at its average historical value. This scenario implies that we simulate the impulse responses starting at every point in the sample, which are then averaged to produce the impulse response functions conditional on the average initial history, as described in Taylor et. al. (2001). Second, that the real exchange rate starts the forecast period at its equilibrium value.

- Second, estimate another set of impulse responses with a shock in the initial period.

- Third, calculate the difference between the impulse responses with and without the shock in the initial period. This difference is taken as the estimated impulse response of the non-linear model.

- Fourth, calculate the half-life of the shock as the time it takes the real exchange rate to revert back 50% towards its trend value.19

Table 9 compares the relative speed of mean reversion implied by our EPSTAR and BS-EPSTAR specifications, and Chart 1 provides a graphic visualization of these data.20 Half-lives are calculated from the simulated impulse responses on the basis

19 Our approach requires that the impulse response function is monotonic, which is the case.
20 Our use of the word "relative" is intended to emphasize that real exchange rates in the two model specifications mean-revert to different equilibria, as is implicit in the discussion throughout this paper. For the EPSTAR specification, this equilibrium is a long term average value, whereas for the BS-EPSTAR specification it is a stochastic trend.
of eight different shocks ranging from 1% to 60%, where the magnitude of shocks has been chosen on the basis of the observed standard deviations of the real exchange rates in our sample. In Chart 1, the horizontal axis plots, for the EPSTAR model, deviations from the PPP-implied equilibrium real exchange rate, and for the BP-EPSTAR specification deviations from the average productivity-adjusted equilibrium; the vertical axis plots the estimated exponential transition functions for both specifications. Consistent with Table 9, Chart 1 indicates that the speed of mean reversion is generally slightly faster under the BS-EPSTAR model specification that augments the basic linear PPP specification with non-linear adjustment around a productivity-adjusted equilibrium than it is under the alternative of simply augmenting the basic PPP specification with a non-linear functional form (EPSTAR). This result suggests that the speed of exchange rate adjustment depends not only upon the magnitude of an initial shock, but also on the relative position of the real exchange rate compared to its productivity-adjusted equilibrium. From Table 9 and Chart 1, the estimated half-life for a 1% shock is 4 years in the EPSTAR model and 3.7 years in the BS-EPSTAR; for a 60% shock, the half-life of both models is estimated to be equivalent, at two years. These two augmentations to the basis linear PPP specification can therefore explain much of the Obstfeld-Rogoff (2000) puzzle.

5 Conclusions

This paper has reappraised the half-life persistence of shocks to real exchange rates around linear PPP-implied equilibria. Consistent with the large existing literature, we find that these shocks persist for approximately four years. Two approaches—the introduction of a non-linear functional form, and real shocks to exchange rate equilibria within a linear framework—have been adopted in the literature in an effort to shorten half-lives to a length more consistent with the observed volatility of nominal and real exchange rates. Although not mutually exclusive, in practice these augmentations have typically been considered in isolation of one another. By contrast, we consider both together, within the context of an innovative panel estimation procedure.

We report three important findings that extend the existing literature. First, a substantial reduction in half-life persistence, to approximately one half the level

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21 The standard deviation of real exchange rates ranges from 0.078 for Ireland to 0.30 for Portugal, with a sample average of 0.16. Thus, shocks of up to 60% correspond to 2 years.

22 For very small shocks, it is also the case that the half-life of shocks under the BP-ESTAR and EPSTAR specifications are similar to the basic linear PPP model. Consequently, for small deviations of real exchange rates away from equilibrium, the puzzle of slow adjustment still applies.
found using linear PPP. Second, that estimated half-lives are generally shortened by adoption of both augmentations rather than employing either in isolation, indicating that functional form and the incidence of productivity shocks are both important in determining the speed at which real exchange rates revert back towards equilibrium following a shock. Our results suggest, however, that adoption of a non-linear functional form is the most important augmentation. And third, that the sign of the relationship between OECD real exchange rates and productivity shocks contradicts the prediction of the Balassa-Samuelson hypothesis, and instead is consistent both with a rapid assimilation of technological advances that bears down on non-traded sector price levels, and a growing proliferation of PTM strategies in the traded sector that contradict the LOOP.

Further extension of these results should include the development of a rigorous testing framework for the panel STAR estimation methodology that we have employed in this paper, similar to the univariate STAR testing framework developed, \textit{inter alia}, by Granger and Teräsvirta (1993), Teräsvirta (1994) and Teräsvirta (1998). We leave this task to future research.
References


**Table 1. Countries**

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**Table 2. Traded and Non-Traded Sector Definitions**

**Traded Sector**

- AGRICULTURE, HUNTING, FORESTRY & FISHING
- MINING & QUARRYING
- TOTAL MANUFACTURING
- TRANSPORT, STORAGE & COMMUNICATION

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**Non – Traded Sector**

- ELECTRICITY, GAS & WATER SUPPLY
- CONSTRUCTION
- WHOLESALE & RETAIL TRADE, RESTAURANTS & HOTELS
- FINANCE, INSURANCE, REAL ESTATE & BUSINESS SERVICES
- COMMUNITY, SOCIAL & PERSONAL SERVICES

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Notes: Sector weights calculated for country i as the share of nominal value added of a sector in the total value added of that economy.
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Table 3.2. Summary statistics: Productivity Data

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<td>0.640613</td>
<td>3.42564</td>
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<tr>
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<td>0.544784</td>
<td>2.559349</td>
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<tr>
<td>FRANCE</td>
<td>0.063663</td>
<td>0.048105</td>
<td>0.129113</td>
<td>2.777537</td>
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</tr>
<tr>
<td>GERMANY</td>
<td>-0.07622</td>
<td>0.054012</td>
<td>0.839148</td>
<td>3.038387</td>
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<tr>
<td>GREECE</td>
<td>-0.01952</td>
<td>0.033689</td>
<td>0.686099</td>
<td>3.698576</td>
<td>21</td>
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<tr>
<td>ICELAND</td>
<td>-0.05235</td>
<td>0.053444</td>
<td>-0.52792</td>
<td>1.707079</td>
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</tr>
<tr>
<td>ITALY</td>
<td>0.076126</td>
<td>0.085189</td>
<td>0.054092</td>
<td>2.253233</td>
<td>10</td>
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<tr>
<td>JAPAN</td>
<td>0.043602</td>
<td>0.042877</td>
<td>-0.3944</td>
<td>2.688569</td>
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<tr>
<td>KOREA</td>
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<td>0.095162</td>
<td>0.353997</td>
<td>2.006159</td>
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</tr>
<tr>
<td>LUXEMBOURG</td>
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<td>0.149451</td>
<td>0.971803</td>
<td>2.700556</td>
<td>22</td>
</tr>
<tr>
<td>MEXICO</td>
<td>-0.10814</td>
<td>0.12943</td>
<td>-0.23648</td>
<td>1.746633</td>
<td>17</td>
</tr>
<tr>
<td>NETHERLANDS</td>
<td>0.081714</td>
<td>0.050441</td>
<td>-0.51132</td>
<td>1.848942</td>
<td>14</td>
</tr>
<tr>
<td>NEW ZEALAND</td>
<td>0.053658</td>
<td>0.064019</td>
<td>1.042962</td>
<td>3.372541</td>
<td>22</td>
</tr>
<tr>
<td>NORWAY</td>
<td>0.056827</td>
<td>0.066098</td>
<td>-0.97851</td>
<td>3.426027</td>
<td>22</td>
</tr>
<tr>
<td>PORTUGAL</td>
<td>-0.04867</td>
<td>0.057114</td>
<td>0.426521</td>
<td>1.866271</td>
<td>12</td>
</tr>
<tr>
<td>SPAIN</td>
<td>0.074728</td>
<td>0.057829</td>
<td>-0.27805</td>
<td>2.12094</td>
<td>22</td>
</tr>
<tr>
<td>SWEDEN</td>
<td>-0.03252</td>
<td>0.057353</td>
<td>0.139856</td>
<td>1.439002</td>
<td>9</td>
</tr>
<tr>
<td>UK</td>
<td>0.055921</td>
<td>0.055114</td>
<td>0.283941</td>
<td>1.640444</td>
<td>22</td>
</tr>
</tbody>
</table>
### Table 4. Linear PPP Regressions

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>EMU/EMS</th>
<th>All excl. commodity currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.0077 (0.2137)</td>
<td>0.0101 (0.2399)</td>
<td>0.0073 (0.2787)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.7930 (0.0000)</td>
<td>0.7854 (0.0000)</td>
<td>0.7947 (0.0000)</td>
</tr>
<tr>
<td>CRISIS</td>
<td>−0.0709 (0.1109)</td>
<td>0.0118 (0.8980)</td>
<td>−0.0465 (0.3922)</td>
</tr>
<tr>
<td>EMU</td>
<td>−0.0032 (0.8651)</td>
<td>−0.0032 (0.8659)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.61</td>
<td>0.59</td>
<td>0.60</td>
</tr>
<tr>
<td>Sum squared residuals</td>
<td>4.92</td>
<td>3.78</td>
<td>4.64</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>320.98</td>
<td>186.01</td>
<td>271.00</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.31</td>
<td>24.67</td>
<td>1.30</td>
</tr>
<tr>
<td>Schwarz Info Criterion</td>
<td>−1.21</td>
<td>−1.28</td>
<td>−1.14</td>
</tr>
<tr>
<td>Countries included</td>
<td>22</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td># observations</td>
<td>407</td>
<td>235</td>
<td>359</td>
</tr>
</tbody>
</table>

Notes: The table reports estimation results from panel regressions of the form:

\[
q_{it} = \alpha + \beta q_{it-1} + u_{it}
\]

where \( q_{it} \) is log of the real exchange rate for currency \( i \) versus the US dollar at time \( t \). See country group definitions in Table 1, and definitions for CRISIS and EMU dummy variables in Section 3. The models are estimated using ordinary least squares with country fixed effects, and with serial correlation and heteroscedasticity robust standard errors. P-values reported in parentheses.
Table 5. Linear Balassa-Samuelson Regressions

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>EMU/EMS</th>
<th>All excl. commodity currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.1032</td>
<td>0.1387</td>
<td>0.1046</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.5969</td>
<td>-0.8272</td>
<td>-0.6351</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>CRISIS</td>
<td>-0.0508</td>
<td>0.2059</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td>(0.4675)</td>
<td>(0.1442)</td>
<td>(0.9202)</td>
</tr>
<tr>
<td>EMU</td>
<td>-0.0617</td>
<td>-0.0622</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0335)</td>
<td>(0.0376)</td>
<td></td>
</tr>
</tbody>
</table>

Adjusted R-squared   | 0.22         | 0.12         | 0.19                          |
S.E. of regression   | 0.18         | 0.19         | 0.18                          |
Sum squared residuals | 12.22        | 8.80         | 11.47                         |
Log likelihood       | 135.90       | 74.38        | 108.61                        |
Schwarz Info Criterion | -0.30      | -0.20        | -0.24                         |
Countries included   | 22           | 12           | 19                            |
# observations       | 407          | 235          | 359                           |

Notes: The table reports estimation results from panel regressions of the form:

$$ q_{it} = \alpha + \beta \mu_{it} + u_{it} $$

where $q_{it}$ is log of the real exchange rate and $\mu_{it}$ is the relative productivity differential between traded and non-traded goods and services in the home country versus the US, $\mu_{it} = \gamma(\sigma^T - \sigma^N) - \gamma_{US}(\sigma^T_{US} - \sigma^N_{US})$. See country group definitions in Table 1, and definitions for. CRISIS and EMU are dummy variables in Section 3. The models are estimated using ordinary least squares with country fixed effects, and with serial correlation and heteroscedasticity robust standard errors. P-values reported in parentheses.
Table 6. Test for non-linearity and asymmetry

\[ H_0: \text{no nonlinearities} \quad H_0: \text{no asymmetry} \]

\[
d
\]

\[
\begin{array}{ccc}
1 & 0.1963 & 0.8402 \\
2 & 0.0001 & 0.3871 \\
3 & 0.0898 & 0.6139 \\
4 & 0.0427 & 0.8285 \\
5 & 0.0866 & 0.1079 \\
\end{array}
\]

Notes: The table reports the p-values of the tests for remaining non-linearities and asymmetry in the EPSTAR BS-EPSTAR equations for different values of the delay parameter \(d\). Results based upon the full panel.

Table 7. EPSTAR Models

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>All</th>
<th>EMU/EMS</th>
<th>All excl. commodity currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1)</td>
<td>1.1410 (0.0000)</td>
<td>1.1016 (0.0000)</td>
<td>1.1396 (0.0000)</td>
</tr>
<tr>
<td>(\beta_1)</td>
<td>-0.6690 (0.0000)</td>
<td>-0.6397 (0.0000)</td>
<td>-0.6918 (0.0000)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>14.0284 (0.0151)</td>
<td>11.6908 (0.0669)</td>
<td>12.003 (0.0238)</td>
</tr>
<tr>
<td>CRISIS</td>
<td>-0.0735 (0.0820)</td>
<td>0.0269 (0.7601)</td>
<td>-0.0562 (0.2769)</td>
</tr>
<tr>
<td>EMU</td>
<td>0.0241 (0.1838)</td>
<td>0.0254 (0.1762)</td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.65</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td>Sum squared residuals</td>
<td>4.42</td>
<td>3.42</td>
<td>4.16</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>343.01</td>
<td>199.26</td>
<td>290.65</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>1.58</td>
<td>1.53</td>
<td>1.59</td>
</tr>
<tr>
<td>Schwarz Info Criterion</td>
<td>-1.29</td>
<td>-1.13</td>
<td>-1.22</td>
</tr>
<tr>
<td>Countries included</td>
<td>22</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td># observations</td>
<td>407</td>
<td>235</td>
<td>359</td>
</tr>
</tbody>
</table>

Notes: The table reports estimation results from panel regressions of the form:

\[
\tilde{q}_{it} = \alpha + \beta_1 \tilde{q}_{it-1} + \beta_1 [1 - \exp(-\theta \tilde{q}_{it-2})] \tilde{q}_{it-1} + u_{it}
\]

where \(\tilde{q}_{it}\) is the demeaned real exchange rate (\(\tilde{q}_{it} = q_{it} - \mu_i\), where \(\mu_i = \sum_{t=1}^{T} q_{it}\)), and \(\theta\) is the parameter of speed of transition, as defined above. See country group definitions in Table 1, and definitions for. CRISIS and EMU are dummy variables in Section 3. The models are estimated using non-linear least squares (NLLS) with serial correlation and heteroscedasticity robust standard errors. P-values reported in parentheses.
Table 8. BS-EPSTAR Models

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>EMU/EMS</th>
<th>All excl. commodity currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-0.0059</td>
<td>-0.0066</td>
<td>-0.0066</td>
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<tr>
<td></td>
<td>(-0.3159)</td>
<td>(0.4221)</td>
<td>(0.3110)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-0.7408</td>
<td>-0.9725</td>
<td>-0.7442</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.1950</td>
<td>1.1769</td>
<td>1.1827</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-0.7668</td>
<td>-0.7849</td>
<td>-0.7880</td>
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<tr>
<td></td>
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<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>16.2104</td>
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<td>13.428</td>
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<tr>
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<td>(0.0281)</td>
<td>(0.0099)</td>
</tr>
<tr>
<td>CRISIS</td>
<td>-0.0715</td>
<td>0.0341</td>
<td>-0.0509</td>
</tr>
<tr>
<td></td>
<td>(-0.0798)</td>
<td>(0.6888)</td>
<td>(0.3096)</td>
</tr>
<tr>
<td>EMU</td>
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<tr>
<td></td>
<td>(-0.2766)</td>
<td>(0.3876)</td>
<td>(0.2596)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.67</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>Sum squared residuals</td>
<td>4.10</td>
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<tr>
<td>Log likelihood</td>
<td>358.23</td>
<td>212.26</td>
<td>303.23</td>
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<tr>
<td>Durbin-Watson stat</td>
<td>1.63</td>
<td>1.58</td>
<td>-1.55</td>
</tr>
<tr>
<td>Schwarz Info Criterion</td>
<td>-1.35</td>
<td>-1.19</td>
<td>1.62</td>
</tr>
<tr>
<td>Countries included</td>
<td>22</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td># observations</td>
<td>407</td>
<td>235</td>
<td>359</td>
</tr>
</tbody>
</table>

Notes: The table reports estimation results from panel regressions of the form:

\[
\tilde{q}_{it} = \alpha + \gamma \tilde{\nu}_{it} + \beta_1 (\tilde{q}_{it-1} - \gamma \tilde{\nu}_{it-1}) + \beta_2 [1 - \exp(-\theta(\tilde{q}_{it-2} - \gamma \tilde{\nu}_{it-2})^2)](\tilde{q}_{it-1} - \gamma \tilde{\nu}_{it-1}) + u_{it}
\]

where \( \tilde{q}_{it} \) is the demeaned real exchange rate (\( \tilde{q}_{it} = q_{it} - \frac{\sum_{t=1}^{T} q_{it}}{T} \)) and \( \tilde{\nu}_{it} \) is the demeaned productivity differential, \( \tilde{\nu}_{it} = \nu_{it} - \frac{\sum_{t=1}^{T} \nu_{it}}{T} \), \( \nu_{it} = \gamma [(a^T_{it} - a^N_{it}) - (a^T_{i,U,S} - a^N_{i,U,S})] \). See country group definitions in Table 1, and definitions for. CRISIS and EMU are dummy variables in Section 3. The models are estimated using non-linear least squares (NLLS) with serial correlation and heteroscedasticity robust standard errors. P-values reported in parentheses.
Table 9. *Estimated Half-Lives for Nonlinear Models*

<table>
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<th>Shock (%)</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>10</th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPSTAR</td>
<td>1.99</td>
<td>2.03</td>
<td>2.35</td>
<td>2.43</td>
<td>3.52</td>
<td>3.75</td>
<td>3.94</td>
<td>4.00</td>
</tr>
<tr>
<td>BS-EPSTAR</td>
<td>1.99</td>
<td>1.99</td>
<td>2.06</td>
<td>2.23</td>
<td>3.46</td>
<td>3.45</td>
<td>3.56</td>
<td>3.71</td>
</tr>
</tbody>
</table>

*Notes:* The table reports half-lives of real exchange rate shocks calculated by Monte Carlo methods (conditional on average initial history) based on the EPSTAR and BS-EPSTAR equations estimated in Tables 6 and 7. Results are based upon estimation of the panel with all countries.
Notes: Chart 1 plots on the vertical axis the adjustment speed of the real exchange rate back to PPP-implied equilibrium (in the case of the EPSTAR estimation equation; that is, $\Phi(.) = 1 - \exp(-\theta (q_{it-2} - \mu_i)^2)$) and to PPP-implied equilibrium adjusted by average productivity-differentials (in the case of the BS-ESPTAR estimation equation; that is, $\Phi(.) = 1 - \exp(-\theta (q_{it-2} - \gamma \nu_{it-2})^2)$) in the wake of a shock, as measured by the estimated transition function in each case. The vertical axis represents the speed of non-linear adjustment given by the transition function.
Chart 2. Real Effective Exchange Rates and productivity differentials by country

Notes: Real Effective Exchange Rate in red and productivity differential in blue.
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