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## An economic capital model integrating credit and interest rate risk in the banking book

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EUROPEAN CENTRAL BANK

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## WORKING PAPER SERIES

NO 1041 / APRIL 2009

# **AN ECONOMIC CAPITAL MODEL INTEGRATING CREDIT AND INTEREST RATE RISK IN THE BANKING BOOK**

by Piergiorgio Alessandri  
and Mathias Drehmann



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by Piergiorgio Alessandri<sup>2</sup>  
and Mathias Drehmann<sup>3</sup>



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## Abstract

Banks typically determine their capital levels by separately analysing credit and interest rate risk, but the interaction between the two is significant and potentially complex. We develop an integrated economic capital model for a banking book where all exposures are held to maturity. Our simulations show that capital is mismeasured if risk interdependencies are ignored: adding up economic capital against credit and interest rate risk derived separately provides an upper bound relative to the integrated capital level. The magnitude of the difference depends on the structure of the balance sheet and on the repricing characteristics of assets and liabilities.

**Keywords:** Economic capital, risk management, credit risk, interest rate risk, asset and liability management

**JEL Classification:** G21, E47, C13



## Non-technical summary

According to industry reports, interest rate risk is after credit risk the second most important risk when determining economic capital in the banking book. However, no unified economic capital model exists which integrates both risks in a consistent fashion. Therefore, regulators and banks generally analyse these risks independently from each other and derive total economic capital by some rule of thumb. Indeed, the most common rule arguably consists in simply “adding up”. A serious shortcoming of this procedure is that it obviously fails to capture the interdependencies between both risks. The framework developed in this paper captures the complex dynamics and interactions of credit and interest rate risk. First, we condition on the systematic macroeconomic risk drivers which impact on both risk classes simultaneously. Second, we model net-interest income dynamically taking not only account of the repricing of assets and liabilities in line with changes in the risk free yield curve but also of the impact of changes in the riskiness of credit exposures. This allows us to capture the margin compression due to the repricing mismatch between long term assets and short term liabilities. However, not only liabilities but also assets get repriced over time. This implies that credit risk losses are gradually offset once more and more assets reflect the change in the risk-free yield curve as well as changes in the credit quality.

The conceptual contribution of the paper is the derivation of an economic capital model which takes account of credit and interest rate risk in the banking book in a consistent fashion. The way credit and interest rate risk are modelled individually is in line with standard practices. The credit risk component is based on the same conceptual framework as Basel II and the main commercially available credit risk models. Interest rate risk, on the other hand, is captured by earnings at risk, the approach banks use traditionally to measure this risk type. In contrast to standard models we integrate both risks using the framework proposed by Drehmann, Sorensen and Stringa (2008) taking account of all relevant interactions between both risks. We show that changes in net-interest income can be decomposed into two components: the first one captures the impact of changes in the yield curve, while the second accounts for the crystallisation of credit risk, which implies a loss of interest payments on defaulted loans. Conditionally on the state of the macroeconomy, these two sources of income risk are independent. This is an important insight as it significantly

simplifies our analysis. But it also underlines that conditioning on the macroeconomic environment is crucial for an economic capital model aiming to integrate credit and interest rate risk.

Using our model, we determine capital in line with current regulatory practices. We then derive capital based on the integrated approach and compare it to simple economic capital, ie the sum of capital set separately against credit and interest rate risk. For a hypothetical but realistic bank, we find that the difference between simple and integrated economic capital is often significant but it depends on various features of the bank, such as the granularity of assets, the funding structure of the bank or the bank's pricing behaviour. However, simple capital exceeds integrated capital under a broad range of circumstances. A range of factors contribute to generating this result. A relatively large portion of credit risk is idiosyncratic, and thus independent of the macroeconomic environment, and the correlation between systematic credit risk factors and interest rates is itself not perfect. Furthermore, if assets in the bank's portfolio are repriced relatively frequently, increases in credit risk can be partly passed on to borrowers.



## 1 Introduction

*“The Committee remains convinced that interest rate risk in the banking book is a potentially significant risk which merits support from capital”* (Basel II, §762, Basel Committee, 2006).

The view expressed by the Basel Committee in the Basel II capital accord receives strong support from the data. According to industry reports, interest rate risk is after credit risk the second most important risk when determining economic capital for the banking book (see IFRI-CRO, 2007). However, no unified economic capital model exists which integrates both risks in a consistent fashion for the banking book. Therefore, regulators and banks generally analyse these risks independently from each other and derive total economic capital by some rule of thumb. Indeed, the most common rule arguably consists in simply “adding up”. A serious shortcoming of this procedure is that it obviously fails to capture the interdependencies between both risks. For example, the literature has shown consistently that interest rates are a key driver of default frequencies, *i.e.* interest rates risk drives credit risk.<sup>1</sup> And as we will show, credit risk also drives interest rate risk in the banking book. This raises several questions: what is the optimal level of economic capital if the interdependencies are captured? Do additive rules provide a good approximation of the true integrated capital? More importantly, is the former approach always conservative or can both risks compound each in some circumstances? In order to answer these questions, we derive integrated economic capital for a traditional banking book (where exposures are assumed to be non-tradable and held to maturity) and we compare it to economic capital set against credit as well as interest rate risk when interdependencies are ignored. We show that this is only possible by using an economic capital model, developed in this paper, which consistently integrates credit and interest rate risk taking account of the complex repricing characteristics of asset and liabilities.

The dynamic interactions between credit and interest rate risk that lie at the core of our model can be illustrated with a simple example. Consider a risk-neutral bank which fully funds an asset  $A$  with some liability  $L = A$ ; assets and liabilities are held to maturity and subject to book value accounting as we assume that there is no market where they can be traded. Assume that  $A$  and  $L$  have a time to repricing<sup>2</sup> of one year, and that  $L$  gets remunerated at the risk-free rate  $r_0$ . Under risk neutrality, the interest rate charged on  $A$  is  $r_0$  plus a spread equal to the probability of default (PD) times the loss given default (LGD). Net interest income, *i.e.* income received on assets minus income paid on liabilities, is therefore equal to expected losses ( $EL = PD * LGD * A$ ). If capital is set in the standard fashion against credit risk (*i.e.* as the difference between the expected loss and the

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<sup>1</sup> The literature on modelling default is by now so large that an overview can not be given in this paper. For recent examples showing a link between interest rates and credit risk see Carling *et al.* (2006), Duffie *et al.*, (2007) or Drehmann *et al.* (2006).

<sup>2</sup> Time to repricing, not maturity, is the key driver for interest rate risk. The two need not coincide. For example, a flexible loan can have a maturity of 20 years even though it can be repriced every three months. Throughout the paper we make the simplifying assumption that maturity and time to repricing are the same.

VaR), capital and net interest income indeed cover expected and unexpected losses up to the required confidence level. However, one of the key characteristic of banks is that they borrow short and lend long, and hence there is a repricing mismatch between assets and liabilities. This repricing mismatch is the key source of interest rate risk for banks as changes in the yield curve impact more quickly on interest paid on liabilities than interest earned on assets.

This effect can also be seen in our example. Assume now that interests on liabilities are re-set daily rather than annually. If interest rates increase permanently by e.g. 50% after assets are priced, interest income from assets remains unaffected (and equal to  $(r_0 + PD * LGD) * A$ ) as coupon rates of assets are locked-in until the end of the year. However, interest payments on liabilities increase in line with the risk-free rate and margins between short term borrowing and long term lending get squeezed. In our example, net interest income drops to  $EL - 0.5 * r_0 * L$ . If total economic capital is only set as the difference between expected losses and VaR for the credit loss distribution, losses due to interest rate risk already eat into capital before any credit risk crystallises. Therefore, capital is also required against random fluctuations in net-interest income or, as it is often referred to, against earnings at risk.

Reality is clearly more complex than our example. First, as has already been pointed out, interest rates are an important determinant of the riskiness of credit exposures. Hence, not only does a rise in interest rates impact negatively on net interest income, but it also implies higher credit risk losses. That said, for a lumpy portfolio, a portion of credit risk is idiosyncratic, and thus independent of the macroeconomic environment: the larger the idiosyncratic component, the weaker the overall correlation between defaults and interest rates. Second, the crystallisation of credit risk reduces interest income: when a loan defaults, the bank loses interest payments as well as the principal. Third, the repricing structure of banks' balance sheets is more complex. A substantial fraction of assets (as well as liabilities) mature or can be re-priced during a one year horizon. This implies that higher credit risk and higher interest rates can be passed on to borrowers, leading to an increase in net interest income. Finally, any change in interest rates and credit risk will generally affect the mark-to-market value of the banks' exposures. The model we propose captures the first three channels but not the last one, because we focus on a traditional banking book containing non-tradable assets which are valued using book value accounting. Therefore, in line with the current regulatory approach, we set capital against realised losses but not against changes in the mark-to-market value of the balance sheet.<sup>3</sup> In other words, in our model "credit risk" is exclusively determined by default risk and "interest rate risk" is determined by net interest income fluctuations stemming from adverse yield curve movements (*i.e.* the earnings implications of repricing, yield curve and basis risk).

Traditionally it would be argued that the sum of economic capital set against credit risk and interest rate risk separately is a conservative upper bound in comparison to economic capital set against both risks jointly. Breuer *et al.* (2008) discuss this problem in the context of market and

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<sup>3</sup> Our concluding section discusses the implications of this choice.

credit risk assessment for the banking and trading book. Here a similar argument is often made that the risk measure of the total portfolio, i.e. the whole bank, is less than the sum of the risk measures for the banking and trading book. Breuer *et al.* show that this argument is based on two premises. One is that, under a subadditive risk measure, the risk of a portfolio is smaller or equal than the sum of the risks of its components. The other is that the aggregate portfolio of the bank can be decomposed into two sub-portfolios – the banking and the trading book – such that credit risk is only impacting on the banking book and market risk only on the trading book. In reality, this last premise does not necessarily hold – not even approximately. Many positions depend simultaneously on both credit and market risk factors. Breuer *et al.* clarify this in the context of foreign currency loans, which depend on classic credit risk factors as well as a market risk factor (the exchange rate). The authors show empirically as well as theoretically that, if some positions depend on both market and credit risk factors, assuming that the portfolio is separable may result in an under- or over-estimation of the actual risk.

This result has strong implications for our work. Regulators and practitioners typically set capital against credit and interest rate risk independently, and obtain a measure of total capital by simply adding these up (we label this “simple economic capital” for convenience). If risks were separable and a sub-additive measure of risk is used, this procedure would always deliver a conservative level of capital. But this is *a priori* unclear, given the highly non-linear interactions between credit and interest rate risk. Simple economic capital may actually turn out to be lower than “integrated economic capital”, i.e. the capital level implied by a consistent, joint analysis of credit and interest rate risk.

The conceptual contribution of the paper is to derive an economic capital model which takes account of credit and interest rate risk in the banking book. The way we set capital against credit and interest rate risk individually is fully in line with standard practices. The credit risk component is based on the same conceptual framework as Basel II and the main commercially available credit risk models. Interest rate risk, on the other hand, is captured by earnings at risk, the approach banks commonly use to measure this risk type (see Basel Committee, 2008a). In other words, we focus on a traditional banking book where exposures are not marked-to-market and interest rate risk arises due to volatility in the bank’s net interest income. In contrast to standard models, however, we integrate credit and interest rate risk using the framework proposed by Drehmann, Sorensen and Stringa (2008) (henceforth DSS) taking into account all relevant interactions between both risks. These are threefold: (a) both risks are driven by a common set of risk factors; (b) interest rates are an important determinant of credit risk; and (c) credit risk impacts significantly on net-interest income. In the conceptual part of the paper, we show that changes in net-interest income can be decomposed into two components: the first one captures the impact of changes in the yield curve, while the second accounts for the crystallisation of credit risk, which implies a loss of interest payments on defaulted loans. Conditionally on the state of the macroeconomy, these two sources of income risk are independent. This important insight

significantly simplifies their aggregation. It also underlines that conditioning on the macroeconomic environment is crucial for an economic capital model aiming to integrate credit and interest rate risk.

Using our model, we determine capital in line with current market and regulatory practices as “the amount of capital a bank needs to absorb unexpected losses over a certain time horizon given a confidence interval” (p. 9 Basel Committee, 2008). We then derive capital over a one year horizon based on the integrated approach and compare it to simple economic capital, *i.e.* the sum of capital set separately against credit and interest rate risk. The main result of our empirical analysis is a reassuring one: for our stylised bank, which is only subject to credit and interest rate risk in the banking book, simple economic capital always seems to provide an upper bound. The quantitative difference between simple and integrated economic capital, though, depends on the structure and repricing characteristics of the bank’s portfolio.

The remainder of the paper is structured as follows. Section 2 provides a short overview of the literature. In Section 3 we derive the integrated economic capital model. Section 4 discusses our implementation and Section 5 presents the results. Section 6 undertakes some sensitivity tests. Section 7 concludes.

## 2 Literature

There is by now a large and well known literature on economic capital models for credit risk (for an overview see e.g. Gordy, 2000, or McNeil *et al.*, 2005). These models are based on the idea that there is one or a set of common systematic risk factors which drive default rates of all exposures, but that conditional on a draw of systematic risk factors, defaults across exposures are independent. Various models then differ in the way they link default rates and systematic risk factors and whether they analytically solve for the loss distribution or simulate it. Our approach to credit risk modelling follows this tradition. However, contrary to most models, we condition credit risk and the yield curve on a common set of systematic risk factors. Furthermore, we account for the loss in coupon payments if assets default.

In contrast to credit risk, no unified paradigm has yet emerged on how to best measure interest rate risk in the banking book (e.g. see Kuritzkes and Schuermann, 2007). The Basel Committee points to this as an important reason why interest rate risk in the banking book is not treated in a standardised fashion in the Basel II capital framework (see §762, Basel Committee, 2006).

One of the simplest interest risk measures is gap analysis, where banks or regulators assess the impact of a parallel shift or twist in the yield curve by purely looking at the net repricing mismatch between assets, liabilities and off-balance sheet items.<sup>4</sup> By now the literature has identified several

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<sup>4</sup> Generally, gap analysis allocates assets, liabilities and off-balance sheet items to time buckets according to their repricing characteristics and calculates their net difference for each bucket. Because of this netting procedure, gap analysis may fail to consider non-linearities and, consequently, underestimate the impact of interest rate risk. For example, some short-term customer deposit rates track the risk-free rate plus a negative spread. Hence, for large falls

problems with standard and more sophisticated gap analysis (e.g. see Staikouras, 2006). Therefore, there has been a shift to more sophisticated methods based on either static or dynamic simulation approaches (see Basel Committee, 2004, 2008). Interest rate risk in the banking book can either be measured by earnings at risk or using an economic value approach. The latter measures the impact of interest rate shocks on the value of assets and liabilities (e.g. see OTS, 1999, or CEBS, 2006), whereas the former looks at the impact of the shocks on the cashflow generated by the portfolio (i.e. a bank's net interest income). This paper follows the traditional earnings at risk approach which is heavily used in the industry and for regulatory purposes (see Basel Committee, 2008).

For capital purposes, regulators only require banks to look at a few specific interest rate risk shocks. For example, the standard stress test scenario is a 200bp parallel up-and downward shift of the yield curve (see Basel Committee, 2006). Alternatively, the 1<sup>st</sup> and 99<sup>th</sup> percentile of a five years historical distribution can be used as a stress scenario (see Basel Committee, 2004). It is interesting to note that the tails of the five year historical distribution generally include either a positive or negative 200bp shock but not both.<sup>5</sup> This is already a clear indication that it is impossible to explicitly set capital against a few specific scenarios as the probability of these scenarios crystallising is changing over time. Furthermore, it is impossible to consider all relevant scenarios: individual stress tests cannot by construction cover the full distribution of possible outcomes, something we assess using a simulation approach.

From the perspective of an integrated risk management framework, standard interest rate risk analysis used at banks and for regulatory purposes has another important drawback: implicitly, these methods assume that shocks to the risk-free yield curve have no impact on the credit quality of assets. But clearly this assumption does not hold: interest rates risk and credit risk are highly interdependent and, therefore, need to be assessed jointly.

Jarrow and Turnbull (2000) are among the first to show theoretically how to integrate interest rate (among other market risks) and credit risk. They propose a simple two factor model where the default intensity of borrowers is driven by interest rates and an equity price index, which in turn are correlated. Their theoretical framework is backed by strong empirical evidence that interest rate changes impacts on the credit quality of assets (see Duffie *et al.*, 2007, or Grundke, 2005).

If papers integrate both risks, they look at the integrated impact of credit and interest rate risk on assets only, for example by modelling bond portfolios without assessing the impact of interest and credit risk on liabilities or off-balance sheet items. Barnhill and Maxwell (2002) and Barnhill *et al.* (2001) measure credit and market risk for the whole portfolio of banks. In contrast to our paper they take a mark-to-market perspective. However, they ignore one of the most important sources

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in the risk-free term structure, banks may not be able to lower deposit rates in line with the risk-free rate because they face a zero bound on coupons.

<sup>5</sup> Given long interest rate cycles a -(+) 200bp shock is well within the 1% (99%) percentile of the distribution, often even well within the 5% (95%), but only in very rare cases are both shocks observed. This observation is based on weekly data for the 3 month and 10 year interest rates from the beginning of 1992 to July 2007 for US, UK, Germany and the Euro, and five years of observations of annual changes in the interest rates (as suggested by Basel Committee, 2004).



of interest rate risk – repricing mismatches between assets and liabilities.<sup>6</sup> Our work focuses on the latter, providing a thorough description of how a bank’s maturity structure and pricing behaviour affects its risk profile.

The approach of DSS is possibly closest to the operations research literature discussing stochastic programming models for dynamic asset and liability management.<sup>7</sup> Early papers include Bradley and Crane (1972) or Kusly and Ziemba (1986), which aim to determine the optimal dynamic asset and liability allocation for a bank. However, computational limitations imply that these authors can only look at three period binary tree models where assets and liabilities are tradable and defaults do not occur. The literature on portfolio optimisation allowing for defaults is so far limited. For example, Jobst *et al.* (2006) look at dynamic optimal portfolio allocation for a corporate bond portfolio. They simulate correlated interest rates and credit spreads as well as defaults and track future portfolio valuations. As they are interested in optimal portfolio allocation they do not assess economic capital even though this should be possible with their framework.

Dynamic optimal portfolio allocation is beyond the scope of this paper. But rather than looking at a portfolio of tradable assets, we consider non-tradable exposures in the banking book of a hypothetical bank and model corporate and household credit risk directly. Further, and more importantly, by following DSS we capture the complex cash flows from liabilities with different repricing characteristics rather than assuming a simple cash account as Jobst and his co-authors do.

While we use the framework of DSS to derive net interest income, our implementation differs. For their stress test, DSS use the structural macroeconomic model built for forecasting purposes at the Bank of England. This model cannot be easily simulated, so the authors focus on central projections and expected losses. Instead, we use a two country global macroeconomic vector autoregression model (GVAR) to model the macro environment in the spirit of Pesaran *et al.* (2004), which allows us to undertake stochastic simulations and therefore enables us to derive the full net profit distribution. Furthermore, in contrast to DSS, we look not only at expected losses conditional on the macro scenario but also at unexpected credit risk losses for individual exposures in the portfolio

So far there has been a limited discussion how interdependencies across risks impact on economic capital. Decomposing net income into its components (i.e. market, credit, interest rate risk in the banking book, operational and other risks) and computing returns on risk weighted assets, Kuritzkes and Schuermann (2007) find that interest rate risk in the banking book is after credit risk the second most important source of financial risks. Furthermore, they show that there are diversification benefits between risks.

Significant diversification benefits are also found in studies which use simple correlations between different risks (Kuritzkes *et al.*, 2003, or Dimakos and Aas, 2004). However, as Breuer *et al.*

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<sup>6</sup> The papers look at a maturity mismatch of +/- one year and conclude that this is important. But +/- one year is clearly too simplistic to capture the full impact of the maturity mismatch on the riskiness of banks.

<sup>7</sup> For an overview of this literature Zenios and Ziemba (2007).



(2008) point out, the latter approaches implicitly assume that risks are separable which in the case of market (and hence interest rate risk) and credit risk is not necessarily true. As already discussed in the introduction, in the context of foreign currency loans the authors find that total risk can be under- as well as overestimated if market and credit risk are wrongly assumed to be separable.

This is consistent with the findings in Kupiec (2007). The paper extends a single-factor credit risk model to take into account stochastic changes in the credit quality (and hence the market value) of non-defaulting loans. The value of the resulting portfolio is a non-separable function of market and credit risk factors. The author compares an integrated capital measure to additive measures calculated under a range of credit and market risk models, and finds that no general conclusion can be reached on whether additive rules under- or overestimate risk.

It is worth stressing that the diversification issue should ideally be examined within a model that integrates all relevant risks, and that such a model is not available to date. For instance, Kupiec (2007) or Breuer *et al.* (2008) focus on the asset side, abstracting from any issues related to maturity mismatch and net interest income volatility, whereas in this paper we model these in detail but do not consider changes in the economic value of the portfolio. Therefore, the literature can currently only provide partial answers to the general question of when and why additive rules can underestimate risk.

### 3 The framework

Throughout the framework discussion, we assume that the bank holds a portfolio of  $N$  assets with  $A=[A^1, \dots, A^N]$ . Each exposure  $A^i$  has a specific size, a time to repricing  $b^i$ , a default probability  $PD_t^i(X)$ , loss given default  $LGD_t^i$ , and coupon rate  $C_t^i(X)$ .  $X$  is a vector of systematic risk factors affecting both interest rates and defaults. To keep the discussion general, we assume that  $X \sim F$  is randomly distributed with an unspecified distribution function  $F$ . Following the literature, we also assume that conditional on  $X$ , defaults across different assets  $A^i$  are independent.

As indicated in the introduction, a risk-neutral bank conditions the pricing of loans on current and future credit conditions. This is one of the key links between interest rate and credit risk. At origination loans are priced in such a way that, given current and expected risk factors, their face value coincides with their market value (*i.e.* the present value of future payments and the principal). Under risk neutrality, this implies that expected interest income covers expected credit losses. In the multi-period setting, assets are priced not only at the beginning of the simulation but at each point in time. However, not all assets get repriced in each period. The time of repricing of an individual asset is determined by its repricing maturity. In our empirical implementation this



ranges for example from zero (*i.e.* the asset gets repriced every period) up to ten years.<sup>8</sup> For the derivation of the one period set-up, however, we initially assume that coupons  $C^i$  are fixed.

The bank is funded by  $M$  liabilities  $L=[L^1, \dots, L^M]$ . Each liability  $L^j$  falls into a repricing bucket  $b^j$  and pays a coupon rate  $C_t^j(X)$ . In line with assets, coupon rates are assumed to be fixed and equal to  $C^j$  in the single period framework but, depending on the repricing characteristics of liabilities, change endogenously in the multi-period set-up. All assets and liabilities are held in the banking book, using book value accounting.

### 3.1 Single period framework

To highlight the main mechanism of our framework, we use a very general formation of a portfolio loss distribution (e.g. McNeil *et al.*, 2005). However, we slightly change the basic set-up to account for the impact of defaults on net interest income. We will focus first on a one period model and later extend the analysis to a multi-period set-up.

In a standard portfolio model the total loss  $L$  of the portfolio is a random variable and can be characterised by

$$L(X) = \sum_i^N \delta_i(X) A_i LGD_i \quad (1)$$

where  $\delta_i(X)$  is a default indicator for asset  $i$  taking the value 1 with probability  $PD^i(X)$  and the value 0 with probability  $(1-PD^i(X))$ . We assume conditional independence.<sup>9</sup> Therefore, conditional on the state of systematic risk factors  $X$ , the default indicators  $\delta_i(X)$  are *i.i.d.* Bernoulli random variables. Hence, our set-up is in the tradition of Bernoulli mixture models. It has been shown that all standard industry models such as CreditRisk+, CreditMetrics, Moody's KMV and CreditPortfolioView but also Basel II can be formulated in this fashion (e.g. see Frey and McNeil, 2002). Note that generally these models, and in particular Basel II, do not take changes in the mark-to-market value into account. The models only differ because of their assumptions on the distribution of the systematic risk factors, the mapping between risk factors and PDs, and whether they are solved analytically or numerically. Given the complexity of the multi-period framework, we will do the latter for our empirical application; we also identify macro factors as the systematic risk drivers of PDs in the spirit of Wilson (1997a/b) (see Section 4.4 for details). The unconditional probability of incurring  $l$  losses  $P(L=l)$  is then given by

$$P(L=l) = \int P(L(X)=l) F(X) dX \quad (2)$$

<sup>8</sup> For details of the balance sheet structure used in the empirical implementation see Section 4.1.

<sup>9</sup> This is the standard assumption used in credit risk models implemented for day to day risk management, even though recent research has shown that this assumption does not necessarily hold (see Duffie *et al.*, 2007).

A graphical representation of the unconditional loss distribution is given in the Annex Figure A1.

So far this is in line with standard credit risk portfolio models. It is very easy to take account of net interest income in this framework. Net interest income is simply interest payments received on assets minus interest payments paid on liabilities. Given coupon rates are fixed for the moment, the only stochastic component of net interest income in the one period set-up is whether assets default or not.

Take an asset  $A^i$ . If no default occurs, the cash flow contribution to interest income is  $C^i A^i$ . In case of default, however, the cash flow contribution is only  $(1-LDG^i)C^i A^i$  as we assume that coupon payments can be partially recovered with the same recovery rate  $(1-LGD^i)$  as the principal. Total realised net interest income  $RNI$  is therefore

$$\begin{aligned} RNI(X) &= \sum_i [C^i A^i - \delta_i(X) LGD^i C^i A^i] - \sum_j C^j L^j \\ &= \sum_i C^i A^i - \sum_j C^j L^j - \sum_i \delta_i(X) LGD^i C^i A^i \\ &= NI - \sum_i \delta_i(X) LGD^i C^i A^i \end{aligned} \quad (3)$$

As can be seen from equation (3), realised net interest income can be decomposed into a component which excludes the effect of default,  $NI$ , and a term which sums over coupon losses due to crystallised credit risk; the latter depends on the state of systematic risk factors.  $NI$  is defined as

$$NI = \sum_i C^i A^i - \sum_j C^j L^j \quad (4)$$

Given that coupon rates are pre-determined, the first component in equation (3) ( $NI$ ) is not stochastic, whilst the latter is. As coupons only default when the underlying asset defaults, the latter random component can be simply incorporated into the loss distribution by defining the loss including defaulted coupons  $L^*$  as

$$L^*(X) := \sum_i^N \delta_i(X) (1 + C^i) A_i LGD_i \quad (1')$$

The corresponding unconditional loss distribution is analogous to that in equation (2). Ultimately we are interested in net profits  $NP(X)$  which are the sum of credit risk losses and net interest income:

$$NP(X) = RNI(X) - L(X) = NI - L^*(X) \quad (5)$$

The second equality of equation (5) simply takes account of the fact that realised net interest income can be decomposed into  $NI$  and defaulted coupon payments which are included into  $L^*$ . Since  $NI$  is non-stochastic, only  $L^*$  introduces randomness into net profits. Therefore, the net

profit distribution is identical to the distribution of  $L^*$  bar a mean shift of the size of  $NI$ . Put differently, the unconditional probability of realising net profits  $np$  is

$$\begin{aligned} P(NP = np) &= P(L^* = -(np - NI)) \\ &= \int P(L^*(X) = -(np - NI)) dF(X) \end{aligned} \quad (6)$$

Figure A2 in the Annex provides a graphical representation of this argument. Credit risk losses enter negatively into net profits, hence the negative sign in equation (6) before  $(np - NI)$ . Since defaulted coupon payments increase losses,  $L^* \geq L$  and the distribution of  $-L^*$  is to the left of the distribution of  $-L$ . Note that  $L^*$  is not linear in the coupons, so this is not a pure mean shift.  $NI$  is non-stochastic and positive. Therefore, the net profit distribution is equal to the distribution of  $-L^*$ , except a mean shift of the size of  $NI$ . Overall, the mean of the  $NP$  distribution is equal to  $NI - E(L^*)$ .

Standard economic capital models for credit risk assume that the expected loss is covered by income. Expected net profits are therefore zero. As an aside, it is interesting to observe that this assumption holds if (a) the bank is fully funded by liabilities ( $\sum^i A^i = \sum^j L^j = L$ ) paying the risk-free rate  $r$  ( $C^j = r \forall j$ ), (b) assets and liabilities have a repricing maturity of one period, and (c) banks price assets in a risk neutral fashion. This can indeed be seen in our framework. Under the conditions stated above, the one-period ahead expected net profits are given by

$$\begin{aligned} E(NP) &= NI - \sum_i PD^i LGD^i (1 + C^i) A^i \\ &= \left( \sum_i C^i A^i - rL \right) - \sum_i PD^i LGD^i (1 + C^i) A^i \end{aligned} \quad (7)$$

As shown in Annex 1 the risk-neutral coupon rate in discrete time for an asset with a time to repricing of one is

$$C^i = \frac{(r + PD^i * LGD^i)}{(1 - PD^i * LGD^i)} \quad (8)$$

Inserting (7) into (8), expected net profits can be re-written as

$$\begin{aligned} E(NP) &= \left( \sum_i C^i A^i - rL \right) - \sum_i PD^i LGD^i (1 + C^i) A^i \\ &= \left( \sum_i (r + PD^i LGD^i) A^i - PD^i LGD^i A^i \right) - rL \\ &= 0 \end{aligned} \quad (9)$$

As will become apparent from our simulation results expected net profits need not be zero if the bank is not fully funded by liabilities, assets have different maturities than liabilities, or the bank is not pricing assets in a risk-neutral fashion.<sup>10</sup>

<sup>10</sup> We also implemented this simple example (fully matched bank, risk neutral pricing, one-quarter horizon) in our simulation set-up. Mean net profits are indeed zero. Results are available on request.

### 3.2 *The multi-period framework*

In contrast to the single period framework,  $NI$  is random in the multi-period set-up because coupon rates on assets and liabilities adjust in line with fundamentals. To determine coupon rates in a dynamic setting we apply the pricing framework discussed in Annex 1. But in order to account for bank and depositors' behaviour we need to introduce further assumptions:

- (i) Depositors are passive: once deposits mature, depositors are willing to roll them over maintaining the same repricing characteristics.
- (ii) The bank does not actively manage its portfolio composition: if assets mature or default, the bank continues to invest into new projects with the same repricing and risk characteristics as the matured assets. At the end of each period, the bank also replaces defaulted assets with new assets which have the same risk and repricing characteristics.

These assumptions are essential to ensure that the bank's balance sheet balances at each point in time. Whilst this is a fundamental accounting identity which must hold, risk management models often ignore it as profits and losses are not assessed at the same time. This is a crucial innovation in the framework of DSS.

Assumption (ii) is often used in practice by risk managers, who call this “ever-greening” the portfolio: once an asset matures, the bank issues a new loan with the same repricing and risk characteristics. For example, a matured loan to the corporate sector which originally had a one year repricing maturity is replaced by a loan to the corporate sector with a one year repricing maturity. Similarly, once an asset defaults, the bank invests in a similar asset with the same repricing and risk characteristics. This new asset is funded by reinvesting the recovery value of the defaulted loan and the remainder out of current profits or shareholder funds.<sup>11</sup> This implies that at the beginning of each quarter the bank holds the same amount of risky assets on its balance sheet. We assume that, if the bank makes positive profits, it holds the profits in cash until the end of the year without expanding its lending activity.<sup>12</sup> The stock of cash is used to buffer negative net-profits. Whenever the buffer is not sufficient and capital falls below initial levels, we assume that shareholders inject the necessary capital at the end of the quarter. Assumption (i) implies that the volume and source of deposits does not change over time. Together with assumption (ii), this also means that at the beginning of each period the overall portfolio is the same in terms of risk and repricing characteristics. Clearly, our behavioural assumptions are to a certain degree arbitrary. But we restrict ourselves to a simple behavioural rule rather than re-optimising the bank's

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<sup>11</sup> Whilst recovery may not be instantaneous, it is sufficient to assume that the bank can sell the defaulted loan to an outside investor who is paying the recovery rate.

<sup>12</sup> By holding profits in cash the bank foregoes potential interest payments. However, given the one-year horizon, these are immaterial. As a sensitivity test we replicated our baseline results under the assumption that profits earn the risk-free rate of return, and the changes turned out to be negligible. Details are available upon request.

portfolio in a mean-variance sense in each period as this would be beyond the scope of this paper. As said, our rules are also commonly assumed by risk managers.<sup>13</sup>

**Figure 1: Timeline of the multi-period framework**

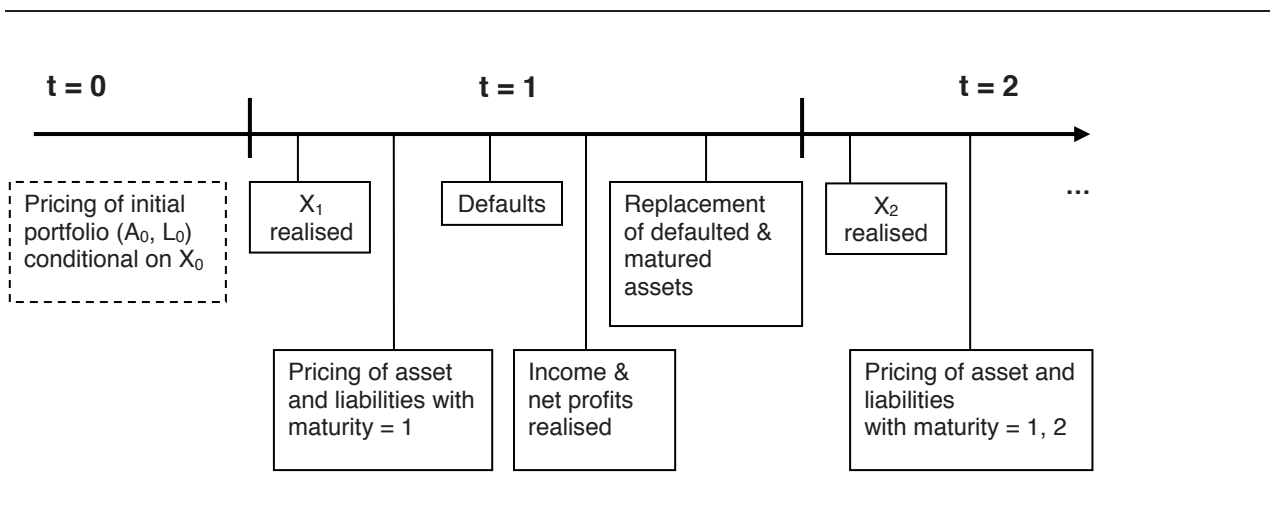


Figure 1 clarifies the time line of the multi-period framework. The bank starts with an initial portfolio  $A_0 = \sum_i A_0^i$  and  $L_0 = \sum_j L_0^j$ . Initial coupon rates for assets/liabilities ( $C_0^i(X_0)/C_0^j(X_0)$ ) are priced based on macroeconomic conditions  $X_0$  at time 0. At the beginning of  $t=1$  a shock hits the economy changing the macro conditions to  $X_1$ , which can already be taken into account when the bank reprices all assets and liabilities with a time to repricing of 1. After repricing, credit risk losses are realised; then interests on assets and liabilities are paid, and time-1 net profits are calculated. Finally, the bank replaces the defaulted assets and re-invests matured assets and liabilities. At the beginning of  $t=2$ ,  $X_2$  is drawn. At this stage the bank can reprice assets and liabilities with a repricing maturity of 1 and 2. Then, as in  $t=1$ , credit risk losses materialise and net profits are calculated. The latter period is repeated until the end of the simulation horizon (in our case  $t=1, \dots, 4$ ). However, the repricing mechanism becomes increasingly complex over time, as different assets mature at different points in time: in  $t=3$  the bank reprices assets with repricing maturity 1 and 3, while in  $t=4$  it reprices assets with a repricing maturity of 1, 2, and 4 quarters. Annex 2 provides a stylised example of how the repricing mechanism works.

<sup>13</sup> DSS make similar behavioural assumptions, and provide an extensive discussion on how changes in these assumptions may affect their results. Their discussion largely applies to our framework as well.

### 3.2.1 NI in the multi-period framework

In the single period framework,  $NI$  was non-stochastic. However, coupons are now changing depending on the repricing characteristics of the underlying assets and liabilities. It is also important to recall that every asset (liability) has a different time to repricing  $b_i$  ( $b_j$ ). The contribution of assets ( $NI_t^A(X_t)$ ) to  $NI_t(X_t)$  conditional on  $X_t$  at time  $t$  is:

$$NI_t^A(X_t) = \sum_{i=1}^N \sum_{p=0}^t I_p^i C_p^i(X_p) A^i \quad (10)$$

where  $I_p^i = 1$  in period  $p$ , when asset  $A^i$  has been repriced the last time prior to time  $t$ , and  $I_p^i = 0$  otherwise. Equation (10) sums across coupon incomes from different assets which have been repriced at different periods.  $I_p = 1$  for assets which have been repriced the last time in period  $p$  and therefore earn a coupon rate  $C_p^i(X_p)$ , which was set taking account of time- $p$  macro conditions  $X_p$ . Note that assets which had an initial time to repricing of  $b_i > t$  have not been repriced, so they still earn coupon rates  $C_0^i(X_0)$ .<sup>14</sup>

Similarly, given that we assume that borrowers are willing to roll over the bank's liabilities, the liability contribution  $NI_t^L(X_t)$  to  $NI(X)$  is:

$$NI_t^L(X_t) = \sum_{j=1}^N \sum_{p=0}^t I_p^j C_p^j(X_p) L^j \quad (11)$$

where  $I_p^j = 1$  in period  $p$ , when liability  $L^j$  has been repriced the last time prior to  $t$ , and  $I_p^j = 0$  otherwise. In line with equation (10), equation (11) sums over all liabilities taking into account the last time  $p \leq t$  when liabilities have been repriced. Total conditional net interest income  $NI_t(X_t)$  in period  $t$  is therefore

$$NI_t(X_t) = NI_t^A(X_t) - NI_t^L(X_t) \quad (12)$$

Equation (12) does not account for the impact of defaults on net interest income: it only reflects the impact of repricing on interest income.

Equations (10) and (11) are at the heart of the model. They imply that for every macro scenario we need to track coupon rates for all asset and liability classes with different repricing maturities. Coupon rates in turn are set in different time periods and depend on the prevailing and expected

<sup>14</sup> For example in period 4, all assets which had initially a time to repricing  $b_i > 4$  continue to carry the initial coupon rates and hence  $I_p^i$  is only equal to 1 if  $p=0$ . Assets with repricing maturities of less than 4 periods have been repriced prior to or at the beginning of period 4. In particular, assets with  $b_i = 1, 2, 4$  have been repriced in period 4, so for all these assets  $I_p^i = 1$  for  $p=4$ , whereas assets with  $b_i = 3$  were last repriced in period  $p=3$  and hence  $I_p^i = 1$  for  $p=3$ .

macro factors at that point in time. In comparison to standard credit risk model, this increases the computational complexity enormously.

### 3.3 The multi-period profit and loss distribution

Given our timing (Figure 1),  $NI_t(X_t)$  is non-random in period  $t$  as coupon rates of assets and liabilities which can be repriced in that period already reflect macro conditions  $X_t$ . Therefore, we can apply the framework developed in Section 3.1 on an iterative basis. This is a crucial insight of our framework and it facilitates the computation significantly as it allows us to disentangle interest income and credit risk losses including defaulted coupons. In each period,  $NI$  is determined by equation (12), and losses due to the default of coupons and principals are determined by equation (1'). Note that coupon rates between periods may change and need to be incorporated into (1') in the dynamic set-up. Therefore credit risk losses including defaulted coupons conditional on  $X$  at time  $t$  are

$$L_t^*(X_t) = \sum_{i=1}^N \sum_{p=0}^t I_p^i \delta_i(X_t) (1 + C_p^i(X_p)) A^i LGD_i \quad (13)$$

where  $I$  and  $\delta$  are again indicator functions.  $I_p^i = 1$  in period  $p$ , when asset  $A^i$  has been repriced the last time prior to time  $t$ , and  $I_p^i = 0$  otherwise.  $\delta_i(X_t) = 1$  if asset  $i$  defaults at time  $t$ , and  $\delta_i(X_t) = 0$  otherwise. The interpretation is again similar to equation (10) and (11). Note however that the default indicator does not depend on the repricing maturity but only on credit conditions at time  $t$ . As in the one period framework (see equation (5)), net profits  $NP_t(X_t)$  equal the sum of  $L_t^*(X_t)$  and  $NI_t(X_t)$ . We simulate the model dynamically, so total net profits in period  $T$  depend on the history of macro risk factors  $\bar{X}_T = [X_0, X_1, \dots, X_T]$ :<sup>15</sup>

$$NP_T(\bar{X}_T) = \sum_{t=1}^T NP_t(X_t) = \sum_{t=1}^T (NI_t(X_t) - L_t^*(X_t)) \quad (14)$$

Analogous to the single period framework, the ex-ante distributions of credit risk losses, net-interest income and net-profits for time  $T$  is the integral over all possible states. Since we cannot explicitly derive them, we obtain these distributions by simulation techniques. The specific implementation is discussed in the next section, but the mechanism follows our time line. In each period, we first draw  $X_t$ , then determine  $NI_t$ , simulate defaults of individual assets and coupons and finally calculate  $NP_t$ . After reinvestment, this process is repeated for the next quarter and so on up to time  $T$ . In the end we sum across all quarters. We repeat the simulation ten thousands times to

<sup>15</sup> As excess profits are invested in cash rather than a risk free asset, we can add up net-profits across time without taking account of the time value of money. As pointed out in footnote 11 we also undertook a sensitivity test investing net-profits in risk free assets. Results differ only marginally and do not change the main message of the paper.



derive the full unconditional distributions. Note that our horizon of interest is one year; since  $T=4$  throughout the analysis, we drop the time index  $T$  in the remainder of the paper.

### 3.4 Economic Capital

As discussed in the introduction, in line with current market and regulatory practices, we set the level of capital such that it equals the amount a bank needs to absorb unexpected losses over a certain time horizon at a given confidence level (Basel Committee, 2008 or Kuritzkes and Schuermann, 2007). In our framework, unexpected losses can arise because of credit risk or adverse interest rate shocks.

For credit and interest rate risk, we follow standard convention and measure unexpected losses as the difference between the Value at Risk (VaR) and expected losses. More precisely, the VaR of the credit risk loss distribution  $VaR_{CR}^y$  at a confidence level  $y \in (0,1)$  is defined as the smallest number  $l$  such that the probability of  $L$  exceeding  $l$  is not larger than  $(1-y)$ :

$$VaR_{CR}^y = \inf[l, P(L \geq l) \leq (1-y)] \quad (15)$$

For risk management purposes the confidence level is generally high with  $y \geq 0.9$ . In line with risk management practices, we set economic capital against credit risk  $EC_{CR}^y$  at the confidence level  $y$  so that it covers the difference between expected and unexpected losses up to  $VaR_{CR}^y$ . Or formally

$$EC_{CR}^y = VaR_{CR}^y - E(L) \quad (16)$$

$VaR_{CR}^y$  and  $EC_{CR}^y$  are shown in Figure A1 in the Annex. Analogous to credit risk, we define the VaR of the  $NI$  distribution  $VaR_{NI}^z$  at a confidence level  $z \in (0,1)$  as the smallest number  $ni$  such that the probability of  $NI$  exceeding  $ni$  is not larger than  $(1-z)$ . Or

$$VaR_{NI}^z = \inf[ni, P(NI \geq ni) \leq (1-z)] \quad (17)$$

$NI$  provides positive contributions to net profits, so we are interested in the left tail of the distribution. Therefore  $z$  is in this case below or equal to 0.1. Given that the focus is on the left tail, economic capital ( $EC_{NI}^{(1-z)}$ ) is meant to cover unexpectedly low  $NI$  outcomes at the confidence level  $(1-z)$ :

$$EC_{NI}^{(1-z)} = E(NI) - VaR_{NI}^z \quad (18)$$

Given this definition, economic capital set at the 99% confidence level covers all unexpected low outcomes of  $NI$  between the  $VaR_{NI}$  at the 1% level and expected  $NI$ . Note that  $VaR_{NI}$  and  $EC_{NI}$  do not incorporate defaulted coupons. As we argue in Section 3, these are an important part of the analysis and they can be accounted for equivalently in the income calculation (3) or in the credit risk loss calculation (1'). We follow the first route and construct VaR and EC statistics for realised

net interest income  $RNI$ , which incorporates the loss of payments on defaulted assets. The definitions of  $VaR_{RNI}$  and  $EC_{RNI}$  are analogous to equations (17) and (18).

Ultimately, we are interested in risk measures for the net profit distribution. Risk managers do not focus on the right tail of this distribution, which constitutes the up-side risk for a bank, but on the left tail. In line with  $VaR_{NI}$ , we define the VaR of the net profit distribution  $VaR_{NP}^z$  at a confidence level  $z \in (0,1)$  as the smallest number  $np$  such that the probability of  $NP$  exceeding  $np$  is not larger than  $(1-z)$ . Or

$$VaR_{NP}^z = \inf[np, P(NP \geq np) \leq (1-z)] \quad (19)$$

Mechanically we could set capital against net profits such that it buffers all unexpected low outcomes; i.e. we could set it as the difference between  $E(NP)$  and  $VaR_{NP}^z$ . Mathematically this definition would make sense. Economically, however, it does not because it implies that the bank also holds capital against low but positive profits, even though banks hold, as discussed above, capital to buffer (unexpected) losses. To clarify this, say a bank manages its capital to a 95% confidence level and  $VaR_{NP}^{5\%} > 0$ . Such a bank would not hold any capital as it knows that it makes positive profits with a 95% likelihood. Even if it manages capital to a confidence level of 99% and  $VaR_{NP}^{1\%} < 0$ , the bank would not set capital as the difference between  $E(NP)$  and the VaR because it does not make sense to “buffer” positive profits. Insofar as the bank only holds capital against net losses, a more sensible definition of the economic capital  $EC_{NP}^{(1-z)}$  at a confidence level  $(1-z)$  is

$$EC_{NP}^{(1-z)} = \begin{cases} 0 & \text{if } VaR_{NP}^z \geq 0 \\ -VaR_{NP}^z & \text{if } VaR_{NP}^z < 0 \end{cases} \quad (20)$$

The intuition behind equation (20) is illustrated in Figure A2 in the Annex. Here  $VaR_{NP}^z > 0$  at a confidence level  $(1-z)$ , so no capital is needed. Using a higher confidence level  $(1-y)$  some unexpected negative net profits (*i.e.* net losses) can materialise and the bank would set capital to buffer the possible negative outcomes.

As discussed in the introduction, we are ultimately interested in assessing whether setting economic capital in a naïve fashion by adding economic capital against credit risk and economic capital against net interest rate risk (including defaulted coupons) provides a conservative bound in comparison with setting capital against net profits. We assess this by looking at the following measure for confidence level  $y$

$$M_{EC}^y = \frac{(EC_{CR}^y + EC_{RNI}^y) - EC_{NP}^y}{(EC_{CR}^y + EC_{RNI}^y)} \quad (21)$$

The larger  $M_{EC}$ , the more conservative simple economic capital is. Conversely, if  $M_{EC}$  is negative then simply adding up the two capital measures independently would underestimate the risk of the total portfolio.<sup>16</sup>

In our framework,  $EC_{NP}$  covers negative net profits (*i.e.* net losses) rather than looking at the difference between expected net profits and unexpected net profits as  $EC_{RNI}$  and  $EC_{CR}$  do. This is economically sensible, because profit fluctuations have a direct impact on bank capital independently of whether they are expected or not. With perfect competition and risk neutral pricing, average profits would be zero and the difference would not be material. However if banks earn rents (for example by pricing customer deposits below the risk-free rate, as we can observe empirically) expected profits are positive, which increases  $M_{EC}$ . In other words, rents may introduce a further wedge between “simple” and “integrated” economic capital.

We maintain that  $M_{EC}$  is the most appropriate measure in this context, but to control for this issue we also provide an alternative measure  $M_2$  that takes into account the mean of the net profit distribution:

$$M_2^y = \frac{(EC_{CR}^y + EC_{RNI}^y) - (E(NP) - Var_{NP}^{(1-y)})}{(EC_{CR}^y + EC_{RNI}^y)} \quad (22)$$

Given that we model a banking book,  $EC_{CR}$  and  $EC_{RNI}$  do not take account of changes in the mark-to-market valuations of the exposures; hence, they do not capture aspects of (and interactions between) credit and interest rate risk which arise when assets are marked to market (we briefly discuss this in the conclusions). It can also be argued that  $EC_{CR}$  and  $EC_{RNI}$  do not fully disentangle credit and interest rate risk, in the sense that the former incorporates the effect of higher interest rates on default probabilities and the latter the effect of higher (actual or expected) credit risk on income. These issues should be certainly kept in mind throughout the discussion of our results. The key point, though, is that our framework represents a plausible description of how current capital models for the banking book capture these risks. As already discussed, the current regulatory approach to credit risk and the commonly used “earnings at risk” approach to interest rate risk do not take changes in market valuations into account. Furthermore, some credit risk models include a set of macroeconomic risk factors and hence capture (directly or indirectly) some of the links between interest rates and credit risk. This is for instance the case for CreditPortfolioView (Wilson 1997a, b), the classic example of such an economic capital model. To the extent that our  $EC_{CR}$  and  $EC_{RNI}$  definitions reflect limitations and ambiguities that are common to many widely used risk management tools, the model should provide a plausible benchmark for our “simple economic capital” setting. Our pricing model represents of course a

<sup>16</sup> It is well known that VaR is not a coherent risk measure. However, Expected Shortfall is not coherent in our set-up either as credit and interest rate risk interact in a non-linear fashion. Therefore we only report economic capital numbers based on VaR measures. The insights from all results remain when using expected shortfall instead. Results are available on request.

departure from standard modelling practices. Most interest rate risk models do not take account of the possible repricing of assets beyond changes in the risk-free rate. Hence, by modelling endogenous spreads we add a layer of realism and complexity to the analysis. However, in line with standard approaches to model interest rate risk, we also undertake a sensitivity test where all spreads are excluded (see Section 6.1).

## 4 Implementation

Most quantitative risk management models currently used can be described as a chain starting with shocks to systematic risk factors feeding into a model that describes the joint evolution of these factors and finally a component that calculates the impact on banks' balance sheets (see Summer, 2007). Depending on the distributional assumptions and the modelling framework, the loss distribution can be derived either analytically or by simulating this chain repeatedly. Our implementation follows in this tradition.

For the discussion it is important to recall the timing shown in Figure 1. In the first quarter ( $t=0$ ) the balance sheet is fixed and all initial coupons are priced based on the observed macroeconomic conditions. Figure A3 in the Annex shows how the simulation works for every subsequent quarter  $t=1, \dots, 4$ . At the beginning of  $t=1$ , we first draw a vector of random macroeconomic shocks and determine the state of the macroeconomy using a Global VAR (in the spirit of *e.g.* Pesaran *et al.*, 2004). The GVAR also allows us to derive a forward risk-free yield curve. Using a satellite model, we then obtain PDs conditional on the new macro conditions. At this point the bank can reprice all assets and liabilities in the first repricing bucket, which already allows us to calculate *NI*. We then simulate (conditionally independent) defaults to derive *L* and *RNI* and hence net profits *NP*. At the end of the quarter the bank rebalances its balance sheet in line with the behavioural assumptions presented in Section 3.2. The remaining forecast periods follow the same structure, except that the repricing mechanism becomes increasingly complex as different assets and liabilities are repriced at different points in time as discussed in Section 3.2.

### 4.1 The hypothetical bank

Table A1 in Annex 3 provides an overview of the balance sheet used for the simulation. It represents the banking book of a simplified average UK bank as exposures in various risk and repricing buckets are derived by averaging the published balance sheets of the top ten UK banks.

In order to limit the number of systematic risk factors we have to model, we assume that the bank only has exposures to UK and US assets. This reduces the complexity of the simulation considerably without diminishing the insights of the paper. We look at seven broad risk classes in both the UK and the US: interbank; mortgage lending to households, unsecured lending to households; government lending; lending to PNFCs (private non financial corporations); lending

to OFCs (other financial corporations, *i.e.* financial corporations excluding banks); and “other”. Exposures within an asset class are homogenous with respect to PDs and LGDs. We assume that the bank is fully funded by UK deposits. These consist of interbank, household, government, PNFC, OFC, subordinated debt, and “other”.

Contrary to DSS, we model a portfolio which is not infinitely fine grained. Since no data are available on the size of the exposures, we construct a hypothetical loan size distribution for each asset class. Anecdotal evidence suggests that loan size distributions are approximately log-normal. Therefore, we assume that asset sizes are log-normally distributed with variance one and a mean of £300.000 for household mortgage exposures, £50.000 for unsecured household lending, £100mn for PNFC and £200mn for OFC. The resulting distributions are shown in Figure A4 in the Annex. This parameterisation is very much “back of the envelope” based on the limited information we have. But it delivers a size distribution which looks similar to the size distribution in other countries where detailed data are available. We will also undertake a sensitivity test to assess the implications of an infinitely fine-grained portfolio.

All exposures are assumed to be non-tradable and held to maturity using book value accounting. In line with accounting standards, assets and liabilities are allocated to five repricing buckets as shown in Table A1. For the actual analysis assets, liabilities and off-balance sheet items in the last three buckets are assumed to be uniformly distributed over quarters within each bucket. For the last bucket we assume that the maximum time to repricing is ten years. The interest rate sensitivity gap is the difference between assets and liabilities in each repricing bucket.

It is important to stress that we are using repricing buckets rather than maturity buckets in order to correctly capture the impact of changes in the macro-economic environment on the bank’s net interest income and hence profits. This means that, for example, a flexible mortgage with a 20-year maturity that reprices every three months is allocated to the three-month repricing bucket. As DSS show, the repricing characteristics are the key determinant of interest rate risk in the banking book. The interest rate sensitivity gap relative to total assets of our balance sheet is fully in line with the average interest rate sensitivity gap of the top ten UK banks in 2005.<sup>17</sup> Given that in the UK mortgage borrowers predominantly borrow on a flexible rate basis, a high proportion of assets is allocated to the 0-3 months repricing bucket (see Table A1).<sup>18</sup>

In contrast to DSS we do not look at interest sensitive off-balance sheet items. UK banks on average use these items to narrow the repricing gap between short term borrowing and long term

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<sup>17</sup> Under UK accounting standards known as FRS 13, UK banks have to publish the interest sensitivity gap of on-balance and off-balance sheet exposures in their annual accounts.

<sup>18</sup> The average interest rate sensitivity gap relative to total assets in the UK is stable over time, but given varying economic and institutional conditions there are differences across countries. For example, given a much higher proportion of fixed rate mortgages 50.2% of loans and securities have a remaining time to repricing greater than one year for the average US bank, in comparison to 20.7% for the average UK bank (at the end of 2005). The liability side looks more similar for the average UK and US bank. For the latter 12.5% of liabilities have a remaining time to repricing of more than a year, whereas the proportion in the UK is 8.3% (for US data see FFIC, 2006).

lending. Hence, the interest rate risk estimated in this paper should be more significant than for the actual average UK bank. The repricing structure of the balance sheet is crucial in determining interest rate risk, so we perform a number of sensitivity tests on our baseline assumptions.

#### 4.2 Shocks, the macro model and the yield curve

To model the macro environment, we implement a two-country version of Pesaran, Schuermann and Weiner's (2004) Global VAR model. Within a generic GVAR, each country is modelled as a standard vector autoregression (VAR) augmented by a set of contemporaneous and lagged 'foreign' factors, constructed as weighted averages of the other countries' variables. Pesaran and co-authors show that, under fairly general conditions, foreign variables are weakly exogenous within each country-specific VAR. Hence, the VARs can be estimated individually and then combined to generate mutually consistent forecasts for the whole world economy. Following Pesaran *et al.* (2006), we use the GVAR as a reduced-form model of systematic (national and international) risk factors.

In our simplified two-country framework, the UK is a small open economy and the US a closed economy. The model has the following form:

$$\begin{aligned} x_t^{uk} &= a_0^{uk} + a_1^{uk}t + \Phi_1^{uk} x_{t-1}^{uk} + \Phi_2^{uk} x_{t-2}^{uk} + \Lambda_0 x_t^{us} + \Lambda_1 x_{t-1}^{us} + \varepsilon_t^{uk} \\ x_t^{us} &= a_0^{us} + a_1^{us}t + \Phi_1^{us} x_{t-1}^{us} + \Phi_2^{us} x_{t-2}^{us} + \varepsilon_t^{us} \end{aligned} \quad (23)$$

(where  $x_t^i$  is a vector of risk factors specific to country  $i$ ). From a UK perspective, this can be interpreted as a GVAR based on a degenerate weighting scheme: we implicitly construct the 'rest of the world' by assigning weights of one to the US and zero to all other countries.<sup>19</sup>

Variables and data are the same as in Dees *et al.* (2007). For the UK,  $x_t$  contains real output ( $GDP_t$ ), CPI inflation ( $CPI_t$ ), real equity prices ( $EQP_t$ ), an overnight nominal interest rate ( $SR_t$ ), a 20-year synthetic nominal bond interest rate ( $LR_t$ ) and the real exchange rate against the dollar ( $EX_t$ ). For the US, the real exchange rate is replaced by oil prices ( $OIL_t$ ). The sample is 1979Q1–2005Q4.

We estimate all equations in (23) by OLS. The systems appear to contain stochastic trends and cointegrating relationships (see Dees *et al.*, 2007), so this procedure may not be fully efficient. However, OLS deliver (super)consistent estimates of all parameters. Furthermore, given the focus of the paper, imposing theoretical restrictions on the macroeconomic data is unnecessary. Diagnostic tests (not reported for brevity) show that the estimation generates approximately normal *i.i.d.* residuals.

<sup>19</sup> Pesaran *et al.*'s (2004) weak exogeneity is an asymptotic result obtained assuming a large number of countries, and would not hold in a symmetric two-country world. Assuming that the US is a closed economy allows us to circumvent this problem.



The pricing of coupons requires a yield curve which is conditional on macroeconomic factors. We use a very simple specification and assume that the yield curve is a linear interpolation of the short ( $SR_t$ ) and long ( $LR_t$ ) interest rates. This is the source of all risk-free rates used in the model.

Shocks are simulated by randomly drawing vectors  $\varepsilon_t = [\varepsilon_t^{uk}, \varepsilon_t^{us}]'$  from a multivariate normal distribution with mean zero and the variance-covariance matrix estimated in equation (23).

#### 4.3 Modelling PDs and LGDs for different asset classes

To estimate the impact of macro factors on PDs we use simple equations in the spirit of Wilson (1997,a,b).<sup>20</sup> As it has already been mentioned, we assume that loans within a particular asset class are homogenous with respect to their risk characteristics, *i.e.* they all have the same PD and LGD. This assumption is dictated by data limitations, as only aggregate default frequencies for corporate and household lending are available in the UK. Because of informal debt restructurings, recorded data on bankruptcies tend to underestimate the true scale of default, especially in relation to household unsecured debt: even if a loss-given-default (LGD) of 100% is chosen, implied write-offs calculated on the basis of PD\*LGD are significantly below the recorded figures. To adjust for this, bankruptcy data is scaled up.

Given this adjustment, default probabilities within an asset class are then estimated as a function of the macroeconomic outputs of the GVAR model. For each asset class  $ac$  the equation for PDs is

$$OD_t^{ac} = \alpha^{ac} + \beta_1^{ac} \Delta GDP_{t-1} + \beta_2^{ac} \Delta EQP_{t-1} + \beta_3^{ac} SRR_{t-1} + \varepsilon_t^{ac} \quad (24)$$

where  $OD_t^{ac}$  is the quarterly log-odds transform of the (adjusted) default ratio for asset class  $ac$ .<sup>21</sup> The literature suggests that GPD and equity returns should have a negative effect on credit risk whilst interest rate should increase the riskiness of an asset class. Overall this is a very simplified PD model, which nonetheless allows us to forecast average PDs in every scenario in a consistent fashion.

We assume that *LGDs* are fixed. Broadly in line with average industry numbers, we assume that the *LGDs* are 40% for interbank loans, 30% for mortgage loans, 100% for credit card loans and 80% for corporate loans.

#### 4.4 Pricing of assets

We calculate coupons using the risk-neutral pricing model proposed in DSS (see Annex 1). Given the non-linearity of the model, we can only implement the framework by introducing two

<sup>20</sup> The equations were developed as part of measurement model for the UK financial stability; for an overview over this model see Alessandri *et al.* (2009).

<sup>21</sup> The log-odds transformation ensures that aggregate default frequencies remain within the 0-1 boundary. It is equivalent to assuming that PDs follow the more standard formulation in the spirit of Wilson *i.e.*  $PDs = 1/(1 + \exp(-\beta X))$ .



approximations. These are discussed in detail in Annex 3, where we also show that they do not bias our results.

It is well known that there is no simple mapping from actual *PDs*, which we simulate, into risk-neutral *PDs*, which we require for pricing (see e.g. Duffie and Singleton, 2003). The empirical literature has found that the jump-to-default risk premia defined as the ratio of risk neutral over actual *PDs* varies between 1 and 4 (see Driessen, 2005, Saita 2006 or Berndt *et al.* 2005).<sup>22</sup> At this stage it is hard to derive firm conclusions from the literature that could be easily implemented in our already complex model. But acknowledging the fact that the jump-to-default risk may be greater than one, we include the fixed risk premia presented in Table A2 in Annex 3.

By not modelling the risk premia explicitly we may introduce a downward bias in the bank's net interest income, as our coupon rates on assets are likely to be implausibly low at times of stress. Implicitly, we also assume perfectly competitive markets: the bank prices every new loan issue fairly and individually rather than with respect to the risk contribution to its portfolio. Overall, we conjecture that an economic capital framework looking at the lower tail of the net profit distribution without any spreads provides a conservative assessment of the actual risk profile of the bank.

#### 4.5 Pricing of liabilities

In theory, the bank's liabilities should be priced similarly to assets using the bank's own *PD* and *LGD*. While this seems to be the case for banks' debt instruments, it is well known that shorter-term customer deposit rates are generally below the risk-free interest rate even when accounting for non-interest costs net of fees. This may be the result of deposit insurance schemes or barriers to entry limiting competition (see e.g. Corvoisier and Gropp, 2002). While an economic rationalisation of negative spreads can be found for short maturities, it is not convincing for medium to long maturities. We assume that, as the time-to-repricing increases, the interest paid by the bank on deposits gradually converges to the risk-free interest rate. Other liabilities pay the risk-free interest rate or in case of sub-ordinate debt, interbank and other liabilities the risk-free interest rate plus a fixed 15bp spread. All liability spreads are summarized in Table A3 in the Annex. Essentially all deposits can be seen as fully insured by a deposit insurance scheme.

#### 4.6 The simulation

Our initial macroeconomic and balance sheet data are end-2005. The forecast horizon is one year. The simulation follows the time line described in Figure 1. We simulate 10,000 macro scenarios. In each of these scenarios, we draw one realisation per quarter of the portfolio loss distribution using Monte Carlo methods.

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<sup>22</sup> For example, Saita (2006) estimates the actual one-year *PD* for Xerox in December 2000 was 4.8% whilst he extracts 13% as the one-year risk neutral *PD*. This implies a jump-to-default risk premium of 2.7.

## 5 Results

### 5.1 Macro factors, PDs and interest rates

Figure A5 in the Annex provides an overview over the distribution of macroeconomic variables in the UK after four quarters. US variables look generally similar. Several things are noteworthy. First, all macro variables are roughly normally distributed. Given we simulated the GVAR using normally distributed innovations this is not surprising. Second, after four quarters, several recessionary scenarios emerge where interest rates fall and inflation picks up. However, we also observe scenarios where interest rates increase substantially. Third, there are several scenarios where short term as well as long interest rates changed by more than 200bp at the end of the simulation – the stress test scenarios generally used to measure interest rate risk in the banking book. From a regulatory perspective this is very interesting as it suggests that a 200bp shock may not be very severe. The distributions of quarterly default rates for UK corporate and household borrowers in Q4 are shown in Figure A6 in Annex 4. It is apparent that PDs remain very low in all simulations. This is partly due to the initial conditions: PDs were very low in 2005 by historical standards. But it also reflects the relatively weak impact of macro factors on default rates in the PD model used for the simulation.

### 5.2 The impact on the bank

Figure 2 and Table 1 provide an overview over various components of the profit and loss distribution. Even though macro variables and PDs are roughly normally distributed, credit risk losses show the characteristic fat tail (Panel A). Credit risk losses range from a minimum of 0.8bn to a maximum of 16bn. Interestingly, mean credit risk losses are around 1.37bn, which fits reasonably closely with the reported average provisions of UK banks of 1.59bn for 2006 – the year we forecast – even though our balance sheet is highly stylised and losses do not map one-to-one into provisions.

In line with the distribution of simulated interest rates, net interest income (Panel B) is roughly normally distributed and it shows a much smaller variance than the credit risk loss distribution.<sup>23</sup>

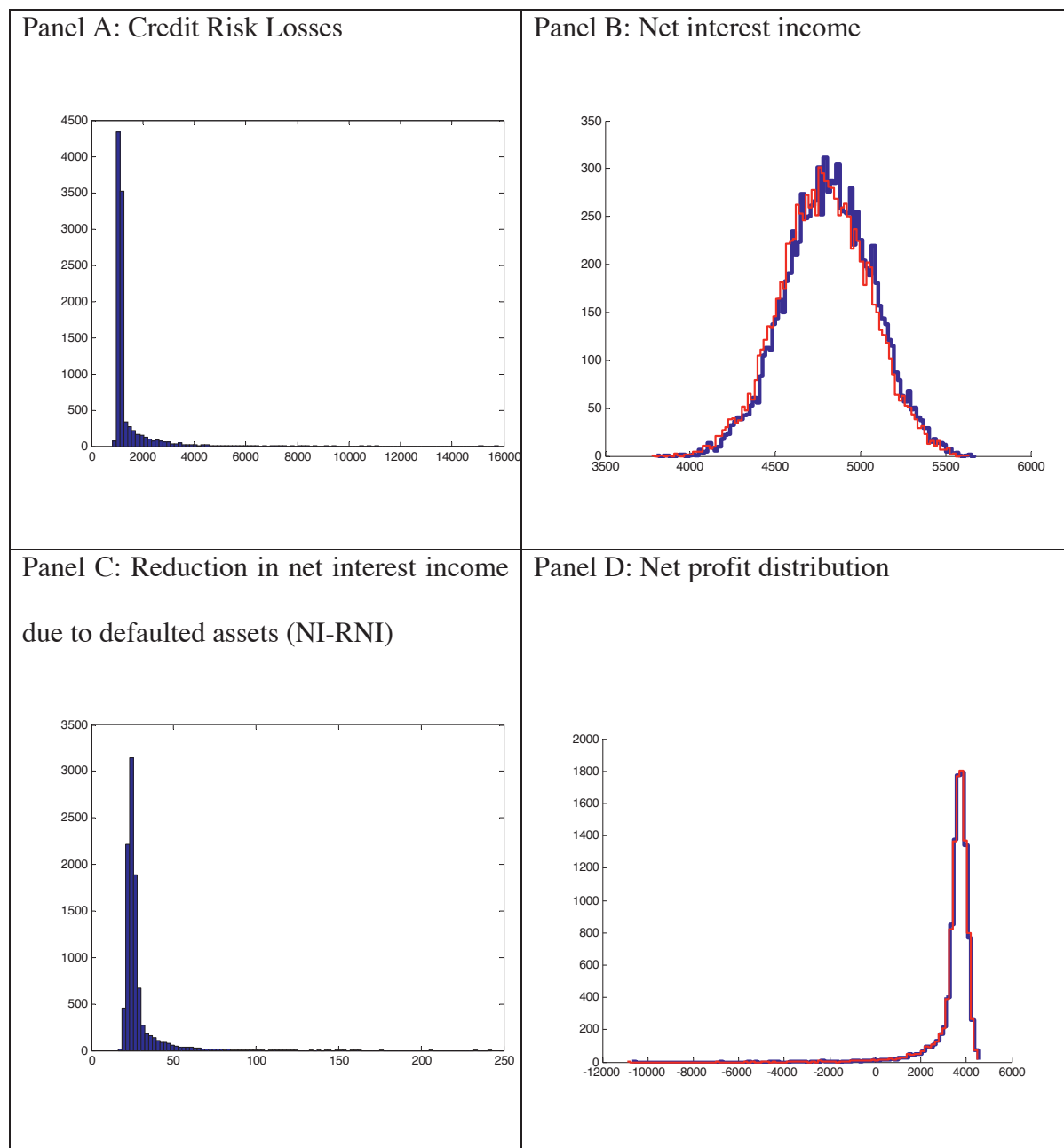
The mean realised net interest income, which accounts for defaulted coupons, is 4.8bn. This is lower than the reported average net interest income of 6.32bn for 2006, possibly because the spreads we add to the risk-neutral coupon rates are not high enough. Clearly, this may also be a result of our assumed balance sheet as our bank is only funded in one currency.

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<sup>23</sup> Throughout the paper we assume that assets and liabilities are priced fairly at the beginning of the simulation. As an additional robustness check we also ran simulations assuming that assets and/or liabilities are 20% over or under-priced. The initial mis-pricing changes only the mean of the net-profits distribution but not its shape in line with the results shown in Section 6.1 therefore not discussed further. Results are available on request.

Panel B also shows that the impact of defaulted coupons on realised net interest income (*RNI*) is relatively small in absolute terms. As expected, the reduction in net interest income due to defaulted coupon rates (Panel C) is exactly in line with the credit risk loss distribution (Panel A). Overall, the net profit distribution (Panel D) shows a significant negative fat tail, even though net profits are positive in more than 98% of the simulations.

**Figure 2: Annual profit and loss distributions**



Note: in millions. In panels B and D, the blue (red) line shows the distribution excluding (including) the impact of defaulted coupon payments.

**Table 1: Losses, income and profits**

	mean	median	st.dev.	min	max	.1 %tile	1 %tile	5% tile	95 %tile	99 %tile	99.9 %tile
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,871
Net interest income (NI)	4,810	4,810	259	3,793	5,680	4,014	4,199	4,386	5,233	5,395	5,535
Net interest income including losses due to defaulted coupons (RNI)	4,782	4,781	260	3,764	5,647	3,973	4,170	4,353	5,206	5,371	5,514
Net-Profits	3,404	3,581	815	-10,991	4,570	-4,183	-112	2,031	4,061	4,251	4,434

Note: in millions.

### 5.3 Economic Capital

Table 2 provides an overview over economic capital against different risks at different confidence levels. Given the skew of the credit risk loss distribution, economic capital against credit risk ( $EC_{CR}$ ) is non-linearly increasing in the confidence level. This is less pronounced for economic capital against changes in net interest income ( $EC_{NI}$  and  $EC_{RNI}$ ) because the underlying net income distributions only show a slight skew. The ratio of  $EC_{RNI}$  to  $EC_{CR}$  therefore decreases from around 30% at the 95% confidence level to 11% at the 99.9% confidence level. These numbers seem broadly in line with banks' practices. For example, the IFRI-CRO (2007) report suggests that for an average bank the ratio of the capital set against interest rate risk relative to capital set against credit risk is 16%. But given different balance sheet structures, this ratio exhibits a significant variance and can reach 50% or more.

**Table 2: Economic Capital**

	Confidence Level		
	95%	99%	99.9%
$EC_{CR}$	1,348	3,412	7,493
$EC_{NI}$	424	611	797
$EC_{RNI}$	429	612	809
$EC_{CR} + EC_{RNI}$	1,777	4,024	8,302
$EC_{NP}$	0	112	4,183
$M_{EC}$	100.00%	97.21%	49.62%
$E(NP) - VaR_{NP}$	1,372	3,516	7,586
$M_2$	22.76%	12.62%	8.62%

Note: in millions.  $EC_{CR}$  is the economic capital against credit risk;  $EC_{NI}$  is the economic capital against changes in net interest income excluding the impact of defaults on coupon payments;  $EC_{RNI}$  is the economic capital against changes in net interest income including the impact of defaults on coupon payments;  $EC_{NP}$  is the economic capital against changes in net profits.  $M_{EC}$  is the ratio of  $[(EC_{CR} + EC_{RNI}) - EC_{NP}]$  over  $(EC_{CR} + EC_{RNI})$ .  $E(NP)$  are expected net profits.  $VaR_{NP}$  is the VaR of net profits at confidence interval (1-y) where y is the confidence level stated in the table.  $M_2$  is the ratio of  $[(EC_{CR} + EC_{RNI}) - (E(NP) - VaR_{NP})]$  over  $(EC_{CR} + EC_{RNI})$ . See Section 3.4.

The key question of this paper is whether simple economic capital provides a conservative upper bound. Simple economic capital is just the sum of  $EC_{RNI}$  and  $EC_{CR}$ . This is significant at all confidence levels. However, taking the complex dynamic interactions of credit and interest rate risk into account, the bank makes positive net profits in more than 95% of the scenarios.

Therefore, integrated economic capital ( $EC_{NP}$ ) at this confidence level is zero. Even at the 99% confidence level, integrated economic capital would be minimal and less than 3% of simple economic capital. Only at the 99.9% percentile economic capital against net profits reaches a substantial amount, but is still only around 50% of simple economic capital.

The difference between simple and integrated economic capital is very large. The bottom of the table shows that this gap is mostly due to the fact that integrated capital covers only unexpected negative profits. As we explain in Section 3.4,  $M_2$  is an alternative measure that treats profits in the same way as credit risk losses, assuming that capital is set aside against unexpectedly low profits independently of whether these are positive or negative. By this metric, integrated capital is again lower than simple capital but only by an 8% to 20% margin (depending on the confidence level). In Section 3.4 we argue that  $M_2$  is not an economically sensible indicator, so in the reminder of the discussion we focus on  $M_{EC}$  and report  $M_2$  purely for completeness. In any case, our main result proves to be extremely robust: in all the cases we consider, simple economic capital provides an upper bound independently of whether we look at  $M_{EC}$  or  $M_2$ .

## 6 Sensitivity analysis

### 6.1 The impact of pricing

To assess the impact of different pricing assumptions, we first drop all negative spreads on deposits, then we drop all additive spreads on assets as well as liabilities. Finally, we assess the implication on net interest income if all assets and liabilities are priced as risk-free instruments. The results of the latter test can be seen as roughly equivalent to the outcome of a simple gap analysis.

Table A4 in the Annex shows the summary statistics for the profit and loss distributions of these tests. As more than 75% of the bank's deposits are corporate and household deposits with a time to repricing of one year or less, the rents earned by negative spreads are significant. Mean  $NI$  and  $RNI$  drop by around 30% in comparison to the base case (see Table 1). Given the additive nature of the spreads, the whole  $NI$  and  $RNI$  distribution by and large shifts linearly down. This can be seen in Table A5, where we compare the distributions after de-meaning them to net out the downward shift.

The impact of additionally dropping additive spreads on assets and all other liabilities is similar. The shape of the  $NI$  and  $RNI$  distributions remains essentially the same as above. However, the mean  $NI$  and  $RNI$  are now roughly 50% lower than in the base case (see Table A4 and Table 1).

A bank earns a substantial part of its net interest income by (re-)pricing assets according to their risk. Hence,  $NI$  is even lower if the bank charges only the risk-free rate. However, on average the

bank still earns a positive net interest income, which is driven by the original repricing mismatch of its book and the fact that equity funding is non interest rate bearing (see Section 6.2.3).

**Table 3: Economic capital under different pricing assumptions.**

	<i>No negative spreads on liabilities</i>			<i>No additive spreads</i>			<i>Risk free pricing</i>		
	Confidence Level			Confidence Level			Confidence Level		
	95%	99%	99.9%	95%	99%	99.9%	95%	99%	99.9%
$EC_{CR}$	1,348	3,412	7,493	1,348	3,412	7,493	1,348	3,412	7,493
$EC_{RNI}$	415	593	783	414	593	781	420	597	797
$EC_{CR} + EC_{RNI}$	1,763	4,005	8,276	1,763	4,005	8,275	1,768	4,009	8,290
$EC_{NP}$	0	1,621	5,686	344	2,483	6,543	836	2,955	7,050
$M_{EC}$	100.00%	59.52%	31.30%	80.48%	38.01%	20.93%	52.69%	26.30%	14.97%
$E(NP)-VaR_{NP}$	1,376	3,518	7,582	1,374	3,513	7,573	1,379	3,497	7,593
$M_2$	21.97%	12.18%	8.39%	22.04%	12.29%	8.49%	21.97%	12.76%	8.42%

See note to Table 2.

As long as defaulted coupon rates are taken into account in  $RNI$ , spreads do not alter the credit risk loss distribution (see Table 3). Hence,  $EC_{CR}$  is exactly the same as in the base case. As we saw above, additive spreads generate a mean shift in the net interest income distribution but virtually do not affect its shape (see Table A4). Therefore,  $EC_{RNI}$  never changes by more than 3.5% relative to the base case under any pricing assumptions. However, given the downward shift in its mean income, the bank incurs net losses more often; the lower the spreads, the more likely negative net profits are. Hence  $EC_{NP}$  is higher than in the base case at all confidence levels and the difference between simple and integrated economic capital is less pronounced, though it remains large and positive even under risk-free pricing.

## 6.2 The impact of granularity

It is well known that the granularity of a portfolio changes the shape of the credit risk distribution. A comparison of Table 1 with Table A6 in the Annex illustrates this point clearly. Whilst the mean credit risk losses are equal, the standard deviation of credit risk losses drops enormously for an infinitely fine-grained portfolio (from 765 to 34). Less default variability maps one to one into a lower default frequency for coupon rates, which in turn impacts  $RNI$ . On the other hand, given that the pricing is based on expected losses,  $NI$  is unaffected by the changes in the granularity of the underlying portfolio.<sup>24</sup> Given the positive rents earned on deposits and liabilities, the bank always makes positive net profits. Hence, integrated capital is zero at all confidence levels (see Table 4).

<sup>24</sup> Computational limitations imply that quantiles of the  $NI$  distribution in the granular case differ by around 0.02%-0.05% from the base case.

**Table 4: Economic capital for an infinitely fine-grained portfolio**

	Confidence Level		
	95%	99%	99.9%
$EC_{CR}$	58	83	108
$EC_{RNI}$	428	598	787
$EC_{CR} + EC_{RNI}$	486	680	895
$EC_{NP}$	0	0	0
$M_{EC}$	100.00%	100.00%	100.00%
$E(NP)-VaR_{NP}$	438	605	806
$M_2$	9.71%	11.04%	9.93%

See note to Table 2. All exogenous spreads are set as in the baseline simulation.

### 6.3 The impact of the repricing mismatch

Whilst the granularity of assets only changes the shape of the credit risk loss distribution, shifts in the time to repricing of assets and liabilities impact only on net interest income. To see the implications of different repricing characteristics, we look at two extreme cases. In the first (which we call “all short”) we assume that the bank is fully funded by short liabilities. In this case all liabilities are shifted to the 1-3 months bucket. In the second (which we refer to as “all long”) all liabilities are assumed to have a time to repricing of more than one year. Given that our simulation has a one year horizon, this implies that liabilities do not get repriced and generate fixed net interest payments for the bank.<sup>25</sup>

Table A7 in the Annex provides an overview over the profit and loss distribution in both cases. As expected the volatility of  $NI$  is in both cases significantly higher than in the base case. However, the volatility for the “all long” case is three times higher than in the “all short” case. And the minimum  $NI$  for the former is -3.1bn in comparison to 2.4bn for the latter or 3.8bn in the base case. This result may seem surprising: one would expect income volatility to decrease if the funding costs are fixed. A look at Table A8 explains the result: in absolute terms, the interest rate sensitivity gap at short repricing maturities is actually highest in the “all long” case.<sup>26</sup> This implies much higher volatility in  $NI$ , so  $EC_{RNI}$  is at all confidence levels around seven times larger than in the base case (see Table 5 and Table 2). In comparison, in the “all short” case  $EC_{RNI}$  is only about twice as large as in the base case.

<sup>25</sup> In both cases the repricing characteristics of assets, the portion of non interest bearing assets and liabilities and all additive spreads are kept as in the baseline.

<sup>26</sup> The interest rate sensitivity gaps for the 1-3 months bucket are -23% for “all short”, -10% for the base and +52% for “all long”.



**Table 5: Economic capital under alternative funding assumptions**

	<i>All short</i>			<i>All long</i>		
	Confidence Level			Confidence Level		
	95%	99%	99.9%	95%	99%	99.9%
$EC_{CR}$	1,348	3,412	7,493	1,348	3,412	7,493
$EC_{RNI}$	978	1,386	1,841	3,166	4,521	5,949
$EC_{CR} + EC_{RNI}$	2,326	4,798	9,335	4,514	7,934	13,442
$EC_{NP}$	0	386	4,471	217	1,897	5,056
$M_{EC}$	100.00%	91.95%	52.11%	95.19%	76.09%	62.39%
$E(NP) - VaR_{NP}$	1,525	3,668	7,753	3,493	5,173	8,332
$M_2$	34.41%	23.55%	16.94%	22.63%	34.80%	38.02%

See note to Table 2. All exogenous spreads are set as in the baseline simulation.

The “all long” bank has a positive sensitivity gap: contrary to standard banks, it borrows long and lends short. This repricing mismatch makes it vulnerable to drops (rather than increases) in the interest rate, which is good news for integrated risk management, as low interest rates imply low credit risk. Therefore, integrated economic capital could be lower than in the base case. On the other hand, the larger absolute interest rate gap leads to higher income volatility. Given these conflicting channels, the overall effect on  $EC_{NP}$  is *a priori* unclear. Table 5 shows that the second effect dominates: integrated economic capital is at all confidence levels higher than in the base case. The same is true for the “all short” bank.  $M_{EC}$  does not give a clear message: it can be higher or lower than in the baseline depending on the confidence level and the assumed pricing maturity. We do not have an explanation for these differences. Nonetheless, we stress that simple economic capital proves to be an upper bound in all cases.

#### 6.4 The impact of equity

The discussion so far begs the question of what is the optimal capital level for the bank. Since shareholder funds are non-interest bearing, this does not simply coincide with the  $EC_{NP}$  figure calculated in the baseline case. We assume that the bank does not pay any dividends during the year and that equity simply receives the accumulated profits at the end of the year. This implies that no cash flows have to be paid on equity during our simulation horizon. Hence, the higher the equity, the lower the interest payments on liabilities. In the extreme, if a bank were fully funded by equity, interest payments on liabilities would be zero and income would be only based on interest income from assets.

Table A9 in the Annex provides an overview of the simulated statistics for 0%, 4% and 8% equity levels. All additional spreads for assets and liabilities are set to zero to isolate the impact of changing equity levels. Indeed, decreasing the amount of equity changes the  $NI$  distribution

substantially. Dropping equity from 8% to 4% decreases the mean  $NI$  by 22% and increases the standard deviation by more than 50%. Similarly, diminishing equity from 4% to 0% decreases the mean  $NI$  by nearly 40% and increases the standard deviation by another 26%. As has been shown by several previous sensitivity tests, the distribution of credit risk losses is unaffected by the funding structure of the bank. Changes in the  $RNI$  distribution are therefore driven by shifts in the  $NI$  distribution.

**Table 6: Economic capital for initial equity levels of 0%, 4% and 8%**

	0% equity			4% equity			8% equity		
	Confidence Level			Confidence Level			Confidence Level		
	95%	99%	99.9%	95%	99%	99.9%	95%	99%	99.9%
$EC_{CR}$	1,348	3,412	7,493	1,348	3,412	7,493	1,348	3,412	7,493
$EC_{RNI}$	576	820	1,086	429	614	810	281	407	537
$EC_{CR} + EC_{RNI}$	1,924	4,232	8,580	1,778	4,026	8,303	1,630	3,819	8,031
$EC_{NP}$	1,092	3,203	7,326	408	2,549	6,615	0	1,838	5,954
$M_{EC}$	43.25%	24.32%	14.61%	77.06%	36.68%	20.33%	100.00%	51.86%	25.86%
$E(NP)-VaR_{NP}$	1,395	3,506	7,630	1,371	3,513	7,578	1,366	3,461	7,577
$M_2$	27.48%	17.15%	11.08%	22.88%	12.76%	8.73%	16.20%	9.36%	5.65%

See note to Table 3. All exogenous spreads are set to zero.

The impact of initial equity levels on  $EC_{RCI}$  and  $EC_{NP}$  can be seen in Table 6. The table clearly shows that the higher the initial equity, the lower the necessary capital to buffer against credit and interest rate risk losses. This is purely driven by the fact that expected net interest income is higher for higher equity because the bank does not need to pay any dividends or interest to equity holders over the course of the year. The difference between simple economic capital and integrated economic capital is therefore also larger for higher levels of initial equity levels.

According to  $EC_{NP}$ , the 99.9% economic capital for the bank should only be 6.6bn, which is approximately 2% of total assets (see Table 6). This would suggest that the optimal capital level is lower than 4%, but this conclusion should be taken with a large grain of salt. Our result is obviously conditional on specific assumptions on the granularity and pricing characteristics of the balance sheet. We also do not take account of any other sources of risk, such as other market risk factors or operational risk. Importantly, in our simulation we only consider fully insured customer deposits. Hence, depositors do not demand a higher risk premium for higher leverage; this would decrease net interest income and raise its volatility, which in turn impact on integrated capital.

## 7 Conclusion and discussion

This paper provides a consistent framework to derive economic capital against credit and interest rate risk in the banking book. We formulate an economic capital framework where interest and credit risk interact in a non-linear, dynamic fashion. We apply this framework to a stylised UK bank, comparing a “simple economic capital” measure that purely adds economic capital against

credit and interest rate risk to an “integrated” measure that takes into account the interactions between them. We find that the difference between the two measures depends on various features of the bank, but that simple capital exceeds integrated capital under a broad range of circumstances, providing an upper bound relative to the bank's overall risk.

A range of factors contribute to generating this result. A relatively large portion of credit risk is idiosyncratic, and thus independent of the macroeconomic environment, and the correlation between systematic credit risk factors and interest rates is itself not perfect. Furthermore, assets in the bank's portfolio are repriced relatively frequently, and hence increases in credit risk can be partly passed on to borrowers. Our analysis also rests on a number of assumptions: for instance we do not account for prepayment risk (which is negligible in the UK but quite substantial in the US), hedging, or subordinated debt. Given the magnitude and robustness of our results, though, our conjecture is that extending the model in these directions would not change our main conclusion.

More importantly, we emphasise that, since we focus on traditional banking book risks, relating our insights to the recent crisis is not trivial. Securitisation, derivatives and liquidity management - which were at the core of the turmoil - remain outside the scope of this paper. Furthermore, changes in the economic value of the portfolio are not taken into account as all exposures are assumed to be non-tradable and therefore valued using book value accounting. It is difficult to speculate on whether “integrated capital” would remain lower than “simple economic capital” in a context where these restrictions are relaxed. In fact, the findings in Breuer et al. (2008) and Kupiec (2007) suggest that there may be no general answers to this type of questions. As a consequence, risk managers and regulators should work on the presumption that interactions between risk types may be such that the overall level of capital is higher than the sum of capital derived for risks independently. Our paper shows that this is unlikely for credit and interest rate risk in the banking book, but also that additive rules are in this case potentially very inefficient. From a risk management perspective, this should provide another strong enough incentive to move towards an integrated analysis of risks.

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## Annex 1: Endogenous coupon rates

This annex is based on Drehmann, Stringa and Sorensen (2007). The economic value  $EVA^i$  of a generic asset  $i$  with time to repricing of  $T$  (which is also for simplicity equal to its maturity) is simply the risk-adjusted discounted value of future coupon payments  $C_s^i(X_s)$  and the principal  $A$ . Hence at time  $t$ :

$$EVA_t^i(X_t) = \sum_{k=1}^T D_{t+k}^i(X_t) C_s^i(X_s) A^i + D_{t+T}^i(X_t) A^i \quad (A1)$$

For simplicity we assume that all assets are equivalent to bullet bonds – *i.e.* repay the principal only at maturity and pay a constant coupon  $C_s^i$  that is determined at time  $t=s$  based on the observed macroeconomic variables  $X_s$ . For example, such an asset could be a fixed-interest rate bond with no embedded options or a simple bank loan. The discount function conditional on current conditions is given by:

$$D_{t+k}^i(X_t) = E \left( \prod_{l=1}^k \frac{1}{1 + R_{t+l-1,t+l}^i(X_t)} \right) \quad (A2)$$

where  $R$  is the risk-adjusted interest rate. In continuous time,  $R$  equals the risk-free rate plus a credit risk premium equal to  $PD \times LGD$ . However, as our application is set up in discrete time, we follow Duffie and Singleton (2003):

$$R_{t+l-1,t+l}^i(X_t) = \left( \frac{r_{t+l-1,t+l}(X_t) + PD_{t+l-1,t+l}^i(X_t) \times LGD^i}{1 - PD_{t+l-1,t+l}^i(X_t) \times LGD^i} \right) \quad (A3)$$

where  $r_{t+l-1,t+l}$  is the forward risk-free interest rate between  $t+l-1$  and  $t+l$  known at time  $t$ .  $LGD^i$  is the expected loss given default for borrower  $i$  which, for simplicity, we assume here to be constant.  $PD_{t+l-1,t+l}^i$  is the risk neutral probability of default of borrower  $i$  between  $t+l-1$  and  $t+l$  conditional on surviving until  $t+l-1$ . PDs and yields depend on the same set of systematic risk factors  $X_t$ .

We do not observe empirical coupon rates and need to reprice assets and liabilities according to their contractual repricing characteristics. To do so we assume that at the time  $t=0$  of issuance the economic value equals the face value of the asset. This implies that  $EVA_{t=0}^i(X_0) = A^i$  in equation (A1). Solving for  $C_0^i$  we obtain:

$$C_0^i(X_0) = \frac{1 - D_T^i(X_0)}{\sum_{k=1}^T D_k^i(X_0)} \quad (A4)$$

## Annex 2: A simple multi-period example

To provide some intuition for the dynamic set-up, it is useful to consider a simplified bank with two asset classes  $A^i, A^j$ , one liability class  $L$  with  $L=A^i+A^j$ . Asset  $A^i$  ( $A^j$ ) has  $PD^i$  and  $LGD^i$  ( $PD^j$  and  $LGD^j$ ) and gets repriced after one (two) period. Each asset class consists of an infinitely fined grained portfolio of assets so that realised losses equal expected losses conditional on  $X$ .

Liabilities are repriced every period and pay a coupon rate  $C^L$  equal to the risk-free interest rate  $r$ . We also assumed that the risk-free yield curve is flat and the macro environment is such that  $E(PD_1)=E(PD_2)$  for both assets.

Following equation (A4), the initial risk-free yield curve and expected  $PD$ s in period one and two determine coupon rates  $C_0^i$  and  $C_0^j$  for each asset class. Assume that the realisation of  $X_1$  is such that  $PD$ s and interest rates do not change and  $PD_1=E(PD_1)$  for both assets. Hence, there will be no repricing and given a well diversified portfolio within the two asset classes,  $PD^i LGD^i A^i$  assets default. If an asset with unit size defaults, the bank loses  $LGD^i (1+C^i)$  as discussed above. Hence, losses accounting for defaulted coupons and principals are

$$L_1 = PD_1^i LGD^i (1+C_0^i) A^i + PD_1^j LGD^j (1+C_0^j) A^j \quad (A5)$$

Given that the bank does not re-price any assets and liabilities  $NI_1$  is just

$$NI_1 = C_0^i A^i + C_0^j A^j - C_0^L L \quad (A6)$$

where the first and second term are cash flow contributions from asset  $i$  and asset  $j$  and the third term are interest payments on liabilities  $L$ . Net profits are therefore

$$NP_1 = NI_1 - L_1 \quad (A7)$$

Given our assumptions that the term structure of interest rates and  $PD$ s is flat, net interest income  $NI$  exactly offsets credit risk losses which equal expected losses in this case. Hence  $NP_1=0$ .

Coupon rates for period two can be forecasted by following the same line of argumentation as above. But assume that the realisation of  $X_2$  is such that  $\overline{PD}_2 > PD_1$  for both asset classes even though risk-free interest rate remains unchanged so that  $\overline{C}_1^L = C_0^L = r$ . As asset class  $A^i$  has a one period maturity, the bank is able to reprice  $A^i$  to reflect the higher credit risk and  $\overline{C}_1^i > C_0^i$ .

However, the bank can not reprice asset class  $A^j$  as coupon rates are locked-in for another period given the assumed maturity of 2 periods. Net interest income for period two is

$$NI_2 = \overline{C}_1^i A^i + C_0^j A^j - \overline{C}_1^L L. \quad (A8)$$

Even though  $NI_2 > NI_1$  the bank will be expected to make a loss in this period as cash flows earned on asset  $j$  will not offset write-offs in this asset class given higher  $PD$ s. Table A2.1 summarises the dynamic example.

**Table A2.1: Development of key variables in the dynamic example**

	$t_1$	$t_2$
<i>Coupon Rates</i>	$C_0^i$ $C_0^j$ $C_0^L$	$\bar{C}_1^i$ , $C_0^j$ $\bar{C}_1^L$
<i>Losses</i>	$L_1 = PD_1^i LGD^i (1 + C_0^i) A^i$ $+ PD_1^j LGD^j (1 + C_0^j) A^j$	$L_2 = \bar{PD}_2^i LGD^i (1 + \bar{C}_0^i) A^i$ $+ \bar{PD}_2^j LGD^j (1 + C_0^j) A^j$
<i>NI</i>	$NI_1 = C_0^i A^i + C_0^j A^j - C_0^L L$	$NI_2 = \bar{C}_1^i A^i + C_0^j A^j - \bar{C}_1^L L$
<i>Net Profits</i>	$0$	$< 0$

### Annex 3: Empirical implementation of the pricing framework

In order to implement our framework, we rely on two approximations. The first one consists of assuming that banks use a random walk model to form expectations on future  $PD$ s, *i.e.* they assume that  $E_t(PD_{t+k}) = PD_t$ . Using model-consistent expectations is possible but computationally very cumbersome given the high dimensionality of the model.<sup>27</sup> In order to assess the implications of this approximation, we replicated the baseline case using model-consistent expectations as a sensitivity test.<sup>28</sup> This indicates that wrongly formed expectations slightly bias income levels downwards and decrease the variance of  $RNI$ , which was to be expected as model-consistent expectations are less volatile. Most importantly, the error margins introduced by this approximation for  $M_{EC}$  are small and below 2% at all confidence levels.

Second, when calculating the discount factors  $D_{t+k}^i$ , we approximate equation (A2) as follows

$$D_{t+k}^i = E_t \left( \prod_{l=1}^k \frac{1}{1 + R_{t+l-1,t+l}^i} \right) \approx \prod_{l=1}^k E_t \left( \frac{1}{1 + R_{t+l-1,t+l}^i} \right) = \prod_{l=1}^k \frac{1 - E_t(PD_{t+l-1,t+l}^i) LGD^i}{1 + r_{t+l-1,t+l}} \quad (A9)$$

<sup>27</sup> For each quarter  $t=1, \dots, 4$  and scenario  $s=1, \dots, 10,000$  we need expectations for 6  $PD$ s over a 10 year horizon; the implied total number of expectations to be calculated is  $4 \times 10,000 \times 6 \times 40 = 9,600,000$ .

<sup>28</sup> Results are available on request.

The last equality holds as the forward yield curve is known at the time of pricing and LGDs are fixed. By looking at the product of expectations rather than the expectation of the product, though, we ignore any conditional cross-correlations between discount factors at different points in time. It is hard to quantify the bias this introduces as the simulation becomes too complex to calculate coupon rates correctly. However, we would argue that the bias does not affect our results in a significant fashion. As pointed out above, at the time of pricing the forward yield curve is known and LGDs are fixed. Therefore, the conditional correlation is driven by the conditional correlation between PDs. All PDs are by construction conditionally homoscedastic - a property they inherit from the GVAR (see equations 23-24). Hence, their conditional auto-correlations are constant over time. Furthermore, the realised unconditional autocorrelations are small and positive, and decline rapidly to zero for lag lengths greater than one.<sup>29</sup> This would suggest that the bias is not substantial and that the coupons we calculate are too low on average. Given the robustness tests in Section 6.1, this means that  $EC_{NP}$  in our base case is likely to be too high in comparison to the case where the approximation would not be made.

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<sup>29</sup> We assessed correlation coefficients of PDs for 6 asset classes empirically by looking at the distribution of correlation coefficients implicit in the simulation. Results are available on request.

## Annex 4: Additional Tables

**Table A1: Balance Sheet**

<b>Assets</b>		<b>Repricing buckets:</b>						<b>Total</b>
		<b>1-3 m</b>	<b>3-6 m</b>	<b>6-12 m</b>	<b>1-5 y</b>	<b>&gt;5 y</b>	<b>non i.b.</b>	
<b>Bank</b>	UK	12,783	697	560	130	249	1,378	14,418
<b>HH.Mort</b>	UK	41,331	4,137	3,736	16,678	1,886	134	67,767
<b>HH.Unsec</b>	UK	7,278	692	607	3,320	1,000	653	12,896
<b>Gov</b>	UK	954	94	68	242	302	872	1,660
<b>PNFC</b>	UK	21,374	1,701	1,357	1,318	523	14	26,273
<b>OFC</b>	UK	15,769	1,635	1,429	5,757	4,402	1,545	28,992
<b>Other</b>	UK	16,256	1,596	1,265	3,708	6,693	24,806	29,517
<b>Bank</b>	US	19,537	1,065	855	198	381	2,106	22,037
<b>HH.Mort</b>	US	25,722	2,574	2,325	10,379	1,173	83	42,174
<b>HH.Unsec</b>	US	4,529	431	378	2,066	622	406	8,026
<b>Gov</b>	US	1,292	127	97	310	475	1,609	2,301
<b>PNFC</b>	US	13,302	1,059	844	820	325	8	16,351
<b>OFC</b>	US	9,814	1,018	889	3,583	2,740	961	18,043
<b>Other</b>	US	31,050	3,048	2,416	7,083	12,783	47,381	56,379
<b>Total assets</b>								428,789
<b>Liabilities</b>		<b>Repricing buckets:</b>						<b>Total</b>
		<b>1-3 m</b>	<b>3-6 m</b>	<b>6-12 m</b>	<b>1-5 y</b>	<b>&gt;5 y</b>	<b>non i.b.</b>	
<b>Bank</b>	UK	38,050	2,069	1,229	680	902	1,035	43,965
<b>HH</b>	UK	69,472	2,838	2,881	2,377	350	5,409	83,327
<b>Gov</b>	UK	1,651	106	114	68	10	160	2,110
<b>PNFC</b>	UK	22,177	695	677	622	172	2,758	27,101
<b>OFC</b>	UK	57,146	1,957	1,779	1,556	367	7,324	70,129
<b>Sub</b>	UK	11,889	948	683	2,506	8,491	10,199	34,716
<b>Other</b>	UK	61,240	4,195	3,483	7,892	7,917	63,828	148,555
<b>Total liabilities</b>								409,902
<b>Shareholder funds</b>							18,887	

Note: in millions.

**Table A2: Pricing of Assets**

<b>Asset Class<sup>30</sup></b>	<b>Modelling of Cash Flow</b>
UK interbank unsecured <sup>31</sup>	Risk-free rate +15bps
UK household secured (mortgage)	Coupon from net interest income model +50bps
UK household unsecured	Coupon from net interest income model +50bps
UK government	Risk-free rate
UK PNFC	Coupon from net interest income model +50bps
UK OFC	Risk-free rate +15bps
UK other assets <sup>32</sup>	Risk-free rate
US interbank unsecured	Risk-free rate +15bps
US household secured (mortgage)	Coupon from net interest income model +50bps
US household unsecured	Coupon from net interest income model +50bps
US government	Risk-free rate
US PNFC	Coupon from net interest income model +50bps
US OFC	Risk-free rate +15bps
US other assets	Risk-free rate

**Table A3: Pricing of Liabilities**

<b>Liability Class</b>	<b>Modelling of Cash Flow</b>
Unsecured interbank <sup>33</sup>	Risk-free rate +15bps
Household	Risk-free rate minus variable negative spread <sup>34</sup>
Government	Risk-free rate
PNFC	Risk-free rate minus variable negative spread <sup>35</sup>
OFC	Risk-free rate
Subordinated liabilities	Risk-free rate +15bps
Other liabilities <sup>36</sup>	Risk-free rate +15bps

<sup>30</sup> All footnotes referring to UK asset classes also apply to US asset classes.

<sup>31</sup> Unsecured interbank loans + derivatives + certificates of deposit.

<sup>32</sup> Includes reverse repos.

<sup>33</sup> Unsecured interbank deposits + derivatives.

<sup>34</sup> The negative spread on household deposits is 200bps in the 0-3 months repricing bucket, 150bps in the 3-6 month bucket, 100bps in the 6-9 month bucket, 50bps in the 9-12 month bucket and 0bps at longer maturities.

<sup>35</sup> The negative spread on corporate deposits is 100bps in the 0-3 months repricing bucket, 75bps in the 3-6 month bucket, 50bps in the 6-9 month bucket, 25bps in the 9-12 month bucket and 0bps at longer maturities.

<sup>36</sup> Includes debt securities and repos.



**Table A4: Losses, income and profits under alternative pricing assumptions**

	mean	median	st.dev.	min	max	.1 %tile	1 %tile	5% tile	95 %tile	99 %tile	99.9 %tile
<b>No negative spreads on liabilities</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,871
Net interest income (NI)	3,303	3,302	252	2,318	4,337	2,532	2,711	2,892	3,716	3,891	4,079
Net interest income including losses due to defaulted coupons (RNI)	3,274	3,273	254	2,290	4,315	2,492	2,681	2,860	3,690	3,867	4,057
Net-Profits	1,897	2,074	813	-12,506	3,146	-5,686	-1,621	520	2,545	2,733	2,956
<b>No additive spreads</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,872
Net interest income (NI)	2,435	2,434	252	1,451	3,469	1,665	1,844	2,024	2,849	3,023	3,211
Net interest income including losses due to defaulted coupons (RNI)	2,408	2,407	254	1,423	3,449	1,627	1,815	1,994	2,823	3,001	3,191
Net-Profits	1,030	1,208	813	-13,354	2,279	-6,543	-2,483	-344	1,678	1,867	2,089
<b>Pricing of all assets and liabilities as risk free instruments</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,871
Net interest income (NI)	1,939	1,940	256	964	2,974	1,147	1,348	1,523	2,358	2,538	2,721
Net interest income including losses due to defaulted coupons (RNI)	1,921	1,923	258	944	2,964	1,124	1,324	1,502	2,343	2,523	2,711
Net-Profits	543	719	815	-13,849	1,814	-7,050	-2,955	-836	1,200	1,397	1,636

Note: in millions.

**Table A5: De-meaned loss, income and profit distributions under alternative pricing assumptions**

	min	0.1%	1%	5%	mean	95%	99%	99.9%	max
<b>Base simulation</b>									
Credit losses	-543	-445	-388	-345	0	1,348	3,412	7,493	14,410
NI	-1,017	-797	-611	-424	0	422	585	725	870
RNI	-1,018	-809	-612	-429	0	424	589	733	865
Net-Profits	-14,394	-7,586	-3,516	-1,372	0	657	847	1,030	1,166
<b>No spreads on liabilities</b>									
Credit losses	-543	-445	-388	-345	0	1,348	3,412	7,493	14,410
NI	-984	-770	-591	-410	0	414	588	776	1,034
RNI	-985	-783	-593	-415	0	416	593	783	1,041
Net-Profits	-14,402	-7,582	-3,518	-1,376	0	649	837	1,060	1,250
<b>No additive spreads</b>									
Credit losses	-543	-445	-388	-345	0	1,348	3,412	7,493	14,410
NI	-984	-770	-591	-410	0	414	588	776	1,034
RNI	-985	-781	-593	-414	0	415	593	783	1,040
Net-Profits	-14,384	-7,573	-3,513	-1,374	0	648	837	1,059	1,249
<b>Pricing of all assets and liabilities as risk free instruments</b>									
Credit losses	-543	-445	-388	-345	0	1,348	3,412	7,493	14,410
NI	-975	-792	-591	-417	0	419	599	781	1,035
RNI	-977	-797	-597	-420	0	422	602	790	1,043
Net-Profits	-14,392	-7,593	-3,497	-1,379	0	657	855	1,093	1,271

Note: in millions.

**Table A6: Losses, income and profits for a granular portfolio**

	mean	median	st.dev.	min	max	.1 %tile	1 %tile	5% tile	95 %tile	99 %tile	99.9 %tile
Credit risk losses	1,383	1,382	34	1,282	1,533	1,288	1,308	1,328	1,441	1,465	1,491
Net interest income (NI)	4,811	4,809	257	3,841	5,604	4,030	4,217	4,386	5,233	5,407	5,554
Net interest income including losses due to defaulted coupons (RNI)	4,783	4,781	259	3,807	5,579	3,996	4,185	4,355	5,208	5,382	5,531
Net-Profits	3,400	3,400	263	2,380	4,216	2,594	2,795	2,962	3,834	4,003	4,160

Note: in millions. Spreads are paid on deposits and assets as in the base simulation.

**Table A7: Losses, income and profits if all liabilities are short term or long term**

	mean	median	st.dev.	min	max	.1 %tile	1 %tile	5% tile	95 %tile	99 %tile	99.9 %tile
<b>Only short term liabilities</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,871
Net interest income (NI)	4,689	4,690	596	2,393	6,814	2,855	3,308	3,712	5,664	6,065	6,435
Net interest income including losses due to defaulted coupons (RNI)	4,660	4,660	597	2,364	6,780	2,819	3,274	3,683	5,637	6,044	6,416
Net-Profits	3,282	3,407	974	-11,511	5,544	-4,471	-386	1,757	4,457	4,873	5,281
<b>Only long term liabilities</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,871
Net interest income (NI)	4,682	4,681	1,932	-3,116	12,159	-1,273	168	1,512	7,820	9,136	10,509
Net interest income including losses due to defaulted coupons (RNI)	4,654	4,654	1,930	-3,138	12,130	-1,295	132	1,488	7,789	9,107	10,479
Net-Profits	3,276	3,338	2,087	-12,778	11,006	-5,056	-1,897	-217	6,554	7,901	9,342

Note: in millions. Spreads are paid on deposits and assets as in base simulation.

**Table A8: Interest rate gaps for the base case, if all liabilities are short or long**

	Repricing buckets					
	1-3 m	3-6 m	6-12 m	1-5 y	>5 y	non i.b.
IR gap base	-9.5%	1.6%	1.4%	9.3%	3.6%	-2.0%
IR gap short	-22.9%	4.6%	3.9%	13.0%	7.8%	-2.0%
IR gap long	51.5%	4.6%	3.9%	-57.2%	3.6%	-2.0%

**Table A9: Losses, income and profits for changing equity levels**

	mean	median	st.dev.	min	max	.1 %tile	1 %tile	5% tile	95 %tile	99 %tile	99.9 %tile
<b>Equity = 0%</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,872
Net interest income (NI)	1,708	1,707	352	334	3,149	631	891	1,135	2,288	2,528	2,794
Net interest income including losses due to defaulted coupons (RNI)	1,682	1,681	354	307	3,129	595	862	1,106	2,262	2,501	2,774
Net-Profits	303	463	848	-14,103	1,910	-7,326	-3,203	-1,092	1,101	1,363	1,650
<b>Equity = 4%</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,872
Net interest income (NI)	2,368	2,367	261	1,348	3,439	1,569	1,756	1,942	2,797	2,978	3,173
Net interest income including losses due to defaulted coupons (RNI)	2,341	2,340	263	1,321	3,419	1,531	1,728	1,912	2,772	2,955	3,153
Net-Profits	963	1,140	815	-13,414	2,245	-6,615	-2,549	-408	1,624	1,821	2,049
<b>Equity = 8%</b>											
Credit risk losses	1,378	1,146	765	835	15,788	933	990	1,033	2,726	4,790	8,872
Net interest income (NI)	3,028	3,027	170	2,362	3,730	2,503	2,628	2,748	3,308	3,425	3,551
Net interest income including losses due to defaulted coupons (RNI)	3,001	3,000	172	2,335	3,709	2,464	2,595	2,720	3,284	3,402	3,533
Net-Profits	1,623	1,821	792	-12,823	2,580	-5,954	-1,838	257	2,153	2,286	2,447

Note: in millions. No additive spreads are paid on deposits and assets.

## Annex 5: Additional figures

Figure A1: A stylised credit risk loss distribution

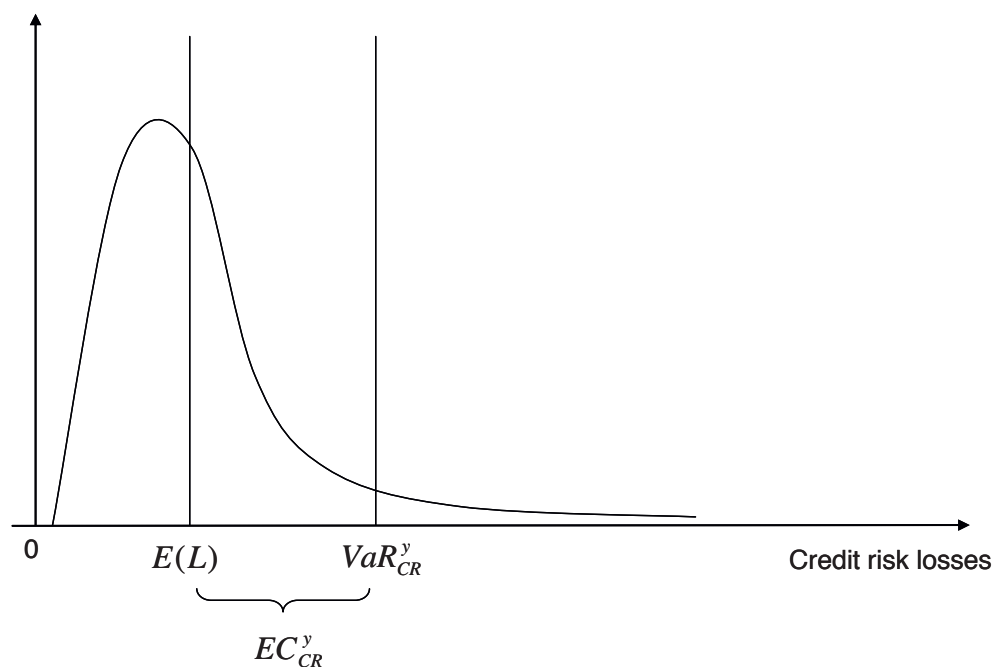
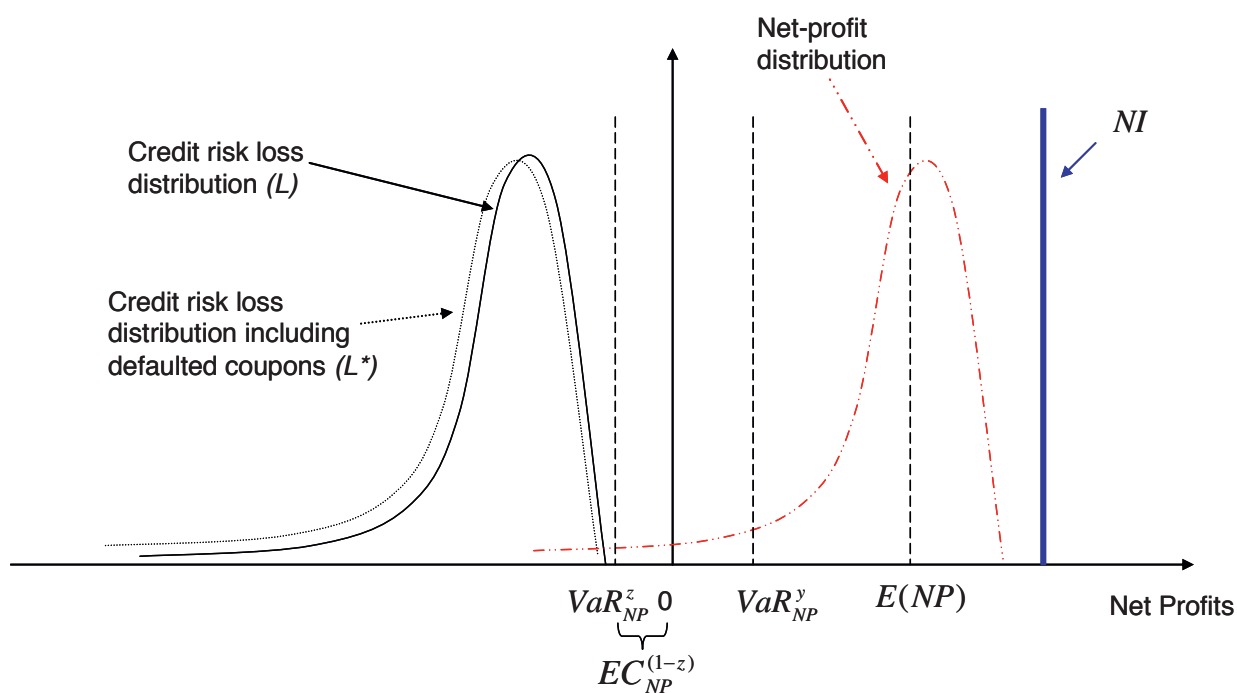
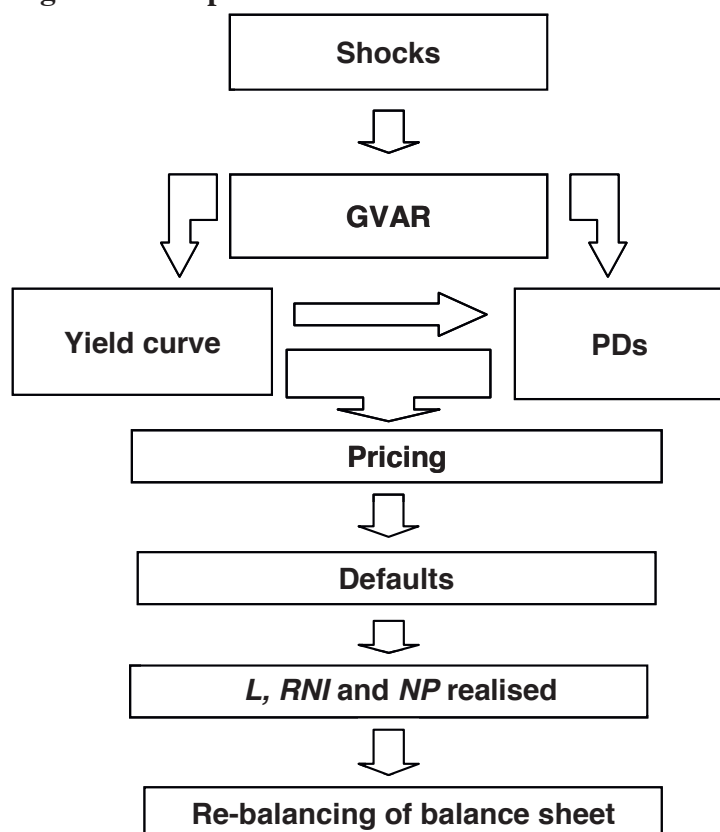


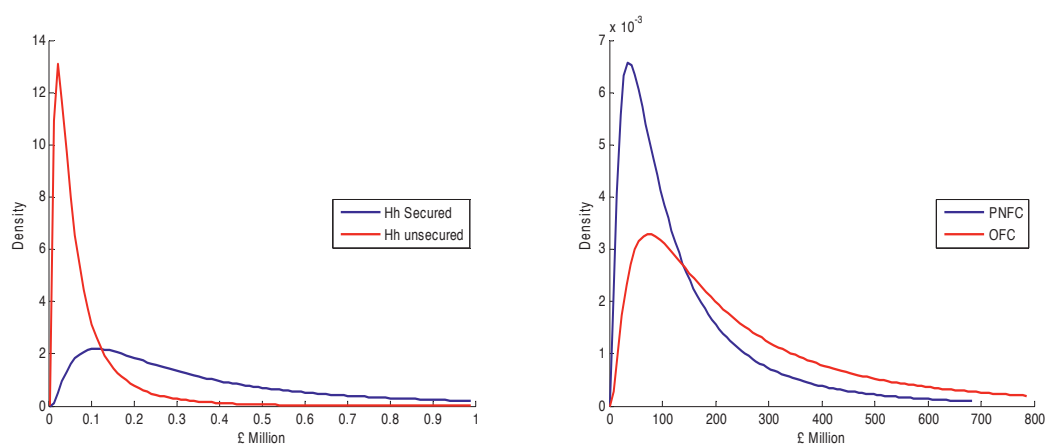
Figure A2: A stylised net profit distribution in the one period set-up with fixed coupons



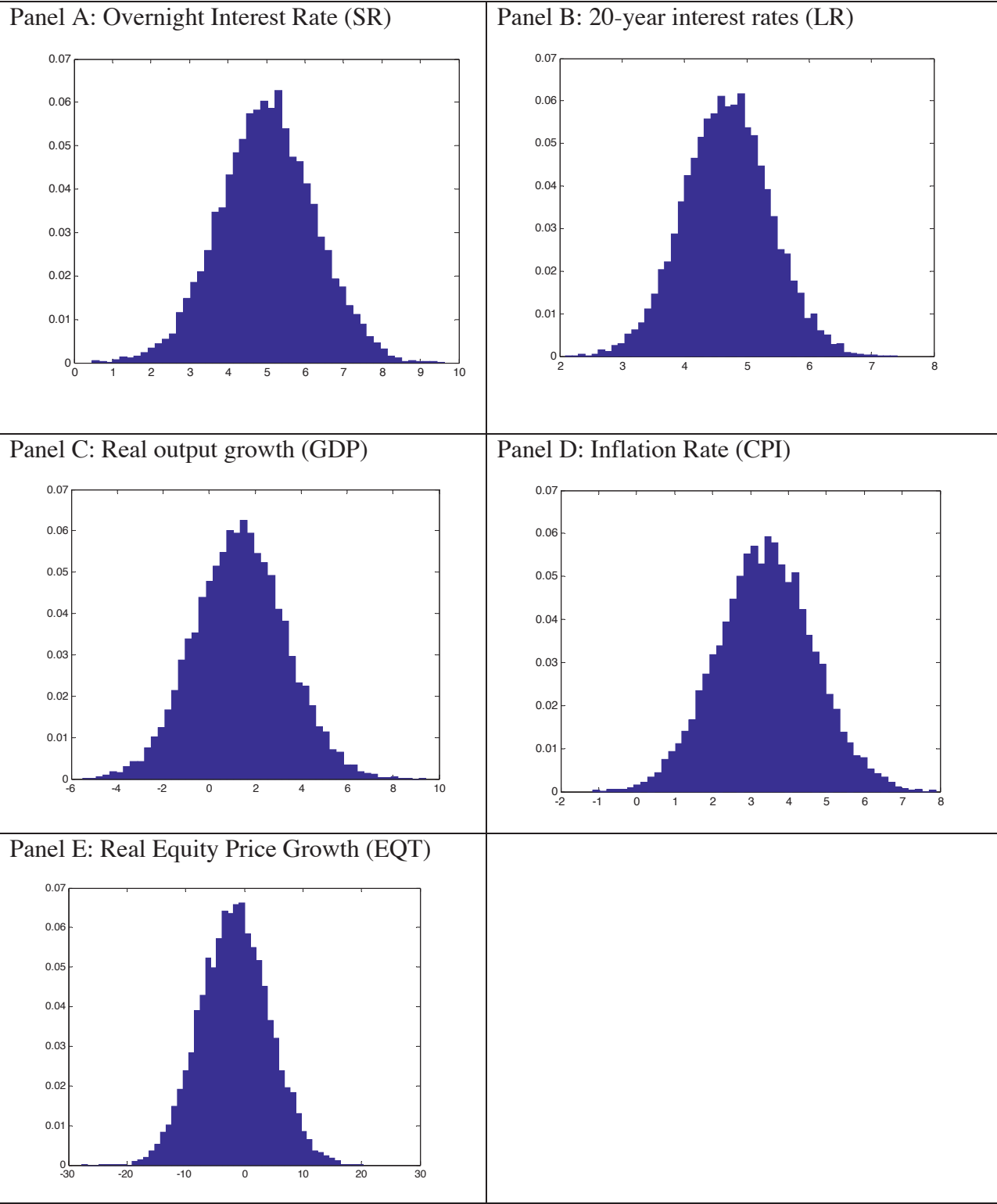
**Figure A3: Implementation of the framework**



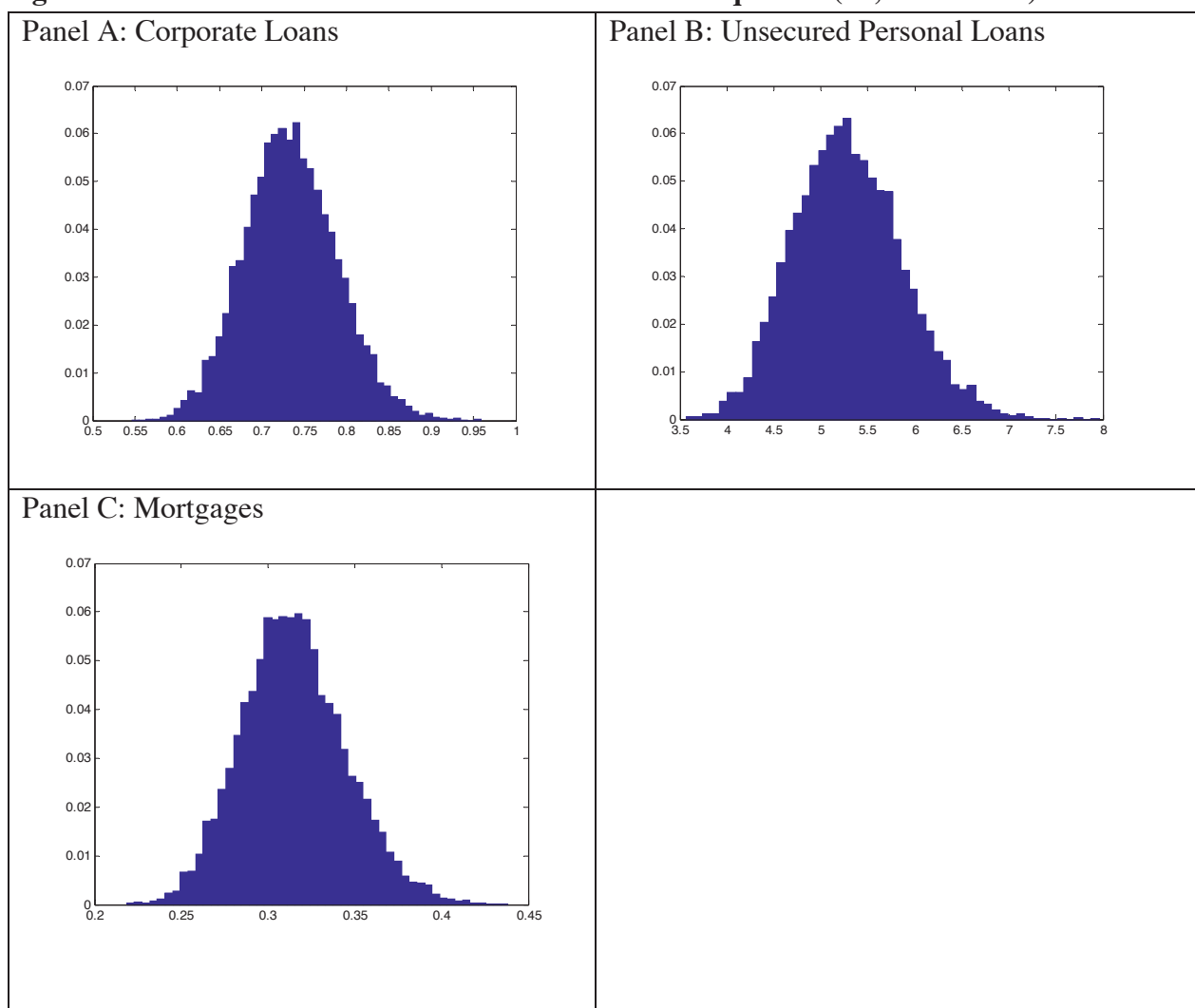
**Figure A4: Size distribution of the hypothetical portfolio**



**Figure A5: Distribution of UK macro variables in the final quarter (% , annualised)**



**Figure A6: Distribution of UK default rates in the final quarter (% , annualised)**





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