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## When does lumpy factor adjustment matter for aggregate dynamics?

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**WORKING PAPER SERIES**

**NO 1016 / MARCH 2009**

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LUMPY FACTOR  
ADJUSTMENT  
MATTER FOR  
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by Stephan Fahr  
and Fang Yao



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by Stephan Fahr<sup>2</sup> and Fang Yao<sup>3</sup>

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# CONTENTS

Abstract	4
1 Non-technical summary	5
2 Introduction	6
3 Basic model	7
3.1 Firms	7
3.2 Households	9
3.3 Aggregation	10
4 Model comparisons	10
4.1 The lumpy labour model	10
4.2 The lumpy investment model	11
4.3 The lumpy capital-labour ratio	12
5 Calibration	13
6 Simulation results	14
7 Impulse responses	15
8 Sensitivity analysis	16
8.1 Role of the elasticity of labour supply	17
8.2 Role of the elasticity of intertemporal substitution	17
8.3 Role of the depreciation rate	17
9 Conclusion	18
References	19
Tables and figures	21
European Central Bank Working Paper Series	27

## Abstract

We analyze the dynamic effects of lumpy factor adjustments at the firm level onto the aggregate economy. We find that distinguishing between capital and labour as lumpy factors within the production function result in very different dynamics for aggregate output, investment and labour in an otherwise standard real business cycle model. Lumpy capital leaves the RBC mainly unchanged, while lumpy labour allows for persistence and an inner propagation within the model in form of hump-shaped impulse responses. In addition, when modeling lumpy adjustments on both investment and labour, the aggregate effects are even stronger. We investigate the mechanisms underlying these results and identify the elasticity of factor supply as the most important element in accounting for these differences.

*JEL classification:* E32; E22; E24

*Key words:* Lumpy labor adjustment, Lumpy investment, Business cycles, Elasticity of supply.

# 1 Non-technical Summary

The main objective of this paper is to analyze the transmission mechanism of the micro-lumpiness onto the aggregate dynamics in a Dynamic Stochastic General Equilibrium (DSGE) model. In doing so, we intend to study the effects of the lumpy production factor adjustments in a unified framework, so that one can shed some light on the underlying mechanism from a broader perspective.

With recent microeconomic data collected from the firm's level, convincing evidence has been documented for both labour and capital adjustments, showing that adjustments of production factors at the plant level exhibit a lumpy pattern in response to shocks, and furthermore they are strongly coordinated in timing. This evidence brings difficulty for the widely used convex adjustment costs model that implies a smoothing adjustment at the firm level. The main theme running in the macroeconomic theory is whether modeling the micro lumpiness explicitly changes the model's implication for the aggregate dynamics. In contrast to the (S,s) literature, in which the effects of lumpy factor adjustments are normally studied separately, we view both lumpy factor adjustments as intrinsically close-related issues, and hence in this paper we present a tractable theoretical framework to study them together. The main questions we intend to address are: why does lumpiness in different production factors lead to different effects on aggregate dynamics and what are the mechanisms through which these effects work? Does coordinated lumpy adjustments of both production factors matter for the aggregate dynamics?

The answers to those questions from our model's perspective are: first, lumpy labour and capital lead to different effects regarding the dynamics of output and other aggregate variables. In particular, lumpy labour adjustment leads to a hump-shaped response of aggregate real variables, which cannot be obtained by the lumpy capital model. Moreover, when the lumpy factor adjustments coordinate with each other, the effects on the aggregate dynamics are even strengthened. Second, we investigate the underlying mechanism for these results and identify that sufficiently elastic factor supply is the prerequisite for the lumpy adjustment to have aggregate effects, and furthermore that the *intra*temporal substitution as opposed to the *inter*temporal margin is important for business cycle dynamics.

Finally, to further explore the mechanism of this channel, we conduct three theoretical experiments in our model. First, we try to eliminate the lumpy labour effect from the lumpy labour model by decreasing the elasticity of labour supply. Second, we want to reestablish aggregate effects of firm's lumpy capital by raising the elasticity of capital supply. Hereby we can either decrease the elasticity of substitution  $\eta$  to weaken the consumption smoothing motive, or alternatively we increase the depreciation rate  $\delta$  allowing for a more immediate response of capital supply to the aggregate state. All results from those experiments confirm our claim that elasticities of factor supply and the intertemporal elasticity to be the core driving forces for micro-lumpiness to have an aggregate effect.

## 2 Introduction

A recurrent question when bringing together microeconomic evidence and macroeconomic effects is to what degree the investment and hiring dynamics at the firm level translate into aggregate effects. In this paper we analyze the dynamic effects of lumpy adjustments in capital and labour onto the aggregate economy. Investigating the effects of non-convexities in capital adjustment, Veracierto (2002) and Khan and Thomas (2008) found no effect from firm level lumpiness on aggregate variables in a general equilibrium setting. This finding has been further confirmed by Reiter et al. (2008) in a monetary model. By contrast, King and Thomas (2006) found that lumpy labour adjustment enhances persistence of aggregate employment, but its aggregate implications are virtually no different to those obtained from a standard quadratic adjustment cost model. In a recent paper, Yao (2008) introduced lumpy labour adjustment by integrating increasing-hazard labour adjustment process into a DSGE model and finds that this extension helps to enhance both volatility and persistence of employment dynamics in the model. These results seem to suggest that the macroeconomic effects of micro frictions depend on the source of rigidity. It is therefore important to understand the mechanism which translates microeconomic dynamics to the macroeconomic level. Especially with increasing quality of microeconomic data sources, the question is under what conditions macroeconomic analysis needs to take this firm-level activity into account for understanding macroeconomic fluctuations and under what conditions the rich activity at the firm level can be abstracted from. Due to this reason, the main questions we address in this paper are: why does lumpiness in different production factors lead to different effects on aggregate dynamics, what are the mechanisms through which these effects work and does coordinated lumpy adjustments of both production factors matters for the aggregate dynamics?

The empirical evidence accumulated over the last decade shows that adjustments of production factors at the plant level exhibit a lumpy pattern in response to shocks, whereby discrete adjustments are followed by long periods of inactivity. Convincing evidence has been documented for both labour and capital adjustments, such as Doms and Dunne (1998), Cooper et al. (1999), Nilsen and Schiantarelli (2003) for investment. In addition Cooper et al. (1999) find the hazard function for capital adjustments to be increasing over time following the last adjustment. For labour adjustments, Hamermesh (1989), Caballero and Engel (1993) and Caballero et al. (1997) find strong support for lumpy and asynchronous changes in firm-level employment. More recently, Letterie et al. (2004) investigate the complementarity between labour and capital demand using plant-level data for the Dutch manufacturing sector and observe lumpy adjustment for both factors and a strong degree of coordination between the two. Varejao and Portugal (2007) reveal the importance of lumpy labour adjustments for the economy, measured as adjustments of more than 10% of the plant's labour force to account for about 66% of the total job turnover, and on average around 75% of all observed Portuguese employers do not change employment over an entire quarter. Related to this is the issue of indivisibility for hiring for small firms whereby the economic role of this is not entirely assessed. Finally, Vermeulen (2006) finds stickiness in labour adjustment due to indivisibility for small firms.

The evidence suggests a strong degree of non-convexities for adjustments in both production factors, and more importantly, it is evident that they are coordinated between each other in time as found by the correlation by Letterie et al. (2004). However, in the literature these



closely related issues are studied separately and within different frameworks. To fill this gap, we use a unified framework in this paper allowing for lumpy adjustments in both, capital and labour and in the joint adjustment through the capital/labour ratio. In particular, we follow the Calvo (1983) staggering setting of investment and/or recruitment decisions in a prototypical RBC model. This allows to replicate the results that are shown in the literature in a simplified way and comparing the underlying mechanism.

The principal results we find are that lumpy labour and capital lead to very different effects regarding the dynamics of output and other aggregate variables. Most importantly, lumpy labour adjustment allows under sufficiently elastic labour supply a hump-shaped response of output, which cannot be obtained by the lumpy capital model. Capital, by contrast, is not sufficiently elastic due to its intertemporal link through the consumption Euler-equation and the effects are largely neutralized by price changes. Moreover, when the lumpy factor adjustments coordinate with each other, the effects on the aggregate dynamics are even strengthened. In a subsequent step we investigate the underlying reasons for these effects and identify the elasticities of labour supply and the intertemporal elasticity to be the core driving forces. We conclude that sufficiently elastic factor supply is the prerequisite for the lumpy adjustment to have aggregate effects, and furthermore that the *intra*temporal substitution as opposed to the *inter*temporal margin is important for business cycle dynamics. It is the elasticity of factor supply that adjusting firms face which determines the dynamics. This needs to be sufficiently large without affecting factor prices too much in order for firm level dynamics to transmit to the aggregate level.

The remainder of the paper is organized as follows: section 3 introduces the baseline model, in which we formulate the firm's adjustment process as a Poisson process in either production factor. In section 4, we introduce lumpy labour adjustment into the RBC model and present the resulting dynamic labour demand equations and compare this to two different specification in the same framework: the lumpy capital adjustment and the lumpy capital-labour ratio adjustment. In section 6, we present the simulation results for all three models. Section 8 uses the simulation results of both lumpy adjustment models to unveil the necessary conditions that underlie the origins of persistent dynamics and analyze the sensitivity of the model.

### 3 Basic Model

In this section we introduce a general framework to allow for lumpy adjustment in different factors. At this stage we do not specify which production factors are under concern, but apply only general terms  $(X, Y)$  for the employed production factors, whereby  $X$  is the production factor subject to the Calvo restriction of adjustment, whereas  $Y$  is the factor that can be re-optimized freely.

#### 3.1 Firms

We assume that firms in the economy are subject to a staggered adjustment scheme a la Calvo (1983) that is induced by some frictions in the factor market. The economy is populated by a large number of firms that are differentiated by their stocks of the factor  $X$ . There is one

commodity in the economy that can be either consumed or invested. In each period, all firms can adjust factor  $Y$ , but only a fixed fraction of firms are allowed to re-optimize factor  $X$ .

Another distinctive feature of our model is that the decreasing-return-to-scale technology is used to produce output<sup>1</sup>. This assumption allows us to derive the optimal adjustment at the firm level, for in the case of constant-return-to-scale only the capital–labour ratio is determinate. However, decreasing-return-to-scale production technology implies positive profits for the firm, and the smaller the size of the firm is, the more efficient the firm becomes in terms of profits per production factor employed. Hence, firms have an incentive to be small. In order to set a minimum firm size, we introduce this fixed cost of operation  $\varphi$ , which is equal to the profits earned at steady state. Given this cost, all firms in a stochastic environment make positive profits in some periods and negative profits in others. Since firms expect zero profits in the long run, no entry and exit occur in this economy and hence the number of firms are constant.

Every firm that obtains the Calvo signal in order to re-optimize factor  $X$  choose the same value for the adjustment, as this is purely a forward-looking decision. Therefore we can index firms by  $j$  referring to the firms that adjusted the lumpy factor  $j$  periods in the past. Given the distribution of vintage groups in the economy  $\Omega(j)$ , we can retrieve the competitive equilibrium allocations by solving the following planner’s problem:

$$V(Z_t, X_t^j) = \max_{\{X_t^0, Y_t^j\}_{j=1}^{\infty}} \left\{ \sum_{j=0}^{\infty} \Omega(j) [F(Y_t^j, X_t^j, Z_t) - p_t^X X_t^j - p_t^Y Y_t^j - \varphi] + E_t[\tilde{\beta}_{t+1} V(Z_{t+1}, X_{t+1}^j)] \right\} \quad (1)$$

subject to the production technology:

$$F(Z_t, X_t^j, Y_t^j) = Z_t (X_t^j)^a (Y_t^j)^b \quad (2)$$

and the distribution of vintage groups in the aggregate scheme induced by the Poisson distribution is:

$$\Omega(j) = \alpha^j (1 - \alpha)$$

where  $\alpha$  represents the probability that a firm do not adjust the frictional factor. We denote  $p_t^X$  and  $p_t^Y$  as the prices of factor  $X_t$  and  $Y_t$  respectively, and  $Z_t$  is the total productivity shock, following and AR(1) process:  $\ln Z_t = \varsigma \ln Z_{t-1} + \epsilon_t$ ,  $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ ,  $0 < \varsigma < 1$ .  $\tilde{\beta}_{t+1}$  is the stochastic discount factor, which equals  $\beta E_t(U'(C_{t+1}))/U'(C_t)$ .  $\varphi$  denotes the fixed cost of production that dissipates profits of firms entailed from the decreasing-return-to-scale production technology. When adjusting the frictional factor  $X_t$ , the firm takes into account future factor prices  $p_t^X, p_t^Y$  and the fact that the factor  $X_t$  may not be adjusted again until the next Calvo signal arrives.

The first-order conditions of the problem are:

$$p_t^X = F_X^{(0)}(t) + \alpha(1 - \alpha) E_t \left\{ \tilde{\beta}_{t+1} [F_{X_{t+1}}^{(1)} - p_{t+1}^X] \right\}, \quad (3)$$

<sup>1</sup>The diseconomy of scale can be theoretically motivated in several ways. e.g. Howitt and McAfee (1988) emphasizes the role of externalities, i.e. the marginal adjustment cost faced by a firm is positively related to the activity level already attained by its rivals.

$$F_Y^{(j)}(t) = p_t^Y \quad \forall j \quad (4)$$

Equation (3) tells us that under the uncertainty of factor adjustment, due to  $\alpha \in (0, 1)$ , the price for the optimal level of  $X$  does not necessarily equate its marginal product, but is affected by the expected difference between future productivity and prices. Besides, the degree of rigidity  $\alpha$  influences to what degree the forward-looking component is important for today's factor demand decision. For the freely adjustable factor, the price equals marginal productivity.

To obtain the optimal factor demand of the frictional factor  $X$ , we employ equation (2) in combination with the first order conditions:

$$\left\{ X_t^{(0)} \right\}^{\frac{1-a-b}{1-b}} = \frac{ab^{b/1-b} \sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t+j} z_{t+j}^{1/1-b} (p_{t+j}^Y)^{b/b-1}]}{\sum_{j=0}^{\infty} \alpha^j E_t[\tilde{\beta}_{t+j} p_{t+j}^X]}, \quad (5)$$

Equation (5) characterizes optimal factor demand of an adjusting firm in period  $t$ . At the individual level the optimal factor demand reacts to all future shocks and equilibrium prices. In particular, when the covariance between the productivity level and the inverse of the price level is small enough, optimal factor demand is increasing in all expectations of future shocks  $z_{t+j}$  and decreasing in all expectations of future factor prices  $p_{t+j}^X$  and  $p_{t+j}^Y$ . This implies a 'front-loading' or factor hoarding effect to insure against future adjustment restrictions. If we assume prices being constant, it is easy to see that a positive persistent shock will make the adjustment higher than it would be in a frictionless economy. Firms acquire more of factor  $X$  than what they currently need in order to hedge the future adjustment risk, vice versa for negative shocks. Additionally equation (5) also shows that the higher the value of  $\alpha$ , the higher the weight attached on future shocks. Factor demand is more sensitive to the future shocks when adjustment frictions are severer in the market. These results are generally in line with the implications of models in the (S,s) literature such as Thomas (2002).

### 3.2 Households

There is a continuum of identical households, who are endowed with  $K_0$  units of capital at  $t = 0$  and one unit of time for each subsequent period, which can be spent on either working or leisure. The infinitely-lived representative household chooses consumption, labor supply and investment to maximize the expected discounted utility:

$$\max \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

where  $u(C_t, L_t) = \frac{C_t^{1-\eta}}{1-\eta} - \chi \frac{L_t^{1+\phi}}{1+\phi}$  is instant utility of the representative household, with  $C_t$  representing aggregate consumption and  $L_t$  as the aggregate labour supply. The budget constraint of the household is

$$C_t + K_{t+1} + N\varphi = F(K_t, L_t, Z_t) + (1 - \delta) K_t$$

where  $N$  denotes the number of firms in the economy, which is exogenous for simplicity.

The optimality conditions of the household are obtained by:

$$1 = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\eta} (r_{t+1} + 1 - \delta) \right] \quad (6)$$

$$w_t = \chi L_t^\phi C_t^\eta \quad (7)$$

The first equation represents the intertemporal Euler equation and the second one relates real wage to the marginal rate of substitution between consumption and leisure.

### 3.3 Aggregation

Factor market prices  $p_t^X$ ,  $p_t^Y$  clear the markets for aggregate variables  $X_t$  and  $Y_t$ <sup>2</sup>, defined as the aggregation of firm level quantities:

$$X_t = \sum_{j=0}^{\infty} \Omega(j) X_t^j, \quad Y_t = \sum_{j=0}^{\infty} \Omega(j) Y_t^j$$

Output at the aggregate level consists of the contributions from every single firm weighted by the distribution of vintages.

$$F(X_t, Y_t, Z_t) = \sum_{j=0}^{\infty} \Omega(j) F(X_t^j, Y_t^j, Z_t) \quad (8)$$

## 4 Model comparisons

### 4.1 The Lumpy labour Model

In this section we assume the rigid factor  $X$  is labour, together with the optimal behavior of households and firms, we obtain the lumpy labour version of the model. Equation (5) for the lumpy factor becomes

$$(l_t^0)^{\frac{1-a-b}{1-b}} = \frac{ab^{b/1-b} \sum_{j=0}^{\infty} (\alpha_l)^j E_t [\tilde{\beta}_{t+j} z_{t+j}^{1/1-b} r_{t+j}^{b/b-1}]}{\sum_{j=0}^{\infty} (\alpha_l)^j E_t [\tilde{\beta}_{t+j} w_{t+j}]}, \quad (9)$$

where  $l_t^0$  denotes labour decision at the firm level and the relevant factor prices  $p_t^X$  and  $p_t^Y$  are already substituted with the wage  $w_t$  and the rental rate  $r_t$  respectively. The price information combined with the expected evolution of productivity contains all relevant information of future optimal factor allocation.

Aggregate labour  $L_t$  evolves according to

$$L_t = (1 - \alpha_l) l_t^0 + \alpha_l L_{t-1}, \quad (10)$$

<sup>2</sup>The counterparts of  $X_t$  and  $Y_t$  in the supply side of the economy are either capital ( $K_t$ ) or labour ( $L_t$ ).

combining aggregate labour from  $t - 1$  with the individual optimal decision of adjusting firms. Due to the fact that labour is lumpy at the firm level, only the optimizing firms have effects on aggregate labour, while all other firms introduce persistence in the aggregate evolution of labour.

After log-linear approximation around the steady state, we obtain the following dynamic labour demand equations at the firm and at the aggregate level:

$$\hat{l}_{0,t} = \alpha_l \beta E_t[\hat{l}_{0,t+1}] - b \frac{1 - \alpha_l \beta}{1 - a - b} \hat{r}_t - (1 - b) \frac{1 - \alpha_l \beta}{1 - a - b} \hat{w}_t + \frac{1 - \alpha_l \beta}{1 - a - b} \hat{z}_t \quad (11)$$

$$\alpha_l \beta E_t[\hat{l}_{t+1}] - (1 + \alpha_l^2 \beta) \hat{l}_t + \alpha_l \hat{l}_{t-1} - b \kappa \hat{r}_t - (1 - b) \kappa \hat{w}_t + \kappa \hat{z}_t = 0, \quad (12)$$

where the hat on variables represents log deviations from steady state and  $\kappa = \frac{(1 - \alpha_l)(1 - \alpha_l \beta)}{1 - a - b}$ . These two equations reveal the key difference between the demand behavior at different levels. At the firm level, demand is purely forward-looking, while at the aggregate level, not only the forward-looking component but also the lagged counterpart play a role in forming aggregate dynamics. In particular, observing equation (11), we know that at the firm level persistence of the labour demand depends mainly on the Calvo parameter  $\alpha_l$  and the subjective discount factor  $\beta$ . Labour demand depends negatively on real prices and positively on the aggregate technology shock. By contrast, the aggregate labour demand (12) exhibits more complex dynamics, which involves an AR(2) process. Again, the labour market rigidity parameter  $\alpha_l$  determines the persistence of the labour dynamics. In addition, note that both equations require a decreasing-returns-to-scale technology ( $1 - a - b > 0$ ) to ensure that the size of labour demand is determined at the firm level.

Using the two equations (11) and (12), we can demonstrate why changes in prices can undo the lumpy labour adjustment at the aggregate level, but not at the firm level. The key difference between these two equations is that, at the firm level (11), the optimal factor demand is determined by taking prices as given. It amounts to ignoring any responses of aggregate prices to shocks ( $\hat{w}_t = \hat{r}_t = 0$ ), so that  $\hat{l}_{0,t} = \alpha_l \beta E_t[\hat{l}_{0,t+1}] + \frac{1 - \alpha_l \beta}{1 - a - b} \hat{z}_t$ . As a result, only the movements in the aggregate shocks is reflected in the optimal labour demand. By contrast, the aggregate labour demand equation (12) is affected by the changes in both shocks and prices. A change in productivity has no employment effect if the shock translates into equal changes in prices. As seen in the equation (12), if 1% rise in the shock leads to 1% increases in the interest rates and wages, then these effects on labor are exactly cancelled out. These equations thereby explain the different results for the partial and general equilibrium in the literature.

## 4.2 The Lumpy Investment Model

By assuming that the lumpy factor is capital, we obtain a version of the model which has similar implications as Khan and Thomas (2008). Equation (5) for the lumpy factor then becomes

$$(k_t^0)^{\frac{1-a-b}{1-b}} = \frac{ba^{a/1-a} \sum_{j=0}^{\infty} (\alpha_k)^j E_t[\tilde{\beta}_{t+j} z_{t+j}^{1/1-a} w_{t+j}^{a/a-1}]}{\sum_{j=0}^{\infty} (\alpha_k)^j E_t[\tilde{\beta}_{t+j} r_{t+j}]}, \quad (13)$$



where  $k_t^0$  denotes the capital stock of the adjusting firms, while aggregate capital  $K_t$  evolves according to

$$K_t = (1 - \alpha_k)k_t^0 + \alpha_k(1 - \delta)K_{t-1}. \quad (14)$$

According to this equation non-adjusting firms loose capital due to depreciation and any investment is concentrated within the firms that are enabled to adjust capital.

Similarly to the lumpy labour model, we can obtain the log-Linearized dynamic capital demand equation as follows:

$$\alpha_k \beta E_t[\hat{k}_{t+1}] = (1 + \alpha_k^2 \beta(1 - \delta))\hat{k}_t - \alpha_k(1 - \delta)\hat{k}_{t-1} + \kappa(1 - a)\hat{r}_t + \kappa a \hat{w}_t - \kappa \hat{z}_t, \quad (15)$$

where  $\kappa = \frac{(1 - \alpha_k(1 - \delta))(1 - \alpha_k \beta)}{1 - a - b}$ , and the rigidity parameter  $\alpha_k$  determines the persistence of capital dynamics in this equation.

### 4.3 Lumpy Capital-labour ratio

In this section, we consider a model, in which firms use a kind of "putty-clay" technology to produce output similar to Gilchrist and Williams (2000) and Gilchrist and Williams (2005). This is motivated by the evidence that both lumpy labour and capital adjustments coordinate each other in time. We can still use the same framework to model this phenomenon. Here we assume that only the firms that receive the Calvo signal can re-optimize over both capital and labour, while other firms have to keep using their old vintages of capital and labour. Consequently, the capital-labour ratio becomes a new state variable, which may differ among firms that make the adjustment in the different periods of time. Firms that receive the Calvo signal re-optimize their capital and labour contemporaneously to maximize their discounted profits stream:

$$V_t = \max_{L_t^0, K_t^0} \left\{ \sum_{j=0}^{\infty} \alpha^j \tilde{\beta}_{t+j} [F(L_t^0, K_t^0, Z_t) - w_{t+j} L_t^0 - r_{t+j} K_t^0 - \varphi] \right\} \quad (16)$$

subject to the same production technology and the law of motion of the technology shock as in the basic model.<sup>3</sup>

We obtain two first-order conditions with respect to capital and labour respectively,

$$b (L_t^0)^a (K_t^0)^{b-1} = \frac{\sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} r_{t+j}]}{\sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} z_{t+j}]} \quad \text{and} \quad a (L_t^0)^{a-1} (K_t^0)^b = \frac{\sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} w_{t+j}]}{\sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} z_{t+j}]}, \quad (17)$$

Combining these two optimal conditions, the optimal capital-labour ratio is as follows,

$$\left(\frac{K}{L}\right)_t^0 = \frac{b \sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} w_{t+j}]}{a \sum_{j=0}^{\infty} (\alpha)^j E_t[\tilde{\beta}_{t+j} r_{t+j}]}, \quad (18)$$

<sup>3</sup>We require that the initial distribution of capital-labor ratios is uniform across firms. Otherwise the adjustment would be characterised by an adjustment phase with non-trivial dynamics.

Intuitively the optimal capital-labour ratio is increasing when expected wages in the future are high, and it is decreasing in the expected future interest rates. Because the aggregate technology shock exerts the same effect on labour and capital adjustments, the total effects on the optimal ratio is cancelled out<sup>4</sup>.

Log-linearizing the equation (18), we obtain,

$$k_{-l_{0,t}} = \alpha\beta E_t[k_{-l_{0,t+1}}] - (1 - a\beta)\hat{r}_t + (1 - a\beta)\hat{w}_t \quad (19)$$

where  $k_{-l_{0,t}}$  denotes the log deviation of the optimal capital-labour ratio from the steady state at the firm level. From this equation we can see that the predictable prices movements are important in determining the optimal capital and labour ratio.

To aggregate the capital labour ratio, we use the fact that  $1 - a$  percent of firms adjust their capital-labour ratio to the level defined in 18 and the rest of firms use the old capital-labour ratio adjusted by the depreciation rate. As a result, after log-linearization, we obtain the following aggregate equation for the capital-labour ratio,

$$k_{-l_t} = (1 - \alpha(1 - \delta))k_{-l_{0,t}} + \alpha(1 - \delta)k_{-l_{t-1}} \quad (20)$$

where  $k_{-l_t}$  denotes the log deviation of the aggregate capital-labour ratio from the steady state.

Finally, by combining equations (19) and (20) we obtain the dynamic equation for the aggregate capital-labour ratio.

$$\alpha\beta E_t[k_{-l_{t+1}}] - (1 + \alpha^2\beta(1 - \delta))k_{-l_t} + \alpha(1 - \delta)k_{-l_{t-1}} - (1 - \alpha(1 - \delta))(1 - \alpha\beta)(\hat{r}_t - \hat{w}_t) = 0 \quad (21)$$

## 5 Calibration

We use this model as a laboratory to analyze the impact of employment and investment rigidity on business cycles. We follow the tradition of the RBC literature to calibrate our model such that it is consistent with the long-run growth facts in the U.S. data, and then study its short-run dynamics by investigating the statistical properties of simulated time series and impulse response functions.

Note that the features we introduce in the model do not affect the steady state equations, thus, in steady state we have the same relationships among the variables as in the standard RBC model. For this reason we can safely use many standard parameter values in the RBC literature. For quarterly data the discount rate  $\beta$  we use is 0.99 to reflect a real rate of interest of around 4% per annum. The depreciation rate  $\delta$  is set to 2.5% indicating an annual depreciation rate of 10%. We select the capital share  $b$  to be 0.33 to match the average annual capital-output ratio of 2.4 as used in Khan and Thomas (2008) and the labour share of output  $a$  is set to be 0.58, consistent with direct estimates for the U.S. economy as found in King et al. (1988). We set the fixed operating cost  $\varphi$  to exactly offset pure profits the firm may make in steady state. Hence, the per period profit ratio in the long-run is  $\varphi/F(K, L) = (1 - a - b)$ .

<sup>4</sup>Note that this result does not depend on the rate of return to scale of the production technology. A constant returns to scale production function would obtain identical results.

Regarding the parameters of the utility function we assume different values and compare the outcomes to shed light on the mechanism leading to differing results between lumpy labour and capital adjustment.

For the parameter  $\eta$  affecting the curvature of the utility function with respect to consumption our baseline case uses  $\eta = 1$ , representing log-utility in consumption which is consistent with long-run growth facts, but for the inspection of the mechanism we use 0.1 and 5 to unveil the conditions under which aggregate effects appear. Regarding the elasticity of supply in hours  $\phi$ , we apply the indivisible labour assumption as in Hansen (1985) and Rogerson (1988), implying  $\phi = 0$ . The relative weight between consumption and leisure is determined by  $\chi$  to obtain that 20% of time is dedicated to market activities in steady state as used in Thomas (2002).

The adjustment parameters for capital and labour are set to account for the observed net investment and labour flows at the firm level. The labour adjustment parameter is calibrated according to empirical work on estimating hazard functions using aggregate net flow data. Caballero and Engel (1993) used U.S. manufacturing employment and job flow data (1972:1-1986:4) to estimate constant hazard functions. Their result suggests that, on average, 22.9% of firms in the U.S. adjust their employment per quarter. As a result we choose 0.77 for  $\alpha$ , implying a mean duration of employment of 4.35 quarters. The capital adjustment parameter is set to the same level as the labour adjustment parameter to facilitate comparison of simulation results from both lumpy factor models.

Finally, we set  $\rho = 0.95$  and  $\sigma_\nu = 0.007$  for aggregate technology shocks to match the estimated parameters of Solow residuals commonly used in the RBC literature following King and Rebelo (1999)<sup>5</sup>.

Category	Preferences				Technology			Rigidity			Shock	
Var.	$\phi$	$\chi$	$\beta$	$\eta$	$a$	$b$	$\delta$	$\alpha_l$	$\alpha_k$	$\alpha$	$\varsigma$	$\sigma_\epsilon$
Values	0	3.614	0.99	1	0.58	0.33	0.025	0.77	0.77	0.77	0.95	0.007

Table 1: Parameter values used in the baseline calibration of the plain RBC, the lumpy capital and the lumpy labor model.

## 6 Simulation Results

To evaluate the quantitative performance of the lumpy RBC models, we compare impulse responses and second moments generated by the lumpy labour (LL) model the lumpy Capital (LC) model and the lumpy capital-labour ratio (LKL) model with the standard RBC model by Hansen (1985). Our baseline model uses the parameter values of table 1, with  $\phi = 0$  implying an infinite Frisch elasticity of labour supply identical to the indivisible labour model, and  $\eta = 1$  representing a log utility function for consumption.

<sup>5</sup>Veracierto (2002) suggests that the standard deviation of shocks should be smaller to account for the decreasing returns to scale assumption. He chooses 0.0063 given his parameter values of labor and capital shares. However, since we are interested in the relative volatilities between variables to output in a linearized model, the scale of the standard deviation is not important.



In Table (5) and (6) we report second moments of U.S. business cycles data and those generated by the lumpy factor models. In all cases, the moments are calculated from HP(1600)-filtered time series<sup>6</sup>.

We can observe that the dynamics of the different aggregate variables in the LC model are very similar to those generated by the RBC model, volatility and persistence measures from both models are close to each other. The LL and lumpy capital-labour ratio model produces more persistent dynamics for output and labour than the other two models. Cogley and Nason (1995) have shown that the standard RBC models fail to account for the observed positive serial correlation in growth rate of output. As seen in the tables, the LL model enhances persistence of business cycles by introducing lagged labour in the dynamics. By contrast, both the LC model and the RBC model only generate two thirds of the persistence observed in the data. As to cyclical volatility, we find that all three models can capture the general pattern of volatilities in the data. However, the LL model dampens the volatility of employment, because its setting gives basically the same aggregate dynamic equation as the quadratic adjustment cost model shown in Yao (2008). Regarding the LKL model, we find that its effects on aggregate variables are stronger than those of lumpy labour, while it is closer to the LC and RBC response at the micro-level. This is because the strong complementarity between the two production factors makes firms very difficult to change their production technology, so that the whole economy needs more time to fully digest the technology shock. This effect is presented when either production factor is subject to an inelastic supply condition.

In short, the LC model's aggregate implication is not different to the standard RBC model, while the LL model can generate high persistence combined with a lower degree of volatility<sup>7</sup>. With these results we confirm the finding by Thomas (2002) that firm level lumpy capital adjustment plays only a minor role for aggregate variables. By contrast, lumpy labour adjustment is important in shaping aggregate dynamics of output that is found in King and Thomas (2006).

## 7 Impulse Responses

In this section we use variations in the parameters to analyze the mechanism that entails different results of the lumpy models. We find that the necessary condition in order to obtain the aggregate effects of lumpiness is the sufficiently high elasticity of supply associated with the lumpy production factor. We will focus on the role played by the parameter of relative risk aversion  $\eta$ , the elasticity of labour supply  $\phi$  and the depreciation rate  $\delta$ , because these parameters can either directly or indirectly influence the supply elasticity of the production factors. By changing the parameters we can either eliminate the aggregate effects of lumpy labour or establish them for the lumpy capital and lumpy capital-labour ratio models.

In the benchmark case (Figure 1), we present the impulse response functions for the baseline parameterization. General results are that depending on the firm's lumpy factor different

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<sup>6</sup>Each statistics is based on a 10,000-period simulation, so that the moments statistics for the simulated time series can roughly converge to their population values.

<sup>7</sup>We also simulate a version of our model with the habit formation in consumption, and find that adding the habit formation to this framework enhances the aggregate effects of lumpy adjustment even further. However, since habit formation mechanism is well understood and combining them brings no new insights, we do not discuss this case in the paper.

aggregate dynamics are generated. It becomes apparent that the standard RBC model and the model with lumpy investment have very similar dynamics, while the version with lumpy labour generates a hump-shaped response in output and labour. The explanation for these differing dynamics lies in the elasticity of supply of the lumpy factor. Following a shock in productivity the adjusting firms are the only ones to reset the amount of the lumpy factor. If supply is not elastic enough to adjust to the new demand for these factors, aggregate effects will not be materialized. This is the case for lumpy capital as its supply is restricted by the smoothing effects of the intertemporal Euler equation on consumption. With an intertemporal elasticity of substitution of one from the household's side, interest rates vary substantially if demand from the firm's side increases. Higher capital demand after a technology shock therefore induces higher interest rates and contains the overall capital increase. Consequently adjusting firms cannot vary the capital margin sufficiently strongly for lumpy capital adjustment to have an aggregate effect.

By contrast, in the case of lumpy labour with an infinitely elastic Frisch elasticity, where labour supply is free to be adjusted, the labour demand of the adjusting firms can be accommodated by the quantity supplied instead of the changes in real wage. While the non-adjusting firms hold labour size constant, the adjusting firms face little constraint in adjusting by a large margin. As a result, we observe that, at the firm level, labour surges immediately after a positive shock, however aggregate labour rises sluggishly because of its partial adjustment nature.

The "putty-clay" economy with lumpy capital-labour ratios has very different impulse responses to the other models for aggregate variables. On the one hand, aggregate dynamics are smoother and the humped response of the LL model is reinforced. On the other hand, lumpy adjustments at the firm level are weakened by this technology. Even though labour elasticity is high in this case, firm's level labour adjustment is much lower than that in the LL model. The reason is that because labour and capital adjustments are highly complementary in this model, inelastic supply of capital affects also the adjustment of labour at the firm level. In order to obtain strong aggregate effects within the "putty-clay" economy both production factors need to be elastic. This can be achieved by an elastic labour supply combined with a variable capital utilization.

These results seem to suggest that for microeconomic lumpiness to have aggregate effects it is required that the adjusting firm faces a high elasticity of the lumpy factor to exploit the new conditions in its technology.

## 8 Sensitivity Analysis

To further reveal the mechanism of the described channel, we conduct three experiments. First, we eliminate the lumpy labour effect from the LL model by decreasing the elasticity of labour supply. Second, we generate aggregate effects of firm's lumpy capital by raising the elasticity of capital supply. Hereby we can either decrease the elasticity of substitution  $\eta$  to weaken the consumption smoothing motive, or alternatively we increase the depreciation rate  $\delta$  allowing for a more immediate response of capital supply to the aggregate state. The objective in this sensitivity analysis is not to reproduce realistic parameterization, but instead to further inspect and understand the mechanism.

### 8.1 Role of the elasticity of labour supply $\phi$

The elasticity of labour supply ( $\phi^{-1}$ ) has a direct effect on the responsiveness of labour supply to shocks. In the first case, we set  $\phi$  to be 5, an arbitrary large value that gives rise to an inelastic labour supply. As depicted in Figure 2 all three models are characterized by observationally similar aggregate dynamic effects. Most effects of the aggregate shock are absorbed by the dynamics of the real wage, and aggregate labour has a very modest reaction to the shock. An elastic labour supply is at the core of the transmission from lumpy labour adjustments to have persistent aggregate effects.

### 8.2 Role of the elasticity of intertemporal substitution $\eta$

Decreasing the elasticity on labour supply weakens the transmission from firm level to aggregate effects of lumpy labour adjustments. The question is if we can generate aggregate effects in a situation with lumpy investment by altering the elasticity of substitution of capital supply. The intertemporal substitution is a very important propagation mechanism in the RBC model as it affects consumption and thereby also growth in aggregate capital. By setting  $\eta = 0.1$  households are characterized by low degrees of risk aversion, which may generate large fluctuations in consumption and output. As can be seen in figure 3, the higher elasticity of capital amplifies the response for investment and capital of adjusting firms and leads to the aggregate effects on output characterized by the hump-shaped response in all three versions. The household is willing to substitute consumption intertemporally to take advantage of the positive productivity shock and hence high investment and higher interest rates are generated. In the presence of lumpy labour this is further amplified.

### 8.3 Role of the depreciation rate $\delta$

The depreciation rate can affect the supply condition of capital indirectly. By setting the depreciation rate to a large value ( $\delta = 0.18$ ), re-optimizing firms adjust by a large margin, but aggregate investment is identical to the frictionless RBC case as depicted in figure 4. Total investment is strongly conditioned by the intertemporal elasticity from the household first order condition, but the allocation of this investment between the different firms is altered. In the RBC case investment serves to replace depreciated capital and as equal investment for all firms, whereas in the case of lumpy capital, only those firms that obtained the Calvo signal are able to benefit from investment, all remaining firms see their capital stock decline due to depreciation. The adjusting firms thereby benefit entirely from the increased investment and hence adjust by a large margin.

In conclusion, the predetermination of capital stock and the elastic labour supply are the key factors that result in the different implications of the two lumpy adjustment models. All hypothetical experiments conducted in this section illustrate that elastic supply is prerequisite for lumpy factor having an aggregate effect.

## 9 Conclusion

The central theme of this study is to analyze under which conditions frictions in the factor market could have a significant effect on the aggregate dynamics. By using a unified and tractable DSGE model of staggered factor adjustment as a laboratory device, we study the mechanism transmitting micro-distortions in the factor market to aggregate dynamics. We find that elastic factor supply is essential in order for individual firm adjustments to have aggregate effects. Without this precondition, lumpy factor demand at the firm level will be neutralized by the movement of prices in market clearing. In addition, we find that the elasticities of intertemporal substitution and the Frisch elasticity of labour supply are both highly important for the dynamics. When presenting adjustment frictions in the factor market, these parameters will determine whether the distortion at the firm level has a significant impact on the aggregate level. The same also holds for the depreciation rate. We used sensitivity analysis to show that all these parameters have impact on the aggregate effect of lumpy factor adjustment through their influence on the elasticity of factor supply. This highlights again the importance of the intratemporal adjustment for the dynamics of business.

Although analyzed in a very specific context, the mechanism at hand is quite general. We used a Calvo setting to create lumpiness in factor adjustment, but menu costs would be an alternative specification. The important element is that the adjusting firms may adjust by a large margin without affecting factor prices. In addition the mechanism may be applied to any factor. Last but not least, this exploration of the mechanism in our simple model has also significance in more sophisticated DSGE models. A prominent example where this mechanism is at work is by Christiano et al. (2005), where capacity utilization is introduced to allow for elastic capital services without affecting the price of capital.

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## 10 Tables and Figures

The lumpy labour model is summarized in the following 9 equations.

Aggr. labour	$\alpha_l \beta E_t[\hat{l}_{t+1}] = (1 + \alpha_l^2 \beta) \hat{l}_t - \alpha_l \hat{l}_{t-1} + \frac{b\bar{R}}{\bar{r}} \kappa \hat{R}_t + (1 - b) \kappa \hat{w}_t - \kappa z_t$ $\kappa = \frac{(1 - \alpha_l)(1 - \alpha_l \beta)}{1 - a - b}$
Optim. labour	$\hat{l}_t = (1 - \alpha_l) \hat{l}_{0,t} + \alpha_l \hat{l}_{t-1}$
Capital	$\bar{R} \hat{R}_t = \bar{r} \hat{y}_t - \bar{r} \hat{k}_t$
Labour supply	$\hat{w}_t = \phi \hat{l}_t + \hat{c}_t$
Euler equation	$0 = E_t[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}]$
Capital accum.	$\hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t$
Production	$\hat{y}_t = z_t + a \hat{n}_t + b \hat{k}_t$
Resource	$\delta \hat{i}_t = \bar{Y} / \bar{K} \hat{y}_t - \bar{C} / \bar{K} \hat{c}_t$
Technology	$z_{t+1} = \rho z_t + v_{t+1}$

Table 2: Collection of log-linearized equilibrium equations in the lumpy labor model

The lumpy capital model is summarized in the following 9 equations.

Aggr. capital	$\alpha_k \beta E_t[\hat{k}_{t+1}] = (1 + \alpha_k^2 \beta (1 - \delta)) \hat{k}_t - \alpha_k (1 - \delta) \hat{k}_{t-1} + \kappa \frac{(1 - a) \bar{R}}{\bar{r}} \hat{R}_t + \kappa a \hat{w}_t - \kappa z_t$ $\kappa = \frac{(1 - \alpha_k (1 - \delta))(1 - \alpha_k \beta)}{1 - a - b}$
Optim capital	$\hat{k}_t = (1 - \alpha_k (1 - \delta)) \hat{k}_{0,t} + \alpha_k (1 - \delta) \hat{k}_{t-1}$
Wage	$\hat{w}_t = \hat{y}_t - \hat{l}_t$
Labour supply	$\hat{w}_t = \phi \hat{l}_t + \hat{c}_t$
Euler equation	$0 = E_t[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}]$
Capital accum.	$\hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t$
Production	$\hat{y}_t = z_t + a \hat{n}_t + b \hat{k}_t$
Resource	$\delta \hat{i}_t = \bar{Y} / \bar{K} \hat{y}_t - \bar{C} / \bar{K} \hat{c}_t$
Technology	$z_{t+1} = \rho z_t + v_{t+1}$

Table 3: Collection of log-linearized equilibrium equations in the lumpy capital model

The Lumpy capital-labour ratio model is summarized in the following 10 equations.

Aggr. ratio	$\alpha \beta E_t[k_{-l,t+1}] - (1 + \alpha^2 \beta (1 - \delta)) k_{-l,t} + \alpha (1 - \delta) k_{-l,t-1} - \kappa (\hat{r}_t - \hat{w}_t) = 0$ $\kappa = (1 - \alpha (1 - \delta)) (1 - \alpha \beta)$
Optim ratio	$k_{-l,0,t} = \alpha \beta E_t[k_{-l,0,t+1}] - (1 - a \beta) \hat{r}_t + (1 - a \beta) \hat{w}_t$
Wage	$\hat{l}_t = \hat{k}_{t-1} - k_{-l,t}$
Labour supply	$\hat{w}_t = \phi \hat{l}_t + \eta \hat{c}_t$
Interest rate	$\bar{R} \hat{R}_t = \bar{r} \hat{y}_t - \bar{r} \hat{k}_t$
Euler equation	$0 = E_t[\hat{c}_t - \hat{c}_{t+1} + \hat{R}_{t+1}]$
Capital accum.	$\hat{k}_{t+1} = \delta \hat{i}_t + (1 - \delta) \hat{k}_t$
Production	$\hat{y}_t = z_t + a \hat{n}_t + b \hat{k}_t$
Resource	$\delta \hat{i}_t = \bar{Y} / \bar{K} \hat{y}_t - \bar{C} / \bar{K} \hat{c}_t$
Technology	$z_{t+1} = \rho z_t + v_{t+1}$

Table 4: Collection of log-linearized equilibrium equations in the lumpy capital model

$\sigma_x/\sigma_y$	U.S. data		RBC	Lumpy Capital	Lumpy Labour	Lumpy KL ratio
Variables	$\sigma_x$	$\sigma_x/\sigma_y$	$\sigma_x/\sigma_y$	$\sigma_x/\sigma_y$	$\sigma_x/\sigma_y$	$\sigma_x/\sigma_y$
Hours	1.69	0.98	0.74	0.72	0.62	0.53
Employment	1.41	0.82	0.74	0.72	0.62	0.53
Real wage	0.76	0.44	0.31	0.32	0.32	0.35
Consumption	1.27	0.74	0.31	0.32	0.32	0.35
Output	1.72	1.00	1.00	1.00	1.00	1.00
Investment	5.34	3.10	3.38	3.35	3.33	3.21
labour productivity	0.73	0.42	0.31	0.32	0.41	0.56

Table 5: Comparison of the volatilities, Data source :Cooley(1995) Table 1.1

$Corr(x_t, x_{t+1})$	U.S. data	RBC	Lumpy Capital	Lumpy Labour	Lumpy KL ratio
Hours	0.89	0.70	0.70	0.84	0.93
Employment	0.91	0.70	0.70	0.84	0.93
Capital	0.89	0.96	0.96	0.96	0.97
Real wage	0.59	0.81	0.80	0.82	0.81
Consumption	0.87	0.81	0.80	0.82	0.81
Output	0.86	0.72	0.72	0.78	0.80
Investment	0.48	0.70	0.70	0.78	0.81
labour productivity	0.85	0.81	0.80	0.65	0.61

Table 6: Comparison of persistence, Data source : OECD MEI database (From 1965,Q1 to 2007, Q1);All time series are in logarithms and have been detrended by Hodrick-Prescott filter.



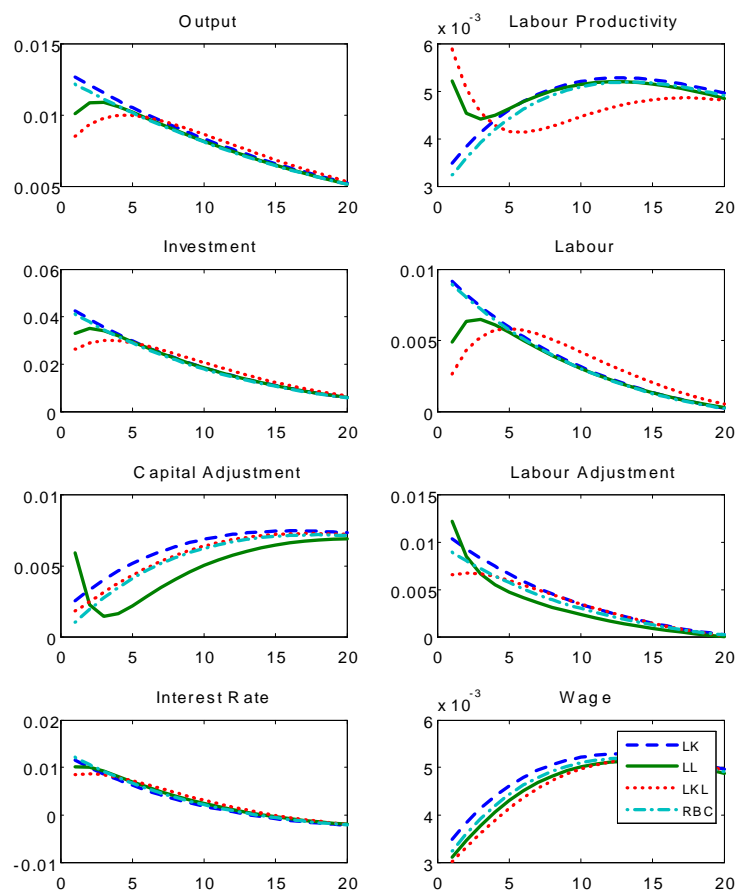


Figure 1: The benchmark specification: comparing the IRFs of lumpy capital (LK), lumpy labour (LL), lumpy KL ratio (LKL) and the RBC model (RBC) with parameters  $\eta = 1$  and  $\phi = 0$ .

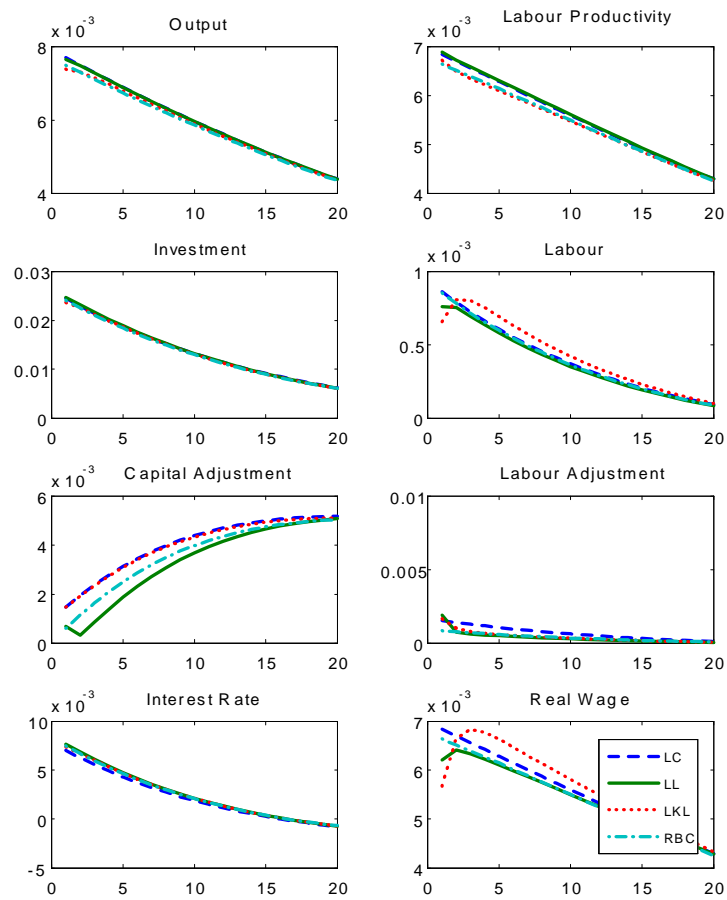


Figure 2: Comparing the IRFs of lumpy capital (LK), lumpy labour (LL), lumpy KL ratio (LKL) and the RBC model (RBC) with elastic labour supply:  $\eta = 1, \phi = 5$

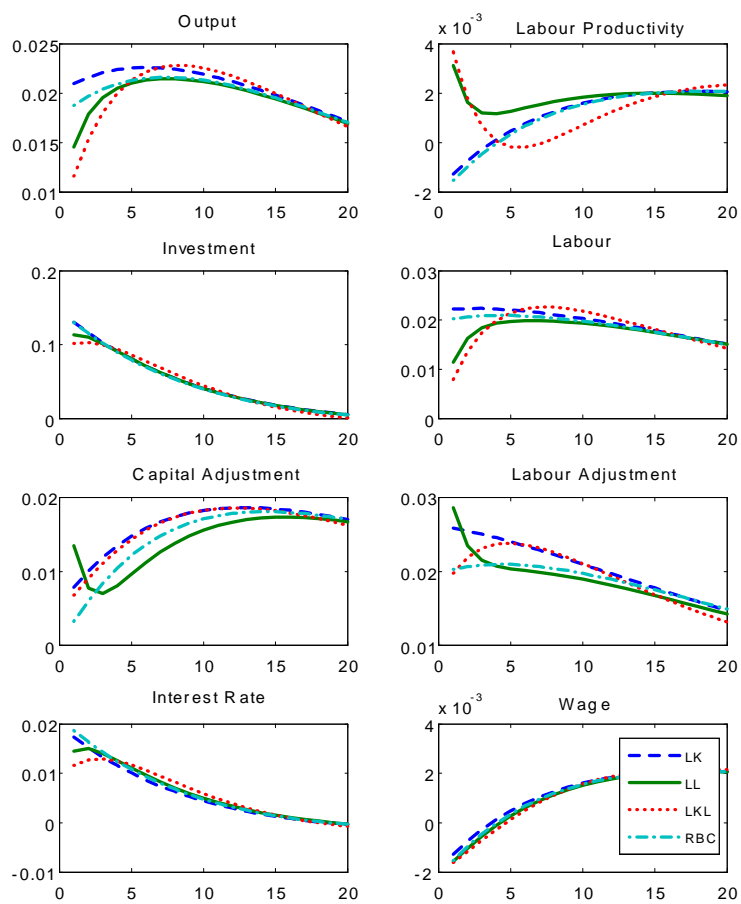


Figure 3: Comparing the IRFs of lumpy capital (LK), lumpy labour (LL), lumpy KL ratio (LKL) and the RBC model (RBC) with elastic capital supply:  $\eta = 0.1$ ,  $\phi = 0$

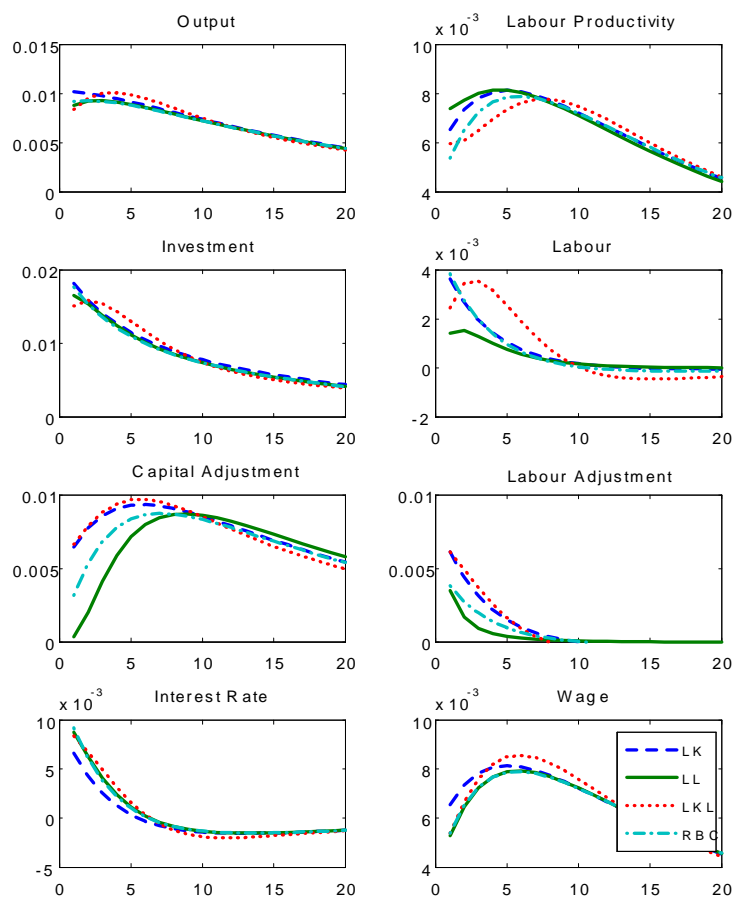


Figure 4: Comparing the IRFs of lumpy capital (LK), lumpy labour (LL), lumpy KL ratio (LKL) and the RBC model (RBC) with a high depreciation rate:  $\eta = 1, \phi = 0, \delta = 0.18$

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