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## Growth accounting for the euro area: a structural approach

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EUROPEAN CENTRAL BANK

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**WORKING PAPER SERIES**

**NO 804 / AUGUST 2007**

**GROWTH ACCOUNTING  
FOR THE EURO AREA**

**A STRUCTURAL  
APPROACH**

by Tommaso Proietti  
and Alberto Musso



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### GROWTH ACCOUNTING FOR THE EURO AREA

#### A STRUCTURAL APPROACH <sup>1</sup>

by Tommaso Proietti <sup>2</sup>  
and Alberto Musso <sup>3</sup>



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## Abstract

This paper is concerned with the estimation of euro area potential output growth and its decomposition according to the sources of growth. The growth accounting exercise is based on a multivariate structural time series model which combines the decomposition of total output according to the production function approach with price and wage equations that embody Phillips type relationships linking inflation and nominal wage dynamics to the output gap and cyclical unemployment, respectively.

Assuming a Cobb-Douglas technology with constant returns to scale, potential output results from the combination of the trend levels of total factor productivity and factor inputs, capital and labour (hours worked), which is decomposed into labour intensity (average hours worked), the employment rate, the participation rate, and population of working age. The nominal variables (prices and wages) play an essential role in defining the trend levels of the components of potential output, as the latter should pose no inflationary pressures on prices and wages.

The structural model is further extended to allow for the estimation of potential output growth and the decomposition according to the sources of growth at different horizons (long-run, medium run and short run); in particular, we propose and evaluate a model-based approach to the extraction of the low-pass component of potential output growth at different cutoff frequencies. The approach has two important advantages: the signal extraction filters have an automatic adaptation property at the boundaries of the sample period, so that the real time estimates do not suffer from what is often referred to as the "end-of-sample bias". Secondly, it is possible to assess the uncertainty of potential output growth estimates with different degrees of smoothness.

*Keywords:* Potential output, Output gap, Euro area, Unobserved components, Production function approach, Low-pass filters.

*JEL classification:* C32, C51, E32, O47

## Non-technical summary

The main purpose of this paper is to propose an extended empirical approach to estimate and analyse potential output growth and to apply it to the case of the euro area. This contribution can be also seen as proposing a structural approach to growth accounting. The reference framework adopted is a model based approach: we specify and estimate a multivariate structural time series model embodying the decomposition of output according to a production function approach and two Phillips type relationships relating price and wage inflation to the output gap and the unemployment gap, respectively. Assuming a Cobb-Douglas technology with constant returns to scale, potential output results from the combination of the trend levels of total factor productivity and factor inputs, capital and labour (hours worked), which is decomposed into labour intensity (average hours worked), the employment rate, the participation rate, and population of working age. The nominal variables (prices and wages) play an essential role in defining the trend levels of the components of potential output, as the latter should pose no inflationary pressures on prices and wages. Typically, estimates of potential output growth based on this framework, as well as on simpler approaches, tend to exhibit a marked procyclical pattern, unless some smoothness prior is imposed. As shown in the application, this is the case also for the euro area. Against this background, one of the key contributions of the paper is to propose an extension of the basic statistical framework allowing for a formal analysis of the degree of smoothness of the growth rate of potential output and its components. More precisely, we propose a model-based filtering approach for estimating potential output growth at different horizons, namely in the medium and long run. For this purpose the band-pass decomposition of potential output is embedded within the original parametric model so that we are able to estimate the underlying growth at any relevant horizon also in real time and to assess its reliability using standard optimal signal extraction principles. Finally, we provide a novel way of estimating the level of smoothness that is consistent with the definition of potential output and the NAIRU as those components of output and unemployment that exerts no inflationary pressure on prices and wages. The approach we propose has two important advantages. First, the signal extraction filters have an automatic adaptation property at the boundaries of the sample period, so that the real time estimates do not suffer from what is often referred to as the

”end-of-sample bias”. Second, it allows for an assessment of the uncertainty surrounding potential output growth estimates with different degrees of smoothness. The application focuses on the case of the euro area. Using our extended framework, we provide a discussion of potential output growth developments and its main sources since 1970. Moreover, we illustrate to which extent the reliability of potential output growth estimates for the euro area decreases as the imposed degree of smoothness increases. A finding of the applied exercise is that the estimates of potential output resulting from our original model do not carry additional information that is relevant for explaining the behaviour of the nominal variables, although they have a procyclical appearance. Overall, the application makes clear that the proposed extended framework allows for a formal analysis of various key aspects of potential growth, thereby representing a potentially important methodological contribution in the empirical analysis of growth and its sources.



# 1 Introduction

The notion of potential output, defined by Okun (1962) as the maximum level of output the economy can produce without inflationary pressures, plays an important role in macroeconomic analysis. In the European context, estimates of potential output and the deviations of actual output from potential, known as the output gap, provide relevant information for the conduct of economic policy. From a monetary policy perspective, these estimates are one of the factors from which a reference value for monetary growth is derived (see ECB, 2004). As for fiscal policy, they are instrumental for deriving measures of structural budget deficits, which play a key role in the context of the Stability and Growth Pact. Moreover, from a structural policy perspective, they can provide indications on the sustainability of growth developments as well as on the need for further reforms in the labour and product market, also against the background of the targets of the Lisbon strategy.

The main purpose of this paper is to estimate and analyse potential output developments in the euro area during the period 1970-2005. We perform a growth accounting analysis that emerges directly from fitting a multivariate structural time series model which combines the decomposition of total output obtained by the production function approach with two price and wage equations that embody a Phillips type relationship relating inflation and nominal wage dynamics to the output gap and cyclical unemployment, respectively.

The structural model extends that entertained by Proietti, Musso and Westermann (2007) (henceforth referred to as PMW) in two directions: first, the measure of labour input that is adopted is hours worked rather than the number of employed persons. This enriches the framework of the analysis, allowing for a breakdown of this production factor into four components: labour intensity (average hours worked), the employment rate, the participation rate, and a demographic factor, concerning the evolution of the working age population. This choice is also more in line with the traditional production function analysis, and bears important consequences on the estimation of total factor productivity growth. Secondly, an additional equation is specified relating nominal wages to the deviation of unemployment from structural unemployment, or NAIRU (non-accelerating inflation rate of unemployment, i.e. the rate of unemployment that is consistent with a stable rate of inflation), or, as it is

sometimes called, the NAWRU (non-accelerating wage inflation rate of unemployment). As a result, we base our analysis on a multivariate structural time series model that is formulated in terms of seven variables, namely, total factor productivity, average hours worked, the participation rate, the contribution of the unemployment rate, a capacity utilisation measure, the consumer price index, and nominal wages.

Assuming a Cobb-Douglas technology with constant returns to scale, potential output results from the combination of the trend levels of total factor productivity and factor inputs, labour and capital. The nominal variables (prices and wages) play an essential role in defining the trend levels of the above mentioned variables, as they should pose no inflationary pressures on prices and wages.

The structural model is further extended to allow for the estimation of potential output growth and its decomposition into sources at different horizons (long-run, medium run and short run); in particular, we propose and evaluate a model-based approach to the extraction of the low-pass component of potential output growth at different cutoff frequencies. The approach has two important advantages: the signal extraction filters have an automatic adaptation property at the boundaries of the sample period, so that the real time estimates do not suffer from what is often referred to as the "end-of-sample bias". Secondly, it is possible to assess the uncertainty of potential output growth estimates with different degrees of smoothness.

Discussions of the appropriate or desirable degree of smoothness of potential output estimates most often are undertaken in an informal way, e.g. by setting to an ad hoc value a particular parameter which regulates the smoothness. Several studies, for example with reference to the NAIRU, follow the approach of Gordon (1998) and apply a smoothness prior without a formal analysis to justify it. In this paper we show how it is possible to extend the statistical framework adopted to allow for a formal discussion of the degree of smoothness of potential output and its components.

The paper is structured as follows. Section 2 summarises the production function approach and illustrates the specification of the structural model. Section 3 reports and discusses in detail the estimation results. Section 4 discusses the estimation of potential output growth at different time horizons by model-based low-pass filtering. Section 5 elaborates on the growth accounting analysis

allowed for by the structural approach we propose. Finally, section 6 summarises the conclusions that can be drawn from the analysis.

## 2 The model

This section describes the multivariate structural time series model upon which our growth accounting analysis is based. We begin by reviewing the method of decomposing output fluctuations known as the production function approach.

### 2.1 The production function compositional approach

The production function approach (PFA) is a multivariate method that obtains potential output from the "non-inflationary" levels of its structural determinants, such as productivity and factor inputs.

Let  $y_t$  denote the logarithms of output (gross domestic product), and consider its decomposition into two components,

$$y_t = \mu_t + \psi_t,$$

where  $\mu_t$ , potential output, is the expression of the long run behaviour of the series and  $\psi_t$ , denoting the output gap, is a stationary component, usually displaying cyclical features. Potential output is the level of output consistent with stable inflation, whereas the the output gap is an indicator of inflationary pressure.

We assume that the technology can be represented by a Cobb- Douglas production function with constant return to scale on labour and capital:

$$y_t = f_t + \alpha h_t + (1 - \alpha)k_t. \quad (1)$$

where  $f_t$  is the Solow residual,  $h_t$  is hours worked,  $k_t$  is the capital stock (all variables expressed in logarithms), and  $\alpha$  is the elasticity of output with respect to labour ( $0 < \alpha < 1$ ).

To achieve the decomposition  $y_t = \mu_t + \psi_t$ , the variables on the right hand side of equation (1) are broken down additively into their permanent (denoted by the superscript  $P$ ) and transitory (denoted

by the superscript  $T$ ) components, giving:

$$f_t = f_t^{(P)} + f_t^{(T)}, \quad h_t = h_t^{(P)} + h_t^{(T)}, \quad k_t = k_t^{(P)}. \quad (2)$$

It should be noticed that potential capital is always assumed to be equal to its actual value; this is so since capacity utilisation is absorbed in the cyclical component of the Solow's residual. Only survey based measures of capacity utilisation for the manufacturing sector are available for the euro area.

Hence, potential output is the value corresponding to the permanent values of factor inputs and the Solow residual, while the output gap is a linear combination of the transitory components:

$$\begin{aligned} \mu_t &= f_t^{(P)} + \alpha h_t^{(P)} + (1 - \alpha)k_t, \\ \psi_t &= f_t^{(T)} + \alpha h_t^{(T)}. \end{aligned} \quad (3)$$

Under perfect competition the output elasticity of labour,  $\alpha$ , can be estimated from the labour share of output. For the euro area the average labour share obtained from the national accounts (adjusted for the number of self-employed) is about 0.65.<sup>1</sup>

Hours worked can be separated into four components that are affected differently by the business cycle, as can be seen from the identity  $h_t = n_t + pr_t + er_t + hl_t$ , where  $n_t$  is the logarithm of working age population (i.e., population of age 15-64),  $pr_t$  is the logarithm of the labour force participation rate (defined as the ratio of the labour force over the working age population),  $er_t$  is that of the employment rate (defined here as the ratio of employment over the labour force), and  $hl_t$  is the logarithm of labour intensity (i.e., average hours worked). Each of these determinants is in turn decomposed into its permanent and transitory component in order to obtain the decomposition:

$$h_t^{(P)} = n_t + pr_t^{(P)} + er_t^{(P)} + hl_t^{(P)}, \quad h_t^{(T)} = pr_t^{(T)} + er_t^{(T)} + hl_t^{(T)}. \quad (4)$$

The idea is that population dynamics are fully permanent, whereas labour force participation, employment and average hours are also cyclical. Moreover, since the employment rate can be restated in

<sup>1</sup>Although the choice of a Cobb-Douglas production function with constant factor income shares is to some extent controversial and the evidence for the euro area in this respect is scarce, Willman (2002) provides some evidence in favour of such a production function for the euro area. See Musso and Westermann (2005) for adjusted estimates of the euro area labour share. The greatest advantage of the Cobb-Douglas specification is its additivity on the logarithmic scale.

terms of the unemployment rate, we can relate the output gap to cyclical unemployment and potential output to structural unemployment. As a matter of fact, the unemployment rate being one minus the employment rate, the variable  $cur_t = -er_t$  (the contribution of the unemployment rate, using a terminology due to Rünstler, 2002), is the first order Taylor approximation to the unemployment rate. Thus,  $cur_t^{(P)}$  can be assimilated to the NAIRU and  $cur_t^{(T)}$  to the unemployment gap.

As it is well known, there are several alternative ways of obtaining the trend components of the individual determinants; our approach will provide a parametric dynamic representation for the components and will relate them to nominal variables, prices and wages, so as to enforce the definition of potential output as the level that is consistent with stable inflation. The introduction of the nominal variables is essential for discriminating the permanent (supply) from the transitory (demand) variations. In our application we shall consider both the consumer price index and nominal wages, and relate their variation to the output and the unemployment gap, respectively.

## 2.2 The Multivariate Model

The multivariate unobserved components model for the estimation of potential output and the output gap, implementing the PFA outlined in the previous sub-section, is formulated in terms of the seven variables already mentioned

$$[f_t, hl_t, pr_t, cur_t, c_t, p_t, w_t]' = [y_t', p_t, w_t]'$$

The variable  $c_t$  is the logarithm of capacity utilisation. The variables are divided into two blocks. The first block defines the permanent-transitory decomposition of  $y_t = [f_t, hl_t, pr_t, cur_t, c_t]'$ , and yields potential output and the output gap according to the PFA. The second block is constituted by the price and wage equations, which relate underlying inflation to the output gap and nominal wages dynamics to the unemployment gap.

For  $y_t$ , we specify the following system of time series equations:

$$y_t = \mu_t + \psi_t + \Gamma X_t, t = 1, \dots, T, \quad (5)$$



where  $\mu_t = \{\mu_{it}, i = 1, \dots, 5\}$  is the  $5 \times 1$  vector containing the permanent levels of  $f_t, hl_t, pr_t, cur_t$ , and  $c_t$ ,  $\psi_t = \{\psi_{it}, i = 1, \dots, 5\}$  denotes the transitory component in the same series, and  $\Gamma X_t$  are fixed effects.

The permanent component is specified as a multivariate integrated random walk:

$$\Delta^2 \mu_t = \zeta_t, \quad \zeta_t \sim \text{NID}(\mathbf{0}, \Sigma_\zeta). \quad (6)$$

Here  $\Delta = 1 - L$  denotes the difference operator, and  $L$  is the lag operator, such that  $Ly_t = y_{t-1}$ ; NID stands for normally and independently distributed. It is assumed that the disturbance covariance matrix has rank 4. This restriction enforces the stationarity of  $c_t$  around a deterministic trend, possibly with a slope change, and amounts to zeroing out the elements of  $\Sigma_\zeta$  referring to  $c_t$ , and introducing a slope change variable in  $X_t$ . For more details about the trend in capacity see PMW.

The matrix  $X_t$  contains interventions that account for a level shift both in  $pr_t$  and  $cur_t$  in 1992.4, an additive outlier (1984.4) and a slope change in 1975.1 in capacity utilisation,  $c_t$ ;  $\Gamma$  is the matrix containing their effects.

The specification of second-order trends postulates that the underlying growth changes slowly over time if the size of  $\Sigma_\zeta$  is small compared to the variance of the cyclical components. PMW discuss some of the most relevant specification issues that arise with respect to the characterisation of the trend components in the variables under analysis and the isolation of the transitory component of unemployment rates and labour participation rates. The various specifications are compared in PMW on the grounds of their data coherency, predictive validity and the reliability of the corresponding output gap.

With respect to the cyclical components,  $\psi_{it}, i = 1, \dots, 5$ , among the various alternative specifications considered by PMW, in this paper we adopt the pseudo-integrated cycles model. The key aspect of this specification is that it is assumed that the cyclical component of each variable is driven by both the economy-wide business cycle and an idiosyncratic cycle. In particular, we take the cycle in capacity as the reference cycle, writing  $\psi_{5t} = \bar{\psi}_t$ , where  $\bar{\psi}_t$  is the stationary second order autoregressive process

$$\bar{\psi}_t = \phi_1 \bar{\psi}_{t-1} + \phi_2 \bar{\psi}_{t-2} + \kappa_t, \quad \kappa_t \sim \text{NID}(0, \sigma_\kappa^2). \quad (7)$$



The roots of the autoregressive polynomial are a pair of complex conjugates. This restriction is enforced by the following reparameterisation:  $\phi_1 = 2\rho \cos \lambda_c$ ,  $\phi_2 = -\rho^2$ , with  $\rho \in (0, 1)$  and  $\lambda_c \in [0, \pi]$ . For the cycle in the  $i$ -th variable ( $i = 1, 2, 3, 4$ ), where  $i$  indexes  $f_t, hl_t, pr_t, cur_t$ ,

$$\psi_{it} = \rho_i \psi_{i,t-1} + \theta_i(L) \bar{\psi}_t + \kappa_{it}, \quad \kappa_{it} \sim \text{NID}(0, \sigma_{\kappa,i}^2) \quad (8)$$

where  $\kappa_{it}$  is an idiosyncratic disturbance,  $\rho_i$  is a damping factor. We refer to (8) as a *pseudo-integrated cycle*. It encompasses several leading cases of interest:

1. If  $\theta_i(L) = 0$ , it defines a fully idiosyncratic AR(1) cycle with autoregressive coefficient  $\rho_i$  and disturbance variance  $\sigma_{\kappa,i}^2$ .
2. If  $\rho_i = 0$  the  $i$ -th cycle has a common component and a white noise idiosyncratic one, that is  $\psi_{it} = \theta_i(L) \psi_t + \kappa_{it}$ .
3. If  $\rho_i = 0$  and  $\sigma_{\kappa,i}^2 = 0$  the  $i$ -th cycle reduces to a model with a common cycle, that is  $\psi_{it} = \theta_i(L) \psi_t$ .

The rationale of (8) is that the cycle in the  $i$ -th series is driven by a combination of autonomous forces and by a common cycle; cyclical shocks, represented by  $\bar{\psi}_t$  are propagated to other variables according to some transmission mechanism, which acts as a filter on the driving cycle. As a result, the cycle  $\psi_{it}$  is more persistent, albeit still stationary, than  $\bar{\psi}_t$ . This framework is particularly relevant for extracting the cycle from the labour variables.

We are now capable of defining potential output and the output gap as linear combinations of the cycles and trends in (5):

$$\mu_t = [1, \alpha, \alpha, -\alpha, 0]' \boldsymbol{\mu}_t + \alpha n_t + (1 - \alpha) k_t; \quad \psi_t = [1, \alpha, \alpha, -\alpha, 0]' \boldsymbol{\psi}_t.$$

The specification of the model is completed by two structural equations for prices and wages,  $p_t$  and  $w_t$ , respectively, linking the changes in these two nominal variables to  $\psi_t$  and the unemployment gap  $\psi_{4t}$  respectively. The measurement equation is specified as follows:

$$\begin{aligned} p_t &= \mu_{pt} + \gamma_t + \delta_C(L) compr_t + \delta_N(L) neer_t \\ w_t &= \mu_{wt} + \theta_{lp} lp_t + \delta_T(L) ttrade_t \end{aligned} \quad (9)$$

where  $\mu_{pt}$  and  $\mu_{wt}$  are the underlying levels of prices and wages, which are specified below,  $\gamma_t$  is a seasonal component affecting prices, which has a trigonometric representation, see Harvey (1989),  $compr_t$  and  $neer_t$  refer to a commodity price index and the nominal effective exchange rate of the euro, respectively. Nominal wages are a function of labour productivity,  $lp_t = y_t - h_t$ , which can be expressed in terms of the unobserved components as  $lp_t = [1, (\alpha - 1), (\alpha - 1), 1 - \alpha, 0]' \mu_t + (\alpha - 1)n_t + (1 - \alpha)k_t + [1, (\alpha - 1), (\alpha - 1), 1 - \alpha, 0]' \psi_t$ , and a variable measuring terms of trade,  $ttrade_t$ , defined as the difference between the euro area GDP deflator and the deflator of imports. The lag polynomials in (9) are given respectively by  $\delta_C(L) = \delta_{C0} + \delta_{C1}L$ ,  $\delta_N(L) = \delta_{N0} + \delta_{N1}L$  and  $\delta_T(L) = \delta_{T0} + \delta_{T1}L$ .

The dynamic specification for the unobserved components  $\mu_{pt}$  and  $\mu_{wt}$  is the following:

$$\begin{aligned} \mu_{pt} &= \mu_{p,t-1} + \pi_{p,t-1} + \eta_{pt}, & \begin{pmatrix} \eta_{pt} \\ \eta_{wt} \end{pmatrix} &\sim \text{NID} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{p,\eta}^2 & \sigma_{pw,\eta} \\ \sigma_{pw,\eta} & \sigma_{w,\eta}^2 \end{pmatrix} \right], \\ \mu_{wt} &= \mu_{w,t-1} + \pi_{w,t-1} + \eta_{wt}, & \\ \pi_{pt} &= \pi_{p,t-1} + \theta_p(L)\psi_t + \zeta_{pt}, & \begin{pmatrix} \zeta_{pt} \\ \zeta_{wt} \end{pmatrix} &\sim \text{NID} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{p,\zeta}^2 & \sigma_{pw,\zeta} \\ \sigma_{pw,\zeta} & \sigma_{w,\zeta}^2 \end{pmatrix} \right]. \\ \pi_{wt} &= \pi_{w,t-1} + \theta_w(L)\psi_{4t} + \zeta_{wt}, & \end{aligned} \quad (10)$$

The price equation is a generalisation of the univariate Gordon triangle model (Gordon, 1997), featuring three essential ingredients: an exogenous component driven by the nominal effective exchange rate of the euro and commodity prices, inflation *inertia* associated with the unit root in inflation and its MA(1) feature, and the presence of demand shocks, as  $\pi_{pt}$  depends dynamically on the current and past values of the output gap, via the lag polynomial  $\theta_\pi(L)$ . The wage equation helps in identifying the NAIRU via a Phillips curve relationship, which links nominal wages to labour productivity, prices and the unemployment gap,  $\psi_{4t} = cur_t^{(T)}$ .

The components  $\pi_{pt}$  and  $\pi_{wt}$  represent core prices and wages inflation. It is assumed that the disturbances are mutually independent and independent of any other disturbance in the output equations, so that the only link between the nominal variables and the output equations is due to the presence of the output gap as a determinant of inflation, and the unemployment gap as a determinant of wage change; the order of the lag polynomials  $\theta_p(L)$  and  $\theta_w(L)$  is one, and we write  $\theta_p(L) = \theta_{p0} + \theta_{p1}L$ ,  $\theta_w(L) = \theta_{w0} + \theta_{w1}L$ . The equations are related via the cross-correlations of the disturbances driving

the underlying prices and wages.

Ignoring the seasonal component in prices, the reduced form of the equations (9)-(10) is:

$$\begin{aligned}\Delta^2 p_t &= \theta_p(L)\psi_{t-1} + \delta_C(L)\Delta^2 compr_t + \delta_N(L)\Delta^2 neer_t + \xi_{pt} \\ \Delta^2 w_t &= \theta_w(L)\psi_{4,t-1} + \theta_{lp}\Delta^2 lp_t + \delta_T(L)\Delta^2 ttrade_t + \xi_{wt}\end{aligned}\quad (11)$$

where the bivariate random vector  $(\xi_{pt}, \xi_{wt}) = (\zeta_{p,t-1} + \Delta\eta_{pt}, \zeta_{w,t-1} + \Delta\eta_{wt})$  has a vector MA(1) representation.

Gordon (1997) stresses the importance of entering more than one lag of the output gap in the triangle model, which allows to distinguish between level and change effects; this follows from the decomposition  $\theta_p(L) = \theta_p(1) + \Delta\theta_p^*(L)$ . In our case  $\theta^*(L) = -\theta_{p1}$ ;  $\theta_p(1) = \theta_{p0} + \theta_{p1} = 0$  would imply that the output gap has only transitory effects on inflation. Long-run neutrality is a testable restriction. The same applies to the lag polynomial  $\theta_w(L)$ .

### 3 The empirical analysis

#### 3.1 Database description

The time series used in this paper, listed in table 1, are quarterly data for the euro area covering the period from the first quarter of 1970 to the fourth quarter of 2005. As far as possible euro area wide data are drawn from official sources such as Eurostat or the European Commission. Historical data for euro area-wide aggregates were largely taken from the Area-Wide Model (AWM) database (see Fagan, Henry and Mestre, 2001).

The plot of the series is available from figure 1. All the series are seasonally adjusted except for  $p_t$  and  $compr_t$ . Residual seasonal effects were detected for the labour market series, especially  $cur_t$ ;  $pr_t$  and  $cur_t$  are subject to a downward level shift in the fourth quarter of 1992, consequent to a major revision in the definition of unemployment.

The series on hours worked,  $h_t$ , results from the interpolation of the euro area aggregate annual time series derived from the country data of the Total Economy Database of The Conference Board and Groningen Growth and Development Centre (January 2006 vintage; for Germany, data before

1991 were approximated on the basis of the growth rates of data for West Germany). The quarterly series was estimated using the Fernández method using employment as an indicator variable (see Proietti (2006) for further details on this method).

The capital stock at constant prices is constructed from euro area wide data on seasonally adjusted fixed capital formation by means of the perpetual inventory method. As in Rünstler (2002) and PMW, we define the contribution of the unemployment rate ( $cur_t$ ) as minus the logarithm of the employment rate ( $er_t$ ).  $cur_t$  enables modelling the natural rate of unemployment without breaking the linearity of the model, the only consequence for the measurement model being a sign change in (4) <sup>2</sup>.

Seasonally adjusted survey based rates of capacity utilisation in manufacturing were obtained from the European Commission starting from 1980.1 and self compiled (GDP-weighted average of available national indices) for previous years. The logarithm of capacity utilisation in the manufacturing sector,  $c_t$ , is slightly trending. The evidence arising from the Buseti and Harvey (2001) test is that we cannot reject stationarity when the trend is linear and subject to a level shift and slope break occurring in 1975.1.

### 3.2 Estimation results

The model is estimated by maximum likelihood using the support of the Kalman filter. Estimation and signal extraction were performed in Ox 3.3 (Doornik, 2001) using the Ssfpack library, version beta 3.2; see Koopman, Doornik and Shephard (1999). The maximum likelihood estimate of the covariance matrix of the trend disturbances resulted

$$10^7 \cdot \tilde{\Sigma}_{\zeta} = \begin{bmatrix} 3.260 & -0.383 & 0.091 & -0.731 & 0.000 \\ -2.555 & 13.640 & -0.793 & -0.175 & 0.000 \\ 0.283 & -5.057 & 2.980 & 0.250 & 0.000 \\ -2.568 & -1.257 & 0.840 & 3.789 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$$

<sup>2</sup>Denoting with  $U_t$  the unemployment rate, then  $cur_t = -\ln(1 - U_t) \approx U_t$  is the first order Taylor approximation of the unemployment rate.

(the upper triangle reports correlations). The estimated cycle in capacity is

$$\begin{aligned} \bar{\psi}_t = & 1.62 \quad \bar{\psi}_{t-1} - 0.71 \quad \bar{\psi}_{t-2} + \kappa_t, \quad \kappa_t \sim \text{NID}(0, 408 \times 10^{-7}), \\ & (.02) \quad \quad \quad (.03) \end{aligned}$$

and implies a spectral peak at the frequency 0.28 corresponding to a period of about five to six years. The specific damping factors,  $\rho_i$ , are large for  $pr_t$  and  $cur_t$  (0.94 and 0.89, respectively), for the Solow's residual  $f_t$  we have  $\rho_1 = 0.42$ , whereas  $\rho_2$ , associated to  $hl_t$  is not significantly different from zero.

Table 2 reports the parameter estimates of the loadings and the pseudo-integrated cycles parameters. The table also reports the LjungBox test statistic, using four autocorrelations, computed on the standardised Kalman filter innovations, and the Bowman and Shenton (B-S, 1975) normality test. Significant residual autocorrelation is detected for  $hl_t$ . It must however be remarked that the residual displays a highly significant lag 4 autocorrelation, which may as well be the consequence of the temporal disaggregation of hours worked.

All the loadings parameters are significant, with the exception of those for average hours worked,  $hl_t$ , for which the cyclical component has a very small amplitude. Among the possible explanations of this result, we cannot ignore that the series on hours worked was derived by disaggregating the annual series into a quarterly series, so that part of the short run variation of hours could not be recovered. The Solow residual and participation rates loads positively on the contemporaneous values of  $\bar{\psi}_t$  of the common cycle, whereas  $cur_t$  loads negatively, as expected.

The price equation has an excellent fit, and the output gap has a significant effect on underlying inflation. The Wald test of the restriction  $\theta_{\pi 0} + \theta_{\pi 1} = 0$  (long run neutrality of inflation to the output gap) is not significant. As a result, the change effect is the only relevant effect of the gap on inflation.

As for the wage equation, the null of long run neutrality  $\theta_w(1) = 0$  cannot be rejected, as the Wald test takes the value 2.59 with  $p$ -value 0.11. Changes in wages are negatively related to the unemployment gap in the short run. The estimated lag polynomial can be rewritten  $\tilde{\theta}_w(L) = 0.028 - 0.649\Delta$ , which makes it clear that the most relevant effect is the change effect, which is negative and takes the value -0.649; the level effect, 0.028, is not significantly different from zero.

The estimated covariances between the level and slope disturbances in equation (10) were respectively  $\tilde{\sigma}_{pw,\eta} = 10^{-7}$  (corresponding to a 0.05 correlation coefficient) and  $\tilde{\sigma}_{p,\eta} = 11 \times 10^{-7} = \tilde{\sigma}_{p,\eta}\tilde{\sigma}_{w,\eta}$ , i.e. we estimated a positive perfect correlation between the two disturbances.

The maximum likelihood estimates of the parameters associated to the exogenous variables are (standard error in parenthesis);  $\tilde{\delta}_{N0} = -0.035$  (0.010),  $\tilde{\delta}_{N1} = -0.016$  (0.010),  $\tilde{\delta}_{C0} = 0.005$  (0.003),  $\tilde{\delta}_{C1} = 0.008$  (0.003),  $\tilde{\delta}_{T0} = -0.002$  (0.018),  $\tilde{\delta}_{T1} = 0.034$  (0.018). While terms of trade has no significant effect on wages, the coefficients of the nominal effective exchange rate of the euro and commodity prices, which enter the prices equation, have the expected sign and are significant.

The potential output and output gap estimates are plotted in figure 2, along with the decomposition of potential output quarterly growth (at annual rates,  $400 \cdot \Delta\tilde{\mu}_t$ ) into its three sources (bottom right panel).

Figure 3 displays the smoothed estimates, obtained by the Kalman filter and smoother (see Durbin and Koopman, 2001) applied to the estimated state space model, of the NAIRU, that is the trend in  $cur_t$ ,  $\tilde{\mu}_{4t}$ , the unemployment gap,  $\tilde{\psi}_{4t}$ , and the core components of price and wage inflation,  $\tilde{\pi}_{pt}$  and  $\tilde{\pi}_{wt}$ , respectively. It is worth remarking that the estimates of the NAIRU and the unemployment gap appear to be fairly accurate, in the sense that the final estimation standard error is small compared to that presented for the US in Staiger, Stock, and Watson (1997a,b). Also, the amplitude of the unemployment gap is smaller than that of the output gap, as it should be expected. These results are consistent with the view that structural change in the labour market has been the main driving force of changes in the unemployment rate over the past four decades, as opposed to cyclical dynamics. As a result the largest portion of changes in the unemployment rate are estimated to be permanent, at the expense of the cyclical component. As regards the relatively limited width of the confidence bands, these findings are in line with previous studies which found that multivariate system estimates of the NAIRU tend to be significantly less uncertain compared to univariate or uniequational estimates (Schumacher, 2005).



## 4 The procyclicality of potential output estimates

### 4.1 Issues related to the procyclicality of potential output estimates

The smoothed estimates of potential output growth,  $400 \cdot \Delta \tilde{\mu}_t$ , displayed in the bottom left panel of figure 2, reveal that this component features a certain degree of short run volatility. If we further compare them with the estimated output gap, presented in the top right panel of the same figure, we notice a distinctive degree of concordance between them, especially with respect to the expansionary and recessionary patterns and the turning points. This behaviour, often referred to as the *procyclicality* of potential output growth estimates, may appear at odds with the implicit idea that the underlying factors driving it should change slowly over time or even change rarely, if at all. We shall argue that this is not the case.

Note that potential output was estimated as the component of production that has no effect on inflation and no smoothness prior was imposed on the representation of  $\mu_t$ , except for the fact that it is specified as an I(2) process such that no level disturbances are present. The variance parameters, which regulate the evolution of the components, were estimated by the maximum likelihood principle, so that in principle there is no guarantee that the resulting estimates are not procyclical. We mention in passing that the alternative trend specifications explored by PMW and in particular the damped slope specification, which featured I(1) trends, faced us the same procyclicality problem.

Procyclicality raises two related important issues that we address in the next sections: the first concerns the possibility of conducting a growth accounting analysis at a long-run temporal horizon; the second, which will be addressed in section 4.3, is whether potential output carries additional information that is relevant for explaining inflation.

As far as the first issue is concerned, we believe that nothing prevents from investigating potential output growth at different, usually longer, horizons; on the contrary, useful insight on the sources of growth can be obtained by such analyses, whose need and relevance is attested by a large number of attempts and different solutions.

There are various alternative ways of conducting the analysis; some of these (including the in-

roduction of smoothness priors) are discussed in Appendix A. The strategy that we propose in the next section consists of a novel application of the theory of model based band-pass filtering set forth in Gómez (2001), Kaiser and Maravall (2005) and Proietti (2007). Conditional on the maximum likelihood parameter estimates we address the issue of measuring potential output growth and its components at medium and long run horizons by embedding a band-pass decomposition of potential output in the model based framework and using optimal signal extraction principles. This has two important advantages: on the one hand, it is possible to assess the statistical reliability of the estimates, on the other, in the absence of model misspecification, there is an automatic adaptation of the signal extraction filters at the boundaries of the sample space, and consequently the estimates are not affected by what is customarily referred to as the "end of sample bias". As a result growth accounting at a long run horizon is a descriptive analysis that does not interfere and at the same time is not inconsistent with the estimation of the model, which embodies behavioural relationship between the real and nominal economic variables.

## 4.2 A model-based low-pass filtering of potential output

This section defines a class of low-pass filters for the separation of the long run movements in potential output growth. In particular, we propose a model based decomposition of the process  $\mu_t$  into a low-pass and a high-pass components, that enables to extract a smoothed potential output series (and the corresponding decomposition into the sources of growth) using standard optimal signal extraction principles. As a result the components can be estimated and their reliability assessed by the Kalman filter and smoother applied to the a modified state space model. The latter is observationally equivalent with respect to the parameters of the original structural form in section 2.

The starting point is the following decomposition of the multivariate white noise disturbance  $\zeta_t$ :

$$\zeta_t = \frac{(1 + L)^m \zeta_t^\dagger + (1 - L)^m \kappa_t^\dagger}{\varphi(L)}, \quad (12)$$

where  $m \geq 1$  is an integer whose value is chosen a priori, defining the order of the decomposition,  $\zeta_t^\dagger$  and  $\kappa_t^\dagger$  are two mutually and serially independent Gaussian disturbances,  $\zeta_t^\dagger \sim \text{NID}(\mathbf{0}, \Sigma_\zeta)$ ,  $\kappa_t^\dagger \sim$

$\text{NID}(\mathbf{0}, \lambda \Sigma_\zeta)$ , and the scalar polynomial  $\varphi(L)$  is such that:

$$|\varphi(L)|^2 = \varphi(L)\varphi(L^{-1}) = |1 + L|^{2m} + \lambda|1 - L|^{2m}, \quad (13)$$

where  $|1 - L|^m = (1 - L)^m(1 - L^{-1})^m$ ,  $|1 + L|^m = (1 + L)^m(1 + L^{-1})^m$ , and  $L^{-1}y_t = y_{t+1}$ .

The non negative scalar  $\lambda$ , chosen a priori, is the smoothness parameter which, along with  $m$ , defines uniquely the decomposition. The existence of the polynomial  $\varphi(L) = \varphi_0 + \varphi_1 L + \dots + \varphi_m L^m$ , satisfying (13), is guaranteed by the fact that the Fourier transform of the right hand side is never zero over the entire frequency range; see Sayed and Kailath (2001). The decomposition (12) was originally applied by Proietti (2007) to the innovations of a univariate time series; we now apply it to the multivariate disturbances of the trend component of the variables entering the production function.

According to (12) a multivariate white noise is decomposed into two orthogonal vector ARMA( $m, m$ ) processes with scalar ARMA polynomials and common AR factor, given by  $\varphi(L)$ . The decomposition (12) is illustrated by the left panel of figure 4. For a white noise process, the contribution of fluctuations defined at the different frequencies is constant. The high frequency components play the same role as low frequency ones. The rectangle with height 1 and base  $[0, \pi]$  can be thought of as the normalised spectral density of the univariate white noise disturbance  $\zeta_{it}, i = 1, \dots, N$ , that drives the potential output dynamics. According to the representation (6), the disturbance would be doubly integrated to form the level of potential output,  $\Delta^2 \mu_{it} = \zeta_{it}$ .

Replacing (12) in the trend equations  $\Delta \mu_t = \zeta_t$ , the process  $\mu_t$  can be correspondingly decomposed into orthogonal low-pass and high-pass components:

$$\mu_t = \mu_t^\dagger + \psi_t^\dagger,$$

where the components have the following representation:

$$\begin{aligned} \varphi(L)\Delta^2 \mu_t^\dagger &= (1 + L)^m \zeta_t^\dagger, & \zeta_t &\sim \text{NID}(\mathbf{0}, \Sigma_\zeta) \\ \varphi(L)\psi_t^\dagger &= \Delta^{m-2} \kappa_t^\dagger, & \kappa_t &\sim \text{NID}(\mathbf{0}, \lambda \Sigma_\zeta). \end{aligned} \quad (14)$$

The low-pass component,  $\mu_t^\dagger$ , has the same order of integration as  $\mu_t$  (regardless of  $m$ ); in particular, it is a vector ARIMA( $m, 2, m$ ) process with a scalar AR polynomial  $\varphi(L)$ , whereas the MA

component features  $m$  unit roots at the frequency  $\pi$ . The high-pass component,  $\psi_t^\dagger$ , has a stationary representation provided that  $m \geq 2$ , and will present  $m - 2$  unit roots at the zero frequency in the moving average representation. It should also be noticed that the covariance matrices of the low-pass and high-pass disturbances,  $\zeta_t^\dagger$  and  $\kappa_t^\dagger$ , are proportional,  $\lambda \geq 0$  being the proportionality factor. Obviously, if  $\lambda = 0$ ,  $\mu_t = \mu_t^\dagger$ . As  $\lambda$  increases, the smoothness of the low-pass component also increases, since a larger portion of high-frequency variation is removed.

For given values of  $\lambda$  and  $m$ , the decomposition (14) defines a new potential output disturbance that uses only the low frequencies whereas the remainder will contribute to the high-pass component. The spectral density of the disturbances of the low-pass component has two poles at the frequency  $\pi$ ; on the contrary, the spectral density of the high-pass component has two poles at the zero frequency. The normalised spectrum of the low-pass disturbance is plotted in the left panel of figure 4 for  $m = 2$  and for the values  $\lambda = 26065$  and  $\lambda = 1$ . The complement to one gives the normalised spectral density of the high-pass disturbance in (12).

The role of the smoothness parameter  $\lambda$  is better understood if we relate it to the notion of a cut-off frequency. For this purpose, it is useful to derive the analytic expression of the Wiener-Kolmogorov signal extraction filter for the low-pass component (Whittle, 1983). Assuming a doubly infinite sample, and denoting by  $\tilde{\mu}_t$  the minimum mean square estimators (MMSE) of  $\mu_t$ , the MMSE estimator of the low-pass component is

$$\tilde{\mu}_t^\dagger = w_\mu(L) \tilde{\mu}_t, \quad w_\mu(L) = \frac{|1 + L|^{2m}}{|1 + L|^{2m} + \lambda |1 - L|^{2m}} = \left[ 1 + \lambda \left( \frac{|1 - L|}{|1 + L|} \right)^{2m} \right]^{-1} \quad (15)$$

Hence, the estimator results from the application of a linear filter to the final estimates of the permanent components. The filter  $w_\mu(L)$  is known in the literature as an  $m$ -th order Butterworth filter, see Gómez (2001). It should be remarked that the estimator (15) is different from applying a low-pass filter to the original time series  $y_t$ . In finite samples the estimator  $\tilde{\mu}_t^\dagger$  is computed by the Kalman filter and smoother applied to the state space model with measurement equation  $y_t = \mu_t^\dagger + \psi_t^\dagger + \psi_t + \Gamma X_t$ .

Let  $w_\mu(\omega)$  denote the gain of the signal extraction filter in (15), where  $\omega$  is the angular frequency in radians takes values in the interval  $[0, \pi]$ . The gain is a monotonically decreasing function of  $\omega$ , with unit value at the zero frequency (being a low-pass filter it preserves the long run frequencies)

and with a minimum (zero, if  $m \geq 0$ ) at the  $\pi$  frequency. Let us then define the cut-off frequency of the filter as that particular value  $\omega_c$  in correspondence of which the gain halves. The parameter  $\lambda$  is related to the cut-off frequency of the corresponding signal extraction filter: solving the equation  $w_\mu(\omega_c) = 1/2$ , we obtain:

$$\lambda = \left( \frac{1 + \cos \omega_c}{1 - \cos \omega_c} \right)^m = \left[ \tan \left( \frac{\omega_c}{2} \right) \right]^{-2m}, \quad (16)$$

which expresses the parameter  $\lambda$  as a function of  $\omega_c$  and the order  $m$ . For interpretative purposes the cut-off frequency can be translated into a cut-off period,  $p = 2\pi/\omega_c$ , e.g.  $\omega_c = \pi/2$  implies that the filter selects those fluctuations with periodicity equal or greater than 4 observations (1 year of quarterly data).

In the sequel we concentrate on the case  $m = 2$ , i.e. on the class of Butterworth filters of order 2. Increasing  $\lambda$  we obtain smoother estimates, as, for given values of  $m$  and  $n$ , the cut-off frequency of the filter decreases, and the amplitude of higher frequency fluctuations is further reduced.

The gain of the filter (15) is presented in the right panel of figure 4 for  $m = 1, 2, 3$  and for two different cut-offs; the first is  $\pi/2$ , which corresponds to a period of 4 observations (one year of quarterly data) and the second is  $\pi/20$ , corresponding to 10 years of quarterly data. For higher values of  $m$  we have a sharper transition from 1 to zero. However, as argued in Proietti (2007), the flexibility of the filter is at odds with the reliability of the estimates. The analytical expression of the gain is the following:

$$w_\mu(\omega) = \left\{ 1 + \left[ \frac{\tan(\omega/2)}{\tan(\omega_c/2)} \right]^{2m} \right\}^{-1},$$

and depends solely on  $m$  and  $\omega_c$ . As  $m \rightarrow \infty$  the gain converges to the frequency response function of the ideal low-pass filter, that is

$$w_\mu(\omega) = \begin{cases} 1, & \omega < \omega_c \\ 1/2, & \omega = \omega_c \\ 0, & \omega_c < \omega < \pi \end{cases}$$

The weighting function (15) expresses the signal extraction filter given the availability of a two-sided infinite sample: the filter depends only on  $\lambda$  and  $m$ . On the contrary, at the boundary of the

sample the sample weights depend on the features of the series, i.e. are adapted to it, and are computed by the Kalman filter and smoother for state space representation of the model  $\mathbf{y}_t = \boldsymbol{\mu}_t^\dagger + \boldsymbol{\psi}_t^\dagger + \boldsymbol{\psi}_t + \boldsymbol{\Gamma}\mathbf{X}_t$ , augmented by the prices and wages equations. The main advantage of performing a model-based decomposition is that the no special treatment of the end values is necessary, since the optimal estimates of the components are automatically provided by the Kalman filter and smoother associated to the model featuring the band-pass components (14), whose state space representation can be constructed by using the results in Proietti (2007).

Applying the same univariate filter  $w_\mu(L)$  (using the same value of  $\lambda$ ) to the capital stock and population series, the low-pass component of potential output can then be defined as follows:

$$\tilde{\mu}_t^\dagger = [1, \alpha, \alpha, -\alpha, 0]' \tilde{\boldsymbol{\mu}}_t^\dagger + \alpha n_t^\dagger + (1 - \alpha) k_t^\dagger;$$

whereas the high-pass component is

$$\tilde{\psi}_t^\dagger = [1, \alpha, \alpha, -\alpha, 0]' \tilde{\boldsymbol{\psi}}_t^\dagger + \alpha(n_t - n_t^\dagger) + (1 - \alpha)(k_t - k_t^\dagger).$$

Figure 5 displays the estimates of the low-pass component of potential output growth, also decomposed according to its sources, for  $m = 2$  and two different values of the smoothness parameter,  $\lambda = 26065$ , corresponding to a cut-off frequency  $\omega_c = 0.16$ , and  $\lambda = 419631$ , corresponding to a cut-off frequency  $\omega_c = 0.08$ . The first defines a low-pass component retaining all the potential output fluctuations with a periodicity greater than 10 years, the second has a cutoff period of 20 years. The corresponding annualised potential output growth point estimates,  $400\Delta\tilde{\mu}_t^\dagger$  along with the 95% confidence intervals are displayed in the top and bottom left hand panels. As expected, the confidence intervals become wider as  $\lambda$  increases. Thus, the reliability of potential output growth estimates decreases as the smoothness increases (see Proietti, 2007).

As can be seen from the central panels of Figure 5, most components of potential growth exhibit a significant procyclicality, although with varying degree. This seems to be more pronounced for the trend components of the Solow residual (TFP) and the employment rate, and to a minor extent the participation rate. Thus, the procyclicality of potential output growth does not seem to arise from one specific factor and is on the contrary broadly based. Nevertheless, our extended framework allows



analysing developments in potential growth and its components from a longer term perspective, by abstracting from the shorter term dynamics. An illustration of such application to the case of the euro area is presented in section 5.

### 4.3 Does the procyclicality of potential output matter?

We turn now our attention to the second issue that was raised in section 4.1, concerning as to whether the procyclicality of the potential output estimates also implies that the latter contain information that is relevant for explaining price and wage inflation.

In section 4.2 potential output was decomposed into two parts: a low-pass component and a high-pass one. The intent was descriptive and aimed at extracting the component of potential output growth prevailing at a chosen horizon. The horizon depended upon a smoothness parameter,  $\lambda$ , or equivalently on a cut-off frequency, that is determined outside the model. For given  $m = 2$  and  $\lambda$ , and conditional on the maximum likelihood parameter estimates presented in section 3.2, the Kalman filter and smoother for the relevant state space model produces the MMSE of the components.

To address the above issue we propose to use the decomposition from a different perspective: the high-pass component arising from the combination of the levels of  $\psi_t^\dagger$  will be supposed to contribute to the output gap and thus will enter the prices and wages equations. The output gap and the unemployment gap will depend on the smoothness parameter  $\lambda$ , which is treated as an unknown parameter to be estimated along with the others, and within this framework we can address the issue of the optimal level of smoothness that characterises potential output to be compatible with stable price and wage inflation.

The multivariate model implementing the production function approach is now formulated as follows. The time series equations for  $\mathbf{y}_t$  are

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu}_t^\dagger + \boldsymbol{\psi}_t^\dagger + \boldsymbol{\psi}_t + \boldsymbol{\Gamma}\mathbf{X}_t, \\ \varphi(L)\Delta^2\boldsymbol{\mu}_t^\dagger &= (1+L)^2\boldsymbol{\zeta}_t^\dagger, & \boldsymbol{\zeta}_t &\sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\zeta) \\ \varphi(L)\boldsymbol{\psi}_t^\dagger &= \boldsymbol{\kappa}_t^\dagger, & \boldsymbol{\kappa}_t^\dagger &\sim \text{NID}(\mathbf{0}, \lambda\boldsymbol{\Sigma}_\zeta). \end{aligned} \tag{17}$$

The transitory component  $\boldsymbol{\psi}_t$  has the pseudo-integrated cycles specification discussed in section 2.2.

Potential output is defined as

$$\mu_t = [1, \alpha, \alpha, -\alpha, 0]' \mu_t^\dagger + \alpha n_t^\dagger + (1 - \alpha) k_t^\dagger,$$

whereas the output gap results from the linear combination:

$$\psi_t = [1, \alpha, \alpha, -\alpha, 0]' (\psi_t + \psi_t^\dagger) + \alpha(n_t - n_t^\dagger) + (1 - \alpha)(k_t - k_t^\dagger), \quad (18)$$

where it should be noticed that capital and population are no longer considered as entirely permanent. As a result, their high-pass components contribute to the output gap. Recall that in our case,  $m = 2$  implies that the high-pass component  $\psi_t^\dagger$  has a stationary VAR(2) representation with scalar AR polynomial,  $\varphi(L)$ . The model is completed by the two equations for prices and wages, which are specified as in section 2.2, with the output gap and the unemployment gap that are defined in (18).

In the new measurement framework the decomposition of output into potential and output gap depends on the parameter  $\lambda$ . The latter is identifiable and it can be estimated by maximum likelihood, along with the other parameters; the equations for the nominal variables will indicate what value is most likely, and what smoothness is required of potential output to be consistent with stable inflation. It must be remarked that for  $\lambda = 0$  the corresponding cut-off frequency is  $\omega_c = \pi$  and  $\mu_t^\dagger = \mu_t$  ( $\psi_t^\dagger = \mathbf{0}$ ) so that decomposition corresponds to the original one.

Table 3 reports the value of the likelihood as a function of the smoothness parameter (or, equivalently, of the corresponding cut-off frequency and period, reported in the second and third line), maximised with respect to the remaining parameters. Figure 6 plots the profile likelihood against the cut-off frequency  $\omega_c$ , and complements the evidence resulting from the table, since  $\omega_c$  is a monotonic non linear transformation of  $\lambda$  according to (16).

The value of  $\lambda$  maximising the likelihood is 18.59, corresponding to a cut-off frequency of 0.90 and a period of seven quarters. However, the likelihood is very flat for  $\lambda$  small, as it is evident also from figure 6, where the horizontal line is drawn at the maximum value of the likelihood minus the 95% percentile of a chisquare random variable with one degree of freedom. The values of  $\omega_c$  with respect to which the likelihood is below that threshold would be rejected according to the likelihood ratio test of the hypothesis that  $\omega_c$  equals that particular value. The likelihood ratio test of the hypothesis

$H_0 : \lambda = 0$  is not significant at the 1% level. Since the null hypothesis is that this parameter is on the boundary of the parameter space, the distribution of the likelihood ratio test statistic is the mixture  $LR = \frac{1}{2}\chi_0 + \frac{1}{2}\chi_1$ , where  $\chi_0$  takes the value zero with probability 1 and  $\chi_1$  is a chi-square random variable with one degree of freedom. The test with size  $a$  has critical region  $LR > c$ , where  $P(X > c) = 2a$ , and  $X \sim \chi_1$ . See Gouriéroux *et al.* (1982) for details. For  $a = 0.01$ ,  $c = 5.41$ , but the LR statistic is only 0.57.

In conclusion, the estimates of potential output resulting from our original model do not carry additional information that is relevant for explaining the behaviour of the nominal variables, although they have a procyclical appearance.

## 5 Stylised facts of potential output growth in the euro area based on the structural growth accounting approach

Using our extended framework presented in the previous sections, we provide in this section an application and a discussion of potential output growth developments and its main sources in the euro area since 1970.

The decomposition according to the various sources of potential output growth considered by the production function approach based on the low-pass filter for  $m = 2$  and two different values of the smoothness parameter,  $\lambda$  is presented in the central panels of Figure 5. As already mentioned, the two values of  $\lambda$  define low-pass components retaining all the potential output fluctuations with a periodicity greater than 10 years and greater than 20 years, respectively. Note that at each time  $t$  potential output growth results from the sum of the contributions of the Solow's residual ( $f_t$ ), capital stock ( $k_t$ ), hours per worker ( $hl_t$ ), the employment rate ( $er_t$ ), labour participation ( $pr_t$ ), and population growth ( $n_t$ ). The right hand panels also present the contribution of labour productivity and capital deepening (i.e., the change in capital per unit of labour), that can be derived from our model-based framework in terms of the smoothed estimates of low-pass components of output and labour.

According to the estimates shown in figure 5, the evolution of euro area potential output growth

since 1970 appears to have resulted from the combination of two forces working in opposite directions: the contributions to potential growth of the growth in labour productivity and working age population have been gradually decreasing, while the opposite pattern can be observed for the growth in labour utilisation (hours worked per head of the working age population).

The gradual increase in labour utilisation since the mid-1990s, reversing a steady deterioration over the preceding two decades, was undeniably in support of higher potential output growth. These more positive developments can at least partially be associated with successful labour market policies towards higher participation and a protracted period of wage moderation which gradually raised the rate of employment in the euro area. The contribution from labour utilisation however continued to be negatively affected for most of the period under analysis by the trend decline in average hours worked per person employed. From a long-run perspective, the contribution to growth from labour utilisation reflected similar trends in the contributions from more specific factors, average hours worked, the employment rate (or the contribution from the unemployment rate) and the participation rate. Average hours worked continued to decline, but the rate of decline gradually slowed on average and in the most recent years the trend level of average hours worked remained broadly unchanged or even increased slightly. However, these developments were partially compensated for by a stronger average increase in the trend participation rate (mainly driven by increases in the trend participation rate for women) and, during the most recent cycle, a stabilisation of the trend unemployment rate (NAIRU). As a result, during the most recent cycle the contribution of labour utilisation growth to potential growth became positive on average.

Over the last decade hourly labour productivity decelerated significantly, representing a major force causing a tendency towards lower potential output growth. For the first two decades under analysis this evidence is consistent with the "productivity growth slowdown" which followed the oil shocks of the 1970s. A possible interpretation of more recent developments is that the labour productivity growth slowdown over the last decade could largely be attributed to more robust job creation, supported by a sustained period of wage moderation and the impact of labour market reforms. In this respect, the slowdown in productivity growth could have resulted to some extent from a trade-off

with increased labour utilisation, as the latter mechanically induced a slower pace of capital deepening. Beyond the higher "job-intensity" of growth, however, other factors may have played a role in the slowdown of labour productivity growth. This view appears to be confirmed by analysis of estimates of the trends of the main components of labour productivity growth, discussed below. Despite favourable economic conditions, hourly labour productivity growth declined in the second half of 1990s as well as during the first half of the first decade of the new millennium (2001-2005). Developments over the last decade represent not only a downward shift from the first half of the 1990s, but also compared to average developments in the previous three decades. These developments stand in stark contrast to the corresponding ones for the US economy, which experienced a turning point in labour productivity growth trend, often associated with the widespread adoption of the advances in Information and Communication Technology (ICT). Not only did the euro area not experience such a positive turning point, but the gradual declining trend continued during the last decade, and possibly accelerated during the last five years. Labour productivity growth can usefully be decomposed into contributions from total factor productivity (TFP, defined as real output per unit of all -combined-inputs) and capital deepening (i.e., the increase in capital per unit of labour). It is often assumed that TFP, sometimes called equivalently multi-factor productivity and typically measured as the Solow residual, is a measure closer to capturing technological progress. However, TFP is a catch-all term that captures the impact of several factors, such that it is not immediate to associate its evolution to technological advances. Measurement problems imply that estimates of TFP growth and capital deepening are surrounded by significant uncertainty. For example, the lack of accurate measures of euro area capital and labour quality for a prolonged period of time implies that available estimates of TFP would also capture changes in factor quality. Nevertheless, available estimates suggest that the trend decline in labour productivity growth resulted from both lower trend capital deepening and lower trend TFP growth. As regards the more recent decade, the former can partly be associated with the robust pace of job creation since the mid-1990s, while the latter might be partly explained by higher utilisation of lower skilled workers. However, these declining trends can be observed since at least the 1970s. Moreover, available estimates of trend TFP growth do not point to a change in the underlying pattern in the most recent years.

## 6 Conclusions

The main purpose of this paper was to propose a number of improvements relating to the main empirical approach available in the literature to estimate and analyse potential output growth and illustrate these improvements with respect to the case of the euro area. This contribution can also be seen as proposing a structural approach to growth accounting. The reference framework adopted is a model-based approach: we specified and estimated a multivariate structural time series model embodying the decomposition of output according to the production function approach and two Phillips type relationships relating price and wage inflation to the output gap and the unemployment gap, respectively.

Typically, estimates of potential output growth based on this framework, as well as on simpler approaches, tend to exhibit a marked procyclical pattern, unless some smoothness prior is imposed. As shown in the application, this is the case also for the euro area. Against this background, one of the key contributions of the paper was to propose an extension of the abovementioned statistical framework allowing for a formal analysis of the degree of smoothness of the growth rate of potential output and its components. More precisely, we have proposed a model-based filtering approach for estimating potential output growth at different horizons, namely in the medium and long run. For this purpose the band-pass decomposition of potential output is embedded within the original parametric model so that we are able to estimate the underlying growth at any relevant horizon also in real time and to assess its reliability using standard optimal signal extraction principles. Finally, we provided a novel way of estimating the level of smoothness that is consistent with the definition of potential output and the NAIRU as those components of output and unemployment that exerts no inflationary pressure on prices and wages. The approach we propose has two important advantages. First, the signal extraction filters have an automatic adaptation property at the boundaries of the sample period, so that the real time estimates do not suffer from what is often referred to as "the end-of-sample bias". Second, it allows for an assessment of the uncertainty surrounding potential output growth estimates with different degrees of smoothness.



The application focused on the case of the euro area. Using our extended framework, we have provided a discussion of potential output growth developments and its main sources since 1970. Moreover, we illustrated to which extent the reliability of potential output growth estimates for the euro area decreases as the imposed degree of smoothness increases. A finding of the applied exercise was that the estimates of potential output resulting from our original model do not carry additional information that is relevant for explaining the behaviour of the nominal variables, although they have a procyclical appearance. Overall, the application makes clear that the proposed extended framework allows for a formal analysis of various key aspects of potential growth, thereby representing a potentially important methodological contribution in the empirical analysis of growth and its sources.

Interesting future related research includes both methodological extensions and further applications of the proposed approach. A potentially insightful methodological extension consists in adopting a sectoral perspective by estimating alternative production functions for the different sectors of the economy. It is likely that such perspective could shed some light on important issues such as the role of the changing sectoral composition of the economy in determining aggregate productivity developments. As regards further applications of the proposed extended framework, a key issue that could be analysed more formally is to what extent recent improvements in labour productivity growth observed in the euro area can be associated to underlying rather than temporary dynamics. However, in order to shed light on these important conjunctural issues such analysis would require not only the application of the proposed framework but also a thorough consideration of other issues, such as the relevance of various alternative data sets. These questions could be fruitfully addressed in future research based on the extended framework presented in this paper.

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## Appendix A - Approaches to deal with the procyclicality of potential output estimates

An approach often adopted to deal with the procyclicality of potential output estimates consists in introducing smoothness priors in the representation of the components of output. More precisely, to avoid this feature of (unrestricted) estimates the variance of disturbances driving the trend components is restricted, see for instance Gordon (1997), i.e. a smoothness prior is enforced, somewhat arbitrarily. The estimates obtained using the Hodrick-Prescott filter (Hodrick and Prescott, 1997, HP henceforth) follow suit. The limitation of this approach, an instance of which is the use of the HP filter, is that nothing guarantees that the corresponding output and unemployment gap estimates are relevant for explaining and forecasting the dynamics of prices and wages, i.e. they may lack content and predictive validity (see PMW for the issue of measuring predictive validity).

Another very popular strategy is that of smoothing the estimates obtained by the model, or the quarterly observed growth rates  $\Delta y_t$ , by running some unweighted moving average of current and past values, giving  $\frac{1}{h} \sum_{j=0}^{h-1} j \Delta \tilde{\mu}_{t-j} = \tilde{\mu}_t - \tilde{\mu}_{t-h}$ ; e.g.  $h = 4$  if the horizon chosen for the analysis is annual and the data are quarterly. While this strategy is amenable for its simplicity, it has also some drawbacks, such as relating the choice of  $h$  to a particular horizon, the asymmetric nature of the moving average filter, which induces a phase shift, thereby complicating the positioning of signals along time, and the assessment of the reliability of the estimates of underlying growth, which is not immediately available from the Kalman filter and smoother.

Yet another idea that will not be explored in this paper is to use multistep estimation, also known as adaptive estimation, which amounts to estimate the parameters of our model specified in section 2 by minimising the variance of the  $h$ -step-ahead prediction errors. If  $h$  is large, then the potential output notion that would be entertained is that consistent with stable inflation in the long run. See Proietti (2005) for univariate applications of this idea. Maximum likelihood estimation is close to minimising the one-step-ahead prediction error variance, and thus it aims at the parameter configuration which optimises the predictive performance in the short run.

The issue of estimating the medium- long-run component of GDP growth, by removing the fluctuations of period shorter or equal to one year, is tackled, from a perspective different from ours and using a different methodology, in Altissimo et al. (2006). See also Denis et al. (2002) on the definition of potential output at short, medium and long run horizons.

Table 1: Time series used for the estimation of potential output.

Series	Description	Transformation
$y_t$	Gross Domestic Product at constant prices	Log
$k_t$	Capital Stock at constant prices	Log
$h_t$	Hours worked, Total	Log
$l_t$	Employment, Total	Log
$hl_t$	Hours per worker	$(h_t - l_t)$
$f_t$	Total Factor Productivity	$(y_t - 0.65h_t - 0.35k_t)$
$pr_t$	Labour Force Participation Rate	Log
$er_t$	Employment rate	Log
$cur_t$	Contribution of Unemployment Rate	$(-er_t)$
$n_t$	Population	Log
$c_t$	Capacity Utilisation (Survey based)	Log
$p_t$	Consumer prices index	Log
$w_t$	Nominal wages	Log
$compr_t$	Commodity prices index (both oil and non-oil)	Log
$neer_t$	Nominal effective exchange rate of the euro	Log
$ttrade_t$	Terms of trade	Log

Table 2: Parameter estimates and diagnostics for the multivariate PFA model with pseudo-integrated cycles

Equation	$\tilde{\theta}_{i0}$	$\tilde{\theta}_{i1}$	$\tilde{\rho}_i$	$10^7 \cdot \tilde{\sigma}_{\kappa,i}^2$	$Ljung - Box$	$B - S$	
$f_t$	0.342 (.045)	-0.210 (.039)	0.40 (.11)	128	3.38	12.58	
$hl_t$	-0.003 (.013)	0.002 (.014)	0.45 (.962)	0	15.72	1.96	
$pr_t$	0.054 (0.015)	-0.030 (.015)	0.94 (.08)	14	7.43	2.65	
$cur_t$	-0.057 (0.012)	0.025 (0.012)	0.89 (.06)	0	4.84	3.53	
$c_t$	—	—	—	—	4.12	3.49	
	$\tilde{\theta}_{p0}$	$\tilde{\theta}_{p1}$	$10^7 \cdot \tilde{\sigma}_{p,\eta}^2$	$10^7 \cdot \tilde{\sigma}_{p,\zeta}^2$	$Ljung - Box$	$B - S$	
$p_t$	0.161 (.037)	-0.153 (.038)	60	12	3.84	5.11	
	$\tilde{\theta}_{w0}$	$\tilde{\theta}_{w1}$	$10^7 \cdot \tilde{\sigma}_{w,\eta}^2$	$10^7 \cdot \tilde{\sigma}_{w,\zeta}^2$	$\tilde{\theta}_{pl}$	$Ljung - Box$	$B - S$
$w_t$	-0.621 (.182)	0.649 (.181)	200	9	0.238 (.082)	13.34	5.96
Log Lik.	4469.58						

Table 3: Profile likelihood for the smoothness parameter.

$\lambda$	0.00	0.11	1.00	3.59	9.00	18.59	33.97	56.98
$\omega_c$	3.14	2.09	1.57	1.26	1.05	0.90	0.79	0.70
$p = 2\pi/\omega_c$	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00
log lik	4469.58	4469.71	4469.76	4469.84	4469.87	4469.87	4469.82	4469.76



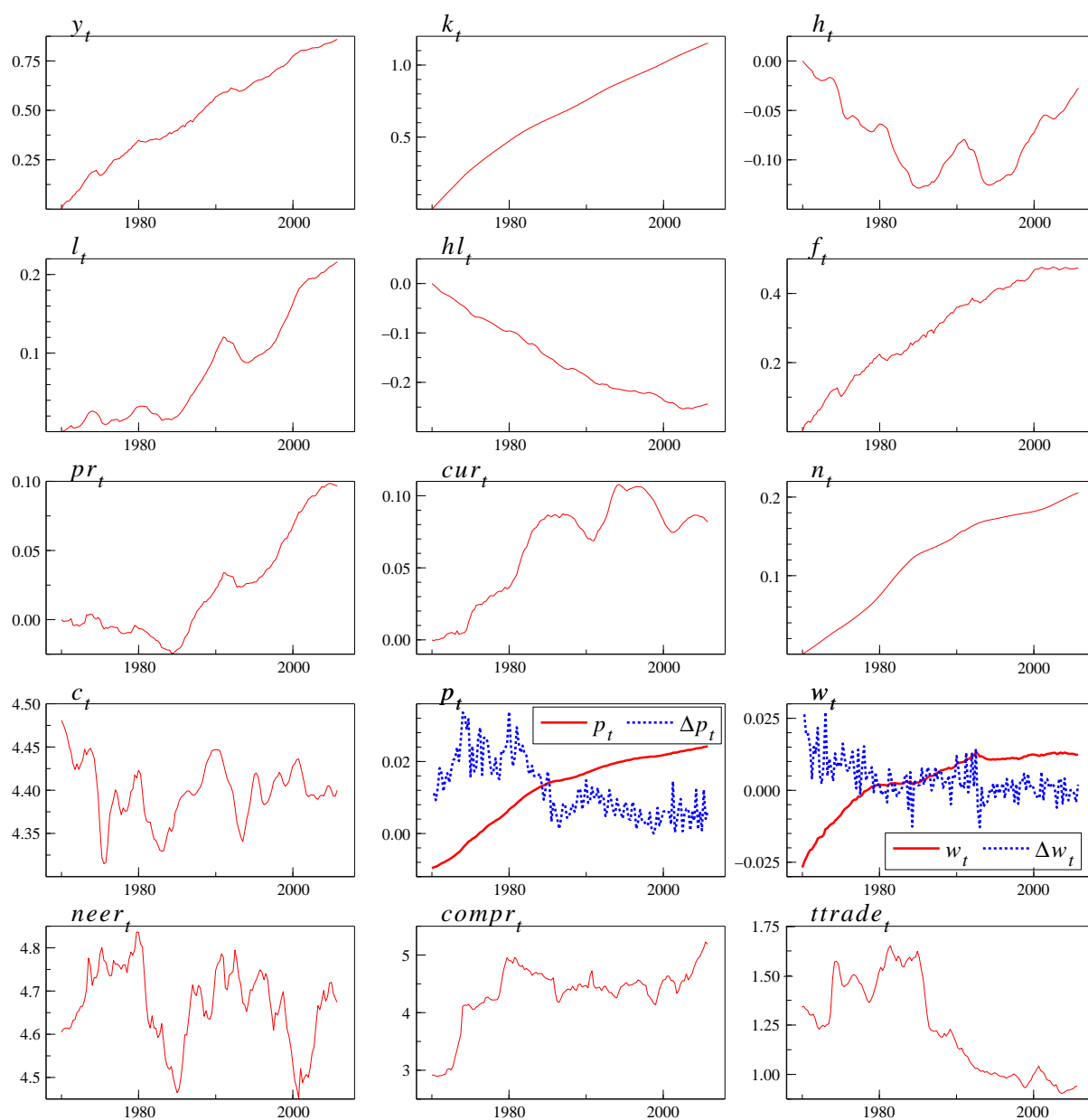


Figure 1: Plot of the available time series.

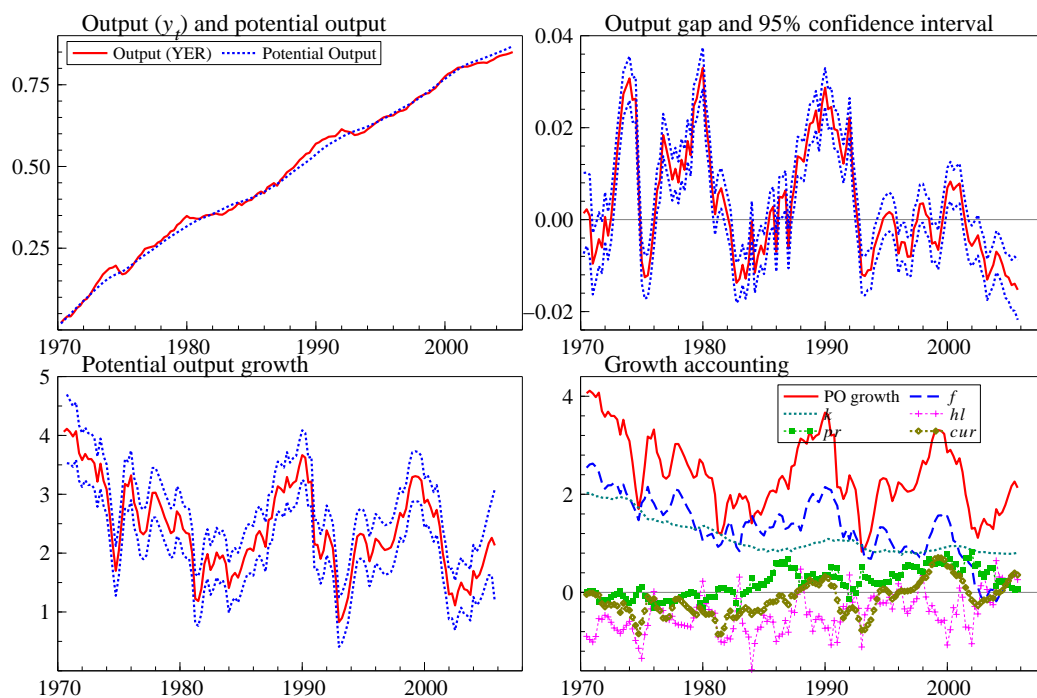


Figure 2: Multivariate structural PFA model. Smoothed components of potential output, output gap estimates (with 95% confidence bounds) and decomposition of potential output growth.

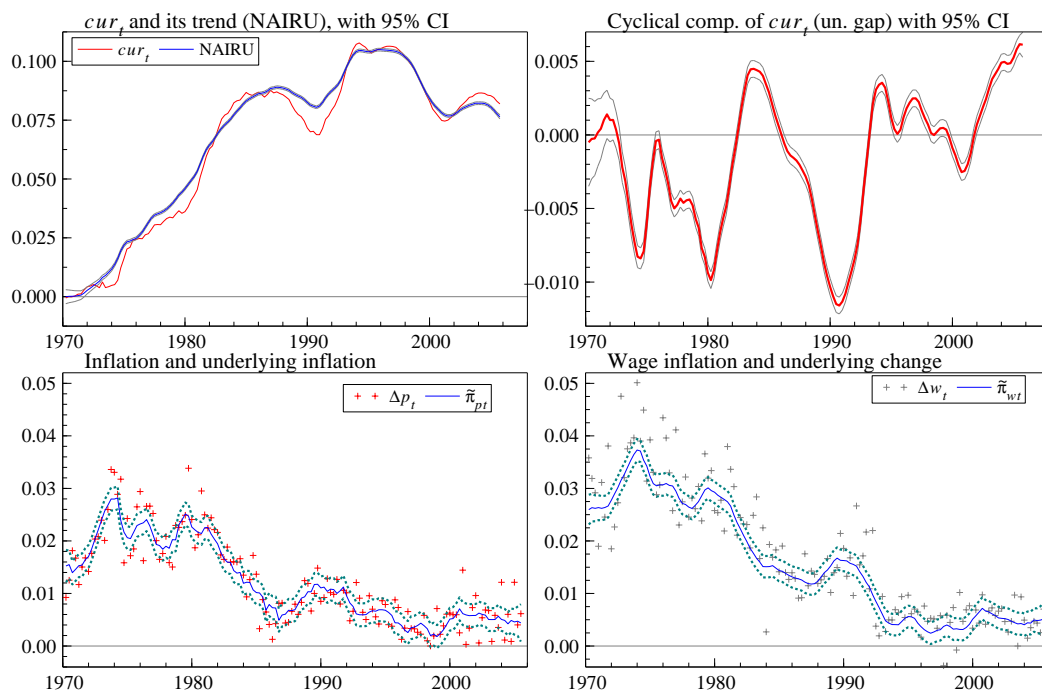


Figure 3: Multivariate structural PFA model. Smoothed estimates of NAIU and the unemployment gap (with 95% confidence bounds), core inflation and nominal wage growth.

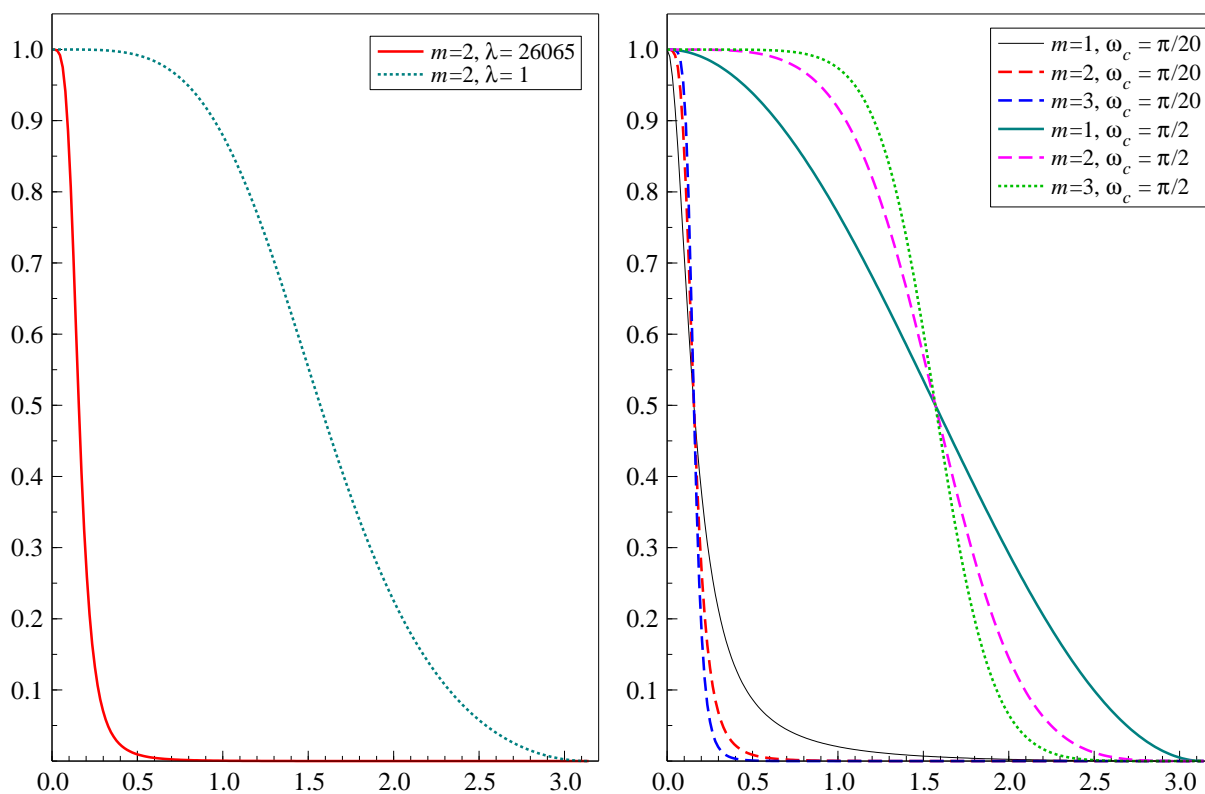


Figure 4: Gain of model-based low-pass filters for different cut-off frequencies and different values of  $m$ .

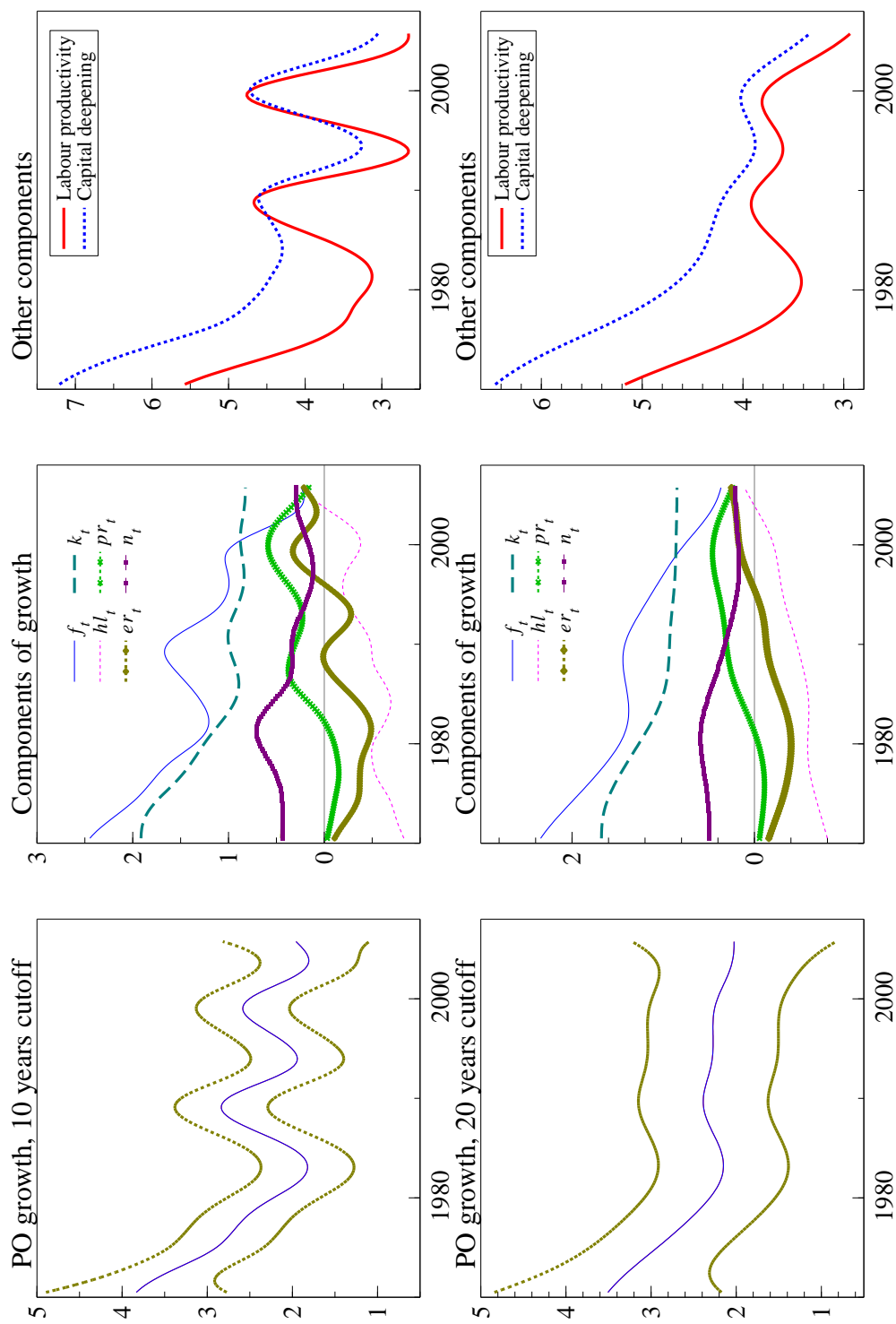


Figure 5: Low-pass estimates of potential output growth and growth accounting for various values of the smoothness parameter.

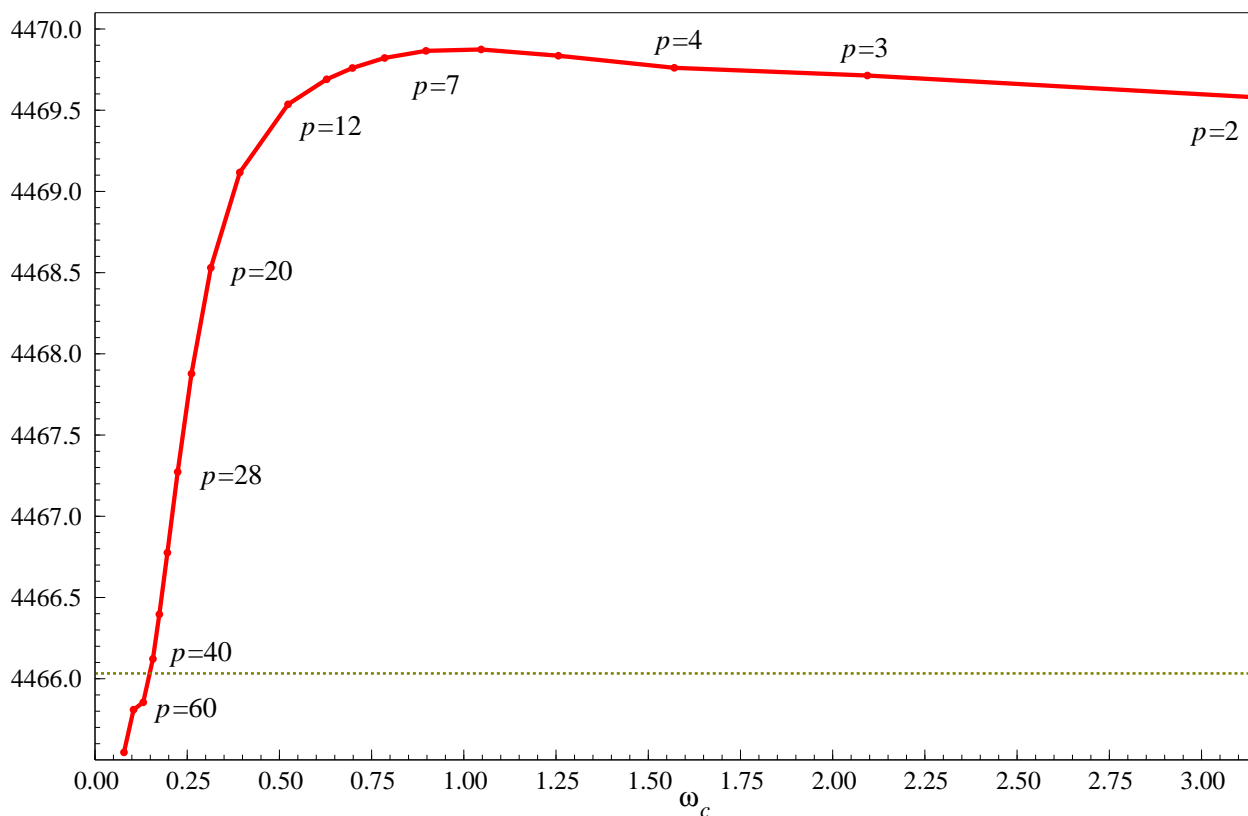


Figure 6: Profile likelihood for cut-off parameter  $\omega_c$ . The value  $p = 2\pi/\omega_c$  is the corresponding period in quarters.

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