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EUROPEAN CENTRAL BANK

## WORKING PAPER SERIES

NO 673 / SEPTEMBER 2006

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### **OPTIMAL MONETARY POLICY IN THE GENERALIZED TAYLOR ECONOMY**

by Engin Kara



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by Engin Kara



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## Abstract

In this paper we use the Generalized Taylor Economy (*GTE*) framework in which there are many sectors with overlapping contracts of different lengths to analyze the design of monetary policy. We derive a utility based objective function of a central bank for this economy and use it to evaluate the performance of alternative simple rules. We find that a simple rule that targets an index that gives more weight to the sectors which have longer contracts and are more important in the aggregate index yields a welfare outcome nearly identical to the optimal policy. However, we find that potential gains in targeting sector specific inflation rates rather than the aggregate inflation rate is very sensitive to the shape of the distribution. We show that except for the cases where prices/wages are reoptimized very frequently, the performance of the sectoral rule can be closely approximated by a simple rule that targets aggregate inflation.

*Keywords:* Inflation targeting, Optimal Monetary Policy.

*JEL:* E32, E52, E58.



## Non-Technical Summary

The implications of sectoral heterogeneity for the design of monetary policy is an important question for policy makers. While there is a growing literature assessing this question, attention has mainly focussed on models with two sectors. By using models with two sectors, many authors have argued that targeting the economy wide inflation is not optimal and can result in significant welfare losses. Instead, they suggest an inflation measure that puts more weight on the sector where there is a longer contract. However, this result may be misleading. The reason is that more realistic cases would require to go beyond the simple case of two-sector economies and instead consider economies in which there are many sectors with different contract lengths. Therefore, it remains an open question as to how monetary policy should be set in such an environment.

We address this question by using the Generalized Taylor Economy (*GTE*) set out in Dixon and Kara(2005b), in which there are potentially many sectors, each with a Taylor contract of a particular length. The unique feature of the *GTE* framework is that it allows us to model any distribution of contract lengths, including the one generated by the Calvo model. Following the literature, we start with the assumption that the central bank adopted an inflation targeting regime and use the *GTE* framework to extend this literature by considering the design of welfare-maximizing inflation targeting monetary policy rules in a setting where there are multiple sectors.

We first examine the monetary policy implications of alternative assumptions regarding the distribution of contract lengths and explore how to assign weights to different sectors in an index for a central bank to target. We then compare the performance of the aggregate inflation targeting relative to the sectoral rule and ask whether it is necessary for a well-designed monetary policy to respond to sector-specific inflations.

We also address the issue of welfare function in such a setting. We derive a utility based objective function of a central bank in our model by following the procedure described in Rotemberg and Woodford(1998) and illustrate the challenge facing the central bank in an environment in which there are many sectors. In particular, we show that welfare in the *GTE* depends on the variances of output gap and on the cross-sectional price dispersion in each sector. We find that in the *GTE* framework it is impossible for the central bank to simultaneously stabilize all the objectives and therefore the first-best allocation cannot be achieved. Given the existence of a trade-off,

under this objective function, we employ Lagrangian methods to determine the optimal policy and use it as a benchmark to evaluate the performance of alternative simple rules.

We find that a simple rule that responds to appropriately weighted average of the sectoral wage inflation rates yields a welfare outcome nearly identical to the optimal policy. However, we find that the result that "the optimal weight is an increasing function of the contract length" does not generalize beyond the case of two-sector economies. Instead, we show that sectoral share of the sector is as potentially important as the contract length in determining the optimal weight for that sector. More specifically, the optimal weight increases with the contract length but decreases if the sector's share is small. However, in sharp contrast to that of earlier studies, in which it is argued that targeting aggregate inflation can result in substantial welfare losses, we find that in a setting where there are multiple sectors the increase in expected welfare relative to the aggregate inflation targeting can be fairly small suggesting that it may not be necessary for a well-designed monetary policy to respond to sector-specific inflations.



# 1 Introduction

”How to deal with heterogeneity in a diversified currency area has been an issue at the heart of the debate in Europe all along the road towards deeper economic integration and already well before the single currency was introduced.”

Otmar Issing (2004)

What are the implications of sectoral heterogeneity for the design of monetary policy is an important question for policy makers. A rapidly growing literature assesses this question in DGE models which allows for two sectors, such as Woodford (2003) and Aoki (2001), or with two-countries such as Benigno (2004). By analyzing different inflation targeting policies in the presence of two sectors with different contract lengths and then ranking these policies using the welfare function based on the representative household’s utility function, these studies conclude that targeting the economy wide inflation is not optimal and can result in significant welfare losses. Instead, they suggest an inflation measure that puts more weight on the sector where there is a longer contract.

Woodford (2003) and the other authors who arrive at the same conclusion, however, use DGE models which have only two sectors. More realistic case would require to go beyond the simple case of two-sector economies and consider economies in which there are many sectors with different contract lengths<sup>1</sup>. Therefore, it remains an open question how monetary policy should be set in such a setting. In addition, much attention in this literature devoted to the models which assume Calvo style contracts in each sector<sup>2</sup>. As shown by Dixon and Kara (2005a), the Calvo model has a distribution of contract lengths which has important implications regarding the persistence properties and the welfare cost estimates of the model. This is because the presence of long contracts in the Calvo model leads to more persistence and price dispersion. This finding suggests that a model with Calvo distribution may overstate the gains from different inflation stabilization policies. As Kiley

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<sup>1</sup>One exception is the model by Mankiw and Reis (2002). However, Mankiw and Reis use a model in which nominal rigidities arise due to the assumption of sticky information rather than staggered contracts.

<sup>2</sup>This excludes the model by Erceg and Levin(2005), who focus on a model which has two sectors; each with Taylor style contracts. However, they only consider the special case in which sectors have identical contract lengths.

(2002) states "it may be preferable to make welfare comparisons in a model where the tail of the distribution is not so large (hence so important)."

Dixon and Kara (2005b) argue that Taylor's model can be generalized to allow for a distribution of contract lengths in different sectors which can be used to model any distribution of contract lengths, including the one generated by the Calvo model. In this paper, following the literature, we start with the assumption that the central bank adopted an inflation targeting regime and use the *GTE* framework to extend this literature by considering the design of welfare-maximizing inflation targeting monetary policy rules in a setting where there are multiple sectors. We examine the monetary policy implications of alternative assumptions regarding the distribution of contract lengths and explore how to assign weights to different sectors in an index for a central bank to target. We then compare the performance of the aggregate inflation targeting relative to the sectoral rule and ask whether it is necessary for a well-designed monetary policy to respond to sector-specific inflations.

We derive a utility based objective function of a central bank in our model by following the procedure described in Rotemberg and Woodford (1998) and illustrate the challenge facing the central bank in an environment in which there are many sectors. In particular, we show that welfare in the *GTE* depends on the variances of output gap and on the cross-sectional price dispersion in each sector. We find that in the *GTE* framework it is impossible for the central bank to simultaneously stabilize all the objectives and therefore the first-best allocation cannot be achieved. Given the existence of a trade-off, under this objective function, we employ Lagrangian methods to determine the optimal policy and use it as a benchmark to evaluate the performance of alternative simple rules.

We find that a simple rule that responds to appropriately weighted average of the sectoral wage inflation rates yields a welfare outcome nearly identical to the optimal policy. We find that the result that "the optimal weight is an increasing function of the contract length" does not generalize beyond the case of two-sector *GTEs*. Instead, we show that sectoral share of the sector is as potentially important as the contract length in determining the optimal weight for that sector. More specifically, the optimal weight increases with the contract length but decreases if the sector's share is small. However, in sharp contrast to that of earlier studies, in which it is argued that targeting aggregate inflation can result in substantial welfare losses, we find that in a setting where there are multiple sectors the increase in expected welfare relative to the aggregate inflation targeting can be fairly small sug-

gesting that it may not be necessary for a well-designed monetary policy to respond to sector-specific inflations.

The remainder of this paper is organized as follows. Section 2 presents the model and section 3 describes equilibrium dynamics. Section 4 derives a welfare function for a central bank based on the representative household's utility function. Section 5 characterizes optimal policy and Section 6 analyzes the implications of various assumptions regarding the distribution of contract lengths and compares the performance of alternative simple rules. Section 7 concludes.

## 2 The Model

This section outlines the Generalized Taylor Economy (*GTE*) set out in Dixon and Kara (2005b), in which there are potentially many sectors, each with a Taylor contract of a particular length. The *GTE* approach allows us to model any distribution of contract lengths, including the one generated by the Calvo model.

In the model economy, there is a continuum of firms  $f \in [0, 1]$ , each producing a single differentiated good combined to produce a final consumption good. The production of intermediate goods requires labour as the only input. Corresponding to the continuum of firms  $f$  there is a unit interval of household-unions. The economy is divided into many sectors where the  $i$ -th sector has a simple Taylor contract length of  $T = i$  periods. The share of each sector is given by  $\alpha_i$  with  $\sum_{i=1}^N \alpha_i = 1$ . Within each sector, each firm is matched with a firm-specific union and there are  $N_i$  cohorts of equal size. The share of each cohort  $j$  within the sector  $i$  is given by  $\lambda_{ij}$  where  $\sum_{j=1}^{N_i} \lambda_{ij} = 1$ . The interval of firm-unions corresponding to cohort  $j$  in sector  $i$  can formally be expressed as:  $f \in [\hat{\alpha}_{i-1} + \hat{\lambda}_{ij-1}\alpha_i, \hat{\alpha}_{i-1} + \hat{\lambda}_{ij}\alpha_i]$  where  $\lambda_{ij} = N_i^{-1}$  and  $\hat{\lambda}_{ij} = jN_i^{-1}$ . Since the cohorts are of equal size and there are as many cohorts as period,  $\alpha_i i^{-1}$  contracts are reset in each period in sector  $i$ . The representative household derives utility from consumption, real money balances and leisure. The representative household-union in each sector chooses the reset wage to maximize lifetime utility given labour demand and the additional constraint that nominal wages will be fixed for  $T_i$  periods in which the aggregate output and price level are given.

## 2.1 Firms

A typical firm in the economy produces a differentiated good which requires labour as the only input, with a CRS technology represented by

$$Y_{ft} = A_{it}L_{ft} \quad (1)$$

where  $a_{it} = \log A_{it}$  is a productivity shock in sector  $i$  and follows the  $AR(1)$  process:  $a_{it} = \rho_i a_{it} + \varepsilon_{it}$ .  $f \in [0, 1]$  is firm specific index.

Single differentiated good  $Y(f)$  is combined to produce a final consumption good  $Y$ . The production function here is  $CES$  with constant returns and corresponding unit cost function  $P$

$$Y_t = \left[ \int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

$$P_t = \left[ \int_0^1 P_{ft}^{1-\theta} df \right]^{\frac{1}{1-\theta}} \quad (3)$$

The demand for the output of firm  $f$  is given by

$$Y_{ft} = \left( \frac{P_{ft}}{P_t} \right)^{-\theta} Y_t \quad (4)$$

The firm chooses  $\{P_{ft}, Y_{ft}, L_{ft}\}$  to maximize profits subject to (4,??), yields the following solutions for price, output and employment at the firm level given  $\{Y_t, W_{ft}, P_t\}$ .

$$P_{ft} = \frac{\varepsilon}{(1+\tau)} \frac{W_{ft}}{A_{it}} \quad (5)$$

$$Y_{ft} = \left( \frac{\varepsilon}{(1+\tau)} \right)^{\theta} \left( \frac{W_{ft}}{A_{it}P_t} \right)^{-\theta} Y_t \quad (6)$$

$$L_{ft} = \left( \frac{\varepsilon}{(1+\tau)} \right)^{\theta} \left( \frac{1}{A_{it}} \right) \left( \frac{W_{ft}}{A_{it}P_t} \right)^{-\theta} Y_t \quad (7)$$

Where  $\varepsilon = \frac{\theta}{\theta-1}$  measures the markup.  $\tau$  denotes a subsidy to employment, which reflects our assumption that the labour income is subsidized in

order to eliminate monopolistic distortion<sup>3</sup>. Therefore, in the steady state of the model prices are equal to marginal cost, as in the perfectly competitive economy. Price is an effective markup (adjusted for subsidy) over marginal cost.

## 2.2 Household-Unions and Wage Setting

The representative household  $h$  has a utility function given by

$$U_h = E_t \left[ \sum_{t=0}^{\infty} \beta^t [U(C_{ht}) + V(1 - H_{ht})] \right] \quad (8)$$

where  $C_{ht}$ ,  $H_{ht}$  are household  $h$ 's consumption and hours worked respectively,  $t$  is an index for time,  $0 < \beta < 1$  is the discount factor, and  $h \in [0, 1]$  is the household specific index.

$$P_t C_{ht} + \sum_{s_{t+1}} Q(s^{t+1} | s^t) B_h(s^{t+1}) \leq B_{ht} + (1 + \tau_h) W_{ht} H_{ht} + \Pi_{ht} - T_{ht} \quad (9)$$

where  $B_h(s^{t+1})$  is a one-period nominal bond that costs  $Q(s^{t+1} | s^t)$  at state  $s^t$  and pays off one dollar in the next period if  $s^{t+1}$  is realized.  $B_{ht}$  represents the value of the household's existing claims given the realized state of nature.  $W_{ht}$  is the nominal wage,  $\Pi_{ht}$  is the profits distributed by firms and  $W_{ht} H_{ht}$  is the labour income.  $\tau_{hw}$  denotes the fixed rate at which labour income is subsidized. Finally,  $T_t$  is a lump-sum tax.

The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} u_{ct+1} \right) \quad (10)$$

$$\sum_{s_{t+1}} Q(s^{t+1} | s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \quad (11)$$

$$X_{it} = \frac{\varepsilon}{(1 + \tau)} \left[ \frac{E_t \sum_{s=0}^{T_i-1} \beta^s [V_L (1 - H_{t+s}) (K_{t+s})]}{E_t \sum_{s=0}^{T_i-1} \beta^s \left[ \frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s} \right]} \right] \quad (12)$$

<sup>3</sup>Similar assumption can be found in Erceg and Levin (2005), Huang and Liu (2005), Gali (2003).

Equation (10) is the Euler equation, (11) gives the gross nominal interest rate. Equation (12) shows that the optimal wage is a constant "mark-up" (given by  $\varepsilon$  which is adjusted for subsidy) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration  $s = t \dots t + T_i - 1$ . When  $T_i = 2$ , this equation reduces to the first order condition in Ascari (2000). As mentioned before,  $\tau$  denotes a subsidy to household-unions. Therefore, the steady state of the model satisfies the efficiency condition that the marginal rate of substitution equals the real wage, as in a perfectly competitive economy. Note that the index  $h$  is dropped in equations (10) and (12), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in every period ( $C_{ht} = C_t$ ).

## 2.3 The Government

The labour income subsidy is financed by lump sum taxes so that the government's budget is balanced every period. In particular,

$$T_t = \tau W_t H_t$$

We do not specify a particular policy at this stage since our objective is to find an optimal monetary policy in the economy. However, given any monetary policy, an equilibrium can be computed.

# 3 Equilibrium Dynamics

We consider an equilibrium in which each sector  $i$  is identified by the contract length  $T_i$  and a household-union in each sector is identified by the time at which it can set a new wage. We follow the standard approach of log-linearizing around the steady state of the model. We follow the notational convention that lower-case symbols represent log-deviations of variables from the steady state and variables with astericks denote the equilibrium value of variables under flexible wages.

## 3.1 The flexible wage equilibrium and the natural level

For comparative purposes, we first characterize the equilibrium of the model under flexible wages. As discussed earlier, with the assumption that the

labour income subsidy exactly offsets the distortions associated with the monopolistic competition, the equilibrium allocation under flexible wages and prices coincides with the efficient allocation, as in a perfectly competitive economy. Therefore, in what follows, we will henceforth referred to as the Pareto optimum<sup>4</sup>

Using equation (5) and aggregating for sector  $i$ , along with the fact that the marginal cost is a constant, we can obtain the solution for the sectoral real aggregate wage of each sector. The aggregate real wage can then be obtained by simply summing up the real aggregate wage (weighted by sector share  $\alpha_i$ ). By using the solution for the aggregate real wage and the optimal wage setting rule (12), we obtain:

$$\frac{V_L (1 - H_{t+s}^*)}{u_c(C_{t+s})} = \frac{W_t^*}{P_t^*} = \sum_{i=0}^N \alpha_i A_{it} \quad (13)$$

To solve for aggregate employment, we first use the equation (7) and aggregate for sector  $i$  to obtain sectoral employment. Given the solution for sectoral employment, aggregate employment is then obtained by summing up the sectoral employment (weighted by  $\alpha_i$ ):

$$L_t^* = \frac{Y^*}{\sum_{i=0}^N \alpha_i A_{it}} \quad (14)$$

The linearized version of (13) is given by

$$\eta_{ll} h_t^* + \eta_{cc} c_t^* = a_t \quad (15)$$

where  $a_t = \sum_{i=0}^N \alpha_i a_{it}$  is the weighted average of productivity across sectors,  $\eta_{cc} = \frac{-U_{cc}C}{U_c}$  is the parameter governing risk aversion,  $\eta_{ll} = \frac{-V_{ll}L}{V_l}$  is the inverse of the labour elasticity.

The linearized version of (14) is

$$l_t^* = y_t^* - a_t \quad (16)$$

Combining (15) and (16) gives the solution for the natural level of output,  $y_t^*$ ,

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<sup>4</sup>See Gali (2003) for a discussion.



$$y_t^* = \frac{(1 + \eta_{ll})}{(\eta_{ll} + \eta_{cc})} a_t \quad (17)$$

which implies that the natural level of output is a weighted average of productivity across sectors.

Finally, we can solve for the equilibrium real interest rate by using the household's intertemporal Euler equation (10) which is given by

$$rr_t^* = E_t y_{t+1}^* - y_t^* \quad (18)$$

where  $rr_t^* = r_t^* - E_t \pi_{t+1}^*$  denotes the real interest rate and  $\pi_t^*$  is the aggregate inflation rate.

### 3.2 The Sticky wage equilibrium

We now turn to characterize the sticky wage equilibrium of the economy. We render nominal variables such as wage level and price level as stationary by reexpressing them in terms of log-deviations from the aggregate price level. For example,  $\tilde{x}_{it}$  and  $\tilde{p}_{it}$  denote the logarithmic deviation of the reset wage and price level in sector  $i$  from the aggregate price level, respectively.

The linearized wage decision equation (12) for sector  $i$  is given by

$$\tilde{x}_{it} = \sum_{s=0}^{T_i-1} \Phi_k \pi_{t+s} + \frac{1}{\sum_{s=0}^{T_i-1} \beta^s} \left[ \sum_{s=0}^{T_i-1} \beta^s [\gamma \tilde{y}_{t+s} + \gamma y_{t+s}^* + (1 - \gamma) a_{it+s}] \right] \quad (19)$$

with

$$\gamma = \frac{(\eta_{cc} + \eta_{LL})}{(1 + \theta \eta_{LL})} \quad \Phi_k = \frac{\sum_{k=s+1}^{T_i-1} \beta^k}{\sum_{k=s}^{T_i-1} \beta^k} \quad (20)$$

Where  $\tilde{y}_t = y_t - y_t^*$  is the output gap,  $\pi_t$  is the aggregate inflation rate and  $\theta$  is the elasticity of substitution of consumption goods.

In each sector  $i$ , the sectoral inflation is related to the wage level in sector  $i$  through a relation of the form

$$\sum_{s=1}^{T_i-1} \sum_{j=s}^{T_i} \lambda_{ij} \pi_{it-s-1} = \sum_{j=1}^{T_i-1} \lambda_{ij} [\tilde{x}_{it-j} - \tilde{p}_{it-j-1} - a_{it}] \quad (21)$$

Where  $\tilde{p}_{ijt} = \tilde{x}_{it-j} - a_{it}$  is the logarithmic deviation of the price level in sector  $i$  cohort  $j$  from the aggregate price level<sup>5</sup>.

We then use the log-linearized version of the household's intertemporal Euler equation (10) and subtract the flexible-wage version to obtain the Euler equation in terms of output gap, which is given by

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \eta_{cc}^{-1} (r_t - E_t \pi_{t+1} - rr_t^*) \quad (22)$$

Using equation (4) and aggregating for sector  $i$  yields

$$y_{it} = \theta(p_t - p_{it}) + y_t \quad (23)$$

By using the fact that the linearized price level in the economy is the weighted average of the ongoing prices in the economy, we obtain the following identity:

$$\sum_{i=1}^N \alpha_i \tilde{p}_{it} = 0 \quad (24)$$

where  $\tilde{p}_{it}$  can also be expressed as

$$\tilde{p}_{it} = \tilde{p}_{it-1} + \pi_{it} - \pi_t \quad (25)$$

Using these identities, aggregate inflation can be expressed as

$$\pi_t = \sum_{i=1}^N \alpha_i (\tilde{p}_{it-1} + \pi_{it})$$

which implies that aggregate inflation depends on both sectoral lagged relative prices and inflation levels.

Finally, since nominal rigidities arise due to the assumption of staggered wages and prices are perfectly flexible in the model, wage inflation is given by

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<sup>5</sup>Note that in the sector in which wages are completely flexible, that is  $i = 1$ , wage decision does not depend on the inflation level, as wages can adjust every period. Therefore, in the flexible wage sector, (21) reduces to

$$\tilde{p}_{1t} = \tilde{x}_{1t} - a_{1t}$$

$$\pi_{it}^w = \pi_{it} + \Delta a_{it} \quad (26)$$

where  $\pi_{it}^w$  denotes the wage inflation in sector  $i$  and  $\Delta a_{it} = a_{it} - a_{it-1}$ .

## 4 The welfare function for the *GTE*

This section generalizes the analysis of Rotemberg and Woodford (1998) and derives a utility based objective function of a central bank to provide a benchmark for evaluating the performance of alternative inflation targeting monetary policy rules. The welfare function is given by the sum of all households' utility function:

$$W_t = \sum_{t=0}^{\infty} \beta^t U_t$$

where

$$U_t = \left[ U(C_t) + \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} V(1 - H_{ijt}) \right]$$

Consumption is identical across all households in every period, which reflects our assumption of complete contingent, whilst labour can vary across cohorts.

As shown in the appendix, by taking the second-order logarithmic approximation to this utility function around a steady state, welfare function can be expressed as

$$W = \sum_{t=0}^{\infty} \beta^t U_t = -\frac{U_c(C)C}{2} \sum_{t=0}^{\infty} \beta^t L_t \quad (27)$$

where the loss function is given by

$$L_t = \left[ (1 + \eta_{LL}) \tilde{y}_t^2 + \theta \sum_{i=1}^T \sum_{j=1}^{T_i} \alpha_i \lambda_{ij} \text{var}_{ij} \tilde{p}_{ijt} \right]$$

where  $\text{var}_{ij}(\cdot)$  denotes the unconditional variance. This expression implies that welfare loss depends on the variance of the output gap and on the magnitude of the cross-sectional dispersion in prices in the economy, as in

the case of a standard one sector model. In fact, when  $T_i = 4$ , the function reduces to the welfare function in a standard one sector model, as in Paustian (2005). In the case of the *GTE*, however, in contrast to the one sector model, where there is only one type of contract length, the central bank also cares about the variability of price dispersion in different sectors where sectors differ from their contract length and their budget share. The share of each sector determines how much the central bank should care about the variability of price dispersion in that sector and the contract length determines the magnitude of the variability of price dispersion in that sector.

Note that the welfare function is roughly analogous to those obtained under the popular specification of Calvo contracts. In the case of Calvo contracts, as discussed in Dixon and Kara (2005a), there is a distribution of contract lengths from 1 to infinity. Therefore, a *GTE* can be setup to give exactly the same distribution of contracts as in the Calvo model, where the share of sector,  $\alpha_i$ , is the same as generated by the Calvo model. Although, they have exactly the same distribution of contracts, there is one main difference between the Calvo and the Calvo-*GTE* welfare functions. In the case of Calvo contracts, in each period there is one reset wage and therefore the welfare function reduces to

$$W = -\frac{U_c(C)C}{2} \left[ (1 + \eta_{LL})\tilde{y}_t^2 + \theta \sum_{i=1}^T \alpha_i \text{var}_i \tilde{p}_{it} \right]$$

where

$$N = \infty \quad T_i = i \quad \alpha_i = \omega (1 - \omega)^{i-1} \quad i = 1 \dots \infty$$

As shown by Woodford (2003), the welfare costs of cross sectional dispersion can be summarized in terms of variability of inflation.

However, in the Calvo-*GTE* there is a distribution of sector specific reset wages,  $\tilde{x}_{it}$ , in each period and therefore in addition to the distribution of prices across cohorts as in the Calvo model, the *GTE* has a distribution across sectors within the cohort. Therefore, as pointed out by Erceg and Levin (2005), the welfare costs of cross sectional dispersion cannot be summarized in terms of variability of inflation and must be given explicitly in terms of variances of relative prices.

## 5 Optimal Monetary Policy in the *GTE*

We turn now to examine the issue of optimal monetary policy. We begin with considering if the Pareto optimal allocation can be achieved in the *GTE*. As the welfare function shows, an equilibrium allocation is Pareto optimal in the *GTE* only if the relative prices and output gap is zero in every period, that is  $\tilde{y}_t = \tilde{p}_{it} = 0$  for all  $t$ .

We demonstrate that in the *GTE*, it is impossible to satisfy all the stabilization objectives at the same time and therefore Pareto optimal allocation is not attainable. This is most easily seen by considering the sector in which the wages are allowed to adjust in every period, where sector  $i = 2 \dots n$  have contract lengths  $i$ . In particular, by combining (19) and (21), the relative prices in flexible wage sector can be expressed as

$$\tilde{p}_{1t} = \gamma \tilde{y}_t + \gamma(a_t - a_{1t}) \quad (28)$$

A trade-off arises because of the final term  $\gamma(a_t^* - a_{1t})$ . Consider, for example, the case when  $\tilde{y}_t = 0$ . When  $\tilde{y}_t = 0$ , (28) implies that  $\tilde{p}_{1t} = \gamma(a_t^* - a_{1t})$ , which is inconsistent with the requirement that  $\tilde{p}_{1t} = 0$  unless there is only one shock in the economy which hits the flexible wage sector only.

This simple example illustrates the challenge which the central bank faces in an environment where there are many sectors with different contract lengths.

What happens more generally? As the special case already suggests, in the *GTE* framework both output gap and sectoral relative prices fluctuate in response to the shocks and these fluctuations in output gap and relative prices lead to a trade-off. Therefore, it cannot be possible to satisfy all the stabilization objectives at the same time.<sup>6</sup>

### 5.1 Second-Best Optimal Policy

As the discussion in the previous section reveals, in the *GTE* it is not possible to achieve the first best allocation. Accordingly, in this case one must consider second best optimal policy. We use numerical methods to characterize

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<sup>6</sup>To obtain such a tradeoff, one approach taken in the literature has been to add an exogenous shock, which is referred to as "cost-push shock", to the optimal wage setting rule. In the *GTE*, policy tradeoff arises endogenously.

second best optimal monetary policy. In particular, we compute the optimal policy that can be obtained by maximizing the welfare level defined in (27) subject to the equilibrium conditions (19) - (26). While this is an useful reference, as discussed in Huang and Liu (2005), it is difficult to implement, as it requires the knowledge of leads and lags of the inflation rates and the output gap. Therefore, we use the central bank's first order conditions along with the equilibrium conditions for the model to solve and calculate the level of welfare under optimal monetary policy. We then use it as a benchmark to compare the performance of alternative simple rules with the coefficients in front of the targeting variables are chosen optimally to maximize welfare, which are considered as feasible and effective tools to implement monetary policy.

## 5.2 Alternative Simple Policy Rules

We assume that the central bank has adopted an inflation targeting regime. We consider two policy rules, which are as follows: In the first case, the central bank responds to an appropriately weighted average of the sectoral wage inflation rates; henceforth referred to as the optimal index<sup>7</sup>.

$$r_t = \sum_{i=1}^N \phi_i \pi_{it}^w$$

where  $\pi_t^w$  represents the wage inflation in sector  $i$ . The weight that sector  $i$  receives in the optimal index is given by<sup>8</sup>

$$c_i = \frac{\phi_i}{\sum_{i=1}^N \phi_i}$$

---

<sup>7</sup>Given the fact that nominal rigidities in our model arise due to the assumption of sticky wages and therefore wage stickiness plays a crucial role in determining welfare cost of business cycles, here we consider the cases in which the inflation measure that the central bank chooses to target is the wage inflation rather than the price inflation, as it is already suggested by Erceg, Henderson and Levin (2000). In another exercise which we do not report here, we consider policy rules which responds to the price inflation. We find that, not surprisingly, the performance of the wage inflation rule is always better compared to the price inflation rule.

<sup>8</sup>To ensure that all the sectoral weights in the optimal index are nonnegative, we impose the constraint that all coefficients in this rule are nonnegative.

In the second case, the central bank responds to the aggregate wage inflation, which takes the following form:

$$r_t = \phi^a \pi_t^w$$

where  $\pi_t^w = \sum_{i=1}^N \alpha_i \pi_{it}^w$  is the aggregate wage inflation.

### 5.3 Choice of Parameters

We use a discount factor  $\beta$  of 0.99 which corresponds to the annual real interest rate in the steady state of 4%. As discussed in Dixon and Kara (2005b), we set  $\eta_{LL} = 4.5$ , which implies that intertemporal labour supply elasticity,  $1/\eta_{LL}$ , is 0.2,  $\theta = 6$ , which measures the elasticity of substitution between goods and the relative aversion in consumption,  $\eta_{CC}$ , as unity. Finally, we set the  $\rho_i$  to be 0.95 and the standard deviations of innovations to productivity shocks  $\sigma_i$  to 0.02, which is a standard assumption in the literature (see for example Huang and Liu (2005)).

## 6 Results

We now proceed to examine how the policy rules perform under alternative assumptions regarding the distribution of contract lengths. To do this, we allow for different distributions of contracts with the same mean in the *GTE* framework and for each case, we evaluate the performance of the alternative simple rules in comparison with that of under optimal policy. In order to illustrate the nature of the problem faced by the monetary authority and the implications sectoral heterogeneity for policy design, we start with the simple case of a two sector *GTEs*, which is a common specification in the literature. We then explore monetary policy implications of the model when we allow for a range of contract lengths. For this, we use the empirical distribution employed in Dixon and Kara (2005b) based on Taylor (1993). This has a distribution of wage contract durations from 1 to 8 quarters based on the US economy and the average duration contracts is 3.6 quarters<sup>9</sup>. We compare this economy with the Calvo-*GTE* which has a wider range of contract durations and the share of each duration is the same as generated by

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<sup>9</sup>In Taylors US economy, the sizes of the sectors are  $\alpha_1 = 0.07$ ,  $\alpha_2 = 0.19$ ,  $\alpha_3 = 0.23$ ,  $\alpha_4 = 0.21$ ,  $\alpha_5 = 0.15$ ,  $\alpha_6 = 0.08$ ,  $\alpha_7 = 0.04$ ,  $\alpha_8 = 0.03$ .



the Calvo model<sup>10</sup>. We assume that  $\omega = 0.43$ , which gives a mean contract length of 3.6 quarters as in Taylor's US Economy. Welfare costs are expressed in terms of the equivalent percentage decline in steady-state consumption.

## 6.1 Simple *GTEs*

We start with the simple case of a two sector *GTEs*,  $\{\mathbf{T}, \boldsymbol{\alpha}\} = \{(2, T_2), (0.5, 0.5)\}$  : In sector 1 there are two period contracts and in sector 2, we allow the assumed contract length to vary between complete flexibility ( $T_2 = 1$ ) and 8-period contracts ( $T_2 = 8$ ). We assume that sectors have equal shares.

We begin with comparing the performance of the alternative rules. As figure 1 indicates, the sectoral rule outperforms the aggregate rule yields a welfare outcome nearly identical to the optimal policy, except for the case when sectors have identical contract lengths. Thus, a simple rule that puts more weights on the sector that has longer contracts brings the welfare level not far from the optimal policy. In addition, as figure 2 shows, optimal weight is an increasing function of the contract length. These results are line with the findings of Benigno (2004) and Woodford (2003). In addition, if one sector has fully flexible wages, then the central bank should react to the sector which has sticky wages, which is in line with the findings of Aoki (2001).

What is the intuition behind this finding? As the figure indicates, welfare losses increase with the duration of the contract. This is because longer contracts will adjust sluggishly in response to technology shocks and this sluggish adjustment leads to a higher degree of price dispersion and welfare cost of fluctuations in the economy. In addition, as it is pointed out by Dixon and Kara (2005b), the presence of the longer contracts influences the wage-setting behaviour of short-term contracts and therefore there is a spillover effect from the sluggish long-contract sectors to the short-contract sectors via the price level. Given the fact that the long contracts will adjust sluggishly in response to technology shocks means that the shorter contracts will also adjust sluggishly. As a result, the presence of longer contracts would be more disruptive and would also lead to higher welfare loss. Hence, by putting more weight on the sector that has longer contracts, the central bank can reduce the degree of price dispersion and minimize the disruptive effect of the longer contract sector.

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<sup>10</sup>For computational purposes, the distribution is truncated at  $i = 16$  with the 16 period contracts absorbing all of the weights from the longer contracts.

The aggregate inflation targeting is less effective due to the reason that it implicitly puts too much emphasis on the sector which has short contracts, which can adjust more frequently in response to the shocks. This can perhaps be best seen in the case in which one sector has a flexible contract and the other has two period contracts. As Table 1 shows, the gain from switching to the sectoral rule is as high as 84%.

## 6.2 Distribution of Contract Lengths

Thus far, we have considered the simple case in which there are two sectors. We will next investigate how monetary policy should be set in an economy in which there are more than two type of contract.

### 6.2.1 Taylor's US Economy

Figure 3 reports the optimal weight in the optimal index which minimizes welfare loss<sup>11</sup>. The result in the previous section, that the sectors with have relatively higher contracts get higher weights than their sectoral share, still holds. In addition, our analysis indicates that the central bank should completely ignore the sector which has flexible contracts, as in the case of simpler *GTE*. But, the result that the optimal weight is an increasing function of contract length does not generalize beyond the case of simple *GTEs*. As the figure shows, the longer contracts which are more important in the optimal index gets higher weights. The optimal weights are less than the corresponding sectoral weight for the sectors which have relatively short contracts but as the contract length increases, the sectors with longer contracts start to get higher weights compared with the sectoral weights. But, as contract length further increases, due to the reason that the sectoral shares fall, the optimal weights decline. Thus, the policy implication of Taylor's US Economy is that sectoral share of the sector is as potentially important as the contract length in determining the optimal weight in an optimal index.

We now examine how the aggregate rule performs in an economy in which there is a distribution of contract lengths. As table 2 reports, the net benefit

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<sup>11</sup>Note that a model with a distribution of contract lengths can increase the welfare cost of fluctuations. More specifically, a model with distribution of contract lengths can make the welfare costs of fluctuations almost triple than what the simpler *GTE* suggests. However, our main interest is not comparing welfare losses.

of shifting to the sectoral rule falls dramatically. In fact, welfare gains of shifting to the sectoral rule is quite modest and is only 1% relative to the aggregate rule which suggests that the existence of heterogeneity of contract length is almost irrelevant for monetary policy design.

One might suspect that the relatively low share of shorter contracts in this distribution is mainly responsible for this, perhaps surprising, result. For example, the share of flexible contracts is only 7%. As our analysis in the case of simple *GTE* indicates, the gains in targeting a sectoral rule is substantial when one sector has flexible wages but otherwise it is relatively small. Given this concern, in Figure 4 we perform the same exercise as in table 1 but we consider several cases in which the flexible sector share is higher. In particular, we assume that the share of the flexible sector in the economy is varied from 0.07 to 0.5 and reallocate the remaining shares to the other sectors according to their relative importance in the sectoral index. As the figure reveals, the performance of sectoral rule drops substantially when the flexible sector share is small. This result suggests that the shape of distribution matters significantly for the monetary policy design. In particular, the gain from targeting sector specific inflations rather than the aggregate inflation would be large in an economy where wages/prices reoptimized frequently<sup>12</sup>.

It is important to note that the gain in switching to the sectoral rule is limited when there is a distribution of contract durations. This is because in such a setting unavoidable losses are higher due to the presence of many longer contracts. For example, as we found earlier in the case of two-sector *GTE* where one sector has a flexible contract and the other one has two-period contracts, the gain in switching to the sectoral rule is as high as 84% when sectors have equal shares, whereas in a model with a wider range of contract lengths this gain is only 20%, even when both specification flexible sectors have the same shares in the economy.

### 6.2.2 Calvo-*GTE*

Finally, we consider the implications of Calvo-*GTE* which the share of each sector is generated by Calvo distribution with a wider range of contract lengths than Taylor's US Economy. We assume that  $\omega = 0.43$ , which the mean of completed contract lengths in the economy is 3.6 quarters, as in

<sup>12</sup>Note that Levin, Onatski, Williams and Williams (2005), by using a model which substantially differs from ours, also find that the shape of distribution matters significantly for monetary policy design.

Taylor's US Economy. The main results still hold. As can be seen from Figure 4, the Calvo-*GTE* and Taylor US economy have identical implications for optimal weights. In particular, the optimal weights increase with the contract length but decrease if the share of the sector decreases. As table 1 shows, the net benefit of the sectoral rule is still fairly small. Note that the gain from the sectoral rule is somewhat higher than what the Taylor US Economy suggests, even though both distributions has the same mean. This is because the share of shorter contracts is higher than the Taylor US Economy.

## 7 Conclusions

In this paper, we have presented a general framework to analyze the design of monetary policy rules, in which there can be many sectors with different contract lengths. We have generalized the analysis of Rotemberg and Woodford (1998) and derived a utility based objective function of a central bank to provide a benchmark for evaluating the performance of alternative inflation targeting monetary policy rules in an economy in which there are many sectors with different contract lengths. Our findings can be summarized as follows.

We find that a simple rule that responds to an appropriately weighted average of the sectoral wage inflation rates yields a welfare outcome nearly identical to the optimal policy and the performance of this rule is insensitive to the assumptions regarding the distribution. The optimal weights assigned to sectors depend on the sector share as well as the contract duration.

We find that the performance of the aggregate rule falls behind the sectoral rule because this rule implicitly puts more emphasis on stabilizing price dispersion in sectors in which there are relatively shorter contracts. However, our analysis indicates that the performance of this rule is sensitive to the assumptions regarding the shape of the distribution of contract lengths. We show that the performance of this rule almost identical to the performance of the sectoral rule when we allow for a range of contract durations, suggesting that it may not be necessary for a well designed monetary policy to respond to sector-specific inflations. However, our analysis shows that the benefits of switching to a sectoral rule can be greater in economies in which the share of short contracts is high.

In future work, it would be interesting to explore monetary policy impli-

cations of the model by allowing a different degree of substitutability across goods and therefore different  $\gamma_s$ , measure of the sensitivity of the optimal wage to output in each sector, which we take as identical in this analysis.

## References

- Aoki, K.: 2001, Optimal monetary policy response to relative price changes, *Journal of Monetary Economics* **48**(1), 55–80.
- Ascari, G.: 2000, Optimising agents, staggered wages and the persistence in the real effects of money shocks, *The Economic Journal* **110**, 664–686.
- Benigno, P.: 2004, Optimal monetary policy in a currency area, *Journal of International Economics* **63**(2), 293–320.
- Dixon, H. and Kara, E.: 2005a, How to compare Taylor and Calvo contracts: a comment on Michael Kiley, *Journal of Money, Credit and Banking* (forthcoming).
- Dixon, H. and Kara, E.: 2005b, Persistence and nominal inertia in a generalized taylor economy: how longer contracts dominate shorter contracts?, *Working paper series 489, European Central Bank*.
- Erceg, C. J., Henderson, D. and Levin, A.: 2000, Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* **46**(2), 281–313.
- Erceg, C. J. and Levin, A. T.: 2005, Optimal monetary policy with durable consumption goods, *Journal of Monetary Economics* (forthcoming).
- Gali, J.: 2003, *New Perspectives on Monetary Policy, Inflation, and the Business Cycle*, in *Advances in Economic Theory*, edited by: M. Dewatripont, L. Hansen, and S. Turnovsky, Vol. III, Cambridge University Press.
- Huang, K. X. and Liu, Z.: 2005, Inflation targeting: What inflation rate to target?, *Journal of Monetary Economics* **52**(8), 1435–1462.
- Issing, O.: 2004, Monetary policy implications of heterogeneity in a currency area.
- Kiley, M. T.: 2002, Partial adjustment and staggered price setting, *Journal of Money, Credit, and Banking* **34**(2), 283–98.

- Levin, A. T., Onatski, A., Williams, J. C. and Williams, N.: 2005, Monetary policy under uncertainty in micro-founded macroeconomic models, *NBER Working Paper No. 11523* .
- Mankiw, N. G. and Reis, R.: 2002, Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve, *The Quarterly Journal of Economics* **117**(4), 1295–1328.
- Paustian, M.: 2005, The role of contracting schemes for the welfare costs of nominal rigidities over the business cycle, *Deutsche Bundesbank Discussion papers Series No. 22/2005* .
- Rotemberg, J. J. and Woodford, M.: 1998, An optimization-based econometric framework for the evaluation of monetary policy: Expanded version, *NBER Working Paper No. t0233* .
- Taylor, J. B.: 1993, *Macroeconomic Policy in a World Economy: From Econometric Design to Practical Operation*, W W Norton.
- Woodford, M.: 2003, Interest and prices: Foundations of a theory of monetary policy, *Princeton University Press, Princeton, NJ* .



## 8 Appendix: Derivation of the welfare function

A second-order approximation of the period utility  $U(C_t)$  around steady state yields:

$$U_t(C) = U_C(C)C(c_t + \frac{1-\sigma}{2}c_t^2) + t.i.p + O(\|a\|^3), \quad (29)$$

where  $c_t$  denotes the log-deviation of consumption from steady state,  $t.i.p$  collects all the terms that are independent of policy and  $O(\|a\|^3)$  summarizes all terms of the third or higher orders.

Using the fact that  $\tilde{c} = \tilde{y}_t$  in our model and the definition  $\tilde{y}_t = y_t - y_t^*$ , (29) can be expressed in terms of the output gap

$$U_t(C) = U_C C \left( \tilde{y}_t + \frac{1-\sigma}{2}\tilde{y}_t^2 + (1-\sigma)\tilde{y}_t^2 y_t^* \right) + t.i.p + O(\|a\|)^3$$

Similarly, taking a second order approximation of  $V(1-L_t)$  around steady state yields

$$V(1-L_t) = -V_L(1-L_t)L \left( \tilde{l}_t + \frac{(1+\eta_l)}{2}\tilde{l}_t + (1+\eta_l)\tilde{l}_t l_t^* \right) + t.i.p + O(\|a\|)^3 \quad (30)$$

where  $\tilde{l}_t = l_t - l_t^*$ . Using (7) and aggregating for cohort  $j$  in sector  $i$  gives

$$\tilde{l}_t = \tilde{y}_t + u_t \quad (31)$$

where  $u_t$  is given by

$$u_t = \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} \left( \frac{P_{ij}}{P_t} \right)^{1-\theta} \quad (32)$$

A second order approximation of  $u_t$  yields

$$u_t = \frac{1}{2}\theta \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij} \tilde{p}_{ijt}^2 + O(\|a\|)^3$$

**Proof.**

$$\left(\frac{P_{ft}}{P_t}\right)^{-\theta} = e^{-\theta\tilde{p}_{ft}}$$

where  $\tilde{p}_{ft} = p_{ft} - p_t$ .

Taking a second-order approximation around the steady state yields

$$e^{(1-\theta)\tilde{p}_{ft}} = 1 - \theta\tilde{p}_{ft} + \frac{1}{2}\theta^2\tilde{p}_{ft}^2$$

Integrating over the goods  $f$  belonging to cohort  $j$  in sector  $i$  yields

$$\int_{N_{ij}} \left(\frac{P_{ft}}{P_t}\right)^{1-\theta} = 1 - \theta E_{ij}\tilde{p}_{ijt} + \theta^2 \frac{1}{2} E_{ij}\tilde{p}_{ijt}^2$$

Summing over the cohorts in sector  $i$

$$\sum_{j=1}^{N_i} \lambda_{ij} \left(\frac{P_{ij}}{P_t}\right)^{1-\theta} = 1 - \theta \sum_{j=1}^{N_i} \lambda_{ij} E_{ij}\tilde{p}_{ijt} + \frac{1}{2}\theta^2 E_{ij}\tilde{p}_{ijt}^2$$

Finally, summing over the sectors yields

$$\sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} \left(\frac{P_{ij}}{P_t}\right)^{1-\theta} = \{1 - \theta \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij}\tilde{p}_{ijt} \quad (33)$$

$$+ \frac{1}{2}\theta^2 \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij}\tilde{p}_{ijt}^2\} \quad (34)$$

Note that a second order approximation of  $\left(\frac{P_{ft}}{P_t}\right)^{1-\theta}$  is given by

$$\begin{aligned} \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} \left(\frac{P_{ij}}{P_t}\right)^{1-\theta} &= \{1 + (1-\theta) \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij}\tilde{p}_{ijt} \\ &+ \frac{1}{2}(1-\theta)^2 \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij}\tilde{p}_{ijt}^2\} \end{aligned}$$

Since by definition  $1 = \sum_{i=1}^N \sum_{j=1}^{N_i} \lambda_{ij} \left(\frac{P_{ij}}{P_t}\right)^{1-\theta}$ , we obtain

$$\sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij} \tilde{p}_{ijt} = -\frac{1}{2} (1 - \theta) \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij} \tilde{p}_{ijt}^2 \quad (35)$$

Combining (33) and (35), we obtain

$$\sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} \left( \frac{P_{ij}}{P_t} \right)^{1-\theta} = 1 + \frac{1}{2} \theta \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij} \tilde{p}_{ijt}^2 \quad (36)$$

■

Plugging (32), (31) into (30) and summing the resulting equation with (29), and using the steady state relation  $U_C(C)C = V_L(1 - L)L$  and  $(1 - \sigma)y_t^* = (1 - \eta_l)l_t^*$ , we obtain

$$U_t(C) + V(1 - L_t) = -U_C(C)C \frac{1}{2} \left( \frac{(\sigma + \eta_l)}{2} \tilde{y}_t^2 + \theta \sum_{i=1}^N \sum_{j=1}^{N_i} \alpha_i \lambda_{ij} E_{ij} \tilde{p}_{ijt}^2 \right) + t.i.p + O(\|a\|)^3$$

Note that we assume that  $\sigma = 1$ .

$T_2$ (When $T_1 = 2$ qrts)	Aggregate Rule	Sectoral Rule
1	84.31	0.23
2	0.00	0.00
3	8.18	2.42
4	14.68	3.00
5	18.18	3.22
6	20.05	3.35
7	21.07	3.47
8	21.65	3.68

Table 1: Welfare Losses relative to those under Optimal Policy

	Optimal Policy	Sectoral Rule	Aggregate Rule
Taylor's US Economy	0.1615	0.1624	0.1646
Calvo- <i>GTE</i>	0.1621	0.1621	0.1663

Table 2: Welfare comparisons with different distributions

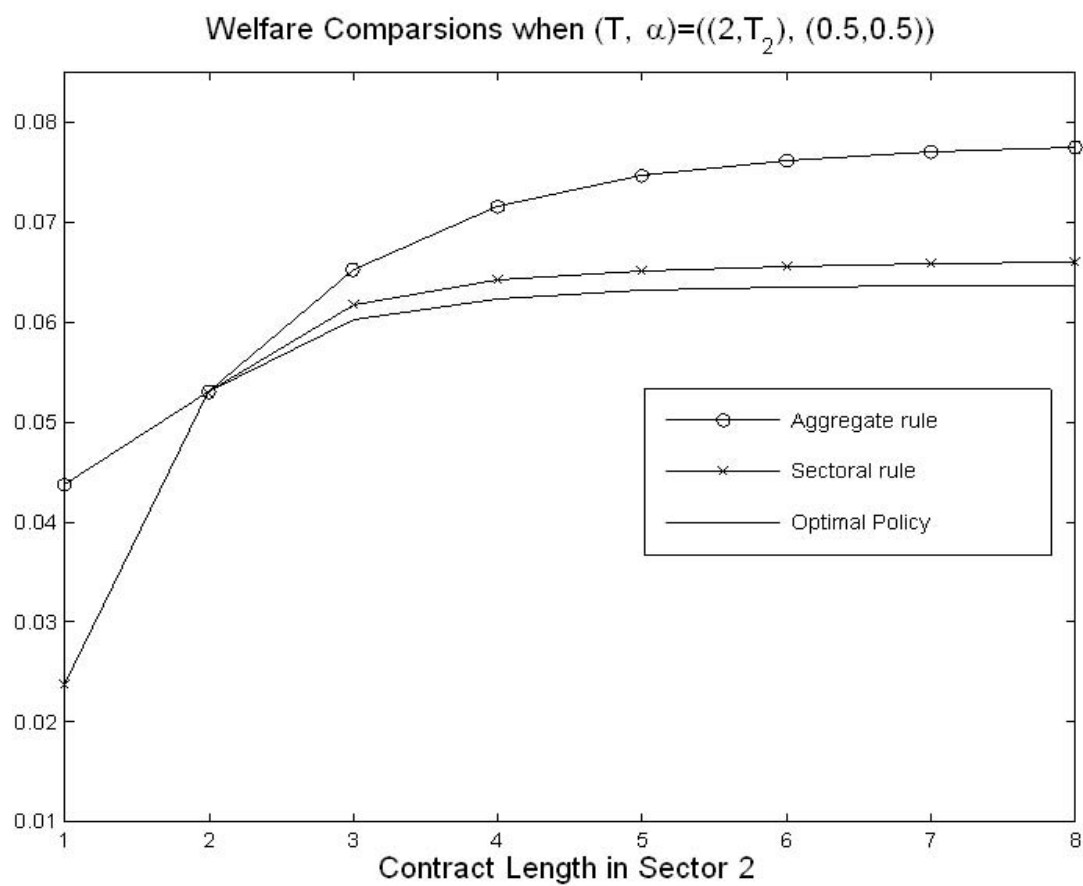


Figure 1: Welfare comparisons in terms of the equivalent % decline in steady state consumption.

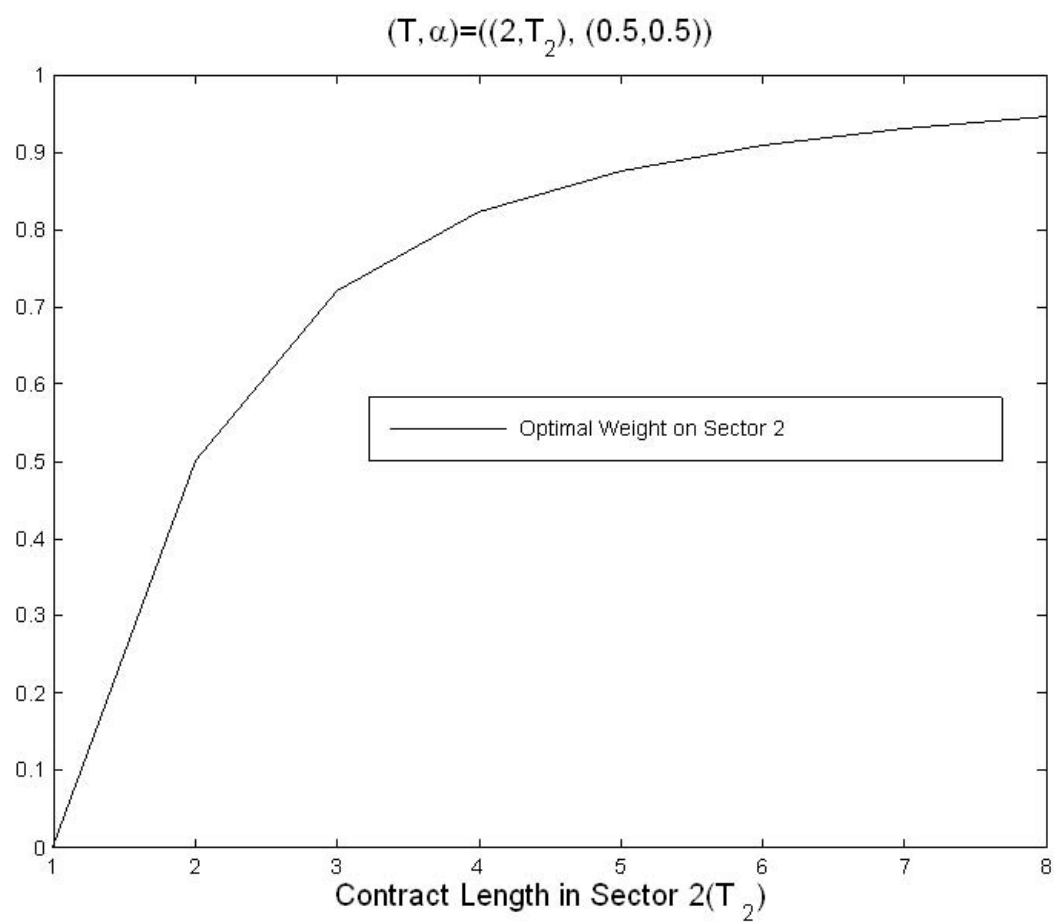


Figure 2: The weight on Sector 2 in the optimal index

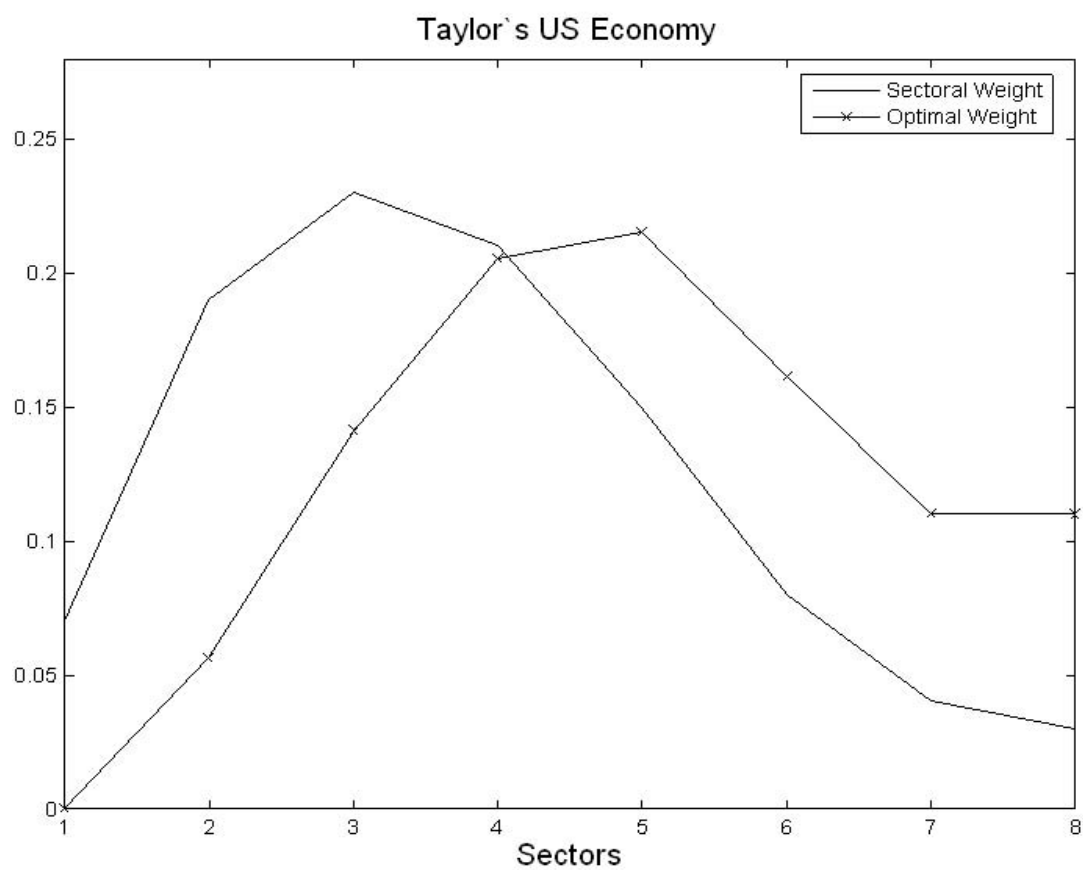


Figure 3: The weight on different sectors in Taylor's US Economy



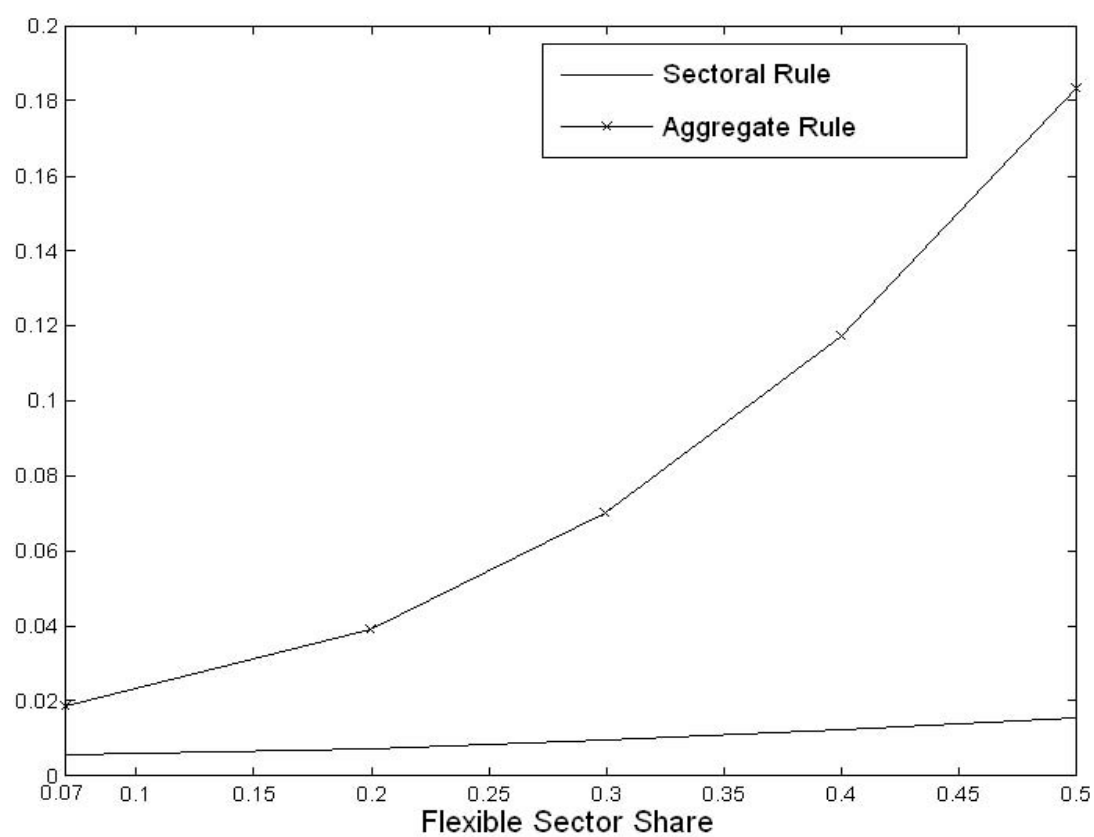


Figure 4: The welfare losses relative to those under optimal policy

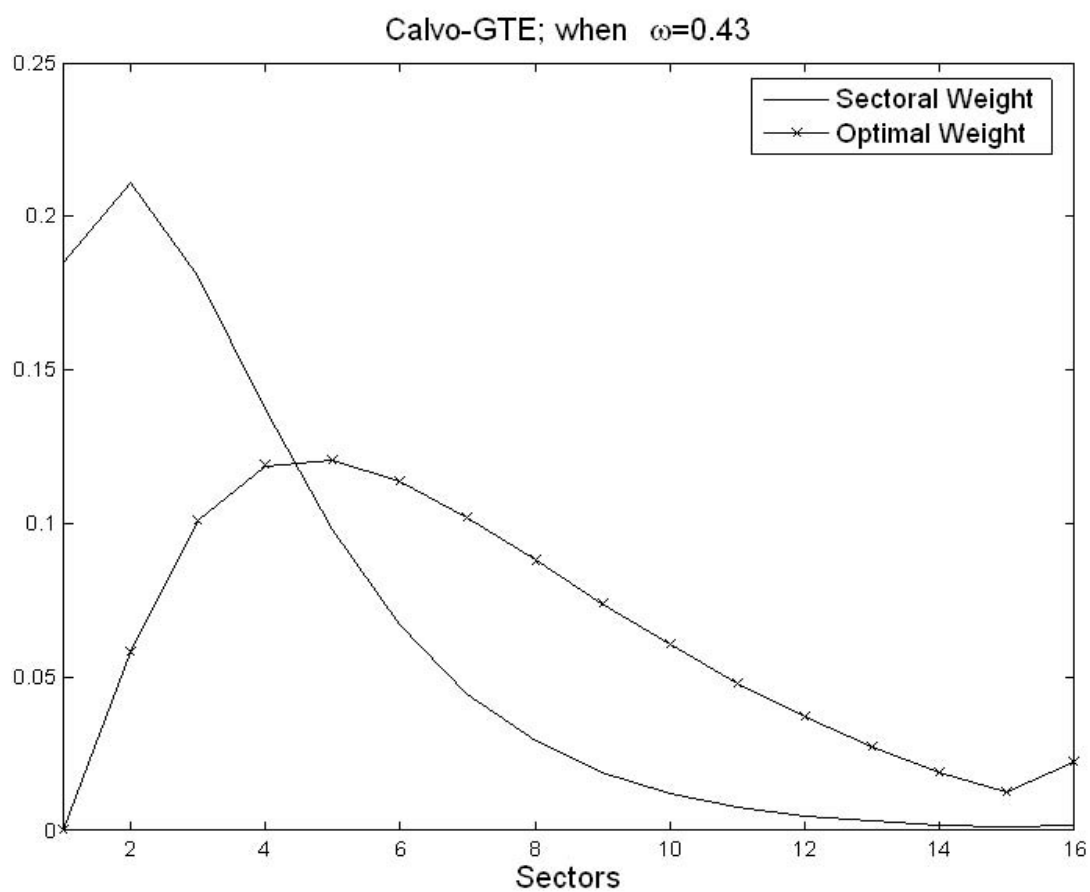


Figure 5: The weight on different sectors in Calvo Economy

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