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THE REAL EFFECTS OF MONEY GROWTH IN DYNAMIC GENERAL EQUILIBRIUM

by Liam Graham
and Dennis J. Snower

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THE REAL EFFECTS OF MONEY GROWTH IN DYNAMIC GENERAL EQUILIBRIUM [1]

by Liam Graham [2]

and Dennis J. Snower [3]

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1 *This research was undertaken whilst Dennis Snower was a research visitor at the ECB. Any errors remain our responsibility.*

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Abstract

We analyse the effects of money growth within a standard New Keynesian framework and show that the interaction between staggered nominal contracts and money growth leads to a long-run trade-off between output and money growth. We explore the microeconomic mechanisms that lead to this trade-off, and show that it remains even when the contract length is endogenised.

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Non-technical summary

Macroeconomic modelling has converged over the past decade onto what is often referred to as a "new neo-classical synthesis": a type of model derived from the microeconomic behaviour of firms and households, but with temporary price and wage rigidities which lead to a role for monetary policy. Such models are widely used to analyse the full range of macroeconomic questions. However such models generally assume an inflation rate of zero. Since inflation rates are usually positive, it is important to explore the consequences of relaxing this assumption. In this paper we ask whether such models can generate a long-run tradeoff between inflation and unemployment.

The real effects of inflation in our model occur as a result of the existence of staggered wage contracts – different cohorts of workers renegotiate their wage contracts at different times, and when they do so hold the (nominal) wage fixed for a set period of time (in our baseline calibration we assume wage contracts last for one year).

Since workers hold their nominal wage fixed while the price level is rising, firms will want less of their labour at the start of the wage contract (when the wage is relatively high compared to the price level), and more at the end of the wage contract (when the wage is relatively low compared to the price level). This results in what we call employment cycling: and the higher is inflation, the greater is the difference between the real wage at the start of the contract and that at the end, so the greater is the degree of employment cycling. This cycling is inefficient for firms so as inflation increases firms hire less labour as inflation rises. Further, if workers prefer smooth labour supply they will dislike employment cycling so will supply less labour as inflation rises. Both these effects lead to employment and output *falling* as inflation rises.

A third effect is the consequence of households having a positive rate of time preference. When setting their wage, households do so as a markup over an average of current prices and expected future prices, putting more weight on current prices because of discounting. As inflation increases, the gap between current and future prices increases, and the wage the household chooses becomes closer to the current than the future wage, so the real wage falls. This results in employment and output *rising* as inflation rises.

We show that except at low rates of inflation (and particular calibrations of our model) the first two effects dominate the third, so higher inflation causes a fall in employment and output. At low rates of inflation, the third effect is stronger, and results in higher inflation increasing employment and output.

We go on to allow the length of the contract to be chosen optimally by households and find that this extra choice does not change our results significantly, at least over the sort of inflation rates observed in the OECD over the past 50 years.

1 Introduction

Although dynamic general equilibrium (DGE) models with nominal rigidities have become standard in the macroeconomic literature,[1] these models usually neglect money growth, as their dynamics are derived from linearizations around a steady state in which the money supply is constant. In practice, of course, the money supply in market economies is usually growing. Thus it is important to inquire whether the presence of money growth would change the properties of these models.

The central long-run issue is whether the classical dichotomy holds in the long run, i.e. whether real variables (such as output, employment or unemployment) are independent of the inflation rate in the long run. Most textbook models of the Phillips curve[2] presuppose this property, thereby making the Phillips curve consistent with a NAIRU or natural rate of unemployment. By contrast, the standard DGE models with nominal rigidities, commonly based on Calvo pricing, lead to a New Keynesian Phillips curve of the form $\pi_t = \beta E_t \pi_{t+1} + \gamma y_t + \varepsilon_t$, where π is inflation, y is output, β is the discount factor, ε_t is an error term, and E_t is the expectations operator. Although the long run inflation-output tradeoff implied by this Phillips curve is not vertical, the mainstream view is that it is negligibly close to vertical, since the discount factor is close to unity in practice.

This paper indicates that the tradeoff between inflation and real variables could be significant, depending on the calibration of the underlying DGE model. This issue is of central importance to macroeconomics. There is a broad consensus in the current macroeconomic literature that real phenomena can be analyzed independently of nominal phenomena in the long run; and this helps explain the appeal of the New Neo-Classical Synthesis and the NAIRU hypothesis. The purpose of this paper is to examine whether this consensus rests on tenuous foundations.

As far as we are aware, the only studies which examine the influence of steady-state money growth in a DGE framework are Ascari (1998, 2000) and Devereux and Yetman (2002). Whereas they find that steady-state output and employment are indeed affected by money growth, they do not examine the microeconomic mechanisms leading to this result. This paper identifies the various channels whereby money growth affects the real economy in the long run, and explores the influence of each. This provides an intuitive

[1] Recent surveys include Goodfriend and King (1997), Roberts (1997), Estrella and Fuhrer (1998), Mankiw (2000), and Gali (2002).

[2] See, for example, Blanchard and Fischer (1989) and Romer (1996).

understanding of the determinants of the long-run tradeoff between real variables and inflation.

In particular, we analyze a DGE model with staggered wage contracts (of the Taylor (1980a) type). An analogous analysis, with the similar qualitative conclusions, can be derived for Taylor price contracts and for Calvo wage and price contracts (see Graham and Snower (2003)). Our results may be summarized as follows.

- First, we show that, for a given length of nominal wage contracts, real variables are not independent of inflation. This is so even though we make the standard assumptions that agents have rational expectations, there is no money illusion, and there are no permanent nominal rigidities.

- Second, the long-run relation between real variables and inflation does not disappear when the contract frequency is endogenous.

- Third, we show that, in the context of a standard DGE model with nominal rigidities, the relation between real variables and inflation depends on three basic phenomena: (i) "employment cycling:" firms' ability to substitute among different labor types over the contract period, (ii) "labor supply smoothing:" households' preference for stable employment paths (rather than ones that are intertemporally volatile), and (iii) time discounting: households have a positive rate of time preference. Time discounting generates a positive relation between output and inflation, whereas employment cycling and labor supply smoothing give rise to a negative relation.

- Fourth, we show that, for standard calibrations of the model's parameters, the time discounting effect is dominant at low inflation rates, but the employment cycling and labor supply smoothing effects come to dominate at higher inflation rates. Thus the employment-inflation relation is upward-sloping at low inflation rates and downward-sloping at high inflation rates. The same is true of the output-inflation tradeoff.

The intuition underlying our results is reasonably straightforward. In our model, households are differentiated by labor types, which are imperfect substitutes in firms' production functions, i.e. there are diminishing returns to each labor type. Thus each household faces a downward-sloping demand curve for its labor type. The households are divided into wage-setting cohorts. Different cohorts set their nominal wage at different

times, and each cohort holds its nominal wage fixed over a given contract period. The wage is set so as to maximize the household's utility over the contract period, given its labor demand curve.

When the money supply grows, the aggregate price level rises through time. Since the nominal wage of each cohort is fixed, its real wage is relatively high at the beginning of the contract period (when the price level is relatively low) and relatively low at the end of that period (when the price level is relatively high). Since there are diminishing returns to each labor type, each firm has an incentive to demand relatively little of the household's labor at the beginning of the contract period, and relatively much at the end. Moreover, since the contract periods are staggered, cohorts with relatively high real wages (whose contracts have been set recently) are matched by cohorts with relatively low real wages (whose contracts have been set some time ago) i.e. there is real wage dispersion. Thus each firm keeps substituting among different labor types, in response to the swings in relative real wages. This is the employment cycling phenomenon.

In practice, employment cycling is of course much more likely to occur in form of hours variations than through hiring and firing. There is evidence that overtime work is sensitive to the wage, and thus it seems plausible that a limited degree of employment cycling may occur when a significant degree of inflation occurs over the contract period, so that significant variations in the real wage take place over this period. It is a well-known empirical regularity that, as the aggregate inflation rate rises, relative prices become more volatile, and this phenomenon is often taken to be the origin of a major real cost of inflation. The inefficiency of employment cycling could possibly be viewed as a simple analytical characterization of this cost.

The faster the money supply grows, the greater is the steady state inflation rate (since in the long run the two are equal we use the terms interchangeably). Thus the greater are the fluctuations of a cohort's real wage over a given contract period, and the more employment cycling the firms will do. Since the different labor types are imperfect substitutes, more employment cycling means lower average productivity of labor (i.e. a lower ratio of output to the total number of workers employed by the firm). On this account, higher inflation is associated with lower output and employment.

Next, observe that firms' employment cycling implies that households' supply of labor services is not constant through time. If a household's marginal disutility of labor rises

with the amount of labor supplied, implying a preference for labor supply smoothing, then employment cycling makes the household worse off. To compensate the household for this welfare loss, it requires a higher wage. The faster the money supply grows (and thus the higher the inflation rate and greater the degree of employment cycling), the higher is the real wage that the household sets, for a given average supply of labor over the contract period. The higher the real wage, the lower is the firm's demand for the household's labor. Thus, once again, higher inflation is associated with lower employment, and thus lower output as well.

Finally, under staggered wage setting, the current contract wage depends on the current price level (which prevails at the beginning of the contract period) and the expected future price level (which prevails later in the contract period). If the households' rate of time preference is positive, then the contract wage is influenced more strongly by the current price level than the expected future price level. Thus, the faster the money supply grows, the more the contract wage lags behind the price level; consequently, the lower is the average real wage over the contract period. The lower the real wage, the more labor will firms demand. In this way, discounting generates a positive relation between inflation and employment (and the corresponding output).

The overall degree to which real variables depend on inflation clearly depends on the relative strengths of the employment cycling and labor supply smoothing effects (on the one hand) and the discounting effect (on the other). At low rates of inflation, there is little employment cycling and thus the supply of labor services is reasonably stable through time. Consequently the discounting effect is dominant, so that output is positively related to inflation in the long run. But at higher inflation rates, employment cycling becomes more pronounced and thus the supply of labor services becomes more volatile over the contract period. When these effects become dominant, output becomes negatively related to inflation in the long run.

These results hinge crucially on employment cycling because, as noted, when this phenomenon is absent, it can be shown that there is no long-run relation between real variables and inflation.

Underlying our analysis is the assumption that households do not index their wages to steady state money growth. Were there to be indexation, the superneutrality of money would be restored since there would be none of the wage dispersion on which the three

effects described above depend. However empirical evidence suggests that indexation only becomes widespread at high rates of inflation.[3] A further important assumption is our choice of Taylor contracts to model the nominal rigidity. Were we to choose Fisher contracts (pre-set wages that are allowed to vary over the contract period), there would be no wage dispersion and hence money would be superneutral. Although Fisher contracts (with cost-of-living adjustments during the contract period) are not uncommon in practice, this clearly does not imply that Taylor contracts are irrelevant in these cases. A world without Taylor contracts - or other rigidities that keep wages fixed over finite time periods - would be one in which wages could be adjusted instantaneously and continuously through time, and this is obviously counterfactual except under hyper-inflation.

Another possibility, and one widely used in the literature, is to use Calvo contracts. However in the context of a model with money growth, Calvo contracts are not appropriate. The reason is straightforward. With Calvo contracts, some households keep their nominal wage unchanged for a very long period of time, which means the real value of this wage approaches zero. This implies that the firm will wish to hire as much of the labor of these households as possible, and as little of the other households. This extreme form of employment cycling is very inefficient so output approaches zero. Since we live in a world in which money growth and inflation are significantly positive, this difficulty points to a major limitation of Calvo contracts as a useful approximation to Taylor contracts - a limitation that has thus far been ignored in the mainstream literature.

The remainder of this paper is organized as follows. Section 2 presents our underlying DGE model with staggered wage setting under an exogenously given contract period. Section 3 derives results and discusses in detail the intuition behind them. In section 4 we allow the wage contract period to be determined endogenously, and show that a long-term relationship between real and nominal variables continues to exist. Section 5 concludes.

[3] For example, Cecchetti (1984) found that nominal wages in the union sector in the U.S. remained unchanged for 7 quarters on average, and that the degree of indexing rose with the inflation rate. Taylor (1993) indicates that annual contracts are the most common span over which nominal wages are set. For a review of the evidence, see Taylor (1999).

2 The model

The model consists of three types of agent. First, there is a continuum of households supplying differentiated labor. Second, a large number of identical firms produce a homogeneous output by means of labor, as described by a production function in which the households' labor types are imperfect substitutes. Third, a government prints money and nominal bonds, and rebates the seigniorage proceeds to households as a lump sum.

Each household makes its wage and employment decisions in a decentralized way, facing a downward-sloping labor demand curve for its services. The households are grouped into N wage-setting cohorts, and each cohort sets a nominal wage contract for N periods. Wage setting is staggered, with different cohorts setting wages at different periods, spread uniformly through time.

2.1 Firms

Firms produce a homogeneous output and there is perfect competition in the product market. Each firm uses all labor types in a CES production function:[4]

$$y_t = n_t = \left[\int_{h'=0}^{1} n_t\left(h'\right)^{\frac{\theta_n - 1}{\theta_n}} dh' \right]^{\frac{\theta_n}{\theta_n - 1}} \tag{1}$$

where y_t is output, $n_t(h)$ is the amount of labor chosen from household h and θ_n is the elasticity of substitution between different labor types. This production function is linear in composite labor n_t. Since all firms are identical, we normalize the number of firms to one (the representative firm).

The representative firm's cost-minimization leads to a downward-sloping demand curve for each household's labor (see Dixit and Stiglitz (1977)):

$$n_{t+i}(h) = \left(\frac{\mathsf{W}_t(h)}{\mathsf{W}_{t+i}} \right)^{-\theta_n} y_{t+i} \tag{2}$$

[4] We use the following notational convention:

	nominal	real
trended	X_t	x_t
detrended	X_t	x_t
steady state	X	x
linearized	$\hat{X}_t$	$\hat{x}_t$

x_t refers to an aggregate variable, $x_t(f)$ and $x_t(h)$ to the value of that variable for a specific firm or household.

where $\mathsf{W}_t(h)$ is the wage set by household h. The corresponding aggregate wage index is W_{t+i} is a wage index defined as

$$\mathsf{W}_t = \left[\int_{h'=0}^{1} \mathsf{W}_t(h')^{1-\theta_n} dh' \right]^{\frac{1}{1-\theta_n}} \tag{3}$$

2.2 Households

Suppose that at time t household h resets its nominal wage, which is then constant for N periods until it can be reset again. Let c_t be its consumption of goods, W_t its nominal wage, M_t its nominal money holding, P_t the aggregate price index, R_t the gross real interest rate on its bond holdings B_t, and T_t its net lump-sum transfers from government.

The household's infinite-horizon problem can be split into a series of N-period sub-problems:

$$\max_{\{c_{t+i}, \mathsf{W}_{t+Ni}(h), B_{t+i}, \mathsf{M}_{t+i}\}} E_t \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} \beta^i U\left(c_{t+i}(h), n_{t+i}(h), \frac{\mathsf{M}_{t+i}(h)}{\mathsf{P}_{t+i}} \right) \tag{4}$$

subject to a series of real budget constraints:

$$\begin{aligned} & \sum_{i=0}^{N-1} \rho_{t,t+i} \left[c_{t+i}(h) + \frac{\mathsf{M}_{t+i+1}(h)}{\mathsf{P}_{t+i}} \right] + R_t \frac{\mathsf{B}_t(h)}{\mathsf{P}_t} \\ = & \sum_{i=0}^{N-1} \rho_{t,t+i} \left[\frac{\mathsf{W}_t(h)}{\mathsf{P}_{t+i}} n_{t+i}(h) + \frac{\mathsf{T}_{t+i}(h)}{\mathsf{P}_{t+i}} + \frac{\mathsf{M}_{t+i}(h)}{\mathsf{P}_{t+i}} \right] + \rho_{t,t+N-1} \frac{\mathsf{B}_{t+N}(h)}{\mathsf{P}_{t+N-1}} \end{aligned} \tag{5}$$

and labor demand constraints (derived above):

$$n_{t+i}(h) = \left(\frac{\mathsf{W}_{t+i}(h)}{\mathsf{W}_{t+i}} \right)^{-\theta_w} y_{t+i}, i = 1, ..., N-1 \tag{6}$$

The discount factor from period t to $t+i$ is given by:

$$\rho_{t,t+i} = \prod_{j=0}^{i} \frac{1}{R_{t+j}} \tag{7}$$

We choose a utility function that has desirable long-run properties:

$$U(...) = \log c_t(h) + \varsigma \frac{(1-n_t(h))^{1-\eta}}{1-\eta} + V\left(\frac{\mathsf{M}_t(h)}{\mathsf{P}_t} \right) \tag{8}$$

The first-order condition with respect to consumption is:

$$\frac{\beta^i}{c_{t+i}(h)} = \rho_{t,t+i}\,\lambda_t(h) \tag{9}$$

For the remainder of the paper, we consider various steady states of this economy and do not analyze transitions between them. A steady state has the property that each N period sub-problem of the household's infinite-horizon optimization is identical, and that each household faces an identical problem when it comes to reset its wage.

The first-order condition for the next N period problem is:

$$\frac{\beta^{i+n}}{c_{t+i+N}(h)} = \rho_{t,t+N+i}\,\lambda_{t+N}(h) \tag{10}$$

Since households face identical N period sub-problems $c_{t+i+N}(h) = c_{t+i}(h)$ and $\lambda_t(h) = \lambda_{t+N}(h)$. So (9) and (10) imply that $\rho_{t,t+i} = \beta^i$, i.e. the steady-state interest rate is constant and tied to the discount factor. Further, (9) then states that (in the steady-state) the household chooses a smooth path of consumption *within* the contract period[5].

What level of consumption does the household choose? The budget constraint can be rearranged to give

$$\begin{aligned} &\sum_{i=0}^{N-1}\beta^i\overline{c}_t - \sum_{i=0}^{N-1}\beta^i\frac{\mathsf{W}_t^*(h)}{\mathsf{P}_{t+i}}n_{t+i}(h) \\ = &\sum_{i=0}^{N-1}\beta^i\left[\frac{\mathsf{T}_{t+i}(h)}{\mathsf{P}_{t+i}} + \frac{\mathsf{M}_{t+i}(h)}{\mathsf{P}_{t+i}} - \frac{\mathsf{M}_{t+i+1}(h)}{\mathsf{P}_{t+i}}\right] + \frac{\mathsf{B}_{t+N}(h)}{\mathsf{P}_{t+N-1}} - R_t\frac{\mathsf{B}_t(h)}{P_t} \end{aligned} \tag{11}$$

where $\overline{c}_t$ is the constant level of consumption chosen in the contract period starting at time t, and $\mathsf{W}_t^*(h)$ is the optimum wage. The right-hand side is the present value of the real net payments from government to household consisting of transfers, seigniorage and revenue from issuing bonds. Since all households behave in the same way, and assuming that there is no government spending, the value of these net transfers must be zero in the

[5] To model off-steady state behaviour in a tractable way, further assumptions would need to be made such as the existence of perfect insurance markets which household use to insure themselves against shocks and so keep consumption constant. Without constant consumption, aggregation becomes technically challenging.

steady state. If this were not the case, the government's accumulation of assets or debt over the infinite life of the economy would be unbounded. So consumption is given by:

$$\bar{c}_t(h) = \frac{1}{\sum_{i=0}^{N-1} \beta^i} \sum_{i=0}^{N-1} \beta^i \frac{\mathsf{W}_t^*(h)}{\mathsf{P}_{t+i}} n_{t+i}(h) \tag{12}$$

Deriving the remaining first order conditions and rearranging gives an expression for the household's optimal wage

$$\mathsf{W}_t^*(h) = \frac{\varsigma\theta_n}{\theta_n - 1} \frac{E_t \sum_{i=0}^{N-1} \beta^i (1 - n_{t+i}(h))^{-\eta} n_{t+i}(h)}{\frac{1}{c_t} E_t \sum_{i=0}^{N-1} \beta^i \frac{n_{t+i}(h)}{\mathsf{P}_{t+i}}} \tag{13}$$

Substituting for consumption from (12) and assuming a symmetric equilibrium in which all households resetting their wage at time t make identical decisions, gives the following labor supply function:

$$E_t \sum_{i=0}^{N-1} \beta^i (1 - n_{t+i})^{-\eta} n_{t+i} = \frac{(\theta_n - 1)}{\varsigma\theta_n} \sum_{i=0}^{N-1} \beta^i \tag{14}$$

Intuitively, the optimal contract wage sets the present value of the marginal disutility of labour (the left side of this equation) equal to the corresponding present value of the marginal utility of consumption (the right side of the equation). Since households smooth consumption but not employment, the present value of the marginal disutility of labour depends on time discounting, but the present value of the marginal utility of consumption does not. In fact, the latter present value is constant. Thus, the optimal contract wage is set so that the present value of the marginal disutility of labour over the contract period remains constant through time.

2.3 The steady state

Since we are concerned with the long run relation between real variables and inflation, we let the steady-state growth rate of money be a constant,

$$\mu = \frac{\mathsf{M}_{t+1}}{\mathsf{M}_t} \tag{15}$$

so that different steady-state values of money growth identify different points on the real variable-inflation tradeoff.

To derive the influence of money growth on real variables, it is convenient to detrend all nominal variables with respect to the money supply. (This detrending process is familiar from the real business cycle literature (King, Plosser and Rebelo (1988)), although of course the trend being removed there is one of technology growth.) For example, the wage index W_{t+i} detrended to time t is

$$W_{t+i} = \frac{\mathsf{W}_{t+i}}{\mathsf{M}_t} = \frac{\mathsf{W}_{t+i}}{\mathsf{M}_{t+i}}\frac{\mathsf{M}_{t+i}}{\mathsf{M}_t} = \mathsf{W}_{t+i}\mu^i \tag{16}$$

where μ is steady state money growth, W_{t+i} is the detrended wage, and W_{t+i} is its trended counterpart.

Consequently the detrended wage index (3) is

$$W_t = \left(\frac{1}{N}\sum_{i=0}^{N-1}\left(W^*_{t-i}\mu^{-i}\right)^{1-\theta_n}\right)^{\frac{1}{1-\theta_n}}$$

Thus, in the steady state, the ratio of the contract wage to the aggregate wage is

$$\frac{W^*}{W} = \left(\frac{1}{N}\frac{1-\mu^{N(\theta_n-1)}}{1-\mu^{\theta_n-1}}\right)^{\frac{1}{\theta_n-1}} \tag{17}$$

Observe that steady-state money growth uniquely determines the ratio of the contract wage to the aggregate wage (given the parameters of the model).

The household's labor supply function (14) determines steady-state household labor supply. Given this labor supply and the wage ratio (17), we obtain aggregate steady-state output y from the labor demand function (6). In short, (6), (14) and (17) comprise a system of three equations in the three unknowns $(n(h), \frac{W^*}{W}, y)$ which define the steady state of the system. We solve this system numerically.[6]

3 Results

We calibrate our model with standard values: the annual real interest rate is 4%; the elasticity of labor substitution is $\theta_n = 5$; the contract period is one year. There are 52 wage-setting cohorts,[7] so that each week one cohort sets its nominal wage and then keeps it fixed for a year. Furthermore, we let households' preferences over leisure be linear,

[6] Details of the solution method are available from the authors on request

[7] The results are quite insensitive to the number of cohorts.

$\eta = 0$ meaning that households are indifferent to the path of labor supplied . Although there is little doubt that the marginal disutility of labor rises with the amount of labor provided per working day, it seems doubtful that most people have strong preferences in favor of labor smoothing over their wage contract period. On the contrary, many people prefer to bunch their overtime work, in order to make room for free time (shopping days, holidays). The intertemporal substitution of labor relevant to our analysis is that which occurs over the contract period, and in this context there may be little preference for labor smoothing[8].In any case, our results under positive values of η are given in the next section.

Figure 1 shows aggregate steady-state employment and output (on the vertical axis) for different values of steady-state money growth (on the horizontal axis). Observe that employment increases monotonically with money growth. Thus if we define unemployment as the difference between total available labor time and employment, we obtain a downward-sloping Phillips curve in inflation-unemployment space. Note, however, that output is positively related to steady-state inflation (money growth) at low inflation rates, but negatively related at higher inflation rates. In other words, the output-inflation trade-off is backward-bending.

3.1 Intuition

To gain an intuitive understanding of these relations, consider the simple case in which there are just two wage-setting cohorts ($N = 2$) and each nominal wage is set for two periods. Let cohort 0 set its wage at time t while cohort 1 sets its wage at time $t + 1$. Suppose that steady-state money growth - and thus steady-state inflation - is positive. Since the price level rises from period to period whereas each cohort's nominal wage is readjusted every second period, it follows that each cohort's real wage is high at the beginning of its contract period and low at the end of it. Specifically, at time t, the real wage of cohort 0 is high and that of cohort 1 is low, and vice versa at time $t + 1$.

The firm at time t has a relatively low demand for cohort 0 and relatively high demand for cohort 1, and vice versa at time $t + 1$. This substitution towards labor types with low real wages (and away from labor types with high real wages) is the essence of what we have called *employment cycling*.

[8] In the real business cycle literature, $\eta = 0$ in the indivisible labor case. This is straightforwardly reconcilable with our argument that employment cycling takes place primarily in terms of hours variations. After all, a one-year wage contract period is sufficiently long to permit workers to bunch their labor services over the contract period even though these services may come in discrete units.

Since different labor types are imperfectly substitutable, employment cycling is inefficient in the sense that, for given aggregate employment n, more employment cycling is associated with lower aggregate output. The greater is steady-state inflation rate, the more employment cycling firms do, and thus the lower the average productivity of labor. In this way, the employment cycling effect leads to a negative relation between output and inflation.

To see this formally, note that since aggregate output y is linear in aggregate employment n, the aggregate wage W is equal to the aggregate price level P. Thus, by the wage ratio (17), the real contract wage of the cohort that sets its wage in period t (namely, cohort 0) is

$$\frac{W_t^*(0)}{P_t} = \left(\frac{1+\mu^{\theta_n-1}}{2}\right)^{\frac{1}{\theta_n-1}} \tag{18}$$

Substituting this into the labor demand function (6), we obtain

$$n_t(0) = \left(\frac{1+\mu^{\theta_n-1}}{2}\right)^{\frac{-\theta_n}{\theta_n-1}} y_t \tag{19}$$

For a given employment level $n_t(0)$, we can see that the higher the rate of money growth μ (and thus the higher the inflation rate), the lower will be the corresponding level of aggregate output y_t. This is the employment cycling effect.

The other determinant of the relation between real variables and inflation curve is the *time discounting effect*.[9] By (13), observe that, for a two-period contract, the optimal contract wage $W_t^*(h)$ is a weighted average of the current price level P_t and the future price level P_{t+1}. The lower the discount factor β (i.e. the higher the discount rate), the smaller the weight on the future price relative to the present one. Thus, the faster the money supply grows, the more the contract wage lags behind the average price level over the contract period, i.e. the lower the average real contract wage over the contract period. As a result, the firm employs more labor and produces more output over the contract period, *ceteris paribus*.

Another view of discounting effect emerges from the labor supply function (14). Recall that this implies that the household sets its contract wage so that the present value of the marginal disutility of labor is a constant. Thus, the lower the discount factor β, the more labor it supplies over the contract period. Formally, note that whereas cohort A's

[9] Note that since households' preferences over leisure are linear, there is no labor supply smoothing effect

real contract wage and employment at time t are given by (18) and (19), respectively, its real contract wage and employment at time $t+1$ are

$$\frac{W_{t+1}^{*}(A)}{P_{t+1}} = \frac{1}{\mu}\left(\frac{1+\mu^{\theta_n-1}}{2}\right)^{\frac{1}{\theta_n-1}} \tag{20}$$

and

$$n_{t+1}(A) = \mu^{\theta_n}\left(\frac{1+\mu^{\theta_n-1}}{2}\right)^{\frac{-\theta_n}{\theta_n-1}} y_{t+1} \tag{21}$$

Substituting the employment equations (19) and (21) into the labor supply function (14) and recalling that steady-state output is constant, we obtain

$$n_t(A)\left(1+\beta\mu^{\theta_n}\right) = k \tag{22}$$

Thus aggregate employment is

$$n = n_t(A) + n_t(B) = n_t(A) + n_{t+1}(A) = \frac{k\left(1+\mu^{\theta_n}\right)}{1+\beta\mu^{\theta_n}} \tag{23}$$

For positive real interest rates ($\beta < 1$) this expression is increasing in μ so steady state employment increases with money growth.

Putting the employment cycling and discounting effects together, we find that as money growth (and steady-state inflation) increases, employment rises (due to the discounting effect) but output for any given level of employment falls (due to employment cycling). At low inflation rates, there is little employment cycling, and thus the discounting effect dominates. As result, there is a positive long-run relation between output and inflation. However, at higher inflation rates, firms engage in lots of employment cycling, and once this cycling effect dominates, the long-run relation between output and inflation turns negative.

3.2 Linearizations

Since we are working in the familiar framework of Taylor contracts, it is instructive to obtain a wage setting equation which we can compared with that in Taylor (1980a). To do this, consider a simplified case where there are just two cohorts i.e. $N = 2$. Substituting

(6) into (14) and linearizing gives a wage-setting equation:

$$k_0\hat{W}_t^* = \frac{1}{1+\beta}k_1\hat{W}_{t-1}^* + \frac{\beta}{1+\beta}(1-k_1)\hat{W}_{t+1}^* - k_2\hat{\mu}_t - k_3\hat{\mu}_{t+1} + k_3(\hat{y}_t + \beta\hat{y}_{t+1}) \tag{24}$$

where

$$k_1 = \frac{\mu^{\theta_n - 1}}{1+\mu^{\theta_n-1}} \tag{25}$$

$$k_0 = \frac{1-\beta}{1+\beta}k_1 + \frac{\beta}{1+\beta} \tag{26}$$

$$k_2 = \frac{1}{1+\beta}k_1 \tag{27}$$

$$k_3 = \frac{1}{\theta_n(1+\beta)}(1-k_1) \tag{28}$$

This is a microfounded analogue of Taylor (1980a) wage-setting equation.

Consider first the case of $\mu = 1$, i.e. we are linearizing around a steady state money growth of zero. Then the wage setting equation becomes

$$\hat{W}_t^* = \alpha\hat{W}_{t-1}^* + (1-\alpha)\hat{W}_{t+1}^* - (2\alpha - 1)\hat{\mu}_t + \frac{1}{\theta_n}(\hat{y}_t + \beta\hat{y}_{t+1}) \tag{29}$$

where

$$\alpha = \frac{1}{1+\beta} \tag{30}$$

In the long-run steady state, $\hat{W}_t^* = \hat{W}_{t-1}^* = \hat{W}_{t+1}^*$, $\hat{\mu}_t = \hat{\mu}_{t+1} = \hat{\mu}$ and $\hat{y}_t = \hat{y}_{t+1} = \hat{y}$ so the equation simplifies to a long-run relation between output and money growth:

$$\hat{y} = \theta_n\frac{1-\beta}{1+\beta}\hat{\mu} \tag{31}$$

and since $\beta < 1$ output is positively related to money growth. This is the case examined in Graham and Snower (2002). It isolates the discounting effect, because in the absence of inflation there is no employment cycling.

Now consider the general case in which money growth need not be zero. Then, given that steady-state inflation is equal to money growth ($\widehat{\pi} = \widehat{\mu}$), the long-run relation between output and inflation is

$$\hat{y} = 2\theta_n\left(\frac{\mu^{\theta_n-1}}{1+\mu^{\theta_n-1}} - \frac{\beta\mu^{\theta_n}}{1+\beta\mu^{\theta_n}}\right)\widehat{\pi} \tag{32}$$

Note that the term $\frac{\beta\mu^{\theta_n}}{1+\beta\mu^{\theta_n}}$ increases faster with μ than does the first term $\frac{\mu^{\theta_n-1}}{1+\mu^{\theta_n-1}}$, so that the output-inflation relation is positive at low inflation rates, but negative at high inflation rates. For $\beta = 0.98$ (corresponding to a real interest rate of 4% per year), the change-over occurs at a steady-state inflation rate of 6%.

3.3 Sensitivities

The key parameters of the model are (a) the discount factor, $\beta = \frac{1}{1+r}$, where r is the real interest rate, (b) the elasticity of intertemporal substitution of labor, η, (c) the elasticity of substitution between different labor types, θ_n, and Let us consider the sensitivities of the relation between real variables and inflation to each of these parameters in turn.

3.3.1 The Interest Rate

Figure 2 shows the sensitivity to the real interest rate r for the indivisible labor case. The greater is r, the stronger is the time discounting effect. Thus, for any positive money growth rate, the more the contract wage lags behind the average price level over the contract period, and this leads the firms to employ more labor and produce more output. In Figure 2, consequently, increases in the real interest rate move the employment-inflation and output-inflation curves upwards.

3.3.2 The Elasticity of Intertemporal Substitution

Consider a more general utility function in which the elasticity of utility with respect to leisure may be greater than unity: $\eta > 0$. In that case, households have a preference for smoothing their labor through time. Figures 3a and 3b show how steady-state employment and output vary with money growth for several illustrative values of η. Observe that increases in the elasticity η shift the steady-state employment-inflation and output-inflation curves downwards. Even raising η from zero to 0.2 significantly reduces the range over which employment and output rise with money growth and the output-inflation relation turns negative at a lower inflation rate.

The reason is what we have called the *labor-supply smoothing effect*. Due to employment cycling, individual households cannot provide a constant stream of labor services through time. Since the marginal disutility of labor rises with labor, these fluctuations in hours worked makes them worse off. In response, they set their contract wage so as to

ensure that they supply less labor, on average, over the contract period. Greater money growth leads to greater employment cycling, and thus lower average labor supply and, via the production function, lower output. In this way, the labor-supply smoothing effect imparts a negative relation to the steady-state employment-inflation tradeoff and, via the production function, to the output-inflation tradeoff.

To see the intuition underlying this result, consider the case of two wage-setting cohorts ($N = 2$) and set $\beta = 1$ to eliminate the discounting effect. Then the labor supply function (14) becomes

$$(1 - n_t)^{-\eta} n_t + (1 - n_{t+1})^{-\eta} n_{t+1} = k \tag{33}$$

Note that the present value of the marginal disutility of labor (the right side of this equation) is convex in labor, so the household prefers to smooth its labor through time. Equation (33) indicates that the greater the difference between n_{t+1} and n_t (i.e. the greater the degree of employment cycling), the lower the average level of employment over the contract period. Thus, in the absence of the discounting effect, the employment-inflation relation is always negative, on account of the labor-supply smoothing effect.

Furthermore, output falls faster than employment as money growth increases, since the effect of money growth on output depends not only on the labor-supply smoothing effect, but also on employment cycling.

There is always a region around zero money growth where discounting effects dominates leading to an increase in labor and output with money growth, but this region is tiny for the standard calibrations.

3.3.3 Elasticity of Substitution in Production

Figure 4 shows sensitivity to the elasticity of substitution in production, θ_n. Increasing θ_n has two effects. First, different labor types become closer substitutes and thus a given amount of employment cycling is associated with higher output. In other words, holding the degree of employment cycling constant (i.e. constant variation of employment over the contract period), an increase in θ_n raises output.

Second, an increase in θ_n induces the firm to raise employment cycling, since it is now less costly to substitute among labor types over the contract period. For a given degree of labor substitutability in production, an increase in employment cycling leads to a fall in output, since different labor types are imperfect substitutes.

For the above calibrations of the other parameters, it turns out that for all but very low rates of inflation, the second effect dominates the first. Thus in Figure 4 the output-inflation curve is shifted downwards. Moreover, the employment-inflation curve shifts upwards, because the greater is the degree of cycling, the stronger is the time discounting effect.

The low values of θ_n (up to 10) correspond to the usual microeconomic parameterizations, whereas the high values (over 10) correspond to parameter values backed out from standard macro estimates of Taylor wage equations (e.g. Taylor (1980b) and Sachs (1980)).

3.3.4 The Contract Multiplier

Our analysis indicates that the contract multiplier has a negligible influence on the long-run response of real variables to money growth. In other words, changing the number of wage-setting cohorts, while holding the length of the contract period constant, has little effect on the long-run employment-inflation and output-inflation relations.[10] What matters for the long-run relation between real and monetary variables, as we will see in the following section, is the length of the contract period, rather than the degree of staggering.

4 Endogenising the Frequency of Nominal Adjustments

Thus far we have assumed that the frequency of nominal adjustments (the length of the wage contract period) remains constant regardless of the rate of money growth. In this regard, our analysis is susceptible to a version of the Lucas critique, since agents may have an incentive to set wages more frequently as the rate of money growth rises. We now consider the following question: When the frequency of nominal adjustments is endogenised, does the long-run tradeoff between inflation and real variables disappear, so that a NAIRU or natural rate (of output, employment, etc.) is restored? Our answer, perhaps surprisingly, is no.

Intuitively, our analysis generates a tradeoff between real variables and inflation on

[10] In a related area, Chari, Kehoe and McGrattan (1996) show that the contract multiplier has a negligible effect on the persistence of real effects from temporary monetary shocks. Note, of course, that this persistence is a different phenomenon from the slope of the long-run relation between real and nominal variables, investigagted here.

account of a nominal rigidity (wage staggering). But if an increase in inflation leads agents to adjust wages more frequently, then the nominal rigidity falls as inflation rises. If the nominal rigidity were to fall sufficiently wages would become fully flexible and the tradeoff between real and nominal variables would be eliminated. We will show, however, that this does not happen unless inflation is extremely high. In the absence of hyper-inflation, the real and monetary sides of the economy remain inextricably interdependent.

To model the frequency of nominal adjustments, we now allow households to choose the length of their wage contract period (N), assuming there is a fixed cost F to changing wages. In practice, this fixed costs generally involves much more than a negotiation cost (e.g. the cost of the time spent negotiating, the expected cost of a breakdown in negotiations, etc.), since wage adjustment are typically accompanied by performance and salary reviews. Formally, the household's problem is to choose N so as to maximize its discounted stream of utility, subject to its constraints:[11]

$$\max_{N} \sum_{j=0}^{\infty} \sum_{i=Nj}^{N(j+1)-1} \beta^{i} \left[\log c_{t+i} + \varsigma \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} - F \right] \tag{34}$$

subject to the budget constraints (5) and the labor demand functions (6).

Note that in the steady state this becomes an infinite sum of identical N period problems, so it can be rewritten as:

$$\max_{N} \frac{1}{1-\beta^{N}} \left[\sum_{i=0}^{N-1} \beta^{i} \left(\log c_{t+i} + \varsigma \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} \right) - F \right] \tag{35}$$

which can be solved numerically.

We calibrate the fixed cost by assuming that at a steady state inflation rate of 4%, wages are set for one year. This is roughly in line with the evidence given by Backus (1984), Benabou and Bismut (1987), Levin (1991), and Taylor (1993, 1998).[12]

[11] In this problem, the household splits the future into a series of N-period problems. The first N-period problem is $\sum_{i=0}^{N-1} \beta^{i} \left[\log c_{t+i} + \varsigma \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} - F \right]$, the second problem is $\sum_{i=N}^{2N-1} \beta^{i} \left[\log c_{t+i} + \varsigma \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} - F \right]$, and so on, to the infinite future. The household then choose the size of the partition N so as to maximize the present value of its utility. See Devereux and Yetman (2002) for an analogous problem.

[12] Much of this evidence is indirect, in the sense that it is inferred from autocorrelation functions of aggregate wage data. Direct evidence on the union sector in the US, presented in Taylor (1983) and Cecchetti (1984), suggest longer average contract periods. However, much more work needs to be done in exploring the frequency of wage and price change.

When the frequency of nominal adjustments is endogenized, the labor supply decision needs to be

Solving the model for an elasticity of labor supply $\eta = 0$, we obtain Figure 5a. These results are striking and may be summarized as follows.

First, that *the endogenisation of the contract period has not eliminated the long-run relation between inflation and real variables.* The reason is that changing wages is costly, in real terms. Specifically, the higher the rate of money growth (and the higher the associated rate of steady-state inflation), the more employment cycling firms do and the lower the average employment over the contract period for any given contract wage. The household can reduce employment cycling and raise average employment over the contract period (for any given contract wage) by reducing the contract length. This is the household's benefit from reducing the contract length N in response to an increase in steady-state inflation. However, reducing N means expending the fixed cost F more frequently. Consequently, when steady-state inflation rises, the contract period does not fall by enough to eliminate the tradeoff between inflation and real variables.

Figure 5b shows the effect of endogenising N for a wider range of inflation. At sufficiently high rates of inflation, the output-inflation and employment-inflation relations eventually bend backward so that a further increase in money growth *reduces* employment. At money growth rates that are substantially higher, the benefits of wage adjustment become so large relative to the costs that wages and prices become perfectly flexible, and there is no longer a trade off between real variables and inflation. However, this outcome only obtains under extreme hyper-inflations.

Second, *when the contract period is endogenous, a rise in steady-state inflation rate leads to shorter contracts and, as a result, the employment-inflation relation becomes weaker (closer to the horizontal axis) than when the contract period remains constant.*

To explain this result intuitively, we show that shorter contracts dampen the time-discounting effect, and thus employment is lower at any given level of inflation. As we have seen, the current contract wage depends on the current price level and all the expected future price levels over the span of the contract. The shorter the contract, the shorter is the range of expected future price levels on which the current contract wage depends. If households have a positive rate of time discount, then the less heavily will future price

recalibrated for each frequency. Specifically, as inflation rises, the frequency of wage adjustments rises and thus the contract length falls. The households wage setting decision (14) sets the present value of the marginal disutility of labor equal to the associated present value of the marginal utility of consumption. The shorter the contract length, N, the lower is the present value of the marginal utility of consumption, *ceteris paribus*.

levels be discounted. Thus, for any given rate of money growth (steady-state inflation) the current contract wage will lag less far behind the price level and, by implication, the higher is the average real wage over the contract period and thus the lower is employment.

To see why this is so, consider two adjacent wage-setting cohorts, i.e. two cohorts who set their wages in immediate succession: for example, one in time period $t-1$ and the other in period $t-2$, and let the wage contract be more than two periods long. The ratio of period-t employment across these cohorts depends on the difference in their real wages. In steady state, the real wage of cohort A in period $t-2$ is the same as the real wage of cohort B in period $t-1$. Under positive inflation, the real wage of each cohort falls through time. Thus, since cohort A set its wage before cohort B, the real wage of A at time t is lower than the real wage of B at time t. The greater the inflation rate, the lower is A's real wage relative to that of B. The ratio of employment across these cohorts depends on this divergence in real wages.

Formally, rewrite the labor demand function (6) in terms of detrended wages: $n_{t+i}(h) = \left(\frac{W_t^*}{W_{t+i}}\mu_t^{-i}\right)^{-\theta_w} y_{t+i}$. Thus, in the steady state, $n_{+i} = \left(\frac{W^*}{W}(N)\,\mu^{-i}\right)^{-\theta_w} y$, where n_{+i} is steady-state employment i periods after the wage is reset. Now consider two adjacent cohorts in the steady state: $n_{+1} = \left(\frac{W}{W_{t+i}}\mu^{-1}\right)^{-\theta_w} y$ and $n_{+2} = \left(\frac{W}{W_{t+i}}\mu^{-2}\right)^{-\theta_w} y$. Then the ratio of employment across adjacent cohorts depends only on the inflation rate (which determines their relative real wages):

$$\frac{n_{+1}}{n_{+2}} = \mu^{-\theta_w} \tag{36}$$

Thus, the steady-state employment of the various cohorts at a particular time may be depicted by the sequence $(n_{+1}, n_{+2}, ..., n_{+N})$, which is a rising geometric series. If a change in the contract period N would lead to a mean-preserving spread on cohort employment, then aggregate employment would remain unchanged. Specifically, suppose that contract period falls from 5 to 3, so that the employment vector contracts from $\left(n'_{+1}, n'_{+2}, n'_{+3}, n'_{+4}, n'_{+5}\right)$ to (n_{+1}, n_{+2}, n_{+3}). Then, in a mean-preserving spread, the fall in the size of the largest cohort (from n'_{+5} to n_{+3}) would be equal to the rise in the size of the smallest cohort (from n_{+1} to n'_{+1}).

So, when the contract period N shortens in response to a rise in inflation, will households adjust their wages so as to induce a mean-preserving spread in cohort employment?

The answer is no, on account of the discounting effect. When the contract period shortens, households have less opportunity to discount the future, and thus the wages it receives are valued more highly. On this account, households set their contract wage higher than the wage that would yield a mean-preserving spread in cohort employment. From the time the contract wage is set, their real wage falls over the contract period, raising employment (with an elasticity greater than unity) and consumption as the contract period progresses. Due to the curvature of the households' utility function, the households' disutility of labor will rise relative to the utility of consumption. Moreover, since the shortening of the contract periods leads households to discount the future less heavily, households' attach more importance to the increase in the disutility of labor relative to the utility of consumption. Thus, they set their wage higher, and induce lower employment.

In short, the firm's employment decisions depend on the real wages of different cohorts at the *same point in time*, whereas the household's wage setting decisions depend on the real wages they receive *through time*. And since households discount the future, a shortening of the contract period leads them to adjust their wages so as to sacrifice some present utility for the sake of future utility. Thus aggregate employment falls.

Formally, for the simple case in which $\eta = 0$, the labor supply relation (14) is

$$\sum_{i=0}^{N-1} \beta^i n_{t+i} = \frac{(\theta_w - 1)}{\varsigma \theta_w} \sum_{i=0}^{N-1} \beta^i \tag{37}$$

and in the steady state, this may be expressed as

$$\sum_{i=0}^{N-1} \left(1 + \beta\mu^{\theta_w} + ... + \beta\mu^{N\theta_w}\right) n_{+0} = \frac{(\theta_w - 1)}{\varsigma \theta_w} \sum_{i=0}^{N-1} \beta^i = \sum_{i=0}^{N-1} \left(\beta\mu^{\theta_w}\right)^i n_{+0} \tag{38}$$

Moreover, aggregate employment is

$$n = \frac{1}{N} \sum_{i=0}^{N-1} n_{+i} = \frac{1}{N} \left(1 + \mu^{\theta_w} + ... + \mu^{N\theta_w}\right) n_{+0} \tag{39}$$

$$= \frac{\left(1 - \mu^{N\theta_w}\right)\left(1 - \beta\mu^{\theta_w}\right)}{\left(1 - \mu^{\theta_w}\right)\left(1 - \left(\beta\mu^{\theta_w}\right)^N\right)} \frac{(\theta_w - 1)}{\varsigma \theta_w} \tag{40}$$

Thus, aggregate employment is increasing in N when there is a discounting effect (i.e. when $\beta < 1$).

Third, *the output-inflation relation is weaker (closer to the horizontal axis) under en-*

dogenous contract length than when the contract period remains constant. When inflation increases, and the contract period shortens as result, there are two effects on output: (a) As noted above, employment falls (for any given level of inflation). Thus, for any given degree of employment cycling, aggregate output falls. (b) As the contract period shortens, the degree of employment cycling falls. Since different types of labor are imperfect substitutes, less employment cycling means more output. As shown in Figure 5, the second effect dominates.[13] Intuitively, it is clear that the second effect must dominate when inflation is sufficiently high, because eventually gives wage setters have the incentive to set wage flexibly ($N = 1$). Then money superneutrality is reestablished and the output-inflation curve coincides with the horizontal axis.

5 Conclusions

This paper has investigated how the long-run employment-inflation and output-inflation tradeoffs depend on microeconomic behavior patterns. In particular, we have shown that these tradeoffs are the outcome of three interrelated influences: (i) The greater the degree of time discounting, the more the contract wage depends on current rather than future prices, and thus the more the contract wage lags behind the price level when there is inflation. (ii) Staggered wage contracts in an inflationary environment lead to employment cycling, i.e. firms substitute among different labor types in responses to their relative real wage movements. Since different labor types are imperfect substitutes, the greater the degree of employment cycling, the lower will be the level of output (*ceteris paribus*). (iii) Households prefer stable employment paths to those that vary over time. Thus greater the degree of employment cycling, the greater will be the average real wage that workers will require. The first influence imparts a positive relation to the employment-inflation and output-

[13] Formally, the production function may be expressed as follows:

$$
\begin{aligned}
y &= \left(\frac{1}{N}\sum_{i=0}^{N-1} n_{+i}^{\frac{\theta_w-1}{\theta_w}}\right)^{\frac{\theta_w}{\theta_w-1}} \\
&= n_{+0}\left(\frac{1}{N}\sum_{i=0}^{N-1} \mu^{\theta_w-1}\right)^{\frac{\theta_w}{\theta_w-1}} \\
&= n_{+0}\left(\frac{1}{N}\frac{1-\mu^{N(\theta_w-1)}}{1-\mu^{\theta_w-1}}\right)^{\frac{\theta_w}{\theta_w-1}}
\end{aligned}
$$

Thus, aggregate output rises less fast than aggregate employment with the contract period N.

inflation tradeoffs, whereas the latter two influences impart a negative relation to these tradeoffs. The relative strength of the three influences, of course, depends on the relative magnitudes of the underlying microeconomic parameters: the rate of time discount, the elasticity of substitution between different labor types, and the elasticity of labor supply.

We have shown that when the contract length is endogenised, the long-run levels of output and employment may be sensitive to changes in money growth (and thus inflation) at low levels of inflation, but at higher levels of inflation real variables become independent of inflation. This is so even at contract periods of significant length, i.e. the relation between real variables and inflation disappears before inflation becomes so high that wages and prices become fully flexible.

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Figure 1 : Output, employment and money growth

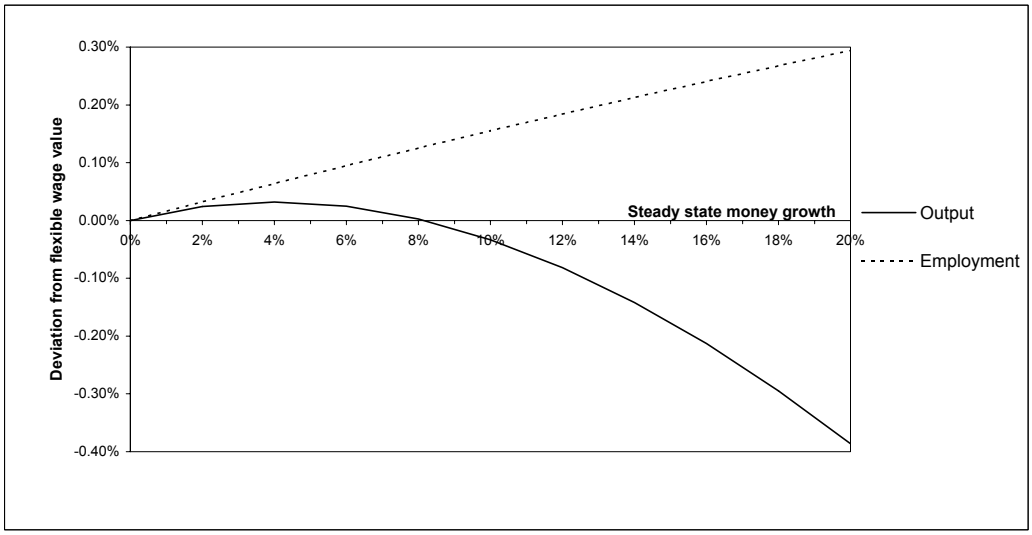


Figure 2 : Sensitivity to the real interest rate

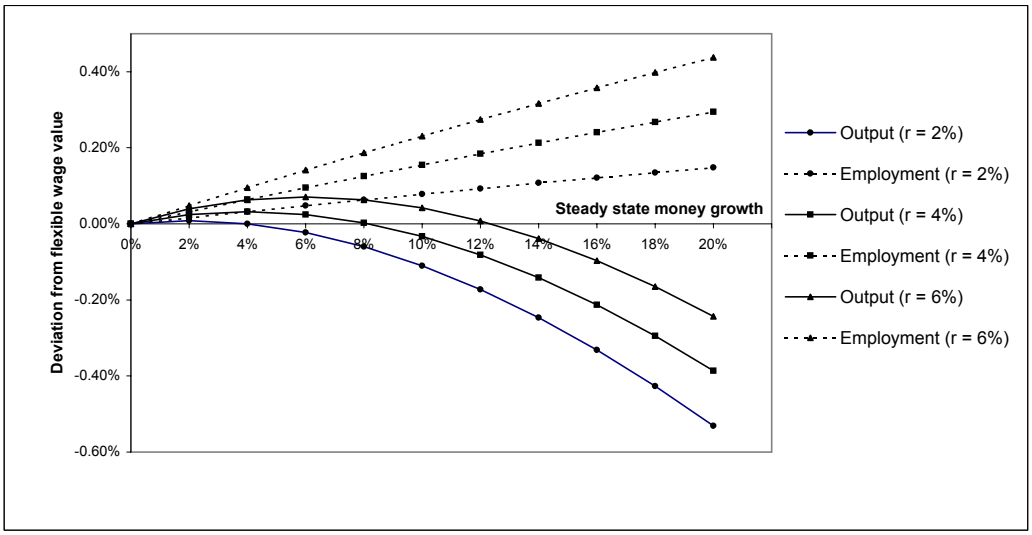


Figure 3a : Sensitivity to the elasticity of labour supply (eta)

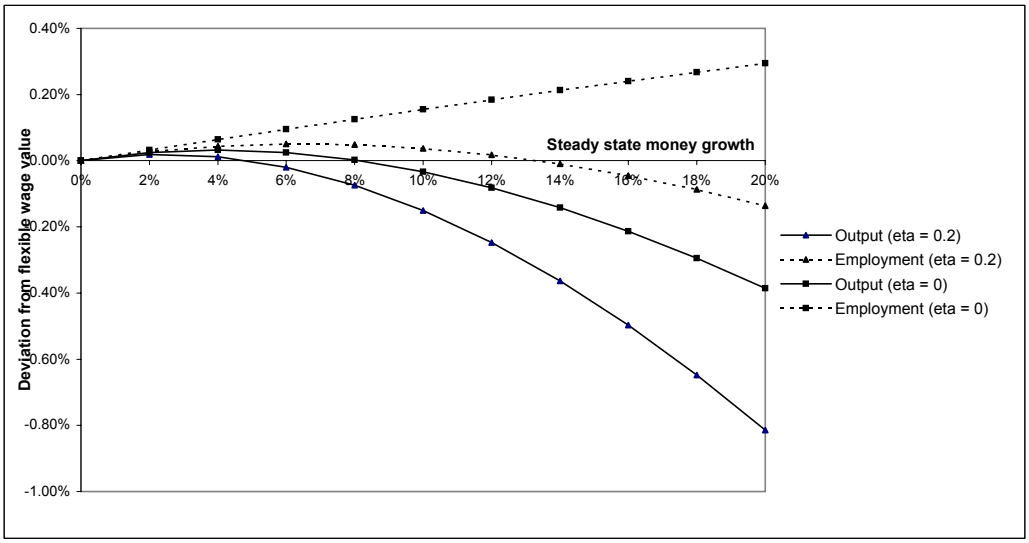


Figure 3b : Sensitivity to the elasticity of labour supply (eta)

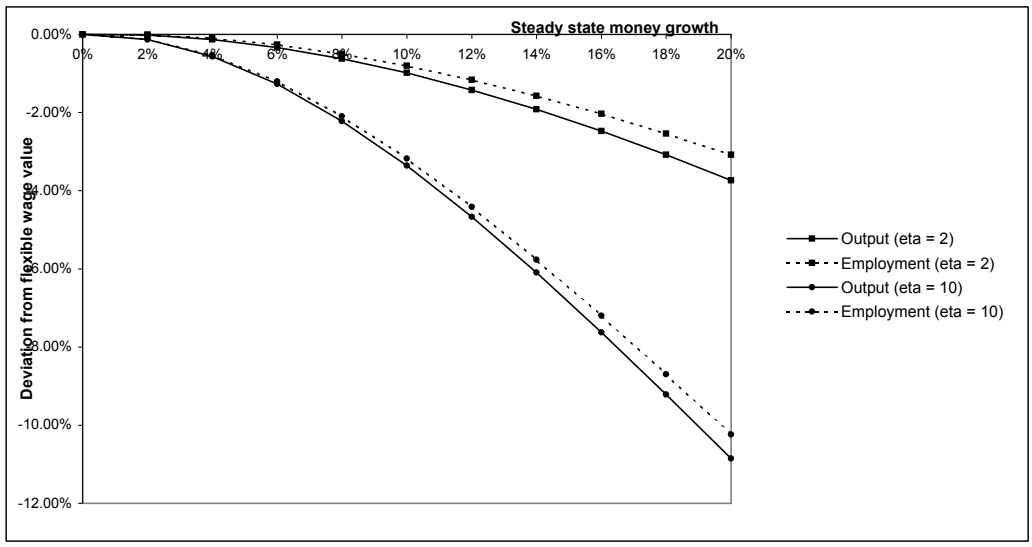


Figure 4 : Sensitivity to the elasticity of substitution between labour types (theta)

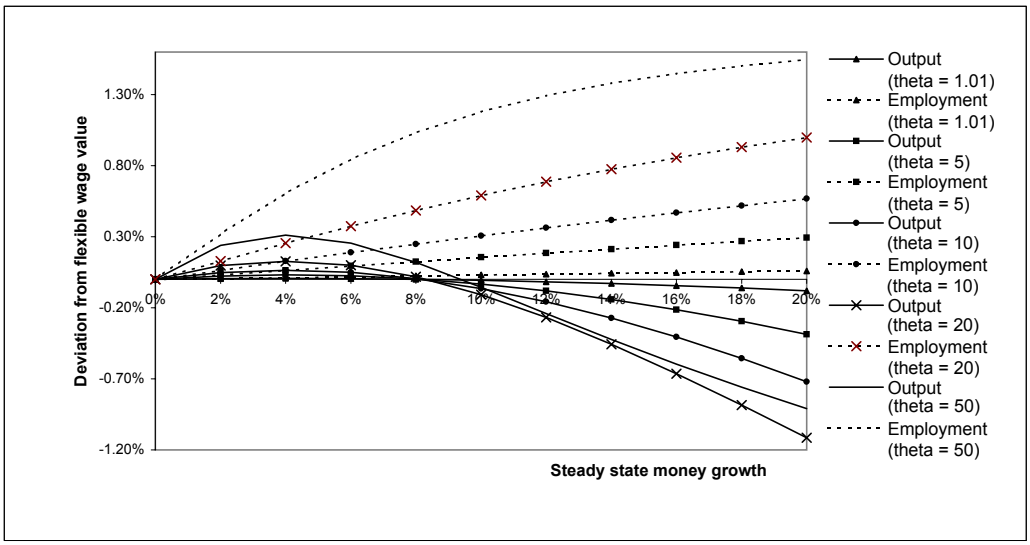


Figure 5a : Endogenising N : 0% - 20% inflation

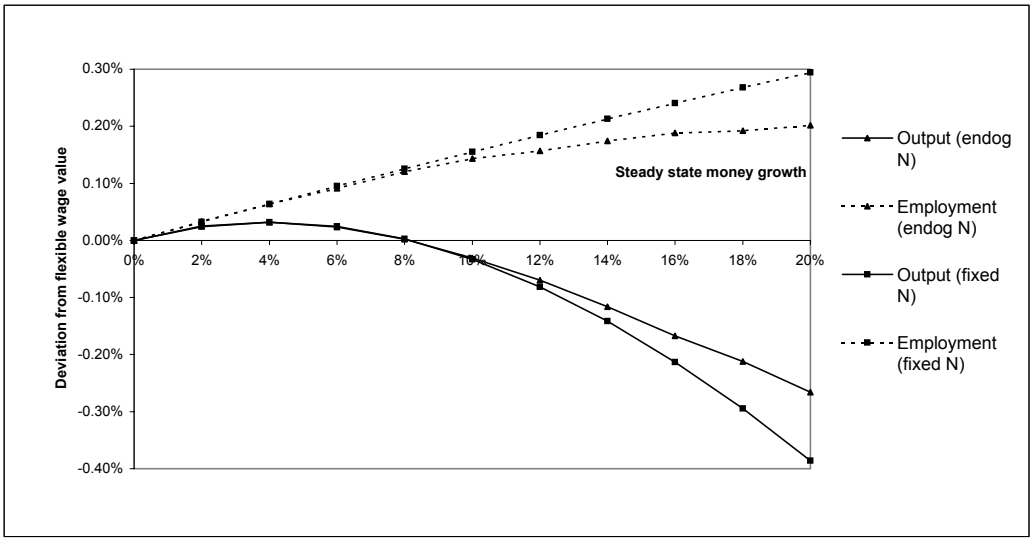
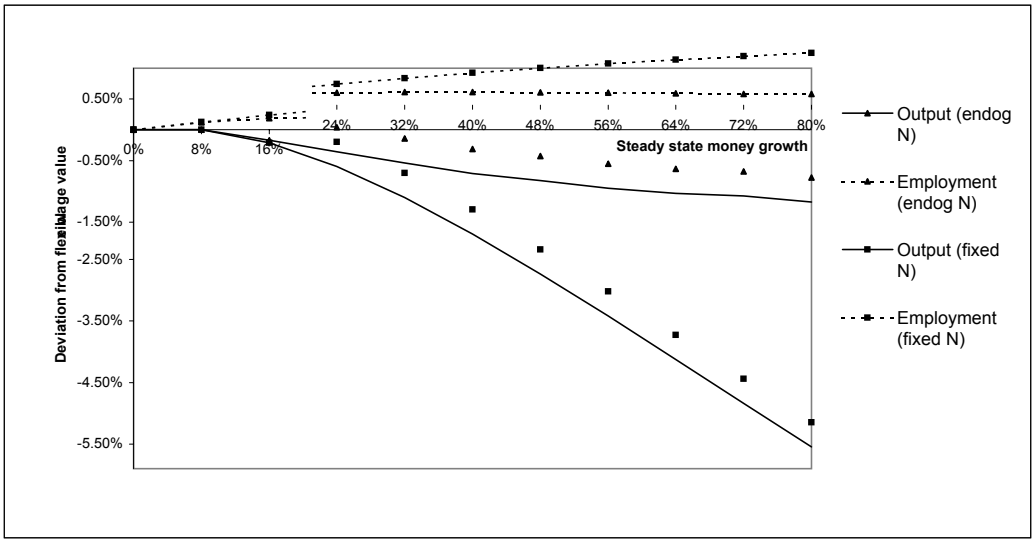


Figure 5b : Endogenising N : 0% - 80% inflation



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