

Moschitz, Julius

**Working Paper**

## The determinants of the overnight interest rate in the euro area

ECB Working Paper, No. 393

**Provided in Cooperation with:**

European Central Bank (ECB)

*Suggested Citation:* Moschitz, Julius (2004) : The determinants of the overnight interest rate in the euro area, ECB Working Paper, No. 393, European Central Bank (ECB), Frankfurt a. M.

This Version is available at:

<https://hdl.handle.net/10419/152827>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



**NO. 393 / SEPTEMBER 2004**

# EURO AREA

by Julius Moschitz



EUROPEAN CENTRAL BANK



## WORKING PAPER SERIES

NO. 393 / SEPTEMBER 2004

# THE DETERMINANTS OF THE OVERNIGHT INTEREST RATE IN THE EURO AREA <sup>1</sup>

by Julius Moschitz <sup>2</sup>



In 2004 all publications will carry a motif taken from the €100 banknote.

This paper can be downloaded without charge from <http://www.ecb.int> or from the Social Science Research Network electronic library at [http://ssrn.com/abstract\\_id=586764](http://ssrn.com/abstract_id=586764).

<sup>1</sup> I am very grateful to Steen Egerskov, Clara Martin Moss, Livio Stracca and especially Nuno Cassola for many helpful discussions, detailed comments and inspiring thoughts. Special thanks to Gabriel Pérez Quirós and Hugo Rodríguez Mendizábal for introducing me to the topic and their comments and suggestions, and Thomas Eife for stimulating discussions. The paper also benefited from comments of the editor and an anonymous referee of the ECB Working Paper series.

All remaining errors are mine. A fellowship from the Spanish Ministry of Education, as well as the hospitality of the European Central Bank is gratefully acknowledged.

<sup>2</sup> Universitat Autònoma de Barcelona, Dept. d'Economia i d'Història Econòmica, 08193 Bellaterra, Barcelona, Spain; tel.: +34 93 581 1813, fax: +34 93 581 2012, e-mail: [moschitz@idea.uab.es](mailto:moschitz@idea.uab.es)

© European Central Bank, 2004

**Address**

Kaiserstrasse 29  
60311 Frankfurt am Main, Germany

**Postal address**

Postfach 16 03 19  
60066 Frankfurt am Main, Germany

**Telephone**

+49 69 1344 0

**Internet**

<http://www.ecb.int>

**Fax**

+49 69 1344 6000

**Telex**

411 144 ecb d

*All rights reserved.*

*Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.*

*The views expressed in this paper do not necessarily reflect those of the European Central Bank.*

*The statement of purpose for the ECB Working Paper Series is available from the ECB website, <http://www.ecb.int>.*

ISSN 1561-0810 (print)

ISSN 1725-2806 (online)

## CONTENTS

|   |    |
|---|----|
| Abstract                                      | 4  |
| Non-technical summary                         | 5  |
| 1 Introduction                                | 7  |
| 2 A model of the reserve market               | 10 |
| 2.1 Demand side                               | 12 |
| 2.2 Supply side                               | 14 |
| 2.3 Equilibrium                               | 18 |
| 2.4 Expected and unexpected changes in supply | 19 |
| 2.5 Underbidding                              | 22 |
| 3 Empirical analysis                          | 23 |
| 3.1 Model specification                       | 23 |
| 3.2 Estimation results and discussion         | 28 |
| 4 Conclusions and further research            | 33 |
| References                                    | 35 |
| A Basic statistics and estimation results     | 38 |
| B Data description                            | 45 |
| C Figures                                     | 46 |
| European Central Bank working paper series    | 55 |

## Abstract

The overnight interest rate is the price paid for one day loans and defines the short end of the yield curve. It is the equilibrium outcome of supply and demand for bank reserves. This paper models the intertemporal decision problems in the reserve market for both central and commercial banks. All important institutional features of the euro area reserve market are included. The model is then estimated with euro area data. A permanent change in reserve supply of one billion euro moves the overnight rate by eight basis points into the opposite direction, hence, there is a substantial liquidity effect. Most of the predictable patterns for the mean and the volatility of the overnight rate are related to monetary policy implementation, but also some calendar day effects are present. Banks react sluggishly to new information. Implications for market efficiency, endogeneity of reserve supply and underbidding are studied.

**JEL classification:** E52; E58; E43.

**Keywords:** Money markets; EONIA rate; Liquidity effect; Central bank operating procedures.



## Non-technical summary

This paper studies the determinants of the overnight interest rate and quantifies them. The overnight interest rate is the equilibrium outcome of supply and demand for bank reserves. The here developed structural model for both supply and demand for reserves allows a detailed analysis of the interactions between the central bank, as the sole net supplier of reserves, and commercial banks, on the demand side. The precise set-up of this market, i.e. institutional details of the reserve market, has important implications for the behavior of the overnight rate, both for conditional mean and variance. These implications are derived from a theoretical model and their magnitudes are estimated for the euro area overnight rate.

The behavior of the overnight interest rate is important for several reasons. Firstly, in most monetary models the central bank is assumed to have perfect control over the interest rate. The transmission mechanism of monetary policy in these models starts at the short-term interest rate. A change in the short-term rate works through to long-term interest rates. These long-term rates are the relevant variables for firms' investment and households' savings decisions. Investment and saving then influence output and prices, the final objectives of a central bank. However, the control of the short-term interest rate is far from perfect in practice. Interest rates are determined on markets, being influenced by both supply and demand side factors. The central bank has a strong influence on the supply side, but is not able to control it perfectly. This paper studies the, widely overlooked, first step in the monetary transmission mechanism, the relation between reserves and the overnight rate. In particular, the assumption made in many models that the central bank has perfect control over the interest rate is analyzed. The ways in which the details of monetary policy implementation affect the behavior of the interest rate are documented.

Secondly, the short-term rate is an important explanatory variable for long-term interest rates. According to the expectation hypothesis the N-period yield is the average of expected future one-period yields, possibly adjusted for a risk premium. Therefore, understanding better the behavior of the short end of the yield curve - the overnight rate - helps explaining other interest rates further out the term structure as well.

Thirdly, in efficient markets there are no (long-lasting) arbitrage opportunities. Predictable patterns usually provide such arbitrage opportunities. Both mean and volatility of the overnight rate are tested for predictable patterns and implications for market efficiency are investigated.

Finally, central banks have a natural interest in studying the determinants of the overnight rate. This is particularly true nowadays as the operating target of many central banks is a short-term interest rate. The behavior of the overnight rate depends on reserve supply, but equally important on the institutional framework for the reserve market.

It is documented that the overnight rate reacts to expected future changes in the policy rate and to permanent changes in supply of reserves. In fact, a substantial liquidity effect is estimated: a change in reserve supply of one billion euro, expected to prevail till the end of the reserve maintenance period, moves the interbank rate eight basis points into the opposite direction. The theoretical model relates the magnitude of the liquidity effect to the distribution of supply shocks, which is confirmed by the data. Interestingly, banks do not react immediately to supply changes. This sluggish reaction to supply changes is not easily explained for rational agents. Temporary supply changes have no effect on the overnight rate.

Predictable patterns are found for the overnight rate. The mean is high at the last day of a month, even higher on the end of a semester or a year. The end of the month, semester and year increases are completely reversed at the first day of the following month. End of month effects are most likely due to window dressing operations. The mean of the overnight rate does not vary systematically throughout the reserve maintenance period. Therefore, the short-term money market does not contain clear arbitrage opportunities, with the possible exception of the sluggish reaction to supply shocks.

The conditional volatility of the overnight rate is closely related to monetary policy implementation. Conditional volatility is especially high at the allotment day of the last open market operation in a reserve maintenance period, and even higher at days afterwards. Volatility increases at the day of a change in the policy rate and around the end of a month.



# 1 Introduction

This paper studies the determinants of the overnight interest rate and quantifies them. The overnight interest rate is at the short end of the yield curve and the equilibrium outcome of supply and demand for bank reserves. The here developed structural model for both supply and demand for reserves allows an in-depth analysis of the interaction between the central bank, as the sole net supplier of reserves, and commercial banks, on the demand side. The precise set-up of this market, i.e. institutional details of the reserve market, has important implications for the behavior of the overnight rate, both for conditional mean and variance. These implications are derived from a theoretical model and their magnitudes are estimated for the euro area overnight rate.

The behavior of the overnight interest rate is important for several reasons. Firstly, in most monetary models the central bank is assumed to have perfect control over the interest rate. The transmission mechanism of monetary policy in these models starts at the short-term interest rate.<sup>1</sup> A change in the short-term rate works through to long-term interest rates. These long-term rates are the relevant variables for firms' investment and households' savings decisions. Investment and saving then influence output and prices, the final objectives of a central bank. However, the control of the short-term interest rate is far from perfect in practice. Interest rates are determined on markets, being influenced by both supply and demand side factors. The central bank has a strong influence on the supply side, but is not able to control it perfectly. This paper studies the, widely overlooked, first step in the monetary transmission mechanism, the relation between reserves and the overnight rate. In particular, the assumption made in many models that the central bank has perfect control over the interest rate is analyzed. The ways in which the details of monetary policy implementation affect the behavior of the interest rate are documented.

Secondly, the short-term rate is an important explanatory variable for long-term interest rates. According to the expectation hypothesis the N-period yield is the average of expected future one-period yields, possibly adjusted for a risk premium.<sup>2</sup> Therefore, understanding better the behavior of the short end of the yield curve - the overnight rate - helps explaining other interest rates further out the term structure as well.<sup>3</sup>

Thirdly, in efficient markets there are no (long-lasting) arbitrage opportunities. Predictable patterns usually provide such arbitrage opportunities. Both mean and volatility of

---

<sup>1</sup>See for example Walsh (1998) for a book-length treatment of monetary models.

<sup>2</sup>Cochrane (2001) discusses extensively the expectation hypothesis and reviews models for the term structure of interest rates.

<sup>3</sup>See e.g. Fabozzi and Modigliani (1996) for a general analysis of money markets. More specifically, Cassola and Morana (2003 and 2004) and Cassola and Moschitz (2004) analyse the transmission of volatility along the euro area yield curve.

the overnight rate are tested for predictable patterns and implications for market efficiency are investigated.

Finally, central banks have a natural interest in studying the determinants of the overnight rate. This is particularly true nowadays as the operating target of many central banks is a short-term interest rate.<sup>4</sup> The behavior of the overnight rate depends on reserve supply, but equally important on the institutional framework for the reserve market.

With these issues in mind the overnight rate is analyzed and the reserve market is discussed with respect to market efficiency, the importance of institutional features and the ability of the central bank to control the interest rate.

In the literature so far the overnight interest rate has not been analyzed extensively, especially in the euro area. One of the earliest statistical descriptions of the daily behavior of the US overnight rate is given by Hamilton (1996 and 1997). More recently, also Bartolini et al. (2001 and 2002) develop models for the US overnight rate, which is known as the federal funds rate. Although the basic set-up in the US and euro area reserve markets are similar, there are important institutional differences making these models not very good descriptions of the euro area overnight rate. Pérez and Rodríguez (2003) provide an optimizing model for reserve demand in the euro area. Gaspar et al. (2004) expand this model to heterogeneous banks. Bindseil and Seitz (2001) model the supply of reserves in close relation to the institutional set-up in the euro area, but the demand side is not derived explicitly. Välimäki (2002) is the first one to provide a model of optimizing behavior for both supply and demand side. However, he makes the simplifying assumption of daily supply of reserves. Under normal circumstances reserves are supplied only once a week in the euro area. Würtz (2003) proposes an econometric model of the overnight rate, focusing mainly on an empirical description. On the contrary, the present paper derives the empirical formulation from a structural model of both supply and demand for reserves, which allows to pin down precisely the effects of implementation issues on the interest rate. Furthermore, the exact supply measure relevant for demand decisions is used and possible endogeneity of reserve supply is tackled.

The present analysis starts with a theoretical model for both supply and demand in the euro area reserve market. The central bank is the sole net supplier of reserves and commercial banks represent the demand side. The model is set up in an intertemporal optimization framework. Not only the current situation in the market is relevant for decisions, but also expected future events. The demand side follows closely Pérez and Rodríguez (2003), augmenting it in order to allow changes in the policy rate. The policy rate is the target rate for the overnight rate.<sup>5</sup> Since banks are forward looking expected changes in the policy rate are important for

---

<sup>4</sup>Borio (1997) offers a detailed discussion of monetary policy operating procedures in industrial countries.

<sup>5</sup>The minimum bid rate of variable rate tenders and the rate applied to fixed rate tenders for the euro area

the behavior of the current overnight rate. Furthermore, a detailed description of the supply side, including all main institutional features of the central bank's operating procedure, is necessary to characterize adequately the determination of the overnight rate. Therefore, the supply of reserves is modeled with a weekly frequency.

Special attention is paid to distinguish expected, unexpected, temporary and permanent supply changes and their effects on the overnight rate. The weekly frequency of the central bank's supply of liquidity implies reserve holdings to change expectedly throughout the week. In addition there are unexpected changes, the so-called supply shocks. In general, these supply shocks are temporary. However, if they occur after the last regular liquidity supply in a reserve maintenance period, these supply shocks have a permanent effect. In this case there is no further (regular) supply of liquidity within the same maintenance period to make up for past supply shocks. Accordingly, supply shocks accumulate until the end of the maintenance period and become permanent supply changes.

The equilibrium in the reserve market is discussed extensively. The model also allows to analyze a special situation in the reserve market, the so-called underbidding. If the policy rate is expected to decrease in the near future total demand for bank reserves decreases immediately. In this case the central bank is not able to supply the desired amount of reserves. The total amount of reserves is then determined at the demand side, by commercial banks. Since reserves are supplied via auctions, this situation has been labelled underbidding. Underbidding is the consequence of some specific characteristics in the reserve market and will be discussed below.

The theoretical model is then taken to the data. Great care is applied in dealing with non-standard statistical properties of the overnight rate. Numerous specification tests are performed and sub-sample stability is analyzed.

One of the main issues in this paper is to determine the effect of a change in reserve supply on the interest rate. A negative relation between reserves and the interest rate is expected. This negative relation is usually called the liquidity effect.<sup>6</sup> However, it is necessary to clarify what exactly is meant in the present paper by the liquidity effect.

Empirical evidence for a liquidity effect comes from Christiano (1991), Gordon and Leeper (1992), Galí (1992), Strongin (1995), Bernanke and Mihov (1998), Kim and Ghazali (1998) and Thornton (2001b), among others. Most of those works use monthly or quarterly data, and so the main difficulty is the identification of the relevant money supply and demand equations. Hamilton (1997) proposes an alternative by using daily data giving way for other identifying

---

main refinancing operations can be interpreted as such a target rate.

<sup>6</sup>Ewerhart et al. (2004) show that under some circumstances the liquidity effect in the money market can be reversed; a low overnight rate may be associated with a scarce liquidity situation, or correspondingly a high overnight rate may be associated with ample liquidity.

assumptions. However, as pointed out by Thornton (2001a) and Gilchrist (2001), not all papers identify the same effect. There are two different, although not unrelated, mechanisms at work. On the one hand, there is a daily demand for reserves in order to fulfill reserve requirements. If this demand is interest rate elastic, a reaction of the overnight rate to a change in liquidity is found. On the other hand, there is a longer-term interest rate elasticity of reserves. Banks have to hold a certain proportion of demand deposits as reserves. Those demand deposits are assumed to depend on an interest rate as opportunity cost. Therefore, if the interest rate changes, demand for deposits changes, and proportionally also reserve requirements. Whether this reaction happens contemporaneously depends on institutional features of reserve fulfillment. In the euro area required reserves are calculated from the previous month's deposits. This is to say that a change in today's interest rate affects next month's reserve requirement and next month's demand for reserves. Hence, the relationship between demand deposits and interest rate cannot be identified on a contemporaneous basis. Following this argumentation, the present work identifies the first effect, the liquidity effect on a daily basis. In other words, the responsiveness of the interbank rate to daily changes in the supply of reserves is analyzed. Although a possible relation between both effects is recognized, the further analysis of this issue is left for future research.

The next section provides a theoretical model for the reserve market. Both supply and demand for reserves are carefully modeled. The equilibrium overnight rate is derived. The effects of expected and unexpected supply changes on the interest rate are discussed. Underbidding is found to be an equilibrium outcome in the present set-up of the reserve market. Section 3 takes the model to the data. Numerous specification tests are performed and the determinants of the EONIA rate, a volume-weighted average of interbank overnight rates in the euro area, are analyzed extensively. Section 4 concludes and outlines further research. The appendix contains all graphs, figures and tables. In particular, it includes an illustration of the reserve market and a graphical summary of the theoretical model, as well as a detailed description of the data used and a review of predictable patterns in mean and volatility of the overnight rate.

## 2 A model of the reserve market

The reserve market is a money market where overnight, unsecured loans of reserves are exchanged.<sup>7</sup> In what follows a model for both, demand and supply side of this particular interbank market is set up. There are two types of agents in the market, the central bank on one hand and commercial banks on the other hand. The key ingredients of the model are the

---

<sup>7</sup>The very short-term money market in the US is called the federal funds market.

optimizing behavior of all agents and the inclusion of the main institutional features of the euro area interbank market. Both issues have important implications. Firstly, demand and supply equations are not simply postulated, rather they are derived from the first order conditions of the maximization problem, and so reflecting optimizing behavior of agents. Secondly, the institutional set-up of the interbank market influences the behavior of agents, therefore, the exact representation of institutional key features is necessary for an adequate model.

Commercial banks are obliged to hold deposits of a certain amount at the central bank, i.e. to hold a certain amount of reserves. However, this reserve requirement does not have to be fulfilled on a daily basis, rather it has to be fulfilled on average over a period of one month, which is called the reserve maintenance period (RMP).<sup>8</sup> The allowance of fulfilling reserves on average leads banks to face an intertemporal decision problem. Banks have to decide on an optimal path of daily reserve holdings. Given that banks have a certain amount of liquidity, it follows that the amount not desired to be held as reserves can be lend to other banks through the interbank market. In case a bank wants to hold more reserves than it has liquidity available, it can borrow at the interbank market. The price paid at the interbank market is the interbank rate. In addition, liquidity can be obtained from (or deposited at) the central bank, where the price for borrowing from the central bank is called the marginal lending rate, and the price for depositing at the central bank is called the deposit rate. To sum up, each bank decides every day on how much reserves to hold, how to act on the interbank market and what recourse to take to the standing facilities, i.e. how much to borrow from or deposit at the central bank. These decisions are made by maximizing profits from reserve management, taking the reserve requirement as a constraint. Profits are revenues minus costs, where costs of reserve management are given by borrowing from the central bank (at the lending rate) and at the interbank market (at the interbank rate), and revenues are interests earned by depositing at the deposit facility and lending to other banks.

The central bank in the model supplies liquidity in order that commercial banks can fulfill demand for reserves at an interest rate consistent with the policy rate  $i_t^*$ . Loosely speaking, the central bank can be seen as minimizing deviations of the interbank rate  $i_t$  from the policy rate  $i_t^*$ . Furthermore, the central bank also provides liquidity for the so-called autonomous factors. Examples of autonomous factors are banknotes in circulation and Treasury deposits. Figure 1 summarizes the above described interactions among central and commercial banks.

The timing of the model is represented in figure 2. When the market opens the central bank decides how much liquidity to supply, taking into account expected demand for reserves (at the policy rate) and the expected size of autonomous factors. Afterwards, commercial banks decide on how much reserves to hold and the interbank rate results. The market closes

---

<sup>8</sup>The length of the reserve maintenance period in the US is two weeks.



and the size of the autonomous factors for that day becomes known. Finally, the reserve position at the central bank and profits are determined. In general the central bank supplies liquidity only once a week, on Wednesday. On the following days up till the next Wednesday liquidity supply stays constant.<sup>9</sup> Although supply of total liquidity is constant throughout a week, reserve supply moves daily in response to shocks hitting the market.

The central bank's balance sheet can be summarized in a very stylized way as showing liquidity supply on the assets side and reserves holdings and autonomous factors on the liabilities side. From the balance sheet identity and given the supply of liquidity, it is easy to see that a change in the autonomous factors must be matched by an equal change of opposite sign in the reserve position. It follows that a forecast error in the autonomous factors affects directly the reserve position of commercial banks, hence, can be interpreted as a shock to supply of reserves. This shock changes banks' end of the day reserve positions. When making their decisions on reserve holdings banks take the existence of this supply shock into account.

## 2.1 Demand side

The demand side follows closely Pérez and Rodríguez (2003), being adapted to allow changes in the policy rate as well as in lending and deposit rates. The economy consists of a continuum of banks with measure one. Each bank maximizes expected profits from reserve management within each maintenance period, subject to the reserve requirement. The timing for any day within the reserve maintenance period is outlined in figure 2. The objective function for bank  $j$  is

$$\max_{\{B_t^j\}_{t=1}^T} E_1 \left[ \sum_{t=1}^T \pi_t^j \right]. \quad (1)$$

Reserves lent to other banks in the interbank market are described by  $B_t^j$  and  $\pi_t^j$  is the profit from reserve management at day  $t$ . Reserves deposited at the central bank are denoted by  $M_t^j$ ,  $i_t$  represents the interbank rate and  $u_t^j$  the supply shock.  $A_t^j$  is the amount of reserves a bank obtains from the central bank and it holds that  $A_t^j = M_t^j + B_t^j$ . The amount of reserves needed at  $t$  to fulfill the requirement for the whole maintenance period is denoted by  $R_t^j$ , with  $R_1^j \equiv rr$  being the size of the reserve requirement:

$$R_{t+1}^j = \max \left\{ 0, R_t^j - \max \left[ 0, M_t^j + u_t^j \right] \right\}. \quad (2)$$

---

<sup>9</sup>In practice most of the liquidity is indeed supplied weekly through open market operations (see the next section for details). However, the maturity of these open market operations is two weeks. Note that from March 2004 onwards the maturity of open market operations will be reduced to one week (see e.g. ECB, 2004).

Note that no overdrafts are allowed, in other words banks cannot run a negative reserve balance (i.e.  $M_t^j + u_t^j \geq 0$ ). In case of a potential overdraft an automatic recourse to the lending facility takes place in order to bring the bank's daily reserve position back to zero. Similarly, once the reserve requirements are fulfilled for the whole maintenance period (i.e.  $R_t^j = 0$ ), all liquidity is put automatically at the deposit facility, which is to say banks do not hold more reserves than strictly necessary. The reserve requirement has to be fulfilled throughout the RMP. It is not important at which day contributions to the reserve requirement are made, but it has to be fulfilled at the end of the RMP, i.e. the reserve requirement can be written as  $R_{T+1}^j = 0$ .

The model is solved backwards from the last day of the maintenance period,  $T$ , since on that day reserve requirements have to be fulfilled at any cost and in consequence future expected variables are not relevant for banks' demand decisions.<sup>10</sup> The resulting first order conditions describe the interbank rate  $i_t$  as a function of the bank's reserves,  $A_t^j$ . At the last day of the reserve maintenance period the demand equation is given by:

$$i_T = i_T^d + (i_T^l - i_T^d) * F(R_T^j + B_T^j - A_T^j), \quad (3)$$

where  $F(\cdot)$  is the distribution function of the supply shock,  $f(\cdot)$  its density function,  $i_t^l$  the marginal lending rate and  $i_t^d$  the deposit rate. Market clearing implies that aggregate borrowing and lending in the interbank market equals zero, i.e.  $B_T = \int_0^1 B_T^j dj = 0$ . Therefore, banks' aggregate reserves equal reserves deposited at the central bank, i.e.  $A_T = M_T$ . Aggregate reserve deficiencies at the last day in a RMP are described by  $R_T = \int_0^1 R_T^j dj$ . The demand curve for all other days,  $t = 1, 2, \dots, T-1$ , is given by:

$$i_t = i_t^l * F(B_t^j - A_t^j) + i_t^d * \left[ 1 - F(R_t^j + B_t^j - A_t^j) \right] - \int_{B_t^j - A_t^j}^{R_t^j + B_t^j - A_t^j} \frac{\partial V_{t+1}(R_{t+1}^j, A_{t+1}^j; I_{t+1})}{\partial R_{t+1}^j} f(u_t) du_t, \quad (4)$$

with the aggregate state variable defined as  $I_t = \{i_t, i_{t+1}, \dots, i_T\}$ . The value function at the last day of the RMP is

$$V_T(R_T^j, A_T^j; I_T) = \max_{B_T^j} E_T [\pi_T^j] \quad (5)$$

<sup>10</sup>The derivation of the first order conditions follows closely Pérez and Rodríguez (2003).



and for all other days

$$V_t(R_t^j, A_t^j; I_t) = \max_{B_t^j} E_t \left[ \pi_t^j + V_{t+1}(R_{t+1}^j, A_{t+1}^j; I_{t+1}) \right]. \quad (6)$$

Given the central bank's supply of reserves, the above first order conditions determine the equilibrium interbank rate. These conditions are derived from optimizing behavior in the reserve management and describe the typical path for the interbank rate. Before discussing the behavior of the interbank rate further, the central bank's supply of reserves is analyzed.

## 2.2 Supply side

The institutional details of the interbank market are crucial for understanding the behavior of the interbank rate. So the supply side of the model closely matches the actual structure of the liquidity management in the euro area.<sup>11</sup>

The central bank supplies liquidity in order to fulfill (expected) demand for reserves at an interest rate consistent with the policy rate  $i_t^*$ . Loosely speaking, the central bank can be seen as minimizing deviations of the interbank rate  $i_t$  from the policy rate  $i_t^*$ . Liquidity is supplied only once a week, with a maturity of two weeks. The main refinancing operations of the European Central Bank (ECB) have exactly these characteristics and almost all the liquidity provided in the euro area is supplied through main refinancing operations.<sup>12</sup>

The central bank's balance sheet identity requires at each day that

$$ca_t = omo_t + nsf_t - af_t = er_t + rr_t \quad (7)$$

or,

$$er_t = omo_t - af_t - rr_t + nsf_t, \quad (8)$$

where  $ca_t$  stands for current account holdings,  $omo_t$  for outstanding open market operations,  $nsf_t$  for net recourse to standing facilities,  $af_t$  for autonomous factors,  $er_t$  for excess reserves and  $rr_t$  for required reserves.<sup>13</sup> Note that current account holdings are the reserves commercial

<sup>11</sup>In what follows the benchmark liquidity policy is modelled. For a discussion of various liquidity policies see e.g. Bindseil (2002).

<sup>12</sup>Besides main refinancing operations also fine tuning and long-term refinancing operations are used by the ECB to supply liquidity. However, fine tuning operations are executed only under special circumstances. Indeed, such fine tuning operations have been performed very few times, namely at 21/6/2000, 30/4/2001, 12 and 13/9/2001, 28/11/2001, 4 and 10/1/2002, 18/12/2002 and 23/05/2003. Long term refinancing operations are structural measures and usually constant throughout the maintenance period.

<sup>13</sup>Note that, strictly speaking, the division into required reserves and excess reserves is defined only at the last day of the maintenance period. However, excess reserves at the last day of the maintenance period are largely constant across maintenance periods  $j = 1, \dots, J$ , that is  $\frac{1}{J} \sum_{j=1}^J er_{T,j} \approx 0.7 * T$  billion euro (see the box on liquidity conditions in the ECB's Monthly Bulletin, various issues). Thus, it seems reasonable to assume

banks hold at the central bank. Furthermore,

$$omo_t = mro_t + ltro_t + fto_t \quad (9)$$

where  $mro_t$  is the outstanding amount from main refinancing operation,  $ltro_t$  from long-term refinancing operations and  $fto_t$  from fine tuning operations. It is assumed that  $ltro_t$  and  $fto_t$  are constant throughout the maintenance period, that is  $ltro_t = ltro$  and  $fto_t = fto$  for all  $t = 1, \dots, T$ .<sup>14</sup>

At an allotment day, normally Tuesday, the size of  $mro_t$  is decided such that the expected excess reserve holdings in seven days are equal to the target level  $er^*$ . An amount sufficiently large in order to provide for the expected autonomous factors and expected demand for reserves, taking into account the expected recourse to standing facilities, is allotted.

Days throughout the maintenance period are denoted by  $t = 1, \dots, T$ . At  $t = s$  a new main refinancing operation is settled, where  $s \in S = \{s_1, s_2, \dots, s_k\}$  with  $s_1$  being the first Wednesday in the maintenance period, and  $s_k$  the last one.<sup>15</sup> The central bank targets average excess reserves, which means, making up for autonomous factor forecast errors of the previous week,  $\{E_{s-8}[\sum_{j=s-7}^{s-1} af_j] - \sum_{j=s-7}^{s-1} af_j\}$ . The target level for excess reserves is given by:

$$er_s^* = E_{s-1}[er_{s+n}] + \frac{1}{m} \left( \sum_{j=s-m}^{s-1} af_j - E_{s-8} \left[ \sum_{j=s-m}^{s-1} af_j \right] \right) \quad (10)$$

with  $m = \min\{7, s-1\}$  and  $n = \min\{6, T-t\}$  and for all  $s \in S$ . At the first allotment in the maintenance period the average excess reserve measure,  $er_{s_1-1}^*$ , takes into account forecast errors only from  $t = 1$  onwards, not including the days from the previous maintenance period. At the last allotment the liquidity situation at  $T$  is targeted, not the liquidity situation at the next allotment day.<sup>16</sup>

Finally, the possibility of changes in the policy rate and the so-called underbidding is included. The size of the open market operation is then:

$$\begin{aligned} mro_s = & er_s^* + (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) + rr \\ & + E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} af_j \right] - E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} nsf_j \right] - ltro - fto. \end{aligned} \quad (11)$$

excess reserves are build up linearly throughout the maintenance period, which leads to define the daily excess reserve,  $er_t$ , to be constant at 0.7 billion euro. It follows that  $rr_t = ca_t - 0.7$ .

<sup>14</sup>See footnote 12.

<sup>15</sup>All days  $t = s$  are called settlement days, whereas  $t = T$  is defined as the last day in the reserve maintenance period.

<sup>16</sup>In general,  $E_{s-1}[er_{s+n}]$  is around  $0.7 * (s+n)$  billion euro.

The central bank provides sufficient liquidity such that targeted excess reserves,  $er_s^*$ , required reserves,  $rr$ , and expected autonomous factors,  $E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} af_j \right]$ , are covered. Long-term and fine tuning operations are subtracted as well as the expected net recourse to standing facilities,  $E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} nsf_j \right]$ . Note that the central bank provides liquidity assuming a linear fulfillment of reserve requirements, that is,  $rr = \frac{\sum_{t=1}^T rr_t}{T}$ . The second term on the right hand side,  $(E_{s-1}[er_{s+m}(i_s^*)] - er_s^*)$ , corrects for the so-called underbidding. Although the central bank wants to provide a certain amount of liquidity, it cannot do so independently of demand. If demand for main refinancing operations is lower than the central bank's desired supply, one speaks of underbidding. Underbidding can be explained as the equilibrium outcome of an expected policy rate decrease together with the interest rate elasticity of reserves. If the policy rate is not expected to change, excess reserves next week are expected to equal this week's excess reserves, hence, the term in parenthesis cancels. If, however, banks expect the policy rate to change, supply of liquidity is determined by the expected demand curve, at the current policy rate. The demand curve shifts with the expected policy rate change, but the current interbank rate does not change, because it is bounded from below by the current policy rate.<sup>17</sup> Therefore, supply is determined by the new demand for excess reserves,  $er_{s+m}$ , at the current policy rate  $i_s^*$ .

Combining equations (8), (9) and (11) defines actual excess reserves on any given day:

$$er_t = \{er_s^* + (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) + rr + E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} af_j \right] - E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} nsf_j \right] - ltro - fto\} + \{fto + ltro - af_t - rr_t + nsf_t\} \quad (12)$$

which can be simplified to:

$$er_t = er_s^* + (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) + E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} af_j \right] - af_t + nsf_t. \quad (13)$$

Note that the relevant settlement day is the most recent one,  $s_l$ . However, for the ease of exposition, the subscript is omitted whenever it is not misleading. Daily total supply of

<sup>17</sup>Liquidity has been allotted up to June 2000 through fixed rate tenders and variable rate tenders afterwards. However, a minimum bid rate is applied, which, in the underbidding case, defines a lower bound for the interbank rate. The minimum bid rate and the rate applied in fixed rate tenders correspond to the mid-point of lending and deposit rate, denoted here as policy rate.

reserves,  $TR_t$ , is then:

$$\begin{aligned}
 TR_t &= rr + er_t \\
 &= rr + er_s^* + (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) \\
 &\quad + \left\{ E_{s-1} \frac{1}{n} \left[ \sum_{j=s}^{s+n} af_j \right] - af_t \right\} + nsf_t.
 \end{aligned} \tag{14}$$

As discussed in the section on demand, in the present model it is assumed that recourse to standing facilities takes place automatically, at the end of the day after the market has closed. In this case  $nsf_t = 0$  throughout the market session, and the relevant supply of reserves,  $\bar{M}_t$ , is given by  $\bar{M}_t = TR_t - nsf_t$ .<sup>18</sup> Splitting up the autonomous factor term leads to:

$$\begin{aligned}
 \bar{M}_t &= rr + er_s^* + (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) \\
 &\quad + \left\{ \frac{1}{n} \left( \sum_{j=s}^{s+n} E_{s-1}[af_j] \right) - E_{s-1}[af_t] \right\} + \{E_{s-1}[af_t] - af_t\}.
 \end{aligned} \tag{15}$$

Three factors shift the daily supply of reserves, namely underbidding, deviations of the actual autonomous factors from its average forecasts and the daily forecast errors itself. The first term in parenthesis on the right hand side represents underbidding, which is demand driven and related to expectations on a changing policy rate. The second term, in braces, denotes divergence of expected autonomous factors from its average forecast, which comes from the fact that liquidity is supplied only once a week. The last term in braces represents daily forecast errors, which are pure supply shocks. The supply shock which occurs at the end of day  $t$  is denoted as  $u_t = \{E_{s-1}[af_t] - af_t\}$ . The relevant supply variable for banks when making their decision is  $M_t = \bar{M}_t - u_t$ , because the size of the supply shock becomes known only after the market closes.

Note that if net recourse to standing facilities is interest rate elastic, total supply of reserves, as given in equation (14), depends on the interest rate. This might be rationalized by the fact that at a very high interest rate banks simply finance themselves by the marginal lending facility, not making use of the interbank market any more. Similarly, if the interest rate is very low, it might be preferable to make use of the deposit facility instead of lending to the interbank market.<sup>19</sup>

The deviation of actual excess reserves from its target is defined as  $b_t \equiv er_t - er_s^*$ . The variable  $b_t$  depicts deviations from the neutral allotment, i.e. from a situation where liquid-

<sup>18</sup>In the US  $\bar{M}_t$  is typically called non-borrowed reserves.

<sup>19</sup>See e.g. Thornton (2001a) for a similar formulation.

ity differs from the amount necessary to keep the interest rate at the policy rate. On all days before the last settlement,  $t = 1, \dots, s_k - 1$ , expected excess liquidity at the end of the maintenance period is:

$$E_t[b_T] = (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*). \quad (16)$$

If there is underbidding, the liquidity shortage created in the underbidding is expected to prevail till the end of the maintenance period. However, forecast errors of autonomous factors are expected to be offset in the next main refinancing operation. After the last allotment, additionally accumulated daily forecast errors of autonomous factors and accumulated recourse to standing facilities affect the expected liquidity situation at the last day of the maintenance period, i.e. for  $t = s_k, \dots, T$ :

$$E_t[b_T] = (E_{s-1}[er_{s+m}(i_s^*)] - er_s^*) + \sum_{j=s_k-1}^{t-1} \{(E_{s_k-1}[af_j] - af_j) + nsf_j\}. \quad (17)$$

## 2.3 Equilibrium

The interbank rate as equilibrium outcome of supply and demand for reserves is illustrated in figures 3 and 4. The exact functional form of the demand curve depends on the distribution function of the supply shocks. For illustrative purposes supply shocks are assumed to be symmetric and drawn from a normal distribution. Figure 3 depicts the demand curve for the last day of the maintenance period. Note that the interbank rate equals the policy rate,  $i_T = i_T^* \equiv (i_T^l + i_T^d)/2$ , whenever reserve deficiencies equal supply of liquidity,  $R_T = M_T$ , in other words, when there is no liquidity shortage throughout the market session. If  $R_T \neq M_T$ , the interbank rate differs from the policy rate. By how much the change in liquidity moves the interest rate depends on the distribution function of the supply shock. During the market session of day  $T$ , banks know that before the end of the maintenance period there is still one supply shock,  $u_T$ , to come. This shock can make up for reserve deficiencies or force a bank to take recourse to marginal lending facility in case of overdraft. The probability of each of these events is determined by the distribution of the supply shock and, hence, the interbank rate reflecting these considerations also depends on the distribution of the shock. Reasons why  $M_T$  might deviate from  $R_T$  are discussed in the following section.

The demand function for all other days is more complicated, since the expected value of a change in the reserve deficiencies,  $\frac{\partial V_{t+1}}{\partial R_{t+1}^j}$ , which in general also depends on supply shocks, is involved. However, from equation (4) it can be seen that for very large  $M_t$  the interbank rate moves towards the deposit rate,  $i_t^d$ , and for very small  $M_t$  the lending rate,  $i_t^l$ , is approached. Besides that, the general model, as presented above, does not lead to a straightforward con-

clusion on the exact shape of the demand curve. Nevertheless, the probabilities for  $M_t$  to be so large (small) that the interest rate reaches the deposit (lending) rate are close to zero, especially at the beginning of the RMP. Therefore, the only important term in the demand equation is

$$\begin{aligned}
 i_t &\approx - \int_{B_t^j - A_t^j}^{R_t^j + B_t^j - A_t^j} \frac{\partial V_{t+1}(R_{t+1}^j, A_{t+1}^j; I_{t+1})}{\partial R_{t+1}^j} f(u_t) du_t \\
 &\approx - \int_{B_t^j - A_t^j}^{R_t^j + B_t^j - A_t^j} -i_{t+1} f(u_t) du_t.
 \end{aligned} \tag{18}$$

Making use of a simplifying assumption on the supply side allows to approximate the middle part of the demand curve. Suppose that the central bank performs open market operations daily, opposed to weekly as assumed above. In this case expected interest rates do not depend on supply shocks, because the central bank corrects daily for these supply shocks, and consequently the expected interest rate simply depends on the expected policy rate and the expected liquidity situation. The policy rate is by definition independent of daily supply shocks and, in the simplified model, the expected liquidity situation is independent of supply shocks, too. The demand curve then has a flat part around the expected interest rate. Demand and supply curves for this approximation are plotted in figure 4.

The supply function in this model is rather simple. During the market session, i.e. before the realization of the shock, supply equals the sum of required reserves, targeted excess reserves, and the difference between the average forecast of autonomous factors and the present day forecast. This follows from equation (15) and defines the vertical part of the supply curve. Furthermore, via the two standing facilities the central bank provides (and absorbs) an unrestricted amount of liquidity at the lending (deposit) rate. Hence, there are two horizontal parts, being equal to the deposit rate for small values of  $M_t$  and equal to the lending rate for large values.

## 2.4 Expected and unexpected changes in supply

The main purpose of this section is to illustrate the effects supply changes have on the interbank rate. There are fundamental differences whether these changes happen at the last day(s) of the maintenance period, or at some earlier days, as well as whether these changes are expected or unexpected. For the ease of exposition and to concentrate on the effects of supply changes it is assumed that no underbidding occurs.

Recalling equation (15) and noting that the size of the autonomous factors,  $af_t$ , becomes known at the end of each day, the supply of reserves relevant for commercial banks, i.e. the expected amount of reserves available during the market session,  $M_t$ , is then given by:

$$\begin{aligned} M_t &= M + v_t \text{ with} \\ v_t &= \left\{ \frac{1}{n} \left( \sum_{j=s}^{s+n} E_{s-1} [af_j] \right) - E_{s-1} [af_t] \right\}, \\ M &= rr + er_s^* \text{ and } n = \min\{6, T - t\}. \end{aligned} \tag{19}$$

The variable  $v_t$  denotes the daily deviation of the expected autonomous factors from its expected average value. In other words, the weekly provision of liquidity implies an expected daily fluctuation for the supply of reserves, which is represented by  $v_t$ .

At the last day of the reserve maintenance period even a non-zero  $v_T$  has usually no impact on the overnight rate,  $i_T$ . Recall that the central bank allots liquidity such that liquidity provision is neutral at  $T$ , i.e.  $\sum_{t=s_K}^{T-1} v_t + v_T = 0$ . The overnight rate at the last day of the maintenance period,  $i_T$ , is determined by  $(R_T - M_T) = - \left( \sum_{t=s_K}^{T-1} v_t + v_T \right) + \vartheta_T$ . The last term,  $\vartheta_T$ , summarizes other variables potentially influencing the overnight rate apart from the sum of expected supply changes. This term  $\vartheta_T$  includes supply shocks,  $u_t$ , and the effects of underbidding. Since the sum of expected supply changes,  $\sum_{t=s_K}^{T-1} v_t + v_T$ , is zero the exact size of  $v_T$  does not matter for the determination of the overnight rate at  $T$ . Under certain assumptions the term  $\vartheta_T$  equals zero and the overnight rate equals then the policy rate,  $i_T = i_T^*$ . These assumptions are that 1) all supply shocks having occurred since the last allotment day sum up to zero, i.e.  $\sum_{t=s_K-1}^{T-1} u_t = 0$ , 2) the boundary conditions given in equation (2) have not been hit and 3) supply shocks are distributed symmetrically.

In fact, whenever the central bank makes its allotment decision such that liquidity provision is neutral at  $T$ , the interbank rate at  $T$  is not affected by expected moves in the autonomous factors.<sup>20</sup> Nevertheless, if the central bank differs expectedly from the neutral allotment, the interbank rate at  $T$  is likely to react.

Unexpected changes in reserves - supply shocks - enter the demand function at  $T$  via the variable  $R_T$ . Shocks that occurred before the last allotment of the maintenance period are neutralized by the central bank latest at the last allotment, hence, do not enter  $R_T$ . However, all shocks which occur after the last allotment do enter the variable  $R_T$  in the following

<sup>20</sup>In practice, however, if the last settlement day happens to fall at day  $T$ , it is not so clear whether the liquidity provision at  $T$  is made caring only about the liquidity situation at  $T$ . Put differently, liquidity provision at  $T$  might not be totally independent of the expected liquidity situation in the following maintenance period, and, therefore, creating a non-neutral liquidity situation at  $T$ .



non-linear way:

$$R_T = \max\{0, R_{T-1} - \max\{0, M_{T-1} + u_{T-1}\}\}. \quad (20)$$

Suppose for simplicity that the last allotment takes place at  $T - 1$ , which implies  $M_{T-1}$  is such that the sum of supply shocks contained in  $R_{T-1}$  are neutralized. As long as  $u_{T-1}$  is small enough (in absolute values) not to hit the restrictions imposed by equation (20), its effect on  $R_T$  is linear. However, a shock larger than  $(R_{T-1} - M_{T-1})$  affects  $R_T$  only up to the point that it makes  $R_T = 0$ . Similarly, a very large negative shock,  $u_{T-1} \leq -M_{T-1}$ , leads to an automatic recourse to the marginal lending facility, since overdrafts are not allowed. The only effect that shock has is to neutralize the impact the liquidity supply  $M_{T-1}$  has on the fulfillment of the reserve requirement, that is, to make  $R_T = R_{T-1}$ .

The discussion of supply changes for other days than the last day of the maintenance period is based on a simplified version of the model. The simplified version includes daily, not weekly, supply of reserves.<sup>21</sup> The demand curve for other than the last day shows a horizontal part, besides those ones at the lending and deposit rate. Reserves changing within a certain range do not affect the interest rate. However, for small or large values of  $M_t$ , the interest rate  $i_t$  moves away from the expected future interest rate  $E_t[i_{t+1}]$ . Supply shocks have no impact on the interest rate at all. Recall that a supply shock at  $t$  enters the demand equation at  $t + 1$ . In the simplified version of the model liquidity is provided every day, neutralizing all past shocks, hence, the supply shock  $u_t$  does not have any effect neither on  $i_t$  nor on  $i_{t+1}$ . The only exception is a very large positive supply shock, big enough to fulfill the reserve requirements for the entire banking sector for the whole maintenance period. In this case the interest rate on the following day jumps to the deposit rate, i.e.  $i_{t+1} = i_{t+1}^d$ .

The demand curves, as presented in figures 3 and 4, serve as benchmark for the empirical investigation, described in the next section. The exact size of the slopes is estimated and the assumed functional form is tested for. Furthermore, it is checked whether expected and unexpected supply changes have the same impact on the interbank rate. It is important to distinguish between both types of supply changes. As seen above, expected supply changes are the result of weekly supply of liquidity, hence, an institutional features, whereas unexpected supply changes are pure forecast errors.

---

<sup>21</sup>The graphical representation of the demand curve at  $t$  assumes that the central bank provides liquidity daily, making up for past shocks every day. Therefore the expected interest rate,  $E_t[i_{t+1}]$ , does not depend on shocks and can be taken out of the integral. As described above, liquidity in the euro area is provided only once a week, and consequently the assumption does not hold in general. However, this simplification might be close to true on a day which happens to be an allotment day and the penultimate day in the maintenance period at the same time, i.e. for  $t = s_k - 1 = T - 1$ . Nevertheless, the simplified version of the model should be useful for highlighting the basic differences between the last day of the maintenance period (or, more generally, the days after the last allotment of a maintenance period) and the days before the last day.

## 2.5 Underbidding

Underbidding refers to a situation in which the central bank cannot allot its desired amount of reserves due to insufficient demand.<sup>22</sup> If reserves are supplied through fixed rate tender procedures, or variable rate tenders with a minimum bid rate, an expected interest rate cut makes current supply relatively expensive, hence, shifting demand into the future. In the euro area several episodes of underbidding have occurred so far. In general, underbidding is the equilibrium outcome of rational agents.

In case liquidity not demanded in one week is supplied the following week, underbidding is definitely an optimal choice for commercial banks: If expectations are correct and the interest rate will be cut, reserves will be bought at a lower rate. If interest rates are not cut, the price in the following week is simply this week's price. However, if the central bank does not make up in the following week for liquidity deficiencies due to underbidding, the outcome depends on the demand elasticity. Suppose the supply curve is vertical between the two rates of the standing facilities, and the demand curve is also vertical at the last day of the maintenance period. Any supply shortage due to underbidding is not offset in the following main refinancing operation, hence, it moves the supply curve at the last day of the RMP. This implies that the interbank rate jumps to the marginal lending rate. Since the interest rate on a given day is a function of the expected rate at the last day of the RMP, the current interest rate jumps as well, making underbidding not an optimal choice.<sup>23</sup>

In the previous section it has been shown that the demand curve at the last day of the maintenance period is downward sloping. Consequently, a small amount of underbidding does not push the expected interbank rate to the marginal lending rate. It does increase the expected rate and therefore also the current interbank rate, but the amount of the increase depends on both the size of underbidding and the slope of the demand curve. There is then an equilibrium amount of underbidding, equalizing the current minimum bid rate with the expected interest rate at the last day of the RMP. Note that the only way to avoid underbidding in this model is to fine those banks which underbid. If all banks are penalized in the same way by simply allotting less liquidity than necessary, it is always profitable for one bank to underbid, given the others do not underbid. Then, in equilibrium all banks will underbid. However, if a bank has to pay a fine being larger than its potential gains from

---

<sup>22</sup>Ewerhart (2002) develops a game theoretic model of liquidity provision to study underbidding and he discusses ways of eliminating it.

<sup>23</sup>This holds for any sensible interest rate cut expectation. However, it does not hold, if the interest rate cut is expected to be more than  $(i_t^l - i_t^d)/2$ , i.e. more than 100 basis points. In other words, if the expected marginal lending rate is lower than the current minimum bid rate. In this case obtaining liquidity in the future from the marginal lending facility is expected to be cheaper than obtaining it now from the current main refinancing operations.

underbidding, i.e. the underbidding amount times the expected interest rate cut, this bank will not underbid. Nevertheless, the implementation of such a scheme is very complicated. An easier way to avoid underbidding is to change the policy rate, as a rule, only at the beginning of each RMP. This is part of a reform in the operating procedure proposed recently by the ECB.<sup>24</sup>

### 3 Empirical analysis

#### 3.1 Model specification

The empirical model is heavily based on the demand equations derived from the theoretical model. In other words, the functional form and the variables included in the estimated equations are not assumed, rather they come from the first order conditions of the theoretical model, representing optimizing behavior of agents. Recall that at the last day of the maintenance period the aggregate demand equation is given by:

$$i_T = i_T^d + (i_T^l - i_T^d) * F(R_T - M_T). \quad (21)$$

In order to estimate this equation a functional form for the distribution function of the supply shocks,  $F(\cdot)$ , has to be chosen. The distribution function  $F(\cdot)$  is proxied by a linear function, which is justified since the interest rate throughout the whole sample reached the upper bound, the lending rate, only at three very special occasions, the so-called underbidding episodes. These underbidding episodes are modeled separately, because the behavior of the interest rate at these days was very different from other days. At all other days the relation between the interest rate,  $i_T$ , and  $(R_T - M_T)$  is well described by the linear part of the distribution function.

Reserve deficiencies,  $R_T$ , are easy to compute, and the end of the day supply of reserves,  $\bar{M}_T = M_T + u_T$ , are published on a daily basis by the ECB. Nevertheless, the relevant decision variable for a commercial bank are the supply of reserves during the market session  $M_T$ , that is, expected reserves, which do not include the supply shock  $u_T$ . Making use of autonomous factor forecast errors allows the computation of the relevant supply variable,  $M_T$ . Note that  $R_T - M_T$  equals the sum of autonomous factor forecast errors and net recourse to standing facilities from the last allotment on up to  $T - 1$ ,  $R_T - M_T = \sum_{t=s_k-1}^{T-1} (u_t + nsf_t)$ .<sup>25</sup> In the following estimations a series  $\tilde{b}_t$  containing accumulated forecast errors and accumulated

<sup>24</sup>See the public consultation "Measures to improve the efficiency of the operational framework for monetary policy" at [www.ecb.int](http://www.ecb.int) or ECB (2004).

<sup>25</sup>This holds strictly only in case of neutral allotment. Note, however that this assumption is indeed fulfilled for almost all days, except allotments around the underbidding episodes.

recourse to standing facilities is used, where  $\tilde{b}_t \approx \sum_{j=s_t-1}^{t-1} (u_j + nsf_j)$  with  $s_t$  being the most recent settlement day.<sup>26</sup> Figure 7 shows a plot of this series.<sup>27</sup>

On all other days, the demand equation does not depend only on reserve deficiencies and reserve supply, but also the expected interest rate is important for the determination of the interbank rate. The expected interest rate depends basically on two factors, the expected policy rate and the expected liquidity situation. The expected policy rate is proxied by a forward rate  $fw_t$ , with

$$fw_t = 2 * r_t^{(2)} - r_t^{(1)}, \quad (22)$$

where  $r_t^{(2)}$  and  $r_t^{(1)}$  are the two and one-week EONIA swap rates, respectively.<sup>28</sup> This forward rate reflects the expected one-week rate in one week's time, which, in general, provides a good assessment of the expected policy rate.<sup>29</sup> The benchmark case, as illustrated in figure 4, assumes daily liquidity provision and the demand curve is characterized by a horizontal part. However, banks might not consider reserve holdings of different days as perfect substitutes, which implies a downward sloping demand curve. Furthermore, the weekly provision of liquidity may introduce non-linearities into the demand curve. From the general model above, these non-linearities are not precisely defined. The following, testable, specification for the demand curve is proposed. Its main features are: 1) For very large (small)  $M_t$ , the interbank rate equals the deposit (lending) rate; 2) In the absence of a) supply shocks, b) expected temporary deviations of  $M_t$  from its average values, c) expected net recourse to standing facilities and d) expected policy rate changes, i.e.  $u_t = v_t = nsf_t = (i_{t+1}^* - i_t^*) = 0$  for all  $t$ , the interbank rate equals the policy rate,  $i_t = i_t^*$ . Note that this is exactly the scenario described in the benchmark case. The interbank rate is then formulated as a function of deviations from

<sup>26</sup>This information is not publicly available. I am very grateful to Clara Martin Moss and Steen Ejerskov from the Monetary Policy Stance Division of the European Central Bank who compiled this series and made it available to me. Their series shows the deviation of the liquidity situation from neutral, expected to prevail at the next settlement day or the last day of the RMP, whatever comes first. In general, this deviation equals the sum of accumulated forecast errors and accumulated net recourse to standing facilities since the last allotment day.

<sup>27</sup>Commercial banks can proxy this variable fairly well.

<sup>28</sup>Approximating the expected policy rate by other forward rates does not seem to change the results. In the previous version of the paper forward rates constructed from both Euribor and EONIA swap rates with maturities of one and two months have been used, but parameter estimates are very similar.

<sup>29</sup>Short-term money market rates follow the policy rate quite closely, in particular this holds for the one month rate. Hence, the expected one month rate should follow closely the expected policy rate. For the predictive power of forward and future rates see e.g. Poole and Rasche (2000) or Gaspar et al. (2001). The variable needed for the estimation of  $i_t$  is the expected policy rate at  $t + 1$ , or more generally, the expected policy rate within this maintenance period. If the interest rate is expected to change in e.g. five weeks, the forward rate changes, but the expected policy rate for this period does not change. In this case, the forward rate does not provide a good proxy for the expected policy rate. Nevertheless, it is assumed that changes in the forward rate reflect expected changes in this maintenance period's policy rate, mainly, because agents are likely to make forecasts at short horizons due to the low precision of long horizon forecasts.

the benchmark.

The liquidity situation at each day, given by  $R_t$  and  $M_t$ , differs from the benchmark due to supply shocks and anticipated supply changes. Deviations of reserve supply,  $M_t$ , and reserve deficiencies,  $R_t$ , from the benchmark,  $M_t^{bench}$  and  $R_t^{bench}$ , change the liquidity situation at  $t$ , and potentially move the interest rate away from the policy rate. Liquidity variables expressed as deviations from the benchmark case are given by:

$$(R_t - M_t) - (R_t^{bench} - M_t^{bench}) = - \sum_{j=s_l-1}^{t-1} \{\max(-M_j, u_j) + nsf_j\} - \sum_{j=s_l}^{t-1} v_j - v_t \text{ and} \quad (23)$$

$$M_t - M_t^{bench} = v_t. \quad (24)$$

It follows that supply shocks and anticipated deviations from the average supply of reserves have the potential to drive a wedge between the interbank and the policy rate, either directly, via  $R_t - M_t$  and  $M_t$ , or indirectly via

$$E_t[i_{t+1}] = E_t[\Psi(R_t, R_{t+1}, \dots, R_T, M_t, M_{t+1}, \dots, M_T, i_t^*, i_{t+1}^*, \dots, i_T^*)]. \quad (25)$$

$\Psi(\cdot)$  is a general function which needs not be further specified for the moment. Note that at all allotment days and at the last day of the maintenance period the sum of expected supply changes is zero, i.e.  $\sum_{j=s_l}^{t-1} v_j = 0$  for  $t \in \{s_1 - 1, s_2 - 1, \dots, s_k - 1, T\}$ . Furthermore, liquidity supply is such that reserve deficiencies at any settlement day,  $R_t$  for  $t \in \{s_1, \dots, s_k\}$ , do not depend on past supply shocks other than  $u_{t-1}$ . Therefore, supply shocks occurring before the last allotment day,  $t = s_k - 1$ , are expected to affect the liquidity situation only temporarily, but are not relevant for the total liquidity situation of the entire reserve maintenance period. Equally, expected supply changes,  $v_t$  for all  $t$ , affect the liquidity situation temporarily only. In contrast, supply shocks occurring after the last allotment day have an effect on the liquidity situation at  $T$ , the last day of the RMP.

One of the central questions in this paper is if temporary changes in supply have an effect on the interest rate, in other words, if a daily liquidity effect exists. The two sources of temporary changes are different in style and can have different implications. If expected supply changes have an effect on the interest rate on a daily basis, then there exists a daily liquidity effect. However, if supply shocks have an effect, it might be due to a daily liquidity effect, but also that commercial banks do not expect supply shocks to be fully offset in the next allotment decision. A daily liquidity effect results whenever banks do not see daily reserves as perfect substitutes. Whereas, even if there is no daily liquidity effect, supply shocks affect

the interest rate if the allotment strategy of neutralizing supply shocks is not fully credible.

Recall that deviations of the liquidity situation from its benchmark are measured by the sum of forecast errors and net recourse to standing facilities,  $\tilde{b}_t \approx \sum_{j=s_l-1}^{t-1} (u_j + nsf_j)$  with  $s_l$  being the most recent settlement day. However, net recourse to standing facilities is very close to zero on most days, except for some days near the end of the maintenance period, as can be seen in figure 5. Therefore, supply shocks are the main driving forces of the liquidity situation.

In figure 6 the interbank rate together with the lending and deposit rate are plotted and some basic statistics are given in table 1. Normally the interbank rate follows the policy rate, which is the mid-point of lending and deposit rate, quite closely, but occasionally there are large spikes. As discussed above, the deviation of the interbank rate from the policy rate can be caused by changes in liquidity or changes in the expected policy rate. A series for changes in liquidity and the forward rate, a proxy for the expected policy rate, are plotted in figures 8 and 9, respectively.

Standard unit root tests confirm that the interest rate, within the sample, is integrated of order one. Furthermore, it is co-integrated with the policy rate,  $i_t^*$ . Therefore, the interest rates,  $i_t$ , is modelled in first differences,  $\Delta i_t \equiv (i_t - i_{t-1})$ , and a unit co-integrating vector,  $(i_{t-1} - i_{t-1}^*)$ , is imposed.<sup>30</sup> The model then is:

$$\begin{aligned}\Delta i_t &= c + \phi(i_{t-1} - i_{t-1}^*) + x_t\beta + h_t\eta_t \\ x_t &= \{b_t - b_{t-1}, b_{t-1} - b_{t-2}, \dots, b_{s_l}, fw_t - fw_{t-1}, d_t\} \\ \ln(h_t^2) &= z_t\lambda + \sum_{j=1}^q \delta_j (\ln(h_{t-j}^2) - z_{t-j}\lambda) + \alpha \{|\eta_{t-1}| - E|\eta_{t-1}| + \gamma\eta_{t-1}\} \\ \eta_t &\sim iid(0, p + (1-p) * \sigma^2).\end{aligned}\tag{26}$$

The parameter  $\phi$  captures how fast the interest rate,  $i_t$ , returns to its long-run value, the target rate  $i_t^*$ . The mean equation includes a constant,  $c$ , and other explanatory variables,  $x_t$ . Deviations of liquidity from the neutral allotment are given by the variable  $b_t$ .<sup>31</sup> The most recent settlement day is indexed by  $t = s_l$ . The autocorrelation function in figure 10 shows clear evidence for conditional heteroskedasticity, which is modeled with an EGARCH specification.<sup>32</sup> The conditional standard deviation of the interest rate is given by  $h_t$ . The

<sup>30</sup> Results on tests for the order of integration and co-integration are not reported. All test results are available from the author.

<sup>31</sup> Both, the actual liquidity situation at each day, that is,  $b_t = \tilde{b}_t \approx \sum_{j=s_l-1}^{t-1} (u_j + nsf_j)$  and  $b_t \approx \sum_{j=s_l-1}^{t-1} u_j$ , the sum of autonomous factor forecast errors alone, are used. Estimation results are practically identical.

<sup>32</sup> An EGARCH model has some advantages over more standard GARCH models, notably restrictions on some parameters are not necessary in order to ensure nonnegativity of conditional variances. See for example Bollerslev et al (1992).

vector  $z_t$  contains explanatory variables for the conditional volatility equation. Of particular interest are variables related to the operating procedure and calendar days. Standardized residuals are denoted by  $\eta_t$ . Frequent small changes and occasionally large spikes characterize the interbank rate, suggesting the underlying distribution to be a mixture of two normal distributions.<sup>33</sup> The probability to come from the first distribution with variance one is  $p$ , and the probability to come from the second distribution with variance  $\sigma^2$  is  $(1 - p)$ . The exponential GARCH model applied here allows to estimate the different impact positive and negative surprise changes of the interest rate have on the volatility, which is given by the parameter  $\gamma$ .

The vector  $d_t(z_t)$  may include further explanatory variables for the conditional mean (variance). This specification allows to test for a wide range of possible effects related to the central bank's operating procedure and calendar days.

One of the main issues of this paper is the analysis of the liquidity effect. Hence, the parameters of main interest are those related to the liquidity variables  $b_t$ . These parameters can be interpreted as determining the slopes of the demand curves. Note that also lagged liquidity variables are included in  $x_t$ , which permits to analyze how fast banks react to changes in supply. If there is an immediate reaction only  $b_t - b_{t-1} \approx u_{t-1}$  should be significant. On the contrary, if other liquidity variables are also significant one can conclude that banks react sluggishly to new information. This sluggish reaction might be banks' choice, or simply reflect the slow diffusion of information.

The liquidity variables used here are those which reflect precisely the liquidity situation banks are faced with when taking their demand decisions.<sup>34</sup> For example, Würtz (2003) uses the accumulated recourse to standing facilities at the last day of the maintenance period, and average reserve surplus on other days. Those variables do not measure the prevailing liquidity situation exactly. The accumulated recourse to standing facilities includes the supply shock which occurs at the end of the last day of the maintenance period, but banks do not know the size of this shock when making their decisions. Furthermore, as seen above, it is not only recourse to standing facilities which defines the liquidity situation, but also the sum of forecast errors. In addition, by using average reserve surplus it is not taken into account that the central bank makes up for past forecast errors and, again, that the end of the day shock is not known to banks. What is more, the recourse to standing facilities might depend on the interest rate (see e.g. Thornton, 2001a). In other words, banks might decide actively on the use of the standing facilities, not only take recourse by force, e.g. in case of overdraft. Then,

<sup>33</sup>The student t-distribution has also been used, but the mixture of normals allows fatter tails together with a larger mass around zero, which is supported by the data.

<sup>34</sup>The same liquidity data is used in Ejerskov et al. (2003). However, they estimate a weekly model for demand and supply of liquidity.



recourse to standing facilities becomes an endogenous variable and cannot be used directly in the estimation of the demand curve. The current model does not suffer from this caveat, since forecast errors are by definition exogenous and, therefore, can be used to estimate the liquidity effect.<sup>35</sup>

### 3.2 Estimation results and discussion

The estimated model is presented in table 2.<sup>36</sup> Residuals, standardized residuals and conditional log volatility are plotted in figures 11, 12 and 13, respectively. Standard tests indicate that the model is well specified. There is no serial correlation left neither in the standardized residuals nor in the squared standardized residuals (see figures 15 and 16) and the empirical distribution of the residuals is very close to its assumed distribution (see figure 17). Lagrange multiplier tests for omitted variables, given in tables 3 to 5, do not show any apparent misspecification. Furthermore, estimated parameters are very stable across sub-samples.<sup>37</sup>

From the theoretical discussion above it has been seen that institutional details have the potential of influencing the interbank rate. Indeed, all key features of the theoretical model are confirmed by the data. In addition the interbank rate is characterized by some other effects not showing up directly in the theoretical model, but clearly being related to the operating procedure. The main results are summarized in table 6, where all predictable patterns of mean and volatility of the overnight rate are stated. Most of these patterns are related to the implementation of monetary policy, but also some calendar day effects are present. In what follows, each of these patterns will be discussed in detail.

It cannot be rejected that the demand curves look like in the benchmark model, as presented in figures 3 and 4. In other words, the demand curve is downward sloping only at the last day of the maintenance period. All four parameters on liquidity at the last day of the maintenance period are negative and significant (panel A in table 2). On all other days the parameter on liquidity is not significant (see panel F in table 5). Hence, on all days other than the last day of the RMP, the demand curve is flat. Recall that this statement holds for not too big deviations from a neutral liquidity situation. Furthermore, note that banks react sluggishly to new information. The interest rate at  $T$  differs from its previous day value also if a change in supply has occurred on the preceding days. It is not only the current

<sup>35</sup>The estimation results given below are obtained by using the actual liquidity situation at each day, that is,  $b_t = \tilde{b}_t \approx \sum_{j=s_t-1}^{t-1} (u_j + nsf_j)$ . Results for  $b_t \approx \sum_{j=s_t-1}^{t-1} u_j$ , the sum of autonomous factor forecast errors alone, are practically identical.

<sup>36</sup>Numerical optimization has to be applied to estimate this model. Several starting values are used to check whether a global maximum has been reached. Standard errors are based on the second derivatives of the log likelihood function. The outer-product estimates are almost identical.

<sup>37</sup>Parameter estimates presented here are very similar to the estimates contained in the previous version of this paper, which uses data up to July 2002.

supply change, which matters. A positive supply change at  $T$  of one billion euro decreases the interest rate, i.e.  $(i_T - i_{T-1})$ , by 7.7 basis points. Note that the supply shock at day  $t$  occurs after the market closes, therefore, affecting supply at  $t + 1$ . Accordingly,  $u_{T-1}$  denotes the unexpected supply change at day  $T$ . If the change in supply occurred before  $t = T$  the interest rate does not react until the last day of the RMP. The effect is significantly smaller than for contemporaneous changes in supply, but still considerable. Lagged supply changes of one billion euro move the interest rate by around 5 basis points into the opposite direction. This sluggish reaction might explain why a permanent change in supply, that is a supply shock after the last allotment day, does not affect the interest rate until the last day. What is more, allowing the interest rate to react also to lagged supply changes permits to pin down the liquidity effect more precisely.

Estimating a model with weekly frequency Ejerskov et al. (2003) find an asymmetric liquidity effect. Positive supply changes imply a larger reaction of the interest rate than negative changes. This asymmetric effect cannot be confirmed in the present analysis, as indicated by panel G in table 5.

An expected change in the future interest rate should move the current interest rate by (almost) the same size. One way to measure this relationship is the use of a forward rate. However, the forward rate at  $t$  is not a perfect signal of the expected interest rate at  $t + 1$ , thus the estimated parameter is likely to be different from one. Indeed, a change in the forward rate moves today's interest rate, but by less than one. Estimated at the first day of a RMP, a change in the forward rate by 10 basis points increases the interbank rate by 6 basis points. The forward rate is best used at this day, since it mostly reflects expected policy rate changes within the current maintenance period. For other days, especially for those close to the end of the maintenance period, expected policy rate changes in the next maintenance period become more important for the determination of the forward rate. However, expected changes in the policy rate in the next maintenance period should not affect the current interest rate.

Summarizing, it can be said that transitory changes in supply do not affect the interbank rate.<sup>38</sup> In other words, there is no daily liquidity effect for temporary supply changes. After the last allotment day all changes in supply as analyzed here are permanent in the sense that they affect the liquidity situation at the last day of the RMP and, accordingly, the reserve position of the whole maintenance period. These permanent changes do not impact on the interest rate till the last day of the maintenance period. The slow diffusion of new information on supply changes, or the low benefits of closely watching total reserve supply

<sup>38</sup> Transitory changes in supply are measured by the supply shock  $u_t$ , occurring before the last settlement day in a reserve maintenance period. However, also deviations of the actual autonomous factors from its average forecasts,  $v_t$ , are transitory. In an earlier version of the paper I included the variable  $v_t$  and found it insignificant, which confirms the results presented here.

in the market are possible reasons for this finding. Another potential explanation has been pointed out by a referee. As outlined in equation (15) both, supply shocks,  $u_t$ , and expected deviations of the actual autonomous factors from its average forecast,  $v_t$ , affect the daily supply of reserves. After the last allotment day in a RMP supply shocks are permanent changes, whereas expected deviations of the autonomous factors,  $v_t$ , are temporary. Commercial banks may have difficulties to distinguish both types of supply changes. As a consequence, banks may conclude wrongly that all changes occurring before the very last day of a RMP, but after the last allotment day, are temporary. Therefore, even permanent changes in supply, represented by  $u_t$ , may not have an immediate effect on the interest rate. Rather, the interest rate may react only at the very last day when it becomes clear which supply changes are permanent.

There is some evidence that the relation between current and future expected interest rate is close to one. All in all, a permanent and fully known change in supply should move the interest rate up to the level expected to prevail at the last day of the maintenance period. The level of the interest rate at the last day depends on the slope of the demand curve and the deviation of liquidity from neutral. Assuming that the liquidity change takes place before the last allotment, the relevant slope is -0.08. Therefore, a liquidity shortage of 13 billion or more moves the interest rate towards the marginal lending rate.

In the underbidding episodes such permanent liquidity shortages were created. Underbidding the weekly allotment by e.g. four billion lower than the neutral amount creates a total liquidity shortage over the whole week of  $4 \times 7 = 28$  billion and, in consequence, leads the interbank rate to touch the upper bound. This is exactly what can be observed in the data, which provides corroboration that the effect of permanent and fully expected supply changes on the interest rate are largely determined by the slope of the demand curve at the last day of the maintenance period.<sup>39</sup> It is important to have in mind that liquidity supply is assumed to be neutral. However, if the central bank expectedly differs from this policy, the above described relationships may change as well.

First differences of the interest rate exhibit slight autocorrelation. This behavior does not come out directly from the theoretical model and contradicts market efficiency. However, in practice it might be costly to obtain information on supply changes directly, so some banks might use past interest rates as a proxy for supply changes.

There is no systematic pattern for the mean of the interest rate throughout the reserve maintenance period, as can be seen in table 3. Various measures are used to test for a possible increase of the interest rate towards the end of the reserve maintenance period, but there is no evidence for such an interest rate hike in the present model. Neither the

---

<sup>39</sup>See for example Bindseil (2002) or Välimäki (2002).

announcement nor allotment or settlement of the last open market operation in each RMP influences systematically the mean of the interest rate (see panel H in table 3). Furthermore, the mean of the interest rate does not behave differently at days of the ECB's Governing Council meeting or press conference than at other days (see panel L in table 3).

Volatility is higher for days after the last allotment day till the last day of the maintenance period, as can be seen very clearly in figure 14. Additionally, there is an increase in volatility at the first day, last day and next to last day in each RMP, as well as at the allotment day of the last open market operation. Volatility increases also at the day of a policy rate change and the day after, as well as during underbidding episodes.<sup>40</sup> As predicted by the theoretical model there is no increase neither in the mean nor in the volatility of the interbank rate for other days before the last allotment day (see table 3).

Positive surprise changes in the interest rate increase volatility more than negative surprise changes. The relevant parameter  $\gamma$  is estimated to be around 9 percent, as can be seen in panel H of table 2. Positive changes in the interest rate indicate an increased probability of ending up the reserve maintenance period with too few reserves, or an expected increase in the policy rate. Banks may be worried more about not fulfilling the reserve requirement than about holding too many reserves, which then can increase volatility. Alternatively, banks are likely to view an increase in the policy rate as less favorable than a decrease, which then also can push up volatility.

One striking difference of monetary policy implementation in the euro area to other countries is the low frequency of open market operations. An important question is then to study the effects of frequency of open market operations. It has been shown that volatility increases after the last allotment day. Hence, for infrequent open market operations, the period after the last allotment day becomes longer and therefore the number of days with high volatility increases. Throughout the sample period the number of days which pass after the last allotment until the last day of the reserve maintenance period varies every month. In general, the last allotment is performed on Tuesday and the last day in a reserve maintenance period is the 23rd of each month. However, there have been some recent changes in the operational framework of the ECB, becoming effective from March 2004 onwards (see e.g. ECB, 2004). Now, there are always five (business) days after the last allotment until the last day of the RMP. It is therefore interesting to test if the volatility increase at the end of the RMP depends

---

<sup>40</sup>The underbidding dummy takes value one at the allotment day of the respective underbidding episode. However, the underbidding dummy may be an endogenous variable. Therefore, a model including the one period lagged underbidding dummy variable is estimated, as well. In other words, the new model includes a dummy taking value one at underbidding settlement days. There should be no, possible, problem of endogeneity with this new variable, because the bidding outcome is announced the day before the settlement day. The estimated parameters are almost identical with both dummy variables, the original one including allotment days, and the new one including settlement days.

on the number of days after the last allotment day. Lagrange multiplier tests, as outlined in panels D to G in table 3, indicate that the number of days after the last allotment day does not matter for volatility. It has to be said that this is only a descriptive analysis, which depends on the current structure of the money market and especially on the current details of open market operations. No general conclusions are drawn on the effects of changing the frequency of open market operations.

All the above characteristics of the interbank rate are related in some way or another to the operating procedure of the central bank. There are some other interesting patterns, which are pure calendar day effects. At the last day of the month the interbank rate increases by 5 basis points. At the end of the second quarter the increase is 18 basis points and 31 basis points at the end of the year. However, this increase is reversed on the following day, the first day of the month, as panel B in table 4 indicates. The end of month spikes may arise from window dressing activities. Volatility of the interbank rate is higher around the end of the month, too. Nevertheless, a central bank cannot do much about avoiding these predictable patterns. It has been shown above that there is no daily liquidity effect, i.e. the effect of a temporary supply changes on the interest rate is insignificant. In particular, the slope of the demand curve at the last or first day of a month is insignificantly different from zero, as can be seen in panel D of table 5. The day of the week does not explain the behavior of the overnight rate. Neither mean nor volatility of the interbank rate depend on the weekday, as can be seen in panel A of table 4.

By comparing the reaction of commercial banks to supply shocks one can test the efficiency of banks' reserve management.<sup>41</sup> The theoretical model motivates changes in the interbank rate as a function of liquidity. The size of the reaction depends on the distribution of supply shocks. The observed standard deviation of supply shocks is around 7 billion euro for days after the last allotment day and around 24 billion euro throughout the whole RMP. Relevant supply shocks are those occurring after the last allotment, because all other shocks are neutralized in subsequent open market operations. It has been shown that the interest rate at the last day of the RMP is given by  $i_T = i_T^d + (i_T^l - i_T^d) * F(R_T - M_T)$ , which can be approximated by  $i_T = (i_T^d + 1) + \tilde{\beta} * (R_T - M_T)$ . Recalling that  $(R_T - M_T) \approx \sum_{t=s_k-1}^{T-1} u_t$  and using the standard deviation of the supply shocks, it follows that  $\tilde{\beta} \approx -0.08$ , which is very close to the estimated parameter on  $u_{T-1}$ .<sup>42,43</sup> In other words, the reaction of banks to supply shocks is

<sup>41</sup>I thank Christian Ewerhart for pointing this out.

<sup>42</sup>The last settlement day can fall at any weekday. Therefore, on average  $(R_T - M_T)$  contains supply shocks from three business days. It follows that  $Var(R_T - M_T) \approx 3 * Var(u_t^{last}) = 3 * (7.02)^2$ , with  $u_t^{last}$  being a supply shock occurring at or after the last allotment day.

<sup>43</sup>Ewerhart et al. (2003) also report the slope of the demand curve to be roughly 8 basis points per billion euro.

fully rational, at least as far as magnitudes are concerned. However, it has been found that banks react sluggishly to new information, a pattern which is not easily explained for rational agents. One possible explanation is that banks do not have timely information on the exact size of the supply shocks. However, banks should be able to proxy the size of the supply shock fairly well. Alternatively, gains from reacting quickly to supply changes might be small. Although the exact size of potential profits is still an open question, a preliminary assessment shows that there may exist some arbitrage opportunities.

From the theoretical model one expects the parameter on  $u_T$  to be zero, that is, a supply shock occurring at the end of day  $T$  should not have any influence on the interest rate at that day. Nevertheless, the estimated parameter is significantly different from zero. One reason can be that during the market session commercial banks have already some clue about the size of the supply shock, thus, they can react to it. This seems to make good sense since this parameter is only different from zero at the last day of the RMP, when banks are supposedly watching their reserve accounts closely.<sup>44</sup>

## 4 Conclusions and further research

This paper studies the determinants of the overnight interest rate and quantifies them. The overnight interest rate is the equilibrium outcome of supply and demand for bank reserves. The here developed structural model for both supply and demand for reserves allows a detailed analysis of the interactions between the central bank, as the sole net supplier of reserves, and commercial banks, on the demand side. The precise set-up of this market, i.e. institutional details of the reserve market, has important implications for the behavior of the overnight rate, both for conditional mean and variance. These implications are derived from a theoretical model and their magnitudes are estimated for the euro area overnight rate.

The overnight rate reacts to expected future changes in the policy rate and to permanent changes in supply of reserves. In fact, a substantial liquidity effect is estimated: a change in reserve supply of one billion euro, expected to prevail till the end of the maintenance period, moves the interbank rate eight basis points into the opposite direction. The theoretical model relates the magnitude of the liquidity effect to the distribution of supply shocks, which is confirmed by the data. Interestingly, banks do not react immediately to supply changes. This sluggish reaction to supply changes is not easily explained for rational agents. Temporary supply changes have no effect on the overnight rate.

---

<sup>44</sup>The alternative interpretation is measurement error. Since the size of the supply shock for  $T$  is not available, it was constructed as:  $u_T = -nsf_T - \sum_{j=s_t-1}^{T-1} (u_j + nsf_j)$ . Although it is in principle possible that  $u_T$  is measured with some error, there is no obvious reason why the above equation should not hold exactly.

Predictable patterns are found for the overnight rate. The mean is high at the last day of a month, even higher on the end of a semester or a year. The end of the month, semester and year increases are completely reversed at the first day of the following month. End of month effects are most likely due to window dressing operations. The mean of the overnight rate does not vary systematically throughout the reserve maintenance period. Therefore, the short-term money market does not contain clear arbitrage opportunities, with the possible exception of the sluggish reaction to supply shocks.

The conditional volatility of the overnight rate is closely related to monetary policy implementation. Conditional volatility is especially high at the allotment day of the last open market operation and even higher at days afterwards. Volatility increases at the day of a change in the policy rate and around the end of a month.

In this paper the relation between operating procedures and the overnight interest rate has been analyzed in great detail. However, equally important is how the here identified effects work through the yield curve and affect other interest rates. As long as these effects are limited to the very short end of the yield curve, implications for the economy as a whole are probably insignificant. On the contrary, if long-term interest rates react strongly as well, implications are far more important. Nevertheless, not much is known about this transmission along the yield curve. Recently Cassola and Morana (2003 and 2004) and Cassola and Moschitz (2004) have made a first step by analyzing volatility transmission along some money market rates.

While the present paper focuses on policy implementation of one particular central bank, an interesting area of research is the comparison of alternative operating procedures and their effects on the behavior of interest rates. However, little work has been done so far in this field.



## References

- [1] Bartolini, L., Bertola, G., Prati, A., 2001. Banks' reserve management, transaction costs, and the timing of federal reserve management. *Journal of Banking and Finance* 25, 1287–1317.
- [2] Bartolini, L., Bertola, G., Prati, A., 2002. Day-to-day monetary policy and the volatility of the federal funds rate. *Journal of Money, Credit and Banking* 34, 137–159.
- [3] Bernanke, B., Mihov, I., 1998. The liquidity effect and long-run neutrality. NBER Working Paper No. 6608.
- [4] Bindseil, U., 2002. Equilibrium bidding in the eurosystem's open market operations. ECB Working Paper No. 137.
- [5] Bindseil, U., Seitz, F., 2001. The supply and demand for eurosystem deposits the first 18 months. ECB Working Paper No. 44.
- [6] Bollerslev, T., Chou, R., Kroner, K., 1992. ARCH modeling in finance. *Journal of Econometrics* 52, 5–59.
- [7] Borio, C. 1997. The implementation of monetary policy in industrial countries: a survey. BIS Economic Papers No. 47.
- [8] Cassola, N., Morana, C., 2003. Volatility of interest rates in the euro area. Evidence from high frequency data. ECB Working Paper No. 235.
- [9] Cassola, N., Morana, C., 2004. Transmission of volatility in the euro area money market. ECB, mimeo.
- [10] Cassola, N., Moschitz, J., 2004. Monetary policy implementation and volatility in the euro area money market. UAB, mimeo.
- [11] Christiano, L., 1991. Modeling the liquidity effect of a monetary shock. Federal Reserve Bank of Minneapolis, Quarterly Review Winter, 3–34.
- [12] Cochrane, J., 2001. Asset pricing. Princeton University Press, New Jersey.
- [13] ECB, 2002. Monthly Bulletin. January 2002.
- [14] ECB, 2004. The monetary policy of the ECB. Frankfurt am Main, Germany.

- [15] Ejerskov, S., Martin Moss, C., Stracca, L., 2003. How does the ECB allot liquidity in its weekly main refinancing operations? A look at the empirical evidence. ECB Working Paper No. 244.
- [16] Ewerhart, C., 2002. A model of the eurosystem's operational framework for monetary policy implementation. ECB Working Paper No. 197.
- [17] Ewerhart, C., Cassola, N., Ejerskov, S., Valla, N., 2003. Optimal allotment policy in the eurosystem's main refinancing operations. ECB Working Paper No. 295.
- [18] Ewerhart, C., Cassola, N., Ejerskov, S., Valla, N., 2004. Liquidity, information, and the overnight rate. ECB Working Paper No. 378.
- [19] Fabozzi, F., Modigliani, F., 1996. Capital Markets. Prentice-Hall, London.
- [20] Galí, J., 1992. How well does the IS-LM model fit postwar U.S. data? Quarterly Journal of Economics May, 709–738.
- [21] Gaspar, V., Pérez-Quirós, G., Rodríguez Mendizábal, H., 2004. Interest rate determination in the interbank market. ECB Working Paper No. 351.
- [22] Gaspar, V., Pérez-Quirós, G., Sicilia, J., 2001. The ECB monetary policy strategy and the money market. International Journal of Finance and Economics 4 (6), 325–342.
- [23] Gilchrist, S, 2001. Commentary to "Identifying the liquidity effect at the daily frequency". Federal Reserve Bank of St. Louis Review July/August, 79–82.
- [24] Hamilton, J., 1996. The daily market for federal funds. Journal of Political Economy 104 (1), 26–56.
- [25] Hamilton, J., 1997. Measuring the liquidity effect. American Economic Review 87, 80–97.
- [26] Kim, B., Ghazali, N., 1998. The liquidity effect of money shocks on short-term interest rates: some international evidence. International Economic Journal 12 (4), 49–63.
- [27] Leeper, E., Gordon, D., 1992. In search of the liquidity effect. Journal of Monetary Economics 29 (3), 341–369.
- [28] Pérez Quirós, G., Rodríguez Mendizábal, H., 2003. The market for funds in Europe: What has changed with the EMU? Forthcoming Journal of Money, Credit and Banking.
- [29] Poole, W., Rasche, R., 2000. Perfecting the market's knowledge of monetary policy. Journal of Financial Services Research 18 (2-3), 255–298.

- [30] Strongin, S., 1995. The identification of monetary policy disturbances: explaining the liquidity puzzle. *Journal of Monetary Economics* 35 (3), 463–497
- [31] Thornton, D., 2001a. Identifying the liquidity effect at the daily frequency. *Federal Reserve Bank of St. Louis Review* July/August, 59–78.
- [32] Thornton, D., 2001b. The federal reserve's operating procedure, non-borrowed reserves and the liquidity effect. *Journal of Banking and Finance* 25, 1717–1739.
- [33] Välimäki, T., 2002. Variable rate liquidity tenders. Bank of Finland. mimeo.
- [34] Walsh, C., 1998. *Monetary theory and policy*. MIT Press, Cambridge, Massachusetts.
- [35] Würtz, F., 2003. A comprehensive model on the euro overnight rate. ECB Working Paper No. 207.

## A Basic statistics and estimation results

Table 1: Basic statistics for selected series.

| Variable                               | Mean  | Std. Dev. | Skewness | Kurtosis |
|--|-------|-----------|----------|----------|
| EONIA rate, in levels                  | 3.343 | 0.931     | 0.283    | 2.043    |
| EONIA rate, in first differences       | 0.000 | 0.143     | 0.884    | 16.745   |
| Forward rate, in levels                | 3.342 | 0.900     | 0.272    | 1.895    |
| Supply shock, $u_t$                    | 0.543 | 24.524    | 1.363    | 50.897   |
| Supply shock, after last allotment day | 0.489 | 7.017     | 1.800    | 11.612   |

NOTE: The EONIA rate is a volume-weighted average of interbank rates in the euro area. See appendix B for a detailed description of the other variables. Sample: All business days from 24/03/1999 to 19/02/2004, both included.

Table 2: Parameter estimates for the Overnight Interest Rate (EONIA).

Model:  $\Delta i_t = c + \phi(i_{t-1} - i_{t-1}^*) + x_t\beta + h_t\eta_t$

$\ln(h_t^2) = z_t\lambda + \sum_j \delta_j \{ \ln(h_{t-j}^2) - z_{t-j}\lambda \} + \alpha \{ |\eta_{t-1}| - E|\eta_{t-1}| + \gamma\eta_{t-1} \}$

$\eta_t \sim \text{iid}(0, p + (1-p)\sigma^2)$ .

Sample: All business days from 24/03/1999 to 19/02/2004, both included.

| Variable   | Parameter | Std. Error | p-value |
|--|-----------|------------|---------|
| <b>Mean equation</b>   |           |            |         |
| <b>(A) Liquidity effects at the last day in a RMP, <math>t = T</math></b>          |           |            |         |
| $u_{T-1}$  | -0.077    | 0.014      | 0.000   |
| $u_{T-2}$  | -0.055    | 0.009      | 0.000   |
| $u_{T-3} + u_{T-4} + u_{T-5}$  | -0.052    | 0.009      | 0.000   |
| $u_T$  | -0.046    | 0.009      | 0.000   |
| <b>(B) Expected future policy rate</b>   |           |            |         |
| $E_t[i_{t+k}^*]$ at the first day in a RMP, $t = 1$                                | 0.628     | 0.060      | 0.000   |
| $E_t[i_{t+k}^*]$ at other days, $t = 2, \dots, T$                                  | 0.000     | 0.007      | 0.946   |
| <b>(C) Calendar day effects</b>  |           |            |         |
| End of month, reversed begin of month; except end of semester                      | 0.051     | 0.002      | 0.000   |
| End of 2nd quarter, reversed begin of 3rd quarter                                  | 0.178     | 0.020      | 0.000   |
| End of 4th quarter, reversed begin of first quarter                                | 0.310     | 0.033      | 0.000   |
| <b>(D) Other variables</b>   |           |            |         |
| First day in a RMP, $t = 1$  | 0.030     | 0.005      | 0.000   |
| dunderbidding  | -0.303    | 0.014      | 0.000   |
| $(i_{t-1} - i_{t-2}) * (1 - \text{first day} - \text{begin of month})$             | 0.067     | 0.011      | 0.000   |
| Constant   | 0.001     | <0.001     | 0.173   |
| Error correction term $(i_{t-1} - i_{t-1}^*)$ at the first day in a RMP, $t = 1$   | -1.000    | -          | -       |
| Error correction term $(i_{t-1} - i_{t-1}^*)$ at all other days, $t = 2, \dots, T$ | -0.040    | 0.008      | 0.000   |

Table 2 (continued)

| Variable   | Parameter | Std. Error | p-value |
|--|-----------|------------|---------|
| <b>Volatility equation</b>                         |           |            |         |
| <b>(E) Days of reserve maintenance period</b>      |           |            |         |
| First day, $t = 1$                                 | 1.516     | 0.194      | 0.000   |
| Last allotment day                                 | 0.841     | 0.250      | 0.001   |
| All days after last allotment                      | 3.045     | 0.381      | 0.000   |
| Next to last day, $t = T-1$                        | 1.850     | 0.393      | 0.000   |
| Last day, $t = T$                                  | 2.315     | 0.510      | 0.000   |
| <b>(F) Calendar days</b>                           |           |            |         |
| End of month and the day before                    | 0.471     | 0.171      | 0.006   |
| Begin and end of a quarter, additionally           | 1.500     | 0.665      | 0.024   |
| Begin and end of a semester, additionally          | 2.170     | 0.455      | 0.000   |
| Policy rate change and the day after               | 1.087     | 0.287      | 0.000   |
| <b>(G) Other dummy variables</b>                   |           |            |         |
| dunderbidding                                      | 1.754     | 0.195      | 0.000   |
| GC meeting after last allotment (Sep and Oct 1999) | 4.028     | 0.291      | 0.000   |
| Underbidding at end of RMP (Dec 2003)              | 1.047     | 0.356      | 0.003   |
| January 2002 (Cash changeover)                     | 3.175     | 0.725      | 0.000   |
| <b>(H) EGARCH parameters</b>                       |           |            |         |
| Constant   | -6.394    | 0.151      | 0.000   |
| $\alpha$   | 2.403     | 0.211      | 0.000   |
| $\delta$   | 0.678     | 0.037      | 0.000   |
| $\gamma$   | 0.089     | 0.033      | 0.007   |
| $\sigma$   | 0.203     | 0.011      | 0.000   |
| $\rho$   | 0.324     | 0.003      | 0.000   |
| <b>Standardised residuals:</b>                     |           |            |         |
| Mean   | 0.019     |            |         |
| Variance   | 0.368     |            |         |
| Skewness   | 0.599     |            |         |
| Kurtosis   | 12.657    |            |         |
| Q(20), p-value                                     | 0.023     |            |         |
| Q(20) for squared residuals, p-value               | 0.970     |            |         |

**NOTE:**  $i_t$  = volume-weighted average of interbank rates in the euro area, the EONIA rate.  $i_t^*$  = policy rate, or target rate, which is defined as the fixed rate (until June 27, 2000) and the minimum bid rate (after June 27, 2000) at which the European Central Bank conducts its weekly open market operations. Any change in the policy rate is assumed to become effective at the day of announcement, not at the day when the next open market operation is settled. All rates are quoted as annual rates, e.g.  $i_t = 5$  means a five percent annual interest rate. Liquidity effects in panel A are estimated using the relevant supply changes, i.e. those occurring at or after the last allotment day in each RMP. See appendix B and the main text for a detailed description of the variables used in the estimation. The parameters in the variance equation represent the effect on the log of the conditional volatility. A zero liquidity effect is tested for and then imposed at two underbidding episodes and after Easter 2003. The respective days are 23/10/2001, 23/12/2002 and 23/04/2003.  $Q(j)$  denotes the Ljung-Box test for serial correlation at lag length  $j$ .

Table 3: Lagrange multiplier test for omitted variables; day of the RMP.

| Omitted variable   | p-value |          |
|--|---------|----------|
|  | Mean    | Variance |
| (A) $D_t = 1$ at days after last allotment and when t equals:            |         |          |
| T  | 0.088   | -        |
| T-1  | 0.016   | -        |
| T-2  | 0.972   | 0.033    |
| T-3  | 0.007*  | 0.102    |
| T-4  | 0.078   | 0.465    |
| (B) $D_t = 1$ at days before last settlement and when t equals:          |         |          |
| T-1  | 0.333   | 0.4      |
| T-2  | 0.034   | 0.000*   |
| T-3  | 0.144   | 0.332    |
| T-4  | 0.608   | 0.528    |
| (C) $D_t = 1$ at all days after last allotment, if last allotment is at: |         |          |
| T-5  | 0.589   | 0.096    |
| T-4  | 0.033   | 0.340    |
| T-3  | 0.666   | 0.171    |
| T-2  | 0.185   | 0.429    |
| (D) $D_t =$ number of days after last allotment minus one and t equals:  |         |          |
| T  | 0.896   | 0.448    |
| T-1  | 0.061   | 0.187    |
| T-2  | 0.025   | 0.250    |
| T-3  | 0.872   | 0.076    |
| (E) $D_t =$ five minus number of days after last allotment and t equals: |         |          |
| T  | 0.010   | 0.121    |
| T-1  | 0.275   | 0.190    |
| T-2  | 0.062   | 0.253    |
| T-3  | 0.835   | 0.041    |
| (F) $D_t = 1$ when t equals T and:                                       |         |          |
| T is a settlement day  | 0.077   | 0.088    |
| T is NOT a settlement day  | 0.276   | 0.106    |
| T-1 is a settlement day  | 0.166   | 0.102    |
| T-1 is NOT a settlement day  | 0.486   | 0.325    |
| T-2 is a settlement day  | 0.137   | 0.317    |
| T-2 is NOT a settlement day  | 0.419   | 0.398    |
| (G) $D_t = 1$ when t equals T-1 and:                                     |         |          |
| T-1 is a settlement day  | 0.043   | 0.387    |
| T-1 is NOT a settlement day  | 0.874   | 0.102    |
| T-2 is a settlement day  | 0.005*  | 0.097    |
| T-2 is NOT a settlement day  | 0.017   | 0.985    |
|  | 0.573   | 0.972    |
| (H) $D_t = 1$ when t falls on:   |         |          |
| The last settlement day in each RMP                                      | 0.147   | 0.237    |
| The last allotment day in each RMP                                       | 0.866   | -        |
| The last announcement day in each RMP                                    | 0.066   | 0.007*   |

Table 3 (continued)

| Omitted variable   | p-value |          |
|--|---------|----------|
|  | Mean    | Variance |
| (I) $D_t = 1$ for $t = T - k$ , with $k$ :                     |         |          |
| 1  | 0.872   | 0.399    |
| 2  | 0.047   | 0.013    |
| 3  | 0.074   | 0.957    |
| 4  | 0.717   | 0.414    |
| 5  | 0.589   | 0.096    |
| 6  | 0.300   | 0.482    |
| 7  | 0.273   | 0.485    |
| 8  | 0.577   | 0.832    |
| 9  | 0.160   | 0.000*   |
| 10   | 0.802   | 0.439    |
| 11   | 0.051   | 0.014    |
| 12   | 0.396   | 0.221    |
| 13   | 0.123   | 0.007*   |
| 14   | 0.407   | 0.568    |
| 15   | 0.135   | 0.503    |
| 16   | 0.105   | 0.276    |
| 17   | 0.950   | 0.760    |
| 18   | 0.081   | 0.749    |
| 19   | 0.546   | 0.052    |
| 20   | 0.192   | 0.020    |
| 21   | 0.515   | 0.605    |
| (J) $D_t = 1$ when $t$ is the first day in a RMP and falls on: |         |          |
| Monday   | 0.346   | 0.712    |
| Tuesday  | 0.798   | 0.751    |
| Wednesday  | 0.877   | 0.239    |
| Thursday   | 0.650   | 0.878    |
| Friday   | 0.628   | 0.669    |
| (K) $D_t = 1$ when $t$ is the last day of a RMP and falls on:  |         |          |
| Monday   | 0.666   | 0.332    |
| Tuesday  | 0.103   | 0.903    |
| Wednesday  | 0.273   | 0.195    |
| Thursday   | 0.980   | 0.488    |
| Friday   | 0.408   | 0.890    |
| (L) $D_t = 1$ when $t$ falls on:                               |         |          |
| The day of a Governing Council meeting                         | 0.316   | 0.36     |
| The day of a press conference                                  | 0.665   | 0.218    |
| The day of a press conference, before December 2001            | 0.154   | 0.195    |
| All days before November 9, 2001 (bi-weekly policy decisions)  | 0.019   | 0.000*   |
| The day of a policy rate change                                | 0.610   | 0.665    |
| The day after a policy rate change                             | 0.237   | 0.660    |

**NOTE:** See appendix B for a detailed description of the abbreviations used. The variable  $D_t$  takes value zero unless otherwise specified.  $H_0$ :  $D_t$  is correctly omitted from the original model specification. \* denotes significance at 1%.

Table 4: Lagrange multiplier tests for omitted variables; calendar days.

| Omitted variable                                   | p-value |          |
|--|---------|----------|
|  | Mean    | Variance |
| (A) $D_t = 1$ when t falls on:                     |         |          |
| Friday   | 0.891   | 0.567    |
| Thursday   | 0.622   | 0.746    |
| Wednesday  | 0.956   | 0.526    |
| Tuesday  | 0.892   | 0.484    |
| Monday   | 0.529   | 0.602    |
| (B) $D_t = 1$ when t is:                           |         |          |
| End of month, except end of semester               | 0.367   | 0.004*   |
| End of 1 <sup>st</sup> quarter                     | 0.255   | 0.219    |
| End of 2 <sup>nd</sup> quarter                     | 0.848   | 0.132    |
| End of 3 <sup>rd</sup> quarter                     | 0.649   | 0.749    |
| End of 4 <sup>th</sup> quarter                     | 0.944   | 0.462    |
| End of any quarter                                 | 0.244   | 0.609    |
| End of 2 <sup>nd</sup> and 4 <sup>th</sup> quarter | 0.849   | 0.170    |
| End of 1 <sup>st</sup> and 3 <sup>rd</sup> quarter | 0.214   | 0.198    |
| Begin of 1 <sup>st</sup> quarter                   | 0.944   | 0.040    |
| Begin of 2 <sup>nd</sup> quarter                   | 0.074   | 0.125    |
| Begin of 3 <sup>rd</sup> quarter                   | 0.848   | 0.416    |
| Begin of 4 <sup>th</sup> quarter                   | 0.847   | 0.405    |
| Begin of any quarter                               | 0.128   | 0.597    |
| (C) $D_t = 1$ for t being the day after:           |         |          |
| Begin of month                                     | 0.408   | 0.166    |
| Begin of month, except begin of quarter            | 0.887   | 0.053    |
| Begin of 1 <sup>st</sup> quarter                   | 0.044   | 0.365    |
| Begin of 2 <sup>nd</sup> quarter                   | 0.460   | 0.177    |
| Begin of 3 <sup>rd</sup> quarter                   | 0.041   | 0.627    |
| Begin of 4 <sup>th</sup> quarter                   | 0.704   | 0.085    |
| Begin of any quarter                               | 0.115   | 0.855    |

**NOTE:** See appendix B for a detailed description of the abbreviations used. The variable  $D_t$  takes value zero unless otherwise specified.  $H_0$ :  $D_t$  is correctly omitted from the original model specification. \* denotes significance at 1%.



Table 5: Lagrange multiplier test for omitted variables; liquidity effects and lagged dependent and explanatory variables.

| Omitted variable  | p-value       |               |
|---|---------------|---------------|
|   | Mean          |               |
| (A) Lagged dependent variable:  |               |               |
| $D_t = \Delta i_{t-2}$ , for all days, $t = 1, \dots, T$                          | 0.052         |               |
| $D_t = \Delta i_{t-22}$ , when $t = T$  | 0.014         |               |
| (B) When $t$ is the first day in a RMP and:                                       |               |               |
| $D_t = \Delta i_{t-1}$  | 0.088         |               |
| $D_t = \Delta i_{t-2}$  | 0.950         |               |
| $D_t = \Delta i_{t-3}$  | 0.959         |               |
| $D_t = i_{t-1} - i_{t-1}^*$   | 0.133         |               |
| $D_t = i_{t-2} - i_{t-2}^*$   | 0.709         |               |
| $D_t = i_{t-3} - i_{t-3}^*$   | 0.805         |               |
| (C) Lagged policy rate changes:   |               |               |
| $D_t = \Delta i_{t-1}^*$  | 0.598         |               |
| $D_t = \Delta i_{t-2}^*$  | 0.022         |               |
| (D) Liquidity effects around end of the month; $D_t = u_{t-1}$ when $t$ falls on: |               |               |
| Begin of month  | 0.779         |               |
| End of month  | 0.524         |               |
| Begin of quarter  | 0.739         |               |
| End of quarter  | 0.616         |               |
| (E) Liquidity effects at the end of a reserve maintenance period:                 |               |               |
| $D_t = u_{t-1}$ , when last allotment was before $t$ and                          |               |               |
| $t$ equals T-1  | 0.976         |               |
| $t$ equals T-2  | 0.903         |               |
| $t$ equals T-3  | 0.280         |               |
| $D_t = u_{t-2}$ , when last allotment was before $t-1$ and                        |               |               |
| $t$ equals T-1  | 0.162         |               |
| $t$ equals T-2  | 0.571         |               |
| $t$ equals T-3  | 0.572         |               |
| (F) Liquidity effects before the last settlement day of a RMP:                    |               |               |
| $D_t = u_{t-1}$ , when $t$ is before the last settlement day                      | 0.503         |               |
| (G) Asymmetric liquidity effects for days after the last allotment:               |               |               |
|   | for $D_t < 0$ | for $D_t > 0$ |
| $D_t = u_{t-1}$ and $t$ equals T  | 0.085         | 0.136         |
| $D_t = u_{t-2}$ and $t$ equals T  | 0.655         | 0.583         |
| $D_t = u_{t-1}$ and $t$ equals T-1  | 0.093         | 0.136         |
| $D_t = u_{t-2}$ and $t$ equals T-1  | 0.397         | 0.047         |
| $D_t = u_{t-1}$ and $t$ equals T-2  | 0.258         | 0.832         |
| $D_t = u_{t-2}$ and $t$ equals T-2  | 0.105         | 0.729         |

**NOTE:** See appendix B for a detailed description of the abbreviations used. The variable  $D_t$  takes value zero unless otherwise specified.  $H_0$ :  $D_t$  is correctly omitted from the original model specification. \* denotes significance at 1%.

Table 6: Predictability of the interbank rate.

| Potential effects                                     | Empirically significant effects |          |
|---|---------------------------------|----------|
|   | Mean                            | Variance |
| <b>Related to operating procedure</b>                 |                                 |          |
| Days of the reserve maintenance period (RMP):         |                                 |          |
| First day in a RMP, i.e. $t = 1$                      | X                               |          |
| Last allotment day                                    |                                 | X        |
| Any day after the last allotment day                  |                                 | X        |
| Next to last day in a RMP, i.e. $t = T-1$             |                                 | X        |
| Last day in a RMP, i.e. $t = T$                       |                                 | X        |
| Any day before the last allotment day, except $t = 1$ |                                 |          |
| Day of policy rate change and the day after           |                                 | X        |
| Liquidity effect at:                                  |                                 |          |
| Last day in a RMP, i.e. $t = T$                       | X                               |          |
| Any day after the last allotment day, except $t = T$  |                                 |          |
| Any day, except $t = 1$ and $t = T$                   |                                 |          |
| Sluggish reaction to supply changes                   | X                               |          |
| Expected supply change, temporary                     |                                 |          |
| Expected supply change, permanent                     | X                               |          |
| Expected policy rate                                  | X                               |          |
| <b>Related to calendar days</b>                       |                                 |          |
| End of month  | X                               | X        |
| Begin of month  | X                               |          |
| End of semester, additional effect                    | X                               | X        |
| Begin of semester, additional effect                  | X                               | X        |
| End of year, additional effect                        | X                               | X        |
| Begin of year, additional effect                      | X                               | X        |
| Weekdays  |                                 |          |

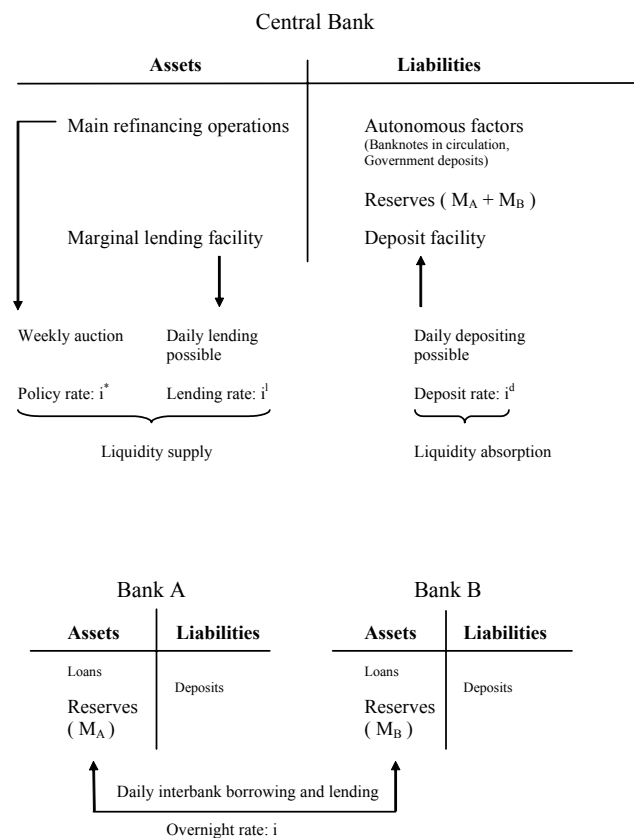
**NOTE:** Empirically significant effects are denoted by X. Results are based on the estimated empirical model and Lagrange multiplier tests. See the relevant tables for details.

## B Data description

Table 7: Description of variables.

| Dummy variable                      | Takes value one at:  |
|-------------------------------------|--|
| T                                   | The last day of each reserve maintenance period (RMP)  |
| T-1                                 | The next to last day of each RMP   |
| First day, $t = 1$                  | The first day in a RMP   |
| Last allotment day                  | The last day in a RMP at which a regular main refinancing operation is allotted (usually a Tuesday)  |
| Last settlement day                 | The last day in a RMP at which a regular main refinancing operation is settled (usually a Wednesday)   |
| Underbidding allotment day          | All allotment days when underbidding occurred. These days are 14/02/01, 11/04/01, 10/10/01, 07/11/01, 04/12/02, 18/12/02, 04/03/03, 04/06/03, 26/11/03   |
| dunderbidding (Volatility equation) | All allotment days when underbidding occurred. Additionally, some underbidding settlement days are also included. Namely, all underbidding settlement days for February, April and October 2001, and both for December 2002 (4th and 18th). Furthermore, this dummy takes value one at days 19/12/02 till 24/12/02, to take into account volatility increase from underbidding close to the end of the RMP |
| dunderbidding (Mean equation)       | This variable takes into account the underbidding effects for the mean, in 2002 and 2003. It takes value one at Wednesdays for underbidding at December 4, 2002, June 4, 2003 (settlement days), the day after settlement March 5, 2003 and the settlement following the underbidding week, March 12, 2003   |
| January 2002                        | The last four days in the first RMP of 2002. Euro cash changeover  |
| GC meeting after last allotment     | Governing Council meeting after the last allotment and policy rate change expectations. Takes value one the days before the last allotment, 20/9/1999 and 18/10/1999 and the days before and after it, i.e. 17/9/99 and 19/10/1999   |
| Underbidding at end of RMP          | Allotment and settlement days of the last regular main refinancing operation in the December 2003 RMP, 16 and 17/12/2003   |
| Policy decisions bi-weekly          | All days until 7th of November 2001. From this time onwards policy decisions are made only once a month (in general)   |
| Press conference                    | The day of the press conference held after the ECB's Governing Council meeting   |
| Governing Council meeting           | The day of the European Central Bank's Governing Council meeting   |
| Policy rate change                  | The day at which a change in the policy rate is announced  |
| Other variables                     |  |
| $i_t$                               | Volume-weighted average of interbank rates in the euro area, the EONIA rate.   |
| $i_t^*$                             | Policy rate, or target rate, which is defined as the fixed rate (until June 27, 2000) and the minimum bid rate (after June 27, 2000) at which the European Central Bank conducts its weekly open market operations. Any change in the policy rate is assumed to become effective at the day of announcement, not at the day when the next open market operation is settled                                 |
| $E_t[i_{t+k}^*]$                    | Expected future policy rate. Proxied by a forward rate constructed with one and two-week EONIA swap rates  |
| $u_t$                               | Supply shock, which is approximately the forecast error on autonomous factors (see main text for details)  |

## C Figures



**Note:**

Total reserves ( $M_A + M_B$ ) = Expected reserves + Supply **shock**

Lending (deposit) rate = Policy rate + (-) 100 bp; E.g.  $i^l = 5\%$ ,  $i^* = 4\%$ ,  $i^d = 3\%$ .

Figure 1: Illustrative summary of demand and supply of reserves. See main text for details and further discussion.

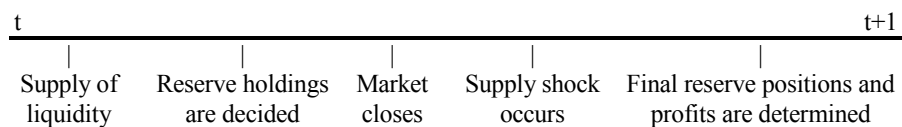


Figure 2: Timing in the interbank market. In general, supply of liquidity is constant throughout a week, changing only on Wednesday.

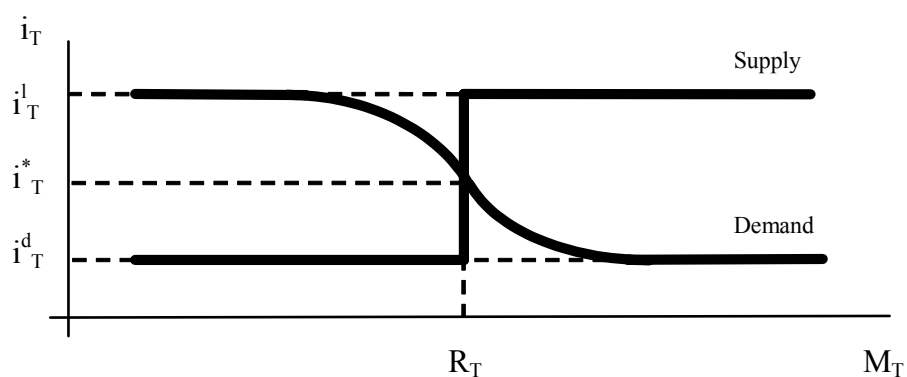


Figure 3: Demand and supply of bank reserves at the last day of a reserve maintenance period.  $M_T$  denotes current reserve holding and  $R_T$  the amount of reserves necessary to fulfill the reserve requirement for the entire reserve maintenance period. The overnight rate is denoted by  $i_T$ , marginal lending and deposit rates by  $i_T^l$  and  $i_T^d$ , respectively, and the policy rate by  $i_T^*$ .

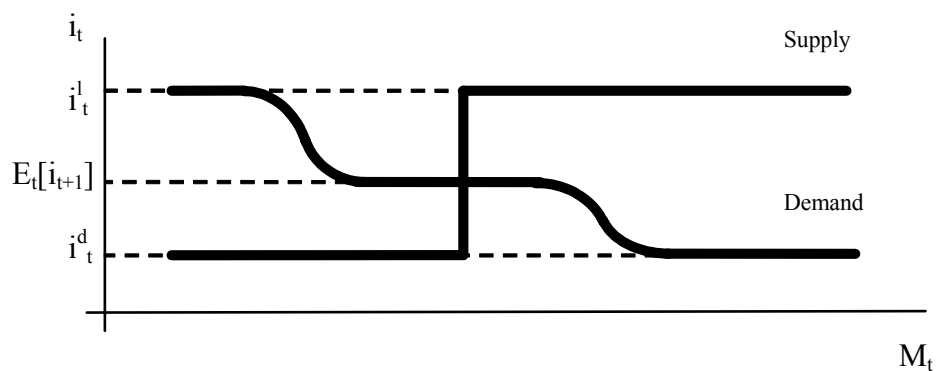


Figure 4: Demand and supply of bank reserves at days other than the last day of the RMP. Simplified model.  $M_t$  denotes current reserve holding. The overnight rate is denoted by  $i_t$  and marginal lending and deposit rates by  $i_t^l$  and  $i_t^d$ , respectively.

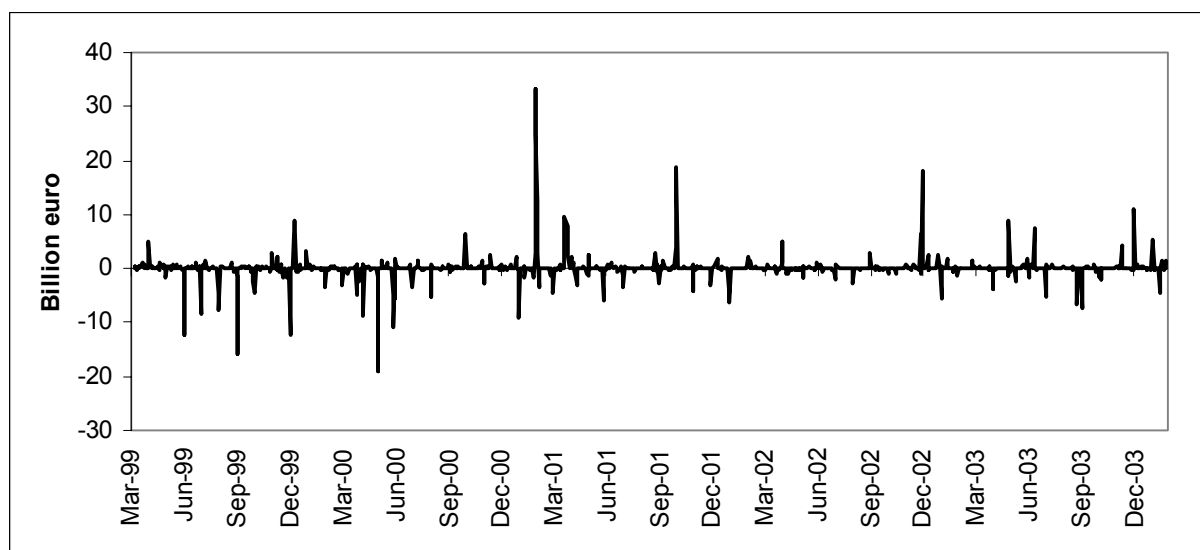


Figure 5: Net recourse to standing facilities. Vertical lines indicate the last day in each reserve maintenance period.

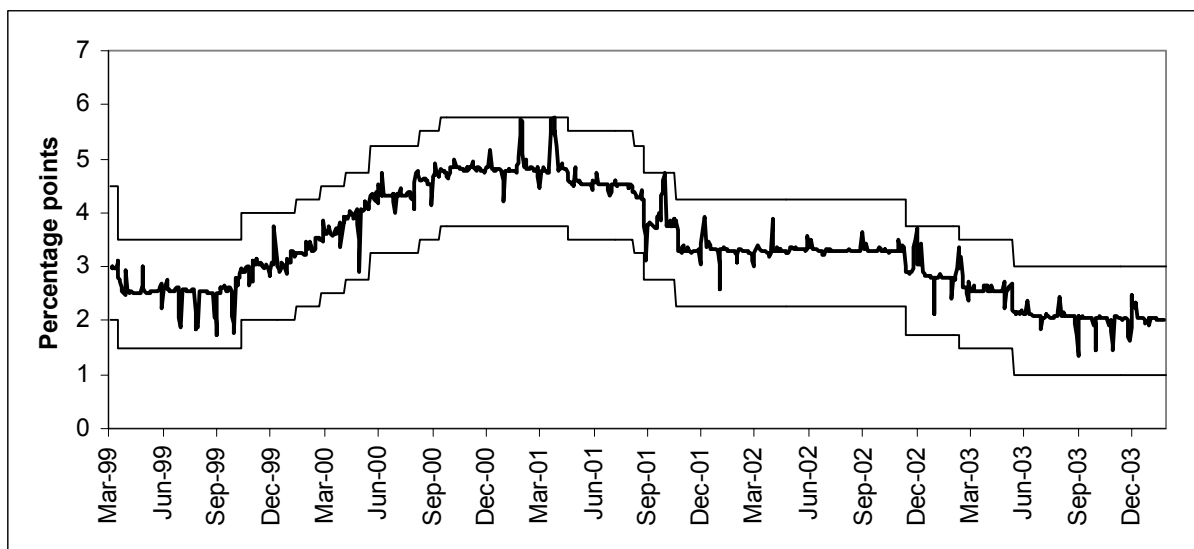


Figure 6: Euro area interbank rate (EONIA) together with deposit and marginal lending rates, which define lower and upper bounds, respectively.

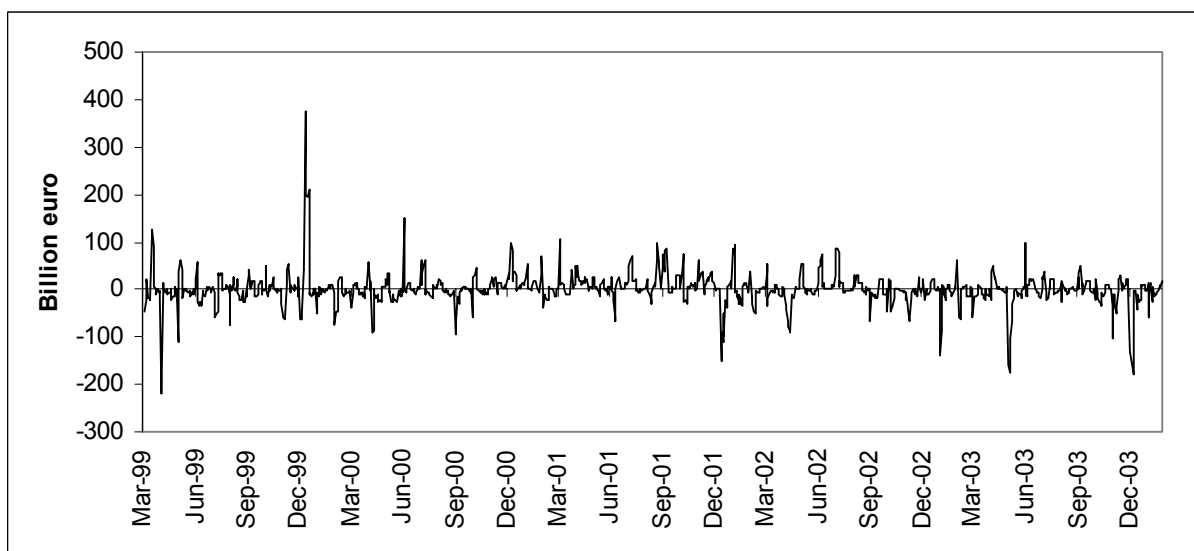


Figure 7: Deviation from neutral liquidity.

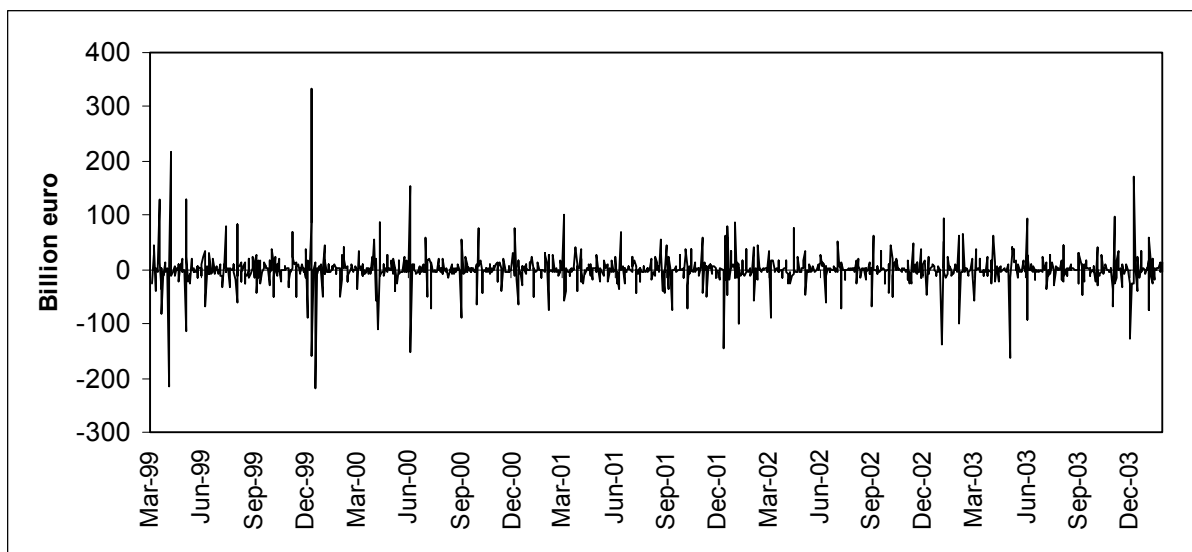


Figure 8: Change in deviation from neutral liquidity.

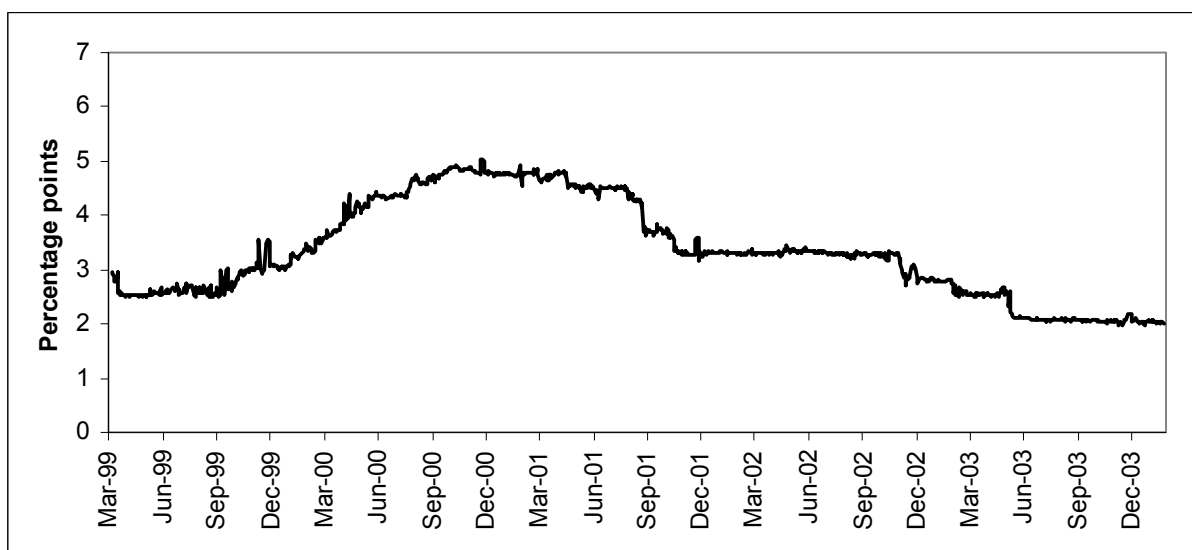


Figure 9: Proxy for expected policy rate. Constructed from two and one-week EONIA swap rates.



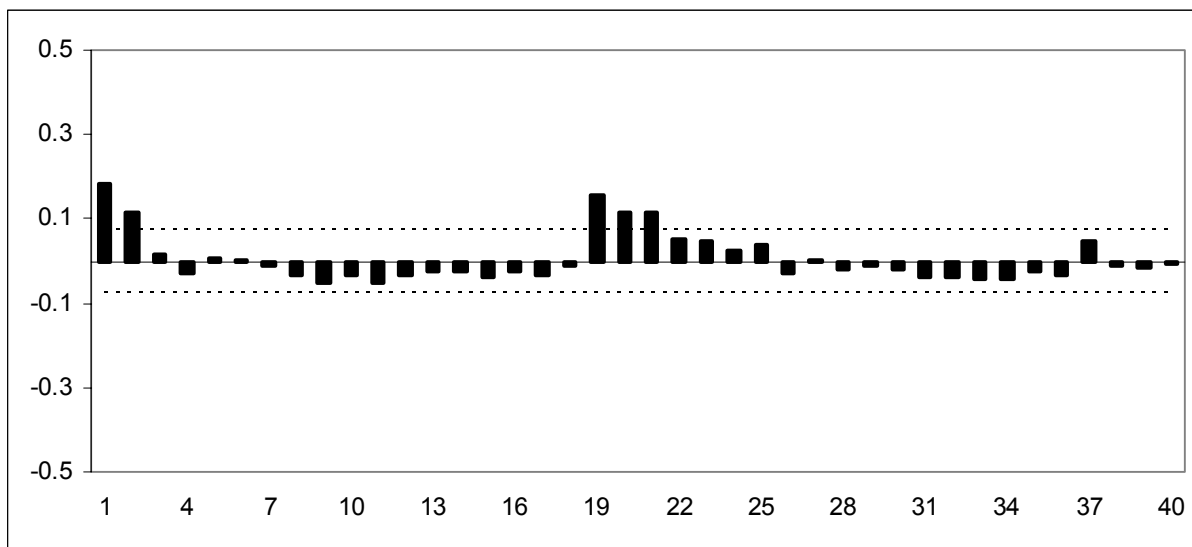


Figure 10: Autocorrelation function for squared residuals from Least Square estimation. Dotted lines represent significance at 1%.

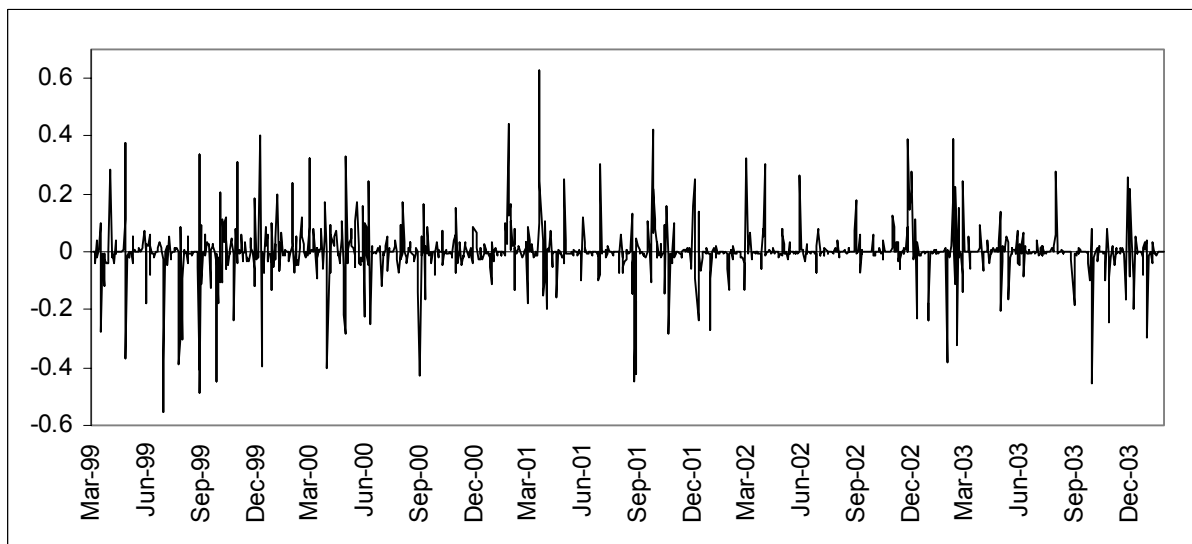


Figure 11: Residuals from EGARCH model.

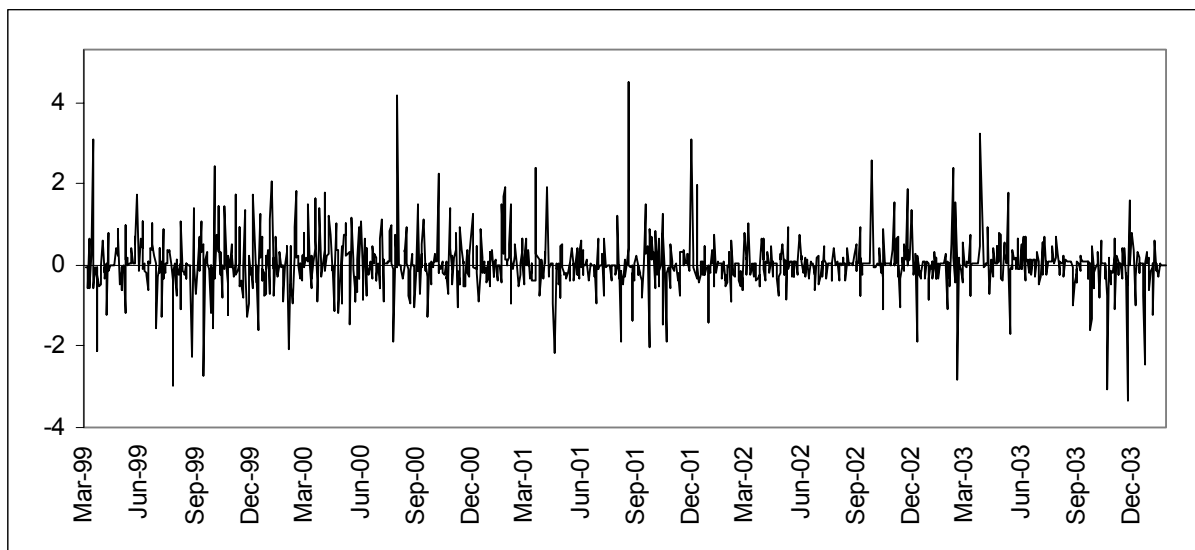


Figure 12: Standardized residuals from EGARCH model.

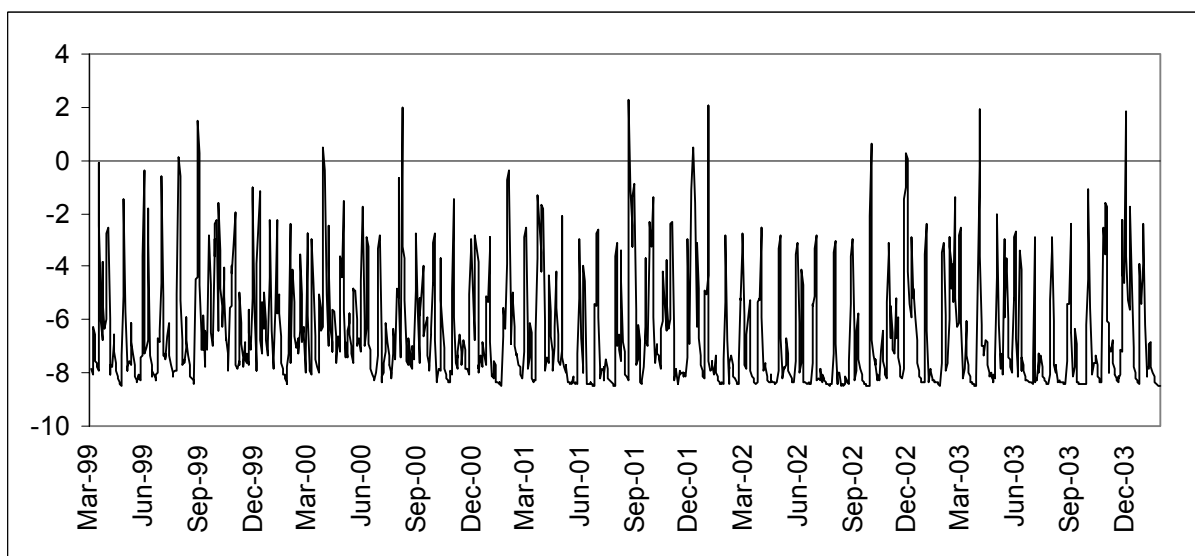


Figure 13: Logarithm of Conditional Volatility from EGARCH model.

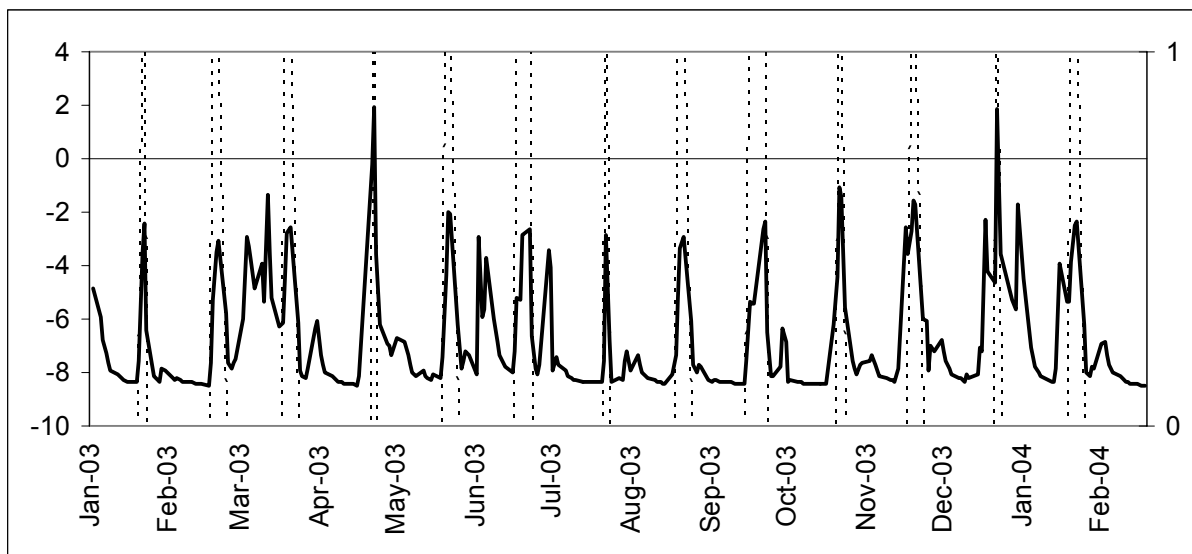


Figure 14: Logarithm of Conditional Volatility from EGARCH model (left scale). Dotted lines represent a dummy variable taking value one on all days after the last allotment day until the last day of a RMP and value zero otherwise (right scale).

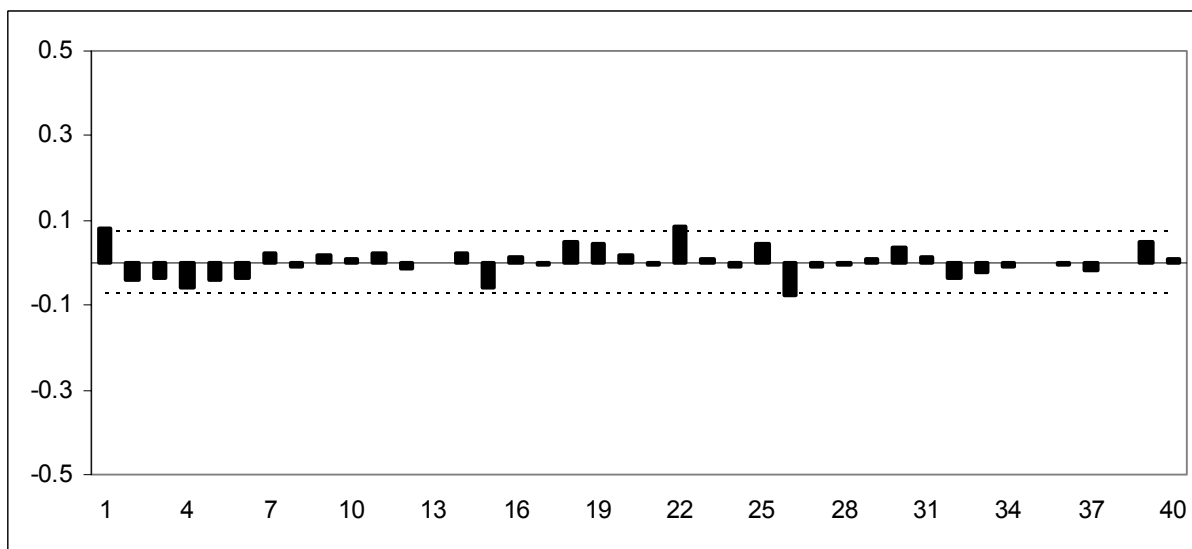


Figure 15: Autocorrelation function for residuals from EGARCH model. Dotted lines represent significance at 1%.

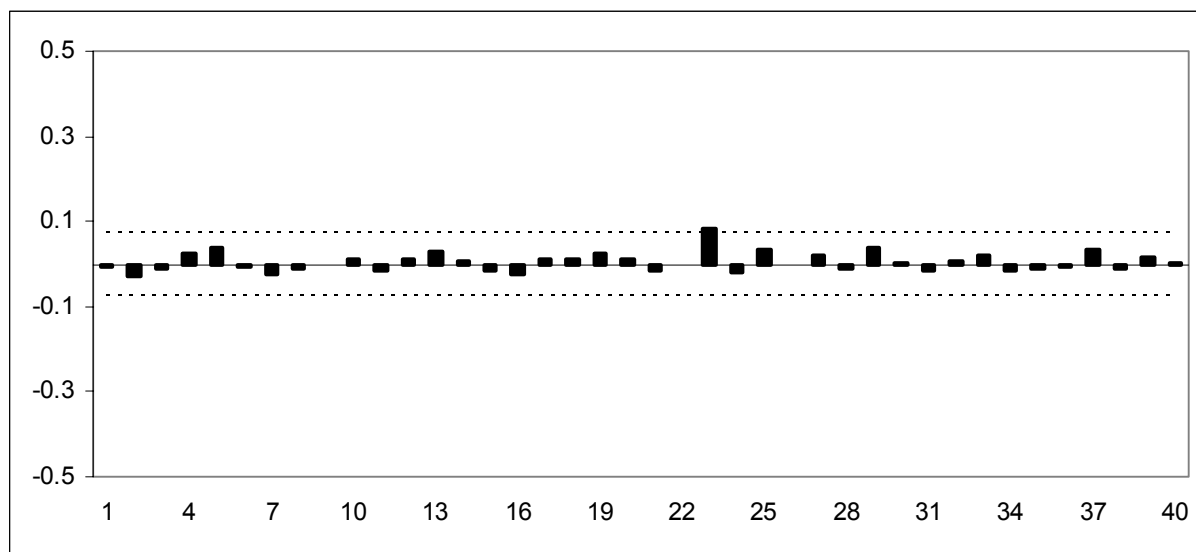


Figure 16: Autocorrelation function for squared residuals from EGARCH model. Dotted lines represent significance at 1%.

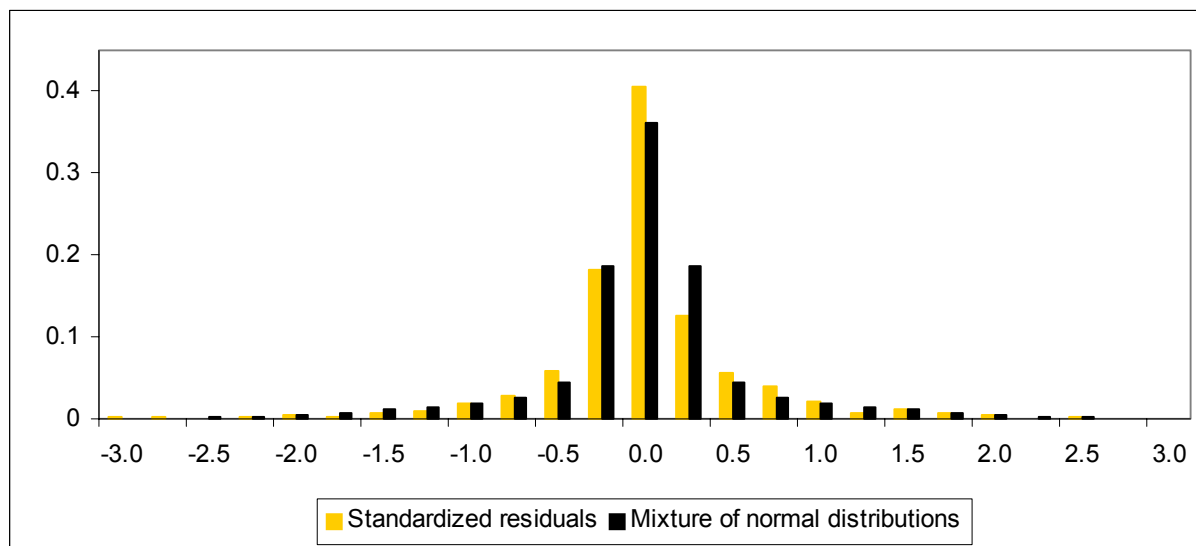


Figure 17: Estimated and assumed distribution of residuals from EGARCH model.

## European Central Bank working paper series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (<http://www.ecb.int>)

- 366 "The informational content of over-the-counter currency options" by P. Christoffersen and S. Mazzotta, June 2004.
- 367 "Factor substitution and factor augmenting technical: progress in the US: a normalized supply-side system approach" by R. Klump, P. McAdam and A. Willman, June 2004.
- 368 "Capital quality improvement and the sources of growth in the euro area" by P. Sakellaris and F. W. Vijselaar, June 2004.
- 369 "Sovereign risk premia in the European government bond market" by K. Bernoth, J. von Hagen and L. Schuknecht, June 2004.
- 370 "Inflation persistence during periods of structural change: an assessment using Greek data" by G. Hondroyannis and S. Lazaretou, June 2004.
- 371 "Inflation persistence: facts or artefacts?" by C. R. Marques, June 2004.
- 372 "The operational target of monetary policy and the rise and fall of reserve position doctrine" by U. Bindseil, June 2004.
- 373 "Technology shocks and robust sign restrictions in a euro area SVAR" by G. Peersman and R. Straub, July 2004.
- 374 "To aggregate or not to aggregate? Euro area inflation forecasting" by N. Benalal, J. L. Diaz del Hoyo, B. Landau, M. Roma and F. Skudelny, July 2004.
- 375 "Guess what: it's the settlements!" by T. V. Koepl and C. Monnet, July 2004.
- 376 "Raising rival's costs in the securities settlement industry" by C. Holthausen and J. Tapking, July 2004.
- 377 "Optimal monetary policy under commitment with a zero bound on nominal interest rates" by K. Adam and R. M. Billi, July 2004.
- 378 "Liquidity, information, and the overnight rate" by C. Ewerhart, N. Cassola, S. Ejerskov and N. Valla, July 2004.
- 379 "Do financial market variables show (symmetric) indicator properties relative to exchange rate returns?" by O. Castrén, July 2004.
- 380 "Optimal monetary policy under discretion with a zero bound on nominal interest rates" by K. Adam and R. M. Billi, August 2004.
- 381 "Fiscal rules and sustainability of public finances in an endogenous growth model" by B. Annicchiarico and N. Giammarioli, August 2004.

- 382 “Longer-term effects of monetary growth on real and nominal variables, major industrial countries, 1880-2001” by A. A. Haug and W. G. Dewald, August 2004.
- 383 “Explicit inflation objectives and macroeconomic outcomes” by A. T. Levin, F. M. Natalucci and J. M. Piger, August 2004.
- 384 “Price rigidity. Evidence from the French CPI micro-data” by L. Baudry, H. Le Bihan, P. Sevestre and S. Tarrieu, August 2004.
- 385 “Euro area sovereign yield dynamics: the role of order imbalance” by A. J. Menkveld, Y. C. Cheung and F. de Jong, August 2004.
- 386 “Intergenerational altruism and neoclassical growth models” by P. Michel, E. Thibault and J.-P. Vidal, August 2004.
- 387 “Horizontal and vertical integration in securities trading and settlement” by J. Tapping and J. Yang, August 2004.
- 388 “Euro area inflation differentials” by I. Angeloni and M. Ehrmann, September 2004.
- 389 “Forecasting with a Bayesian DSGE model: an application to the euro area” by F. Smets and R. Wouters, September 2004.
- 390 “Financial markets’ behavior around episodes of large changes in the fiscal stance” by S. Ardagna, September 2004.
- 391 “Comparing shocks and frictions in US and euro area business cycles: a Bayesian DSGE approach” by F. Smets and R. Wouters, September 2004.
- 392 “The role of central bank capital revisited” by U. Bindseil, A. Manzanares and B. Weller, September 2004.
- 393 “The determinants of the overnight interest rate in the euro area” by J. Moschitz, September 2004.

