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## The role of central bank capital revisited

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## WORKING PAPER SERIES

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# THE ROLE OF CENTRAL BANK CAPITAL REVISITED <sup>1</sup>

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### **Abstract**

This paper explores the role of central bank capital in ensuring that central banks focus on price stability in monetary policy decisions. The paper goes beyond the existing literature on this topic by developing a simple, but comprehensive, model of the relationship between a central bank's balance sheet structure and its inflation performance. The first part of the paper looks at solvency, i.e. under which conditions the "economic" capital (i.e. the discounted long term P&L) of a central bank always remains positive, despite adverse shocks, assuming a stability oriented monetary policy. The second part shows that in practice, capital is important for central banks beyond the issue of positive economic capital, when taking realistic assumptions regarding central bank independence. Capital thus remains a key tool to ensure that central banks are unconstrained in their focus on price stability in monetary policy decisions.

**Keywords:** Central Bank Capital, Central Bank Independence

**JEL classification:** E42, E58



## Non-technical summary

Although it is generally acknowledged that central banks need a certain level of capital in order to achieve their monetary policy objectives, i.e. a low level of inflation, the explanations of the causation and exact nature of the relationship have often remained vague and the underlying assumptions have not been spelled out.

This paper revisits the role of central bank capital using a specific model of the evolution of the central banks' profitability and balance sheet structure in the context of a basic macroeconomic setting. This deliberately simple model of central bank capital intends to illustrate under what conditions a low (or even negative) level of capital would not have harmful effects on the ability of the central bank to achieve its monetary policy target. The model describes the evolution of the central bank's balance sheet within a macroeconomic context, under certain assumptions about both the mechanics of the economy and the institutional constraints faced by the central bank. Under these assumptions, it is shown that a temporary shock creating negative capital and a loss-making situation is always reversed in the long run with the central bank returning to profitability and a positive level of capital. There are two exceptions: when the economy falls into a deflationary trap, from which it is not possible to escape; and, second, when the growth rate of the demand for banknotes falls short of nominal interest rates. However, a central bank with a loss-making balance sheet structure would in this context still be able to conduct its monetary policy in a responsible way, even with a negative long-term profitability outlook. Hence, some other factors not included in such a "mechanical" model must be considered in order to explain the empirical evidence regarding the negative correlation between inflation performance and financial strength of central banks.

In order to do so, we address the relationship between central bank capital and its credibility to perform its monetary policy functions. It is likely that there is another set of factors, related to the institutional environment in which the central bank exists, that is causing a relationship between the weakness in the central bank's financial position and its inability to control inflation. In practice, a central bank can never achieve an absolute, guaranteed institutional independence. In particular, no government can commit future governments (whether they obtain power by election, war, or revolution) not to change the central bank law or abolish its exclusive right to issue legal tender. Other, admittedly unlikely scenarios limiting the infinite solvency of the central bank, like a *de facto* dollarisation, have never a zero likelihood in the eyes of the public. It is hence important to distinguish between a situation where an assumed absence of central bank's concerns about balance sheet strength is shared by the public and the State, and the more realistic situation where prolonged central bank losses would lead to a complete loss of credibility. Even if a central bank is not subject to liquidity constraints,

credibility is indeed positively correlated with the level of capital when some extensions to the model are included, allowing some degree of concern about the profit and loss account to affect the central bank's interest rate setting behaviour.

Finally, the paper briefly discusses profit sharing rules, which are obviously closely linked to the capital issue. The main conclusion is that a fully automated and fully credible rule of re-capitalisation by the government of the central bank in case of losses can be regarded as a substitute for positive capital. Since such rules are however difficult to implement in practice, positive capitals seems to remain a key tool to ensure that independent central bankers always concentrate on price stability in their monetary policy decisions.



## 1. Introduction

In the last two decades the issue of commercial banks' capital adequacy has become one of the most analysed topics in banking and finance. Banks, regulators and academics have devoted substantial resources to developing capital adequacy regulations that could be generally applied to banks everywhere in the world. Central banks, too, normally have a capital position in their balance sheet. However, there are no rules requiring central banks to hold a certain level of capital: indeed, cross-country surveys reveal that they have been taking rather different approaches. The rationale for credit institutions to hold capital obviously cannot be translated one-to-one to central banks. For instance, the central bank normally cannot become illiquid as long as the currency it issues is legal tender. Nevertheless, recent studies have concluded that positive central bank capital is required if the central bank is to perform its tasks successfully, in particular, to achieve price stability. However, this conclusion sometimes seems to be drawn from anecdotal evidence without providing an exact analysis of the nature of the relationship and the underlying assumptions. This paper, therefore, revisits the role of central bank capital on the basis of a specific model of the evolution of the central banks' profitability and balance sheet structure in the context of a basic macroeconomic setting. Using this approach, it is possible to investigate the interrelationship between capital and other balance sheet items and macroeconomic variables with more precision than previous studies. The paper identifies the long-run steady states for different levels of initial central bank capital, using inter alia Monte Carlo simulations.

The paper is structured as follows. Section 2 reviews the existing literature on the topic. While this literature stresses the empirical finding of an inverse relationship between the central bank's financial strength and the country's inflation rate, it lacks a more formal derivation of causality between the two variables. Section 3 thus presents a simple model of central bank capital in order to illustrate under what conditions deficient capital would not have harmful effects on the central bank's policy target. The model explicitly describes the development of the central bank's balance sheet within a macroeconomic context, under the assumption that the central bank is unconstrained in liquidity terms. A central bank is defined here as being liquidity unconstrained if it has the strictly non-revocable privilege to issue legal tender. The model further assumes that the central bank sets interest rates according to a Taylor rule and that there is a Wicksellian relationship between interest rates and inflation. Section 4 provides Monte Carlo simulations and derives a proposition describing the conditions under which central banks in the long run always manage to return to profitability and positive capital. They do not in particular in two cases: when the economy falls into a deflationary trap, from which it is not possible to escape in the basic Wicksellian model; and, second, when the growth in the demand for banknotes falls short of nominal interest rates. In these cases, the central bank never recovers profitability. In Section 5, the relationship between central bank capital or balance sheet strength and its credibility to perform its monetary policy functions is addressed. This section stresses the importance of distinguishing between a situation, where an assumed absence of central

bank's concerns about balance sheet strength is shared by the public and the State, and the more realistic situation where prolonged central bank losses would lead to a credibility crisis. Therefore, even if a central bank is not subject to liquidity constraints, credibility should be positively correlated with the level of capital.

This gives rise to the observed relationship between the level of capital and inflation performance. In essence, the reasons why a central bank might seek to increase profitability at the expense of higher inflation are closely linked to the broader issue of central bank independence. Again, Monte Carlo simulations are used to examine the quantitative effects under a range of different assumptions. Section 6 discusses profit sharing rules between the central bank and the government, which is also obviously linked to capital. Section 7 concludes.

## **2. The literature on central bank capital**

The issue of central bank capital and financial independence has in the past not attracted the level of public attention paid, for example, to the operational and statutory (or “institutional”) independence of central banks. Nevertheless, over the last few years, there has been increasing discussion about whether central banks need capital and, if so, how much. There are a number of possible reasons. First, the trend over the last decade of greater independence for central banks to implement monetary policy has raised the question of central bank financial independence. Second, central banks have been more aware of the risks they face through the use of more sophisticated risk management techniques. Third, there is a pressure, particularly resulting from the International Monetary Fund, for higher levels of transparency in central banks' accounting practices.<sup>2</sup> Finally, interest in central bank financial independence has also been spurred by public debates in a number of countries, such as Finland, Japan and Switzerland. The following section reviews some of the main themes that have been raised in the literature.

Stella [1997], [2002], an IMF staff member, was one of the first to analyse the fact that several central banks had incurred such large losses that they had to be recapitalised by the government, or were in the process of being recapitalised. Their balance sheets had become so weak – i.e., capital was substantially negative – that further losses were inevitable and this consequently interfered with the attainment of the central banks' policy objectives, primarily maintaining a low inflation rate. For instance in Uruguay in the late 1980s, the central bank's losses were equal to 3% of GDP; in Paraguay the central bank's losses were 4% of GDP in 1995; in Nicaragua losses were a staggering 13.8% of GDP in 1989. By end of 2000, the Central Bank of Costa Rica had negative capital equal to 6% of

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<sup>2</sup> See “Code of Good Practices on Transparency in Monetary and Financial Policies” on the IMF website at <http://www.imf.org/external/np/mae/mft/index.htm>

GDP.<sup>3</sup> However, he also found that there were some rather puzzling outliers: for example, the Central Bank of Chile made losses over years but there was no effect on its very good inflation performance.

Stella argues that capital – when defined in the traditional accounting sense that is used for commercial banks – is meaningless when applied to central banks. Instead, the central bank’s net worth – defined as the price a fully informed risk-neutral investor would pay to purchase the bank under normal market conditions – is a much more useful indicator of a central bank’s potential profitability and financial independence. First, net worth takes into account the central bank’s “franchise value” – the value of its special legal status of being able to print money and impose reserve requirements on commercial banks. At the same time, it also takes into account the central bank’s off-balance sheet rights and obligations – such as the obligation to bail out the banking sector in a crisis or defend a fixed exchange rate peg – which would tend to reduce net worth.<sup>4</sup> Therefore, a central bank could have a balance sheet structure such that even though it had “large” capital, it would still make considerable losses; conversely, it could have zero capital yet make very large profits. Thus, Stella preferred to use a more vague concept of a “central bank’s financial strength”. According to this definition, a financially strong central bank is one that possesses sufficient resources to attain its fundamental policy objectives. Stella [1997] concludes:

*“...Eventually, the [central bank’s] balance sheet may deteriorate to a point where they either must abandon control over inflation, repress the financial system, become reliant on constant infusions by the Treasury, or – the last alternative – be recapitalised.”*

However, the link between the central bank’s financial strength and the need to abandon price stability or repress the financial system appears to be drawn from the possibility that the central bank becomes illiquid. It is argued that a financially weak central bank (which is likely to have negative capital and repetitive losses) will consequently have to sell liquid interest-earning assets in order to cover its operating costs and other financial obligations. As it sells these assets, *ceteris paribus*, it also reduces its potential to make profits in the future and “eventually the central bank will exhaust its supply of valuable liquid assets”. At this point, it is argued that the central bank would have three options: issuing its own debt certificates; repression of the financial system (e.g. through high non-remunerated reserve requirements); or the abandonment of price stability by injecting excess liquidity into the banking system. He rules out the first option of issuing debt certificates as it runs the risk of the central bank accumulating an unsustainable debt burden:

*“...The sustainability of central bank debt issuance is a function of the same factors that determine the sustainability of government debt in general. These include expectations of the*

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<sup>3</sup> See also Leone [1993], Dalton and Dziobek [1999]

<sup>4</sup> Blejer and Schumacher [1999] provide a detailed analysis and taxonomy of the different types of central banks’ “contingent liabilities” – defined as financial commitments triggered by the occurrence of an event whose realisation is uncertain, e.g. guaranteeing the stability of the banking sector or holdings of derivatives. These liabilities tend to be off-balance sheet and reduce the transparency of central bank accounts. They argue that this may result in serious problems regarding the proper assessment of the financial position of the monetary authority.

*future income and expenditure stream of the central bank, the growth rate of demand for the securities being purchased from the central bank, the reputation of the issuer of the security, macroeconomic developments, the government's commitment to guarantee obligations of the central bank, budgetary development, etc.”*

However, one could argue that debt sustainability issues are less of a concern to a central bank than to a government. Unlike a government, a central bank can always obtain the funds it needs to cover its expenses simply by crediting a credit institution's current account at the central bank. If this were to create excess liquidity in the banking system (i.e., with aggregate current accounts higher than reserve requirements), overnight interest rates would consequently fall to the deposit facility rate or to zero if a deposit facility were not available. Of course, if interest rates were to fall to zero, it could result in a too “loose” monetary policy and lead to an increase in inflation. In order to avoid this, the central bank could issue debt certificates to absorb the excess liquidity, offering an interest rate at a level which would ensure price stability. Banks would have no alternative except to purchase the debt certificates, as the alternative would be to earn zero interest on the excess reserves. One might argue naively that this process could continue indefinitely, with the losses resulting from the interest paid on the debt certificates being covered by further crediting banks' current accounts and consequently issuing more debt certificates to mop up the excess liquidity. There would be no technical restriction on the amount of debt certificates which could be issued.

In the literature there is understandably no precise definition of what the optimal level of capital should be. Stella identifies four different ways that central banks have used in practice to determine their own level of capital: (1) An absolute nominal value of capital; (2) A target ratio of capital to another central bank balance sheet item; (3) A target ratio of capital to a macroeconomic variable; (4) According to the perceived risks to the “solvency” of the bank. According to the last approach, the optimal level of capital would depend on a large number of qualitative and quantitative factors, such as the macroeconomic economic environment, the central bank's vulnerability to large financial shocks, the bank's historical legacy, the status of institutional relations with the government, the bank's policy obligations (e.g., defence of a currency peg), and its volatility of profits.<sup>5</sup>

Martínez-Resano [2004] also attempts to define the factors which determine the optimal level of capitalisation. He surveys the full range of risks that a central bank's balance sheet is subject to. Then, using a simple VAR model, he analyses the interplay between capitalisation, accounting rules and dividend distribution, so as to determine a simple benchmark for central bank financial strength. He concludes that, in the long run, central banks' financial independence should be secure as long as demand for banknotes is maintained. However, in the short- and medium term, he finds that financial vulnerability could impact on a central bank's effective independence. In order to avoid this possibility, he concludes that adequate capitalisation is key. In the model, the size of the optimal

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<sup>5</sup> A fuller discussion of the process of determining the level of required capital and specific practices is provided by Dalton [1999].

capital buffer depends on risk exposure, volatility, costs of underperformance (i.e. the impact of lack of independence on inflation) and how the central bank's profits are distributed. If a central bank were guaranteed to retain all its profits, the central bank could operate without capital at all. Otherwise, a certain level of capital is required or, as a second-best option, adequate institutional arrangements. He finds that one of the preferred methods for profit distribution is "smoothing", as it is consistent with the assumed natural cycles of central banks' earning generation and is also neutral in terms of income for the Government. Stella [2002] and Sullivan [2002] also stress the importance of a clear set of rules to govern the distribution of both profit and losses in this historically highly opaque area.<sup>6</sup> Sullivan [2002] argues that the central bank law should specify clearly how its profit is calculated and how net losses are allocated, as well as how the bank will be recapitalised in the event of a crisis.

Ernhagen et al [2002] from the Sveriges Riksbank also examine whether having a buffer of capital is of importance for a central bank's independence. The authors argue that the central bank can always pay its debts and operating costs with banknotes, but if the total demand for banknotes among the public has not increased then these newly printed banknotes will quickly be exchanged at the central bank. As argued previously by Stella [1997], the central bank will then be forced either to finance itself by issuing "accelerating" interest bearing debt which would further reduce future expected profitability or by lowering interest rates to levels which would trigger inflation and consequently increase demand for banknotes. Reducing interest rates to increase demand for banknotes may not be compatible with the central bank's monetary policy objectives. However, as already mentioned, the central bank does not need to "finance itself" through debt certificates, but only may want to absorb excess liquidity through debt certificates for monetary policy purposes.

Ernhagen et al [2002] provide some crude calculations regarding the minimum level of capital for the Riksbank, although they admit that it is impossible to provide a precise answer and that it depends to a large extent on the safety margins that the government wishes to provide. First, they calculate the amount of capital – assuming an interest rate of 5% – that would be needed to cover operating costs, i.e. wages, rent, fixed capital etc. under the assumption that they cannot rely on seignorage income. They assume no seignorage income because of the increasing use of e-money and other cashless payments, which may cause an abrupt reduction in demand for banknotes.<sup>7</sup> Second, they add the risks of losses on the central bank's foreign exchange portfolio based on value-at-risk calculations, although it is acknowledged that the risk of losses depends significantly on whether Sweden has a floating or fixed exchange rate. Currently Sweden has a floating exchange rate, but if it were to have a fixed exchange rate, e.g. if it were to join ERM II, the risks of losses resulting from defending the exchange rate could be substantial. Finally, they add the potential losses, which could result from providing

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<sup>6</sup> According to Vaez-Zadeh [1991], p.70, who reviewed central banking laws in some 60 countries, almost one third of central banks do not have any specific provisions regarding the treatment of losses. Pringle and Courtis [1999] find that most of the 27 central banks they surveyed provide rules for the distribution of profits (although there is no explicit mention of what the rules are for losses).

<sup>7</sup> Note that the Bank of England does not receive seignorage income from banknote issue at all. The assets that back the UK banknotes are held by a separate department of the bank (the Issue Department) in an attempt to demonstrate that assets of appropriate quality back the currency. The profits, once the expenses of banknote production and distribution have been taken care of, go directly to the Treasury. The bank's main source of income is from unremunerated cash ratio deposits which commercial banks are required to hold at the Bank of England.



emergency liquidity assistance if there were a banking crisis: this may be much more serious in terms of cost than a loss on the foreign exchange portfolio.<sup>8</sup> They use the losses from the previous banking crisis in the late 1980s as a benchmark although they say there is much uncertainty: the true cost would depend on the quality of the collateral provided, the amount of lending without adequate collateral, and the extent to which the losses would be shared with the government. Having calculated the minimum level of capital required, the authors also assess the costs of having too high a level of capital. They conclude that as long as the return on the “excess” capital is as high as the interest rate paid by the government on its debt, and that the profits of the central bank are transferred to the government, there are no adverse consequences resulting from overcapitalisation of the central bank.

According to Pringle [2003], the Bank of Japan (BOJ) has also thought rather extensively about central bank capital. While conducting monetary policy in recent years, the BOJ has taken on considerable risks, purchasing vast amounts of long-term Government bonds at very low yields.<sup>9</sup> When the economy recovers, bond yields will inevitably rise, sending bond prices – and thus the value of the BOJ’s assets on a “mark-to-market” basis – plummeting. Pringle argues that the BOJ’s capital – until recently only 3% of its total assets – could be wiped out if the BOJ had to report huge paper losses, thus undermining the credibility of the BOJ.<sup>10</sup> The issue has become more urgent as the BOJ’s return on assets declined sharply last year, as a result of lower interest rates in the US and euro area, and the BOJ made an unprecedented request to the Finance Ministry to be allowed to keep 15% of its profit for 2002 to build up its capital (as a rule, the central bank only retains 5%). Although Pringle questions the usefulness of capital when the BOJ balance sheet is exposed to massive “macro” risks and that in the longer term there is unlikely to be a problem as the BOJ is a monopoly supplier of base money, he concludes that “to support its credibility, an independent central bank needs adequate capital as well” without however providing detailed arguments.<sup>11</sup>

Overall, one may conclude that the literature seems to have one main shortcoming: although stressing that capital plays a substantially different role in a central bank than in a commercial bank, it seems to rely on the assumption that a central bank would not be able to issue debt certificates indefinitely. However, as argued above, this kind of “illiquidity” argument does not appear to be technically obvious in the case of a central bank. Furthermore, no model has so far been developed to analyse the exact interaction between the different variables. In the following sections, we will aim at giving more

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<sup>8</sup> Gros and Schobert [1999], in contrast, argue that the lender of last resort function is not a reason to “over-capitalise” central banks. They argue that the only danger that a central bank might face as a lender of last resort is that it would be obliged to create additional money, but this can always be done without limits independently of its capital. Then, even if the central bank lends big amounts to banks in trouble during a financial crisis and has difficulties collecting the loans when the crisis is over, these losses should not be covered through additional money creation, but they are anyway the responsibility of national finance ministries, which would have to make up any losses national central banks incur as lenders of the last resort.

<sup>9</sup> In a speech delivered on 25 October 2003, Kazuo Ueda, a Policy Board member of the Bank of Japan, remarked, “The BOJ has, over the past few years, taken various risks in its operations seen unthinkable under normal circumstances and there are risks that this will bring about consequences...If yields rose sharply and the value of JGBs holdings fell by 10% or so, the BOJ will essentially be in negative net worth.”

<sup>10</sup> See also JP Morgan, *Japan Markets Outlook and Strategy*, 24 January 2002, which raised questions as to the likely duration of the Bank of Japan’s willingness to use its “rinban” operations to support the long end of the government bond yield curve because of the potential losses that could be incurred.

<sup>11</sup> Svensson [2004] also takes up the issue of the need for central bank capital in the context of escaping a deflationary trap. He assumes, referring to Stella, that “a negative capital would require a capital injection and put the bank at the government’s mercy. In order to avoid this, the bank never voluntarily allows its capital to fall below a certain minimum level.”



precision to analysis through a simple modelling approach encompassing both the central bank balance sheet and a stylised macro-economy.

### **3. A simple model based on a central bank with no liquidity constraints**

The mechanisms by which central bank capital can impact on a central bank's ability to achieve price stability can be best illustrated by first developing a simplistic model in which there is a kind of dichotomy between the level of capital and inflation performance. To create this dichotomy requires the strong assumption that the central bank is "liquidity unconstrained", which could be seen to reflect *total certainty that the central bank will maintain the right to issue legal tender over an infinite horizon*. Making such an assumption implies that the level of capital and the profit and loss situation should not become a problem as they could for a private company, which eventually would face illiquidity.

It needs to be highlighted that the model obtains the dichotomy result quasi by assumption, and that to approach reality, some of the assumptions will need refinement, which will lead to a breakdown of the dichotomy result. Still, the model is an appropriate starting point to derive the actual reasons for the relevance of central bank capital in the most transparent way.

Most importantly, the model does not yet encompass the interaction between a central bank's financial strength and the public's expectations, which could lead to a breakdown of the model's hypothesised Wicksellian relationship between inflation and interest rates. Since ultimately the assumption of the central bank as a potentially unlimited source of money depends on the public's trust in this money's worth, it can indeed prove unrealistic that a loss making central bank is believed to ignore balance sheet concerns and rely on a very long-term profitability outlook. In fact, even within the framework of the model presented here, there is still a non-zero probability of the economy falling in a deflationary trap and the central bank sustaining permanent losses. In addition, it could happen, especially after a series of capital losses, that even unjustified private sector expectations of a risk that the central bank might lose the right to issue legal tender at some distant point in time could already today undermine the reputation of the central bank. Hence, a feasible explanation for the importance of central bank capital could well rely on a self-fulfilling expectations kind of argument. The public does not fully give credit to the theoretical possibility – from an accounting point of view – of a central bank conducting a successful monetary policy while incurring repeated capital losses.

Having made this caveat, we now turn to the specification of the model, by considering the following simplistic central bank balance sheet.



*Stylised balance sheet of a central bank*

Assets	Liabilities
Monetary policy operations (“M”)	Banknotes (“B”)
Other Financial Assets (“F”)	Capital (“C”)

Banknotes are assumed to always appear on the liability side, while the three other items can be *a priori* on any side of the balance sheet. For the purpose of the model, a positive sign is given to monetary policy and other financial assets when they appear on the asset side and a positive sign to capital when it appears on the liability side. The following assumptions are taken on each of these items:

- *Monetary policy operations* can be interpreted as the residual of the balance sheet. It is remunerated at  $i_M$  %, the operational target interest rate of the central bank. Assume that the central bank, when setting  $i_M$ , follows a kind of simplified Taylor rule of the type  $i_{M,t} = 4 + 1.5(\pi_{t-1} - 2)$ . According to this rule, the real rate of interest is 2% and the inflation target is also 2%.<sup>12</sup> An additional condition has also been introduced in the Taylor rule, namely that in case it would imply pushing expected inflation in the following year into negative values, the rule is modified so as to imply an expected inflation of 0%. It will later be modelled that for profitability/capital reasons, i.e., reasons not relating directly to its core task, the central bank may also deviate from this interest rate setting rule.
- *Other financial assets* contain foreign exchange reserves including gold but possibly also domestic financial assets clearly not relating to monetary policy. Assume it is remunerated at  $i_F$  %. The rate  $i_F$  % may be higher or lower than  $i_M$  %, which depends *inter alia* on the yield curve, international imbalances in economic conditions, the share (if any) of gold in F, etc. Also, F can be assumed to produce revaluation gains/losses each year. One may assume that  $i_{F,t} = i_{M,t} + \rho + \omega_t$  with normally, but not necessarily,  $\rho > 0$ , implying that the rate of return on F would tend to be higher than the interest rate applied to the monetary policy instruments, and  $\omega_t$  is a random variable with zero mean reflecting the associated risks. F can in principle be determined by the central bank, but it may also be partially imposed on the central bank through its secondary functions or ad hoc requests of the Government. Indeed, F may include, especially in developing countries, claims resulting from bank bailouts or from direct lending to the Government, or also from special industry borrowing programs imposed on the central bank by the Government. Typically, such assets are remunerated at below market interest rates, such that one would obtain  $\rho < 0$ .
- *Banknotes* are assumed to depend on inflation and normally follow some increasing trend over time, growing faster when inflation is high<sup>13</sup>. Assume that

<sup>12</sup> See e.g. Woodford [2003] for a discussion of the properties of such policy rules.

<sup>13</sup> This assumption would not hold in case money velocity accelerates when inflation increases (Cagan-Bresciani Terroni thesis). In such a case, the profitability outlook of the central bank becomes less optimistic, as in the general case when

$B_t = B_{t-1} + B_{t-1}(2 + \pi_t)/100 + B_{t-1}\varepsilon_t$ , whereby  $\pi_t$  is the inflation rate, “2” is the assumed real interest or growth rate and  $\varepsilon_t$  is a noise term. It is assumed that the real interest rate is exogenous.

Despite the development of new retail payment technologies over many years, and speculation that banknotes could vanish in the long run, banknotes have continued to increase in most countries at approximately the rate of growth of nominal GDP. Our stylised balance sheet does not contain reserves (deposits) of banks with the central bank, but it can be assumed alternatively that reserves are implicitly contained in Banknotes (which may thus be interpreted as the monetary base). The irrelevance of the particular distribution of demand between banknotes in circulation and reserves with the central bank would thus add robustness to this assumption on the dynamics of the monetary base.<sup>14</sup>

- *Capital* depends on the previous year’s capital, the previous year’s profit (or loss), and the profit sharing rule between the central bank and the Government. In the basic model setting, it is assumed that the profit sharing rule is as follows: if profit is positive, i.e.  $P_{t-1} \geq 0$  then  $C_t = C_{t-1} + \alpha P_{t-1}$  (with  $0 < \alpha < 1$ ), else  $C_t = C_{t-1} + P_{t-1}$ , and  $\alpha$  is set to 0.5. Profits depend on the returns on the different balance sheet positions and on operating costs. With regard to operating costs,  $q$ , it is assumed that they grow over time at the inflation rate. Profit and thus Capital is likely to contain a further random element, which reflects that extraordinary costs may arise to the central bank when the Government manages to assign additional duties to the bank. In the less industrialised countries, these costs could typically be the support of insolvent banks, or the forced granting of credit to the Government. As mentioned above, such factors can also be modelled as affecting the remuneration rate of financial assets.

To complete the model, one needs an equation that explains the evolution across time of the inflation rate. A Wicksellian relationship between inflation tomorrow and inflation today is assumed, i.e.  $\pi_{t+1} = \pi_t + \beta(2 + \pi_t - i_{M,t}) + \mu_t$ , i.e. inflation normally accelerates if interest rates on monetary policy operations are below the sum of the real rate on capital (2%) and the current inflation rate.<sup>15</sup> The noise term  $\mu_t$  means that inflation is never fully controlled. The equation also implies that there is a risk of ending in a deflationary trap: when  $\pi_t < -2$ , then, due to the zero constraint to interest rates, prices should start falling further and further, even if interest rates are zero. If  $\mu_t \sim N(0, \sigma_\mu^2)$ , then this can always happen theoretically, but of course the likelihood decreases rapidly when the sum of the present inflation and of the real rate is high.

Adding a time index  $t$  for the year, the time series are thus determined as follows over time<sup>16</sup>:

demand for banknotes decreases for technological or other reasons. Note as well that in a macroeconomic context of hyperinflation, the opportunity cost of holding banknotes could reverse this positive relationship between inflation and banknote demand. In extreme situations, hyperinflation could cause the surge of privately issued money substitutes or the return to money-less trade in goods.

<sup>14</sup> A switch from banknotes holdings to reserve holdings would imply that seignorage revenues would in the first case stem from a general tax to the holders of banknotes, while in the second case they would be comparable to a tax on the banking sector.

<sup>15</sup> See Woodford [2003] for a discussion of such Wicksellian inflation functions.

<sup>16</sup> The order of the equations, although irrelevant from a conceptual point of view, reflects how the eight variables can be updated sequentially and thus how simulations can be obtained.

$$\begin{aligned}
\text{Eq. 1} \quad & \pi_t = \pi_{t-1} + \beta(2 + \pi_{t-1} - i_{M,t-1}) + \mu_t \\
\text{Eq. 2} \quad & q_t = (1 + \pi_t / 100)q_{t-1} \\
\text{Eq. 3} \quad & F_t = F \\
\text{Eq. 4} \quad & \text{if } P_{t-1} \geq 0 \text{ then } C_t = C_{t-1} + \alpha P_{t-1} \text{ (with } 0 < \alpha < 1), \text{ else } C_t = C_{t-1} + P_{t-1} \\
\text{Eq. 5} \quad & B_t = B_{t-1} + B_{t-1}(2 + \pi_t)/100 + \varepsilon_t \\
\text{Eq. 6} \quad & \text{if } \max(4 + 1.5(\pi_{t-1} - 2), 0) < \frac{\pi_{t-1}}{\beta} + 2 + \pi_{t-1}, i_{M,t} = \max(4 + 1.5(\pi_{t-1} - 2), 0) \\
& \text{else } i_{M,t} = \frac{\pi_{t-1}}{\beta} + 2 + \pi_{t-1} \\
\text{Eq. 7} \quad & i_{F,t} = i_{M,t} + \rho + \omega_t \\
\text{Eq. 8} \quad & M_t = B_t + C_t - F_t \\
\text{Eq. 9} \quad & P_t = i_{M,t}M_t + i_{F,t}F_t - q_t
\end{aligned}$$

This simple modelling framework captures all basic factors relevant for the profit situation of a central bank and the related need for central bank capital. It can also be used to analyse the interaction between the central bank balance sheet, interest rates and inflation. It should be noted that, from equation 1 and  $i_{M,t} = 4 + 1.5(\pi_{t-1} - 2)$ , a second order differences equation can be derived of the form  $\pi_{t+1} - (1 + \beta)\pi_{t-1} + 1.5\pi_{t-2} = \beta + \mu_t$ . Disregarding the stochastic component,  $\mu$ , this equation has a non-divergent solution whenever  $-2/3 < \beta < 2/3$ . The constant solution  $\pi_t = 2, \forall t$ , is a priori a solution in the deterministic setting. However, it has probability 0 when considering again the shocks  $\mu_t$ .

Simulations can be performed to calculate the likelihood of profitability problems arising under various circumstances. The model can be calibrated for any central bank and for any macroeconomic environment. In this paper, we focus on a few important illustrative scenarios.

Before proceeding with the simulations, the impact of capital on the central bank's profitability and hence financial independence should be described briefly. First, note that as long as we do not foresee the case of bankruptcy of the central bank, then by definition, negative capital is not a problem per se. Indeed, as long as the central bank can issue legal tender, it is not clear what could cause bankruptcy. By substitution using the balance sheet identity, one obtains the profit function:

$$P_t = i_{M,t}(B_t + C_t) + (i_F - i_M).F_t - q_t$$

Therefore, a higher capital means higher profits since it increases the size of the (cost-free) liability side. For given values of the other parameters, one may therefore calculate a critical value of central bank capital, which is needed to make the central bank profitable at a specific moment in time:

$$P_t > 0 \Rightarrow C_t > \frac{(i_F - i_M)}{i_M} F_t - \frac{1}{i_M} q_t - B_t.$$

Unsurprisingly, the higher the monetary policy interest rates, the lower the critical level of capital required to avoid losses, since the central bank does not pay interest on banknotes (or excess reserves, i.e. reserve holdings in excess of the required reserves). A priori this level of capital can be positive or negative, i.e. positive capital is neither sufficient nor necessary for a central bank to make losses. It would also be possible for a central bank with positive capital to suffer losses over a long period, which could eventually result in negative capital. Likewise, a central bank with negative capital could have permanent profits, which would eventually lead to positive capital. Moreover, when considering the longer-term profitability outlook of a central bank in this deterministic set-up, it will turn out that initial conditions for capital and other balance sheet factors are irrelevant and the only crucial aspect is given by the growth rate of banknotes as compared to the growth rate of operating costs. The intuition for this result (stated in proposition 1 below) is that, when considering only the long term, in the end the growth rate of banknotes needs to dominate the growth rate of costs, independently of other initial conditions.

#### 4. Monte Carlo simulations and the long run steady state of the central bank balance sheet and inflation in the base model

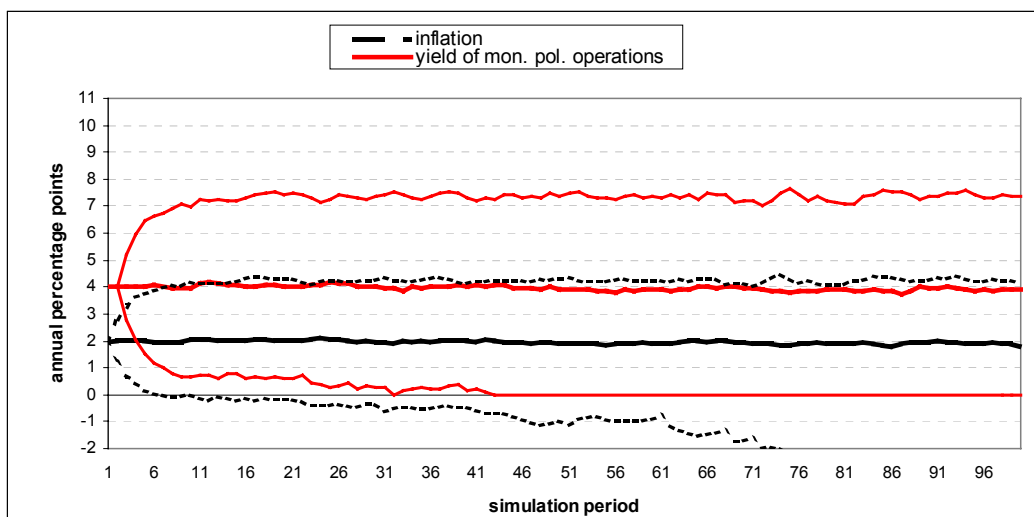
Obviously, the starting value of the array  $(M_0, F_0, B_0, C_0, \pi_0, i_0)$  as well as the level of the parameters  $(\alpha, \beta, \rho, \sigma_\varepsilon^2, \sigma_\omega^2, \sigma_\mu^2)$  will be crucial for determining the likelihood that a central bank will be at a certain moment in time in the domain of positive capital and profitability. Consider the following two contrasting examples: (1) A profitable central bank with positive capital; (2) A non-profitable central bank with negative initial capital. In both cases it will be assumed, for the sake of simplicity, that other financial assets (F) are zero.

##### *Example 1. A profit making central bank with positive capital*

The classical central bank of an industrialised country with a floating exchange rate regime (e.g., US, euro area, UK) has positive capital and monetary policy operations on the asset side. To illustrate the case, assume that  $M_0 = 120$ ,  $B_0 = 100$ ,  $C_0 = 20$ . Assume furthermore that  $\rho = 2\%$ ,  $q = 1$ ,  $\pi_0 = 2\%$ ,  $i_0 = 4\%$  and  $(\sigma_\varepsilon = 1, \sigma_\omega = 0.5, \sigma_\mu = 0.5)$  and that  $\alpha=0.5, \beta=0.2$ .

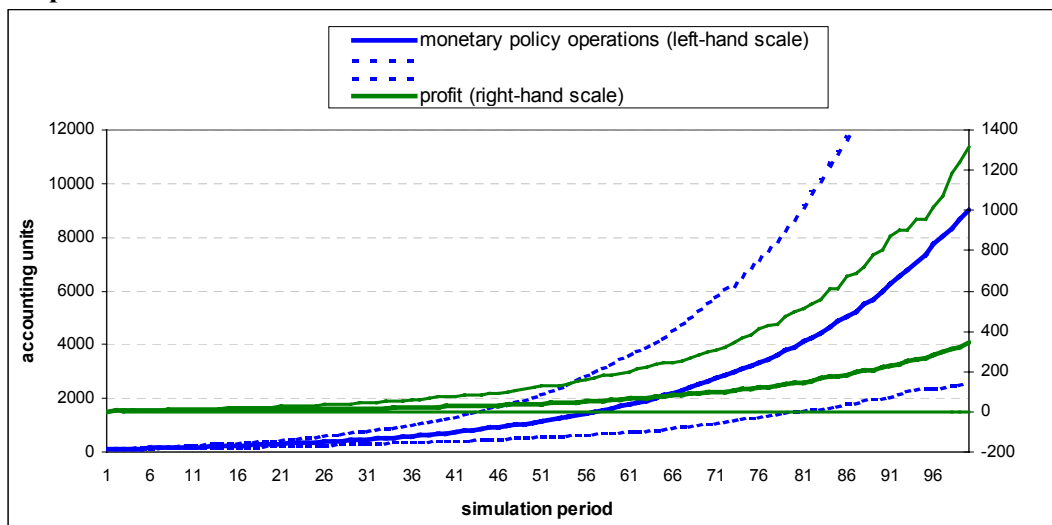
Thanks to its Taylor rule, the monetary policy performance of the central bank is very good: inflation remains close to the target rate of 2% and the nominal interest rate will remain at 4% on average. Banknotes will show steady growth in line with the nominal interest rate of 4%, while monetary policy operations will grow somewhat slower due to the de-leveraging effect of capital. The following two charts (figures 1 and 2) show the median and 95% confidence intervals of interest rates, inflation, the size of outstanding monetary policy operations, and profits for 1000 Monte Carlo simulations over 100 periods.

**Figure 1: Evolution of inflation and interest rates: a profitable bank with positive initial capital**



Note: The paths shown are the median (thick lines) and 0.05, 0.95 quantiles (narrow lines) for the cross-section of the simulated data in each period, with 1000 replications. Initial values were:  $C_0 = 20$ ,  $B_0 = 100$ ,  $F_0 = 0$ ,  $q_0 = 1$

**Figure 2: Evolution of monetary policy operations and profits: a profitable bank with positive initial capital**



Note: The paths shown are the median (thick lines) and 0.05, 0.95 quantiles (narrow lines) for the cross-section of the simulated data in each period, with 1000 replications. Initial values were:  $C_0 = 20$ ,  $B_0 = 100$ ,  $F_0 = 0$ ,  $q_0 = 1$

It is noteworthy that, even in the case of an initially profitable central bank with positive capital, there is a probability of more than 2.5% that the central bank will fall into a deflationary trap from which it is impossible to escape. Their income consequently falls to zero and each year it would make a loss equal to the level of operating costs. The likelihood of ending in a deflationary trap, however, could be reduced by assuming a lower variance of shocks, by modifying further the Taylor rule to make the central bank even more prudent in this respect, or by letting the central bank target a higher inflation rate.

### **Example 2. A non-profitable central bank with negative initial capital**

Assume that the central bank above was at its starting position forced to bail out banks in a crisis and made a loss of 100. Its initial balance sheet is assumed to be as follows  $M_0 = -80, B_0 = 20, C_0 = -100$ . Assume again that  $q = 1, \pi_0 = 2\%, i_0 = 4\%$  and  $(\sigma_\varepsilon^2 = 1, \sigma_\omega^2 = 0.5, \sigma_\mu^2 = 0.5)$ . Now, this central bank will make an expected loss of around 4.2 and it will continue making losses in the foreseeable future. However, after a long period of time, it is still able to return to profitability in most of the cases, unless adverse random shocks push it into a deflationary trap<sup>17</sup>. Before looking again at Monte Carlo simulations, consider first the deterministic case in which economic shocks would be absent  $(\sigma_\varepsilon^2 = 0, \sigma_\omega^2 = 0, \sigma_\mu^2 = 0)$ . It is easy to calculate in a spreadsheet that losses in year 2 will be 4.3, in year 10 will be 5.8, in year 50 will be 22.2. From that, one would expect a further monotonous increase of losses, but one obtains that the maximum loss is reached in year 137 with 162.4. After that, losses decline and profits become positive from year 163 onwards. The reason for this, as pointed out above, lies in the overriding importance of the *growth rates* of banknotes and operating costs, rather than the initial state of the balance sheet. Since eq. 2 assumes that operating costs only grow at the inflation rate and eq. 5 sets the growth of banknotes at the nominal interest rate, the condition of long-term profitability is satisfied in the deterministic context. This property of a long-term return to profitability and positive capital is obtained, in the case of absence of shocks, for the model assumptions made above for any initial values of the capital and operating costs. The following proposition summarises the result.

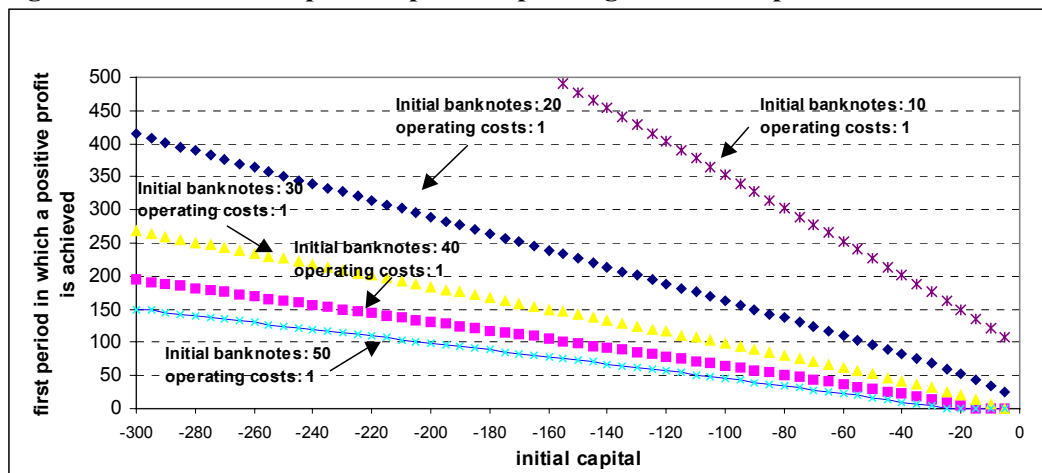
**Proposition (long run return to profitability):** *A central bank in an economy as described by the system of equations (1-9), absence of shocks,  $(\sigma_\varepsilon^2 = 0, \sigma_\omega^2 = 0, \sigma_\mu^2 = 0)$ , and initial values of inflation and key rates at their equilibrium levels of  $\pi_0 = 2, i_0 = 4$  will always return to profitability and positive capital at a certain moment in time, regardless of starting values, i.e.  $\forall C_0 \in \mathfrak{R}, \forall B_0 \in \mathfrak{R}^+, \forall q_0 \in \mathfrak{R} : (\exists \tau \in \mathfrak{R}^+ : \forall t > \tau : P_t > 0), (\exists \tau' \in \mathfrak{R}^+ : \forall t > \tau' : C_t > 0)$ .*

The proof is provided in the annex. It is thus shown that under plausible assumptions, central banks return in the long run to profitability regardless of the starting level of negative capital and regardless of the starting level of operating costs. How long it takes for the central bank to return to profitability depends on operating costs and initial capital. The following charts show, starting from the deterministic specification above, the years of return to profitability for different values of initial capital (figure 3) and different values of operating costs (figure 4).

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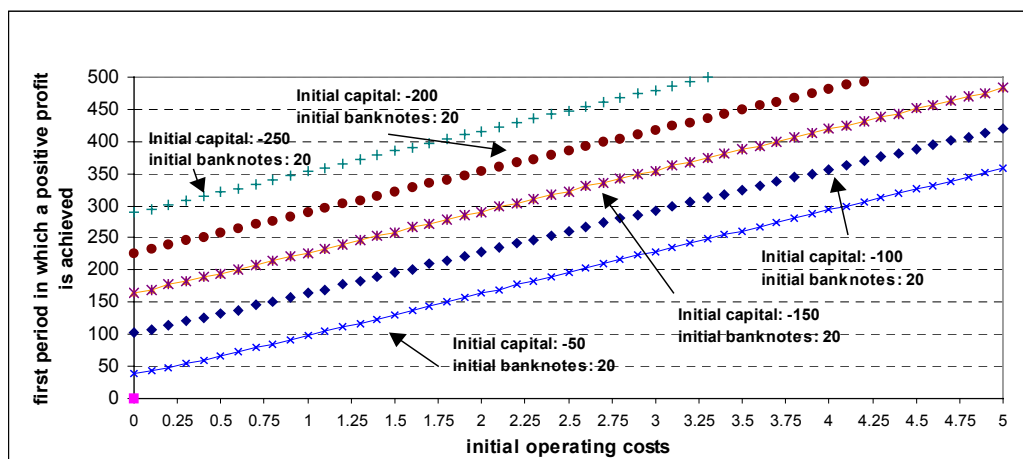
<sup>17</sup> It will return to profitability given the hypothesis made here that banknotes grow by a rate corresponding to the real growth rate plus inflation, which is under normal circumstances approximately equal to the key central bank rate.

**Figure 3: Period of first positive profit depending on initial capital**



Note: The scatterplot shows the first year when profits are achieved in a deterministic context with fixed initial banknotes and varying initial negative capital.

**Figure 4: Period of first positive profit depending on initial capital**

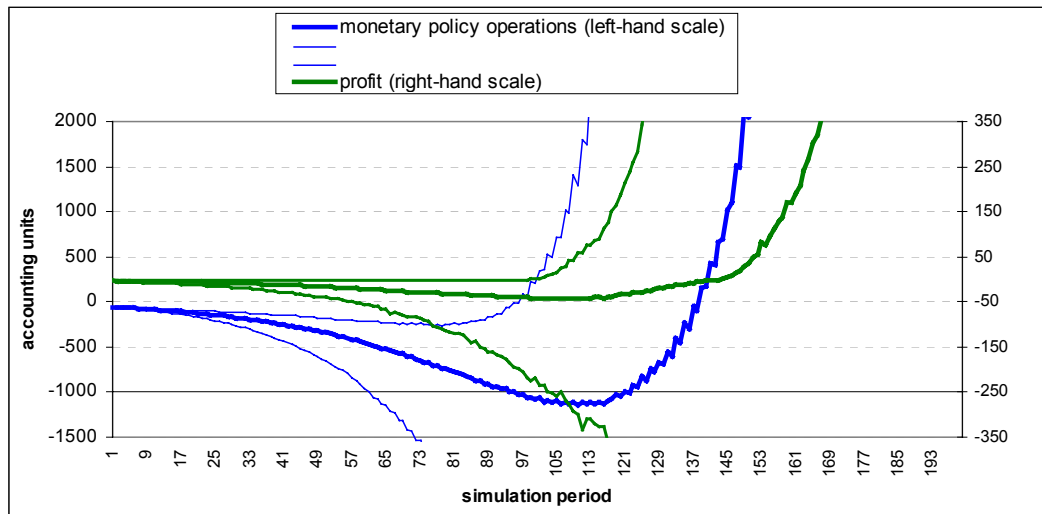


Note: The scatterplot shows the first year when profits are achieved in a deterministic context with fixed initial banknotes and varying initial operating costs.

Monte Carlo simulations tend to confirm these properties of the deterministic case. For instance in the initial specification with initial  $C_0 = -80, B_0 = 20, q_0 = 1$ , the median central bank reaches profitability again after 144 years. Figure 5 displays the evolution of profits and the sum of monetary policy operations over 200 periods.



**Figure 5: Evolution of monetary policy operations and profit: a non-profitable bank with negative initial capital and no preference for returning to profitability**



Note: The paths shown are the median (thick lines) and 0.05, 0.95 quantiles (narrow lines) for the cross-section of the simulated data in each period, with 1000 replications. Initial values were:  $C_0 = -80$ ,  $B_0 = 20$ ,  $F_0 = 0$ ,  $q_0 = 1$

It should be noted, however, that the assumed growth rates of the individual items are key to this result. A return to profitability would no longer be guaranteed if the growth rate of banknotes were assumed to be lower or if the growth rate of operating costs were higher. Furthermore, the point of return to profitability can occur a long time in the future – beyond the work- and life-time expectations of central bankers – and thus, in practice, may be irrelevant for their decision making.

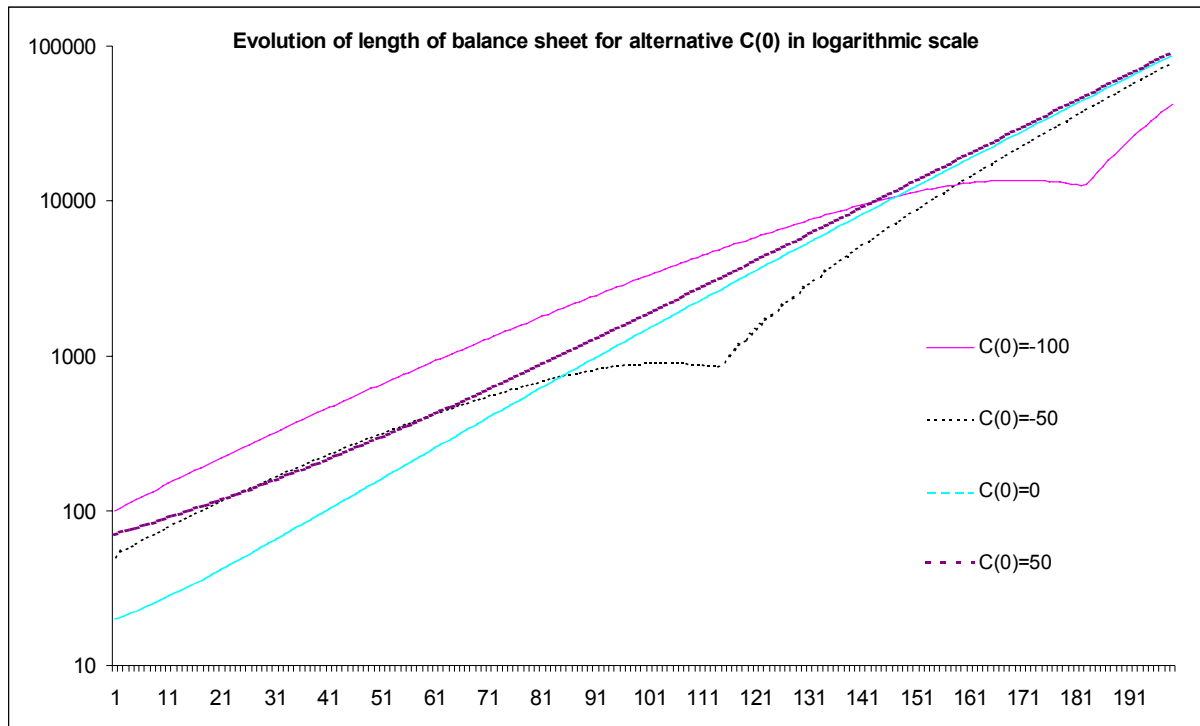
As in the first example, the *monetary policy performance* of the central bank is obviously untouched by its poor profitability and capital outlook, as the model practically by assumption precludes any interrelation in this direction. Proponents of a role of the monetary base in monetary policy implementation may argue that the Wicksellian inflation function on which the present model is based is invalid, or that, at a minimum, it is necessary to check whether its implications do not lead to contradictory results. For instance, they might argue that a sustained growth of the monetary base at rates different from the sum of the real growth rate and the rate of inflation would be counterintuitive. However, banknotes in circulation, which are the only element of the monetary base in our model, are indeed assumed to increase at the rate of inflation plus the real growth rate of the economy, such that no contradiction appears. Moreover, monetarists might argue that liquidity-absorbing open market operations are a kind of substitute to the monetary base, and therefore increase it. However, it can be counter-argued that the central bank would have the option to conduct operations with a very long maturity, such as issuing debt certificates with a maturity of five or ten years. These should definitely not be considered as substitutes for the monetary base.

What is interesting to note in this context is that the length of the balance sheet of a loss-making central bank will always *in the long-run be shorter than that of a profitable central bank*. This follows directly from the reversal towards profitability at some moment in time, which is postulated in the proposition. This reversal implies that there is a point in time after the reversal where capital will be zero, and thus the length of the balance sheet will be exactly equal to banknotes, which is the minimum length of the balance sheet at that moment in time. At that moment, the central bank which started as a profitable one has already built up a huge capital position on the liability side, and the central bank which originally made losses will never be able to catch up.

**Proposition 2 (length of balance sheet).** Consider two central banks which are identical but for their initial capital  $C_0^a, C_0^b$ , with  $C_0^a < C_0^b$ . If  $C_0^a > 0$ , then,  $\forall t \in \mathfrak{R}^+ : L_t^a < L_t^b$ . If  $C_0^a < 0$ , then,  $\exists \tau \in \mathfrak{R}^+ : L_t^a < L_t^b, \forall t > \tau$ .

The proof is given in the annex. One may note again that the point in time in which the reversal occurs may be far in the future. Figure 6 displays for the deterministic case with  $B_0 = 20$  the development of the balance sheet length for alternative levels of initial capital, namely  $C_0 = -100, -50, 0, 50$ . It appears that first, the length of the balance sheet grows fastest for  $C_0 = -100$ , but that this is reversed over time.

**Figure 6: Evolution of length of balance sheet for alternative initial capital (logarithmic scale)**



Finally, it may be interesting to consider the scenario if banknotes in circulation were to disappear due to technological innovations, such as e-money.

Assume that the starting values of  $M$ ,  $B$  and  $C$  are as follows:  $M_0 = -100$ ,  $B_0 = 0$ ,  $C_0 = -100$ ,  $q_0 = 1$ . How would the central bank balance sheet and profit evolve in this even simpler case? Obviously, this central bank could never get out of its loss-making situation, with  $B$  remaining at zero, and  $M$  and the length of the balance sheet both growing at the same rate. If the growth rate of operating costs again equals the rate of inflation, and the nominal interest rate is 4%, then in the long run the weight of operating costs in the growth rate of the positions vanishes, and all quantities grow by 4%, which is simply the nominal growth rate of the economy. So in this case, the “indebtedness” of the central bank is growing at a steady rate.<sup>18</sup> Of course, the fact that the rate of central bank indebtedness with respect to the overall economy would remain constant still does not alleviate the increasing absolute income gap of an eternally loss making central bank.

However, the point so far developed is that, in such a model, central bank capital still does not seem to matter for monetary policy implementation, in essence because negative levels of capital do not represent any threat to the central bank being able to pay for whatever costs it has.<sup>19</sup> Although losses may easily accumulate over a long period of time and lead to a huge negative capital, no reason emerges why this could affect the central bank’s ability to control interest rates. Under this setting, it was shown that in the long run, subject to a central assumption on the growth rate of banknotes, losses and negative capital are always reversed into profits and positive capital unless unexpected shocks lead to a deflation trap. Operating costs can be covered indefinitely provided that their growth rate does not exceed that of banknotes. One could therefore conclude that the model implies a perfect dichotomy between the central bank balance sheet structure and its ability to fulfil its monetary policy tasks. This is however not what experience has suggested so far. The next section tries to find out why the model’s outcome contrasts with the empirical evidence.

## 5. Why central bank capital matters – liquidity constraints, central bank credibility and independence

Having shown that in the model above, a perfect dichotomy exists between the central bank’s balance sheet and its monetary performance, how does one explain the observation, made for instance by Stella (2003), that many financially weak central banks are associated with high inflation rates? It is likely that there is another set of factors, related to the institutional environment in which the central bank exists, that is causing a relationship between the weakness in the central bank’s financial position

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<sup>18</sup> Ernhagen [2002] argues that indebtedness keeps “accelerating” in such a case. In the context of the base model presented, however, it would grow at a comparable rate as the whole economy.

<sup>19</sup> See section 2 for a more detailed discussion on why a central bank is not directly constrained in the amount of credit it can sustain, unlike any other economic agent.

and its inability to control inflation. In many cases, central banks have a weak financial position for one or more of the following reasons: (1) they have been forced to bail out the banking system after a crisis; (2) the economy is heavily dollarised, reducing seignorage revenues; (3) they are large lenders to the Government at low or negligible interest rates in order to finance the budget deficit. Yet, these elements are the direct result of a general lack of either institutional independence of the central bank from the Government or public trust in monetary policy. In fact, one can assume that the degree of public trust in an effective inflation control is to a great extent dependent on the perceived institutional independence of the central bank, whereby the public will generally judge from the historical track record. Lack of public trust and a low level of institutional independence seem to be key factors explaining poor inflation performance. Thus, in order to show the importance of central bank capital, it is necessary to establish a causal link between a central bank's financial strength and its perceived level of institutional independence. This section does not attempt to model fully the relationship between a central bank and the Government, but instead highlights some of the factors that could be important in the present context. These are incorporated into the model developed in section 3 to analyse the quantitative effects.

We start by examining three possible scenarios of the relationship between the central bank and the government: first, total lack of public trust in the central bank (possibly full institutional dependence); second, unlimited public trust (what could be called perfect independence of the State and the central bank); and, third, some point between these two extremes, which is probably the most realistic. First, in case of **dependence**, i.e. if there were no separation between the central bank and the government, the capital of the central bank is obviously irrelevant since one then has to consider only the aggregate capital of the State (including the central bank and the government). This may best be illustrated by comparing the balance sheet of the State and its sub-units in case of a capital-rich central bank and a capital-poor central bank.

*The case of a central bank with negative capital*

Central Bank		Government		Consolidated state	
CB Capital 500	Banknotes 100	Gvt. Assets 1000	Gvt debt 500	Gvt. Assets 1000	Banknotes 100
	MPOs 400		CB Capital 500		Gvt.. Debt 500
					MPOs 400

*The well-capitalised central bank*

Central Bank		Government		Consolidated state	
MPOs 600	Banknotes 100	Gvt Assets 1000	Gvt. Debt 1500	Gvt. Assets 1000	Banknotes 100
	CB Capital 500	CB Capital 500		MPOs 600	Gvt. Debt 1500

Assuming that the central bank and the government are in practice one, such that the accounting separation does not reflect substance, we still obtain the result that the State's balance sheet is longer if the central bank is rich, i.e. well equipped with capital. It could be argued that if there is no separation of substance anyway (which has never appeared to be an appropriate institutional setting), capitalising

the central bank introduces redundancies in the State's financial position and does not increase the public's confidence in the worthiness of money issued by the central bank. It would thus be preferable to not equip the central bank with capital in that case. Of course, the two cases illustrated above also make a difference for financial markets: the second case would lead to a higher amount of Government paper in circulation and, *ceteris paribus*, to a longer balance sheet of the banking system. The minimum length of the State's balance sheet corresponds to the government's assets, and will be obtained whenever central bank capital is equal or lower than the negative of banknotes. When central capital is above this level, the length of the State balance sheet grows proportionally.

At the other end of the spectrum, if unconstrained liquidity and **absolute independence of the central bank from the Government** were generally acknowledged, capital would also be irrelevant. This is the case modelled in section 3. Absolute independence implies that the central bank will never need to approach the government to get additional funds in order to regain the public's trust in the value of the currency. In particular, this condition requires that the Government can never change the central bank's law, i.e. never change the rights and functions of the central bank, and, as defined in section 3, *in particular never can withdraw the central bank's licence to issue legal tender*. Otherwise, the central bank could not risk sustaining repeated losses. This also includes the assumption that external events do not exist which can do so, such as wars or revolutions.<sup>20</sup> As shown in section 3, the central bank can in that case always continue performing its tasks regardless of the level of capital and normally will even return to profitability in the long run. Regardless of its profitability and capital position, it can also fund whatever operating expenses it deems to be necessary. It should be noted that there are exceptions to this assignment of the right to issue legal tender to the central bank, such that one should not regard the threat of losing the licence to issue legal tender as completely irrelevant.

Where central bank capital matters is probably **in between these two extremes**. This, indeed, is probably where to place most of the world's central banks. In practice, *total independence* of the central bank now and in the future, as assumed in our basic model, is impossible, since no government can commit future governments (whether they obtain power by election, war, or revolution) not to change the central bank law, even if that is written in a constitution. *Complete lack of independence*, on the other hand, is undesirable because of the conflict between the Government's short-run objectives at any point in time, and its objectives in the long run (i.e., the problem of time inconsistency developed in the Barro-Gordon model). Thus, *partial independence* of the central bank is the only possible and desirable solution, leaving us with the question why exactly capital matters in this case for monetary policy.

Understanding precisely the way central bank capital becomes relevant in this case would require modelling the relationship between the Government and the central bank. We will not aim here at modelling this relationship thoroughly; nevertheless, some of the key aspects are discussed and their impact on the model developed in the previous section is examined. Consider first what exactly

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<sup>20</sup> Technical innovations that lead to lower growth rates of banknotes or even to a disappearance of banknotes are not the issue here, since the central bank then can still issue deposits. That is in any case the only thing it is doing actively, already today.

happens in case the privilege to issue legal tender is withdrawn as could happen *de facto* as a result of dollarisation. Although the right to issue legal tender is not traditionally seen as a relevant for institutional independence, it would be an important factor when assessing a central bank's financial independence (see Martínez-Resano [2004]). If the central bank lost the right to issue currency, it would still need to pay its expenses (salary, etc.) in a new legal tender that it does not issue. Also, banknotes and outstanding credits would need to be redeemed in the new currency at a certain fixed exchange rate. Consider here the two cases of central banks with positive and with negative capital with a very simple balance sheet consisting only in Capital, Banknotes, and monetary policy operations.

*Two central banks, before their right to issue legal tender is withdrawn*

Positive Capital Central Bank		Negative Capital Central Bank	
Monetary policy operations	Banknotes	Capital (negative)	Banknotes
	Capital		Monetary policy operations

After the withdrawal of the right to issue legal tender, both central banks become normal financial institutions. After liquidating their banknotes and monetary policy operations, their balance sheets take the following shape:

*Two central banks, before their right to issue legal tender is withdrawn*

Positive Capital (former) Central Bank		Negative Capital (former) Central Bank	
Financial assets	Capital	Capital (negative)	Financial debt

Obviously, the second institution is bankrupt, and the holders of its banknotes and of liquidity absorbing monetary policy operations are not likely to recover their claims (as the no Ponzi game condition should hold in this case<sup>21</sup>). Also, the institution will immediately have to stop paying salaries and pensions, etc. In case of a positive probability of withdrawal of the right to issue legal tender, central bank capital and profitability will thus matter. In the case of negative capital, the following issues will arise:

- The staff and decision making bodies of the central bank have incentives to get out of the negative capital situation by lowering interest rates below the neutral level, which in turn triggers inflation, and eventually an increase of the monetary base up till positive capital is restored.

<sup>21</sup> It has also been argued that even for a central bank with certainty about the right to issue legal tender, the no-Ponzi game condition should hold. While in practice, this is probably true, we argue that in theory, if taking indeed the strict assumption of an eternal guarantee of the right to issue legal tender, Ponzi game considerations are not relevant for the central bank as it will with certitude, by definition never encounter liquidity problems.

- Negative capital weakens the bargaining position of the central bank towards the Government, since the Government's admittedly somewhat remote threat to withdraw from the central bank the right to issue legal tender becomes more worrisome for the central bank. Thus, with negative capital, the central bank will tend to be more pliable towards the Government, even if it does not want to ask the Government to re-capitalise it.
- The markets will have reasons to anticipate less stability-oriented behaviour of the central bank, which drives up inflationary expectations.
- The holders of the monetary base may also feel uncomfortable from the mere risk of losing their money in case that the central bank loses its right to issue legal tender – even if this central bank would never be tempted to trigger inflation.

One may thus conclude from this approach that the higher the likelihood of a central bank to lose its right to issue legal tender, the more important central bank capital becomes. As the likelihood of such an event will however never be zero, central bank capital will always matter.<sup>22</sup> Once this conclusion is drawn, one can start deriving, through simulations, which level of central bank capital is adequate to ensure a monetary policy aiming exclusively at maintaining price stability.

Assuming that a financially weak central bank wishes to restore its capital to a positive level, how could this be done? It could get out of a loss-making situation, for instance, by buying back all its outstanding liquidity-absorbing open market operations (e.g. debt certificates). This would lead to corresponding excess reserves and zero interest rates in the money market. Since excess reserves do not need to be remunerated, this frees the central bank immediately from this source of losses. If the zero interest rates implied by the excess reserves are too low from a macroeconomic point of view (which they normally are), they will trigger inflation. If this inflation is not stopped by the central bank through an increase in interest rates, and even accelerates, then the central bank will eventually return to profits since banknotes will increase such as to allow the central bank to again purchase interest rate bearing assets ( $M > 0$ ) after some time. For instance, when inflation rate has reached 1000%, the central bank will soon need to do liquidity-providing operations to provide enough liquidity for banknotes, and it can then again set interest rates at some level which at least avoids that inflation rates accelerate further (which would mean, in the model, setting interest rates at 1002%).

In order to gain the public's trust in the currency, the central bank will thus normally care about profitability and positive capital. Therefore, one may, *in the case of negative capital*, substitute the interest rate generated by the Taylor rule  $i_{M,t}$  by an interest rate  $\tilde{i}_{M,t}$  determined as follows (with  $\theta < 0$  a constant):

$$\tilde{i}_{M,t} = \min(4 + \theta, i_{M,t})$$

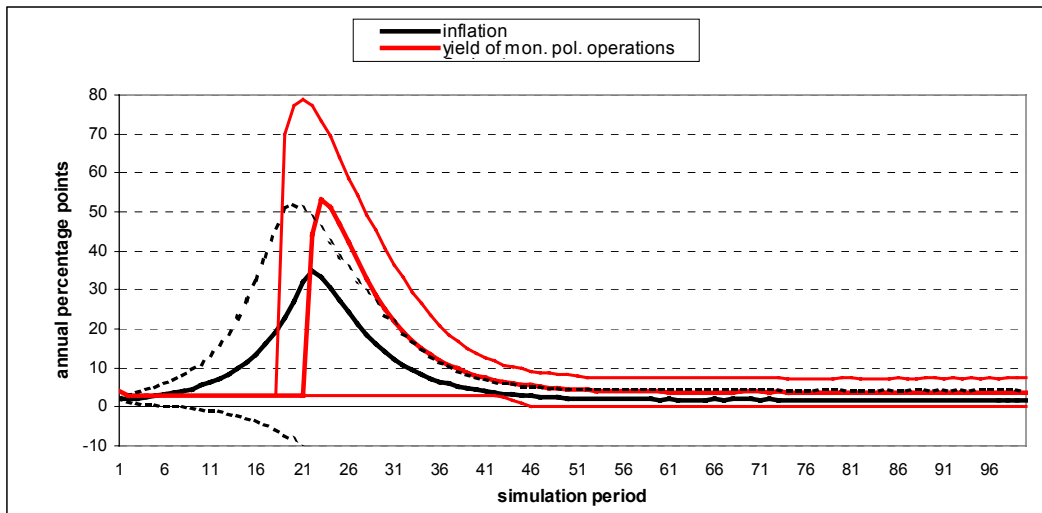
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<sup>22</sup> One may add to that logic softer arguments explaining the preference of central bankers for positive capital, and thus their possible readiness to deviate from a price stability-oriented interest rate policy in the event that capital and profits are negative. For instance, one may conjecture that higher salary levels of central bankers and job-related privileges may be easier to justify in an organisation that is profitable, than in one that makes losses and survives only thanks to the right to issue legal tender that is assigned to it by the Government. There may also be higher levels of non-monetary income, e.g., in form of prestige, if one works for a profitable organisation, rather than a permanently loss-making state bureaucracy.



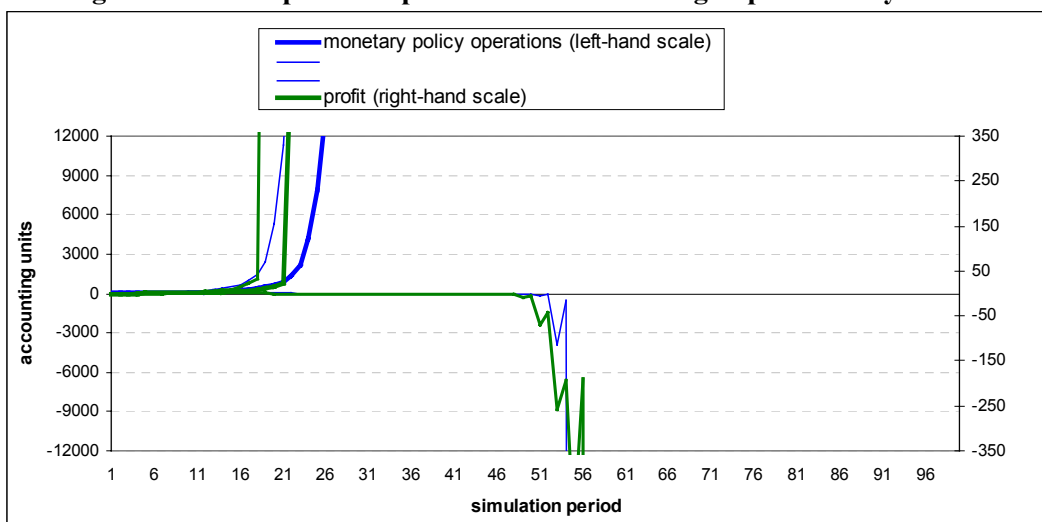
The functional form given to the capital term in this equation is, of course, ad hoc. It implies that if capital is negative, the central bank no longer reacts to an increase of inflation (reflected in the suppression of the inflation term) and even reduces rates further, by an amount corresponding to  $\theta$ . Simulating again the major time series in the case of such a modified policy function yields the following charts (figures 7 and 8), obtained with  $\theta = -1$ :

**Figure 7: Evolution of inflation and interest rates: a non-profitable bank with negative initial capital and preference for returning to profitability**



Note: The paths shown are the median (thick lines) and 0.05, 0.95 quantiles (narrow lines) for the cross-section of the simulated data in each period, with 1000 replications. Initial values were:  $C_0 = -80$ ,  $B_0 = 100$ ,  $F_0 = 0$ ,  $q_0 = 1$

**Figure 8: Evolution of monetary policy operations and profit: a non-profitable bank with negative initial capital and preference for returning to profitability**

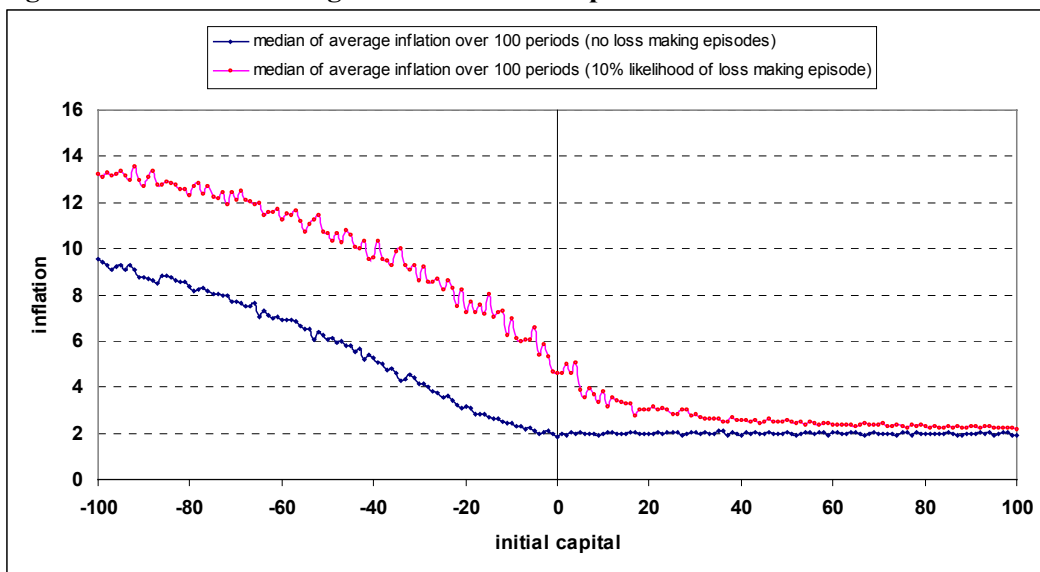


Note: The paths shown are the median (thick lines) and 0.05, 0.95 quantiles (narrow lines) for the cross-section of the simulated data in each period, with 1000 replications. Initial values were:  $C_0 = -80$ ,  $B_0 = 100$ ,  $F_0 = 0$ ,  $q_0 = 1$

Obviously, the central bank ends up much faster in the domain of positive capital and profitability – at the expense of temporarily higher inflation rates.

To see what the initial capital implies for the likelihood of the central bank falling temporarily into the domain of negative capital (triggering inflation), we simulated the evolution of the central bank balance sheet and of interest rates and inflation for initial capital levels between -100 and + 100. The following chart reveals that the initial level of capital is relevant for the expected (more precisely: the median) level of inflation, whereby the point where the inflation curve reaches the 2% target level depends on the parameters of the model. In the case of the standard specification of parameters used so far, such a relevance is only obtained for negative values of capital. This is due to the fact that our central bank over time quickly accumulates positive capital and that there are no direct shocks to profits. If, however, there is a certain likelihood of a large negative shock to profit (due e.g. to a foreign exchange revaluation or “contingent liabilities” as formulated by Blejer and Schumacher [1999]), then the positive relationship between capital and inflation performance extends into positive capital whenever the central bank has a preference for profitability at the expense of higher inflation (modelled by  $\theta < 0$ ). One may then calculate the “value at risk” of the central bank (not only taking into account its risky assets, but also the functions it may have to fulfil), and determine a capital that with, say, a 95% probability ensures that within one year capital will not be exhausted. This is the approach basically taken by Ernhagen et al [2002]. Here, we simply simulate the threat of losses by assuming that, with a likelihood of 10%, the bank makes an annual loss equal to 33% of its banknotes in circulation. Then, as figure 9 reveals, inflation performance becomes satisfactory ex ante only with an initial capital level of around 40.

**Figure 9: Median of average inflation over 100 periods**



Note: For each initial capital, a simulation with 200 replications was run and the median 100-period-average inflation was computed. In the first scenario, the profit rule is given strictly by equation 9 in the system, while the second scenario incorporates a 10% likelihood (independent draw each period) of making a loss equivalent to one third of banknotes and a preference of the central bank for returning fast to profitability ( $\theta = -1$ ). In both simulations,  $B_0 = 100$ ,  $F_0 = 0$ .

Selecting a level of capital that is deemed sufficient to keep the probability of inflation being triggered by lack of capital sufficiently low therefore means choosing some *non-negative* level of capital such that the likelihood that the central bank falls back into negative capital in the future remains negligible. Indeed, central banks tend to have non-negative capital. The implication of temporary huge losses and implied negative capital tend to be mitigated by rules that the government may re-capitalise the central bank in such cases. A re-capitalisation rule is a full substitute for capital in so far as it is unconditionally automatic. The more discretion that is given to the government in its decision to re-capitalise, the less such a rule will work as a substitute for capital to achieve a good inflation performance.

Besides the described relationship between central bank capital and the credibility of monetary policy objectives, it should be noted that the level of capital has also two more direct operational consequences, which might also be integrated into a model:

- If the profitability of a central bank is at risk, there may not only be incentives to neglect monetary policy objectives, but also to choose monetary policy assets or other financial assets with higher expected yield. This may appear undesirable because it implies allocation of resources of the central bank away from its core tasks, or because one may generally feel that public institutions should only engage in the most basic investment activities and take no unnecessary risk, even if this is at the detriment of expected profits.
- One may argue that defining a central bank with no constraint in terms of capital means introducing another state institution without balance sheet restriction, which may thus have leeway to waste scarce resources. On the other hand, if it is clear that whenever capital falls below some threshold, the central bank needs to be bailed out by the government in a somewhat painful procedure, then capital restrictions may provide incentives to the central bank to act efficiently (an effect which is however limited by the fact that under normal conditions, central banks generate profits even if they are not overly efficient).

As a final remark, it can be conjectured that the origin of the public's perception about the necessity of a strong financial position of the central bank may be related to the era of the gold standard when a central bank could indeed become insolvent and negative capital thus indicated a serious problem and should probably have triggered a run on its gold reserves. Financial markets may also *perceive* a reduction of central bank capital as increasing the probability that the relationship (of independence) between the State and central bank will be reviewed and possibly changed.

## **6. Some considerations regarding profit sharing rules**

For most starting points of central bank balance sheets and capital, the rule governing the sharing of profits between the central bank and the government will be critical for determining the evolution of

capital and profitability, and thus the likelihood that central bankers may be tempted to deviate from a stability-oriented interest rate policy due to worries about their bank's capital. So far, we had assumed that the government receives simply one half of the profit of the central bank, as long as profits are positive, and that otherwise, no transfer takes place. However, alternative rules can be built in and simulated easily. A more general class of profit sharing rules would be, for  $G$  being the transfer to the government,  $\lambda$  and  $\psi$  being positive coefficients between 0 and 1 and  $P_t^+, P_t^-$  being positive and negative profits, such that always  $P_t^+ \times P_t^- = 0$ .

$$G_t = \lambda P_t^+ + \psi P_t^-$$

The profit-sharing rule assumed above is a special case of this rule with  $\lambda = 0.5; \psi = 0$ . Many central banks have profit-sharing rules that make reference to some balance sheet indicator, such as the level of capital or a capital adequacy ratio. For instance, often the share of profits to be transferred to the Government increases if the capital has reached a certain level, either in absolute terms, or in relation to some other balance sheet items. There may even be a "bail-out" rule in case of negative capital.

$$\left\{ \begin{array}{l} \text{if } C_t \geq f(X_t) : G_t = \lambda_1 P_t^+ \\ \text{if } 0 < C_t < f(X_t) : G_t = \lambda_2 P_t^+ \\ \text{if } 0 > C_t : G_t = -\rho C_t + \lambda_2 P_t^+ - \psi P_t^- \end{array} \right.$$

with  $\lambda_1 > \lambda_2$  and  $X_t$  being the array of relevant balance sheet variables. If the constant  $\rho$  is set to 1, then the central bank is fully bailed out whenever it reaches negative capital. One simple specific rule is the one which defines a capital ratio, as for instance the capital divided by the sum of net financial assets, including monetary policy assets, and assigns a share  $1 > \lambda_1 > 0$  of the profit whenever this "capital adequacy ratio" is ensured, but leaves all profit with the central bank when it is not. Assuming that the capital adequacy ratio is 8%, one thus obtains the following rule:

$$\left\{ \begin{array}{l} \text{if } C_t \geq 8\% * (M_t + F_t) : G_t = \lambda_1 P_t^+ \\ \text{else: } G_t = 0 \end{array} \right.$$

There are no limitations to the design of profit-sharing rules. Each profit-sharing rule will, for a given initial structure of the balance sheet and an evolution of exogenous shocks, lead to a different further evolution of the balance sheet and of capital and profits in particular. If one adds the assumption that central bankers will be influenced by negative capital in their monetary policy decisions, i.e. that they will tend to be too loose whenever capital is negative, then also the inflation and interest rate time series may be influenced by the choice of the profit sharing parameters. In principle, the optimality of the parameters of the profit-sharing rule can thus be investigated by simulating the model presented above.

## 7. Conclusion

In this paper, the role of central bank capital was revisited by setting up a simple model of the relationship between the central bank balance sheet, interest rates and inflation. It builds on work of previous papers on central bank capital, which did however not attempt to formally model these relationships. The first part of the paper showed that under a series of strong assumptions a dichotomy between the central bank's balance sheet structure and its ability to maintain low inflation could be constructed (what Martinez-Resano [2004], calls a Modigliani-Miller theorem for central banks). The assumptions were analysed under which one can show that in the steady state, central banks always return to profitability in the long run, regardless of starting levels of operating costs and capital (whereby the "long run" was however shown to easily go beyond human life expectancy, and thus having doubtful implications on actual decision making). In this model, the only real dangers of not returning to profitability in the long run are the case of a deflationary trap (from which there is no escape in our simple Wicksellian setting), or if banknote growth slows down too much (namely below the growth rate of operating costs). In addition, it was shown that the length of the central bank balance sheet in the long run is positively correlated with initial capital, contradicting the impression given in the previous literature that loss accumulation and thus the growth of the "debt" of the central bank would be of an exceptionally dynamic nature.

The second part of the paper discusses a model variant under more realistic assumptions. It then becomes rational for central bankers to care about the level of capital, and to use the tools in their hands, namely open market operations and interest rate policy, to influence the evolution of capital. This idea is integrated in a simple way into the model to show how, indeed, the initial level of central bank capital becomes relevant for the long run average inflation performance of a central bank. If shocks to central bank profits are rather limited, a non-negative capital is sufficient. If however also some non-zero probability is assigned to large losses, then the positive relationship between inflation performance and capital is also valid for positive levels of capital. The required level of positive capital ensuring good inflation performance will depend on the risks in the central bank balance sheet and on "contingent liabilities" i.e. possible off-balance sheet obligations. Finally, the paper briefly discusses profit sharing rules, which are obviously closely linked to the capital issue. Although no simulations are performed, it is argued that the impact of profit-sharing rules on balance sheet items, interest rates and inflation can be simulated in the proposed model in exactly the same way as the impact of initial capital. A fully automated and fully credible rule of re-capitalisation by the government of the central bank in case of losses can be regarded as a substitute for positive capital. Since such rules are however difficult to implement in practice, positive capitals seems to remain a key tool to ensure that independent central bankers always concentrate on price stability in their monetary policy decisions.

## Annex 1: Proof of the proposition of long term return to profitability and positive capital

*Proof:* We assume that the dynamics of the system follows equations 1-9 (section 3). Since inflation and monetary policy rate are fixed at 2% and 4% (this follows from equations 1 and 6, respectively) and financial assets are assumed to be constantly zero, we can simplify the system to :

$$(1) \begin{cases} C_t = C_{t-1} + P_{t-1} \\ B_t = B_{t-1} + B_{t-1}(0.02 + \tilde{\pi}) = (1 + \tilde{i})B_{t-1} \\ M_t = B_t + C_t \\ P_t = i_{M,t}M_t - q_0(1 + \tilde{\pi})^t \end{cases}, \text{ where } \begin{cases} \tilde{i} = i/100 \\ \tilde{\pi} = \pi/100 \end{cases}$$

The goal is to prove that from some  $t_0$  onwards, profit,  $P_t$ , becomes positive (and tends to infinite as  $t$  tends to infinite). We will obtain an expression for  $M_t$ , the first summand of  $P_t$  (see fourth equation in system (1)).

Substituting  $P_t$  by its value, we can further reduce the system to equations:

$$(2) \begin{cases} M_t = B_t + C_t \\ (1 - (1 + \tilde{i})L)C_t = \tilde{i}LB_t - q(1 + \tilde{\pi})^{t-1} \\ (1 - (1 + \tilde{i})L)B_t = 0 \end{cases}$$

where  $L$  denotes the backward operator, e.g.  $LB_t = B_{t-1}$ .

Multiplying the first equation in (2) by the operator  $(1 - (1 + \tilde{i})L)$  we obtain:

$$(1 - (1 + \tilde{i})L)M_t = \tilde{i}LB_t - q(1 + \tilde{\pi})^{t-1}$$

Developing the autoregressive part on the left-hand side, we obtain

$$\Rightarrow M_t = (\tilde{i}LB_t - q(1 + \tilde{\pi})^{t-1}) + (1 + \tilde{i})(\tilde{i}LB_{t-1} - q(1 + \tilde{\pi})^{t-2}) + \dots + (1 + \tilde{i})^{t-1}(\tilde{i}LB_1 - q) + (1 + \tilde{i})^t M_0$$

and using using  $B_t = (1 + \tilde{i})^t B_0$

$$\Rightarrow M_t = (\tilde{i}(1 + \tilde{i})^{t-1}B_0 - q(1 + \tilde{\pi})^{t-1}) + (1 + \tilde{i})(\tilde{i}(1 + \tilde{i})^{t-2}B_0 - q(1 + \tilde{\pi})^{t-2}) + \dots + (1 + \tilde{i})^{t-1}(\tilde{i}B_0 - q) + (1 + \tilde{i})^t M_0$$

Now we regroup the summands,

$$\Rightarrow M_t = \tilde{i}t(1 + \tilde{i})^{t-1}B_0 - [(1 + \tilde{\pi})^{t-1} + (1 + \tilde{i})(1 + \tilde{\pi})^{t-2} + \dots + (1 + \tilde{i})^{t-1}]q + (1 + \tilde{i})^t M_0$$

and observe that the geometric sum of the second term can be simplified, obtaining

$$\Rightarrow M_t = \tilde{i}t(1 + \tilde{i})^{t-1}B_0 - \left[ \frac{(1 + \tilde{\pi})^{t-1} \left[ \left( \frac{1 + \tilde{i}}{1 + \tilde{\pi}} \right)^t - 1 \right]}{\frac{1 + \tilde{i}}{1 + \tilde{\pi}} - 1} \right] q + (1 + \tilde{i})^t M_0$$

<sup>23</sup> Since capital is negative, there is no need to make assumptions about the rate at which profit is distributed to the Government, at least for describing the dynamics until profitability is reached.

Returning to the fourth equation in system (1), we obtain the following expression for  $P_t$  :

$$\Rightarrow P_t = \tilde{i} t (1 + \tilde{i})^{t-1} B_0 - \left[ \frac{(1 + \tilde{\pi})^{t-1} \left[ \left( \frac{1 + \tilde{i}}{1 + \tilde{\pi}} \right)^t - 1 \right]}{\frac{1 + \tilde{i}}{1 + \tilde{\pi}} - 1} \right] q + (1 + \tilde{i})^t M_0 - (1 + \tilde{\pi})^t q_0$$

It is clear that the first term (positive part) dominates the other three as  $t$  increases (since its exponential term is multiplied by  $t$ , while the other two summands only have an exponential term with at most the same base). Hence  $P_t$  grows monotonically and  $P_{t_0} > 0$  from some  $t_0$  onwards.

The intuition for this result is as follows: The only source contributing to profit in a positive way is banknotes, which grow each year by a factor of  $(1 + \tilde{i})$ . This growth factor is to be compared to the one of capital, which is in any case lower than  $\tilde{i}(1 + \tilde{i})$ , and to the one of operating costs, namely  $(1 + \tilde{\pi}) < (1 + \tilde{i})$ .

However, it can be noted from the second last expression in the proof, if operating costs grew at any rate higher than  $\tilde{i} = 0.04$ , then profits would inevitably remain negative. For the limit case when operating costs grow at exactly the same rate as banknotes,  $\tilde{i} = 0.04$ , the sign of the profit would only depend on the initial conditions. It can therefore be concluded that, apart from the general underlying assumption in the model of steady growth in banknotes coupled with inflation, productivity gains of the central bank over time (operating costs growing only at the inflation rate, and not at the nominal interest rate) are key to the proposition.

## **Annex 2: Proof of the proposition that if a central bank has larger starting capital it will always have a longer balance sheet**

*Proof:* As shown before, assuming that inflation and monetary policy rate are fixed at 2% and 4% and financial assets are constantly zero and all profit is retained, we can simplify the system to:

$$\Rightarrow M_t = \tilde{i} t (1 + \tilde{i})^{t-1} B_0 - \left[ \frac{(1 + \tilde{\pi})^{t-1} \left[ \left( \frac{1 + \tilde{i}}{1 + \tilde{\pi}} \right)^t - 1 \right]}{\frac{1 + \tilde{i}}{1 + \tilde{\pi}} - 1} \right] q + (1 + \tilde{i})^t M_0$$

Assume that there are two central banks “ $a$ ” and “ $b$ ”, and that  $C_0^a, C_0^b$  is their initial and capital, respectively (etc). First, if  $0 < C_0^a < C_0^b$ , then  $\forall t \in \mathbb{N}^+ : M_t^i = L_t^i, i = a, b$ . It is clear that since



$M_0^a < M_0^b$  then  $\forall t \in \mathbb{N}^+ : L_t^a < L_t^b$ . Second, if  $C_0^a < C_0^b$  and  $C_0^a < 0$ , then  $\forall t \in \mathbb{N}^+ : M_t^i = L_t^i, i = a, b$ .

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