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INTERGENERATIONAL ALTRUISM AND NEOCLASSICAL GROWTH MODELS¹

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CONTENTS

Abstract	4
Non-technical summary	5
1 Introduction	8
2 The behaviour of altruistic households	11
2.1 The two-period overlapping generations model	11
2.2 Modelling the bequest motive	12
2.3 Expectations and optimal choices	14
2.4 Small open economy	18
3 The intertemporal equilibrium	20
3.1 Definitions	21
3.2 The cobb-douglas case	24
3.3 Comparison with the planner's optimal solution	26
4 Steady state	30
4.1 Steady state with positive bequests	31
4.2 Steady state with zero bequests	34
4.3 Existence and multiplicity of steady states	35
5 Fiscal policies	36
5.1 Neutrality of government debt	37
5.2 Neutrality of pay-as-you-go social security	38
5.3 Estate taxation	39
5.4 The neutrality of high debts	40
6 Heterogenous altruistic dynasties	41
6.1 Steady state	42
6.2 Government debt	44
6.3 Pay-as-you-go social security and estate taxation	44
7 Other forms of altruism	46
7.1 Others forms of pure altruism	46
7.2 Ad hoc altruism	49
8 Extensions	53
8.1 Altruism and education	54
8.2 Altruism and the environment	57
9 Conclusion	61
References	62
European Central Bank working paper series	66

Abstract

This paper surveys intergenerational altruism in neoclassical growth models. It first examines Barro's approach to intergenerational altruism, whereby successive generations are linked by recursive altruistic preferences. Individuals have an altruistic concern only for their children, who in turn also have altruistic feelings for their own children. The conditions under which the Ricardian equivalence (debt neutrality) theorem applies are specified. The effectiveness of fiscal policy is further analysed in the context of an economy populated by heterogeneous families differing with respect to their degree of intergenerational altruism. Other forms of altruism, referred to as ad hoc altruism, are also examined, along with their implications for fiscal policy.

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Non-technical summary

Private sector's reaction to fiscal policy is a key determinant to the effectiveness of fiscal policy in stimulating economic activity and growth. Modern macroeconomic theory is based on the assumption of highly rational and reactive agents, who are farsighted and rely on rather complex calculations to take their consumption-saving decisions. There are two main paradigms in modern macroeconomics: the overlapping generations model and the infinitely lived agent models. In the former public debt crowds out private saving and has real effects on economic activity, whereas in the latter it is neutral. It has been shown that the debt neutrality result – often referred to as the Ricardian equivalence theorem – depends on the set of taxpayers and is valid only if the set of taxpayers remain the same over time, which is the case in the infinitely lived agent model. In this respect, death seems to make the Ricardian equivalence theorem a theoretical curiosity, and the overlapping generations model a more realistic abstraction of real economies.

While it is true that individual die, human organisations, among them families, are more permanent and may even be infinitely lived, thereby giving some support to the debt neutrality result. Intergenerational altruism reconciles finite lifetime and infinite horizons. Family affections clearly extend one's economic decision making beyond one's finite lifetime, triggering intergenerational transfers such as education or bequests. Families may then well be able to counter the effects of fiscal policy, exactly as infinitely lived agents are in standard macroeconomic models. This paper is a self-contained survey examining intergenerational altruism in neoclassical growth models, its effects on the economic equilibrium and on the effectiveness of fiscal policy.

In 1974 Barro revived Ricardo's idea of the offsetting of public by private transfers, leading to the neutrality of public debt. Barro's analysis of debt neutrality is based on an assumption that individuals are motivated by a special form of intergenerational altruism (dynastic altruism), whereby individuals have an altruistic concern for their children, who in turn also have altruistic feelings for their own children, and so on. Through such a recursive relation all generations of a single family (a dynasty) are linked together by a chain of private intergenerational transfers, countervailing any attempt by the government to redistribute resources across generations. This offsetting of public by private transfers operates only if bequests are positive. This is an important qualification to Barro's debt neutrality result.

Dynastic altruism, which is the more prominent and conceptually consistent form of altruism developed in dynamic models, is thoroughly reviewed. In models of parent-to-child or descending dynastic altruism, Ricardian equivalence obtains only if bequests are positive. Otherwise, families – albeit altruistic – behave as if they consisted of selfish agents, leaving room for effective fiscal policy. The non-negative bequest condition plays a crucial role in the determination of the economic equilibrium and the validity of the debt neutrality result. While resembling a liquidity constraint in the infinitely lived agent model, it forbids parents to borrow using their children’s future earnings or human capital as collateral. This is consistent with the fact that inherited debts are not enforceable. After characterising the economic equilibria with positive and zero bequests, we analyse the role of fiscal policy in the dynastic model, and mainly expound the debt neutrality result, along with the neutrality of pay-as-you-go social security under lump-sum transfers. Any dynamic path of the economy with dynastic altruists coincides with the social optimum (assuming that the social discount rate equals the intergenerational discount rate in families): any lump-sum redistribution of resources across generations is neutral, provided that bequests are positive all along the equilibrium path. We further analyse fiscal policy by departing from the standard assumption that individuals are homogenous and characterised by the same degree of intergenerational altruism. When individuals are heterogenous with respect to their degree of altruism, public debt does not affect the long-term economic equilibrium, which is pinned down by the degree of altruism of the more altruistic family. Public debt however operates a redistribution of resources away from the less altruistic families who face binding bequest constraints in the long run.

We also examine other forms of dynastic altruism consistent with Barro’s recursive definition of altruism, ascending altruism and two-sided altruism. These forms could be expected to deliver debt neutrality unconditionally, as families leaving zero bequests could be families characterised by child-to-parent gift under ascending altruism. We find that this is not the case and no form of dynastic altruism therefore ensures debt neutrality without condition. Even under two-sided altruism there are cases, in which both bequests and gifts are constrained and fiscal policy remains effective. We then review *ad hoc* forms of altruism and their implications for the debt

neutrality results. Only one specific form of *ad hoc* altruism always guarantees debt neutrality; this form departs from the recursive approach underpinning dynastic altruism, with its objective function being formally equivalent to that of the social planner. Extensions to the fields of education and environmental are presented in a final section.

1 Introduction

How do altruistic sentiments in the family affect economic outcomes and policies? This largely self-contained paper surveys the macroeconomic literature on inter-generational altruism, examining the assumptions underpinning altruistic growth models and their consequences for both the macroeconomic equilibrium and fiscal policy.

Private sector's reaction to fiscal policy is a key determinant to the effectiveness of fiscal policy in stimulating economic activity and growth. Modern macroeconomic theory is based on the assumption of highly rational and reactive economic agents, who are farsighted and rely on rather complex calculations to take their consumption-saving decisions. However, the two main macroeconomic paradigms - the overlapping generations model and the infinitely lived agent model - entail opposite conclusions regarding the impact of fiscal policy on economic activity. Whereas public debt crowds out private savings and results in a lower level of capital accumulation in the Allais (1947) - Samuelson (1958) - Diamond (1965) overlapping generations model, it is neutral in the Ramsey (1928) infinitely lived agent model. Key to the neutrality result is the overlap between the period of time over which the government reimburses public debt by levying taxes and the period of time over which the consumer's budget constraint extends. If consumers die before public debt is redeemed, the financing of a given level of public expenditure from the issuance of public bonds increases their net wealth compared with an equivalent financing from taxation, as death allows them to escape future taxation and to leave the tax burden to future generations. More generally, Buiter (1988) and Weil (1989) proved that the cornerstone of the neutrality result is whether or not new agents enter the economy. Infinitely lived individuals would not support the entire tax burden associated with increases in public debt, were new individuals to be born tomorrow, regardless of their life span. The set of taxpayers must remain the same over time for the neutrality result to apply.

Intergenerational altruism reconciles finite lifetimes and infinite horizons. Family affections extend one's economic decision making beyond one's finite lifetime. The view that wealth is stored up for the purposes of enhancing children's welfare has been advocated by neoclassical economists. In his *Principles of Political Economy*, Marshall points to the concern for children as the main reason for saving. This

concern is mainly expressed by intergenerational transfers, such as bequests. Altruistic families or dynasties, exactly as infinitely lived agents, are able to counter the effects of fiscal policy. If a government takes one euro from children and gives it to their parents, it affects neither parents' nor children's consumption profiles, since the parents compensate for this transfer by increasing their bequests to their beloved children by exactly one euro. This offsetting of public by private transfers is at the heart of the debt neutrality debate, which dates back to Ricardo and has been revived by Barro (1974). Barro's approach to intergenerational transfers is in line with Becker's (1974) theory of social interactions, according to which redistribution between family members is neutral, when the head of the family makes positive gifts to all the members of the family. Barro applies the same logic to the complete sequence of descendants.

Barro's analysis of debt neutrality is based on an assumption that individuals are motivated by a special form of intergenerational altruism, which we refer to as dynastic altruism. Individuals have an altruistic concern for their children, who also have altruistic feelings for their own children, and so on. Through this recursive relation, all generations of a single family - or a dynasty - are linked together by a chain of private intergenerational transfers. This view of altruism is consistent with the succession of generations within a dynasty and therefore fully reconciles finite life and infinite horizon. In this respect, dynastic altruism seems to provide a fully fledged microeconomic foundation for the infinitely lived agent model, insofar as the infinitely lived agent can be interpreted as a dynasty of altruistically linked individuals. A dynasty, however, clearly differs from an infinitely lived agent, insofar as it is a succession of distinct - albeit altruistic - individuals, who are endowed with their own preferences and freedom of choice. This entails serious qualifications to the debt neutrality result - also known as the Ricardian equivalence theorem. Assume for instance that parents are so poor that despite their strong altruistic feelings they cannot afford to leave bequests to their children. If the government takes one euro away from these now relatively wealthy children and gives it to their needy parents, the parents would use this sum to increase their consumption, not to increase their bequests, and the children would end up with a lifetime income lower than prior to the policy intervention. Importantly, this suggests that parents fully agree with this redistributive scheme and would even implement it themselves in the family by leaving debt - negative bequests - to their children, if inherited debt were enforceable.

The non-negative bequest constraint plays a crucial role in the definition of the economic equilibrium and in the analysis of fiscal policy in the dynastic model. Even though it formally resembles a liquidity constraint in the infinitely lived agent model, there is a clear distinction between non-negative bequest conditions and liquidity constraints. While there is no reason for forbidding individuals to borrow over their life-cycle, using future earnings as collateral, children's future labour income - or human capital - is no valid collateral for parents' private borrowing. Altruistic feelings do not always trigger positive transfers between generations. Poor parents love their children but may leave no bequests, which has direct implications for the effectiveness of fiscal policy. Fiscal policy is effective, when successive generations are not linked by a chain of positive private transfers.

Modelling the bequest motive requires several crucial assumptions in a dynastic framework, as described by Barro (1974). When presenting the altruistic individual's utility function in Section 2, we pay particular attention to the modelling of expectations and to the first individual of the altruistic dynasty, two aspects which are usually disregarded in the literature. The behaviour of altruists is illustrated in the case of a small open economy. In Section 3 we examine the closed economy version of altruistic models and characterise the intertemporal equilibrium, which generically features either zero bequests (bequest-constrained equilibrium) or positive bequests (bequest-unconstrained equilibrium). We also compare the intertemporal equilibrium with the social optimum. In Section 4 we characterise the steady state equilibria of the dynastic model, focusing on existence and multiplicity. The neutrality of fiscal policy - public debt, social security and estate taxation - is thoroughly analysed in Section 5, where we provide a theoretical exposition of the Ricardian equivalence theorem.

The baseline altruistic model of economic growth presented in Sections 2 to 5 is built upon the assumption of a representative family or dynasty, in this respect very much similar to the infinitely lived representative agent model. The coexistence of bequest-constrained and bequest-unconstrained families is worth enquiring and seems to be a more appropriate abstraction of real economies, where heterogeneity of behaviours clearly prevails. In Section 6 we consider the altruistic growth models with heterogeneous individuals. It is shown that Ricardian equivalence still holds from a macroeconomic viewpoint, as capital accumulation, which is driven by the

saving behaviour of the more altruistic individuals, is not affected by fiscal policies, but that there are important distributional effects of fiscal policies.

Other forms of altruism, which have been investigated in the literature, are also reviewed in Section 7. First, we review models of ascending altruism and models of two-sided altruism, which stretch Barro's intuitive formulation of dynastic or pure altruism towards its limit. Second, we survey other forms of altruism, which we refer to as *ad hoc* altruism. They are *ad hoc* to the extent that the benefactor's utility does not directly depend on the beneficiary's utility, in contrast to Barro's description of family affections. Extensions of the baseline altruistic growth model to the fields of education and of environmental economics are provided in Section 8. A last section offers some concluding remarks.

2 The behaviour of altruistic households

The overlapping generations model is appropriate for the analysis of intergenerational transfers, owing to its demographic structure. Altruistic transfers are therefore investigated in a dynastic framework underpinned by the baseline two-period overlapping generations model, in which a new generation is born in each period, so that two generations are alive in each period. First, we briefly outline the two-period overlapping generations model, a thorough exposition of which is provided by de la Croix and Michel (2002). Second, we introduce the bequest motive in this model, setting out the utility function of altruistic individuals. Third, we characterise the optimal decisions taken by altruistic individuals. Finally, we consider the small open economy case with a view to illustrating the behaviour of altruists.

2.1 The two-period overlapping generations model

Consider an economy where time is discrete. Individuals who are identical within as well as across generations are indexed by their date of birth, t . An individual's life-cycle consists of two periods, which we refer to as youth and old-age. The number of individuals born in period t is $N_t = (1 + n)N_{t-1}$, where $n > -1$ is the exogenous population growth rate.

Young agents born in period t supply one unit of labour, receive the market wage w_t , consume c_t and save s_t , therefore facing the budget constraint: $w_t = c_t + s_t$.



When old, they consume the proceeds of their savings, $d_{t+1} = R_{t+1}s_t$, where R_{t+1} is the factor of interest.

Agents are selfish and maximise their life-cycle utility¹ $U_t = U(c_t, d_{t+1})$. Their saving function s^D is given by:

$$s_t = \arg \max_s U(w_t - s, R_{t+1}s) \equiv s^D(w_t, R_{t+1})$$

Their optimal consumptions are:

$$\begin{aligned} c_t &= w_t - s^D(w_t, R_{t+1}) \equiv c^D(w_t, R_{t+1}) \\ d_{t+1} &= R_{t+1}s^D(w_t, R_{t+1}) \equiv d^D(w_t, R_{t+1}) \end{aligned}$$

With a neoclassical production sector, the equilibrium of the overlapping generations economy may be dynamically inefficient. Saving decisions are decentralised and individuals may save more than necessary to maintain the golden rule capital stock, defined as the stock of capital maximising net output. In such a case, the economic equilibrium is not Pareto-optimal. There is then room for fiscal policies such as public debt financing or pay-as-you-go social security, which improve welfare by absorbing saving in excess of the golden rule, thereby increasing net output. Regarding the long-run equilibria of the overlapping generations model, standard assumptions on the utility and production function are not sufficient to ensure uniqueness or even existence of positive steady states. Galor and Ryder (1989) have shown that, under fairly standard assumptions, this model can experience no or more than one positive steady state.

2.2 Modelling the bequest motive

Young altruists born in period t supply one unit of labour, receive the market wage w_t , inherit x_t , consume c_t and save s_t . When old, they consume part of the proceeds of their savings, d_{t+1} , and bequeath the remainder, $(1+n)x_{t+1}$, to their $1+n$ children. The budget constraints that individuals face over their life are therefore:

$$x_t + w_t = c_t + s_t \tag{1}$$

¹We assume that the function $U(c, d)$ is strictly concave and twice continuously differentiable over the interior of the set $\mathbb{R}_+^* \times \mathbb{R}_+^*$. Moreover: $U'_c(c, d) > 0$, $U'_d(c, d) > 0$, $\lim_{\varrho \rightarrow 0} U'_c(\varrho, d) = +\infty$ and $\lim_{\varrho \rightarrow 0} U'_d(c, \varrho) = +\infty$.

$$R_{t+1}s_t = d_{t+1} + (1+n)x_{t+1} \quad (2)$$

Bequests² are private intergenerational transfers from the old to the young. Since children are exempted by law from responsibility for parental debts, credit institutions do not accept children's future earnings as collateral for parents' private borrowing. Inherited debt are not enforceable. In bequest models, it is therefore assumed that parents face the following non-negativity constraint:

$$x_{t+1} \geq 0 \quad (3)$$

If this constraint is binding, bequests are zero and bequest motive said to be inoperative. Altruistic households behave as if they were selfish, when the non-negative bequest constraint is binding. The evolution of bequests is obtained by eliminating s_t in the budget constraints (1) and (2):

$$x_{t+1} = \frac{1}{1+n} [R_{t+1}(x_t + w_t - c_t) - d_{t+1}] \quad (4)$$

Parents are assumed to have an altruistic concern for their children. According to Barro's (1974) recursive definition of altruism³, parents care about their children's welfare by weighting their children's utility in their own utility function V_t . Denoting with V_{t+1} the well-being of each of their $1+n$ children, the utility of individuals born in period t is given by:

$$V_t = U_t + \gamma V_{t+1} \quad (5)$$

where $U_t = U(c_t, d_{t+1})$ is the utility from life-cycle consumption.

Parents have two sources of utility: (i) they derive (selfish) utility from consumption; (ii) they derive (altruistic) utility from the welfare of their children. We refer to the parameter γ as the degree of intergenerational altruism⁴. Equation

²The structure of the model is such that parents' and children's life-cycles overlap. It results that bequests could also be interpreted as *inter vivos* gifts. In the absence of incentive and information problems, there is no difference between both types of transfers and we shall only refer to them as bequests.

³Most authors, including Bevan and Stiglitz (1979), Buiter (1979) and Carmichael (1982), who examine Barro's formulation of dynastic altruism, assume separability with respect to the attainable level of children's utility.

⁴An alternative specification consists in writing $\tilde{\gamma}(1+n)$, where $\tilde{\gamma}$ is the factor of pure altruism and $1+n$ the number of children. These two formulations are equivalent, when the number of children per family is exogenous (Buiter and Carmichael, 1984). A refinement of this approach considers that altruism influences fertility (Barro and Becker, 1988).

(5) relates the utility of parents to the utility of each of their children. Although parents have altruistic feelings only for their own children, these children are also concerned for their own children, i.e. $V_{t+1} = U_{t+1} + \gamma V_{t+2}$. It results that parents' utilities depend - albeit not directly - on the utilities of their grand-children, i.e. $V_t = U_t + \gamma U_{t+1} + \gamma^2 V_{t+2}$. We can substitute children's utilities forward for all $T > t$:

$$V_t = \sum_{j=t}^{T-1} \gamma^{j-t} U_j + \gamma^{T-t} V_T$$

If the following condition holds,

$$\lim_{T \rightarrow +\infty} \gamma^{T-t} V_T = 0$$

we can express V_t as a weighted infinite sum of the life-cycle utilities of current and future generations:

$$V_t = \sum_{j=t}^{+\infty} \gamma^{j-t} U_j \quad (6)$$

Altruistic individuals take into account the infinite stream of their descendants' utilities. Their altruistic utility is equal to the discounted sum (with a discounting factor γ) of their own life-cycle utility and the life-cycle utilities of all their descendants. The degree of intergenerational altruism γ is assumed to be smaller than 1. This reflects weights diminishing with the social distance between the altruists and those to whom they are altruistically related, as parents discount less the utility of their children than that of their grand-children. This also implies that the infinite sum (6) is convergent, when life-cycle utilities are bounded.

2.3 Expectations and optimal choices

Individuals belonging to generation $\theta \geq t$ choose c_θ , s_θ , $d_{\theta+1}$ and $x_{\theta+1}$, take prices w_θ , $R_{\theta+1}$ as given and maximise their utility V_θ subject to their budget constraints (1) and (2) and to the non-negative bequest condition (3) evaluated in period $t = \theta$. To decide how much to leave to their children, they need to forecast the choices of all their descendants, whose decisions and utility levels hinge on the bequests they will receive. Individual choices are therefore based on forecasts of all current and future prices.

In each period t , an individual's information set is denoted with $\mathcal{P}_t = \{(w_\theta, R_{\theta+1}); \theta \geq t\}$. This notation makes clear that the expectations of all successive cohorts

are compatible, since we have $\mathcal{P}_t = \mathcal{P}_{t+1} \cup (w_t, R_{t+1})$. By definition the maximum utility of an individual is given by the following recursive relation:

$$V_t^*(x_t, \mathcal{P}_t) = \max_{c_t, s_t, d_{t+1}, x_{t+1}} \left\{ U(c_t, d_{t+1}) + \gamma V_{t+1}^*(x_{t+1}, \mathcal{P}_{t+1}) \right\} \quad (7)$$

subject to (3) and (4).

$V_t^*(x_t, \mathcal{P}_t)$ stands for the maximum level of utility that can be attained by individuals who have inherited x_t from their parents. Importantly, this level depends on the sequence of all current and future prices, $\{w_\theta, R_{\theta+1}\}_{\theta=t}^{+\infty}$, which is the individual's information set. This is the level of utility individuals attain by maximising the sum of the utility they derive from their life-cycle consumption and the utility, γV_{t+1}^* , they derive (out of altruism) from leaving a bequest x_{t+1} to each of their $1+n$ children. Equation (7) is a recursive relation, the solution of which $\{V_t^*(\cdot)\}_{t \geq 0}$ is the sequence of utilities of all members of the altruistic dynasty. This is also the Bellman equation of an infinite horizon problem, relating the value function of parents, V_t^* , to the value function of children, V_{t+1}^* . Two remarks are here in order. First, the value function is generally not independent from the period where it is evaluated, and is therefore indexed by time. Second, recursive utilities are well defined only if the expectations of all generations are compatible. Compatibility of the expectations of successive generations is a crucial assumption of the altruistic model, which is usually not stated in an explicit manner.

2.3.1 The associated infinite horizon optimisation problem

Consider the following infinite horizon problem with an initial state $x_0 \geq 0$ and an exogenously given sequence of positive prices $\mathcal{P}_0 = \{w_t, R_{t+1}\}_{t \geq 0}$:

$$\max_{\{c_t, d_{t+1}, x_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \gamma^t U(c_t, d_{t+1}) \quad (8)$$

subject to: $\forall t \geq 0, x_{t+1} = \frac{1}{1+n} [R_{t+1}(w_t + x_t - c_t) - d_{t+1}]$

$\forall t \geq 1, x_t \geq 0$

To characterise the solution of this maximisation problem⁵, we set up the Lagrangian \mathcal{L}_t of period t , which is equal to the sum of the life-cycle utility $U(c_t, d_{t+1})$

⁵For a thorough presentation of discrete time optimisation, see Mc Kenzie (1986).

and the increase in the shadow value (in terms of utility) of x_t over one period⁶, $\gamma p_{t+1} x_{t+1} - p_t x_t$:

$$\mathcal{L}_t = U(c_t, d_{t+1}) + \frac{\gamma}{1+n} p_{t+1} [R_{t+1}(x_t + w_t - c_t) - d_{t+1}] - p_t x_t$$

For all $t \geq 0$, maximising the Lagrangean with respect to c_t and d_{t+1} gives:

$$U'_c(c_t^*, d_{t+1}^*) = \frac{\gamma}{1+n} p_{t+1} R_{t+1} \quad (9)$$

$$U'_d(c_t^*, d_{t+1}^*) = \frac{\gamma}{1+n} p_{t+1} \quad (10)$$

For all $t \geq 1$, maximising \mathcal{L}_t with respect to x_t subject to the non-negative bequest condition gives:

$$-p_t + \frac{\gamma}{1+n} p_{t+1} R_{t+1} \leq 0 \quad (= 0 \text{ if } x_t^* > 0) \quad (11)$$

The transversality condition states that the limit of the shadow value of bequests tends to zero when time goes to infinity:

$$\lim_{t \rightarrow +\infty} \gamma^t p_t x_t^* = 0 \quad (12)$$

These conditions, along with equations (3) and (4), are necessary and sufficient conditions for optimality⁷. Equivalently, in addition to (3) and (4), the following conditions are necessary and sufficient:

$$\forall t \geq 0, U'_c(c_t^*, d_{t+1}^*) = R_{t+1} U'_d(c_t^*, d_{t+1}^*) \quad (13)$$

$$\forall t \geq 1, U'_c(c_t^*, d_{t+1}^*) - \frac{1+n}{\gamma} U'_d(c_{t-1}^*, d_t^*) \leq 0 \quad (= 0 \text{ if } x_t^* > 0) \quad (14)$$

$$\lim_{t \rightarrow +\infty} (1+n) \gamma^{t-1} U'_d(c_{t-1}^*, d_t^*) x_t^* = 0 \quad (15)$$

Equation (13) is obtained by merging equations (9) and (10) and eliminating the shadow price p_{t+1} . Equation (14) results from plugging (10) into (11). The transversality condition (15) is also obtained by substitution of p_t .

Equations (13) and (14) characterise the optimal life-cycle consumptions and the optimal bequest x_t^* . In period t , old individuals can reduce their own consumption by one unit, suffering a utility loss of $U'_d(c_{t-1}^*, d_t^*)$ and can increase their

⁶The current shadow price p_{t+1} of bequest x_{t+1} in period $t+1$ is discounted by the factor γ in order to calculate the increase in the shadow value in period t .

⁷The necessary condition is satisfied when the objective is finite along a path with zero bequests. See assumption A.2. in Michel (1990a).

bequest x_t^* to each of their children, increasing the utility of their children by $U'_c(c_t^*, d_{t+1}^*)/(1+n)$. This increase in the utility of their children raises their own utility by $\gamma U'_c(c_t^*, d_{t+1}^*)/(1+n)$. If bequests are positive ($x_t^* > 0$), the utility loss from a reduction in parental consumption equals the utility gain from increased bequests. If the utility loss from reduced consumption exceeds the utility gain from increased bequests, altruists leave no bequests ($x_t^* = 0$). Lastly, the transversality condition (15) means that the limit of the shadow value of bequests is equal to zero.

If the optimisation problem (8) has an optimal solution from any date t onwards and any level of x_t , the associated sequence of value functions, $V_t(x)$, which is by definition the maximum of the objective function (8) from t to $+\infty$ starting at $x_t = x$, satisfies the Bellman equation⁸. Thus, this sequence of value functions⁹ is the solution to the altruistic problem (7).

2.3.2 The dynasty's founding father

Despite the fact that the bequest left by the first old generation, x_0 , is usually considered as given and treated as an initial condition of the economic dynamics, it is actually an economic decision taken by the first old generation born in period $t = -1$. The N_{-1} first old agents receive the proceeds of their savings $R_0 s_{-1}$, which they use to consume d_0 and leave the remainder $(1+n)x_0$ to their children. In period $t = 0$, the first-period consumption of the first old individual c_{-1} is given, as it belongs to the past. Old individuals in period 0 therefore solve the following maximisation problem:

$$\max_{d_0, x_0} \left\{ U(c_{-1}, d_0) + \gamma V_0^*(x_0, \mathcal{P}_0) \right\} \quad (16)$$

subject to: $R_0 s_{-1} = d_0 + (1+n)x_0$ and $x_0 \geq 0$

Previously, we have resorted to the optimisation problem (8) to solve (7). Similarly, we set up a new optimisation problem to solve (16), the objective function of which, $\sum_{t=-1}^{+\infty} \gamma^t U(c_t, d_{t+1})$, is maximised under the following set of constraints (c_{-1} ,

⁸The Bellman equation, which defines the behaviour of altruistic individuals, corresponds to an infinite number of optimisation problems.

⁹Standard assumptions ensure that these functions exist; see de la Croix and Michel (2002).

s_{-1} , R_0 and \mathcal{P}_0 are given):

$$\begin{aligned}x_0 &= \frac{1}{1+n}[R_0 s_{-1} - d_0] \\ \forall t \geq 0 \quad x_{t+1} &= \frac{1}{1+n}[R_{t+1}(w_t + x_t - c_t) - d_{t+1}] \\ \forall t \geq 0 \quad x_t &\geq 0\end{aligned}$$

The Lagrangeans of periods $t > 0$ are unchanged and the first-period Lagrangean \mathcal{L}_0 is:

$$\mathcal{L}_0 = \gamma^{-1}U(c_{-1}, d_0) + U(c_0, d_1) + p_0[R_0 s_{-1} - d_0 - x_0] + \frac{\gamma}{1+n}p_1[R_1(x_0 + w_0 - c_0) - d_1]$$

By maximising \mathcal{L}_0 with respect to d_0 and x_0 subject to $x_0 \geq 0$, we obtain:

$$\begin{aligned}U'_d(c_{-1}, d_0^*) &= \frac{\gamma}{1+n}p_0 & (17) \\ -p_0 + \frac{\gamma}{1+n}p_1 R_1 &\leq 0 \quad (= 0 \text{ if } x_0^* > 0)\end{aligned}$$

Note that the first condition corresponds to equation (10) evaluated in period $t = -1$ and the second to equation (11) evaluated in period 0. Eliminating the shadow prices in these two conditions, which characterise the optimal behaviour of the first old altruists, gives equation (14) for $t = 0$.

2.4 Small open economy

It is more difficult to characterise the behaviour of altruists than that of selfish individuals, as an altruist's economic decision making requires relatively sophisticated expectations. In this section, altruistic behaviour is illustrated in the simple case of a small open economy with a constant world interest rate or, alternatively, of an economy where production occurs according to a linear technology. Such an assumption simplifies the maximisation problem a great deal, since (given the wage rate w and the interest factor R) the value function $V_t(x)$ is independent from time:

$$\begin{aligned}V(x_t) &= \max_{c_t, d_{t+1}, x_{t+1}} \left\{ U(c_t, d_{t+1}) + \gamma V(x_{t+1}) \right\} \\ \text{subject to: } x_{t+1} &= \frac{1}{1+n}[R(x_t + w - c_t) - d_{t+1}] \text{ and } x_{t+1} \geq 0\end{aligned}$$

The maximisation problem faced by each generation is the same, which should come as no surprise, since it is assumed that the dynasty's macroeconomic environment is stationary. For any bequest $x \geq 0$, the optimal consumptions $\tilde{c} = \tilde{c}(x)$ and $\tilde{d} = \tilde{d}(x)$, and the bequest passed on to the next generation $\tilde{z} = \tilde{z}(x)$ are the solutions to:

$$V(x) = \max_{c,d,z} \left\{ U(c, d) + \gamma V(z) \right\}$$

subject to: $z = \frac{1}{1+n} [R(x + w - c) - d]$ and $z \geq 0$.

Let us further assume that the value function is concave and differentiable¹⁰. For an interior solution (with positive bequests $\tilde{z} > 0$), the two following optimality conditions are obtained by differentiation:

$$U'_c(\tilde{c}, \tilde{d}) = \frac{\gamma R}{1+n} V'(\tilde{z}) \quad (18)$$

$$U'_d(\tilde{c}, \tilde{d}) = \frac{\gamma}{1+n} V'(\tilde{z}) \quad (19)$$

Comparing these two conditions with the optimality conditions (9) and (10) shows that the shadow price p_{t+1} is equal to the marginal value of bequests x_{t+1}^* . The optimality analysis with the Lagrangean \mathcal{L}_t corresponds to a “marginal form” of the Bellman equation applied to one particular solution. The Lagrangean method is more powerful, because it requires no assumption on the (unknown) value function. Moreover, providing an analytical form of the value function is feasible only in very special cases. In the following example, we calculate a closed-form solution of the value function in the case of log-linear life-cycle utilities.

Example

In the case of a log-linear utility $U(c_t, d_{t+1}) = \ln c_t + \beta \ln d_{t+1}$ with $\beta > 0$, we prove that, under some conditions, there are positive constants a, b, m such that $V(x_t) = a + b \ln(x_t + m)$ is the unique solution of the Bellman equation. With this form of the value function, equations (18) and (19) imply:

$$\tilde{c} = \frac{(1+n)(\tilde{z} + m)}{\gamma b R} \quad \text{and} \quad \tilde{d} = \frac{(1+n)\beta(\tilde{z} + m)}{\gamma b}$$

¹⁰For the concavity and the differentiability of the value function, see Stokey and Lucas (1989) and de la Croix and Michel (2002).

By substitution, the maximum \tilde{M} of $U(c, d) + \gamma V(z)$ satisfies:

$$\tilde{M} = (1 + \beta + \gamma b) \ln(\tilde{z} + m) + \gamma a - (1 + \beta) \ln b + \xi$$

where $\xi = (1 + \beta) \ln((1 + n)/\gamma) - \ln R$ and \tilde{z} is given by:

$$\tilde{z} + m = \frac{\gamma b R}{(1 + n)(1 + \beta + \gamma b)} \left[x + w + \frac{(1 + n)m}{R} \right]$$

The condition $\tilde{M} = V(x) = a + b \ln(x + m)$ is then equivalent to the three following conditions, which pin down m , b and a :

- 1) $\ln(x + m) = \ln\left(x + w + \frac{(1 + n)m}{R}\right)$ implies $m = \frac{Rw}{R - (1 + n)}$.
- 2) $b = 1 + \beta + \gamma b$ implies $b = \frac{1 + \beta}{1 - \gamma}$.
- 3) the identification of the constant term gives:

$$a = \frac{1}{1 - \gamma} \left[b \ln\left(\frac{\gamma R}{1 + n}\right) - (1 + \beta) \ln b + \xi \right]$$

b is positive ($\gamma < 1$), and m is positive¹¹ if and only if $R > 1 + n$. In addition, the condition for an interior solution ($\tilde{z}(x) > 0$ for all $x > 0$) is equivalent to $\gamma R \geq 1 + n$. One can show that, under these assumptions, the value function $V(x) = a + b \ln(x + m)$ is the unique solution to the Bellman equation. When $\gamma R = 1 + n$, the optimal bequest is always equal to the received bequest¹², $\tilde{z}(x) = x$. When the degree of altruism γ is greater than $(1 + n)/R$, $\tilde{z}(x)$ is greater than x . When it is smaller than $(1 + n)/R$, the optimal bequest is necessarily equal to zero from a finite date t onwards.

3 The intertemporal equilibrium

Until now we have focused on the behaviour of altruists, considering prices as given. In this section, we examine the intertemporal equilibrium of the dynastic model, assuming that production occurs according to a neoclassical production function. After characterising the competitive intertemporal equilibrium in the general case, we

¹¹The function V is defined for $x \geq 0$ and the consumptions \tilde{c} and \tilde{d} are positive for $\tilde{z} \geq 0$ if and only if m and b are positive.

¹²As we shall see in Section 3.2, where prices are endogenous, the steady-state equilibrium is characterised by $\gamma R = 1 + n$ (the modified golden rule) when the bequest motive is operative.

thoroughly analyse the economic dynamics under the assumption of a Cobb-Douglas production function. We then consider the social optimum and its decentralisation. We finally spell out the main differences between the infinitely lived agent and the dynastic model.

3.1 Definitions

3.1.1 Production and firms

Production occurs according to a neoclassical technology $F(K, L)$ using two inputs, capital K and labour L . Homogeneity of degree one of the function F allows us to write output per young as a function¹³ of capital per young: $f(k) = F(k, 1) + (1 - \mu)k$ where $k = K/L$ is the capital stock per young (or worker) and $\mu \in [0, 1]$ the depreciation rate of capital.

In each period, there is one representative firm, producing one good, which is either consumed or invested. For given prices, wage rate w_t and interest factor R_t , the maximum of profits, $\Pi_t = F(K_t, L_t) - w_t L_t - R_t K_t$, is obtained when marginal products are equal to prices. The factor prices are given by:

$$w_t = F'_L(\cdot) = f(k_t) - k_t f'(k_t) \equiv w(k_t) \text{ and } R_t = F'_K(\cdot) - \mu = f'(k_t) \equiv R(k_t) \quad (20)$$

3.1.2 Intertemporal equilibrium

Given the initial capital stock K_0 and the initial wealth of the first old altruists $s_{-1} = K_0/N_{-1}$, an intertemporal equilibrium with perfect foresight is a sequence of prices $\{w_t, R_t\}_{t \geq 0}$, of value functions $\{V_t^*\}_{t \geq 0}$, of individual quantities $\{c_t, s_t, d_t, x_t\}_{t \geq 0}$ and of aggregate quantities $\{K_t, L_t, Y_t, I_t\}_{t \geq 0}$ such that in each period t :

- Firms maximise their profits (equation (20))
- Individuals maximise their utility ((16) for the first old and equation (7) for the individuals born in period $t \geq 0$)
- The next period's capital stock K_{t+1} is equal to investment I_t or the sum of individual savings $N_t s_t$:

$$K_{t+1} = I_t = N_t s_t$$

¹³The function f is assumed continuous on \mathbb{R}_+ and twice continuously differentiable on \mathbb{R}_+^* . Moreover, we assume that for all positive k : $f(k) > 0$, $f'(k) > 0$ and $f''(k) < 0$.

- The labour and the good markets clear:

$$L_t = N_t \quad \text{and} \quad Y_t = F(K_t, N_t) = N_t c_t + N_{t-1} d_t + I_t$$

In an economy with dynastic altruism, the assumption of perfect foresight is more stringent than in models with selfish individuals, such as the Diamond (1965) model, where individuals only need to forecast next period's prices, namely the rate of interest. As altruistic individuals have to forecast all future prices to take decisions today, the characterisation of the economic equilibrium entails an infinite dimensional fixed point of the sequence of prices $\{w_t, R_t\}_{t \geq 0}$. Sequences of value functions $\{V_t^*\}_{t \geq 0}$ and of individual optimal decisions $\{c_t^*, d_t^*, s_t^*, x_t^*\}_{t \geq 0}$ are associated with the sequence of prices, while the aggregation of individual optimal decisions determines the macroeconomic variables and ultimately the sequence of prices.

3.1.3 Characterisation of the intertemporal equilibrium

Assuming that an intertemporal equilibrium with perfect foresight exists, a simple method of characterisation consists in replacing the equilibrium prices with their expressions ($w_t = w(k_t)$ and $R_t = R(k_t) = f'(k_t)$) in the individual optimality conditions¹⁴. Under standard assumptions, equations (9) and (10), together with $R_{t+1} = f'(k_{t+1})$, define the optimal consumptions as a function of the capital stock and of the shadow price of bequests:

$$c_t^* = \mathcal{C}(k_{t+1}, p_{t+1}) \quad \text{and} \quad d_{t+1}^* = \mathcal{D}(k_{t+1}, p_{t+1})$$

We also have d_0^* as a function of p_0 and the initial conditions (see equation (17)). Plugging the optimal consumptions into the equation describing the evolution of bequests and that driving the dynamics of capital, we obtain the two following relations:

$$(1+n)k_{t+1} = s_t^* = x_t^* + w(k_t) - \mathcal{C}(k_{t+1}, p_{t+1})$$

$$(1+n)x_{t+1}^* = f'(k_{t+1})s_t^* - \mathcal{D}(k_{t+1}, p_{t+1})$$

Bequests thus are a function of the capital stock and the shadow price:

$$x_{t+1}^* = f'(k_{t+1})k_{t+1} - \frac{\mathcal{D}(k_{t+1}, p_{t+1})}{1+n} \equiv \mathcal{E}(k_{t+1}, p_{t+1})$$

¹⁴Since these optimality conditions are necessary and sufficient, the conditions obtained by substitution are also necessary and sufficient for an intertemporal equilibrium.

Using this equation in period t , we obtain the dynamic equation of capital:

$$(1+n)k_{t+1} = \mathcal{E}(k_t, p_t) + w(k_t) - \mathcal{C}(k_{t+1}, p_{t+1}) \quad (21)$$

When characterising the intertemporal equilibrium, we must distinguish two cases depending on whether or not the optimal bequest in period t is positive.

Positive bequests ($x_t^* > 0$)

If the optimal bequest x_t^* is positive in period t , the optimality condition (11) implies:

$$p_t = \frac{\gamma}{1+n} f'(k_{t+1}) p_{t+1} \quad (22)$$

Equations (21) and (22) implicitly define a two-dimensional dynamics of k_t and p_t . The initial capital stock k_0 is given, but not the shadow price p_0 . These equations define the forward-backward dynamics of the dynastic model. The same expressions hold in each period, provided that bequests are positive all along the transition path. In this case, the following transversality condition pins down the optimal path:

$$\lim_{t \rightarrow +\infty} \gamma^t p_t \mathcal{E}(k_t, p_t) = 0 \quad (23)$$

Zero bequests ($x_t^* = 0$)

If the optimal bequest is equal to zero in period t , the dynamics in period t can be described by a one-dimensional dynamic equation. If $x_t^* = 0$, the equation $\mathcal{D}(k_t, p_t) = (1+n)f'(k_t)k_t$ implicitly defines p_t as a function π of k_t , and we obtain:

$$(1+n)k_t = x_{t-1}^* + w(k_{t-1}) - \mathcal{C}(k_t, \pi(k_t)) \quad (24)$$

We can distinguish two cases depending on whether or not x_{t-1}^* is positive. If x_{t-1}^* is positive, equation (22) in period $t-1$ gives:

$$p_{t-1} = \frac{\gamma f'(k_t)}{1+n} \pi(k_t) \equiv \sigma(k_t)$$

Together with $x_{t-1}^* = \mathcal{E}(k_{t-1}, \sigma(k_t))$, equation (24) implicitly defines (for one period) a one-dimensional dynamic equation.

If bequests are not positive in period $t-1$ ($x_{t-1}^* = 0$), equation (24) defines (for one period) a one-dimensional dynamic equation, which is similar to the dynamics of the baseline overlapping generations model - the Diamond model of Section 2.1.

To check whether this occurs along the dynamic path of the altruistic economy, one must examine equation (11):

$$-p_t + \frac{\gamma}{1+n} p_{t+1} f'(k_{t+1}) \leq 0$$

which holds when $x_t^* = 0$.

In practice, it is only possible to characterise either intertemporal equilibria along which bequests are always positive or equilibria along which bequests are always zero. Analysing dynamics switching between a temporary equilibrium with positive bequests and a temporary equilibrium with zero bequests is an issue for future research.

3.2 The Cobb-Douglas case

We analyse the dynamics of the altruistic model in the Cobb-Douglas case. We look for a solution satisfying (21) and (22) in all periods (i.e., a dynamic path along which bequests are positive) and the transversality condition (23). With a Cobb-Douglas production function $f(k_t) = Ak_t^\alpha$ ($A > 0$ and $\alpha \in (0, 1)$) we have:

$$w_t = w(k_t) = (1 - \alpha)Ak_t^\alpha \quad \text{and} \quad R_t = f'(k_t) = \alpha Ak_t^{\alpha-1}$$

With a log-linear utility function $U(c_t, d_{t+1}) = \ln c_t + \beta \ln d_{t+1}$ ($\beta > 0$) we obtain, according to (9) and (10), the following functions $\mathcal{C}(k_{t+1}, p_{t+1})$ and $\mathcal{D}(k_{t+1}, p_{t+1})$:

$$c_t^* = \mathcal{C}(k_{t+1}, p_{t+1}) = \frac{1+n}{\gamma p_{t+1} f'(k_{t+1})} \quad \text{and} \quad d_{t+1}^* = \mathcal{D}(k_{t+1}, p_{t+1}) = \frac{(1+n)\beta}{\gamma p_{t+1}}$$

We can then calculate x_t^* :

$$x_t^* = \mathcal{E}(k_t, p_t) = \alpha Ak_t^\alpha - \frac{\beta}{\gamma p_t}$$

By multiplying equation (21) by p_t we obtain:

$$(1+n)p_t k_{t+1} = A p_t k_t^\alpha - \frac{\beta}{\gamma} - \frac{(1+n)p_t}{\gamma p_{t+1} \alpha A k_{t+1}^{\alpha-1}} \quad (25)$$

When the bequest motive is operative, condition (22) holds:

$$p_t = \frac{\gamma \alpha A k_{t+1}^{\alpha-1} p_{t+1}}{1+n}$$

Substituting the expression of p_t in equation (25) gives:

$$\alpha\gamma Ak_{t+1}^\alpha p_{t+1} = Ap_t k_t^\alpha - \left(1 + \frac{\beta}{\gamma}\right)$$

Let us define $v_t = Ap_t k_t^\alpha$, the implicit value of output (for the dynasty). The previous equation is linear in this new variable:

$$v_{t+1} = \frac{1}{\alpha\gamma} \left(v_t - 1 - \frac{\beta}{\gamma}\right)$$

This equation admits a unique bounded solution, the constant solution:

$$v_t = \bar{v} = \frac{1}{1 - \alpha\gamma} \left(1 + \frac{\beta}{\gamma}\right)$$

It is the unique solution satisfying the transversality condition $\lim_{t \rightarrow +\infty} \gamma^t p_t x_t^* = 0$. Indeed, we have:

$$p_t x_t^* = \alpha Ap_t k_t^\alpha - \frac{\beta}{\gamma} = \alpha v_t - \frac{\beta}{\gamma}$$

Since $p_t = \bar{v}/(Ak_t^\alpha)$, we obtain:

$$x_t^* = \left(\alpha - \frac{\beta(1 - \alpha\gamma)}{\gamma + \beta}\right) Ak_t^\alpha$$

Thus, bequests are positive if and only if the degree of altruism γ is sufficiently high:

$$\gamma > \frac{\beta(1 - \alpha)}{\alpha(1 + \beta)} \equiv \bar{\gamma}$$

Here again we must distinguish two cases depending on whether or not bequests are positive. In the Cobb-Douglas case, the condition for positive bequests only depends on parameters characterising preferences and technology.

Positive bequests ($\gamma > \bar{\gamma}$)

When bequests are positive, the dynamics (k_t, x_t) of the economy can be fully characterized analytically.

$$\text{If } \gamma > \bar{\gamma}, \text{ then for all } t \geq 0, \text{ we have: } \begin{cases} k_{t+1} = \frac{\alpha\gamma}{1+n} Ak_t^\alpha \\ x_t = \left(\alpha - \frac{\beta(1 - \alpha\gamma)}{\gamma + \beta}\right) Ak_t^\alpha \end{cases}$$

These dynamics converge to the capital stock \hat{k} and the level of bequests \hat{x} :

$$\hat{k} = \left(\frac{\alpha\gamma A}{1+n}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \hat{x} = A \left(\alpha - \frac{\beta(1 - \alpha\gamma)}{\gamma + \beta}\right) \left(\frac{\alpha\gamma A}{1+n}\right)^{\frac{\alpha}{1-\alpha}}$$

Zero bequests ($\gamma \leq \bar{\gamma}$)

When $\bar{\gamma}$ is larger than 1 or individuals are not sufficiently altruistic to leave bequests ($\gamma \leq \bar{\gamma}$), optimal bequests are zero, and we have $\mathcal{D}(k_t, p_t) = (1+n)f'(k_t)k_t$ and $p_t = \pi(k_t) = \beta/(\alpha\gamma Ak_t^\alpha)$. The intertemporal equilibrium with altruistic individuals is then equivalent to that of an economy consisting of selfish individuals, consuming entirely their life-cycle income. When individuals leave zero bequests, the dynamics of the economy can also be expressed in an explicit manner.

$$\text{If } \gamma \leq \bar{\gamma}, \text{ then for all } t \geq 0, \text{ we have: } \begin{cases} k_{t+1} = \frac{(1-\alpha)A\beta}{(1+n)(1+\beta)} k_t^\alpha \\ x_t = 0 \end{cases}$$

The capital stock converges to k^D :

$$k^D = \left[\frac{(1-\alpha)A\beta}{(1+n)(1+\beta)} \right]^{\frac{1}{1-\alpha}}$$

To conclude this example, note that the possibility to switch from a temporary equilibrium with positive bequests to a temporary equilibrium with zero bequests along the transition path is excluded in the Cobb-Douglas economy.

3.3 Comparison with the planner's optimal solution

3.3.1 The central planner's problem

Consider a social planner with a utilitarian objective, that is a discounted sum of generational utilities, with the discount factor reflecting social time preference. What should be the objective function of a central planner in an economy with altruistic individuals? When individuals are altruistic, one faces the issue of whether or not the social planner should ignore this dimension in designing the social objective. In other words, the question is whether or not the social planner should ignore individuals' altruistic feelings, and simply adopt as social objective the discounted sum of generational utilities, after laundering their altruistic components.

In studies on dynastic altruism¹⁵, the social objective usually only includes the selfish component of each generation's utility. If this were not the case, there would

¹⁵As noted by Michel and Pestieau (2001), the same approach can be adopted with other types of altruism, in line with Harsanyi (1995) who wants to “*exclude all external preferences, even benevolent ones, from our social utility function*”. Using a model where bequests are motivated by

be double counting and the social weights would increase over time, thereby leading to a time-inconsistent optimisation problem (see Bernheim, 1989). The most usual specification assumes that the central planner mimics the founding father of the dynasty, but without taking account of non-negative bequest constraints. It is equivalent to the problem of a central planner combining life-cycle utilities. Hence, the central planner problem can be interpreted in two ways. It can be considered either as the command optimum of an economy with selfish agents or as the command optimum of an altruistic economy.

We consider the problem of a benevolent planner, who can allocate the resources of the economy between capital accumulation, consumption of the young and consumption of the old. The resource constraint $F(K_t, L_t) = N_t c_t + N_{t-1} d_t + K_{t+1}$ can be expressed in intensive form $f(k_t) = (1+n)k_{t+1} + c_t + d_t/(1+n)$. The objective of the social planner is to maximise the discounted sum of the life-cycle utilities of all current and future generations with the social discount factor γ under the resource constraints of the economy:

$$\begin{aligned} & \max_{\{c_t, d_{t+1}\}_{t=-1}^{+\infty}} \sum_{t=-1}^{+\infty} \gamma^t U(c_t, d_{t+1}) \\ & \text{subject to: } \forall t \geq 0 \quad f(k_t) = (1+n)k_{t+1} + c_t + \frac{d_t}{1+n} \\ & \quad k_0 \text{ and } c_{-1} \text{ given.} \end{aligned}$$

To characterise the optimal solution, we make use of the method of the infinite Lagrangean¹⁶:

$$\mathcal{L} = \sum_{t=-1}^{+\infty} \gamma^t U(c_t, d_{t+1}) + \sum_{t=0}^{+\infty} \gamma^t q_t [f(k_t) - (1+n)k_{t+1} - c_t - \frac{d_t}{1+n}]$$

For all $t \geq 0$, the maximum with respect to c_t , d_t and k_{t+1} is attained when:

$$U'_c(c_t^*, d_{t+1}^*) = q_t \tag{26}$$

joy of giving, Michel and Pestieau (2001) compare the case where utilities are purged from their altruistic component with the case where they are unaltered. Social discounting may also result from uncertainty. See the discussion of social discounting in Arrow and Kurz (1970) and in Michel (1990b).

¹⁶The two consumptions c_t and d_{t+1} appear in two different resource constraints (in t and $t+1$). In order to apply the method of the Lagrangean \mathcal{L}_t of period t , one can define a modified state variable as in Michel and Venditti (1997).

$$U'_d(c_{t-1}^*, d_t^*) = \frac{\gamma q_t}{1+n} \quad (27)$$

$$q_t = \frac{\gamma q_{t+1} f'(k_{t+1})}{1+n} \quad (28)$$

The transversality condition is:

$$\lim_{t \rightarrow +\infty} \gamma^t q_t k_{t+1} = 0 \quad (29)$$

We can now compare these optimality conditions with those of the altruistic problem ((9) to (12)), thereby analysing the decentralisation of the social optimum.

3.3.2 Decentralisation of the social optimum

The social optimum can be decentralised in a market economy with non-altruistic individuals by means of lump-sum taxes and transfers. This is the Second Welfare Theorem applied to the overlapping generations model - see Atkinson and Sandmo (1980). The optimal transfer τ_t to each young individual in period t is financed by a tax equal to $(1+n)\tau_t$ paid by each old at the same period. Since old individuals consume the profit net of taxes, the condition for decentralisation,

$$d_t^* = R_t s_{t-1} - (1+n)\tau_t = f'(k_t)(1+n)k_t - (1+n)\tau_t,$$

defines the optimal lump-sum tax:

$$\tau_t = f'(k_t)k_t - \frac{d_t^*}{1+n}$$

If all optimal taxes τ_t paid by the old are non-negative, the optimal path is the intertemporal equilibrium of an economy with altruistic individuals; the level of bequests is equal to the lump-sum tax $x_t^* = \tau_t$.

To prove this result, assume that for all t , $x_t^* = \tau_t \geq 0$ and $p_t = q_t$. Hence, the optimality conditions (9) to (11) are satisfied. Moreover, $x_{t+1}^* = f'(k_{t+1})k_{t+1} - d_{t+1}^*/(1+n) < f'(k_{t+1})k_{t+1}$. Since we have $0 \leq p_{t+1}x_{t+1}^* < p_{t+1}f'(k_{t+1})k_{t+1} = (1+n)p_t k_{t+1}/\gamma = (1+n)q_t k_{t+1}/\gamma$, the transversality condition (29) of the planner's problem implies the transversality condition (12) of the altruist's maximisation problem. Hence, the solution of the planner problem is an intertemporal equilibrium of an altruistic economy with positive bequests.

The intuition of this result is simple. When τ_t is always positive, altruistic agents, who have the same utility as the social planner, choose to leave bequests equal to the transfers implemented at the command optimum.

If all bequests are positive and if the transversality condition (29) is satisfied, the intertemporal equilibrium of the dynastic model coincides with the planner's optimal solution. Indeed, since the intertemporal equilibrium satisfies $x_t^* > 0$ in every period, replacing q_t by p_t allows to obtain equations (26) to (28) from equations (9) to (11).

Since $\{c_t^*, d_t^*, k_{t+1}\}_{t=0}^{t=+\infty}$ is the optimal allocation chosen by the planner with a social discount factor γ , the founding father of the dynasty behaves as a family planner, reallocating the resources of the dynasty across generations. A dynasty in which individuals are altruistic and are linked to future generations through a chain of positive bequests can be interpreted as an infinitely lived individual. Alternatively, the altruistic model can be thought of as a realistic interpretation of the infinite horizon representative agent model.

3.3.3 Infinitely lived agents versus altruistic agents

Even though the overlapping generations model with dynastic altruism can be thought of as a microfoundation for the infinite horizon representative agent model, four significant differences between these two models need to be stressed.

First, bequests must be positive. The old generation can never take resources away from future generations; they could do so if inherited debt were enforceable. Such a restriction does not make much sense in a model with infinitely lived agents. In the absence of credit constraints, one can borrow against one's own future labour income, thus shifting resources from the future to the present. It is not always possible to interpret an infinitely lived agent as a dynasty of altruists.

Second, there is the condition that the indirect utility functions of each generation (the value functions) must be defined, as each generation takes their life-cycle decisions, being aware of the effects of their bequests on the welfare of the next generation. In contrast, infinitely lived agent determine their entire consumption path at the outset of their lives, taking prices as given.

Third, in contrast to the standard assumption of time-additively separable utility functions in models of infinitely lived agent, we consider a more general formulation of preferences, which are represented by a non-separable life-cycle utility function. This has implications for the intertemporal substitution effects, which are reinforced, when the current marginal utility depends on future consumption. As shown by

Michel and Venditti (1997), this difference may have important consequences for the equilibrium dynamics.

The fourth difference relates to the transversality condition. In the altruistic model, the discounted value of bequests tends to zero. In the infinitely lived agent model, the discounted value of wealth tends to zero. The wealth of a representative infinitely lived agent includes all the assets of the economy. On the contrary, the bequest of an altruistic agent, who lives a finite number of periods, only includes the wealth transmitted to the next generation. Whereas the transversality condition (29) of an infinitely lived agent implies the transversality condition of an altruist (12), the converse is not true.

4 Steady state

In this section we confine the analysis to steady states. There are two types of steady states: steady state with positive bequests and steady state with zero bequests. After spelling out the steady state equilibrium conditions, we specify the condition under which bequests are positive and address the issue of existence and multiplicity of steady states in the model of dynastic altruism.

In steady state, the marginal utility $U'_d(c, d)$ can be eliminated in equation (13) and the optimality condition (14) becomes $\gamma R \leq 1 + n$ ($=$ if $x > 0$). The following conditions are necessary and sufficient for a steady state equilibrium:

$$x + w = c + s \quad \text{and} \quad Rs = d + (1 + n)x \quad (30)$$

$$U'_c(c, d) = RU'_d(c, d) \quad (31)$$

$$\gamma R \leq 1 + n \quad (= \text{if } x > 0) \quad (32)$$

$$(1 + n)k = s \quad (33)$$

$$w = w(k) \quad \text{and} \quad R = R(k) \quad (34)$$

These conditions fully characterise the steady states of the dynastic model¹⁷. The transversality condition is fulfilled, since the degree of altruism γ is smaller

¹⁷These conditions imply the equilibrium condition of the good market, since we have: $f(k) = R(k)k + w(k)$, $R(k)k = R(k)\frac{s}{1+n} = \frac{d}{1+n} + x$ and $w(k) = c + s - x = c + (1+n)k - x$.

than 1. In a steady state with positive bequests ($x > 0$), the interest factor R is equal to $\hat{R} = (1+n)/\gamma$. The steady state capital intensity k is the so-called modified golden rule, $k = \hat{k} = f'^{-1}((1+n)/\gamma)$.

4.1 Steady state with positive bequests

When bequests are positive, the intertemporal equilibrium is Pareto-optimal, since it coincides with the social optimum (see Section 3.3). As the condition for non-negative bequests plays an important role in the effectiveness of fiscal policies, many economists have investigated the determinants of bequests. In his seminal paper, Barro (1974) mentioned the factors that are likely to influence bequests, and pointed to the need for further analysis:

“The derivation under which the solution for intergenerational transfer would be interior appears to be a difficult problem and would seem to require some specialization of the form of the utility functions in order to make any headway. However it seems clear that bequests are more likely to be positive the smaller the growth rate of the wage rate, the higher the interest rate ...”

However, Barro considered an overlapping generations model with exogenous wage and interest rate (see also Drazen, 1978), thereby disregarding significant general equilibrium effects. Carmichael (1982) analysed a model of dynastic altruism with a neoclassical production sector and emphasised the role of the underlying utility function in the bequest behaviour. Abel (1987) and Weil (1987) were the first to establish a formal condition for the existence of a steady state with positive bequests. Both of them assume¹⁸ that the underlying overlapping generations economy- the Diamond model - has a unique and stable positive steady state capital intensity k^D . The dynamics of the Diamond model are:

$$k_{t+1} = s^D(w(k_t), R(k_{t+1}))$$

where $s^D(., .)$ is defined Section 2.1.

Abel (1987) and Weil (1987) show that bequests are positive if and only if the steady-state equilibrium of the Diamond model, k^D , is smaller than the modified

¹⁸Weil (1987) assumes that the life-cycle utility function $U(c, d)$ is additively separable.

golden rule capital stock \hat{k} . Since \hat{k} is equal to $f'^{-1}((1+n)/\gamma)$, the Abel-Weil condition can be stated as follows: $\gamma > (1+n)/f'(k^D)$, i.e. bequests are positive if the bequest motive is sufficiently strong. This condition implies that over-accumulation of capital in the Diamond model¹⁹ rules out positive bequests in the model of dynastic altruism.

Although the Abel-Weil condition is intuitive, it is obtained under some restrictive assumptions on the Diamond model. Importantly, the characterisation of equilibrium is based on the assumption of existence, uniqueness and stability of the steady state of the Diamond model. Thibault (2000) has established a necessary and sufficient condition for the existence of a steady-state equilibrium with positive bequests, which holds regardless of the number and the stability property of steady states in the Diamond model. This condition is obtained by expressing savings as a function of bequests. The steady-state conditions (30) and (34) imply:

$$c + \frac{d}{\hat{R}} = w(\hat{k}) + \left(1 - \frac{1+n}{\hat{R}}\right)x = w(\hat{k}) + (1-\gamma)x \equiv \Omega$$

The consumptions only depend on the disposable-for-consumption life-cycle income Ω , and satisfy the arbitrage condition (31), i.e. $U'_c = \hat{R}U'_d$. Thus, the first-period consumption can be expressed as follows: $c = \Omega - s^D(\Omega, \hat{R})$. This leads to an expression of savings as a function of bequests:

$$s = w(\hat{k}) + x - c = s^D(w(\hat{k}) + (1-\gamma)x, \hat{R}) + \gamma x \equiv \phi(x)$$

An equilibrium with positive bequests exists if and only if $\phi(x) = (1+n)\hat{k}$ admits a positive solution \hat{x} . Assuming that the second-period consumption d is a normal good, $s^D(w, R)$ is increasing with respect to w and, therefore, $\phi(x)$ is increasing. The existence of a positive \hat{x} solution to $\phi(\hat{x}) = (1+n)\hat{k}$ is then equivalent to $\phi(0) < (1+n)\hat{k}$, or:

$$s^D(w(\hat{k}), \hat{R}) < (1+n)\hat{k} \quad (35)$$

This condition means that, at the modified golden rule, savings in the underlying overlapping generations economy would not be sufficient to maintain the capital stock of the golden rule modified by the degree of altruism γ . Given a level of

¹⁹Over-accumulation of capital occurs when k^D is greater than the golden rule $k^G = f'^{-1}(1+n)$, and thus also greater than the modified golden rule: $k^D > k^G > \hat{k}$.

bequests \hat{x} , the steady-state consumptions are determined by (30):

$$\hat{c} = \hat{x} + w(\hat{k}) - (1+n)\hat{k} \quad \text{and} \quad \hat{d} = (1+n)(\hat{R}\hat{k} - \hat{x})$$

A graphical rule can be used to determine whether or not bequests are positive. Altruists choose to leave positive bequests in the long run if and only if, evaluated at the modified golden rule \hat{k} , the curve representing the saving function in the Diamond model divided by $1+n$ (i.e. $k \rightarrow S^D(k) = \frac{1}{1+n}s^D(w(k), R(k))$) lies below the 45° line.

To illustrate the graphical rule, let us consider four degrees of altruism $\gamma_1, \gamma_2, \gamma_3$ and γ_4 arranged in ascending order. For each degree of altruism γ_i , we define $\hat{k}_i = f'^{-1}((1+n)/\gamma_i)$, the capital stock of the golden rule modified by the degree of altruism γ_i . Let us further assume that Figure 1 depicts the saving function in the Diamond model.

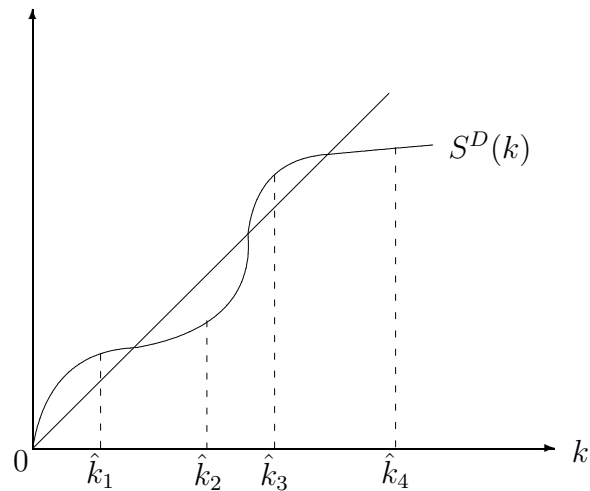


Figure 1: The graphical rule

The graphical rule indicates that the model of dynastic altruism does not experience a steady state with positive bequests if the degree of altruism is γ_1 or γ_3 . Since $S^D(\hat{k}_2)$ and $S^D(\hat{k}_4)$ are respectively smaller than \hat{k}_2 and \hat{k}_4 , the dynastic model has an equilibrium with positive bequests when the degree of altruism is either γ_2 or γ_4 . Interestingly, if the Diamond model has no positive steady state, the dynastic model has a steady state with positive bequests, as the Diamond savings function

always lies below the 45° line (see Figure 2).

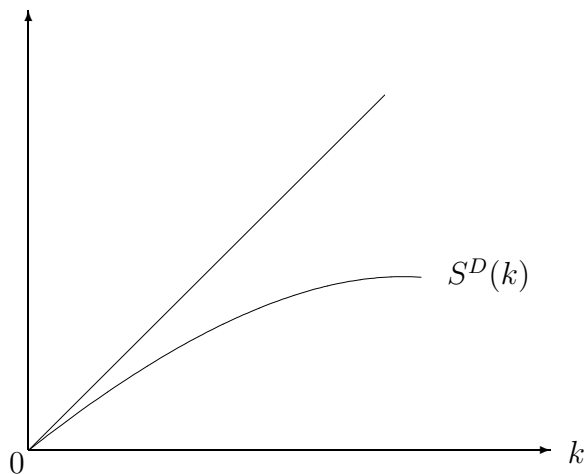


Figure 2: No positive steady state in the Diamond model

Furthermore, we remark that the necessary and sufficient condition (35) on s^D for the existence of a steady state with positive bequests can be equivalently expressed using the life-cycle utility function. This condition is equivalent to $\gamma > (1+n)U'_d(c, d)/U'_c(c, d)$, where the marginal utilities are evaluated at $c = w(\hat{k}) - (1+n)\hat{k}$ and $d = (1+n)\hat{R}\hat{k}$. As the function $\varphi(s) = U'_c(w(\hat{k}) - s, \hat{R}s) - \hat{R}U'_d(w(\hat{k}) - s, \hat{R}s)$ is increasing in s because of the strict concavity of U , $\varphi((1+n)S^D(\hat{k})) = 0$ and (35) are equivalent to $\varphi((1+n)\hat{k}) > 0$.

4.2 Steady state with zero bequests

Altruists who are not sufficiently wealthy to leave a bequest to their children behave as if they were selfish. Any steady state with zero bequests of the economy with dynastic altruism, therefore, is a steady state of the Diamond economy. The zero-bequest steady states of the model of dynastic altruism feature a capital stock which is greater than that of the modified golden rule, since equations (32) and (34) imply that a steady state with zero bequests satisfies the following inequality: $\gamma f'(k) \leq 1+n$, or equivalently $k \geq \hat{k}$. Since the modified golden rule capital stock \hat{k} is smaller than that of the golden rule $k^G = f'^{-1}(1+n)$, regardless of the degree of altruism, dynamically-inefficient equilibria of the Diamond model are equilibria with zero

bequests of the dynastic model. The only dynamically-efficient Diamond equilibria, which are also equilibria of the dynastic model, are located between the modified golden rule capital stock \hat{k} and the golden rule capital stock k^G . According to the graphical rule, the zero-bequest equilibria of the dynastic model are the Diamond equilibria located on the right-hand side of the modified golden rule, \hat{k} . Whereas the steady state with positive bequests is unique, there can be a multiplicity of steady states with zero bequests.

4.3 Existence and multiplicity of steady states

The steady state with positive bequests can coexist with bequest-constrained equilibria, which are formally equivalent to those of the Diamond model²⁰. To illustrate multiple equilibria, let us assume that the saving function $S^D(k)$ is represented in Figure 3.

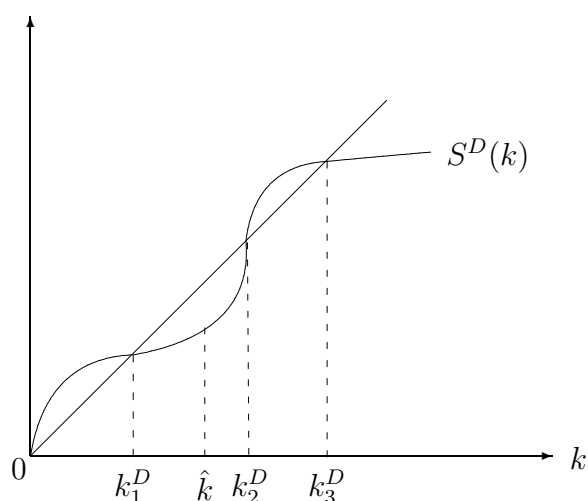


Figure 3: Multiplicity of equilibria

The economy depicted in Figure 3 experiences three steady states. The equilibrium with positive bequests \hat{k} coexists with two bequest-constrained equilibria k_2^D and k_3^D . The steady-state equilibrium k_1^D , which would be a steady state of the Diamond model, is not an equilibrium of the dynastic model, as it is smaller than the modified golden rule ($k_1^D < \hat{k}$). In contrast to the Diamond model, the model of

²⁰Aiyagari (1992) obtains a similar result in a pure exchange economy.

dynastic altruism always experiences at least one steady state with positive capital. We consider two cases. First, if the Diamond model has no positive steady state, we have proved in Section 4.1 (see Figure 2) that the dynastic model has a unique steady state, the modified golden rule \hat{k} . Second, if the Diamond model has several positive steady states, either the highest of these equilibria, k_{max}^D , is greater than, or equal to, \hat{k} and it is a steady state with zero bequests, or k_{max}^D is smaller than \hat{k} and the dynastic model has a steady state with positive bequests, because (35) is satisfied²¹.

Finally, using the graphical rule, it is straightforward to establish that the dynastic model has a unique positive steady state only in two cases:

- If the Diamond model has no positive steady state greater than \hat{k} , the dynastic model has a unique steady state, the modified golden rule.
- If the Diamond model has a unique steady state k^D greater than \hat{k} and if (35) is not satisfied, k^D is the unique equilibrium of the dynastic model.

5 Fiscal policies

Any dynamic path of the economy with dynastic altruists coincides with the social optimum, provided that bequests are positive all along the equilibrium path (see Section 3.3). This means that fiscal policies aimed at redistributing resources between generations have no impact on the intertemporal equilibrium, as long as fiscal policy choices remain compatible with the existence of an equilibrium with positive bequests. Public debt is neutral, as public intergenerational transfers resulting from the issuance and redemption of government bonds are offset by private intergenerational transfers of an equivalent amount. In this section, we illustrate the neutrality of fiscal policies by analysing their effects on the steady state of the dynastic model. First, we present the debt-neutrality result. Second, we extend the neutrality result to unfunded or pay-as-you-go social security schemes. Third, we analyse the effects of estate taxation on the equilibrium of the dynastic model. Finally, we reconsider public debt and its neutrality property, when bequest motive is inoperative before, but not after government intervention.

²¹We have $\lim_{k \rightarrow +\infty} \frac{S^D(k)}{k} = 0$ (since $S^D(k) < \frac{w(k)}{1+n}$). Thus, for $k > k_{max}^D$ we have $S^D(k) < k$.

5.1 Neutrality of government debt

We consider a public debt scheme along the lines of Diamond (1965). The relation between savings and capital accumulation is modified, as savings finance both physical capital and government bonds:

$$K_{t+1} + B_t = N_t s_t$$

In each period the government reimburses the capital and interest of the outstanding debt by issuing new bonds and levying taxes on the young. The government budget constraint is:

$$B_t = R_t B_{t-1} - N_t \tau_t$$

B_t denotes the total level of debt and τ_t is a lump-sum tax paid by each young. We further assume that the debt issued in period 0 was distributed to the old in period 0 and that there is no public spending. We define the debt per young individual $b_t \equiv B_t/N_{t-1}$ and assume that it is constant, $b_t = b$. The path of taxes necessary to maintain this constant debt ratio is given by: $\tau_t = (R_t - (1+n))b$. Henceforth, we restrict the analysis to steady state with a view to explaining the debt neutrality result in a simple framework. In steady state, we have:

$$\tau = [R - (1+n)]b$$

In the absence of government intervention ($b = 0$), $\{c, d, x, k\}$ is a steady state of the dynastic model if and only if the optimality conditions (30), (31), (32), (33) and (34) are satisfied (see section 4). When bequests are positive ($x > 0$), equation (32) pins down the steady state capital stock, the modified golden rule $k = \hat{k}$, and the long-run equilibrium is $\{\hat{c}, \hat{d}, \hat{x}, \hat{k}\}$. To extend the baseline model to public debt, only two optimality conditions have to be modified in steady state.

- The first-period budget constraint becomes:

$$w + x - \tau = c + s \quad \text{where} \quad \tau = [R - (1+n)]b$$

- The relation between the capital stock and savings reads now:

$$s = (1+n)(k + b)$$

Given $k = \hat{k}$ and $x = \hat{x} + \hat{R}b$, the consumptions are $c = \hat{x} + w(\hat{k}) - (1+n)\hat{k} = \hat{c}$ and $d = \hat{R}s - (1+n)x = \hat{d}$. The condition for positive bequests $x > 0$ results

from $\hat{x} > 0$, when debt is positive²². Hence, consumption and production are not modified in the long run with a constant debt per young individual. The Ricardian equivalence theorem applies to the model of dynastic altruism, when the bequest motive is operative before debt is issued. The only changes concern the decision variables s and x . Altruists counter the government intervention by reallocating their bequests and their savings. Increasing their bequests by $\hat{R}b$ allows them to leave their consumption path and their utility unaffected, when the government issues public bonds amounting to b .

5.2 Neutrality of pay-as-you-go social security

An increase in social security benefits makes parents richer and children poorer, since children pay taxes to finance the social security system. Altruistic parents, who leave bequests to their children before the increase in the scale of the social security programme, are aware of the transfer of resources operated by the pension system and react to this policy change by increasing their bequests. Any increase in the scale of the social security programme is thereby offset by an equivalent increase in bequests, provided that bequests are positive before the policy change.

To simplify the exposition, we consider the steady state of an economy without a pay-as-you-go social security system ($\tau = 0$), which we denote with $\{\hat{c}, \hat{d}, \hat{x}, \hat{k}\}$. In steady state, the optimal bequest \hat{x} satisfies equation (31):

$$U'_c(w(\hat{k}) + \hat{x} - \hat{s}, \hat{R}\hat{s} - (1+n)\hat{x}) = \hat{R}U'_d(w(\hat{k}) + \hat{x} - (1+n)\hat{s}, \hat{R}\hat{s} - (1+n)\hat{x})$$

where $\hat{s} = (1+n)\hat{k}$ is the level of savings at the modified golden rule.

Let us consider an unfunded pension scheme consisting of a payroll tax τ paid by the workers and a pension benefit θ given to the retirees. The budget of the public pension system is balanced in each period:

$$\theta = (1+n)\tau$$

If bequests are positive, the steady state capital stock is given by the modified golden rule. The incomes of the young and the old are $x + w - \tau$ and $Rs + (1+n)\tau$,

²²Neutrality is also obtained with a negative debt, i.e. public investment, as long as $\hat{x} + \hat{R}b$ is positive.

respectively. The steady-state bequest x must then satisfy the optimality condition:

$$\begin{aligned} U'_c(x + w - \tau - s, Rs + (1 + n)\tau - (1 + n)x) \\ = RU'_d(x + w - \tau - s, Rs - (1 + n)x + (1 + n)\tau) \end{aligned} \quad (36)$$

When $k = \hat{k}$, $w = w(\hat{k})$, $R = \hat{R}$ and $s = \hat{s}$, $x = \hat{x} + \tau$ is the solution to equation (36). Given a level of bequests $x = \hat{x} + \tau$, the consumption of parents, $d = \hat{R}\hat{s} + (1 + n)\tau - (1 + n)x = \hat{R}\hat{s} - (1 + n)x = \hat{d}$, as well as the consumption of children, $c = w(\hat{k}) - \tau + x - \hat{s} = w(\hat{k}) + \hat{x} - \hat{s} = \hat{c}$, are not affected by the pension system.

The neutrality of a pay-as-you-go system is valid in the dynastic model, provided that bequests are positive before its introduction. The private intergenerational transfers from parents to children exactly offset the public intergenerational transfers operated by the pension system, and the optimality conditions defining the consumption path of the dynasty remain unchanged. Altruistic agents increase their bequests exactly by the amount of taxes paid by the young to finance the social security scheme.

5.3 Estate taxation

Estate taxation affects the intertemporal equilibrium, since it distorts individual choices. A proportional tax rate τ_e applies to bequests, and the tax revenue is redistributed in a lump-sum manner, θ_e , to the young individuals. Thus, the first-period budget constraints are modified as follows:

$$(1 - \tau_e)x + w + \theta_e = c + s$$

The optimality condition regarding bequests (i.e. equation (32)) becomes:

$$R - \frac{1 + n}{(1 - \tau_e)\gamma} \leq 0 \quad (= \text{if } x > 0)$$

If bequests are positive, the steady-state capital stock \hat{k}_e is given by:

$$\hat{k}_e = f'^{-1}\left(\frac{1 + n}{(1 - \tau_e)\gamma}\right)$$

Assuming that the government budget is balanced in each period, we have: $\theta_e = \tau_e x$. Estate taxation reduces the capital stock ($\hat{k}_e < \hat{k}$), while increasing the interest factor (i.e. $R(\hat{k}_e) > R(\hat{k})$). As the net product per young agent (available for consumption) $f(\hat{k}_e) - (1 + n)\hat{k}_e$ is diminished, estate taxation reduces the steady state welfare of altruistic individuals.

5.4 The neutrality of high debts

The neutrality of government debt or public pension hinges on the assumption that bequests are positive all along the equilibrium path. If bequests are constrained before government intervention, the Ricardian equivalence theorem does not hold, and fiscal policy affects the economic equilibrium. Let us reconsider the case of public debt. The steady-state equilibrium with a constant debt ratio, b , is characterised by the following equations:

$$x^b + w^b - \tau = c^b + s^b \quad \text{and} \quad R^b s^b = d^b + (1+n)x^b$$

$$U'_c(c^b, d^b) = R^b U'_d(c^b, d^b)$$

$$\gamma R^b \leq 1+n \quad (= \text{if } x^b > 0)$$

$$(1+n)(k^b + b) = s^b$$

$$w^b = w(k^b) \quad \text{and} \quad R^b = R(k^b)$$

$$\tau = [R^b - (1+n)]b$$

There are two possibilities depending on whether x^b is positive or equal to zero.

- If $x^b = 0$, the steady state $\{c^b, d^b, x^b = 0, k^b\}$ is a steady state of the Diamond economy.

- If x^b is positive, the steady state is given by the modified golden rule, with $k^b = \hat{k}$, $R^b = \hat{R}$, $w^b = w(\hat{k})$ and $s^b = (1+n)(\hat{k} + b)$.

The optimal solution $\{\hat{c}, \hat{d}, \hat{k}\}$ corresponds to the steady state obtained by ignoring the non-negative bequest condition $x \geq 0$. When there is no debt, we denote with:

$$\tilde{x}^0 = \frac{\hat{R}\hat{s} - \hat{d}}{1+n}$$

the transfer (positive or negative) which is desired by the parents. Taking into account the non-negative bequest condition, the optimal bequest chosen by an altruist is given by:

$$x^0 = \max \left\{ 0, \tilde{x}^0 \right\}$$

Given a debt level b , the transfer (positive or negative) which is desired by a parent becomes:

$$\tilde{x}^b = \frac{\hat{R}(1+n)(\hat{k} + b) - \hat{d}}{1+n} = \tilde{x}^0 + \frac{1+n}{\gamma} b$$

When the government issues a positive debt b , the optimal bequest chosen by an altruist is:

$$x^b = \max \left\{ 0, \tilde{x}^b \right\}$$

To examine the effects of public debt, we distinguish different cases depending on the level of debt and the degree of altruism γ . When the desired parent-to-child transfer is non-negative ($\tilde{x}^0 \geq 0$), a positive debt implies $\tilde{x}^b > 0$ and we obtain the neutrality result showed in Section 5.1. When the desired intergenerational transfer is negative ($\tilde{x}^0 < 0$) in the absence of public debt, altruists choose to leave no bequests (i.e. $x^0 = 0$). Altruists then behave as pure life-cyclers and fiscal policy - public debt or social security - is effective.

When fiscal policy is effective, its effects depend on the size of public debt, b . Consider the threshold level of debt \bar{b} equal to $-\gamma\tilde{x}^0/(1+n)$. When the size of debt b is sufficiently low ($b \leq \bar{b}$), public debt does not affect bequests. As bequests are constrained before and after the government intervention, the effect of public debt is the same as in the Diamond model.

However, when b is greater than \bar{b} , bequests x^b become positive. Bequest motive is inoperative before the introduction of debt but not afterwards. Importantly, an increase in b from above \bar{b} has no further effect on the equilibrium. This property has been studied by several authors in voting models (see, e.g., Cukierman and Meltzer, 1989). In this framework, the amount of debt preferred by old agents is the level \bar{b} , which makes individuals free from the non-negative bequest constraint.

6 Heterogenous altruistic dynasties

Up to now we have assumed that the economy consists of a perfectly homogenous population. Such an assumption does not allow to study the distributional effects of fiscal policy, as all individuals are identical within each generation. Departing from this assumption leads us to reconsider the effects of fiscal policy, especially the neutrality result.

We consider an economy consisting of two types of altruistic agents. They have the same life-cycle utility function $U(c, d)$, but different degrees of altruism: $\gamma_1 > \gamma_2$. In each dynasty, all agents have the same degree of intergenerational altruism γ_i , $i \in \{1, 2\}$. We denote with p_i the exogenous proportion of individuals of type i .

First, we study the steady state of this economy. Second, we characterise the effects of fiscal policy.

6.1 Steady state

In steady state, the optimality conditions (30), (31) and (32) apply to the two types of individuals with x_i , c_i , s_i , d_i and γ_i . The equilibrium prices satisfy (34), but the relation between the capital stock and savings needs to be amended to take account of individuals' heterogeneity:

$$(1 + n)k = p_1s_1 + p_2s_2 \quad (37)$$

As we have $\gamma_2 < \gamma_1$, the optimality conditions imply $\gamma_2R < \gamma_1R \leq 1 + n$. Since condition (32) holds for both types, a positive bequest in the less altruistic dynasty ($x_2 > 0$) is ruled out. Only individuals belonging to the dynasty endowed with the higher degree of altruism can leave bequests. In steady state, the less altruistic individuals leave no bequests ($x_2 = 0$) and their saving function is similar to that of selfish individuals: $s_2 = s^D(w(k), R(k))$.

If bequests are positive in the more altruistic dynasty ($x_1 > 0$), according to (32) we have: $\gamma_1R(k) = 1+n$. The steady-state capital-labour ratio is that of the modified golden rule corresponding to the degree of altruism of the more altruistic agents ($k = \hat{k}_1 = f'^{-1}((1+n)/\gamma_1)$). The steady state capital-labour ratio is determined by the degree of altruism of the more altruistic agents, regardless of their relative number. The society is divided into two classes: those who are linked with their children through bequests and those who behave as if they were selfish.

The fact that the steady-state capital-labour ratio \hat{k}_1 is not affected by the share of the more altruistic individuals in the population is consistent with the findings of Ramsey (1928) and Becker (1980), who show that, in an economy with heterogeneous infinitely lived agents, the most patient ones impose their view on the long-run capital accumulation²³.

Let us calculate the savings of the more altruistic individuals, s_1 , in the case of positive bequests $x_1 > 0$. In steady state, the life-cycle budget constraint of the

²³Vidal (1996a) extends this result to heterogeneous dynasties of a closed economy. Vidal (2000) studies capital mobility under the assumption that degrees of intergenerational altruism differ across countries.

more altruistic individuals is:

$$c_1 + \frac{d_1}{R(\hat{k}_1)} = w(\hat{k}_1) + \left(1 - \frac{1+n}{R(\hat{k}_1)}\right)x_1 = w(\hat{k}_1) + (1 - \gamma_1)x_1 \equiv \Omega_1$$

In addition to their wages, altruists consume the difference between the bequest they receive from their parents and the bequest they leave to their children. This, along with the condition $U'_c(c_1, d_1) = R(\hat{k}_1)U'_d(c_1, d_1)$, implies: $c_1 = \Omega_1 - s^D(\Omega_1, R(\hat{k}_1))$. Their consumptions only depend on their disposable-for-consumption life-cycle income. By substitution in the first-period budget constraint, we obtain:

$$s_1 = w(\hat{k}_1) + x_1 - c_1 = s^D(w(\hat{k}_1) + (1 - \gamma_1)x_1, R(\hat{k}_1)) + \gamma_1 x_1 \equiv \phi_1(x_1)$$

Under the assumption that the second-period consumption is a normal good, $s^D(w, R)$ is increasing in w , and thus $\phi_1(x_1)$ is increasing in x_1 . Moreover, ϕ_1 increases from $\phi_1(0) = s^D(w(\hat{k}_1), R(\hat{k}_1))$ to $+\infty$, when x_1 increases from 0 to $+\infty$. The equilibrium condition (37) is at the steady state \hat{k}_1 :

$$p_1 \phi_1(x_1) = (1+n)\hat{k}_1 - p_2 s^D(w(\hat{k}_1), R(\hat{k}_1))$$

and there exists a solution $x_1 > 0$ if and only if the right-hand-side of this expression is greater than $p_1 \phi_1(0)$:

$$(1+n)\hat{k}_1 > s^D(w(\hat{k}_1), R(\hat{k}_1))$$

This is exactly the condition we would have in the model of homogenous altruistic agents with degree of altruism γ_1 . At the modified golden rule \hat{k}_1 , the Diamond saving function lies below the modified golden rule capital stock. In this case, there exists a unique steady state with positive bequests x_1 in the economy with heterogenous altruists. The bequests of the more altruistic individuals compensate for the insufficient savings of the less altruistic individuals. This clearly appears when studying the effect of p_1 on the equilibrium. Even though the capital-labour ratio \hat{k}_1 of the modified golden rule does not depend on the share of more altruistic individuals in the population, the level of bequests does. Interestingly, x_1 is a decreasing function of p_1 , and so is the life-cycle income Ω_1 . The lower the proportion of the more altruistic agents, the more they consume and the higher their utility.

6.2 Government debt

We consider the case of a government debt b that is constant per young individual. The first-period budget constraint of individuals of type i needs to be amended to take account of taxation:

$$w + x_i - \tau = c_i + s_i$$

Physical capital and government bonds are financed by savings of both types of individuals:

$$(1 + n)(k + b) = p_1 s_1 + p_2 s_2 \quad (38)$$

The analysis developed in Section 6.1 still applies. We have $x_2 = 0$, $s_2 = s^D(w - \tau, R)$ and if $x_1 > 0$: $k = \hat{k}_1$, $\Omega_1^b = w(\hat{k}_1) - \tau + (1 - \gamma_1)x_1$ and:

$$\tau = (R(\hat{k}_1) - (1 + n))b \equiv \varepsilon(\hat{k}_1)b$$

The savings of the more altruistic individuals are:

$$s_1 = \phi_1(x_1, b) \equiv \gamma_1 x_1 + s^D(w(\hat{k}_1) - \varepsilon(\hat{k}_1)b + (1 - \gamma_1)x_1, R(\hat{k}_1))$$

Equation (38) becomes:

$$p_1 \phi_1(x_1, b) = (1 + n)(\hat{k}_1 + b) - p_2 s^D(w(\hat{k}_1) - \varepsilon(\hat{k}_1)b, R(\hat{k}_1))$$

Bequests x_1 are positively related to government debt b . When x_1 is positive, public debt is neutral from the aggregate point of view, since it does not modify capital, output and total consumption. In the economy with heterogeneous agents, it has redistributive implications, reducing the income, the consumptions and the welfare of the less altruistic individuals. Since total consumption is unchanged, increasing public debt results in higher levels of consumption and welfare for the more altruistic individuals. This stems from the increase in the bequests of the more altruistic individuals x_1 , compensating for the lower savings of the less altruistic individuals. Public debt has no redistributive implications only in the case of homogenous agents ($p_1 = 1$), provided that bequests are positive.

6.3 Pay-as-you-go social security and estate taxation

A pay-as-you-go system with lump-sum taxes and benefits entails the same effects as government debt. When bequests are positive in the more altruistic dynasty

($x_1 > 0$), the economy is in a situation of under-accumulation of capital with $k = \hat{k}_1$. The life cycle income Ω_2^τ of the less altruistic individuals is reduced by an increase in the scale of the social security programme. With a lump-sum tax τ paid by the young, the benefits received by retirees are $\theta = (1 + n)\tau$, and the steady-state life-cycle income of the less altruistic individuals is given by:

$$\Omega_2^\tau = w(\hat{k}_1) - \tau + \frac{\theta}{R(\hat{k}_1)} = w(\hat{k}_1) - (1 - \gamma_1)\tau < w(\hat{k}_1) = \Omega_2$$

Aggregate variables and prices are unchanged in the long run, whereas there is a welfare loss for the less altruistic individuals and a welfare gain for the more altruistic individuals.

Estate taxation with heterogeneous individuals has been studied by Michel and Pestieau (1998)²⁴. A proportional tax rate τ_e applies to bequests and the tax revenue is redistributed in a lump-sum manner θ_e to the young individuals. Thus, the first-period budget constraints are modified as follows:

$$(1 - \tau_e)x_i + w + \theta_e = c_i + s_i$$

The optimality condition regarding bequests (32) becomes:

$$R - \frac{1 + n}{(1 - \tau_e)\gamma_i} \leq 0 \quad (= \text{if } x_i > 0)$$

This implies that the less altruistic individuals do not leave bequest ($x_2 = 0$), and if x_1 is positive, the steady-state capital stock \hat{k}_e is given by:

$$\hat{k}_e = f'^{-1}\left(\frac{1 + n}{(1 - \tau_e)\gamma_1}\right)$$

Assuming that the government budget is balanced in each period, we have: $\theta_e = \tau_e p_1 x_1$. Estate taxation reduces the capital stock (i.e. $\hat{k}_e < \hat{k}_1$), while increasing the interest factor (i.e. $R(\hat{k}_e) > R(\hat{k}_1)$). The net product per young agent (available for consumption) $f(\hat{k}_e) - (1 + n)\hat{k}_e$ is diminished.

Estate taxation has three effects on the welfare of the less altruistic individuals who do not leave bequests: a negative effect on their labour income $w(\hat{k}_e)$, a positive effect resulting from the redistribution of estate tax revenues $\theta_e = p_1 \tau_e x_1$ and a

²⁴They consider the case in which the less altruistic individuals are pure life-cyclers (i.e., $\gamma_2 = 0$). The value of γ_2 ($< \gamma_1$) has no impact on the steady-state equilibrium; see Vidal (1996a).

positive effect stemming from the decrease in the relative price $1/R(\hat{k}_e)$ of old-age consumption. For the more altruistic individuals, there are two additional effects, the tax on bequests and the induced changes in bequests. Michel and Pestieau (1998) show in a simple case with a log-linear utility and a Cobb-Douglas production function that the negative effect dominates for a sufficiently low level of the estate tax rate τ_e , and that estate taxation worsens the steady-state welfare of both types of agents.

7 Other forms of altruism

The neutrality of fiscal policy hinges on individual reactions. The motive for intergenerational transfers is therefore crucial in analysing the effects of fiscal policy. Dynastic altruism guarantees the neutrality of fiscal policy when bequests are positive, but results are less clear cut, when other motives underpin intergenerational transfers. In this section we present several models of intergenerational altruism and analyse fiscal policy in each of them, thereby making clear the conditions for the neutrality of fiscal policy.

We distinguish two strands of models. In the first one, the utility of the beneficiary is an argument of the utility of the benefactor. Since we have already examined the model of descending dynastic altruism, we focus on others forms of pure altruism: ascending and two-sided altruism. In the second one, altruism is said to be *ad hoc*. Either the altruistic argument in the benefactor's utility function is only some part of the utility of the beneficiary (Burbidge 1983, Abel 1987) or some other variables such as the level of bequests (paternalistic altruism) or the level of income (family altruism).

7.1 Others forms of pure altruism

7.1.1 Ascending altruism

In his 1974 paper, Barro stresses that the neutrality result depends on the existence of positive transfers between parents and children. These transfers can be from parents to children (descending) or from children to parents (ascending). The model of ascending intergenerational altruism is formally similar to the model of descending altruism. Children have an altruistic concern for their parents and face the following

budget constraints:

$$c_t + s_t + g_t = w_t$$

$$d_{t+1} = R_{t+1}s_t + (1+n)g_{t+1}$$

where g_t denotes the gift that individuals born in period t give to their parents and $(1+n)g_{t+1}$ the gifts that they receive in period $t+1$ from their $1+n$ children. Gifts are private intergenerational transfers from the young to the old and are restricted to be non-negative in each period:

$$g_t \geq 0$$

We again consider a recursive definition of altruism. Children care about their parents' welfare by weighting their parents' utility in their own utility function v_t . Denoting with v_{t-1} the well-being of their parents, we assume that the utility of individuals born in period t is given by:

$$v_t = U(c_t, d_{t+1}) + \delta v_{t-1}$$

where $\delta \in (0, 1)$ is the degree of ascending altruism. This formulation is based on several implicit assumptions. We can substitute parental utilities backwards to obtain an infinite sequence of past life-cycle utilities (from $t = 0$ to $t = -\infty$). The optimality conditions are therefore similar to those prevailing in the case of descending altruism ((13) and (14)). Equation (13) is the arbitrage condition driving consumption choices, whereas reversing the direction of transfers leads to replacing (13) with the following condition:

$$-U'_c(c_t, d_{t+1}) + \delta(1+n)U'_d(c_{t-1}, d_t) \leq 0 \quad (= \text{ if } g_t > 0) \quad (39)$$

Since ascending altruism is based on calculations regarding past utilities, this formulation raises some modelling concerns:

- Past variables are given and cannot be modified. In this context, what is the significance of a backward dynamics of the capital stock?
- Assuming that all generations have the same behaviour, the intertemporal equilibrium goes from $t = -\infty$ to $t = +\infty$ and has no initial condition.

- From (13), (20) and (39), the steady state capital stock of the economy with positive gifts satisfies: $f'(k) = R = (1 + n)\delta$. The steady state with positive gifts is characterised by over-accumulation of capital²⁵.

Along the same lines as those we developed when analysing descending altruism, one can show that public debt or pay-as-you-go social security do not affect steady-state consumptions, when long-run gifts are positive. Individuals can counter fiscal policies by adjusting gifts. Ricardian equivalence holds, as long as the chain of positive intergenerational transfers is not broken. Since public debt is an ascending public transfer between generations, an increase in the level of public debt is offset by an equivalent decrease in gifts. There therefore exists a level of public debt, such that gifts are driven down to zero. When public debt is sufficiently high, parents become so wealthy that there is no longer a need for gifts. As gifts are no longer positive, families cannot counter fiscal policies, which are then effective.

7.1.2 Two-sided altruism

Neither descending nor ascending altruism can guarantee the neutrality property, which holds only if bequests or gifts are positive. Some authors have therefore combined both ascending and descending altruism, leading to a new form of altruism known as two-sided or reciprocal altruism.

Since intergenerational transfers operate in both directions, from children to parents (gifts g_t) and from parents to children (bequests x_t), an individual born in t faces the following budget constraints:

$$c_t + s_t + g_t = w_t + x_t$$

$$d_{t+1} + (1 + n)x_{t+1} = R_{t+1}s_t + (1 + n)g_{t+1}$$

In each period, private intergenerational transfers are assumed to be non-negative:

$$g_t \geq 0 \quad \text{and} \quad x_t \geq 0 \tag{40}$$

Assuming that individuals have an altruistic concern for both their parents and their children, one can represent their utility function as follows:

$$v_t = \delta v_{t-1} + U(c_t, d_{t+1}) + \gamma v_{t+1}$$

²⁵O'Connell and Zeldes (1993) analyse the model of ascending altruism under the assumption of strategic behaviours. When parents save less to receive more, the steady state may be characterised by under-accumulation of capital.

where $\delta \in (0, 1)$ and $\gamma \in (0, 1)$ are the degree of ascending altruism and the degree of descending altruism, respectively.

The formulation of two-sided altruism deserves some comments:

- Analysing two-sided altruism is difficult, because the life cycle utility $U(c_t, d_{t+1})$ is both in v_{t-1} and in v_{t+1} , and two key questions therefore arise. When does a solution exist? What is the relation between the degree of ascending altruism and the degree of descending altruism? Kimball (1987) shows that strong assumptions on the degrees of altruism are required to guarantee that an infinite sum of life-cycle utilities is the solution to the functional equation defining the utility of altruists²⁶. Some parametric restrictions are also necessary to ensure that intergenerational transfers are positive.

- Since the intertemporal equilibrium goes from $-\infty$ to $+\infty$, there are no initial conditions.

- In a model where individuals leave bequests to their children and support their parents, three types of steady-state equilibrium are possible. Because of the two inequality constraints (40), there are two first-order conditions (14) and (39), which are not mutually compatible in steady state. The steady state cannot therefore be characterised by both positive bequests and positive gifts. Either bequests are positive and gifts zero, or bequests are zero and gifts positive, or both are zero. There is a wide range of parameters leading to inoperative intergenerational transfers motive (see Vidal, 1996b).

Concerning fiscal policies, the results are straightforward extensions of those obtained under one-sided altruism. The neutrality of government debt is again guaranteed only if the same type of transfers (either gifts or bequests) is positive both before and after the change in the level of government debt.

7.2 *Ad hoc* altruism

There always are restrictions to the neutrality of public debt in models of dynastic altruism. In the literature there is only one specification of altruism ensuring that Ricardian equivalence always holds. This specification departs from the recursive

²⁶Kimball (1987) shows that the sum of both degrees of altruism must be smaller than one, i.e. $\delta + \gamma < 1$.

definition of altruism proposed by Barro and belongs to *ad hoc* forms of the altruistic utility function, which we review in this section. First, we examine the specification of the altruistic utility function ensuring debt neutrality and highlight its caveats. Second, we present paternalistic altruism, whereby bequests are broadly equivalent to consumption goods in the utility of parents. Third, we briefly expound family altruism, which departs from paternalism, but still does not assume that families are infinitely lived decision makers.

7.2.1 A model with debt neutrality

Burbidge (1983) has proposed a particular form of altruism, which always results in debt neutrality. He suggests adding a term of ascending altruism, which relates to an altruistic concern for parents, to a term of descending altruism:

$$v_t = \frac{1}{\gamma}U(c_{t-1}, d_t) + U(c_t, d_{t+1}) + \sum_{j=1}^{+\infty} \gamma^j U(c_{t+j}, d_{t+1+j})$$

This utility function is the sum of the utility of dynastic altruists born in t (see expression (5)) and the life-cycle utility of their parents, which is weighted by an altruistic factor $1/\gamma$. Given c_{t-1} , this implies that the welfare function of the young in t coincides with the central planner's objective:

$$v_t = \sum_{i=-1}^{+\infty} \gamma^i U(c_{t+i}, d_{t+1+i})$$

The intertemporal equilibrium of this model coincides with the command optimum. Transfers to the young are interpreted as bequests and transfers to the old as gifts. Fiscal policies, therefore, are ineffective. Importantly, note that the component of descending altruism appears in the central planner's objective, but not the component of ascending altruism of future generations.

Abel (1987) has extended Burbidge's analysis by assuming that the altruistic concern for parents is weighted by δ , which can differ from $1/\gamma$. For $\delta \neq 1/\gamma$, fiscal policy is not always neutral, because the objective of an altruist, $v_t = \delta U(c_{t-1}, d_t) + \sum_{j=1}^{+\infty} \gamma^j U(c_{t+j}, d_{t+1+j})$, differs²⁷ from that of the social planner.

²⁷In contrast to the model of two-sided dynastic altruism, it is sufficient to assume that the product $\gamma\delta$ is smaller than 1 to guarantee that optimal decisions made by two successive generations are mutually consistent.

This form of *ad hoc* altruism strongly departs from the notion of dynastic altruism. Both Burbidge and Abel make a distinction between the concern for parents and the concern for children, as if future generations had no concern for their parents.

7.2.2 Paternalistic altruism

We examine one of the most popular specification of *ad hoc* altruism. Bequests are said to be paternalistic, when parents derive utility not from their children's utilities, but from the size of the estate they leave to them. The utility function of a paternalistic altruist, who is born in t and consume c_t and d_{t+1} , can be represented by the following function:

$$v_t = U(c_t, d_{t+1}) + \Phi(x_{t+1}) \quad (41)$$

where x_{t+1} is the level of bequests and separability is assumed for the sake of simplicity. Φ is defined on the set of non-negative values of x_{t+1} and the non-negative bequest constraint still applies to this model. With an infinite marginal utility of zero bequests (i.e., $\lim_{x \rightarrow 0} \Phi'(x) = +\infty$), optimal bequests are always positive. As the objective function (41) does not depend on the decisions and budget constraints of children, fiscal policies are effective.

Paternalistic bequests are related to altruistic bequests. Paternalistic parents also accumulate savings for the purposes of leaving bequests to their children. Nevertheless the amount and structure of bequests are not related to their children's preferences, but rather to parental views on what is good for their children, or to the pleasure they derive from giving. Models dealing with paternalistic bequests are therefore often referred to as “bequest-as-consumption models” or “joy-of-giving models”, because bequests enter in the parental utility function as a consumption good (see, for example, Abel and Warshawsky (1988) or Andreoni (1989)).

7.2.3 Family altruism

Models of dynastic altruism consider the family as an infinitely lived entity. By contrast, models of pure life-cyclers feature another extreme view on the family, according to which parents and children are fully distinct economic units. Following Becker (1991), one can envisage a less drastic approach to modelling economic relations within the family.

Models of family altruism assume that a family is neither a dynasty nor an isolated household. Each individual starts a new household, when he becomes adult. In turn, each of an individual's children will also establish a new household, and so on. Individuals are members of two family units: the family founded by their parents and their own household. They play a different role in these two households. They belong to the former during both their childhood and adulthood, where they play the role of children, and to the latter when adult and old, where they play the role of parents. In the former they make no decision, being completely passive when young and being only a descendant and possibly heir when adult. In the latter they are fully fledged decision makers.

Family altruism refers to the sentiments between these two successive households. Altruists born in t take account of their children's adult disposable income denoted with ω_{t+1} . The budget constraints of individuals born in t are the following:

$$\begin{aligned}\omega_t &= w_t + x_t = c_t + s_t \\ R_{t+1}s_t &= d_{t+1} + (1+n)x_{t+1} \\ \omega_{t+1} &= x_{t+1} + w_{t+1} \\ x_{t+1} &\geq 0\end{aligned}$$

The utility of altruists depends on three arguments: their first-period consumption c_t , their second-period consumption d_{t+1} and their children's disposable income ω_{t+1} during adulthood:

$$v_t = U(c_t, d_{t+1}) + \Psi(\omega_{t+1})$$

Altruists can influence the starting position of their grown-up children. They are non-paternalistic, since intergenerational transfers aim at providing children with a good starting position in life. The idea²⁸ behind family altruism is that parents care only about the income of their children and not about how they use their income.

The concept of family altruism leads to interesting fiscal policy conclusions. It can be shown that the introduction of a pay-as-you-go pension system has no real

²⁸Some growth models with human capital use a similar concept of altruism. For example, the preference of altruists in Glomm and Ravikumar (1992) depends on the quality of schools. This variable is directly linked to the adult disposable income of children (see Section 8).

effects, when bequests are positive. In contrast to the model of dynastic altruism, such a neutrality property does not hold for public debt.

To illustrate the neutrality of a pay-as-you-go pension system, we assume that the government levies a tax τ_{t+1} on each young and distributes the tax revenue to the old, born in t , who receive θ_{t+1} . Balancing the pension system in every period implies $N_t\theta_{t+1} = N_{t+1}\tau_{t+1}$, or $\theta_{t+1} = (1+n)\tau_{t+1}$. Following the method developed in Section 5.2, the new optimal bequest is $x'_{t+1} = x_{t+1} + \tau_{t+1}$. The parental consumption and the income of children are not altered by the social security scheme, as we have:

$$d'_{t+1} = R_{t+1}s_t + (1+n)\tau_{t+1} - (1+n)x'_{t+1} = R_{t+1}s_t - (1+n)x_{t+1} = d_{t+1}$$

$$\omega'_{t+1} = w_{t+1} - \tau_{t+1} + x'_{t+1} = w_{t+1} + x_{t+1} = \omega_{t+1}$$

This proves that bequests exactly offset the intergenerational transfers operated by the pension system.

The non-neutrality of public debt is straightforward. Assume that a debt, issued in period $t+1$, is distributed to the old in $t+1$ and is reimbursed by the young in $t+3$. Since altruists born in t do not take into account the utility of their descendants, they do not care about the situation of agents born in $t+3$. As in the Diamond model, but in contrast to models of dynastic altruism, public debt has real effects.

The model with family altruism²⁹ leads to conclusions regarding the effectiveness of fiscal policy, which are less clear-cut and more realistic than those obtained with either the standard overlapping generations model or the model of dynastic altruism.

8 Extensions

Intergenerational altruism significantly influences the economic equilibrium and the effectiveness of fiscal policy. It is worth enquiring, as it most likely underpins a wide range of economic decisions. Selfishness is certainly not a fully satisfactory assumption for the analysis of bequests, gifts, or private education. Altruistic behaviours

²⁹Lambrecht, Michel and Thibault (2002) analyse the equilibrium dynamics of the model with family altruism and show that its dynamical properties are halfway between the overlapping generations model with pure life-cyclers (Diamond, 1965) and the model of dynastic altruism (Barro, 1974). For an analysis of pay-as-you-go social security in a model of family altruism, see Lambrecht, Michel and Vidal (2004).

may also drive economic decisions which have an impact on future generations, such as environmental policies. In this section, we consider two issues that can be analysed under the assumption of altruistic behaviours. First, we consider a model of education, where parents' educational choices are driven by altruism. Second, we turn to environmental economics and present a model, where there is an intergenerational external effect.

8.1 Altruism and education

In growth models, education is closely related to the concept of human capital, which represents a quantity of efficiency units of labour. The production function, F , uses two inputs, physical capital K_t and efficient labour or human capital H_t . This function is assumed to be linearly homogenous and each production factor is paid its marginal product:

$$R_t = F'_K(K_t, H_t) = f'(k_t) \quad \text{where} \quad f(k) = F(k, 1) \quad \text{and} \quad k_t = K_t/H_t$$

$$w_t = F'_L(K_t, H_t) = f(k_t) - k_t f'(k_t) = w(k_t)$$

The labour income of an individual that supplies h_t efficiency units of labour is equal to $w_t h_t$. The human capital of individuals born in t depends on their parents' human capital, h_t , and their parents' educational spending, e_t :

$$h_{t+1} = \varphi(h_t, e_t)$$

Altruistic parents, who maximise $V_t = u(c_t, d_{t+1}) + V_{t+1}$, choose how much to spend on their children's education, along with their consumptions, c_t and d_{t+1} , and the bequest they leave to their children, x_{t+1} . We can then write the altruistic maximisation problem as follows:

$$V_t^*(x_t, h_t) = \max_{c_t, e_t, d_{t+1}, x_{t+1}} U(c_t, d_{t+1}) + V_{t+1}^*(x_{t+1}, h_{t+1})$$

$$\text{subject to : } x_t + w_t h_t = c_t + (1+n)e_t + s_t$$

$$R_{t+1} s_t = d_{t+1} + (1+n)x_{t+1}$$

$$h_{t+1} = \varphi(h_t, e_t)$$

Parents take into account the impact of their educational spending on the welfare of their children, which depends on their level of human capital, h_{t+1} , and their

bequests. This model has two state variables and is therefore more intricate than the baseline model of dynastic altruism. Most authors have assumed that there is no physical capital or that parents have an altruistic concern only for the level of human capital of their children.

Glomm and Ravikumar (1992) have for example developed a simplified altruistic model of education, in which parents are only concerned for their children's human capital, focusing on the distribution of income in the economy. Parents decide on the education of their children. In each period t , children devote u_t units of their time endowment to educate themselves, whereas their parents pay e_t for their education. They also benefit from the level of human capital of their parents, h_t , so that their own level of human capital in period $t + 1$ is:

$$h_{t+1} = Au_t^\alpha e_t^\beta h_t^{1-\beta} \quad \text{with } \alpha > 0 \quad \text{and } 0 < \beta < 1 \quad (42)$$

An individual's income in period $t + 1$ is h_{t+1} . With their income individuals finance their consumption and the education of their children:

$$h_{t+1} = c_{t+1} + e_{t+1} \quad (43)$$

The life-cycle utility is assumed to be log-linear:

$$U_t = \ln(1 - u_t) + \ln c_{t+1} + \ln e_{t+1} \quad (44)$$

Individuals choose u_t , c_{t+1} and e_{t+1} so as to maximise (44) subject to the constraints (42) and (43). In period t , h_t and e_t are given. The solution to this maximisation problem is:

$$u_t^* = \frac{\alpha}{\alpha + 1/2}$$

$$c_{t+1}^* = e_{t+1}^* = \frac{1}{2} h_{t+1}$$

By substituting the optimal decisions into (43), we obtain the dynamics of human capital:

$$\ln h_{t+1}^* = b^* + \ln h_t^* \quad \text{where } b^* = \ln \left(A \left(\frac{\alpha}{\alpha + 1/2} \right)^\alpha \left(\frac{1}{2} \right)^\beta \right)$$

If human capital is initially distributed according to a log-normal distribution of mean μ_0 and variance σ_0^2 , human capital in period t is distributed according to a log-normal distribution of mean μ_t and variance σ_t^2 :

$$\mu_{t+1} = b^* + \mu_t \quad \text{and } \sigma_{t+1}^2 = \sigma_t^2 = \dots = \sigma_0^2$$

The average level of human capital, \bar{h}_t , is defined by:

$$\ln \bar{h}_t = \mu_t + \frac{1}{2} \sigma_t^2$$

When education is publicly financed, all individuals benefit from the same level of educational spending, \bar{e}_t , which is financed by a wage tax, τ_t :

$$\bar{e}_t = \tau_t \bar{h}_t$$

The utility of an individual born in t is then the maximum of (44) under the constraints:

$$c_{t+1} = (1 - \tau_{t+1}) h_{t+1} \quad \text{and} \quad \bar{e}_{t+1} = \tau_{t+1} \bar{h}_{t+1}$$

Individuals make less effort to educate themselves under a public education system. Given τ_{t+1} and \bar{h}_{t+1} , they maximise $\ln(1 - u_t) + \ln h_{t+1}$ under the constraint $h_{t+1} = A u_t^\alpha \bar{e}_t^\beta h_t^{1-\beta}$. Under a public education regime, the optimal effort is $u_t^P = \frac{\alpha}{1+\alpha} < u_t^*$. The optimal effort is smaller than under a private education regime, because individuals can no longer directly influence the education level of their children.

The public educational spending and therefore the level of taxation are the result of a voting equilibrium. The derivative of an individual's utility with respect to τ_{t+1} is equal to $\frac{1}{1-\tau_{t+1}} + \frac{1}{\tau_{t+1}}$ and the maximum level of utility is obtained for $\tau_{t+1} = 1/2$. The result of the voting equilibrium is given by:

$$\tau_{t+1}^P = 1/2, \quad c_{t+1}^P = \frac{1}{2} h_{t+1}^P \quad \text{and} \quad \bar{e}_{t+1}^P = \frac{1}{2} \bar{h}_{t+1}^P$$

Hence,

$$\begin{aligned} \ln h_{t+1}^P &= \ln A + \alpha \ln u_t^\alpha + \beta \ln \bar{e}_t^P + (1 - \beta) \ln h_t^P \\ &= b^P + \beta \ln \bar{h}_t^P + (1 - \beta) \ln h_t^P \end{aligned}$$

where $b^P = \ln \left(A \left(\frac{\alpha}{1+\alpha} \right)^\alpha \left(\frac{1}{2} \right)^\beta \right) < b^*$. With a log-normal distribution $(\mu_t^P, (\sigma_t^P)^2)$ we have:

$$\begin{aligned} \mu_{t+1}^P &= b^P + \frac{\beta}{2} (\sigma_t^P)^2 + \mu_t^P \\ (\sigma_{t+1}^P)^2 &= (1 - \beta)^2 (\sigma_t^P)^2 \end{aligned}$$

We can conclude from this model that:

- A public education system reduces inequality ($\lim_{t \rightarrow +\infty} (\sigma_t^P)^2 = 0$), whereas a private education system maintain inequality ($\sigma_t^2 = \sigma_0^2$).

- In the long run, the mean of the logarithm of human capital grows at a lower rate under a public education regime ($\mu_{t+1}^P - \mu_t^P \simeq b^P$) than under a private education regime ($\mu_{t+1}^* - \mu_t^* = b^* > b^P$). The same conclusion applies to the average level of human capital, since we have: $\ln \bar{h}_{t+1}^P - \ln \bar{h}_t^P = b^P - \frac{1}{2}(1 - \beta)\beta(\sigma_t^P)^2$ and $\ln \bar{h}_{t+1}^* - \ln \bar{h}_t^* = b^*$.

In the model of Glomm and Ravikumar, the effect of taxation on growth is negative, because taxation reduces educational efforts. In another formulation of this model, the educational effort is made by parents, who devote time l_t to the education of their children. Individuals then face the following budget constraints:

$$w_t(1 - l_t) = c_t + e_t$$

Human capital evolves according to:

$$h_{t+1} = A l_t^\alpha e_t^\beta h_t^{1-\beta}$$

In this model, taxation and public education exert opposite effects, because time devoted to the education of children is free from taxation. The growth rate is then higher under a public education regime (see Wigniolle, 1994).

8.2 Altruism and the environment

Dynamic issues relating to the environment, pollution or the depletion of natural resources, have mainly been analysed in the framework of optimal growth models. The main feature of environmental externalities is their double dimension, intra- and intertemporal, as they affect today's generation as well as future generations. Altruistic individuals are concerned for the quality of the environment over their life-cycle, as they directly suffer from pollution or poor environmental quality, but also for the quality of the environment in the future, as they are altruistically linked to their children. Along with physical capital (here bequests), the environment is an asset which is passed on to future generations. Altruistic individuals therefore devote resources to abate pollution and to preserve the quality of the environment. Even individuals who leave zero bequests can contribute to pollution abatement and environmental quality.

Clearly, the environment is a public good shared within as well as between generations. Private contributions to finance public goods typically result in underprovision, as subscription equilibria are non-cooperative. There is a case for public intervention in spite of the altruistic tendencies of private individuals (see Howarth and Norgaard 1995), as subsidies to private contributions can restore efficiency. If pollution stems from industrial activities, there is a tradeoff between the accumulation of physical capital and the quality of the environment. The private return of physical capital differs from its social return, thereby leading to a second inefficiency. In contrast to results obtained in the baseline altruistic model, the market equilibrium is no longer Pareto-optimal, when taking account of environmental externalities.

Jouvet, Michel and Vidal (2000) examine these aspects in a model consisting of altruistic individuals, who only consume during their second period of life, but whose utility is negatively affected by the level of pollution. They can voluntarily contribute to environmental quality. There is no population growth. The utility of individuals born in period t can be written as follows:

$$V_t = U(d_{t+1}, P_{t+1}) + \gamma V_{t+1}$$

subject to : $x_t + w_t = s_t$

$$R_{t+1}s_t = d_{t+1} + z_{t+1} + x_{t+1}$$

$$x_{t+1} \geq 0 \quad \text{and} \quad z_{t+1} \geq 0$$

where the main difference with respect to the maximisation problem set up in Section 2 is the voluntary contribution to pollution abatement z_{t+1} and the pollution term in the utility function. The emission of pollutants in period t is a linear function of the output level, aY_{t+1} , and pollution abatement occurs according to a linear technology, $-bZ_{t+1}$ (where Z_{t+1} is the total contribution to environmental cleaning), whereas pollution absorption takes place linearly, $(1 - h) P_t$. The dynamics of pollution are therefore given by:

$$P_{t+1} = (1 - h) P_t + aY_{t+1} - bZ_{t+1}$$

When choosing their personal contribution z_{t+1} , individuals take other individuals' contributions as given. We have:

$$P_{t+1} = (1 - h) P_t + aY_{t+1} - b(z_{t+1} + \bar{Z}_{t+1})$$

where \bar{Z}_{t+1} is the sum of other individuals' contributions. It is further assumed that the technology of pollution abatement is efficient³⁰, $b > a$.

In steady state, four types of equilibria are possible, depending on whether or not bequests are positive and on whether or not voluntary contributions to pollution abatement are positive. To illustrate these equilibria, we consider the following utility function: $U(d, P) = \ln d + \lambda \ln(\bar{P} - P)$, where \bar{P} is an upper limit on the level of pollution and λ the relative weight of environmental quality, $\bar{P} - P$, in the utility. Figure 4 shows the steady-state equilibria in the plane (λ, γ) .

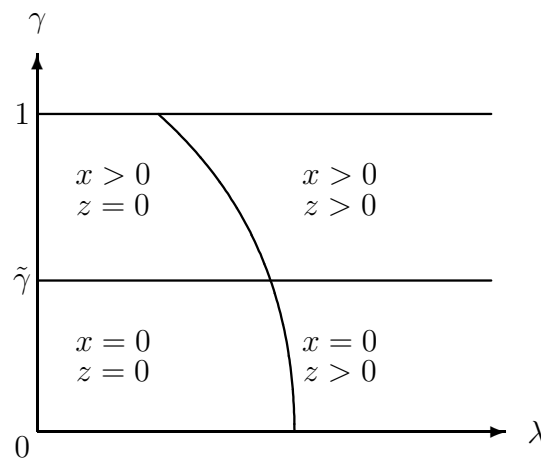


Figure 4: Steady state equilibria

When individuals are not sufficiently altruistic (low γ), bequest motive is inoperative and we have: $k_{t+1} = w_t = f(k_t) - k_t f'(k_t)$. If the steady state \tilde{k} is unique, the condition for positive bequest is:

$$\gamma > \tilde{\gamma} = 1/f'(\tilde{k})$$

If bequests are positive, the steady state is the modified golden rule $k_\gamma = f'^{-1}(1/\gamma)$. In the absence of voluntary contributions, the steady-state level of pollution is $\tilde{P}(\gamma) = aNf(k_\gamma)/h$. There is a threshold³¹, $\tilde{\lambda}(\gamma)$, on the weight of pollution in the utility function above which contributions are positive. Alternatively, when

³⁰Each unit produced devoted to pollution abatement has a negative net effect, $a - b$, on the increase of pollution.

³¹The expression of this threshold is derived in Jouvét, Michel and Vidal (2000).

bequest motive is inoperative, contributions are positive if the weight of pollution in the utility λ exceeds a threshold $\tilde{\lambda}_0(\gamma)$. In both cases, the thresholds triggering positive contributions are lower, the higher the degree of intergenerational altruism.

The competitive equilibrium is suboptimal, because of the two externalities prevailing in the economy. The first externality is well-known in public economics; the Cournot-Nash decision process is inefficient, and individuals under-contribute to pollution abatement. The second externality affects the economy through the production process: altruistic individuals do not take into account the effect of production on pollution, thereby leading to a level of capital that is higher than socially desirable. The central planner takes into account these two externalities and maximises the following social welfare function:

$$\sum_{t=0}^{+\infty} \gamma^t U(d_t, P_t)$$

$$\text{subject to : } f(k_t) = d_t + z_t + k_{t+1}$$

$$P_t = (1 - h) P_{t-1} + aNf(k_t) - bNz_t$$

$$k_0 \text{ and } P_{-1} \text{ given}$$

In the long run, the marginal productivity of capital, which characterises the social optimum, is:

$$f'(k^S) = \frac{1}{\gamma(1 - a/b)}$$

This is the genuine modified golden rule that takes into account the environmental externality of capital accumulation. The social planner chooses to accumulate less capital than altruistic individuals, $f'(k^S) > 1/\gamma$. This is because the social value of capital differs from its private value, as altruistic individuals fail to internalise the impact of production on the environment. Furthermore, the social planner takes into account the social willingness to pay for pollution abatement, leading to higher spending on pollution abatement than in the competitive equilibrium.

Since two externalities have to be internalised by altruistic individuals, the decentralisation of the social optimum can be achieved by using two policy instruments. First, to attain an efficient allocation of resources between consumption, a private good, and the quality of the environment, a public good, the government has to subsidise contributions to pollution abatement. Second, the government has

to limit capital accumulation, for instance by reducing savings, since private altruistic individuals do not take into account the adverse consequences of pollution on environmental quality. This can be done by setting a tax on the return of savings.

9 Conclusion

Altruism is the appropriate microeconomic foundation underpinning the possible ineffectiveness of fiscal policy in stimulating economic activity, referred to as Ricardian equivalence. Our review of altruistic growth models shows that the Ricardian equivalence theorem does not always hold in dynastic models. The debt neutrality result hinges on positive private transfers between successive generations (bequests or gifts). When these transfers are zero, fiscal policy is effective. Barro's intuitive formulation of altruism in macroeconomic models does not always deliver Ricardian equivalence, when taking account of all general equilibrium linkages. Even extending his intuition to two-sided altruism is not enough to ensure debt neutrality without conditions, as fiscal policy is effective when both bequests and gifts are zero.

Dynastic altruism features the view of highly rational economic agents, who are farsighted and see through the government budget constraint, thereby possibly countering the effects of fiscal policies. A specific *ad hoc* form of altruism is needed to deliver the debt neutrality results without conditions. The altruistic utility proposed by Burbidge (1983) is formally equivalent to a central planner's objective and, not surprisingly, delivers Ricardian equivalence, but as any *ad hoc* formulation it suffers from weak theoretical foundations. The model of dynastic altruism remains the benchmark for discussing debt neutrality, as it offers a fully consistent framework to analyse fiscal policy in an intertemporal framework.

As argued by Ricardo, the neutrality result is a point of theory, insofar as individuals certainly suffer from myopia, leaving some room for fiscal policy. Extending the basic framework to heterogeneous individuals provides some insights in this respect. The steady state equilibrium is still a modified golden rule, which depends on the degree of altruism of the more altruistic individuals, but fiscal policy entails important redistributive effects between heterogeneous dynasties. Models consisting of both short-sighted or selfish individuals and far-sighted or altruistic individuals certainly represents a better abstraction of real world economies, and

further progress in the characterisation of the effects of fiscal policy on economic activity requires a better understanding of individual heterogeneity in macroeconomic models. Analysing transition dynamics of heterogeneous economies is key to understanding both the long term and the short term effects of fiscal policy.

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